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A Protocol for Repeated Bargaining

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Abstract

We propose a protocol for repeated bargaining where occasional periods of good outside opportunities yield improved outcomes but also higher breakout probabilities, yet there is a lot of risk sharing. Crucially, we only consider Markov perfect equilibria that have neither non payoff-relevant state variables that are costly to compute nor a contrived process of equilibrium selection.

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1 Introduction

There are many repeated situations without commitment where a pair of risk averse agents with changing circumstances can split a surplus or leave the match. Examples include international borrowing (Kehoe and Perri (2002)), private arrangements in island economies (Attanasio and Rıos-Rull (2000)) or marriages (Mazzocco (2007)). Conventionally, these environments are studied looking at the constrained efficient allocations, maximizing the utility of one agent subject to guaranteeing a certain utility level to the other agent and participation constraints for both (Thomas and Worrall (1988), and Kocherlakota (1996)). In practice, this entails solving a maximization problem with Pareto weights that change whenever the participation constraints bind, an approach proposed by Marcet and Marimon (2019) (henceforth MM).

We propose a new approach to model these environments where simple Markov perfect equilibria generate three desirable properties: *i)* costless agreements are possible, *ii)* outside opportunities can change the terms of the agreements, *iii)* certain circumstances (either changes within the context of the pair or changes on their outside opportunities) may break the arrangement with utility costs for both agents. Our approach can be easily used in complicated environments (many types of pairs of agents, continuous endogenous state variables, market clearing, model estimation). Because of the focus on Markov equilibria, a static consideration of the environment suffices to understand the details of how we model the interaction. The extension to dynamic environments is immediate.

The gist of our approach is to specify the existence of a default arrangement that can be implemented costlessly, and that we think of as a social norm. In the absence of an explicit challenge by any of the agents this is the outcome. Agents can challenge the default arrangement in quest of a more rewarding outcome. When they do so, they propose an alternative arrangement, that will only be in place for one period, in the form of a take-it-or-leave-it offer. We regard challenges as intrinsically costly affairs, that may yield utility loses in the form of negotiation costs for the agents which limits their attractiveness. Further, there is a level of randomness that appears between when the proposals are made and when they are accepted that leads itself to different outcomes for ex ante-identical agents. In these environments, the parameters are often such that elements of *battle of the sexes* appear which yields multiplicity of Nash equilibria of the simultaneous move game. To avoid this possibility we pose the environment as a sequential move game by letting nature choose the identity of the first mover.

We describe a static version of these ideas within a pie sharing environment (Section 2). We also include a comparison with the MM aproach (Section 3).

2 A Simple Static Pie Sharing Environment

Consider two risk averse agents, $\{i, j\}$, that have to split a surplus if they are together, or they can be separate. To avoid corner solutions and to get uniqueness of outcomes we pose extreme value shocks to preferences. Before the shocks are realized, the utilities that the agents get are publicly observable: if they stay together, they get $\{u^i(\lambda^i), u^j(\lambda^j)\}$, with λ^i, and $\lambda^j = 1-\lambda^i$ being the respective shares of the surplus that belong only to a finite set $\lambda^i \in \{\lambda_1, \cdots, \lambda_{N^\lambda}\} = \Lambda \subset [0, 1]$; if they do not stay together, they get $\{V^i, V^j\}$.

Sequence of Events First, nature allocates one of the two agents to be a first mover, call it i for simplicity of notation, who then chooses whether to accept the status quo N^i or to challenge it proposing alternative sharing rule $\lambda^i \in \Lambda$. The latter choice intrinsically generates disutility κ on both agents, the costs of arguing. Then, privately observed extreme value shocks $\left\{\epsilon^{T,i}, \epsilon^{A,i}, \epsilon^{T,j}, \epsilon^{A,j}\right\}$, $\epsilon \sim G(\mu, \alpha)$ with mean zero ($\mu = -\alpha\gamma$), are realized for both agents that affect the utility of staying together or separating. If the choice of i is N^i, then agent j chooses whether to play N^j and accept the status quo or to make a take it or leave it counteroffer λ^j, which also generates disutility κ on both agents, and makes agent i decide whether to accept it a^i and stay together with sharing rule λ^j or reject it r^i and separate. If the choice of i in the first stage is λ^i, then agent j chooses either to accept, a^j, $i's$ proposed arrangement λ^i or to reject it, r^j, in which case agents separate. Figure 1 depicts the game played in extensive form, including the actual payoffs in each branch. This sequence of events ensures that neither agent is sure of the final outcome when they take actions making outcome probabilities continuous functions of the parameters that determine the utilities of being alone or being together; it also avoids the need to infer the realized shocks from the actions of the other player (player i has not seen the shocks when it makes its first offer so player j has nothing to learn; player j has seen its shocks when it makes its offer but they are irrelevant for player i's choice to accept or reject).

We analyze the game backwards to infer its subgame perfect equilibrium.

Third Stage: Accept or Reject Agents choose whether to accept or reject challenging offers λ^i or λ^j. If player i has proposed λ^i, player j compares accepting, a^j, yielding $u(1-\lambda^i) + \epsilon^{T,j} - \kappa$ with rejecting (r^j), yielding $V^j + \epsilon^{A,j} - \kappa$. Its ex-ante utility (before the realization of the shocks) is

$$\Omega^{3j}\left(V^j, \lambda^i\right) = \mathbb{E}\left[\max\left\{u(1-\lambda^i) + \epsilon^{T,j} - \kappa, V^j + \epsilon^{A,j} - \kappa\right\}\right] = \\ \alpha \log\left[\exp(u(1-\lambda^i)/\alpha) + \exp(V^j/\alpha)\right] - \kappa, \quad (1)$$

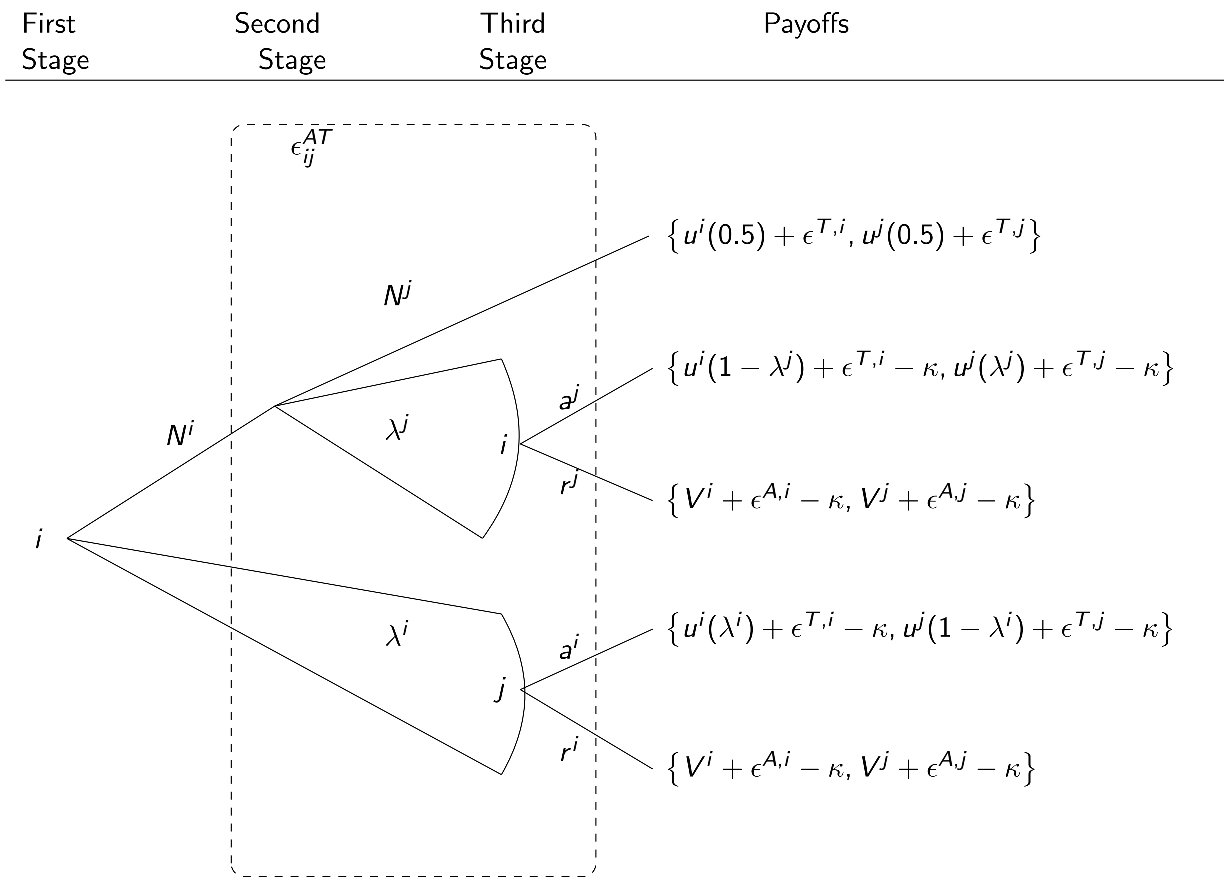

Figure 1: Game Tree when Player i is the First Mover

with acceptance probability

$$q^{3j} = \frac{\exp[\alpha^{-1}\ u(1-\lambda^i)]}{\exp[\alpha^{-1}\ u(1-\lambda^i)] + \exp[\alpha^{-1}\ V^j]}. \tag{2}$$

The expected utility of the proposer, player i, becomes

$$\begin{aligned}\widehat{\Omega}^{2i}(V^i, V^j, \lambda^i) = \mathbb{E}\left\{q^{3j}(\lambda^i, V^j)\ \left[u(\lambda^i)+\epsilon^{T,i}-\kappa\right] + \left[1-q^{3j}(V^j,\lambda^i)\right]\ \left(V^i+\epsilon^{Ai}-\kappa\right)\right\} = \\ q^{3j}(\lambda^i, V^j)\ \left[u(\lambda^i)-\kappa\right] + \left[1-q^{3j}(V^j,\lambda^i)\right]\ \left(V^i-\kappa\right).\end{aligned} \tag{3}$$

Symmetry yields $\left\{\Omega^{3i}, s^{3i}, q^{3i}, \widehat{\Omega}^{2j}\right\}$.

Second Stage: Player j's choice of how to respond when i chose N^i It either accepts social norm N^j yielding $\mathbb{E}\left\{u(0.5)+\epsilon^{T,j}\right\}$, or proposes an alternative λ^j which yields $\widehat{\Omega}^{2j}\left(V^j, V^i, \lambda^j\right)$. It solves

$$\overline{\Omega}^{2j}\left(V^j, V^i, \epsilon^j | N^i\right) = \max \left\{u(0.5)+\epsilon^{T,j}, \max_{\lambda^j} \widehat{\Omega}^{2j}\left(V^j, V^i, \lambda^j\right)\right\}, \tag{4}$$

which yields ex-ante

$$\mathbb{E}\left[\overline{\Omega}^{2j}(V^j, V^i, \epsilon^j | N^i)\right] = \alpha \log\left[\exp\left(\frac{u(0.5)}{\alpha(1-q^{3j})}\right) + \exp\left(\frac{q^{3j}u(\lambda^j)+(1-q^{3j})V^j-\kappa}{\alpha(1-q^{3j})}\right)\right].$$

and associated choice probabilities of which $q^{2j}(V^j, V^i, N^j|N^i) = \frac{\exp\left(\frac{u(0.5)}{\alpha(1-q^{3j})}\right)}{\exp\left(\frac{u(0.5)}{\alpha(1-q^{3j})}\right)+\exp\left(\frac{q^{3j}u(\lambda^j)+(1-q^{3j})V^j-\kappa}{\alpha(1-q^{3j})}\right)}$.

First Stage: After Nature, First Mover, i, Chooses whether to Challenge Player i assesses the utility of each option based only on the observed values of the outside opportunities $\left\{V^i, V^j\right\}$. In doing so it uses the choice probabilities of agent j. It solves

$$\begin{aligned}\Omega^{i1}\left(V^i, V^j\right) = \max\Bigg\{ & q^{2j}\left(V^j, V^i, N^j|N^i\right)\ u(0.5) + \sum_{\lambda^j} q^{2j}\left(V^j, V^i, \lambda^j|N^i\right)\ \Omega^{3i}(V^i, \lambda^j), \\ & \max_{\lambda^i}\ \left[q^{3j}(V^j\lambda^i)\ \left(u(\lambda^i)-\kappa\right) + \left(1-q^{3j}(V^j\lambda^i)\right)\ \left(V^i-\kappa\right)\right]\Bigg\}\end{aligned} \tag{5}$$

Equilibrium It is just the set of probability choices at each stage. As this involves sequential decisions over compact sets, a unique solution always exists.

Equilibrium Properties The allocations that result have the desired properties (see Figure 2 for an example): *i)* costless agreements (blue) are possible, and more likely the lower the value of outside opportunities, the higher the arguing costs, and typically the lower the variance of extreme value shocks; *ii)* better outside opportunities (orange) can improve the terms, which happens more often with lower arguing costs; and *iii)* the arrangement may break and separation ensues (red), which occurs more often when outside opportunities are high and when the variance of shocks is high.

3 A comparison with Marcet and Marimon (2019)

Our approach has various important differences with the standard approach that follows MM. First, from the conceptual side, the environments are slightly different. In ours, negotiation, which we interpret as a departure from the social norm, is inherently costly while MM assume it is costless. A related issue is the restriction to Markov equilibria that can be interpreted as imposing for sufficient conditions to

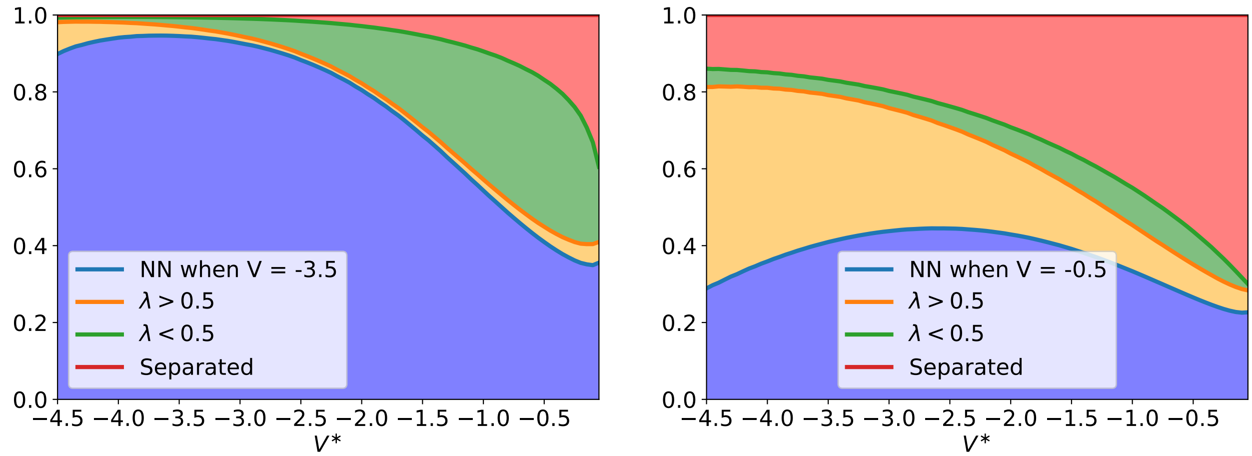

Figure 2: Probability of Outcomes as function of the other's outside value for low (left panel) and high (right panel) own outside value with $\kappa = 0.45$.

obtain uniqueness of the outcome. MM choose the efficient allocation but the implementation requires a particular equilibrium selection, one of infinitely many that is justified only because it is efficient. Our approach also has an explicit notion of social norm or allocation that can be achieved without engaging in costly negotiation, that can be taken to be arbitrary but can be estimated. In MM the initial sharing rule when the match is formed is also arbitrary. There is nothing in their approach that restricts how this initial sharing rule becomes an outcome.

Our setting has extreme value shocks. We think of this as the modern structural way to incorporate a rationale for agents that are observationally equal yet they do different things. Any use of the MM approach that wants to have such property would also need to have some type of unobservable shocks. With respect to other types of shocks, we see no difference between the two approaches where, any stochastic process on shocks that affect either the arrangement if agents remain married or the outside opportunities can be implemented with ease.

On the computational side there are two main differences that weigh in favor of our approach where the equilibrium has a closed form solution while the outcome in MM may not. Any time that the allocation is not incentive compatible a nonlinear equation has to be solved which may be quite costly when the problem has to be solved thousands of time as is in applications.[1]

But the real computational gain comes when it is noted that their approach requires the carryover

[1] We see this advantage as closely related to that one implemented in standard dynamic problems by using the endogenous grid method of Carroll (2006) that substituted via a clever switch from iterating on the decision rule to iterating on its inverse the need to solve a nonlinear equation to a direct use of an explicit closed form.

of a continuous state variable while ours does not. This is really the great reward of our approach. Kato (2022) has used effectively our approach to estimate a model where household wealth is a state variable. The need to carry an additional state in MM would have made the estimation of the many parameters in Kato's model just too costly.

4 Conclusion

The protocol of interaction that we propose is well suited for environments where agents form durable pairs that sometimes break down, where one type of arrangement is common, but not unique, where successful members do often better than their associates, and especially for economies with large numbers of agents of various types. When researchers need to consider other variables such as assets and solve the model many times, perhaps because of the need to estimate many parameters the rewards of our approach really matter.

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