The Ronald O. Perelman Center for Political Science and Economics (PCPSE)
133 South 36th Street
Philadelphia, PA 19104-6297

pier@econ.upenn.edu
http://economics.sas.upenn.edu/pier

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OLIVER DE GROOT
University of Liverpool
and CEPR

C. BORA DURDU
Federal Reserve Board

ENRIQUE G. MENDOZA
University of Pennsylvania
and NBER

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Oliver de Groot[a], C. Bora Durdu[b,*], Enrique G. Mendoza[c]

[a] *University of Liverpool and CEPR*
[b] *Federal Reserve Board*
[c] *University of Pennsylvania and NBER*

Abstract

We compare global (fixed-point iteration) and local (first-order, higher-order, risky-steady-state, and quasi-linear) solutions of open-economy incomplete-markets models. Cyclical moments of a workhorse endowment model are broadly in line with the data and similar across solutions calibrated to the same data targets, but impulse responses and spectral densities differ. Alternative local solutions yield nearly identical results. Calibrating them requires non-trivial interest-rate elasticities that make net foreign assets (NFA) "sticky," causing them to differ sharply from global solutions in experiments altering precautionary savings (e.g., increasing income volatility, adding capital controls). Analytic and numerical results show that our findings are due to the near-unit-root nature of NFA under incomplete markets and imprecise solutions of their autocorrelation. These findings extend to a Sudden Stops model with an occasionally binding collateral constraint. In addition, quasi-linear methods yield smaller financial premia and macroeconomic responses when the constraint binds.

Keywords: Solution methods; Sudden stops; Precautionary savings; Occasionally binding constraints
JEL Classification: *D82, E44, F41*

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*Corresponding author. Email: oliverdegroot@gmail.com

1. Introduction

Incomplete asset markets play a key role in major strands of the international macroeconomics literature (e.g., business cycles, sovereign default, sudden stops, global imbalances, macroprudential regulation, currency carry trade, etc.). Since the dynamics of external wealth (or net foreign assets, NFA) generally lack analytic solutions, researchers rely on numerical methods. However, choosing the appropriate method is difficult for several reasons. First, deterministic models yield stationary equilibria dependent on initial conditions. Second, in stochastic models, the evolution of wealth is state-contingent and driven by precautionary saving (i.e., certainty equivalence fails). Third, with standard preferences, if the interest rate equals the rate of time preference, precautionary saving makes NFA diverge to infinity.

The literature follows two approaches to address these issues. The first, based on the seminal work of Schmitt-Grohé and Uribe (2003), modifies the models by inducing stationarity with one of three assumptions: a debt-elastic interest-rate (DEIR) function, preferences with endogenous discounting (ED), or asset holding costs (AHC).[1] These assumptions support a well-defined deterministic steady state of NFA independent of initial conditions. The models are then solved with a first-order approximation (1OA) around that steady state, recovering certainty equivalence. Innovations to local methods have occurred since then, including higher-order methods (e.g., Schmitt-Grohé and Uribe, 2004; Devereux and Sutherland, 2010; Fernández-Villaverde et al., 2011), the risky steady state (RSS) method (Coeurdacier et al. (2011)), and quasi-linear methods for handling occasionally binding constraints (QLOBC), including OccBin by Guerrieri and Iacoviello (2015) and DynareOBC by Holden (2016, 2021).[2]

Table 1 summarizes the numerical methods used in a set of research papers and policy applications published between 1991 and 2021. Among local methods, 1OA is the most common in research papers and ubiquitous in policy applications, and DEIR is the most common stationarity-inducing method. Most DEIR applications set the debt elasticity parameter, ψ, to an arbitrarily small value so as to prevent the DEIR function from playing a role other than in-

[1] They show that business cycle moments and impulse response functions of an RBC small open-economy model obtained with any of these assumptions are very similar.

[2] Boehl and Strobel (2022) and Kulish et al. (2017) have also produced similar algorithms.

ducing stationarity, with ψ ranging from 0.00001 to 0.01 and the value of 0.001 used by Schmitt-Grohé and Uribe (2003) as the most common.[3] In other cases, ψ is calibrated or estimated.

The second approach, introduced by Mendoza (1991), uses global approximation (GA) methods to solve for the nonlinear decision rules and long-run distribution of external wealth of the models in their original form. These methods are similar to those used in closed-economy models of heterogeneous agents with incomplete markets. The existence of a well-defined stochastic steady state follows from the same condition as in those models: the interest rate must be lower than the rate of time preference (see Ljungqvist and Sargent, 2018, Ch. 18).

This condition is a general equilibrium result in a multicountry setup, because with an interest rate equal to the rate of time preference, Supermartingale convergence of the marginal benefit of saving leads all countries to accumulate infinitely large NFA for self-insurance, which violates world asset market clearing (see Mendoza et al., 2009). Hence, assuming an interest rate lower than the rate of time preference in small-open-economy models is an *implication* of the assumption that the interest rate is a world-determined price. With local methods, the stationarity inducing assumption is constructed so that, at a chosen deterministic steady state, the interest rate equals the rate of time preference.

While global methods solve the models in their original form, capturing the global nature of precautionary saving driving NFA, they suffer from the curse of dimensionality, becoming exponentially inefficient with the number of endogenous state variables. In contrast, local methods solve large-scale models efficiently but require a stationarity-inducing assumption that is not part of the original model. This tradeoff poses three key questions: How different are local and global solutions? Are the differences economically meaningful? Can they be reduced?

This paper answers these questions by using analytic and numerical tools to compare global and local solutions for two small open-economy models: An endowment model and a model of sudden stops (SS), which is an RBC model with an occasionally-binding collateral constraint.[4]

[3]Garcia-Cicco et al. (2010) explain, following Schmitt-Grohé and Uribe (2003), it is standard to set ψ to a small value because the DEIR function aims to obtain independence of the deterministic steady state from initial conditions without affecting cyclical dynamics. They also study a model in which ψ represents a financial friction and is estimated. In the literature, DEIR functional forms vary and hence ψ values are not directly comparable.

[4]In de Groot et al. (2019, App. C), we compared solutions for a standard RBC model.

For the global solutions, we use a fixed-point iteration method.[5] For the local methods, we consider 1OA, second-order approximation (2OA), RSS, and QLOBC.[6] RSS and QLOBC can be used with or without stationarity-inducing assumptions, and we study both cases.

We implement a calibration approach consistent across solution methods. Most parameters take the same values in the global and local solutions, keeping two as "free" parameters calibrated to target the same two data moments (the average NFA-GDP ratio and the cyclical standard deviation of consumption). For the local solutions, we use mainly the DEIR function because of its prevalence in the literature. The two free parameters are the DEIR elasticity ψ and the deterministic steady state of NFA. For the global solutions, the free parameters are the subjective discount factor and an-hoc debt limit.

We compare across solutions statistical moments, impulse response functions (IRFs), spectral densities (in Appendix B.3.4), Euler-equation errors, and solution run times, and provide analytic results explaining their differences. In addition, we study the robustness of our findings to adding interest-rate shocks, using AHC and ED instead of DEIR to induce stationarity, altering the size of the state space and the realization vector of shocks in the global solution and, for RSS and QLOBC, considering variants without DEIR in which the interest rate is lower than the rate of time preference. We also compare results for experiments that alter precautionary saving incentives (increasing income volatility and introducing capital controls).

Several of the long-run moments produced by the calibrated global and local solutions are similar, except the variability of NFA and net exports (nx) and the autocorrelation of consumption, which are generally higher in the local solutions. Relative to the data, both local and global methods approximate well the data moments, including the nx autocorrelation, which Garcia-Cicco et al. (2010) highlighted as important for open-economy models.[7] This result

[5]Specifically, the *FiPIt* algorithm developed by Mendoza and Villalvazo (2020) that modifies the standard iteration-on-Euler-equation approach to avoid both solving simultaneous non-linear equations (as with standard time-iteration) and irregular interpolation (as with endogenous grid methods). For comparison, in de Groot et al. (2019, App. B.1.2), we solve the model with value function iteration.

[6]In de Groot et al. (2019, App. B.3.7), we present third-order-approximation (3OA) results but the higher-order unnecessary unless stochastic volatility is introduced (see de Groot, 2016). For QLOBC, we use the DynareOBC algorithm. First-order DynareOBC and OccBin give the same solution when the equilibrium is unique. DynareOBC has the advantage that it converges in finite time and can test for equilibium multiplicity.

[7]The exception is the countercyclicality of net exports, which are well-known to be procyclical in endowment models because of the lack of investment-driven borrowing incentives.

is well-known for global solutions, but it is a new finding for the local methods that demonstrates the importance of setting the DEIR parameters, specially the DEIR elasticity (or the corresponding parameter if using ED or AHC). The low elasticity often used in the literature ($\psi = 0.001$) yields a net export autocorrelation near 1, as Garcia-Cicco et al. (2010) showed.[8] In contrast, our calibration requires $\psi = 0.042$ and yields nx autocorrelations around 0.8.

The calibrated endowment-model solutions yield different IRFs and distribute volatility differently across time frequencies. Five periods after a negative income shock, the global solution displays a decline in the NFA-GDP ratio and a decreasing consumption path, while NFA falls less and consumption rises in the local solutions.

Experiments that study precautionary saving, by increasing income volatility or introducing capital controls (as taxes on capital inflows), yield very different results across the global and local approximations. This is due to the near-unit root nature of the NFA equilibrium process, a key endogenous state variable in open-economy analysis. Near-unit root asset dynamics are typical in incomplete-markets models due to the persistence of precautionary saving behavior. Indeed, NFA autocorrelations in our solutions generally exceed 0.95. In the local solutions, we provide analytic results showing how ψ and the center of approximation determine this autocorrelation, whereas in the global solution it is a moment of the endogenous ergodic distribution of NFA. Because they are near-unit-roots, small differences in NFA autocorrelations cause large differences in unconditional moments, IRFs and spectral densities.

Two key moments are particularly striking: First, small differences in NFA autocorrelations induce large differences in the unconditional mean of NFA that measures precautionary savings. Second, they induce large differences in net-exports autocorrelations, because nx is a quasi first-difference of the near-unit-root NFA process. In the endowment model, for example, the global solution predicts that increasing income volatility by raising its autocorrelation from 0.5 to 0.95 increases mean NFA by nearly 13 percentage points (from -0.40 to -0.27), while the autocorrelation of NFA rises from 0.955 to 0.997 and that of net exports from 0.43 to 0.98. In contrast, 2OA and RSS predict that mean NFA rises only 3 percentage points, from about -0.37

[8] See de Groot et al. (2019) for a detailed analysis of the results of local solutions with $\psi = 0.001$.

to -0.34, while the autocorrelation of NFA always exceeds 0.975 and that of nx increases from 0.53 to 0.96. Similarly, higher income variability and capital controls yield sharply larger increases in mean NFA in the global solutions. These results also explain why the calibrated local and global solutions yield similar net-exports autocorrelations: The value of ψ implied by the calibration of the local solutions yields autocorrelations of NFA similar to the global solution.

The weak mean NFA response to stronger precautionary-saving incentives in the local solutions is induced by the relatively high ψ value, making NFA "sticky" because it is analogous to making deviations of NFA from steady state costly (see also Schmitt-Grohé and Uribe, 2003). As a result, mean NFA changes little in our local solutions for higher income volatility or capital controls. The high ψ almost neutralizes precautionary saving and certainty equivalence nearly holds. In contrast, the global solutions, in producing sharply higher mean NFA values, reflect the stronger precautionary saving incentives.

We also found a new unexpected result comparing across local methods: 1OA, 2OA and RSS yield very similar second- and higher-order moments, IRFs and spectral densities for all endogenous variables. To explain these results, we provide analytic local solutions of the endowment model showing that i) the coefficient on lagged NFA in the NFA decision rule is nearly the same when ψ is small (less than 0.1), unless the deterministic and risky steady state of NFA differ by a large margin (at least 40 percentage points of GDP); ii) the coefficients in the square and interaction terms of 2OA decision rules are small. Moreover, the stickiness of NFA at the calibrated ψ values keeps mean NFA nearly unchanged, resulting in similar first-order moments.

The local solutions can be re-aligned to stay closer to the global solutions as income volatility or capital controls change, by re-calibrating ψ and the center of approximatio to match the mean NFA and consumption standard deviation of the global solution. This approach, however, has the drawback that it requires solving the model globally first.

The QLOBC method we used to solve the SS model with its occasionally binding collateral constraint works by introducing news shocks that hit every time the constraint is violated to push the relevant variables back to the constraint. For consistency with rational expectations, the news shocks are constructed as if they were expected along a first-order, perfect-foresight path and so are akin to being endogenous.[9] This method, however, ignores precautionary savings; the possibility of alternative future paths in which the constraint may or may not bind; and the equity risk premium.

Our findings from comparing local and global solutions for the endowment model extend to the SS model. In addition, QLOBC yields large differences relative to the global solution in the amount of precautionary savings induced by the collateral constraint, the tightness of the constraint, the probability of hitting it, and its effect on financial premia.[10] Lower equity returns imply higher equity prices and investment when the constraint binds, and hence higher borrowing capacity. As a result, QLOBC both with the constraint binding or not-binding at steady state does not match the macroeconomic effects of sudden stops found in the GA solution.

Comparing computational performance, the small scale of the endowment model allows the global solution to run in 0.1 seconds, using a 200-node non-linear NFA grid and 5 nodes for income shocks. The 2OA solutions and RSS run in 0.3 to 3 seconds. The global solution also yields smaller Euler-equation errors. For the SS model, the QLOBC method is relatively faster than the global solution, but markedly slower than the standard local methods used for the endowment model. QLOBC with the constraint binding (not binding) at steady state runs in 244 (332) seconds, compared with 381 seconds for the global solution. Due to the near-unit-root nature of NFA, QLOBC methods take longer than standard local methods. This is because long perfect-foresight paths and long time-series simulations are needed to solve for the endogenous news shocks that support the constraint and to attain convergence of long-run moments.

Related literature This paper is related to several studies comparing global and local solutions. Rabitsch et al. (2015) compare the local method using ED preferences proposed by Devereux and Sutherland (2010) for solving portfolio allocations in a two-country incomplete-markets model, with a global solution. The solutions are similar for symmetric countries with zero long-run NFA, but differ sharply for asymmetric countries and a center of approximation

[9]Our results changed little using the variant of the QLOBC method proposed by Holden (2016) that integrates over future uncertainty when constructing future paths (see Appendix C.2.3).

[10]The QLOBC solution for the SS model with a DEIR function (henceforth, QLOBC-DEIR) has the same mean NFA-GDP ratio as the global solution by construction. But, the QLOBC solution without DEIR and a rate of time preference higher than the interest rate (henceforth, QLOBC-$\beta R < 1$) yields a much lower mean NFA-GDP ratio than the GA solution.

that differs from the ergodic mean of the global solution.

Global and local solutions with occasionally binding constraints have been compared in the New-Keynesian literature on the zero-lower-bound (ZLB) on interest rates. This literature typically formulates a Taylor rule with a ZLB constraint (rather than constraints on the agents' optimization problems); assumes complete markets; private bonds in zero net supply; and a rate of time preference equal to the steady-state interest rate. Hence, the effects of precautionary saving on asset dynamics and the center of approximation of local solutions, which are essential to our findings, are not at issue in this literature. Fernández-Villaverde et al. (2015) solve a ZLB model using a global (projection) method with one endogenous state (price dispersion).[11] They found that the ZLB yields large nonlinearities that local methods miss. Gust et al. (2017) also solve a ZLB model with projection methods and compare the results with the OccBin method. They found that the solutions differ significantly and affect the propagation of shocks and estimation results.[12] Atkinson et al. (2020) examined model estimation in a ZLB model but, in contrast, conclude there are more accuracy gains from estimating a richer (less misspecified) model using QLOBC methods than estimating a stylized model using global methods.

In the literature on financial frictions, Dou et al. (2019) compared global, 1OA, 2OA and QLOBC (OccBin) methods for closed-economy models and found that the local solutions poorly approximated the nonlinear dynamics and yield biased IRFs. Holden (2016) shows that Dynare-OBC yields similar results as a global solution for a small open-economy endowment model with quadratic utility (which rules out precautionary savings) and NFA adjustment costs to induce stationarity. In contrast, we find that global and QLOBC solutions of our endowment model with an ad-hoc debt limit and CRRA utility (which allows for precautionary savings) differ sharply. We also used QLOBC to solve the SS model, which has two endogenous states (capital and NFA) and a collateral constraint that depends on both states and endogenous as-

[11]In their model, the ergodic mean and deterministic steady state are nearly identical, whereas a key finding of our analysis is that precautionary savings causes large differences in the ergodic mean and steady state of NFA.

[12]Solving our SS model using projection methods is difficult because the global basis functions are not defined in points of the state space where it is infeasible to satisfy the collateral constraint with positive consumption. The boundary varies as capital, NFA and the capital pricing function vary. This problem can be avoided using uneven grids but this is also difficult because the debt limit imposed by the collateral constraint is not a pre-determined value. These hurdles do not arise in ZLB models and models with constant, uni-dimensional debt limits.

set prices, and find the results differ markedly from the global solution. Benigno et al. (2020) propose an alternative perturbation method for solving models with an occasionally binding constraint and applied it to a SS model. Their method induces stationarity with the DEIR function and models constraint regime-switching as driven by draws of regime realizations and regime-transition probabilities determined by parameterized logistic functions.[13]

The rest of the paper is organized as follows. Section 2 presents the endowment model and compares solution methods, providing both analytic and numerical results. Section 3 presents the SS model and compares solution methods. Section 4 concludes.

2. Endowment model

We start with the workhorse small open-economy model with stochastic endowment income, which is useful for deriving analytic results and characterizing NFA dynamics under incomplete markets, as well as comparing numerical solutions.

2.1. Model structure and equilibrium

The economy is inhabited by a representative agent with preferences given by

$$E_0\left\{\sum_{t=0}^{\infty}\beta^t u(c_t)\right\}, \quad u(c_t)=\frac{c_t^{1-\sigma}}{1-\sigma}, \tag{1}$$

where β is the subjective discount factor, c_t is consumption and σ is the CRRA coefficient. The economy's resource constraint is given by

$$c_t = y_t - A + b_t - \frac{b_{t+1}}{R}, \tag{2}$$

where $y_t = e^{z_t}\bar{y}$ denotes income, $\bar{y}$ is normalized to 1, and z_t is an AR(1), $z_t = \rho_z z_{t-1} + \varepsilon_{z,t}$, with $\varepsilon_{z,t}$ i.i.d. from $N(0,\sigma_\varepsilon^2)$.Hence, the variance and autocorrelation of log-income are $\sigma_z^2 = \sigma_\varepsilon^2/(1-\rho_z^2)$ and ρ_z, respectively; b_t denotes NFA in one-period, non-state-contingent discount

[13]Other promising approaches that attempt to adopt the benefits of the global approach while maintaining computational feasibility include Ajevskis (2017) and Mennuni and Stepanchuk (2022).

bonds traded in a global market where R is the gross world interest rate; and A represents constant investment and government spending, necessary for model calibration.

The agent chooses the sequences of bonds and consumption to maximize (1) subject to (2). This optimization problem is analogous to the one solved by a single agent in heterogeneous-agents models (e.g., Aiyagari, 1994). Since the marginal utility of consumption, $u_c(c_t)$, goes to infinity as c_t goes to zero from above, agents never choose NFA lower than a "Natural" Debt Limit (NDL) defined by the (negative of) the annuity value of the worst realization of net income, $b_{t+1} \geq b^{NDL} \equiv -\frac{R}{R-1}\min(e^{z_t}\bar{y} - A)$. Otherwise, they are exposed to sequences in which non-positive consumption has positive probability. Following Aiyagari (1994), we also impose a tighter ad-hoc debt limit, φ, such that $b_{t+1} \geq \varphi \geq b^{NDL}$, which is useful for model calibration.[14]

Using the resource constraint, we can express the Euler equation for bonds as

$$u_c\left(e^{z_t}\bar{y} - A + b_t - \frac{b_{t+1}}{R}\right) = \beta R E_t\left[u_c\left(e^{z_{t+1}}\bar{y} - A + b_{t+1} - \frac{b_{t+2}}{R}\right)\right] + \mu_t, \tag{3}$$

where μ_t is the Lagrange multiplier of the debt limit.

Under complete markets of contingent claims, and assuming income shocks are idiosyncratic to the small open economy, income risk is perfectly diversified. Consumption is constant and the economy's wealth is time- and state-invariant. The solution is akin to that of a perfect-foresight model with $\beta R = 1$ and wealth (the present value of income plus initial NFA) scaled to represent the same wealth as in the complete-markets economy.

With incomplete markets, wealth is state-contingent and consumption fluctuates. Equation (3) implies that $M_t \equiv (\beta R)^t u_c(c_t)$ forms a supermartingale, which converges almost surely to a non-negative random variable because of the Supermartingale Convergence Theorem (see Ljungqvist and Sargent, 2018, Ch. 18). If $\beta R \geq 1$, consumption and NFA diverge to infinity because marginal utility converges to zero almost surely. The economy builds an infinitely large stock of precautionary savings and sustains a consumption process for which M_t converges and $u_c(c_t) \geq \beta R E_t u_c(c_{t+1})$ holds. This is the source of the non-stationarity problem that local

[14]The NDL result requires z_t to be truncated below. However, even setting the truncation at $y_{min} = 0.5$, the probability of hitting it is essentially zero and thus has no effect on the local solutions. This also ensures the ad-hoc debt limit is always tighter than the NDL for the global approximation.

methods address with the stationarity-inducing assumptions. In contrast, if $\beta R < 1$, the economy has a well-defined stochastic steady state with finite unconditional means of assets and consumption. Intuitively, the opposing forces of the pro-saving incentive for self-insurance and the pro-borrowing incentive due to $\beta R < 1$ keep NFA moving within an ergodic set. If NFA falls (rises) too much the first (second) force prevails.

Note, both the NDL and the Supermartingale convergence of NFA dynamics under incomplete markets are global properties of the solution. They are conditions the agent finds optimal to impose on histories of saving and consumption over the infinite future because of the need to self-insure caused by the incompleteness of financial markets.

2.2. *Global solution method*

For the global solution, we solve the model in recursive form over a discrete state space of (b, z) pairs using the *FiPIt* method of Mendoza and Villalvazo (2020).[15] Income follows a discrete Markov process with transition probability matrix $\pi(z', z)$. We solve for the NFA decision rule, $b'(b, z)$, which together with the income process produces a stationary distribution of NFA and income $\lambda(b, z)$. The method solves for $b'(b, z)$ by iterating on the Euler equation (3).

The global method solves the model without imposing assumptions to induce stationarity. If $\beta R = 1$, NFA diverges to infinity, which is undesirable but is the equilibrium solution. However, $\beta R < 1$ is the relevant case because, as discussed above, it is implied by world general equilibrium. Note that with $\beta R < 1$ the *deterministic* stationary state converges to the debt limit, φ, with consumption falling at gross rate $(\beta R)^{1/\sigma}$. Hence, theory predicts that the unconditional mean of NFA in the stochastic, incomplete-markets model can differ significantly from the deterministic steady state and that the difference is due to precautionary savings.

2.3. *Local methods*

The local methods solve a local approximation to the optimality conditions (2)–(3) around the deterministic steady state, b^{dss}, for 1OA and 2OA or the risky steady state, b^{rss}, for RSS. Since

[15] This is in the class of methods that iterate on Euler equations, including endogenous-grid, time-iteration, and projection methods (see Rendahl, 2015, for an overview). *FiPIt* performs better than time-iteration and endogenous-grids for models with two endogenous state variables and an occasionally binding constraint because time-iteration requires solving nonlinear Euler equation systems and endogenous grids require interpolation techniques for irregular grids. *FiPIt* solves Euler equations directly using linear interpolation.

assuming $\beta R = 1$ implies that b^{dss} depends on initial conditions and under uncertainty NFA diverges to infinity, 1OA and 2OA require a stationarity-inducing assumption. As documented earlier, the most common assumption is to introduce the DEIR function

$$R_t = R + \psi \left[e^{b^* - B_{t+1}} - 1 \right], \tag{4}$$

where b^* and ψ are parameters, with ψ determining the elasticity of R_t with respect to NFA, and B_{t+1} is the *aggregate* NFA position (i.e., treated as exogenous by agents). At equilibrium, $b_{t+1} = B_{t+1}$. Since DEIR applications assume $\beta R = 1$, (3) implies $b^{dss} = b^*$.

We implement 1OA, 2OA, and 3OA using Dynare 5.3 and RSS following Coeurdacier et al. (2011).[16] 1OA (2OA) yields local approximations around b^{dss} by solving a first- (second-)order approximation to the decision rules with same-order approximations to the model's optimality conditions. In contrast, RSS solves a linear approximation around b^{rss} and assumes $\beta R < 1$.

RSS takes account of future risk, so the center of approximation may better capture precautionary savings. The value of b^{rss} is obtained from a second-order approximation to the conditional expectation of the steady-state Euler equation, solved jointly with the coefficients of a first-order approximation to the decision rules. This requires a conditional second-order approximation of the full equilibrium conditions' Jacobian, which implies third derivatives of those conditions. de Groot (2014) explains why third derivatives are necessary to obtain stationary NFA dynamics. We also consider a variant of RSS in which b^{rss} is computed as above but is combined with the DEIR function and first-order approximations to the decision rules and equilibrium conditions to obtain stationarity. We denote the original as *full* and the DEIR alternative as *partial* RSS.

2.4. *Calibration*

Table 2 lists the baseline calibration. The parameters common across solution methods ($\sigma = 2$, $R = 1.086$, $\sigma_z = 0.0272$ and $\rho_z = 0.749$) are taken Mendoza (2010). The values of σ_z and ρ_z match the corresponding moments for the cyclical component of Mexico's GDP in

[16] A detailed description of all the methods used is given in Appendix B.

quarterly data for the 1993-2005 period. The local methods use this income process directly. In the global solution, the discrete Markov approximation uses the Rouwenhorst method developed by Kopecky and Suen (2010) with a realization vector of five points ($n_z = 5$). The NFA grid has 200 nodes ($n_b = 200$) and a nonlinear structure with more nodes around the ad-hoc debt limit (see Appendix B.3.1).[17] The mean NFA-GDP and consumption-GDP ratios are also taken from Mendoza's calibration to Mexican data ($E(b/y) = -0.363$, $E(c/y) = 0.65$). Using these and the value of R in the resource constraint yields $A = 0.32$.

The calibration is completed by targeting two free parameters of each solution method to a common pair of moments from the Mexican data: the mean NFA-GDP ratio (-0.363) and the standard deviation of consumption (3.397%). In the global calibration, the free parameters are φ and β and the calibration yields $\varphi = -0.436$ and $\beta = 0.917$. We need to pin down both in the calibration because, while the mean NFA-GDP ratio can be matched by adjusting β alone, this can result in a stochastic steady state in which the NFA distribution is clustered near φ and consumption fluctuates too much, or NFA has a high variance and consumption fluctuates too little. In the local solutions with DEIR (2OA and partial RSS), we follow the standard practice of setting $\beta = 1/R$ so that $b^{dss} = b^*$. This leaves ψ and b^* as the free parameters and the calibration yields $\psi = 0.042$ and $b^* = -0.374$. This ψ is much higher than the 0.001 common in the literature, which has important implications as we will show. Full RSS uses the same β as the global solution and does not have ψ and b^* because it does not need DEIR.

2.5. Comparison of results

NFA decision rules and net exports Two key moments of open-economy models are the autocorrelations of NFA and net exports. The former because it is a key driver of the dynamics of capital flows and their cyclical co-movements, and the latter because of its relevance in the international RBC literature (see Garcia-Cicco et al., 2010). Hence, we start our comparison here.

[17] The Markov process is discrete with bounded support whereas the AR(1) is normally-distributed with unbounded support. Kopecky and Suen (2010) prove their method matches the conditional and unconditional mean and variance, and the first-order autocorrelation of any stationary AR(1) process. To assess the robustness of the global solution to the discrete state-space approximation, we show in Table 4 that increasing n_z to 25 has negligible effects on the results. With $n_z = 25$, the smallest node of z is 4.6 standard deviations below the mean, so tail-events can be well approximated. The NDL tightens but remains much lower than the calibrated debt limit, φ.

Assume for now that b_{t+1} follows an AR(1) process with autocorrelation coefficient ρ_b. Since nx_t is a quasi first-difference of NFA ($nx_t = \frac{b_{t+1}}{R} - b_t$), the autocorrelation of net exports, ρ_{nx}, can be expressed as

$$\rho_{nx}(\rho_b) = \frac{\rho_b(1+R^2) - R(1+\rho_b^2)}{R^2 - 2R\rho_b + 1}. \tag{5}$$

In Appendix B.3.2, we prove that ρ_{nx} is increasing and convex in ρ_b. To assess what this convexity implies, note that $\rho_{nx} \approx -0.5$ when $\rho_b = 0$ (since R is close to 1); turns positive when $\rho_b = 1/R$; and reaches 1 when $\rho_b = 1$. For $R = 1.06$, increasing ρ_b from 0.94 to 0.995 increases ρ_{nx} from 0 to 0.65. If ρ_b is close to 1, as is typical in incomplete-markets models, small differences in ρ_b yield large differences in ρ_{nx} (and other variable moment that depend on b_t).

The global solution determines ρ_b endogenously as a moment of the stationary distribution $\lambda(b,z)$, which in turn reflects the self-insurance incentives embedded in the supermartingale convergence property of the marginal benefit of savings. The local solutions determine ρ_b as one of the coefficients of the NFA decision rule, and (when DEIR is used) the value of ρ_b is determined by ψ and b^*. To show this, the 2OA decision rule can be written as

$$\tilde{b}_{t+1} = h_b\tilde{b}_t + h_z z_t + \frac{1}{2}\left(h_{bb}\tilde{b}_t^2 + h_{zz}z_t^2\right) + h_{bz}\tilde{b}_t z_t + \frac{1}{2}h_{\sigma_z\sigma_z}, \tag{6}$$

where $\tilde{b}_t \equiv b_t - b^{dss}$. The 1OA decision rule only has the first two right-hand-side terms, with identical values of h_b and h_z. The partial RSS decision rules are of the same form as 1OA but with b^{rss} replacing b^{dss} and RSS-specific values of h_b and h_z. The coefficient of interest is h_b because it is the main determinant of ρ_b. This is the case even for 2OA solutions because in all our experiments the nonlinear terms—h_{bb}, h_{zz} and h_{bz}—are small.[18] The term $h_{\sigma_z\sigma_z}$ matters because it isolates the effect of income risk on mean NFA and thus captures precautionary savings in the 2OA solution. Since $h_{\sigma_z\sigma_z}$ is the only quantitatively relevant term that distinguishes 2OA from 1OA, their second- and higher-order moments are very similar.

For RSS, de Groot (2014) shows that income risk matters for determining b^{rss} because the

[18] Appendix B.3.3 shows the robustness of this result. In particular, h_{bb}, h_{bz}, and h_{zz} are irrelevant for the variance and autocorrelation of NFA for a range of ψ, σ and ρ_z values. For mean NFA, these terms are only important if ρ_z is high or ψ is very small.

coefficient of variation of consumption (relative to its risky steady state) is constant and depends on β, r and σ.[19] Intuitively, this captures precautionary savings because, if income risk rises and the share of income allocated to savings remains unchanged, the volatility of consumption would rise. But, by increasing NFA relative to endowment income, more disposable income comes from interest income, so that the coefficient of variation of consumption can remain constant. However, since the RSS decision rule has a linear form, ρ_b differs from the 1OA solution only to the extent that b^{dss} and b^{rss} differ. As we show below, this requires larger differences than those implied by our calibrations. Hence, 1OA, 2OA and RSS moments are likely to be very similar, except for their first moments.

Next, we show how ψ and b^* determine h_b. Assuming log-utility, an i.i.d income process, and $R_t = R\psi e^{b^*-B_{t+1}}$ for tractability, we obtain the following solution for h_b:

$$h_b(\psi, b^*) = \frac{R + e^{b^*\psi}(1 - b^*\psi + \psi) - \sqrt{R^2 + 2e^{b^*\psi}(b^*\psi + \psi - 1)R + e^{2b^*\psi}\left(1 - b^*\psi + \psi\right)^2}}{2e^{b^*\psi}}, \quad (7)$$

where $b^* = b^{dss}$ for 1(2)OA and $b^* = b^{rss}$ for partial RSS. Since we find h_{bb}, h_{zz} and h_{bz} are quantitatively irrelevant, it follows that $\rho_b(\psi, b^*) \approx h_b(\psi, b^*)$. Hence, (7) shows that the calibrated values of ψ and b^* (and R) implicitly determine the autocorrelation of NFA in local solutions. In particular, given b^* and R, the low ψ widely used in the literature implies a ρ_b close to 1, and ρ_b falls for higher ψ.[20]

Condition (7) yields two other results. First, it shows that the h_b obtained with 1OA or 2OA differs from RSS only to the extent that b^{dss} and b^{rss} differ. Second, it illustrates the non-stationarity of local solutions without a stationarity-inducing assumption. If $\psi = 0$, the solution of $h_b(\psi, b^*)$ has two roots, 1 and $R\,(> 1)$. In contrast (and assuming $b^* = 0$ for tractability), if $\psi > 0$ the smaller of the two roots is less than unity, and thus yields a stationary solution.[21]

To numerically study how variations in ψ and b^* alter ρ_b, we solved for ρ_b using the 2OA and partial RSS methods for $\psi \in [0, 0.5]$ and three values of b^*: 0, -0.5 and -0.7. The results,

[19] de Groot (2014, Corollary 5) gives $\frac{var(c)}{(c^{rss})^2} = \frac{2}{\sigma(1+\sigma)}\frac{1-\beta R}{\beta R}$.

[20] For RSS, the mapping is non-trivial since b^{rss} is solved jointly with the coefficients of the decision rule for b_{t+1}, which also depend on ψ.

[21] Schmitt-Grohé and Uribe (2003) show the same for an endowment model with ED preferences.

plotted in Figure 1, show that ρ_b is nearly identical whether b^* is -0.5 or -0.7 for $0 \leq \psi \leq 0.2$, which includes our calibrated ψ value and the values used in all but one of the 76 articles using local solutions included in Table 1.[22] This is a key result, because it means that, for the ψ values used in the literature, approximating around b^{dss} or b^{rss} or solving with 1OA, 2OA or partial RSS makes little difference, unless b^{dss} or b^{rss} differ by wide margins. For $\psi \leq 0.05$, even solving with $b^* = 0$ makes little difference. This explains why the calibration exercise yields the same ψ and b^* values for 2OA and partial RSS. Non-negligible differences between RSS and 2OA require $\psi > 0.15$ or large differences between b^{dss} and b^{rss}. Moreover, since the nonlinear terms of the 2OA decision rule are small, we can expect 2OA and RSS solutions to produce similar variances and correlations for all endogenous variables (as we show below).[23]

The above findings indicate that the implications of ρ_b for ρ_{nx} conjectured in (5) by *assuming* NFA follows an AR(1) apply to the *equilibrium* processes produced by the local methods. The low ψ value commonly used in the literature yields both ρ_b and ρ_{nx} near 1 in local solutions, yet ρ_{nx} is clearly less than 1 in global solutions with slightly lower ρ_b (see de Groot et al., 2019). In the latter, ρ_b and ρ_{nx} are determined endogenously by the NFA-income distribution, $\lambda(b, z)$, the NFA decision rule, $b'(b, z)$, and the definition of nx.

Table 3 compares global and local solutions of ρ_b, ρ_{nx} and $E(b)$ as ρ_z rises from 0 to 0.95 (keeping the rest of the parameters at their calibrated values). 2OA and partial RSS yield similar results, because the gap between b^{dss} and b^{rss}, and the nonlinear terms in 2OA decision rules, are too small to yield large differences.[24] Panel i) shows that for the global solution, as ρ_z rises from 0.5 to 0.95, ρ_b rises slightly from 0.955 to 0.997 but ρ_{nx} rises from 0.426 to 0.983, and $E(b)$ falls from -0.395 to -0.269. Thus, as (5) predicts, small changes in ρ_b near 1 cause large changes in ρ_{nx}. Mean NFA falls because income is more volatile at higher ρ_z and precautionary savings rise. Panel ii) shows the local solutions generally produce slightly higher ρ_b and thus higher ρ_{nx}, and $E(b)$ rises only 3 percentage points (i.e., precautionary savings change little).

[22] The highest ψ was 2.8 from Garcia-Cicco et al. (2010), which they estimated for a model with financial frictions. We study later the implications of local solutions with high ψ values.

[23] While the analytic solution of $h_b(\psi, b^*)$ is for log-utility and i.i.d. shocks, the implications of the analysis hold quantitatively with CRRA utility and AR(1) shocks.

[24] 1OA and 2OA solutions are near-identical, hence we omit 1OA from the table.

Panel iii) shows that the local solutions can be re-aligned to remain close to the GA solutions by re-calibrating ψ and b^* as ρ_z rises to match the mean NFA and standard deviation of consumption in each GA solution. This is true by construction in the original calibration because both solutions were calibrated to match those two moments from the data, but not when the calibrated model is used to examine the effects of parameter variations such as ρ_z. Re-calibrating the DEIR function in this way requires, however, solving the model globally first. Moreover, the implied ψ value rises as ρ_z falls, and this causes NFA to remain close to its center of approximation affecting other results, as we explain later in this Section.

Long-run moments Table 4 compares unconditional cyclical moments across the calibrated global and local solutions and vis-a-vis the moments from Mexican data.[25] Global, 2OA and partial RSS yield moments that approximate well several data moments, particularly the autocorrelation of net exports. The fact that 2OA and partial RSS can do well in this regard is an important result that demonstrates the relevance of a proper calibration of the DEIR function. Note also that 2OA and partial RSS yield near-identical moments, in line with our previous finding showing that the solutions are similar because b^{dss} and b^{rss} do not differ much and the higher-order terms in the 2OA decision rules are quantitatively irrelevant. 2OA and partial RSS differ from the GA solution in that they predict higher volatility in NFA and higher autocorrelation in consumption.

One important moment that all three solutions fail to account for is the countercyclical trade balance. This is a well-known limitation of endowment models that is corrected in models with investment or credit constraints (see Mendoza, 1991, 2010).

Full RSS yields much higher variability in consumption and net exports, lower GDP correlations, and a sharply higher autocorrelation of net exports than the other solutions. This occurs because full RSS has the same β as GA (and hence has $\beta R < 1$) and does not use DEIR. Since it also does not have the debt limit, φ, of the GA solution, the strong borrowing incentives implied by $\beta R < 1$ result in a much lower mean NFA-GDP ratio (-704%) than the other solutions.[26]

[25] We constructed a quarterly series of NFA consistent with the quarterly trade balance flows using initial and terminal conditions from Lane and Milesi-Ferretti (2018) and the net exports data. See Appendix B.3.1 for details.

[26] The global solution without an ad-hoc debt limit (i.e., $\varphi = NDL$) has a mean NFA-GDP ratio of -650%, a

Table 4 also shows execution times and three Euler equation error metrics (average, *L1*, mean square error, *L2*, and maximum, $L\infty$).[27] The global solution with 25 realizations in the income vector yields the smallest errors of all the solutions for all three metrics. Relative to the global solution with 5 realizations, 2OA and Full RSS have larger *L1* and *L2* errors, and 2OA (Full RSS) has a smaller (bigger) $L\infty$ error. Partial RSS has smaller errors than GA (with 5 income realizations) by all three metrics. In terms of execution time, full RSS solves in 0.3 seconds because, given the simplicity of the model, we can split the algorithm into a step that derives the non-linear system of equations in Mathematica and a step that solves it using Matlab. Partial RSS takes longer (3.1 seconds) because it does both steps within Matlab, building on a toolkit developed by Schmitt-Grohé and Uribe (2004). The 2OA solution runs in 0.6 seconds. The global method with 5 and 25 income realizations solve in 0.1 and 3.2 seconds, respectively. The former is faster than the local methods and of comparable accuracy, except for partial RSS that yields smaller Euler errors but takes almost as long as the global method with 25 shock realizations.[28] However, these fast global solutions are possible because the endowment model has only one endogenous state variable (b) and one shock (z).

Precautionary savings Next, we compare solution methods in two experiments that alter incentives for precautionary savings, measured by changes in the long-run average of NFA, relative to the baseline calibration. In the baseline, global, 2OA, and partial RSS were calibrated to match the mean NFA-GDP ratio in Mexican data (-36.3%). We study 1) the effects of increasing the variance of income and 2) introducing capital controls, modeled as a borrowing tax.

Figure 2 plots the value of $E(b)$ as σ_z rises, keeping other parameters unchanged. The solid curve shows the global results, the short-dashed curve 2OA, and the long-dashed curve partial RSS. The curves intersect at $\sigma_z = 2.72\%$ because the three solutions are calibrated to match the same mean NFA at that value.

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similar order of magnitude as full RSS, but yields much larger consumption variability than in the data.

[27] See Table 4 for details on computer hardware and software. Comparable Euler errors were computed using a long time-series simulation of each solution, starting at the means of b and z and with a common set of Gaussian realizations of z_t. At each date t, the right-hand-side of the Euler equation is evaluated using Gauss-Hermite quadrature. See Appendix B.3.3 for details. We thank an anonymous referee for suggesting this methodology.

[28] Since the local model does not feature the nonlinearity created by the ad-hoc debt limit (the region of the state space in which the Euler equation errors are in general largest), the Euler equation error metrics are not strictly directly comparable between the global and local methods.

Global and local methods yield very different results. For 1OA (not shown), certainty equivalence implies no precautionary savings with $E(b)$ remaining at b^{dss} for all values of σ_z. Increasing σ_z from the calibrated value (2.72%) to 8% increases mean NFA in the global solution by 76 percentage points (from the calibrated value of -36.3% to near +40%) while 2OA and partial RSS predict much smaller increases from -36.3% to about -29%. The gap relative to the global solution result widens as σ_z rises. 2OA and RSS solutions are similar because of the reasons explained earlier. Hence, 2OA and partial RSS predict small increases in precautionary savings—approximately consistent with certainty equivalence and the 1OA solution.[29]

Are the above differences in precautionary savings economically meaningful and is one result to be considered more reliable than the others? Building on the analysis by Schmitt-Grohé and Uribe (2003) showing that the DEIR setup is similar to one using instead quadratic costs of deviating from b^{dss} (i.e., $\tilde{\psi}(b_{t+1}-b^{dss})^2/2$) sheds light on these questions. The log-linear Euler equation of the two setups are equivalent if $\tilde{\psi} = \psi/R$.[30] Moreover, by rewriting b_{t+1} as $E(b) + (b_{t+1} - E(b))$ and hence the cost function as $\tilde{\psi}\left((b_{t+1} - E(b)) + \left(E(b) - b^{dss}\right)\right)^2/2$, it is clear that the cost has variable and fixed components. If the fixed cost is larger than the benefit derived from precautionary savings, it is suboptimal to let mean NFA deviate from b^{dss}. Thus, because the calibration of the local solutions requires relatively high ψ values, as opposed to the commonly-used value of 0.001, the implicit cost of moving NFA away from its mean weakens precautionary savings and renders 1OA, 2OA and RSS solutions quite similar.

These results for higher income variance are similar to the ones shown earlier in Table 3, where we increased the variance of income by increasing the persistence of the z shocks. Increasing ρ_z from 0 to 0.95 increases $E(b)$ in the global solution by 14.2 percentage points, compared with about 3.5 percentage points with both 2OA and partial RSS.

Consider next the capital controls policy experiment. We introduce a tax on foreign borrowing common in theoretical and empirical studies (e.g., Bianchi, 2011; De Gregorio et al., 2000). The after-tax net interest rate is $r(1+\tau)$ and the tax revenue is rebated as a lump-sum transfer. The rest of the model and calibration are unchanged. This debt tax strengthens pre-

[29] Appendix B.3.3 shows this analytically for log-utility and i.i.d. shocks.

[30] With DEIR, for $b_{t+1} < b^{dss}$ ($b_{t+1} > b^{dss}$) agents pay more (get less) for borrowing (saving) more.

cautionary saving incentives because it increases the effective interest rate. In the literature, this is a key mechanism driving the effects of macroprudential capital controls (Bianchi and Mendoza, 2018) or the implications of financial integration (Mendoza et al., 2009).

Figure 3 plots $E(b)$ for a range of τ values. The local solutions yield significantly lower mean NFA even with a tax of 2%. With a tax near 5%, the GA solution predicts a mean NFA-GDP ratio of 25% while the local methods predict roughly -25%, a difference of 50 percentage points. For $\tau > 5.3\%$, $r(1+\tau)$ approaches the rate of time preference and NFA diverges to infinity in the global solution, while the local solutions grow approximately linearly.

The rationale behind these results follows from our previous analysis: The local NFA decision rules cannot capture the stronger precautionary saving incentives of capital controls because the relatively high calibrated ψ value implicitly imposes too large a cost of deviating from b^*. As Table 6 shows, ρ_b rises from 0.989 with no debt tax to 0.999 with $\tau = 4.75\%$ in the GA solution, but it rises less in the local solutions, peaking at 0.99. As τ rises, the slightly smaller ρ_b yields significantly lower means of NFA and markedly smaller autocorrelations of net exports and consumption volatility ratios.

Similar results extend to other policy experiments that alter precautionary saving incentives. For instance, income-tax-financed changes in government expenditures alter the variance of after-tax income, with the variance rising as taxes fall. Higher income variance increases precautionary savings, and hence the results shown in Figure 2 indicate that local solutions would yield significantly weaker effects of lower income taxes on the mean NFA position. A similar logic applies to assessments of optimal accumulation of foreign reserves or the effects of financial globalization in models with financial frictions (e.g., Durdu et al., 2009).

Impulse response functions & spectral densities Figure 4 compares IRFs for a negative, one-standard-deviation income shock starting at the unconditional means. Consumption and output are shown in percent mean deviations, while b/y and nx/y are mean deviations. The IRFs for 1OA (not shown), 2OA and RSS are near-identical, in line with the results that the h_b coefficients of NFA decision rules are similar and nonlinear second-order terms are small.

Both global and local yield similarly shaped IRFs for the NFA- and nx-GDP ratios, but the initial declines are larger in the global solution. The consumption IRFs differ in magnitude and shape. In the global solution, consumption falls less on impact and then displays a J-shaped response, while in the local solution it falls more on impact and then rises monotonically.

We also compare global and local solutions in the frequency domain using nonparametric periodograms of simulated data (see Appendix B.3.4). Relative to the global solution, the local solutions predict a higher (lower) contribution of consumption (net export) fluctuations at the business cycle frequencies relative to overall consumption (net export) variance. Moreover, in line with our previous findings indicating that long-run moments and IRFs are almost the same in 2OA and RSS, their spectral densities are also nearly identical.

Interest-rate shocks Next, we add interest-rate shocks to facilitate comparison with the SS model (in the next section) and because of existing results showing that the RSS method yields higher precautionary savings with these shocks (see Coeurdacier et al., 2011; de Groot, 2014).

The gross interest rate is $R_t = e^{\nu_t}\bar{R}$, where ν_t is an exogenous shock and $\bar{R}$ is the mean interest rate. The endowment and interest-rate shocks have a diagonal VAR representation

$$\begin{bmatrix} z_t \\ \nu_t \end{bmatrix} = \begin{bmatrix} \rho_z & 0 \\ 0 & \rho_r \end{bmatrix} \cdot \begin{bmatrix} z_{t-1} \\ \nu_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{z,t} \\ \varepsilon_{r,t} \end{bmatrix}, \quad \Sigma = \begin{bmatrix} \sigma^2_{\varepsilon_z} & \sigma_{\varepsilon_z,\varepsilon_r} \\ \sigma_{\varepsilon_z,\varepsilon_r} & \sigma^2_{\varepsilon_r} \end{bmatrix}, \tag{8}$$

where Σ is the innovation variance-covariance matrix. The DEIR function takes the form

$$R_t = e^{\nu_t}\bar{R} + \psi\left[e^{b^{dss}-B_{t+1}} - 1\right]. \tag{9}$$

As in the original calibration, $\rho_z = 0.749$ and $\sigma_z = 0.0272$ ($\sigma_{\varepsilon_z} = 0.018$). For the global solution, we use 14 nodes in the Markov chain for z and ν each. For simplicity, we use the same autocorrelation on both shocks (see Appendix B.3.5 in de Groot et al., 2019). Hence, $\rho_r = 0.749$. We solve the model with values of σ_{ε_r} and $\sigma_{\varepsilon_z,\varepsilon_r}$ such that σ_ν takes values ranging from 0 to 2.5% and the correlation between income and the interest rate is $\rho_{z,R} = 0$.

A well-defined limiting distribution of NFA now requires $\beta\bar{R} < 1$, otherwise $\beta^t\Pi^t_{j=1}R_j$ diverges to infinity (see Chamberlain and Wilson, 2000). In addition, there are long histories of realizations with R_t lower (higher) than $\bar{R}$, which imply much weaker (stronger) precautionary savings incentives than with a constant interest rate. For example, histories with $\beta R_t > 1$

produce sequences where b_{t+1} can grow very large, since there is no pro-borrowing effect offsetting the precautionary savings incentive.[31] At some point, each of these histories shifts to histories with sufficiently low R_t to induce NFA mean-reversion. The NDL corresponds to the highest realization of R_t, and so is tighter than under $R_t = \bar{R}$. These effects are at work only in the global solution, because they result from expectations of histories of future shocks that take the economy far from $E(b/y)$ and b^{dss}.

Table 5 compares global and local solutions for $\sigma_\nu \in \{0, 0.5, \ldots, 2.5\}$. For the global solution, we show results for the calibrated debt limit ($\varphi = -0.435$) and for the NDL, so as to compare the roles of debt limits and interest-rate shocks in inducing higher mean NFA. The results show that the adjustment-cost-like effect of a hig ψ, keeping NFA close to b^{dss}, continues to affect solutions that use DEIR. Thus, 2OA and partial RSS yield only small increases in $E(b)$. The second- and higher-order moments for RSS and 2OA are still similar, albeit less so, especially for $\sigma_\nu \geq 1$. Hence, the result that a high ψ removes precautionary savings and yields very similar 1OA, 2OA and RSS local solutions is robust to adding interest-rate shocks.

Table 5 also shows that, with interest-rate shocks, full RSS generates higher (lower) consumption (NFA) volatility, near-unitary autocorrelation of nx, and much lower mean NFA-GDP ratio than all of the other solutions. In fact, full RSS yields results closer to the global-NDL solution for second. However, both of these solutions have the shortcoming of producing low mean NFA-GDP ratios, in the -2 to -7 interval. The similarity across full RSS and global-NDL solution is due in part to the NDL being non-binding by definition in the global solution. As such, both solutions never hit a debt limit. But the solutions are not always similar. If $\bar{R}$ is set above the calibrated value of 1.086 such that $\beta\bar{R}$ is almost 1, full RSS yields a much lower mean NFA than the global solution (with either the ad-hoc debt limit or NDL). For a low $\bar{R}$, full RSS frequently violates the NDL.

Endogenous discounting Next, we explore the implications of using the ED approach to induce stationarity instead of DEIR (see Appendix B.3.5 for full details).[32] First, we study an

[31]Reducing $\bar{R}$ while keeping σ_ν constant accentuates these effects, because histories with larger gaps between β and R_t are more probable.

[32]We showed earlier that AHC and DEIR are similar (a higher ψ makes NFA deviations from b^* costlier).

analytic comparison of local DEIR and ED decision rules assuming log-utility and i.i.d. shocks. In line with Schmitt-Grohé and Uribe (2003), DEIR and ED are equivalent to first-order: A non-linear mapping determines the elasticity of the discount factor with respect to consumption, ψ^{ED}, for a given ψ in the DEIR such that the decision rules are the same. 2OA solutions, however, are not equivalent.[33] Varying ψ while adjusting ψ^{ED} so the h_b coefficients of DEIR and ED are equal, yields a DEIR h_{bb} coefficient increasing and concave in ψ while that for ED is slightly decreasing and near-linear. The $h_{\sigma\sigma}$ coefficient is nearly invariant to R and ψ using DEIR, but decreasing and convex in ψ and sensitive to R using ED.

These differences reflect a key theoretical difference between the two approaches: When consumption rises as agents borrow, R_t rises using DEIR but β_t falls using ED. Hence, the marginal benefit of savings, $\beta_t(1+r_t)R_t u'(c_{t+1})$, *rises* in the DEIR solution but *falls* in the ED solution. The latter weakens precautionary saving in the ED solution relative to the global solution with standard preferences and $\beta R < 1$ (see also Durdu et al., 2009). The intuition is that when b rises enough, the discount factor falls and acts as a self-correcting mechanism that weakens saving incentives. Note, however, that agents internalize the dependency of the discount factor on consumption in the global solution but not in the local ones. This introduces an "impatience effect," by which all future utility flows are discounted more heavily as today's consumption rises. The local solutions remove it by assuming that the discount factor depends on aggregate consumption, which agents treat as exogenous.

Next, we compare the long-run moments from the global and 2OA solutions of the ED model (see Appendix Table 3). For global, the table compares the original results for the model with standard preferences and $\beta R < 1$ and the case with ED preferences (GA-ED). Two 2OA solutions are also included: the DEIR solution from Table 4 and a 2OA-ED solution with ψ^{ED} calibrated to match the Mexican data target for NFA-GDP ratio.[34] Hence, all four results yield the same $E(b/y)$ because they were calibrated to match the same data target.

[33]Seoane (2015) compares approaches to induce stationarity using 3OA methods and, in line with our results, finds that different approaches generate large differences when calibrated to Argentina.

[34]ED local and global solutions only have one free calibration parameter, because (a) the GA-ED solution does not require an ad-hoc debt limit, and (b) the steady-state Euler equation yields a relationship determining ψ^{ED} as a function of the steady-state of consumption (or NFA).

2OA-ED and GA-ED yield similar moments. This is due to two important features of the ED case. First, the model already has preferences that support a well-defined deterministic steady state independent of initial conditions. Second, the impatience effect ignored in the 2OA-ED solution is small, in line with results obtained by Schmitt-Grohé and Uribe (2003).

The GA solutions using standard preferences (with $\beta R < 1$) and ED differ in that NFA and net exports are more volatile and their autocorrelations (and the autocorrelation of consumption) are higher in the GA-ED case, and the same is true comparing the 2OA-ED solution with its DEIR counterpart. This occurs because, as explained above, the precautionary-saving motive is weaker with ED preferences than with standard preferences. We also verified this result by repeating the experiment increasing income volatility using the GA-ED and 2OA-ED solutions. Appendix Figure 6 shows significantly smaller increases in $E(b/y)$ as the variance of income rises, compared with those shown in Figure 2.

GA-ED still yields larger increases in NFA than 2OA-ED, but the gap is significantly smaller than in the comparable experiments with standard preferences for the GA and 2OA-DEIR solutions. This result suggests that ED is preferable to DEIR to induce stationarity, but keep in mind that ED preferences weaken precautionary saving incentives. Thus, precautionary saving behavior is weakened with either the DEIR or ED approaches (the former because of the NFA stickiness implied by the calibrated ψ values, the latter because of the self-correcting mechanism reducing the discount factor).

An exact solution We also compared local and global solutions for an alternative endowment model that has an exact solution: the canonical savings model of Levhari and Srinivasan (1969). This model obtains closed-form solutions by assuming that income is a multiplicative return on a risky asset with a log-normal i.i.d process, and that consumption is chosen before the return is observed.[35] The solutions are $c_t = \lambda(\sigma_\varepsilon)b_t$ and $b_{t+1} = (1 - \lambda(\sigma_\varepsilon))R_t b_t$, where the savings rate $\lambda(\sigma_\varepsilon)$ has an analytic solution that is increasing in σ_ε for $\sigma > 1$. Log-NFA follows a random walk with drift, $\ln(b_{t+1}) = \ln(1 - \lambda(\sigma_\varepsilon)) + \ln(b_t) + \ln(R_t)$, and so does consumption, but consumption *growth* is a log-i.i.d. process: $c_{t+1}/c_t = (1 - \lambda(\sigma_\varepsilon))R_t$.

[35] Utility is a constant elasticity function with $c^{1-\sigma}/(1-\sigma)$. The resource constraint is $b_{t+1} = R_{t+1}(b_t - c_t)$, where $\log(R_t) = \mu + \sigma_\varepsilon \varepsilon_{t+1}$, and $\varepsilon_{t+1} \sim N(0,1)$.

Appendix B.3.7 shows results for GA, local solutions up to fourth-order (4OA), and RSS (obtained by detrending the model expressing variables in ratios of b_t). The exact, GA, and 4OA solutions are very close for values of σ_ε in the 0-0.45 interval, but the accuracy of RSS and 2OA deteriorates sharply for $\sigma_\varepsilon > 0.3$. These inaccuracies, however, are not due to the calibration of the DEIR parameters and the implied value of ρ_b, but to the low order of the 2OA and RSS approximations. This is not the case, however, for the endowment model we studied in this Section, where NFA stickiness induced by calibrated ψ values remains a problem regardless of the approximation order.

3. Sudden Stops model

This section compares global and local solutions of the Sudden Stops (SS) model proposed by Mendoza (2010). This is an RBC model augmented with an occasionally binding collateral constraint.[36]

3.1. Model structure

The model's competitive equilibrium is represented as the solution to a representative firm-household problem. Gross output is produced with a Cobb-Douglas technology using capital, k_t, labor, L_t, and imported inputs, υ_t.

$$e^{z_t}F(k_t, L_t, \upsilon_t) = e^{z_t}k_t^{\gamma}L_t^{\alpha}\upsilon_t^{\eta}, \quad 0 \leq \alpha, \gamma \leq 1, \quad \eta = 1 - \alpha - \gamma. \tag{10}$$

Gross output is a tradable good sold at a world-determined price which is the numeraire and set to 1. The relative price of imported inputs is also world-determined and given by $p_t = e^{u_t}\bar{p}$, where $\bar{p}$ is the mean price and u_t is a terms-of-trade shock. The model also includes TFP, z_t, and interest-rate, ν_t, shocks. A standard working capital constraint requires a fraction ϕ of the

[36] In de Groot et al. (2019), we compared solutions of the RBC model itself and found similar differences across global and local solutions for NFA, net exports, and consumption as in the endowment model. However, supply-side variables are similar in the global and local solutions because there is no wealth effect on labor supply and the equity premium is small. As Mendoza (1991) noted, these features render the capital decision rule similar to that implied by the risk-neutral arbitrage of returns on capital and NFA, which implies the Fisherian separation of investment from consumption and savings decisions nearly holds. Hence, in the capital decision rule, the coefficient on lagged NFA in the local solutions and the elasticities of k' with respect to b in the GA solution are negligible.

cost of L_t and υ_t to be paid in advance of sales. Working capital loans are obtained from foreign lenders at the beginning of each period and repaid at the end, so the financing cost of inputs is the net interest rate, $R_t - 1$. Capital is costly to adjust, with adjustment costs per unit of net investment, $k_{t+1} - k_t$, given by $\Psi(\frac{k_{t+1}-k_t}{k_t}) = \frac{a}{2}\left(\frac{k_{t+1}-k_t}{k_t}\right)^2$, with $a \geq 0$. This functional form satisfies Hayashi's conditions so average and marginal Tobin's Q are equal in equilibrium.

The representative firm-household chooses $[c_t, L_t, i_t, \upsilon_t, b_{t+1}, k_{t+1}]_{t=0}^{\infty}$ to maximize

$$E_0\left\{\sum_{t=0}^{\infty}\beta^t\frac{\left(c_t - \frac{L_t^{\omega}}{\omega}\right)^{1-\sigma}}{1-\sigma}\right\}, \tag{11}$$

subject to

$$c_t(1+\tau) + i_t = e^{z_t}F(k_t, L_t, \upsilon_t) - p_t\upsilon_t - \phi(R_t - 1)(w_tL_t + p_t\upsilon_t) - \frac{b_{t+1}}{R_t} + b_t, \tag{12}$$

$$\frac{b_{t+1}}{R_t} - \phi R_t(w_tL_t + p_t\upsilon_t) \geq -\kappa q_t k_{t+1}. \tag{13}$$

The utility function is of Greenwood-Hercowitz-Huffman (GHH) form, which removes the wealth effect on labor supply. The market prices of labor and capital, denoted w_t and q_t, are taken as given by the agent. The left-hand-side of the resource constraint (12) is the sum of consumption, inclusive of an ad-valorem tax τ used to calibrate the ratio of government expenditures to GDP, plus gross investment, i_t, where $i_t = \delta k_t + (k_{t+1} - k_t)\left[1 + \Psi\left(\frac{k_{t+1}-k_t}{k_t}\right)\right]$ and δ is the depreciation rate. The right-hand-side equals total supply, which consists of GDP, $y_t \equiv e^{z_t}F(k_t, L_t, \upsilon_t) - p_t\upsilon_t$, net of foreign interest payments on working capital loans, $\phi(R_t - 1)(w_tL_t + p_t\upsilon_t)$, minus net resources lent abroad, $\frac{b_{t+1}}{R_t} - b_t$. Net exports are given by $nx_t = \frac{b_{t+1}}{R_t} - b_t + \phi(R_t - 1)(w_tL_t + p_t\upsilon_t) = y_t - c_t(1+\tau) - i_t$. The Fisherian collateral constraint (13) prevents debt and working capital credit from exceeding a fraction κ of the market value of capital.

The competitive equilibrium is defined by stochastic sequences of allocations $[c_t, L_t, k_{t+1}, b_{t+1}, \upsilon_t, i_t]_0^{\infty}$ and prices $[w_t, q_t]_0^{\infty}$ such that (a) the agent solves its optimization problem given $[w_t, q_t]_0^{\infty}$ and (k_0, b_0), and (b) $[w_t, q_t]_0^{\infty}$ satisfy the corresponding market-clearing conditions.

3.2. Solution methods

Relative to the endowment model, solving this model involves an occasionally binding constraint that depends on endogenous decisions and market outcomes and an extra endogenous state variable, k_t. For the global solution, we use *FiPIt* defining grids of k and b with 30 and 72 nodes, respectively.[37] For the quasi-local (QLOBC) method, we use the DynareOBC toolkit.

DynareOBC treats the occasionally binding constraint as a source of endogenous news about the future along perfect-foresight paths (see Appendix B.3.6 for details). If the constraint is (is not) binding at the deterministic steady state, the algorithm uses news shocks to solve for unconstrained (constrained) periods along those paths by solving a mixed-integer linear programming problem. Suppose the constraint does not bind at steady state. If agents anticipate the constraint will bind at $t+j$ conditional on the date-t state variables, this provides "news" that b_{t+1} will follow a path higher than otherwise. This approach is akin to assuming that there is no constraint, but whenever agents are on a path that would lead them to borrow more than the constraint allows, a series of news shocks hit that makes them borrow only what is allowed and moderates their borrowing before that happens.[38]

The main output of DynareOBC is a time-series simulation constructed by stitching together the date-t values of perfect-foresight paths conditional on $(k_t, b_t, z_t, u_t, \nu_t)$. Each path is obtained using an extended path algorithm that traces equilibrium dynamics up to period $t+T$. The extended path can be obtained using first- or higher-order approximations, but we report only results based on the former.[39] The path computed for a given starting date t determines the values of (k_{t+1}, b_{t+1}). The rest of the path is discarded and the process is repeated at $t+1$ to generate the values of the time-series simulation for that period.

The efficiency of this method depends on three factors: (a) T: This parameter needs to be

[37]See Mendoza and Villalvazo (2020) for details, including a User Guide and Matlab codes.

[38]The model is similar to the model without the constraint but with sequences of news shocks chosen to yield the same equilibrium as the model with the constraint. This equivalence holds exactly if the model is linear and shock variances are zero, such that the news shocks are unanticipated.

[39]Holden (2016) proposes a variant of QLOBC using Gaussian cubature to integrate over future uncertainty in the extended paths. In Appendix C.2.3, we show the results change little but the execution time rises significantly using this feature. The solutions with or without cubature produce NFA decision rules that remain unconstrained when the global solution is already constrained in a region of the state space, and underestimate the rise in NFA when the constraint binds (see Appendix Figure 12). Hence, the local solutions do not capture the nonlinearities due to precautionary saving incentives near the collateral constraint observed in the global solution.

large enough so that after T no further news shocks are needed (if the constraint does (does not) bind in steady state, after T the constraint must always (never) bind). A model with persistent dynamics, as is the case with high ρ_b under incomplete markets, requires a larger T and a larger T increases the search time for the sequence of shocks that supports the equilibrium; (b) Frequency of binding constraint: In each period for which the perfect foresight path requires news shocks, the search for the equilibrium sequence of news shocks needs to be repeated. A model in which the constraint binds frequently requires more time-costly searches; and (c) Time-series simulation length, N: This parameter needs to be large enough for long-run moments of the endogenous variables to converge. The algorithm is therefore less efficient in models with persistent dynamics (requiring a large T and N), and models in which the news shocks are needed frequently.

Figure 5 illustrates the QLOBC method using the endowment model, with $b_{t+1} \geq \varphi$ as an occasionally binding constraint.[40] Panels (a)-(b) show a stochastic simulation for c_t and b_{t+1} for $t = 90$ to 250 (black-solid lines) and eleven of the perfect-foresight paths (red-dash lines) with the corresponding date-t solution (red circle). In Panel (b), the constraint binds in four of the perfect-foresight paths (the shaded area corresponds to $b_{t+1} < \varphi$). Panels (c)-(d) isolate the path that defines the equilibrium in $t = 141$. The comparable path of b_{t+1} without the collateral constraint is the black-dot line in (d). The constraint first binds along the perfect-foresight path at $t = 144$. Relative to the model without the constraint, agents choose higher b_{t+1} (less debt) earlier, in anticipation of the constraint becoming binding with perfect foresight (i.e., the red-dashed curve is above the black-dotted curve at $t = 142, 143$). Since income rises gradually back to steady state, the constraint continues to bind for several periods, until income is high enough for b_{t+1} to also rise back towards steady state (after $t = 170$).

Standard quasi-linear (QLOBC) methods like first-order DynareOBC (without integrating over future uncertainty) or OccBinn ignore the *risk* of moving between regions of the state space where the constraints binds or not. At each t, QLOBC only considers the perfect-foresight path conditional on the date-t state and ignores histories of future shocks and associated allocations

[40]DEIR is used since the constraint does not bind in the steady state (see Appendix B.3.6 for details).

and prices that can occur. Hence, wealth and precautionary-saving effects of the constraint are ignored, and forward-looking objects like asset prices and returns also abstract from them. These effects are central to SS models, because when the collateral constraint binds, a sudden stop occurs with a deep recession and collapsing prices. The risk of a sudden stop strengthens precautionary savings and is priced in asset markets. (see Mendoza, 2010; Durdu et al., 2009).

3.3. Calibration

Table 7 shows the calibration parameters, most of which were taken from Mendoza (2010). This calibration targeted Mexican data moments using the RBC variant of the model without the collateral constraint. Given those parameters, κ was set to match the observed frequency of Sudden Stops in the model with the constraint, which resulted in $\kappa = 0.2$. The only difference in our GA calibration is that we set φ and β following the same strategy as in the endowment model, targeting them so the RBC model approximates the mean NFA-GDP ratio and the volatility of consumption in Mexican data.[41]

The three shocks have a diagonal VAR representation given by

$$\begin{bmatrix} z_t \\ \nu_t \\ u_t \end{bmatrix} = \begin{bmatrix} \rho_z & 0 & 0 \\ 0 & \rho_r & 0 \\ 0 & 0 & \rho_p \end{bmatrix} \cdot \begin{bmatrix} z_{t-1} \\ \nu_{t-1} \\ u_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{z,t} \\ \varepsilon_{r,t} \\ \varepsilon_{p,t} \end{bmatrix}, \quad \Sigma = \begin{bmatrix} \sigma_z^2 & \sigma_{z,r} & 0 \\ \sigma_{z,r} & \sigma_r^2 & 0 \\ 0 & 0 & \sigma_p^2 \end{bmatrix}. \tag{14}$$

Following empirical evidence in Mendoza (2010), the co-movement between TFP and interest-rate shocks is driven only by the covariance of their innovations and the price shock is independent of the other two. The calibration of the autocorrelation and variance-covariance matrices also follows Mendoza (2010), including the property that $\rho_z = \rho_r$. The discrete approximation to the VAR in the global solution is constructed using the Kopecky-Suen method, following again Mendoza (2010) in assuming that the Markov realization vectors of the shocks have two values, $n_z = n_\nu = n_u = 2$ (see de Groot et al., 2019, App. C.2 for details).[42]

[41] We did this because Mendoza (2010) used ED preferences but we use standard preferences with $\beta R < 1$.

[42] First, a solution with $n_z = n_\nu = n_u = 5$ produces very similar results (see Appendix C.2.2). Second, the NDL is endogenous in this model but under our calibration, the collateral constraint always binds before both the NDL and ad-hoc debt limit.

For the quasi-linear method, we study cases with the collateral constraint binding and non-binding at the deterministic steady state. In the latter case, we use DEIR to induce stationarity (henceforth, labeled QLOBC-DEIR). When the constraint binds at the deterministic steady state, b^{dss} is well-defined without a stationarity-inducing assumption. The bonds Euler equation is $1 = \beta R + \mu(b^{dss})/u'(b^{dss})$, where μ is the multiplier on the constraint. Since $\beta R < 1 \iff \mu^{dss} > 0$, this case requires $\beta R < 1$ (and no DEIR) and thus henceforth labeled QLOBC-$\beta R < 1$.

We calibrate QLOBC-DEIR following a similar strategy to the global solution. First, we find values of ψ and $(b/y)^*$ such that a 1OA solution of the RBC model matches the mean NFA-GDP ratio and the volatility of consumption in the data.[43] Second, mean NFA in the global solution exceeds the RBC-calibrated value because the collateral constraint increases precautionary savings. We therefore align the Sudden Stops global and QLOBC-DEIR solutions by adjusting b^* in the latter to have the same $E(b/y)$. This yields $(b/y)^* = -0.008$ and $\psi = 0.0044$. The rationale for looking at QLOBC-DEIR is that in the global solution the constraint rarely binds and $E(b/y) > b^{dss}/y^{dss}$. Hence, a local approximation around an unconstrained steady state is in line with the unconstrained long-run equilibrium of the GA solution. In contrast, the QLOBC-$\beta R < 1$ version uses the exact calibrated parameters from the global model.

3.4. Results

Long-run moments Table 8 shows results broadly in line with those from the endowment model. In particular, several second-, correlation and autocorrelation moments are similar across global and local solutions, albeit not as close. The global solution yields higher variability and persistence in consumption, NFA, net exports, and leverage. Supply-side variables differ only slightly, because GHH preferences remove the wealth effect on labor supply (preventing precautionary saving from affecting it) and because, around the stochastic steady state, the model still has a near Fisherian separation of saving and investment as in the RBC model.

Looking at first moments, global and QLOBC-DEIR have the same means by construction, because of the calibration of b^*. However, the precautionary-savings effect increasing mean NFA in the global solution is weaker for QLOBC-$\beta R < 1$. Compared with the mean NFA-GDP

[43] We use 1OA to be consistent with the QLOBC methodology, which is quasi-linear.

ratio of -37% in the RBC model, the global solution of the SS model yields -0.03% but QLOBC-$\beta R < 1$ yields -13.5%. This suggests that, as in the case of the endowment model, counterfactual (policy) experiments that alter self-insurance incentives would yield significantly different results under global and QLOBC methods. QLOBC-DEIR could be kept close to the global solution by re-calibrating the DEIR function, but this requires obtaining the global solution first.

These results have implications for both research and policy. For example, quantifying optimal macroprudential regulation or foreign reserves to manage Sudden Stops risk requires determining how NFA responds to this risk without policy intervention and assessing how precautionary saving incentives respond to policy instruments (e.g., Durdu et al., 2009; Bianchi and Mendoza, 2018). By underestimating precautionary savings, QLOBC solutions would result in excessive accumulation of reserves and overly tight macroprudential regulation.

Certainty equivalence does not hold in the quasi-linear solutions even though the perfect-foresight paths are first-order approximations. In the QLOBC-$\beta R < 1$ (QLOBC-DEIR) solution, $b^{dss}/y^{dss} = -0.192$ (-0.008) while $E(b/y) = -0.135$ (-0.003). This, however, is due to asymmetric responses to shocks induced by the constraint, not precautionary saving. This asymmetry is illustrated in Figure 5 (see also Appendix B.3.6). A negative shock that causes the constraint to bind along the perfect-foresight path determining the date-t value of the solution reduces b_{t+1} by less than the increase in b_{t+1} in response to the same size positive shock. Hence, the quasi-linear time-series is "biased" above b^{dss}, implying a mean above b^{dss}/y^{dss}.[44] The global solution has a similar asymmetry but it also has precautionary savings effects due to the risk of future shocks causing the constraint to bind.

Table 8 also reports execution times of the different solutions. The global solution runs in 265 seconds and is faster than both QLOBC-$\beta R < 1$ and QLOBC-DEIR, which take 464 and 356 seconds, respectively. This is due to the three determinants of the efficiency of QLOBC noted earlier and the near-unit-root nature of NFA. Each extended path required at least 250 periods and the full simulation needed $250,000$ periods to converge to invariant moments.[45]

[44] The constraint in this example is a fixed debt limit while in the SS model it depends on $q_t k_{t+1}$.

[45] The estimators of the mean and autocorrelation of an AR(1) process are consistent but biased in finite samples. The bias is higher the closer the true autocorrelation is to 1 but falls as the sample size rises. A near-unit-root process needs a long sample to ensure negligible estimation bias.

These speed comparisons have some caveats. Global methods suffer from the curse of dimensionality and they are slower in models that require a root-finder when the constraint binds.[46] But, once the decision rules are solved, generating time-series simulations is fast. In contrast, the number of state variables is not an issue for QLOBC methods, but execution time rises with the length of perfect-foresight paths; the iterations needed to compute news-shocks sequences that implement the constraint; and the length of the time-series simulation needed for convergence of unconditional moments. In Appendix C.2.4, we show that the speed gap between the global and QLOBC solutions widen when further lengthening the QLOBC simulation; using only TFP shocks; or setting $\kappa = 0.3$. Using DynareOBC with higher-order approximations and/or integrating over future uncertainty further increase run times (e.g., solving the model using first-order DynareOBC with integration over one period of future uncertainty increases execution time by 30 percent, see Appendix C.2.3).

enario Next, we compare the responses of the global and QLOBC solutions to unanticipated declines in the NFA position to shed light on quantitative differences in precautionary saving behavior and Sudden Stop dynamics. In these experiments, the initial capital stock and the exogenous shocks are at their long-run averages (i.e., $z = u = \nu = 0$).

In Figure 6, NFA unexpectedly drops 25bp below its stochastic steady-state value. Since this represents a small increase in debt, the economy remains far from the collateral constraint.[47] For QLOBC-DEIR (red dot-dash), both consumption and investment fall on impact and net-exports rise, allowing NFA to rise gradually. In contrast, for the global solution (solid-blue), a stronger precautionary saving motive causes consumption and investment to fall more on impact and net-exports to rise more, allowing NFA to recover more rapidly. Output, the price of capital and the capital stock also decline more in the global than the QLOBC-DEIR solution.

In Figure 7, NFA also drops 25bp, but this time starting from a point at which the collateral constraint is marginally binding. Since this triggers the collateral constraint, the resulting effects are much larger than in the previous scenario and resemble those of a Sudden Stop

[46]In the SS model without working capital in the constraint, this is not needed, reducing the *FiPIt* run time by 57% (see Mendoza and Villalvazo, 2020).

[47]We exclude QLOBC-$\beta R < 1$ from this scenario as the economy is constrained at its steady state.

event. In the global solution, NFA rises sharply on impact together with a sharp drop in assets pledgeable as collateral (i.e., capital) and their price. Investment and consumprion fall sharply too, and net exports rise. After these impact effects, NFA-GDP declines gradually, but remains above the baseline for several periods, while consumption, investment, net exports and the price of capital adjust sharply in the second period and then remain relatively stable. Qualitatively, the two QLOBC solutions show similar results, but quantitatively they yield weaker Sudden Stop effects (on impact, net exports rise less and consumption, investment and the price of capital fall less). Impact effects with QLOBC-$\beta R < 1$ (green-dash) are closer to the global results than with QLOBC-DEIR, but NFA reverts quickly back to its (steady-state) binding-constraint level, compared with the gradual decline in the global solution. QLOBC-DEIR shows the weakest effects, with smaller falls in consumption, investment and the price of capital, and a smaller rise in net-exports. NFA-GDP rises less and then declines faster than in the other two solutions.

Periodograms Appendix C.2.1 compares periodograms for the global and QLOBC solutions. As in the endowment model, since all of the variables follow AR(1)-like processes, the periodograms are generally downward sloping, indicating that low frequencies account for a larger fraction of the variance of the variables than business cycle and higher frequencies. The QLOBC periodograms for NFA and net exports differ from the global results. Net exports show higher persistence in QLOBC-$\beta R < 1$ while the QLOBC-DEIR and global solutions have similar persistence. For NFA, QLOBC-$\beta R < 1$ has uniformly lower variability at all frequencies relative to global, with QLOBC-DEIR periodogram more similar to the global one.

Collateral constraint multipliers, Sudden Stops, and risk effects The global and QLOBC-$\beta R < 1$ solutions differ sharply in that the collateral constraint binds much more frequently in the latter (20.0% instead of 3.3% of the time).[48] This is partly because QLOBC methods disregard precautionary savings. Moreover, these methods yield smaller credit-constraint multipliers and financial premia than the gloabl solution, and the sudden-stop responses of macro vari-

[48]Global and QLOBC-DEIR have a similar frequency of the constraint binding because the latter was calibrated to the same mean NFA and with $\mu^{dss} = 0$.

ables differ sharply. To show these results, we compare the multipliers, the shadow interest-rate premium (SIP), the equity premium (EP), its components due to unpledgeable capital, $(1-\kappa)SIP$, and risk (RP), and the Sharpe ratio (S). For macro responses in sudden-stop episodes, we compare deviations from unconditional means in c, nx/y, i, y, L and v.

SIP_t is the amount by which the intertemporal marginal rate of substitution, $u_{c,t}/\beta E_t u_{c,t+1}$, exceeds R_t. The bonds Euler equation gives

$$SIP_t = \frac{R_t \mu_t (1+\tau)}{u_{c,t} - \mu_t(1+\tau)}. \tag{15}$$

SIP_t is only relevant when $\mu_t > 0$ and rises as the constraint becomes more binding, because μ_t rises and $E_t u_{c,t+1}$ falls, since the constraint forces agents to defer consumption.

The equity premium is $EP_t \equiv E_t[R^q_{t+1}] - R_t$, where $R^q_{t+1} \equiv (d_{t+1} + q_{t+1})/q_t$ is the return on equity and d_{t+1} is the dividend payment, where $d_t \equiv \exp(\epsilon^A_t) F_{k,t} - \delta + \frac{a}{2}\frac{(k_{t+1}-k_t)^2}{k_t^2}$. Using the Euler equations for bonds and capital it follows that

$$EP_t = (1-\kappa)SIP_t + RP_t, \qquad RP_t \equiv -\frac{COV_t[u_{c,t+1}, R^q_{t+1}]}{E_t u_{c,t+1}}. \tag{16}$$

EP_t has two components: the standard risk premium (RP_t) driven by $COV_t[u_{c,t+1}, R^q_{t+1}]$ and the fraction of SIP_t pertaining to the share of k_{t+1} that cannot be pledged as collateral $(1-\kappa)$. EP_t rises when $\mu_t > 0$ for two reasons: First, SIP_t rises, as explained above. Second, RP_t rises, because $COV_t[u_{c,t+1}, R^q_{t+1}]$ becomes more negative as consumption is harder to smooth and $E_t u_{c,t+1}$ falls as the collateral constraint forces consumption into the future. Thus, EP_t reflects both the tightness of the constraint via SIP_t and the larger *risk* premium that the constraint induces. The Sharpe ratio measures the compensation for risk-taking, defined as $S_t = E[EP]/\sigma(R^q)$. Following standard practice, we compute S_t using unconditional moments.

For the global solution, the financial premia are computed for each triple (b, k, ε) in the state space (see Appendix C.2.3). Means are then computed using the conditional and unconditional distributions of (b, k, ε). For QLOBC, the moments are computed using the time-series simulations. In a first-order approximation, the risk premium is constant, but can be time-varying in a QLOBC setting where the collateral constraint is occasionally binding.

Table 9 reports quintile distributions of μ conditional on $\mu > 0$, the associated within-quintile averages of financial and macro variables, their overall means and medians, and the Sharpe ratios.[49] Consider first the multipliers and financial premia. Results are similar across QLOBC-DEIR and QLOBC-$\beta R < 1$. Relative to the global solution, however, the multipliers and financial premia are markedly smaller in the quasi-local solutions, and the differences grow larger for higher μ (i.e., in the fourth and fifth quintiles).[50] For global, the overall means of SIP, EP, and $(1-\kappa)SIP$ are 2.59, 2.17,and 2.07, respectively, while QLOBC-$\beta R < 1$ (QLOBC-DEIR) yield smaller premia of $1.54, 1.05, 1.23$ $(1.52, 1.04, 1.21)$. The equity premium, EP, increases sharply with μ because $(1-\kappa)SIP$ rises sharply. In the fifth quintile, the global solution yields means for SIP, EP, and $(1-\kappa)SIP$ of 6.59, 5.38, and 5.27%, respectively, while QLOBC-$\beta R < 1$ (QLOBC-DEIR) yields only $3.49, 2.39, 2.79$ $(3.13, 2.14, 2.51)$. Thus, the quasi-local solutions understate SIP and EP. In the global solution, the risk premium, RP, is about 0.1% on average in each of the five quintiles of μ, whereas it is negative in the two QLOBC approaches, and becomes more negative for the higher quintiles. The compensation for risk-taking is also much higher in the global solution, which yields a Sharpe ratio of 1.16, compared with 0.08 and 0.64 for QLOBC-$\beta R < 1$ and QLOBC-DEIR, respectively.

The sizable differences in SIP and EP result in different sudden-stop responses. To explain why, we follow Mendoza and Smith (2006) in expressing the price of capital as

$$q_t = E_t \left(\sum_{i=1}^{\infty} \left[\prod_{j=0}^{i} \frac{1}{E_t R^q_{t+1+j}} \right] d_{t+1+i} \right). \tag{17}$$

Since (16) implies $E_t R^q_{t+1} = (1-\kappa)SIP_t + RP_t + R_t$, lower financial premia with QLOBC implies higher q_t when $\mu_t > 0$, which in turn implies weaker Fisherian deflation effects of the binding collateral constraint. Moreover, since q_t is a monotonic function of investment due to the Tobin-Q investment setup, k_{t+1} is higher and so is borrowing capacity $(\kappa q_t k_{t+1})$, which

[49] Variables are assigned into quintiles according to the quintile distribution of μ. If a given μ_i belongs to a particular quintile of μ, then the corresponding values of the other variables are assigned to the same quintile. μ is small in general because it is in units of marginal utility with $CRRA$ preferences and $\sigma = 2$. For instance, at the unconditional means of c and L, marginal utility is -4.7 in $\log_{10}$. But small μ values do not imply that the constraint is irrelevant for financial and macro outcomes, as Table 9 shows.

[50] As before, the QLOBC results (for financial premia) are largely unchanged if Gaussian cubature is used to integrate over future uncertainty in the extended paths, rather than relying on perfect-foresight paths.

is key for determining allocations when $\mu_t > 0$. This also affects future dividends, creating feedback effects into q_t and borrowing capacity.

The differences in sudden-stop responses reported in Table 9 reflect the above arguments. In the global solution, the responses are in line with standard Sudden Stop features (i.e., large recessions and sharp reversals in the external accounts). The mean percent declines in c, i, y, L, and v (relative to their unconditional means) are -3.6, -4.1, -1.0, -0.7, and -1.8, respectively while nx/y rises 2.6 pp. on average. The responses are generally larger when the constraint binds more, reaching means of -4.9 for consumption and -13.5 for investment and a trade balance reversal of 5.1 pp. in the fifth quintile of μ. QLOBC-$\beta R < 1$ yields smaller mean declines in consumption (-1.03), investment (-0.49), GDP (-0.47), labor (-0.29) and inputs (-0.97) and a smaller mean increase in net exports (0.45). Neither does it match the property that the responses should be larger when the constraint binds more, displaying instead the largest responses in the first quintile of μ. QLOBC-DEIR performs worse, producing *positive* mean responses for i, y, L and v with only a mean decline in c. Moreover, these counterfactual responses grow larger when the constraint binds more, in the fourth and fifth quintiles of μ.

Thus, there are important differences in the two alternative QLOBC approaches we explored. Dynare-$\beta R < 1$ does better at approximating the effects of the collateral constraint, uses the same calibration as the global solution and does not require extra assumptions to impose stationarity, but overstates the probability of hitting the constraint and does poorly at capturing precautionary savings. On the other hand, QLOBC-DEIR (which matches the mean of NFA of the global solution by construction) yields unconditional moments and a frequency of hitting the collateral constraint closer to the global solution but does not produce Sudden Stops when the constraint binds. It actually yields positive mean deviations in investment, output and factor demands that grow larger as μ rises.

4. Discussion and conclusion

We compared global and local solutions of open-economy models with incomplete markets. Our analysis delivers two key findings. First, when the debt-elastic interest rate (DEIR)—used to induce stationarity—is calibrated to match data targets, the local approximations yield similar model moments to a global solution. Second, the required DEIR elasticities are significantly higher than the arbitrarily low value common in the literature. This hampers precautionary saving behavior by counterfactually inducing stickiness in NFA dynamics.

We analytically verify the stickiness of NFA induced by a high elasticity by showing that the DEIR setup is akin to one where deviations of NFA from its mean is costly. Quantitatively, we study the effects of this stickiness in experiments that strengthen precautionary saving incentives by increasing the variability and persistence of income, and introducing capital controls as taxes on foreign borrowing. Global solutions yield large increases in mean NFA while local solutions stay close to their original mean NFA and their calibrated centers of approximation.

The differences across solutions in these experiments originate in the near-unit-root nature of the NFA equilibrium process, a typical feature of incomplete-markets models. We provide analytic and quantitative results showing that small differences in the NFA autocorrelation cause sizable differences not only in the unconditional mean of NFA but also in the means, variances, correlations and autocorrelations of other variables.

A third finding is that 1OA, 2OA and RSS yield similar results. This follows from three properties of the model. One, an analytic derivation of the NFA decision rule shows that its coefficient on lagged NFA is nearly the same for 1OA, 2OA and RSS when the DEIR elasticity parameter or the difference between the deterministic and risky steady states of NFA are small. Two, the coefficients in the square and interaction terms of 2OA decision rules are small. Three, the calibrated ψ values induce enough NFA stickiness to keep mean NFA nearly unchanged even at higher orders of approximation.

Last, we showed that for the Sudden Stops model, the quasi-linear (QLOBC) method has two additional disadvantages. One, they understate the magnitude of the multipliers of the collateral constraint and its effects on financial premia and macro variables. Two, they do not capture *risk* effects of the collateral constraint and their implications for precautionary savings and forward-looking variables like asset prices. These findings can matter for policy analysis. By underestimating the effects of credit constraints on precautionary savings, the local solutions may recommend excessive accumulation of foreign reserves or macroprudential regulation that is too tight.

For the endowment model, local methods did not turn out to be generally faster or more accurate than the global method. The global solution with 5 nodes in the Markov vector of income shocks was faster than the local methods; was of comparable accuracy as proxied by Euler equation errors; and produces similar results when extended to 25 nodes. For the Sudden Stops model, the global solutions were faster than the QLOBC solution calibrated with the constraint binding at the steady state, and significantly faster than the QLOBC solution when the constraint does not bind at steady state. The latter occurs because QLOBC requires a large simulation length to converge to invariant moments, the extended paths needed to construct the simulations are long, and the NFA process is highly persistent. Still, the curse of dimensionality remains a limitation of the global method when increasing the number of grid nodes.

In conclusion, our results highlight the importance, when using local methods, of calibrating parameters of the function used to induce stationarity (e.g., the DEIR elasticity) to match key data moments. But our findings also suggest caution in using calibrated local solutions to conduct policy analysis or explore the effects of structural changes that alter precautionary saving incentives and NFA dynamics (e.g., in studies examining global imbalances, sovereign default, optimal foreign reserves, or macroprudential policy) or when assessing the frequency and magnitude of Sudden Stops. Good practice, in inducing stationarity in local methods, is to examine the robustness of the results to the value of the stationarity inducing parameters. Alternatively, quasi-linear or RSS local methods can be used without inducing stationarity.

Our findings are robust to several modifications. We thus view our findings as suggesting that local and global methods are best seen as complements when used to solve incomplete-markets models. For parsimonious models, a global solution is feasible and desirable, and innovations in hardware and algorithm design are making global solutions of larger models more feasible. But for larger models that cannot be solved globally, it is best to use local methods while being mindful of the limitations we show. Complementing them with global solutions for simplified versions of large models can shed light on the size and direction of those limitations.

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Table 1: Summary of Numerical Methods used in Open-Economy Models

	Global	Local						Total
Research papers	33	68						101
		Stationarity Assumption				Approximation		
		AHC	DEIR	ED	Other	1OA	Higher	
		16	32	8	12	62	6	
Policy models	0	8						8
		Stationarity Assumption				Approximation		
		AHC	DEIR	ED	Other	1OA	Higher	
		0	5	1	2	8	0	

Note: This table presents a survey of 101 research papers and 8 policy models. The stationarity inducing assumptions are asset holding costs (AHC), debt-elastic interest-rate (DEIR), endogenous discounting (ED), and Other. The local approximation are first-order approximation (1OA) and Higher, which includes higher-order perturbation methods and RSS. Appendix A explains the survey methodology and includes comprehensive details of all the papers and models surveyed.

Table 2: Calibration of the Endowment Model

Notation	Description	Value
Common parameters		
σ	Coefficient of relative risk aversion	2
y	Mean endowment income	1
A	Absorption constant	0.321
R	Gross world interest rate	1.086
σ_z	St. dev. of income	0.0272
ρ_z	Autocorrelation of income	0.749
Global solution parameters		
β	Discount factor	0.917
φ	Ad-hoc debt limit	-0.4364
Local solution parameters		
β	Discount factor $(1/R)$	0.921
ψ	DEIR elasticity coefficient (2OA)	0.042
ψ	DEIR elasticity coefficient (partial RSS)	0.042
b^*	DEIR steady-state NFA (2OA)	-0.374
b^*	DEIR steady-state NFA (partial RSS)	-0.374

Table 3: Effects of Higher Income Persistence in the Endowment Model

	ρ_z	0	0.1	0.3	0.5	0.7	0.9	0.95
i)	**Global solution**							
	ρ_{nx}	-0.103	-0.004	0.201	0.426	0.678	0.945	0.982
	ρ_b	0.787	0.834	0.907	0.955	0.984	0.997	0.997
	$E(b)$	-0.411	-0.410	-0.404	-0.395	-0.373	-0.294	-0.269
ii)	**Local solutions**							
	2OA							
	ρ_{nx}	-0.022	0.123	0.327	0.529	0.729	0.920	0.963
	ρ_b	0.927	0.940	0.960	0.975	0.987	0.996	0.998
	$E(b)$	-0.372	-0.372	-0.371	-0.369	-0.364	-0.348	-0.340
	Partial RSS							
	ρ_{nx}	0.022	0.123	0.327	0.529	0.728	0.919	0.962
	ρ_b	0.927	0.940	0.960	0.975	0.987	0.996	0.998
	$E(b)$	-0.373	-0.372	-0.371	-0.370	-0.365	-0.348	-0.336
iii)	**Re-calibrated local solutions for each ρ_z**							
	2OA							
	ρ_{nx}	-0.019	0.081	0.283	0.493	0.729	0.916	0.968
	ρ_b	0.835	0.869	0.923	0.960	0.987	0.996	0.998
	b^*	-0.410	-0.408	-0.404	-0.395	-0.380	-0.309	-0.298
	ψ	0.191	0.172	0.133	0.094	0.042	0.043	0.030
	Partial RSS							
	ρ_{nx}	-0.019	0.080	0.282	0.491	0.721	0.929	0.968
	ρ_b	0.834	0.869	0.922	0.959	0.985	0.997	0.998
	b^*	-0.410	-0.409	-0.404	-0.396	-0.379	-0.320	-0.308
	ψ	0.192	0.173	0.135	0.096	0.048	0.030	0.030

Note: 2OA and RSS denote the second-order and partial risky-steady state solutions, respectively The re-calibrated local solutions for each ρ_z in panel iii) re-calibrate ψ and b^* so as to match two moments of the corresponding GA solution, $E(b)$ (shown in Panel i)) and the standard deviation of consumption (not shown).

Table 4: Long-run Moments: Endowment Model

	Data	Global		Local		
		$n_z = 5$	$n_z = 25$	2OA	Full RSS	Partial RSS
		$n_b = 200$	$n_b = 200$	DEIR	$\beta R < 1$	DEIR
DEIR parameters						
ψ	·	·	·	0.042	·	0.042
b^*	·	·	·	-0.374	·	-0.374
			Cyclical moments			
Standard deviation relative to GDP						
c**	1.247	1.349	1.353	1.363	15.080	1.358
nx/y	0.775	0.541	0.542	0.649	1.799	0.650
b/y	10.302	9.393	9.538	11.457	3.274	11.499
Correlation with GDP						
c	0.895	0.834	0.833	0.759	0.250	0.758
nx/y	-0.688	0.435	0.429	0.458	-0.072	0.464
b/y	0.246	0.537	0.526	0.585	0.364	0.582
First-order autocorrelation						
c	0.701	0.875	0.883	0.947	0.995	0.947
nx/y	0.797	0.762	0.764	0.788	0.999	0.787
b/y	0.974	0.971	0.972	0.984	0.984	0.984
Performance metrics						
Run time (sec)	·	0.1	3.2	0.6	0.3	3.1
Euler errors						
$L1 - norm$	·	2.10e-04	3.23e-05	1.04e-03	2.27e-02	7.55e-05
$L2 - norm$	·	5.03e-04	6.72e-05	1.07e-03	2.85e-02	1.29e-04
$L\infty - norm$	·	1.89e-02	3.83e-04	2.19e-03	1.47e-01	9.42e-04

Note: 2OA and RSS refer to second-order and risky steady state, respectively. Results were obtained using Matlab2024a in PC with a 12th Gen Intel(R) Core(TM) i7-1265U 1.80 GHz processor and 32 GB RAM. Run times include elapsed time up to the solution of decision rules. See Appendix B for details on Euler equation (EE) errors. **This moment was targeted in the calibration of 2OA and partial RSS.

Table 5: Endowment Model with Income and Interest-Rate Shocks

	Interest Rate Standard Deviation (%)					
	0.0	0.5	1.0	1.5	2.0	2.5
Global calibrated						
$E(b/y)$	-0.363	-0.360	-0.351	-0.3335	-0.310	-0.276
$\sigma(c)/\sigma(y)$	1.355	1.371	1.424	1.520	1.661	1.835
$\sigma(b/y)/\sigma(y)$	9.724	10.177	11.672	14.640	19.664	27.077
$\rho(y, nx/y)$	0.428	0.416	0.385	0.348	0.312	0.284
$\rho_{nx/y}$	0.760	0.761	0.763	0.770	0.781	0.789
$\rho_{b/y}$	0.982	0.983	0.986	0.989	0.991	0.990
ρ_c	0.888	0.886	0.879	0.869	0.861	0.854
Global with NDL						
$E(b/y)$	-5.294	-4.612	-3.662	-3.000	-2.510	-2.129
$\sigma(c)/\sigma(y)$	5.757	4.015	3.790	3.689	3.620	3.573
$\sigma(b/y)/\sigma(y)$	2.468	2.738	3.510	4.290	5.041	5.837
$\rho(y, nx/y)$	-0.003	0.061	0.107	0.133	0.151	0.165
$\rho_{nx/y}$	0.981	0.951	0.929	0.910	0.891	0.871
$\rho_{b/y}$	0.981	0.981	0.984	0.986	0.988	0.989
ρ_c	0.979	0.959	0.944	0.928	0.912	0.894
2OA DEIR Baseline calibration						
$E(b/y)$	-0.363	-0.362	-0.358	-0.351	-0.342	-0.331
$\sigma(c)/\sigma(y)$	1.363	1.384	1.445	1.541	1.667	1.814
$\sigma(b/y)/\sigma(y)$	11.458	11.659	12.257	13.249	14.636	16.439
$\rho(y, nx/y)$	0.458	0.445	0.412	0.371	0.329	0.291
$\rho_{nx/y}$	0.788	0.784	0.772	0.759	0.747	0.738
$\rho_{b/y}$	0.984	0.984	0.984	0.985	0.985	0.986
ρ_c	0.947	0.940	0.920	0.893	0.865	0.839
Full RSS ($\beta R < 1$)						
$E(b/y)$	-7.041	-6.232	-4.996	-3.887	-2.937	-2.124
$\sigma(c)/\sigma(y)$	15.080	13.454	11.171	8.982	6.935	5.150
$\sigma(b/y)/\sigma(y)$	3.274	4.838	7.441	9.856	11.743	13.071
$\rho(y, nx/y)$	-0.072	-0.033	-0.003	0.017	0.037	0.062
$\rho_{nx/y}$	0.999	0.995	0.991	0.986	0.975	0.952
$\rho_{b/y}$	0.984	0.992	0.996	0.997	0.998	0.998
ρ_c	0.995	0.994	0.991	0.987	0.977	0.957
Partial RSS (*with DEIR*) *Baseline calibration*						
$E(b/y)$	-0.363	-0.363	-0.361	-0.359	-0.355	-0.351
$\sigma(c)/\sigma(y)$	1.358	1.361	1.369	1.383	1.402	1.427
$\sigma(b/y)/\sigma(y)$	11.499	11.516	11.566	11.650	11.770	11.928
$\rho(y, nx/y)$	0.464	0.463	0.458	0.450	0.439	0.427
$\rho_{nx/y}$	0.787	0.780	0.761	0.731	0.692	0.648
$\rho_{b/y}$	0.984	0.984	0.984	0.984	0.984	0.985
ρ_c	0.947	0.943	0.930	0.910	0.882	0.849

Note: The volatility and persistence of endowment shocks are kept as in Table 2. GA, 2OA and RSS refer to the global, second-order and risky-steady state solutions, respectively.

Table 6: Effects of Capital Controls

	Tax on Capital Flows	0.00	1.00	1.75	2.50	3.25	4.00	4.75
i)	**Global solution**							
	$E(b/y)$	-0.363	-0.331	-0.309	-0.278	-0.229	-0.145	0.031
	ρ_{nx}	0.748	0.768	0.791	0.818	0.849	0.892	0.936
	ρ_b	0.989	0.996	0.996	0.997	0.998	0.999	0.999
	$\sigma(c)/\sigma(y)$	1.349	1.368	1.385	1.422	1.505	1.694	2.182
ii)	**Local solutions**							
	2OA							
	$E(b/y)$	-0.363	-0.343	-0.327	-0.311	-0.295	-0.278	-0.262
	ρ_{nx}	0.777	0.775	0.774	0.772	0.771	0.769	0.768
	ρ_b	0.989	0.989	0.989	0.989	0.989	0.989	0.989
	$\sigma(c)/\sigma(y)$	1.363	1.351	1.341	1.332	1.323	1.314	1.304
	partial RSS							
	$E(b/y)$	-0.363	-0.343	-0.327	-0.311	-0.295	-0.278	-0.262
	ρ_{nx}	0.776	0.774	0.773	0.772	0.770	0.769	0.768
	ρ_b	0.989	0.989	0.989	0.989	0.989	0.989	0.990
	$\sigma(c)/\sigma(y)$	1.358	1.346	1.337	1.327	1.318	1.309	1.300

Note: 2OA and RSS denote the second-order and partial risky-steady state solutions, respectively.

Table 7: Calibration of the Sudden Stops Model

Notation	Description	Value
Common parameters		
σ	Coefficient of relative risk aversion	2
R	Gross world interest rate	1.0857
α	Labor share in gross output	0.592
γ	Capital share in gross output	0.306
η	Imported inputs share in gross output	0.102
δ	Depreciation rate of capital	0.088
ω	Labor exponent in the utility function	1.846
ϕ	Working capital constraint coefficient	0.258
a	Investment adjustment cost parameter	2.750
τ	Consumption tax	0.168
κ	Collateral constraint coefficient	0.2
ρ^A	TFP autocorrelation	0.555
ρ^R	Interest rate autocorrelation	0.555
ρ^p	Input price autocorrelation	0.737
$\sigma^2_{u^A}$	Variance of TFP innovations	1.0273e-04
$\sigma^2_{u^R}$	Variance of interest rate innovations	2.4387e-04
$\sigma^2_{u^p}$	Variance of input price innovations	5.1097e-04
σ_{u^A,u^R}	Covariance of TFP and interest rate innovations	-0.0047
Global solution and QLOBC-$\beta R < 1$ parameters		
β	Discount factor	0.920
QLOBC-DEIR parameters		
β	Discount factor (set to $1/R$)	0.9211
ψ	DEIR elasticity coefficient	0.0044
$(b/y)^*$	DEIR det. steady-state NFA/GDP	-0.008

Note: A first step in calibrating the GA Sudden Stops model uses the RBC model without collateral constraint targeting β and the ad-hoc debt limit φ to match $E(b/y)$ and $\sigma(c)/\sigma(y)$ in the data. For solving the GA Sudden Stops model, however, φ turned out to be irrelevant because the collateral constraint always binds first.

Table 8: Long-run Moments: Sudden Stops Model

	Global	QLOBC	
	GA	$\beta R < 1$	DEIR
Mean relative to GDP			
c	0.695	0.686	0.694
i	0.171	0.171	0.172
nx/y	0.016	0.027	0.017
b/y	-0.003	-0.135	-0.003
$lev.ratio$	-0.110	-0.173	-0.110
υ	0.108	0.108	0.108
Standard deviation relative to GDP			
$\sigma(c)/\sigma(y)$	1.021	0.971	0.944
$\sigma(i)/\sigma(y)$	3.252	3.224	3.409
$\sigma(nx/y)/\sigma(y)$	0.709	0.582	0.681
$\sigma(b/y)/\sigma(y)$	4.714	2.100	3.283
$\sigma(lev.ratio)/\sigma(y)$	2.215	0.979	1.524
$\sigma(\upsilon)/\sigma(y)$	1.496	1.513	1.510
$\sigma(L)/\sigma(y)$	0.596	0.598	0.599
Correlations with GDP			
$\rho(y,c)$	0.854	0.951	0.906
$\rho(y,i)$	0.650	0.685	0.643
$\rho(y,nx/y)$	-0.125	-0.257	-0.131
$\rho(y,b/y)$	-0.077	-0.044	-0.177
$\rho(y,lev.rat.)$	-0.066	0.0085	-0.144
$\rho(y,\upsilon)$	0.830	0.832	0.830
$\rho(y,L)$	0.995	0.995	0.995
First-order autocorrelations			
$\rho(y)$	0.815	0.816	0.817
$\rho(c)$	0.831	0.797	0.797
$\rho(i)$	0.499	0.474	0.499
$\rho(nx/y)$	0.600	0.407	0.513
$\rho(b/y)$	0.990	0.978	0.985
$\rho(lev.rat.)$	0.992	0.986	0.990
$\rho(\upsilon)$	0.772	0.769	0.773
$\rho(L)$	0.792	0.785	0.795
Credit constraint			
Prob.(μ>0)	3.30	20.05	3.48
Execution time			
Runtime in seconds	265	464	356

Note: Results were obtained using Matlab2024a in a PC with a 12th Gen Intel(R) Core(TM) i7-1265U 1.80 GHz processor and 32 GB RAM.

Table 9: Collateral Constraint Multipliers, Macro & Financial Variables Conditional on $\mu > 0$

	$\log(\mu)$		Financial Premia				Macro variables					
	upper limit	mean	means SIP	EP	$(1-\kappa)SIP$	RP	means of deviations from long-run averages c	nx/y	i	y	L	v
Panel a. Global												
Quintiles of μ												
First	-6.78	-7.20	0.18	0.24	0.14	0.10	-1.60	1.27	2.10	0.33	0.34	0.38
Second	-6.35	-6.55	0.80	0.74	0.64	0.10	-3.30	2.17	-4.53	-1.35	-0.77	-3.04
Third	-6.15	-6.24	1.60	1.38	1.28	0.10	-3.28	2.16	-2.56	-0.99	-0.61	-0.66
Fourth	-5.87	-6.01	2.70	2.26	2.16	0.10	-4.22	2.94	-5.98	-1.59	-1.09	-2.79
Fifth	-3.36	-5.68	5.98	4.82	4.72	0.10	-4.81	4.92	-12.58	-1.32	-1.39	-3.26
Overall mean		-6.10	2.24	1.90	1.80	0.10	-3.45	2.67	-4.72	-0.99	-0.71	-1.89
Overall median		-6.24	1.58	1.36	1.26	0.10	-3.56	2.43	-5.94	-1.72	-0.90	-1.89
			Ex-post Sharpe ratio = 1.08									
Panel b. QLOBC-$\beta R < 1$												
Quintiles of μ												
First	-6.93	-7.35	0.14	0.10	0.12	-0.02	-3.34	0.99	-6.76	-2.94	-1.72	-3.65
Second	-6.54	-6.69	0.69	0.47	0.55	-0.08	-0.24	0.16	1.90	0.30	0.24	0.18
Third	-6.31	-6.41	1.30	0.89	1.04	-0.15	-0.44	0.24	1.33	0.12	0.10	-0.22
Fourth	-6.13	-6.22	2.05	1.41	1.64	-0.24	-0.51	0.34	0.90	0.08	0.03	-0.41
Fifth	-5.60	-5.99	3.49	2.39	2.79	-0.40	-0.63	0.54	0.19	0.07	-0.08	-0.77
Overall mean		-6.34	1.54	1.05	1.23	-0.18	-1.03	0.45	-0.49	-0.47	-0.29	-0.97
Overall median		-6.42	1.30	0.89	1.04	-0.15	-0.73	0.36	0.33	-0.17	-0.11	-0.81
			Ex-post Sharpe ratio = 0.08									
Panel c. QLOBC-DEIR												
Quintiles of μ												
First	-6.80	-7.09	0.29	0.20	0.23	-0.03	-1.47	1.15	2.71	0.44	0.35	0.12
Second	-6.52	-6.64	0.83	0.57	0.66	-0.10	-1.32	1.14	3.21	0.64	0.45	0.37
Third	-6.34	-6.42	1.36	0.93	1.09	-0.16	-1.23	1.15	3.66	0.80	0.52	0.32
Fourth	-6.18	-6.26	1.98	1.35	1.58	-0.23	-0.98	1.10	4.52	1.10	0.68	0.77
Fifth	-5.73	-6.06	3.13	2.14	2.51	-0.36	-0.38	1.01	6.43	1.85	1.09	1.48
Overall mean		-6.37	1.52	1.04	1.21	-0.18	-1.07	1.11	4.11	0.97	0.62	0.61
Overall median		-6.42	1.37	0.94	1.09	-0.16	-1.11	1.14	3.99	0.93	0.58	0.57
			Ex-post Sharpe ratio = 0.64									

Note: SIP is the Shadow interest premium, EP is the equity premium, and RP is the risk premium component of EP. The quintile distribution of μ is conditional on $\mu > 0$. Means for other variables are computed using the distribution of μ within each quintile of the overall distribution of μ conditional on $\mu > 0$. $\log(\mu)$ is the base-10 logarithm of the multiplier on the collateral constraint. The moments for the global solution are computed using the recursive decision rules and ergodic distribution of the model. For EP, we compute the equity premium conditional on all date-t states (b, k, s) and then calculate the mean from the ergodic distribution. The Sharpe ratio is computed using the conditional ergodic distribution for $\mu > 0$. The moments for QLOBC are computed using time-series simulation with 250,000 periods.

Figure 1: First-order coefficient of 2OA NFA decision rule

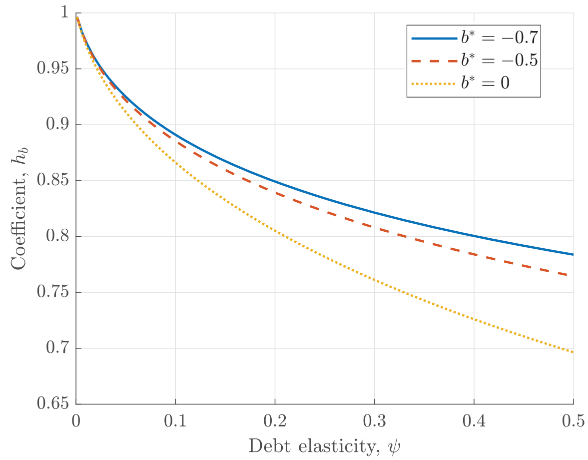
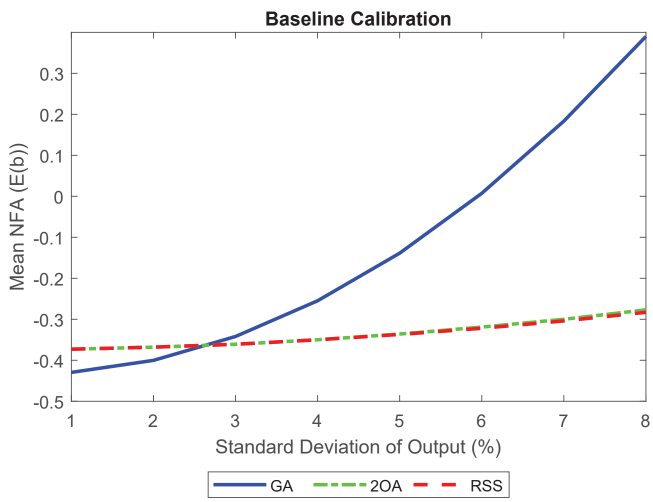
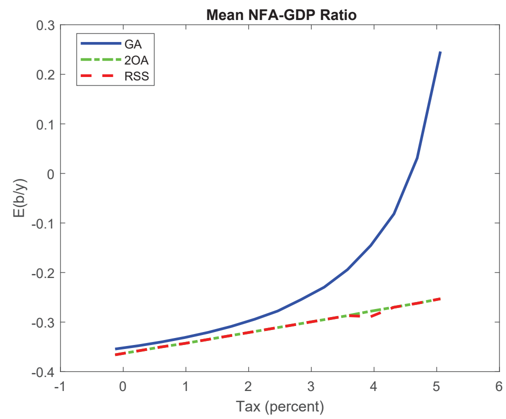


Figure 2: Income Risk and Mean NFA in the Endowment Model



Note: GA refers to global solution, 2OA refers to second-order solution, RSS refers to risky-steady state solution.

Figure 3: Effects of Capital Controls on the Mean NFA-GDP Ratio



Note: GA, 2OA and RSS denote global, second-order and risky-steady state solutions, respectively. Capital controls are modeled as a tax on foreign borrowing with revenue rebated as a lump-sum transfer.

Figure 4: Endowment Model Impulse Responses to a Negative Income Shock

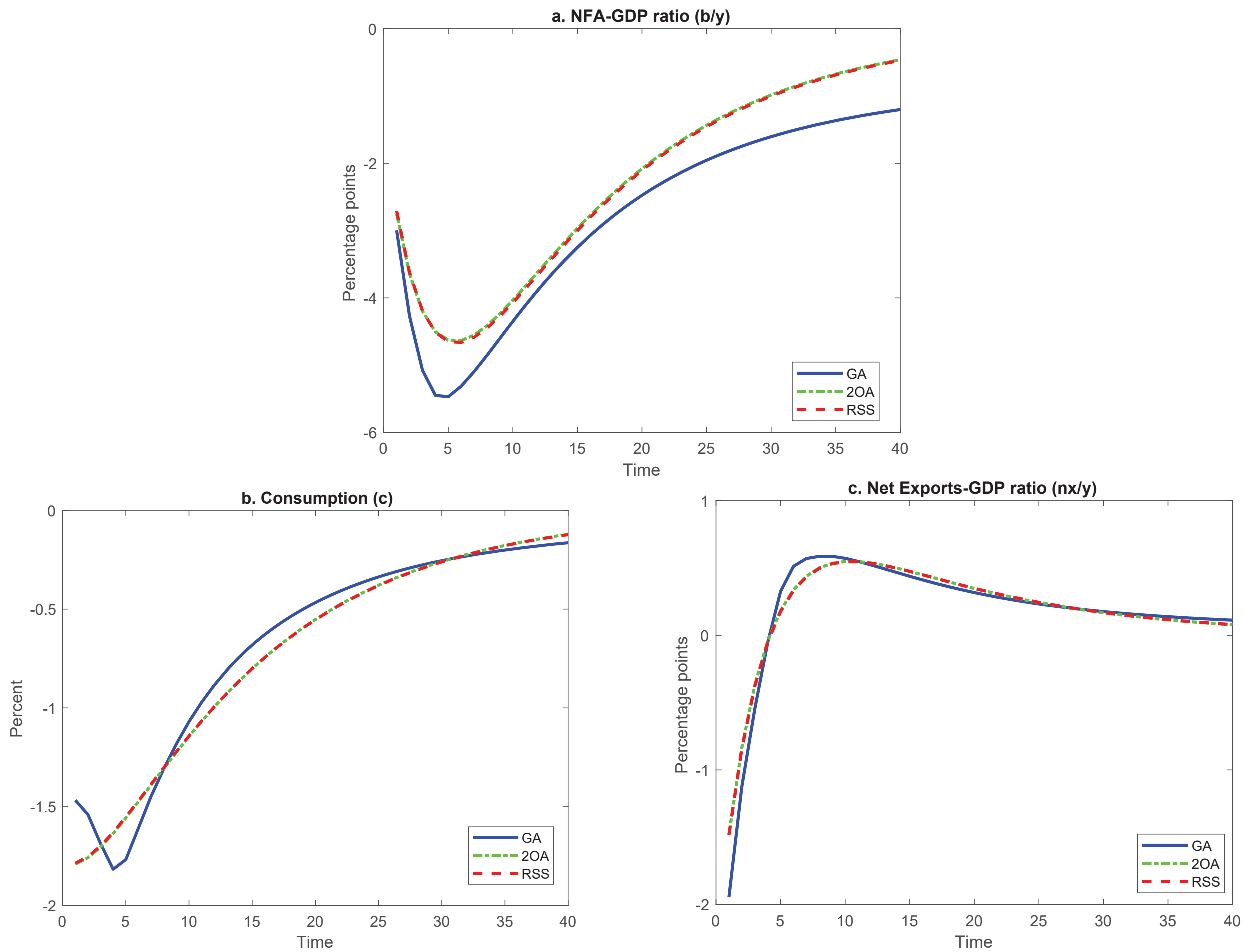
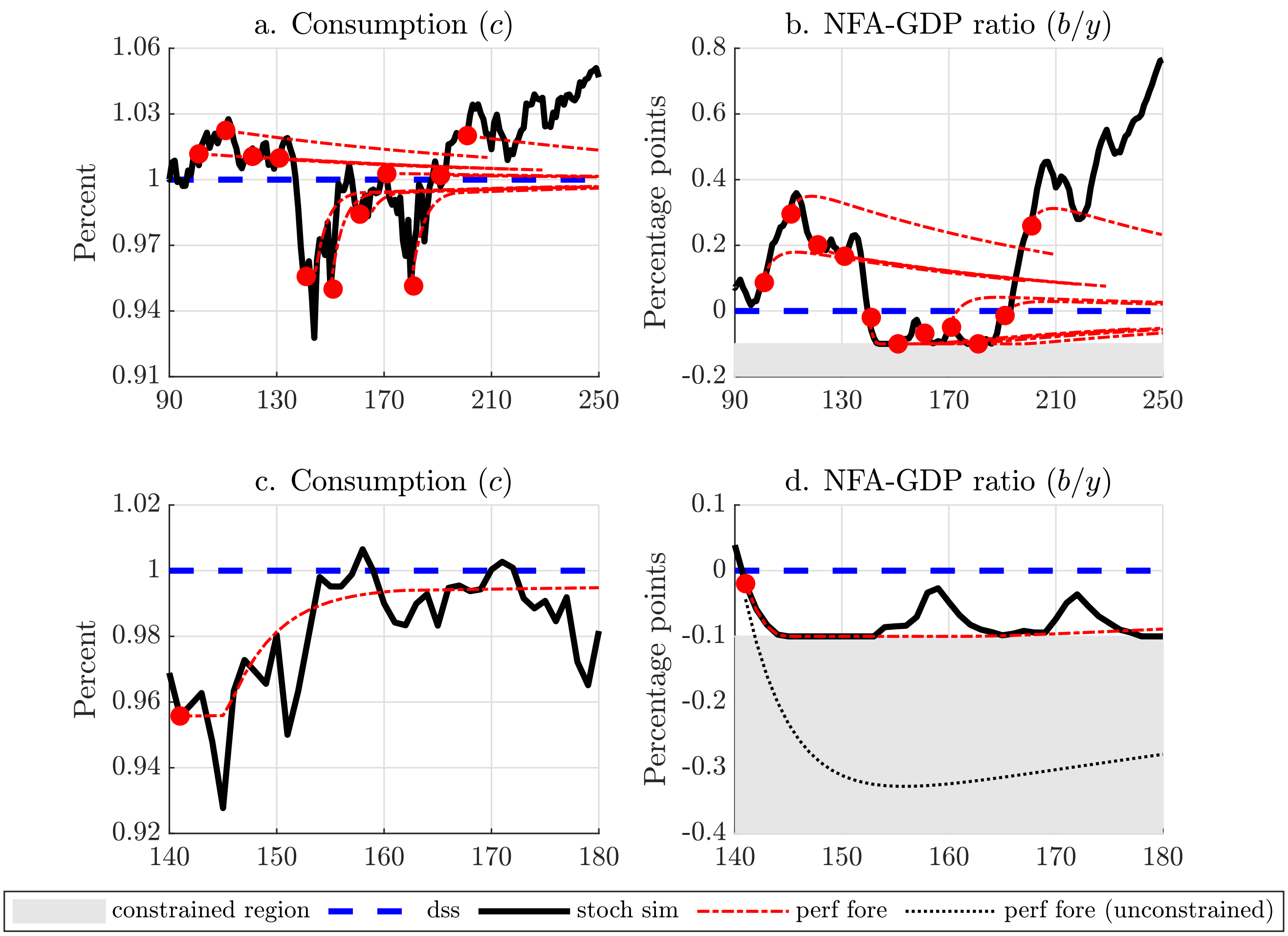


Figure 5: QLOBC Solution for the Endowment Model



Note: The top row plots one draw of the stochastic simulation ("stoch sim") from period 90 to 250 and plots corresponding perfect foresight ("perf fore") paths for select periods. The bottom row focussed on period 140 to 180 and plots both the constrained and unconstrained perfect foresight path from period 141.

Figure 6: SS model responses to an unanticipated NFA drop
(from the stochastic steady state)

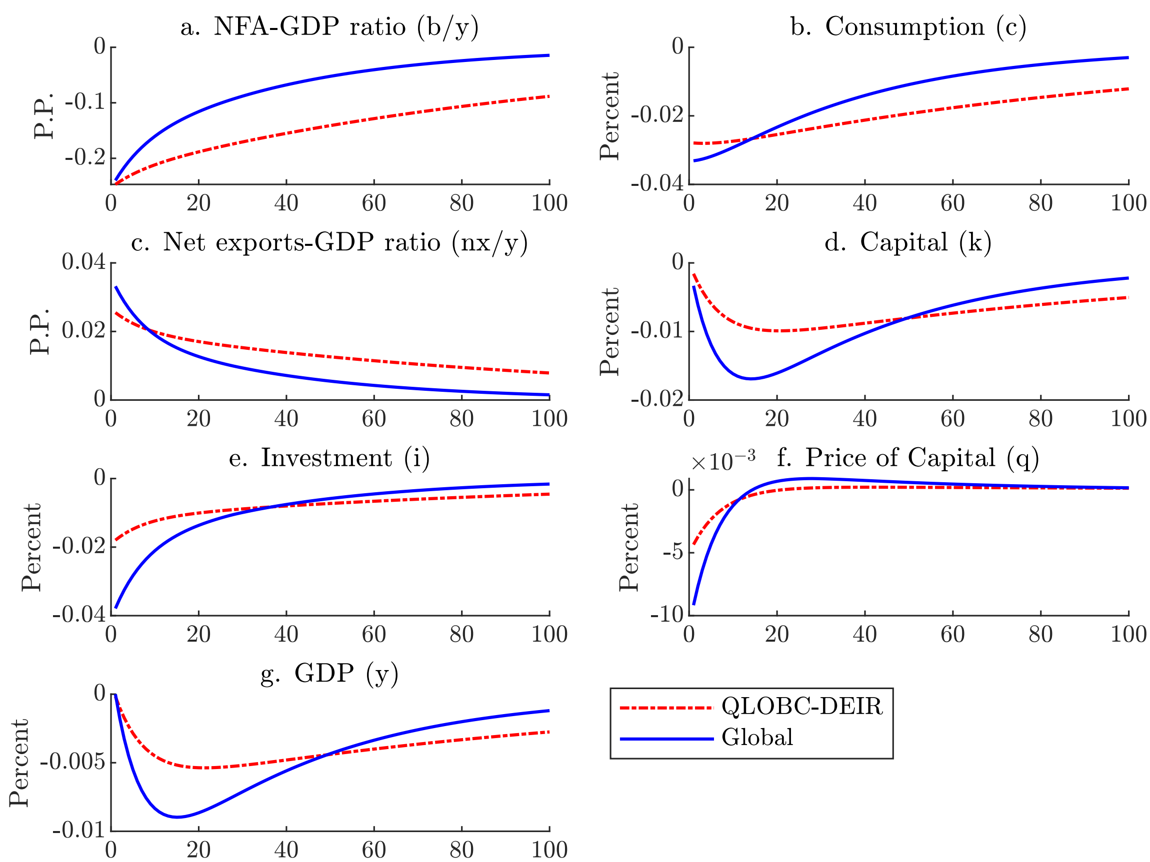


Figure 7: SS model responses to an unanticipated NFA drop
(with a binding collateral constraint)