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Commitment in the Canonical Sovereign Default Model

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Abstract

We study the role of lack commitment in shaping the allocations of the canonical incomplete-markets sovereign default model of Eaton and Gersovitz (1981). We show how the equilibrium with commitment to the circumstances under which default can be undertaken involves a very different set of functional equations than in the equilibrium without commitment. It turns out that, in practice, under commitment default does not exist in all but very extreme quantitative environments. We document how the standard specification of Arellano (2008) displays no default if there is commitment, even in the absence of both utility cost and exclusion from borrowing. While less standard specifications can produce some default under commitment, we provide examples that demonstrate how rare default is. We interpret default as a recourse of last resort.

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1 Introduction

One of the canonical models in economics is the Eaton and Gersovitz (1981) model of sovereign default.[1] This model has two frictions, one of them being that the set of financial instruments is coarse: the only security, debt, is either paid in full or not at all, an event that may be associated to a cost.[2] The other friction is that the circumstances under which default happens are not part of the contract, indeed, they are determined ex-post unilaterally by the borrower.

The main focus of this note is on the second friction, the lack of commitment and its role in the sovereign default environment. To this end, we write a recursive problem with commitment to when (under which realizations of the shock) default can occur and compare its properties with those of the standard no-commitment model.[3] We show that the equilibrium is characterized by quite a different set of functional equations. In particular, the indifference threshold between default and non default disappears and the sovereign commits to repay even in situations where ex-post it would prefer to default. It does so to achieve better terms on loans. We also note that long term debt under commitment has various interpretations and one of them implies the same allocation as short term debt with commitment, while in the absence of commitment they are not the same (Arellano and Ramanarayanan (2012)).

In order to gauge the practical significance of these formal differences, we solve for the equilibrium under commitment within the standard quantitative environment in the literature, that of Arellano (2008), and show that there is no default, even when punishment is eliminated (the sovereign will hold zero assets in the period right after default but is free to return to borrowing and savings afterwards). More generally, however, default is possible but rare. To illustrate this point, we consider an i.i.d. process for the endowment with low-probability bad states. This feature enables the occurrence of default in the model with commitment and we find that it happens at levels of debt that exceed the conventional *natural debt limit*. We conclude that default is only chosen as an instrument of last resort and it allows the sovereign to extend its borrowing beyond the natural debt limit but otherwise looks like a standard borrower that has to pay back in all states of nature. From the above, we can conclude that the lack of

[1] A version of it is widely used in the household bankruptcy literature (Chatterjee et al. (2007), Livshits et al. (2007) and many others).

[2] Mateos-Planas and Seccia (2014) have noted that with full state contingent securities the lack of commitment prevents the issuance of any debt that will not be repaid: lenders know in which state of nature the sovereign would default and consequently will not issue securities that promise to deliver on those states of nature. The economy collapses to the Kehoe and Levine (1993) model. Or rather the decentralised arrangement of state-by-state endogenous borrowing limits in Alvarez and Jermann (1998). Mateos-Planas and Seccia (2014) make precisely the point that default being a zero/one extensive margin decision, rather than a partial intensive margin one, is also needed for this result. With different assumptions on what triggers default such as unobserved preference shocks, obviously the answer is different (Aguiar and Amador (2021)).

[3] When we refer to the standard model we consider only their Markov equilibria, and then, only Markov equilibria that are the limits of finite economies and not those that involve non-continuous decision rules where trigger strategies can be embodied in Markov representations and are not the limit of finite economies (see Krusell et al. (2002)).

commitment is the crucial ingredient for default and not the coarseness of the debt instruments.

We contribute to the literature on sovereign default with commitment in two ways. Grossman and Huyck (1988) develop the concept of "excusable" default in which debt repudiation is not sanctioned by either utility cost or exclusion from borrowing. In this set-up, we show that default does not exist in all but very extreme quantitative environments. Relatedly, Adam and Grill (2017) analyze the decision to partially default from the perspective of a Ramsey plan in a production economy where default is possible but entails output costs that depend on the fraction defaulted. Like in our work, they find that defaults arise only after disaster-like shocks. Unlike in our paper, in Adam and Grill (2017) the shocks are required to be persistent.

We pose the model and the functional equations that characterize the solution for the two cases of lack of commitment and commitment, and compare their properties, in Section 2. In Section 3 we explore quantitatvely the prevalence of default, first in the standard specification of Arellano (2008) (Section 3.1), and then in examples where default starts appearing (Section 3.2).

2 The Model and its Recursive Representations.

Consider an agent, the sovereign, that has preferences over streams of consumption, assessed according to a standard utility function $u(c)$, $c \geq 0$, and discount rate β. It maximizes expected utility. The sovereign has endowment $y \in [\underline{y}, \overline{y}]$ with cdf F and density f. Let $\overline{F}(y) = 1 - F(y)$. The sovereign has access to a market of unsecured debt where it can borrow. A promise to pay b' the following period yields quantity of goods q. Lenders are risk neutral and demand and obtain an expected (inverse) return q^*. In accordance with our Markovian restrictions we only look at payoff relevant outcomes so we do not index events by the history. For this reason, the amount obtained per unit of the good for that quantity of goods (the inverse of the interest rate) is only a function of the total amount borrowed: $q^n(b')$ in the case of no commitment. the promise to pay b' is either honored or not, and, if not, the sovereign cannot save or borrow ever again yielding a value of defaulting of $v^a(y) = u(y) + \frac{\beta}{1-\beta} \int u(y')\, dF(y')$.

We have made two simplifying assumptions on the standard model. That endowment shocks are i.i.d. rather than Markovian and that default entails a perpetual reversion to autarky but carries no utility or income loss. Nothing theoretically relevant is lost with these assumptions, and the quantitative analysis will be on the more general model.

2.1 Lack of Commitment

The state is the pair $\{y, b\}$ and to characterize the problem as a Markov equilibrium it suffices to note that the sovereign takes as given the default location $d^n(b')$, and saving $g(y', b')$ policies followed by the sovereign's successor and chooses whether to default, and if not how much to consume and save/borrow. Lenders also take as given the sovereign's successor policies which instead implies the equilibrium condition that $q(b') = q^* \overline{F}[d^n(b')]$. These considerations imply that the sovereign's problem is

$$v^n(y, b) = \max \left\{ v^a(y),\ \max_{b'} \left\{ u\left[y - b + q^* \overline{F}[d^n(b')]\ b' \right] + \beta \left[\int_{\underline{y}}^{d^n(b')} v^a(y')\ dF(y') + \int_{d^n(b')}^{\overline{y}} v^n[y', b']\ dF(y') \right] \right\} \right\}. \quad (1)$$

The solution of this problem (after imposing the equilibrium condition that the sovereign's choices end up being equal to the decision of its follower) is a pair of functions for default location and for savings that solves (1) and it chooses the lowest value of defaulting that leaves the agent indifferent between defaulting or not:

$$d^n(b) = \min \left\{ \{ y : v^n(y, b) > v^a(y) \} \cup \{ \overline{y} \} \right\}. \quad (2)$$

We now use the results of Clausen and Strub (2020) who show that there exists a level of debt $b^* > 0$, where $v^n(\underline{y}, b^*) = v^a(\underline{y})$ so that for $b \leq b^*$, the sovereign always honours its obligations and prove that the solution has (at most) three areas, one where the solution does not involve default and the standard Euler equation is satisfied, one where the solution does not involve default and the Euler equation is not satisfied, and one where the solution satisfies a Generalized Euler Equation (GEE),[4] this is, an optimality condition in terms of decision functions and their derivatives.[5] We can write the conditions for those three areas using for convenience an explicit consumption function that we obtain by simply applying the budget constraint:

$$u_c\left[c^n(y, b)\right] q^* = \beta \int u_c[c^n(y', g^n(y, b))]\ dF(y'), \qquad \text{if } d^n(g(y, b)) = \underline{y},\ g^n(y, b) < b^*, \quad (3)$$

$$b' = b^*, \qquad \text{if } d^n(g(y, b)) = \underline{y},\ g^n(y, b) = b^*, \quad (4)$$

$$u_c\left[c^n(y, b)\right]\ q^* \left[\overline{F}[d^n(g(y, b))] - g^n(y, b)\ f[d^n(g(y, b))]\ \frac{\partial d^n(b')}{\partial b'} \right] = \beta \int_{d^n(g(y,b))} u_c[c^n(y', g^n(y, b))]\ dF(y'), \qquad \text{if } d^n(g(y, b)) > \underline{y},\ g^n(y, b) > b^*. \quad (5)$$

[4]An equilibrium without default is obviously possible and in this case the sovereign always stays in this area.

[5]See Clausen and Strub (2020) for a discussion of the conditions with short term debt and Mateos-Planas et al. (2022) for the derivation of the GEE with long term debt.

Applying the implicit function theorem we also obtain an expression for the derivative of the defaulting function (outside the critical point b^*):

$$\frac{\partial d^n(b')}{\partial b'} = \frac{u_c\left[c^n(d(b'), g^n(d(b'), b'))\right]}{u_c\left[c^n(d(b'), g^n(d(b'), b'))\right] - u_c\left[d^n(b')\right]}. \tag{6}$$

2.2 Commitment

Under commitment, a sovereign that has promised to pay b if $y \geq y^c$, will default when $y < y^c$ in which case it obtains the value of autarky, $v^a(y)$. If $y \geq y^c$ it pays back and chooses how much to borrow or lend and how much to consume, a problem that can be written as a function of the state, $\{y, b\}$. Note that the price of the debt it faces is independent of its amount given commitment and it is just the probability of being paid back times q^*, $q^c\left(y^{c'}\right) = q^*\ \overline{F}\left(y^{c'}\right)$. If $y \geq y^c$ then the value of the sovereign satisfies

$$v^c(y, b) = \max_{b', y^{c'}}\ u\left\{y - b + q^* \overline{F}\left(y^{c'}\right)\ b'\right\} + \beta\left\{\int_{\underline{y}}^{y^{c'}} v^a(y')\ dF\left(y'\right) + \int_{y^{c'}}^{\overline{y}} v^c(y', b')\ dF(y')\right\}. \tag{7}$$

This is a simple dynamic programming problem that admits differentiability if both the density f and the utility function are differentiable and using functions $y^{c'} = d^c(y, b)$ and $b' = g^c(y, b)$ to denote the solution we write the first order conditions as

$$\overline{F}\left[d^c(y, b)\right]\ q^*\ u_c[c(y, b)] = \beta \int_{d^c(y,b)} u_c[c(y', b')]\ dF(y'), \tag{8}$$

$$b'\ q^*\ u_c[c(y, b)] = \beta\left[v^a\left(d^c(y, b)\right) - v^c\left(d^c(y, b), b'\right)\right], \qquad \text{if } d^c(y, b) > \underline{y}, \tag{9}$$

where Equation (8) uses the envelope condition. Note that strict concavity implies that the solution is unique.

Functional Equations (7) to (9) completely characterize the unique solution to the problem with commitment. They are very different than the functional Equations (1) and (2).

Note that Equation (9) implies that

$$v^a(y^{c'}) = v^c\left(y^{c'}, b'\right) + \frac{b'\ q^*}{\beta}\ u_c\left[c(y, b)\right], \tag{10}$$

which in particular implies ex-post regret of $v^c(y, b)$ at $y^c > \underline{y}$. This is, at the point of default, the value function is not continuous, it jumps downwards, showing that the sovereign is willing to promise to pay even if in those circumstances it would rather not the period later. This is to achieve a lower loan price since it understands the effects of the threshold on the price.

Note also that the first order condition for savings, (8) is not a GEE, nor it is satisfied with inequality as it was in the case with commitment. Here, whenever the sovereign chooses to default on the following period it affects its current marginal utility of consumption. We are seeing here that there are reasons not to choose to default which will be confirmed quantitatively below.

2.3 Comparison of Long and Short Term Debt

Long term debt under commitment can be interpreted in various ways that include a commitment to default only under the same circumstances every period, a commitment to default on prespecified circumstances in each of the future periods and many more of this style. All of these interpretations allow through buybacks the extension of the set of securities that are available and therefore are not strictly a pure description of long term debt as it is done in economies without commitment. An interpretation of the commitment as lasting only one period brings back lack of commitment through issues like dilution and subsequent choices of pervasive default. However, a commitment to pay the coupon and to buyback the debt the following period at a prespecified price except in the circumstances that allow default brings us back to an environment identical to that of short debt.

3 Quantitative Relevance of Default

We turn to the quantitative question of what the allocations under commitment are and how often default is chosen. To this end we first use the standard specification in the literature, the Arellano (2008) economy before moving on to consider some alternative specifications.

3.1 The Arellano 2008 Specification

We extend the models presented in Section 2 with elements from Arellano (2008), namely a persistent Markovian endowment with cumulative distribution $F(y'|y)$ and density $f(y'|y)$, a direct cost to defaulting such that in autarky $c = h(y) \leq y$, and a probability of forgiveness out of autarky θ. In particular, the log of the endowment follows a zero-mean AR(1) process with normally distributed innovations.

We specify all functions, parameter values and the asset grid as in that paper,[6] including a positive

[6] The endowment process can be written $\log y' = \rho_y \log y + \epsilon'$, $\epsilon' \sim N(0, \sigma_\epsilon)$, for an unconditional mean endowment of 1. We approximate it via Tauchen (1986) discretization method with a range for y of μ_y standard deviations, and number of points n_y. The form for h is $h(y) = \min\{1-\pi, y\}$. Parameter values are $\beta = 0.95285$, $\sigma = 2$, $r = .017$, $\rho_y = 0.945$, $\sigma_\epsilon = 0.025$, $\mu_y = 3.0$, $n_y = 21$, $\theta = 0.282$ and $\pi = 0.031$. The implied natural limit for debt is $b^n \equiv (1+r)\underline{y}/r = -47.419$. The only element that is left to determine is the grid for debt b. Throughout we will let the lower bound be -0.50, and the number of negative elements be 25 and evenly spaced. The number of non-negative points is 600. Given the upper

upper bound for debt equivalent to 1.5 times the mean endowment. An important threshold of debt is the natural debt limit, the largest debt that is consistent with positive consumption and the ability to repay in the absence of default, which equals $(1+r)\underline{y}/r > 0$, which is equivalent in size to about 50 times the mean endowment.

Arellano (2008) solved the no-commitment version of this economy by discretization of the state space and the choice sets. The grid size that she uses is 0.25% of average income. We start by replicating her results using the same techniques and precision. We then consider the economy with commitment. It turns out that default is never chosen within her specification. Still, the lower asset bound posed (the credit limit) is binding. We then extend the lower bound up to the natural credit limit while maintaining the grid size. Again, default is never used and the sovereign may reach the level of debt just lower than the natural debt limit (it does so with very low probability). We then proceed to pose a much finer grid to see if there is default at states closer to the natural debt limit. Still, the economy with a grid size of 0.01% of average income (25 times finer than that in Arellano (2008)) yields no default.

Next, we eliminate all punishment from defaulting beyond the requirement that savings is not possible in the period of defaulting. Again, the sovereign chooses never to default and will not progress beyond the point with the largest debt strictly smaller than the natural borrowing limit.[7]

Importantly, this means that in this economy under commitment the equilibrium allocation is the same that what would result if the option to default were not present and the sovereign is just subject to always pay back its debt, this is the sovereign behaves like in a standard incomplete market economy.

3.2 Economies that can Generate Default under Commitment

It is always possible to construct example economies where there is default. It suffices to pose a two period economy with two states in the second period and choose the endowments in both periods so perfect insurance can be implemented via a securities portfolio that perfectly replicates default.[8] This said, we are interested in economies of the type that are used for the sovereign default problem, and for them default under commitment requires some combination of a bad enough state and a low enough probability of the bad enough state. We now proceed to construct two examples, one where we reduce

bound, the top n_U elements are equally spaced at chosen distance. The remaining positive $600 - n_U$ elements are then evenly spaced.

[7] Our characterisation rules out, by construction, default on the upper range of income realizations. It is conceivable, however, that with persistent shocks, default could exist for high realisations of income (persistence may yield such good outlook that access to financial markets is no longer worth a lot and losing access may be worth it). In these examples this is never the case: one-shot deviations to default at any other threshold never dominate the choices made in with a single lower threshold.

[8] Examples with more periods require the additional condition that in the second period, the zero asset choice is repeated one more period.

dramatically the probability of the worst state and the other where we have a large disaster with a relatively low probability. For both examples we give two parameterizations one at each side of the existence of default in equilibrium. We interpret these examples as instances where the default option stretches the natural borrowing limit beyond that that arises without default.

To this end, we pose an economy which, like Arellano's, has CRRA preferences with a risk aversion of 2, discount rate $\beta = 0.95285$ and a risk free interest rate of 1.7%. We pose a grid for debt of size 0.00125 of average endowment, and again no direct costs of default other than current autarky. We also pose shocks that are i.i.d. and have finite support. There are 21 possible states y_j symmetrically posed around 1 separated by $x = 0.02073504$ from each other so that $y_1 = 1 - 10\ x = 0.7926$ and $y_{21} = 1 + 10\ x = 1.207$. The probability of each one of those points is

$$\gamma_j = \lambda(s) \times \begin{cases} e^{s(y_j - 1)} & j \leq 10, \\ e^{-s(y_j - 1)} & \text{otherwise} \end{cases}$$

for some $s \geq 0$ where $\lambda(s)$ is simply a scaling parameter that ensures that γ is indeed a probability distribution and that coincides with the probability of the mean value of the endowment. Note that the case $s = 0$ is the uniform distribution and the larger s is, the more unlikely the extreme states are. Note also that the mean of the process is always one. By scaling s we look for when default starts occurring.

We find that for $s = 30$ which implies a probability of the worst state of 0.06% there is no default, but for a slightly larger value $s = 32$ with probability associated probability distribution of 0.042% there is default once the sovereign reaches a debt position of 47.48. This level of debt is 0.13% larger than the natural debt limit. This is, the sovereign does not use the default option until it has to, once the level of debt is so large that there it is not possible to ensure that the debt could be repaid with probability one. This is a necessary but not sufficient circumstance since the sovereign may not default with debt above the natural limit under the worst shock (recall that there are more grid points between the natural debt limit and the default threshold). As stated above, we interpret the option to default as a way of increasing the natural debt limit beyond what no defaulting yields. In this example, this can be achieved because the probability of the worse state is low enough that the burden of the debt does not increase much when the debt price falls by accounting for the default state.

Figure 1 shows the debt policies in the worst income state for the two economies discussed. The solid blue line shows that of the economy where there is no default and we see how there is a fixed point (the sovereign issues the same debt that it had before in the same state) just below the natural borrowing limit (in the dashed green line). The solid green line is the bond policy in the economy with default for the same worst state that has now a lower probability. We see how it extends beyond the natural borrowing limit where it chooses to default at a higher debt level (in red). The level of debt chosen when

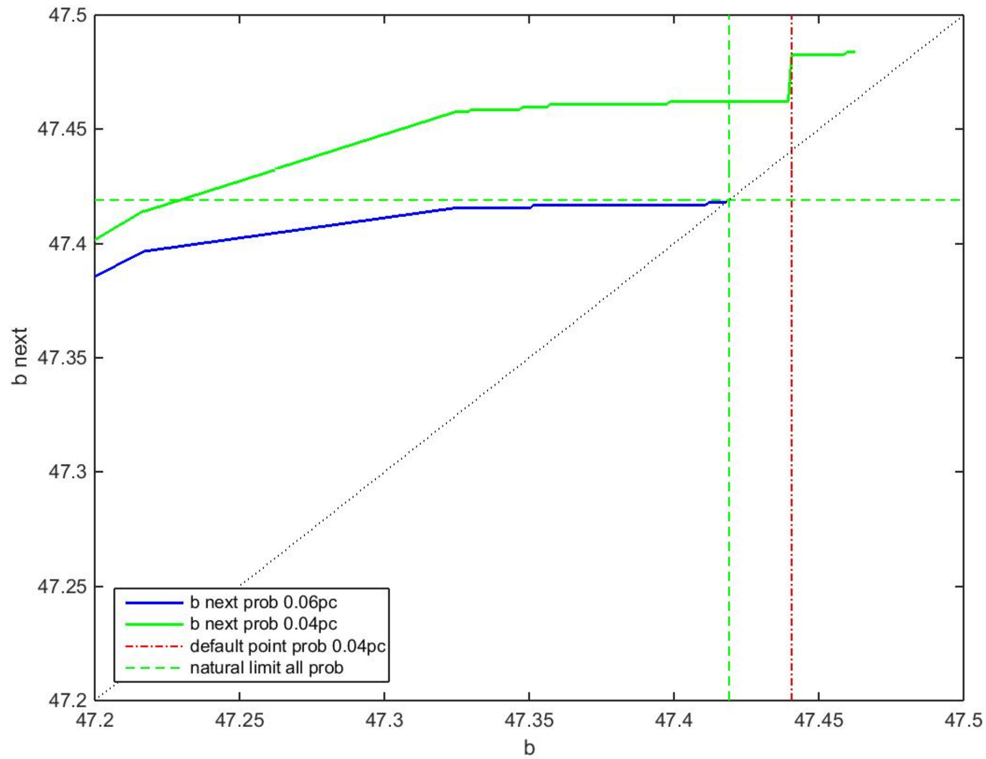

Figure 1: Decision Rules for Debt for Two Economies that Differ in the Probability of the Worst State

default is possible will only be paid if the realization of the state of nature is higher than the threshold.

Disasters Default can also be generated not just by reducing the probability of the most adverse state but also by making this state more adverse. In this case a very bad low state implies quite a tight natural borrowing limit and the option to default in such low state allows to increase the effective borrowing limit. To gauge this margin we fix the probability of the worst state by letting $s = 20$ which implies a probability of the worst state of 0.33% and we vary the value of y_1 while adjusting that of y_{21} to maintain a mean of 1. With a value of $y_1 = 0.6817$ there is still no default but with a value of 0.6737 default appears. Again, default does not show up until the level of debt is larger than the natural borrowing limit and is extremely rare (it takes simulations of millions of periods to obtain an occurrence).

Figure 2 shows the debt policies for the worst income state in the two economies discussed. The blue line shows that of the economy where there is no default and again we see how there is a fixed point (the sovereign issues the same debt that it had before in the same state) just below its natural borrowing limit. The green one is the bond policy in the economy with default which has a worse state

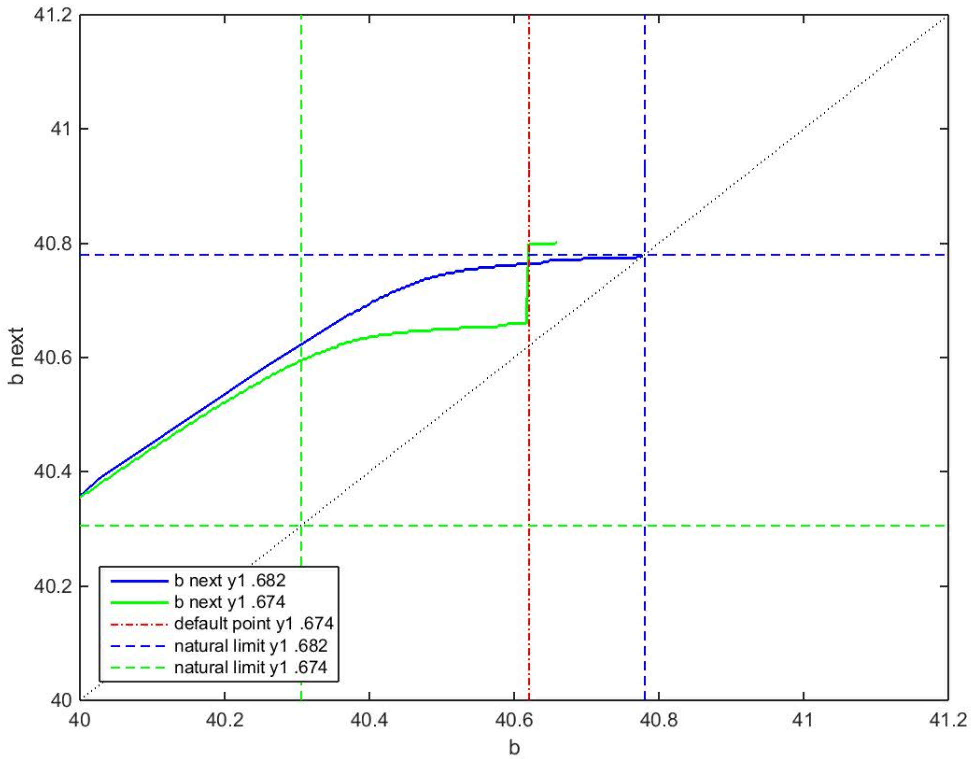

Figure 2: Decision Rules for Debt for Two Economies that Differ in the Size of the Disaster State

and therefore a tighter natural borrowing limit (discontinuous green line). Default is used to allow the sovereign to extend the debt that it can hold.

4 Conclusion

In this paper we have explored the role of lack of commitment to the terms of default in the sovereign default problem. We have characterized theoretically the problem under commitment and we have shown how different the properties of the solution are than those of the environment without commitment. The functional equations are completely different.

We have also explored the quantitative properties of the solution and we find that the lack of commitment is absolutely central to the existence of default. If sovereigns had to comply with the fact that default is only feasible when it is specified in the terms of the loans, they would almost always never choose to default. Operationally, this means that default never happens in the economy parameterized à la Arellano (2008), not even if all punishment beyond the inability to save in the period of default is abandoned. The economy in this case could reach a level of debt arbitrarily close to the natural borrowing limit albeit with very low probability. In these cases, the economy collapses to that where the default option is not present.

Further, we have built examples with default. They require some combination of very rare and very disastrous worse states. Even in these cases, the sovereign will go beyond the natural borrowing limit before using the option to default. In all cases the option to default is extremely rare.

We conclude that lack of commitment, and not the coarseness of the debt instruments is the main rationale for the existence of default. Having a default option is not relevant in itself, but only when there is also lack of commitment to the circumstances under which it will be chosen.

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