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Information Favoritism and Scoring Bias in Contests

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Selective Disclosure and Scoring Bias in Contests*

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Abstract

Two potentially asymmetric players compete for a prize of common value, which is initially unknown, by exerting effort. A designer has two instruments for contest design. First, she decides whether and how to disclose an informative signal of the prize value to players. Second, she sets the scoring rule for the contest, which can be biased in favor of one player. We show that the optimum depends on the designer's objective. An ex post symmetric contest—in which information is symmetrically distributed and the scoring rule offsets the initial asymmetry between players—always maximizes the expected total effort. However, the optimal contest may create dual asymmetry—i.e., the designer discloses the signal privately to one player, while favoring the other in terms of the scoring rule—when the designer is concerned about the expected winner's effort or the expected maximum effort. This could arise even if the players are ex ante symmetric. Our results are qualitatively robust to an endogenous information structure.

Keywords: All-pay Auction; Contest Design; Information Favoritism; Scoring Bias.

JEL Classification Codes: C72, D44, D82.

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1 Introduction

A wide array of competitive activities exemplify a contest, ranging from college admissions, sporting events, competitive procurement, and R&D contests that solicit novel solutions (Taylor, 1995; Fullerton and McAfee, 1999; Che and Gale, 2003) to internal labor markets inside firms (Lazear and Rosen, 1981; Green and Stokey, 1983; Nalebuff and Stiglitz, 1983; Rosen, 1986). Extensive scholarly effort has been devoted to identifying feasible and efficient means of administering such competitions (see, e.g., Fang, Noe, and Strack, 2020; Fu and Wu, 2020; Lemus and Marshall, 2021; Hofstetter, Dahl, Aryobsei, and Herrmann, 2021).

We examine the design of a contest that jointly employs two instruments: (i) an *information disclosure* scheme and (ii) a *scoring bias*. Two observations motivate us. First, discriminatory measures, which treat certain contenders preferentially, are widespread in competitive activities. For instance, preferred contenders competing for promotion to a higher rung on the corporate ladder may be intentionally nurtured by the incumbent CEO and board members. Alternatively, many governments grant small and medium-sized enterprises preferences in public procurement auctions. It is essential that we address questions regarding whether to treat certain contestants preferentially, and if so, whom and to what extent.

Second, participants can encounter uncertainty regarding the contest's nature and the surrounding environment, such as the value of the prize. In a competition for a promotion, employees may not have full information about the nuances of the new role, such as the scope of responsibilities, available resources, and implications for their career trajectory. In another context, contractors vying for a government procurement contract may lack details about the true costs of fulfilling the contract. Thus, contestants' behavior can be influenced by the information available to them regarding the value of the "prize" they are competing for. This highlights the potential for strategic information disclosure: what information to disclose, and to whom.

We address these questions by allowing a contest designer to (i) selectively disclose her information to only one contestant and (ii) bias the contest in favor of one contestant. We show that the two instruments play complementary roles, and their proper combination enhances the contest's performance. Prior studies have typically focused on using only one of the instruments. Our study thus joins the growing trend in joint contest design that employs multiple instruments (e.g., Halac, Kartik, and Liu, 2017; Ely, Georgiadis, Khorasani, and Rayo, 2022).

Snapshot of the Model Two players simultaneously exert effort to vie for a prize of a common value. The prize value is initially unknown and can be either high or low. Player

1 bears a weakly higher marginal effort cost—i.e., $c_1 \geq c_2$—and is thus the underdog. Each player's effort is converted into a score, and the higher scorer wins.

The designer conducts an investigation and acquires an informative binary signal about the true prize value. Prior to the competition, the designer commits to the contest rule, which consists of two elements. First, a disclosure scheme specifies how the signal is disclosed. The disclosure scheme is asymmetric when the signal is conveyed to only one player, which awards the recipient an information advantage. For instance, the organizer of a business pitching competition may brief preferred entrepreneurs more elaborately on the funding opportunities available to winning projects. Second, a multiplier is imposed on each player's effort to generate his score. We normalize the multiplier for the underdog—i.e., player 1—to one and that for player 2 to $\delta > 0$, which is called a scoring bias. The bias can be interpreted as a nominal judging rule, as well as measures that elevate or discount players' (perceived) output.

We mainly consider two objectives for contest design. The first is the usual maximization of expected *total* effort (see, e.g., Moldovanu and Sela, 2001; Moldovanu, Sela, and Shi, 2007). For instance, the government may use R&D challenges to fuel the society's total investment in a certain technological area; e.g., clean energy or AI. The second is the maximization of the expected *winner*'s effort (see, e.g., Moldovanu and Sela, 2006; Fu and Wu, 2022). For instance, in a contest for a corporate leadership role, the capability of the eventual winner is what will drive the value of the company.

Summary of Results The contest game can be viewed as an all-pay auction with interdependent valuations and discrete signal spaces. Siegel (2014) provided the technique for the case with a neutral scoring rule $\delta = 1$. We allow for a scoring bias and adapt Siegel's technique to characterize the equilibrium, which paves the way for contest design.

The optimal contest design crucially depends on whether the designer aims to maximize the expected total effort or the expected winner's effort. Intuitively, the difference is driven partly by the fact that the expected total effort is the sum of the *means*, while the expected winner's effort is the *modified first-order statistic* of the (random) effort choices by the two players.[1]

Results for the maximization of expected total effort affirm the conventional wisdom of leveling the playing field. The optimum is an *ex post symmetric contest*: (i) Players are either symmetrically informed or symmetrically uninformed, and (ii) the contest sets the scoring

[1]It is noteworthy that in our context, the expected winner's effort is not the simple highest effort, except for the case of $\delta = 1$. Under a biased scoring rule ($\delta \neq 1$), the winner may not be the one who exerts the highest effort. We thus call the expected winner's effort a modified first-order statistic to reflect the nuance. In Section 4.2, we also discuss the case in which the designer's objective is to maximize the expected *maximum* effort, i.e., the first-order statistic.

bias to $\delta = c_2/c_1$. Neither player possesses superior information, and the bias $\delta = c_2/c_1$—called the *fair bias* of the contest—precisely offsets the initial advantage of player 2 in terms of bidding efficiency.

However, the optimum can notably depart from the conventional wisdom when the designer maximizes the expected winner's effort. The designer may prefer a *tilting-and-releveling* contest, which distorts the contest in two dimensions and creates *ex post dual asymmetry* between players. Specifically, the designer feeds the signal exclusively to one player, while releveling the playing field by letting the scoring bias deviate from the fair level to favor the other. A tilting-and-releveling contest could enable an upward shift of the winner's effort distribution to elevate the expected winner's effort.

Section 3.2 delves into the logic in depth. Three observations are notable. First, the tilting-and-releveling contest can be optimal even if players are ex ante symmetric—i.e., $c_1 = c_2$ (Theorem 1). Second, with asymmetric players, an optimal tilting-and-releveling contest awards the underdog the information advantage, while compensating the stronger player with a more favorable scoring bias (Theorem 2). Third, the two instruments—i.e., the disclosure scheme and scoring bias—are *complementary*. Specifically, asymmetry never emerges in the optimum if the designer is restricted to deploying only one instrument (Remark 1), and thus the optimal contest requires either ex post symmetry or dual asymmetry.

We extend our model to two alternative settings. First, we endogenize the information structure of the designer's investigation, which corresponds to the concept of the Bayesian persuasion approach pioneered by Kamenica and Gentzkow (2011). Second, we consider contest design when the designer is concerned about the expected maximum effort. Our main results remain qualitatively robust, and the discussion sheds further light on the fundamental trade-off entailed in our context.

Related Literature Our contest model is a variant of the family of all-pay auctions with interdependent valuations, which includes Krishna and Morgan (1997); Lizzeri and Persico (2000); Siegel (2014); Rentschler and Turocy (2016); Lu and Parreiras (2017); and Chi, Murto, and Välimäki (2019). Our study is primarily linked to two strands of the literature on contest design: (i) optimal biases as (identity-dependent) differential treatment of players and (ii) information disclosure. To the best of our knowledge, we are the first to allow the designer to choose their optimal combination.

The literature on optimal biases has conventionally espoused the merits of a level playing field for incentive provision—e.g., Epstein, Mealem, and Nitzan (2011); Franke, Kanzow, Leininger, and Schwartz (2013, 2014); Franke, Leininger, and Wasser (2018). A handful of recent studies—e.g., Drugov and Ryvkin (2017); Fu and Wu (2020); Barbieri and Serena (2022); Wasser and Zhang (2023); Echenique and Li (2022)—identify the contexts in which optimal biases further upset the balance of the playing field. This strand of studies typically abstract away the issue of information disclosure.

The literature has increasingly recognized information disclosure as a valuable addition to the toolkit for contest design. For example, Yildirim (2005); Aoyagi (2010); Ederer (2010); Goltsman and Mukherjee (2011); Halac, Kartik, and Liu (2017); Lemus and Marshall (2021); and Ely, Georgiadis, Khorasani, and Rayo (2022) examine information feedback in dynamic contests. Halac et al. (2017) and Ely et al. (2022) also consider a prize allocation rule. They focus on homogeneous players and symmetric information disclosure. Further, the prize is allocated based on players' outcome and cannot depend on a player's identity. In contrast, we consider a static setting and focus on the interaction between the scoring rule and disclosure scheme, and allow for potentially asymmetric players and selective disclosure; in addition, our scoring rule permits identity-dependent preferential treatment.

Our paper is closely related to studies on disclosing information on contestants' types, including Wärneryd (2012); Lu, Ma, and Wang (2018), Serena (2022); Zhang and Zhou (2016); Chen and Chen (2022); Melo-Ponce (2021); and Antsygina and Teteryatnikova (2023). These studies focus exclusively on disclosure schemes and portray strategic information disclosure as a device that balances competition, which aligns with the conventional wisdom of leveling the playing field. In contrast, we show that a designer can create information asymmetry when she controls both the disclosure scheme and scoring rule.

In the context of private-value auctions, Bergemann and Pesendorfer (2007) consider a joint design problem with which the seller is able to control the accuracy by which bidders learn their valuation and to whom to sell at what price. They demonstrate the optimality of creating informational asymmetry together with an asymmetric follow-up design.

The rest of the paper proceeds as follows. Section 2 sets up the model, and Section 3 characterizes the optimal contest. Section 4 presents further discussions and extensions, and Section 5 concludes. Equilibrium analysis is provided in Appendix A and proofs of our main results are collected in Appendix B.

2 The Model

Two risk-neutral players, indexed by $i \in \mathcal{N} \equiv \{1,2\}$, compete for a prize of a common value $v \in \{v_H, v_L\}$, with $v_H > v_L > 0$. The high value v_H is realized with a probability $\Pr(v = v_H) =: \mu \in (0,1)$, with the low value v_L to be realized with the complementary probability. Players are initially uninformed about v, while its distribution is common knowledge. They simultaneously exert effort $x_i \geq 0$ to win the prize. One's effort incurs a constant marginal cost $c_i > 0$. Without loss of generality, player 2 is assumed to be the stronger contender; i.e., $c_1 \geq c_2$.

Winner-selection Mechanism and Scoring Bias The contest designer imposes a scoring bias $\delta_i > 0$ on each player i's effort entry x_i, which generates his score $\delta_i x_i$. We normalize δ_1 to 1 and set $\delta_2 = \delta > 0$. The scoring rule is biased when δ deviates from 1, which favors player 2 if $\delta > 1$ and player 1 otherwise. We call $\delta = c_2/c_1 (\leq 1)$ the *fair bias*, which perfectly offsets player 1's initial disadvantage.

A player wins if his score exceeds that of the opponent. The winner is picked randomly in the event of a tie in scores. Fixing a set of effort entries $\boldsymbol{x} := (x_1, x_2) \in \mathbb{R}_+^2$, player 1's wining probability is

$$p_1(x_1, x_2) = \begin{cases} 1, & \text{if } x_1 > \delta x_2, \\ \frac{1}{2}, & \text{if } x_1 = \delta x_2, \\ 0, & \text{if } x_1 < \delta x_2, \end{cases}$$

and player 2 wins with a probability $p_2(x_1, x_2) = 1 - p_1(x_1, x_2)$.

Disclosure Schemes The designer conducts an investigation and obtains a verifiable noisy signal $s \in \{H, L\}$ regarding the prize value v. The signal is drawn as follows:

$$\Pr\left(s = H \,\middle|\, v = v_H\right) = \Pr\left(s = L \,\middle|\, v = v_L\right) = q, \tag{1}$$

where $q \in \left(\frac{1}{2}, 1\right]$ indicates the quality or precision of the signal.[2] The signal perfectly reveals the prize value with $q = 1$ and is completely uninformative with $q = 1/2$.

The designer precommits to her disclosure scheme—i.e., how the result of her investigation is to be disclosed. The disclosure scheme can formally be described by $\gamma \in \{CC, CD, DC, DD\}$, where C and D indicate "concealment" and "disclosure," respectively. With a symmetric disclosure scheme $\gamma = CC(DD)$, the realized signal s is conveyed to neither (both) of the players. With $\gamma = CD$, the designer conceals the signal from player 1 while disclosing it to player 2; $\gamma = DC$ is similarly defined.

Contest Design Prior to the contest, the designer chooses (δ, γ) to maximize either (i) the expected total effort of the contest, denoted by $TE(\gamma, \delta; c_1, c_2)$, or (ii) the expected winner's effort, denoted by $WE(\gamma, \delta; c_1, c_2)$.[3] The former design objective has conventionally been adopted in the vast majority of the contest literature, which resembles revenue maximization

[2] We will endogenize the information structure using a Bayesian persuasion approach (e.g., Kamenica and Gentzkow, 2011) in Section 4.1.

[3] By maximizing the expected winner's effort, we assume the designer is committed to adopting the winning product under the context of R&D contests. Alternatively, the designer may lack commitment power, in which case she will adopt the best product regardless of whether the contestant submitting the best product wins the contest prize. For these contests, the designer's objective is to maximize expected maximum effort. We will consider this alternative design objective in Section 4.2.

in the auction literature. The latter, however, is also relevant in a broad array of competitive activities and has attracted increasing attention in recent studies.[4]

Preliminaries: Equilibrium and Notation It is well known that an all-pay auction with complete information or a discrete signal structure, in general, does not possess pure-strategy equilibria (see, e.g., Hillman and Riley, 1989; Baye, Kovenock, and De Vries, 1996; Siegel, 2009, 2010, 2014). Siegel (2014) provides a technique for constructing the unique mixed-strategy equilibrium of an all-pay auction under a neutral scoring rule; i.e., $\delta = 1$. We adapt his result to our context—which allows for an arbitrary scoring bias $\delta > 0$—and fully characterize the equilibrium in the interim bidding stage under each possible (γ, δ). The equilibrium result further enables the solution to $TE(\gamma, \delta; c_1, c_2)$ and $WE(\gamma, \delta; c_1, c_2)$. The analysis is tedious and complicated, so we relegate all details to Appendix A. We discuss the properties of the equilibrium when interpreting the results of optimal contests.

Several types of notation are presented to pave the way for subsequent discussion. Define $\bar{v} := \mu v_H + (1 - \mu) v_L$, which denotes the ex ante expected prize value. Upon receiving a signal $s = H$, a player's expected prize value is updated to

$$\hat{v}_H(q) := \frac{\mu q v_H + (1-\mu)(1-q) v_L}{\mu q + (1-\mu)(1-q)}.$$

Similarly, the posterior upon receiving $s = L$ is

$$\hat{v}_L(q) := \frac{\mu (1-q) v_H + (1-\mu) q v_L}{\mu (1-q) + (1-\mu) q}.$$

A signal $s = H$ is realized with an ex ante probability $\hat{\mu}(q) := \mu q + (1 - \mu)(1 - q)$.

The following observation is worth stating before we lay out the full results of optimal contest design.

Lemma 1 ***(Ex ante Equivalence of Symmetric Disclosure Schemes)*** *Fixing a scoring rule $\delta > 0$, the symmetric disclosure schemes generate the same ex ante equilibrium outcomes in terms of the expected total effort and expected winner's effort—i.e., $TE(CC, \delta; c_1, c_2) = TE(DD, \delta; c_1, c_2)$, and $WE(CC, \delta; c_1, c_2) = WE(DD, \delta; c_1, c_2)$.*

3 Optimal Contest

The solutions to equilibrium expected total effort $TE(\gamma, \delta; c_1, c_2)$ and the expected winner's effort $WE(\gamma, \delta; c_1, c_2)$ are presented in Appendix A, which enable analysis of the op-

[4]See, e.g., Moldovanu and Sela (2006); Barbieri and Serena (2021); Fu and Wu (2022); and Wasser and Zhang, 2023).

timum. We begin with the case of symmetric players—i.e., $c_1 = c_2 > 0$. Discussion of the simple case elucidates the main insight of this paper and lays a foundation for the analysis of asymmetric players.

3.1 Optimal Contest with Symmetric Players

Consider the case of symmetric players with $c_1 = c_2 =: c > 0$. We have the following.

Theorem 1 ***(Optimal Contest with Symmetric Players)*** *Fix $q \in (1/2, 1]$ and suppose that $c_1 = c_2 = c > 0$. The following statements hold.*

(i) If the designer aims to maximize expected total effort, then both $(\gamma^*_{TE}, \delta^*_{TE}) = (CC, 1)$ and $(\gamma^*_{TE}, \delta^*_{TE}) = (DD, 1)$ are an optimal contest scheme.

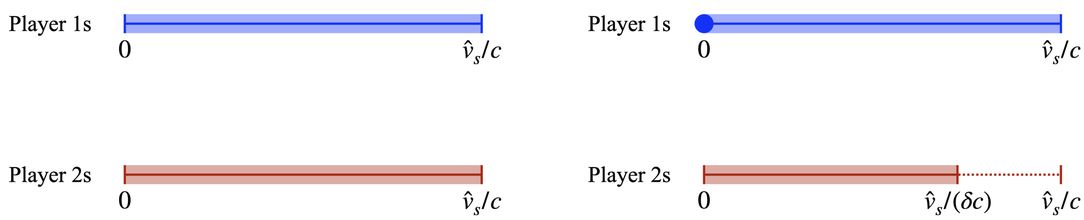
(ii) If the designer aims to maximize the expected winner's effort, then in the case with $\hat{\mu}(q)\hat{v}_H(q) > 4\hat{v}_L(q)$, both $(\gamma^*_{WE}, \delta^*_{WE}) = \big(CD, \hat{\mu}(q)\big)$ and $(\gamma^*_{WE}, \delta^*_{WE}) = \big(DC, 1/\hat{\mu}(q)\big)$ are an optimal contest scheme; in the case with $\hat{\mu}(q)\hat{v}_H(q) \leq 4\hat{v}_L(q)$, both $(\gamma^*_{WE}, \delta^*_{WE}) = (CC, 1)$ and $(\gamma^*_{WE}, \delta^*_{WE}) = (DD, 1)$ are an optimal contest scheme.

The optimal contest depends on the design objectives. Theorem 1(i) is intuitive and echoes the conventional wisdom of the contest literature: A level playing field creates competition and intensifies effort supply. The contest maintains symmetry to maximize the expected total effort: A fair bias $\delta = c_2/c_1 = 1$, together with a symmetric disclosure—i.e., $\gamma \in \{CC, DD\}$. However, Theorem 1(ii) shows that to maximize the expected winner's effort, it may be optimal for the designer to deliberately create ex post dual asymmetry between players: She *tilts* the playing field by awarding information advantage to one player, while *releveling* the playing field by biasing the scoring rule in favor of the other. A *tilting-and-releveling* contest, $\big(CD, \hat{\mu}(q)\big)$ or $\big(DC, 1/\hat{\mu}(q)\big)$, is optimal when the condition $\hat{\mu}(q)\hat{v}_H(q) > 4\hat{v}_L(q)$ is met.

To interpret the result, it is useful to understand the bidding equilibrium under symmetric disclosure vis-à-vis that under asymmetric disclosure.

Equilibrium under Symmetric Disclosure The equilibrium under a symmetric disclosure scheme with discrete signal spaces resembles that in a standard complete-information all-pay auction. Under $\delta = 1$, each player's effort is uniformly distributed over the interval $[0, \hat{v}_s(q)/c]$ under DD, where $\hat{v}_s(q)$ is the updated expected prize value upon receiving a signal $s \in \{H, L\}$. Analogously, one's effort under CC is uniformly distributed over $[0, \bar{v}/c]$.

A symmetric contest, $(DD, 1)$ or $(CC, 1)$, fully extracts their surplus and achieves the first-best outcome for maximization of expected total effort. Suppose instead that a biased



(a) $\delta = 1$.

(b) $\delta > 1$.

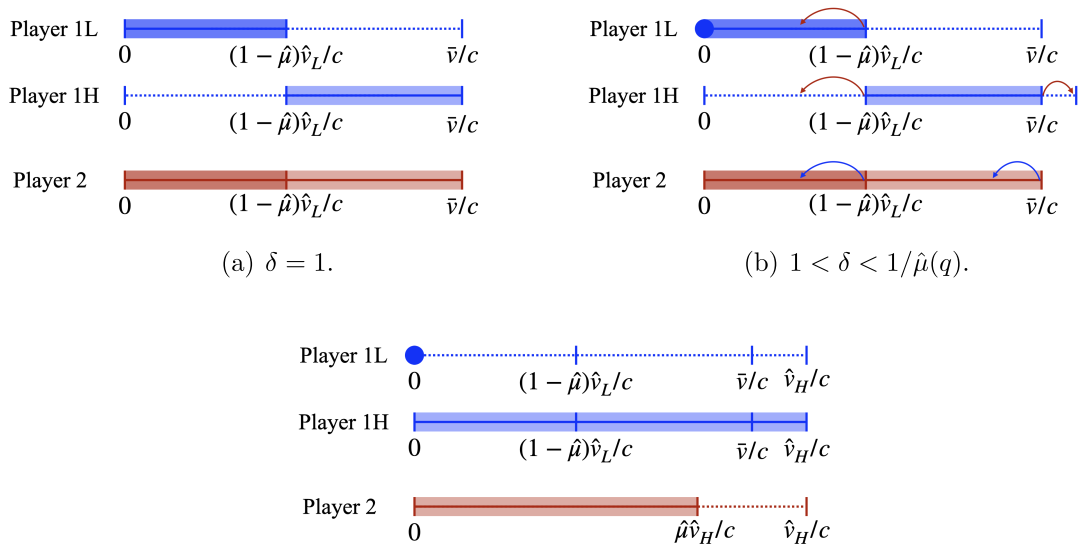
Figure 1: Equilibrium Strategies with Symmetric Players: $\gamma = DD$.

scoring rule is in place, e.g., $\delta > 1$. Player 2 secures a sure win by bidding $\hat{v}_s(q)/(\delta c)$ under DD (or $\bar{v}/(\delta c)$ under CC), which allows him to enjoy a positive surplus. The handicapped player 1 continues to bid up to $\hat{v}_s(q)/c$ under DD (or $\bar{v}/c$ under CC), but he now stays inactive—i.e., exerting zero effort—with a positive probability. The biased scoring rule is obviously suboptimal for either design objective. We visualize this rationale in Figure 1 for the case of $\gamma = DD$.

Equilibrium under Asymmetric Disclosure Asymmetric disclosure fundamentally changes the nature of the equilibrium. We focus on the case of $\gamma = DC$ and begin with $\delta = 1$. Players' equilibrium bidding strategies are depicted in Figure 2(a). Player 1 is informed, and thus his equilibrium bidding strategy is signal-dependent. Specifically, the efforts of player 1 upon receiving signal L, referred to as player $1L$, are uniformly distributed on $\big[0, [1 - \hat{\mu}(q)]\hat{v}_L(q)/c\big]$, while those of player $1H$ are distributed on $\big[[1 - \hat{\mu}(q)]\hat{v}_L(q)/c, \bar{v}/c\big]$ (see Proposition 2 in Appendix A). The equilibrium is monotone, in the sense that the bidding supports of players $1H$ and $1L$ do not overlap. Player 2's efforts are distributed over the interval $\big[0, \bar{v}/c\big]$, but the densities differ in the two contiguous intervals of $\big[0, [1 - \hat{\mu}(q)]\hat{v}_L(q)/c\big]$ and $\big[[1 - \hat{\mu}(q)]\hat{v}_L(q)/c, \bar{v}/c\big]$.

Player $1L$—due to his lower updated expected prize valuation—is effectively an underdog when competing with the uninformed player 2. The distribution of his efforts includes zero, which implies a zero equilibrium payoff for him. In contrast, player $1H$—as a result of receiving H signal—has a higher expected prize valuation, and becomes a favorite in the contest against player 2. The upper support of his efforts remains at $\bar{v}/c$, although he can bid up to $\hat{v}_H(q)/c$ while retaining a positive payoff. He has no incentive to bid more than $\bar{v}/c$ because player 2's prize valuation remains $\bar{v}$ and thus his effort is capped by $\bar{v}/c$.

This observation prompts the natural question of how player $1H$ can be further incentivized, which inspires *tilting and releveling*.



(a) $\delta = 1$.

(b) $1 < \delta < 1/\hat{\mu}(q)$.

(c) $\delta = 1/\hat{\mu}(q)$.

Figure 2: Equilibrium Strategies with Symmetric Players: $\gamma = DC$.

Tilting and Releveling Raising δ above 1 incentivizes player $1H$. We illustrate this rationale in Figure 2(b). A scoring bias $\delta > 1$ favors player 2 and further discourages player $1L$; the upper support of his efforts falls in response. In contrast, player $1H$ continues to enjoy an upper hand for δ in the range of $\left[1, 1/\hat{\mu}(q)\right]$; but an effort $\bar{v}/c$ no longer guarantees a sure win, so the unfavorable scoring rule compels him to step up his effort: The upper support of his effort increases with δ. The bidding support for player 2 shrinks because he is privileged by the favorable bias, which allows him to slack off.

The tilting-and-releveling distortion cannot outperform a fully symmetric contest when the designer's objective is to maximize the total effort because a fully symmetric contest maximizes players' participation and fully dissipates their rents. In contrast, we show below that the tilting-and-releveling contest can be optimal when the designer's objective is to maximize the expected winner's effort, since player $1H$ can be forced to bid more than $\bar{v}/c$.

The main reason different designer objectives can have drastically contrasting implications regarding the optimal mechanism is as follows. When the designer aims to maximize expected total effort, both players' effort evenly accrues to the designer's payoff; however, if the designer is concerned about the expected winner's effort, only the winner's effort (i.e., the modified first-order statistic) matters: The tilting-and-releveling contest "gives up" the low-type informed player 1, but the loss is compensated for by the better-incentivized high-type counterpart. The details are discussed below.

Tilting-and-Releveling as an Optimal Contest By Theorem 1(ii), with $\gamma = DC$, a bias $\delta = 1/\hat{\mu}(q)$ could maximize the expected winner's effort. By Lemma 1, fixing δ, contests under symmetric disclosure, either DD or CC, generate the same ex ante equilibrium outcomes. It thus suffices to compare the tiling-and-releveling contest $\big(DC, 1/\hat{\mu}(q)\big)$ with a fully symmetric contest $(CC, 1)$ to explain the underlying trade-off.

Recall that under $(CC, 1)$, players maintain their prior throughout, so their efforts are uniformly distributed over $[0, \bar{v}/c]$. Players' equilibrium strategies in the contest $\big(DC, 1/\hat{\mu}(q)\big)$ are illustrated in Figure 2(c). Imagine first that a low signal $s = L$ is realized. The negative shock, together with the unfavorable scoring rule, forces player $1L$ to give up—i.e., with his bidding strategy degenerating to a singleton at zero—which clearly causes inefficiency compared with the case of $(CC, 1)$. However, player 2 remains uninformed and is immune to the negative shock; he remains active, which provides insurance for the performance of the contest. Then suppose $s = H$. Player 1—because of the upwardly revised prize expectation and the unfavorable scoring rule—may bid more than $\bar{v}/c$, and the upper support reaches $\hat{v}_H(q)/c$. The contest, when maximizing the expected winner's effort, could outperform $(CC, 1)$.

A trade-off looms large, which ultimately depends on $\hat{\mu}(q)$, $\hat{v}_H(q)$, and $\hat{v}_L(q)$. First, tilting and releveling could yield a gain when a high signal is realized, which occurs with a probability of $\hat{\mu}(q)$. Therefore, $\big(DC, 1/\hat{\mu}(q)\big)$ is more likely to outperform $(CC, 1)$ with a large $\hat{\mu}(q)$. Second, the gain is more significant when the signal prompts substantial upward revision in prize valuation—i.e., from $\bar{v}$ to $\hat{v}_H(q)$—which requires a larger $\hat{v}_H(q)$ relative to $\hat{v}_L(q)$. Summing these gives rise to the condition $\hat{\mu}(q)\hat{v}_H(q) > 4\hat{v}_L(q)$ for the optimality of $\big(DC, 1/\hat{\mu}(q)\big)$. The scoring bias $\delta = 1/\hat{\mu}(q)$ relevels the contest under $\gamma = DC$. This lets the two players, $1H$ versus 2, win with an equal probability when a high signal is realized; it also perfectly eliminates the rent afforded to player 1 by his information advantage.

This rationale continues to apply when players are asymmetric, i.e., $c_1 > c_2$. Tilting and releveling may well emerge as the optimum, which we discuss in Section 3.2. With asymmetric players, the designer should also decide to whom—the strong or the weak player—to award the information advantage and whom to favor in terms of scoring bias.

Complementarity between Information Disclosure and Scoring Bias Before we proceed, it is worth noting that the two instruments, information disclosure and scoring bias, play *complementary* roles. That is, the optimum is either a fully symmetric contest or a tilting-and-releveling contest that embraces dual asymmetry. Suppose the designer is allowed to distort the contest in only dimension, either setting the disclosure scheme while maintaining a neutral scoring rule or biasing the scoring rule while being constrained by symmetric disclosure. The following ensues.

Remark 1 ***(Unidimensional Contest Design with Symmetric Players)*** *Fix* $q \in (1/2, 1]$ *and suppose that* $c_1 = c_2$. *The following statements hold:*

(i) Fix $\delta = 1$. *A symmetric disclosure scheme—i.e.,* $\gamma \in \{CC, DD\}$*—maximizes both expected total effort and the expected winner's effort simultaneously.*

(ii) Fix $\gamma \in \{CC, DD\}$. *The neutral scoring bias—i.e.,* $\delta = 1$*—maximizes both expected total effort and the expected winner's effort simultaneously.*

With a neutral scoring rule $\delta = 1$, an asymmetric disclosure scheme cannot force the high-type player 1 to raise his maximum effort above $\bar{v}/c$, as Figure 2(a) illustrates. Similarly, with a symmetric disclosure scheme in place, biasing the scoring rule only allows the favored player to slack off, as Figure 1(b) shows. Asymmetry is suboptimal when contest can be manipulated in only one dimension.

3.2 Optimal Contest with Asymmetric Players

We now consider the case of asymmetric players with $c_1 > c_2$. The following ensues.

Theorem 2 ***(Optimal Contest with Asymmetric Players)*** *Fix* $q \in (1/2, 1]$ *and suppose that* $c_1 > c_2$. *The following statements hold.*

(i) If the designer aims to maximize expected total effort, then both $(\gamma^*_{TE}, \delta^*_{TE}) = (CC, c_2/c_1)$ *and* $(\gamma^*_{TE}, \delta^*_{TE}) = (DD, c_2/c_1)$ *are an optimal contest scheme.*

(ii) If the designer aims to maximize the expected winner's effort, then in the case with $\hat{\mu}(q)\hat{v}_H(q) > \left(2\frac{c_2}{c_1} + 2\right)\hat{v}_L(q)$, *the optimal scheme is* $(\gamma^*_{WE}, \delta^*_{WE}) = \left(DC, \frac{c_2}{\hat{\mu}(q)c_1}\right)$; *in the case with* $\hat{\mu}(q)\hat{v}_H(q) \leq \left(2\frac{c_2}{c_1} + 2\right)\hat{v}_L(q)$, *both* $(\gamma^*_{WE}, \delta^*_{WE}) = (CC, c_2/c_1)$ *and* $(\gamma^*_{WE}, \delta^*_{WE}) = (DD, c_2/c_1)$ *are an optimal contest scheme.*

Theorem 2(i), again, affirms the conventional wisdom of leveling the playing field under the design objective of maximizing expected total effort. With a symmetric disclosure scheme, the "fair" bias $\delta = c_2/c_1$ perfectly offsets the ex ante asymmetry in bidders' marginal effort costs, which ensures that the designer is able to fully extract the players' surplus.

However, Theorem 2(ii) states that a tilting-and-releveling contest $\left(DC, c_2/\hat{\mu}(q)c_1\right)$ could be optimal when the designer's objective is to maximize the expected winner's effort. The underdog, player 1, is provided with an information advantage. The bias $\delta = c_2/[\hat{\mu}(q)c_1]$ relevels the competition between players $1H$ and 2—as $1/\hat{\mu}(q)$ does in the symmetric case—and discourages player $1L$ entirely. The same trade-off for the designer looms large, as in the case with symmetric players. It is worth noting that the releveling bias $c_2/[\hat{\mu}(q)c_1]$

could remain literally biased against player 2—i.e., $c_2/[\hat{\mu}(q)c_1] < 1$—if players are excessively heterogeneous. However, it is more favorable to player 2 relative to the "fair" bias c_2/c_1—i.e., $c_2/[\hat{\mu}(q)c_1] > c_2/c_1$—to offset player 1's information advantage.

To understand why the *weaker* player 1 receives an information advantage, recall that in a tilting-and-releveling contest, (i) the low-type informed player is fully discouraged—which incurs a loss—and (ii) the uninformed player stays active regardless—which provides insurance. First, giving up the weaker player (player 1 when a low signal is realized) minimizes the loss, since his higher marginal cost limits the forgone effort. Second, keeping the stronger player (player 2) active maximizes the insurance for contest performance. Despite the exit of the low-type informed player with $s = L$, the uninformed player continues to bid actively and his contribution provides insurance for contest performance, which mitigates the loss.

This rationale can further be illustrated by the condition $\hat{\mu}(q)\hat{v}_H(q) > [2(c_2/c_1) + 2]\hat{v}_L(q)$ for the optimality of the tilting-and-releveling contest. It is more likely when players are more asymmetric, i.e., with a smaller c_2/c_1: The loss incurred when $s = L$ is less significant because the forgone effort of player $1L$ is limited by his relatively high cost, while the insurance provided by player 2 is large because of his relatively low cost.

4 Extensions

In this section, we discuss two alternative scenarios that further affirm the main results underlying our model.

4.1 Endogenous Information Structure

We now let the designer flexibly design the information structure of her investigation. She is endowed with full control over the amount of information to be revealed through the investigation and the form of the signal to be disclosed to players. This corresponds to the concept of Bayesian persuasion (Kamenica and Gentzkow, 2011).

An information structure consists of a signal space $\mathcal{S}$ and a pair of likelihood distributions $\{\pi(\cdot|v_H), \pi(\cdot|v_L)\}$ over $\mathcal{S}$.[5] The designer sets $\left(\gamma, \delta, \{\pi(\cdot|v_H), \pi(\cdot|v_L)\}\right)$ to maximize either the expected total effort or the expected winner's effort.

Our exercise remains a limited information design problem. First, we aim to determine who should receive the information. This question becomes moot if the signal structure is fully endogenized. Second, addressing a general information design problem in our context

[5]For instance, the information structure depicted in Section 2 involves a binary signal space $\mathcal{S} = \{H, L\}$ and a conditional likelihood distribution for each underlying state—i.e., v_H or v_L—parameterized by q [see Equation (1)].

is technically challenging: The potential correlation between signals would substantially complicate analysis of common-valued all-pay auctions.

We can show that it suffices to consider a binary signal space in our setting, i.e., $\mathcal{S} = \{H, L\}$. Denote by v_s^π the expected prize value conditional on s, i.e., $\mathbb{E}(v|s)$. Without loss of generality, assume that realization of $s = H$ gives rise to a higher expected prize value, i.e., $v_H^\pi \geq v_L^\pi$. In addition, define $\mu^\pi := \Pr(s = H)$. We can adapt the analysis in Appendix A to obtain the equilibrium by replacing $\hat{v}_H(q)$, $\hat{v}_L(q)$, and $\hat{\mu}(q)$ with v_H^π, v_L^π, and μ^π, respectively. It is straightforward to verify that designing the information structure $\{\pi(\cdot|v_H), \pi(\cdot|v_L)\}$ is equivalent to choosing the tuple $(v_H^\pi, v_L^\pi, \mu^\pi)$ that satisfies the following Bayes-plausibility constraint (Kamenica and Gentzkow, 2011):[6]

$$v_H \geq v_H^\pi > \bar{v} > v_L^\pi \geq v_L, \text{ and } \mu^\pi v_H^\pi + (1 - \mu^\pi) v_L^\pi = \bar{v}. \tag{2}$$

To search for the optimal information structure, we simply express the designer's objective as a function of $(v_H^\pi, v_L^\pi, \mu^\pi)$ and optimize over $(v_H^\pi, v_L^\pi, \mu^\pi)$ subject to constraint (2).[7] The following result ensues.

Theorem 3 ***(Optimal Contest with an Endogenous Information Structure)*** *Suppose that $c_1 \geq c_2$. Consider the joint design of scoring bias $\delta > 0$, disclosure scheme γ, and information structure $\{\pi(\cdot|v_H), \pi(\cdot|v_L)\}$. The following statements hold.*

(i) If the designer aims to maximize expected total effort, then both $(\gamma_{TE}^*, \delta_{TE}^*) = (CC, c_2/c_1)$ and $(\gamma_{TE}^*, \delta_{TE}^*) = (DD, c_2/c_1)$ with any information structure $\{\pi(\cdot|v_H), \pi(\cdot|v_L)\}$ are an optimal contest scheme.

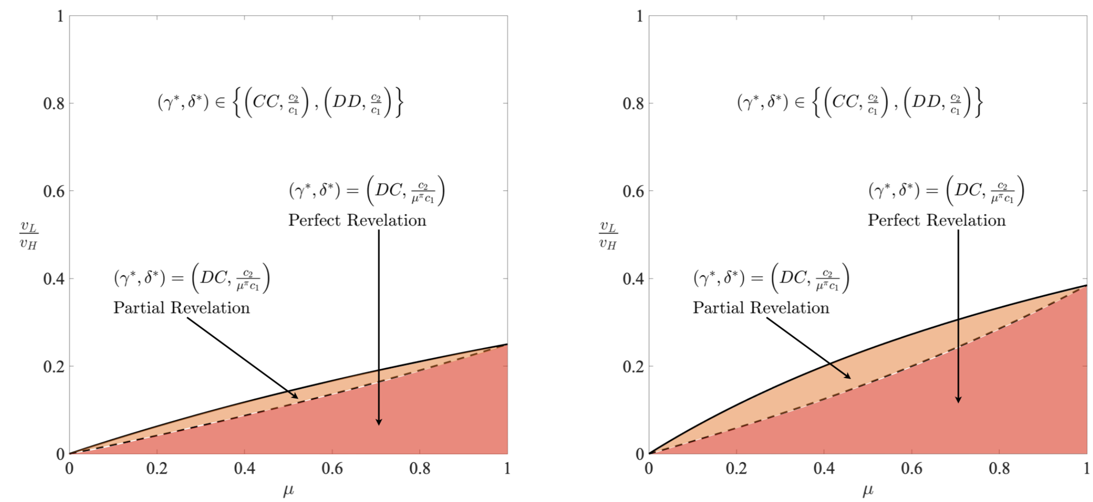
(ii) If the designer aims to maximize the expected winner's effort, then

(a) in the case in which $\bar{v}/v_L > 2c_2/c_1 + 2$, the optimal contest scheme consists of $(\gamma_{WE}^*, \delta_{WE}^*) = (DC, c_2/(\mu^\pi c_1))$ and

$$\pi(H|v_H) = 1, \pi(H|v_L) = \begin{cases} 0, & \text{if } \frac{\bar{v}}{v_L} \geq 4 - 2\mu + 2\frac{c_2}{c_1}, \\ \frac{4-2\mu+2\frac{c_2}{c_1} - \frac{\bar{v}}{v_L}}{2(1-\mu)}, & \text{if } 2\frac{c_2}{c_1} + 2 < \frac{\bar{v}}{v_L} < 4 - 2\mu + 2\frac{c_2}{c_1}; \end{cases}$$

[6] We require that $\pi(s|v)$ not be completely uninformative—i.e., $v_H^\pi > v_L^\pi$. If a completely uninformative information structure is desirable—i.e., $v_H^\pi = v_L^\pi$—the designer can simply choose $\gamma = CC$ to conceal the signal from both players.

[7] The optimization problem can be very involved, given that (i) the designer now optimizes over five dimensions $(\gamma, \delta, v_H^\pi, v_L^\pi, \mu^\pi)$ and (ii) the objective functions $TE(\gamma, \delta; c_1, c_2)$ and $WE(\gamma, \delta; c_1, c_2)$ are piecewise functions with complex expressions. We overcome the difficulty and solve for the optimum by reducing dimensionality step by step. We first pin down the set of biases δ that can be optimal under arbitrary disclosure policy γ and the information structure encapsulated by $(v_H^\pi, v_L^\pi, \mu^\pi)$, then optimize over all information structures. Last, we compare different disclosure policies to obtain the optimum.



(a) Symmetric Players: $(c_1, c_2) = (1, 1)$

(b) Asymmetric Players: $(c_1, c_2) = (1, 0.3)$

Figure 3: Winner's Effort-maximizing Contest Scheme with an Endogenous Information Structure.

(b) in the case in which $\bar{v}/v_L \leq 2c_2/c_1 + 2$*, both* $(\gamma^*_{WE}, \delta^*_{WE}) = (CC, c_2/c_1)$ *and* $(\gamma^*_{WE}, \delta^*_{WE}) = (DD, c_2/c_1)$ *with an arbitrary information structure* $\{\pi(\cdot|v_H), \pi(\cdot|v_L)\}$ *are an optimal contest scheme.*

The implications of our baseline model largely extend. Theorem 3(i) states that when maximizing expected total effort, the optimal contest requires symmetric distribution of information. However, the specific information structure is irrelevant, so the designer does not benefit from the freedom to set the information structure.

By Theorem 3(ii), the designer may, again, tilt and relevel to maximize the expected winner's effort when the condition $\bar{v}/v_L > 2c_2/c_1 + 2$ is met. In contrast to maximizing expected total effort, the prevailing information structure plays a role in achieving the optimum. As in the baseline model, the weaker contender, player 1, exclusively receives the signal. When the ratio $\bar{v}/v_L$ is sufficiently large—i.e., $\bar{v}/v_L \geq 4 - 2\mu + 2c_2/c_1$—the optimum requires perfect revelation, i.e., $\pi(H|v_H) = \pi(L|v_L) = 1$. When the ratio falls in the intermediate range—i.e., $\bar{v}/v_L \in (2c_2/c_1 + 2, 4 - 2\mu + 2c_2/c_1)$—partial revelation emerges.

Theorem 3(ii) can similarly be interpreted in light of the rationale outlined in Section 3. A tilting-and-releveling contest is optimal when a high signal $s = H$ is more likely and the signal triggers substantial revision of expected prize value, which requires large μ^π and v^π_H/v^π_L. Constraint (2) implies that for this purpose, the designer should perfectly reveal the state v_L—i.e., set $v^\pi_L = v_L$—as predicted in Theorem 3(ii). Further, rearranging the

Bayes-plausibility constraint (2) yields

$$\frac{v_H^\pi}{v_L^\pi} = \frac{v_H^\pi}{v_L} = 1 + \frac{\bar{v} - v_L}{v_L} \times \frac{1}{\mu^\pi},$$

which unveils the designer's trade-off of increasing μ^π versus increasing v_H^π / v_L^π. When the term $(\bar{v} - v_L)/v_L$—or the term $\bar{v}/v_L$—is large, an increase in μ^π would lead to a significant decrease in v_H^π / v_L^π. This compels the designer to increase v_H^π / v_L^π, which implies perfect revelation for the optimum, i.e., $v_H^\pi = v_H$. The trade-off leads to partial revelation for a moderate value of $\bar{v}/v_L$, as in Theorem 3(ii). Figure 3 illustrates the optimum in the $(\mu, v_L/v_H)$ space for maximization of the expected winner's effort.

4.2 Expected Maximum Effort

As stated in Footnote 1, with a scoring bias $\delta \neq 1$ the winner of the contest may not exert the higher effort. Maximizing the expected winner's effort presumes that the designer benefits only from the winning entry, which, as pointed out in Footnote 3, is plausible when the designer cannot separate the prize distribution and adoption of the contestant—e.g., admissions contests at elite universities or competitions for promotions inside firms. In some contexts the designer may give the prize to a contestant according to possibly biased scoring rules, but nonetheless benefit from the contestant with the higher effort. For instance, Netflix organized a contest for algorithmic designs to improve its recommendation system and did not adopt the prize winner's algorithm.[8] In this subsection, we consider the case in which the designer's objective is to maximize the expected maximum effort of the contestants, which we denote by $ME(\gamma, \delta; c_1, c_2)$.

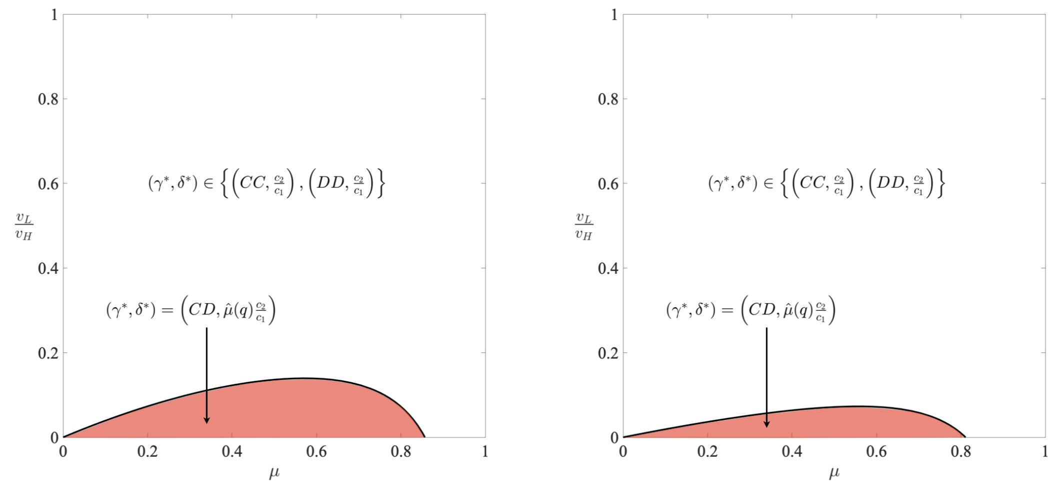
Comparing an ex post symmetric contest $(CC, c_2/c_1)$ to a tilting-and-releveling contest $(CD, \hat{\mu}(q)c_2/c_1)$ leads to the following.

Theorem 4 ***(Expected-maximum-effort-maximizing Contests)*** *Consider two contests $(CC, c_2/c_1)$ and $(CD, \hat{\mu}(q)c_2/c_1)$. The former generates a higher expected maximum effort than the latter if and only if*

$$\frac{\hat{v}_L(q)}{\hat{v}_H(q)} > \frac{\frac{c_2}{c_1}\hat{\mu}(q)\left\{3 - [1 + \hat{\mu}(q)]\frac{c_2}{c_1}\right\}}{3 + (\frac{c_2}{c_1})^2}. \tag{3}$$

A fully general analysis is difficult. However, together with numerical exercises, we can verify that the optimal contest is either $(CC, c_2/c_1)$ or $(CD, \hat{\mu}(q)c_2/c_1)$. Theorem 4 thus establishes the sufficient and necessary condition for the optimality of a tilting-and-releveling

[8] See tinyurl.com/37kdtz74.



(a) Symmetric Players: $(c_1, c_2) = (1, 1)$ (b) Asymmetric Players: $(c_1, c_2) = (1, 0.3)$

Figure 4: Expected-maximum-effort-maximizing Contest Scheme: $q = 0.95$.

contest. Analogous to the case of maximizing the expected winner's effort, dual asymmetry can be optimal if players are ex ante symmetric, i.e., $c_2/c_1 = 1$.

However, in contrast to the previous case, the designer, when tilting and releveling, would give the information advantage to the *stronger* player. Awarding the information advantage to the stronger player requires handicapping him through a sufficiently unfavorable scoring bias. This is suboptimal when the designer's objective is to maximize the expected winner's effort: The stronger player can contribute a relatively high effort because of his lower cost, but a large handicap excessively reduces his winning odds. This concern does not apply when the designer maximizes the expected maximum effort: The designer can benefit from the stronger player's effort even if he does not win.

Figure 4 illustrates the optimum in the $(\mu, v_L/v_H)$ space for maximization of the expected maximum effort, holding fixed $q = 0.95$.

5 Concluding Remarks

This paper studies the optimal design of a contest in which two players compete for a common-valued prize. The designer chooses a combination of two instruments—an information disclosure scheme and a scoring bias—to advance her interests. The optimum depends on the designer's objective. When the designer aims to maximize the expected total effort of the contestants, the optimum embraces the conventional wisdom of leveling the playing field: Information is symmetrically distributed, and a "fair" (i.e., compensating) scoring bias offsets the initial asymmetry between the players in their marginal cost of effort. However, when the designer's objective is to maximize the expected winner's effort or the expected maximum effort, the optimum may feature dual asymmetry: The designer discloses the signal privately to one player only, while a favorable scoring rule compensates the other. We call such contest a *tilting-and-releveling* contest, and show that it could emerge as the optimum even if the players are ex ante symmetric in their marginal costs of effort. We further show that the results are qualitatively robust to extensions to an endogenous information structure.

Our paper is one of the first in the contest literature to examine the optimal combination of multiple design instruments. It is noteworthy that the two instruments demonstrate complementarity, in that the optimal contest requires either ex post full symmetry or dual asymmetry. Asymmetry is thus suboptimal in settings of unidimensional contest design. Our results generate novel implications for contest design and shed fresh light on the debate regarding the relationship between (a)symmetry and the performance of a contest.

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Appendix A Equilibrium Analysis

In this appendix, we characterize the equilibrium under an arbitrary contest scheme (γ, δ), with $\gamma \in \{CC, CD, DC, DD\}$ and $\delta > 0$ and calculate the resulting expected total effort and expected winner's effort. Our analysis is adapted from Siegel (2014), who provides the technique for the case with a neutral scoring rule $\delta = 1$; here we allow for a scoring bias.

A.1 Equilibrium Results

We first characterize the equilibrium under a symmetric information disclosure scheme, i.e., $\gamma \in \{CC, DD\}$, in which case neither player possesses information favoritism.

Proposition 1 ***(Equilibrium Characterization under Symmetric Disclosure)*** *Under $\gamma = DD$, the contest game generates a unique equilibrium, which can be characterized as follows:*

(i) If $\delta < \frac{c_2}{c_1}$, then

$$b_{1s}(x; DD, \delta) = \begin{cases} \frac{c_2}{\delta \hat{v}_s(q)}, & \textit{if } 0 < x \leq \frac{\delta \hat{v}_s(q)}{c_2}, \\ 0, & \textit{otherwise}, \end{cases}$$

$$b_{2s}(x; DD, \delta) = \begin{cases} 1 - \frac{\delta c_1}{c_2}, & \textit{if } x = 0, \\ \frac{\delta c_1}{\hat{v}_s(q)}, & \textit{if } 0 < x \leq \frac{\hat{v}_s(q)}{c_2}, \\ 0, & \textit{otherwise}. \end{cases}$$

(ii) If $\delta \geq \frac{c_2}{c_1}$, then

$$b_{1s}(x; DD, \delta) = \begin{cases} 1 - \frac{c_2}{\delta c_1}, & \textit{if } x = 0, \\ \frac{c_2}{\delta \hat{v}_s(q)}, & \textit{if } 0 < x \leq \frac{\hat{v}_s(q)}{c_1}, \\ 0, & \textit{otherwise}, \end{cases}$$

$$b_{2s}(x; DD, \delta) = \begin{cases} \frac{\delta c_1}{\hat{v}_s(q)}, & \textit{if } 0 < x \leq \frac{\hat{v}_s(q)}{\delta c_1}, \\ 0, & \textit{otherwise}. \end{cases}$$

(iii) The equilibrium bidding strategy under $\gamma = CC$, denoted by $b_i(x; CC, \delta)$, can be obtained by replacing $\hat{v}_s(q)$ with $\bar{v} \equiv \mu v_H + (1-\mu) v_L$ in $b_{is}(x; DD, \delta)$.

Next, we consider the equilibrium under each asymmetric disclosure scheme, i.e., $\gamma = CD$ or DC, in which case one player receives the signal privately.

Proposition 2 ***(Equilibrium Characterization under Asymmetric Disclosure)*** *Under $\gamma = DC$, the contest game generates a unique equilibrium, which can be characterized as follows:*

(i) If $\delta < \frac{c_2}{c_1}$, then

$$b_{1L}(x; DC, \delta) = \begin{cases} \frac{c_2}{\delta[1-\hat{\mu}(q)]\hat{v}_L(q)}, & \text{if } 0 < x \leq \frac{\delta[1-\hat{\mu}(q)]\hat{v}_L(q)}{c_2}, \\ 0, & \text{otherwise,} \end{cases}$$

$$b_{1H}(x; DC, \delta) = \begin{cases} \frac{c_2}{\delta\hat{\mu}(q)\hat{v}_H(q)}, & \text{if } \frac{\delta[1-\hat{\mu}(q)]\hat{v}_L(q)}{c_2} < x \leq \frac{\delta\bar{v}}{c_2}, \\ 0, & \text{otherwise,} \end{cases}$$

$$b_2(x; DC, \delta) = \begin{cases} 1 - \frac{\delta c_1}{c_2}, & \text{if } x = 0, \\ \frac{\delta c_1}{\hat{v}_L(q)}, & \text{if } 0 < x \leq \frac{[1-\hat{\mu}(q)]\hat{v}_L(q)}{c_2}, \\ \frac{\delta c_1}{\hat{v}_H(q)}, & \text{if } \frac{[1-\hat{\mu}(q)]\hat{v}_L(q)}{c_2} < x \leq \frac{\bar{v}}{c_2}, \\ 0, & \text{otherwise.} \end{cases}$$

(ii) If $\frac{c_2}{c_1} \leq \delta \leq \frac{c_2}{\hat{\mu}(q)c_1}$, then

$$b_{1L}(x; DC, \delta) = \begin{cases} \frac{1}{1-\hat{\mu}(q)}\left(1 - \frac{c_2}{\delta c_1}\right), & \text{if } x = 0, \\ \frac{c_2}{\delta[1-\hat{\mu}(q)]\hat{v}_L(q)}, & \text{if } 0 < x \leq \left[1 - \hat{\mu}(q)\frac{\delta c_1}{c_2}\right]\frac{\hat{v}_L(q)}{c_1}, \\ 0, & \text{otherwise,} \end{cases}$$

$$b_{1H}(x; DC, \delta) = \begin{cases} \frac{c_2}{\delta\hat{\mu}(q)\hat{v}_H(q)}, & \text{if } \left[1 - \hat{\mu}(q)\frac{\delta c_1}{c_2}\right]\frac{\hat{v}_L(q)}{c_1} < x \leq \frac{\hat{v}_L(q)}{c_1} + \frac{\delta\hat{\mu}(q)[\hat{v}_H(q)-\hat{v}_L(q)]}{c_2}, \\ 0, & \text{otherwise,} \end{cases}$$

$$b_2(x; DC, \delta) = \begin{cases} \frac{\delta c_1}{\hat{v}_L(q)}, & \text{if } 0 < x \leq \left[1 - \hat{\mu}(q)\frac{\delta c_1}{c_2}\right]\frac{\hat{v}_L(q)}{\delta c_1}, \\ \frac{\delta c_1}{\hat{v}_H(q)}, & \text{if } \left[1 - \hat{\mu}(q)\frac{\delta c_1}{c_2}\right]\frac{\hat{v}_L(q)}{\delta c_1} < x \leq \frac{\hat{v}_L(q)}{\delta c_1} + \frac{\hat{\mu}(q)[\hat{v}_H(q)-\hat{v}_L(q)]}{c_2}, \\ 0, & \text{otherwise.} \end{cases}$$

(iii) If $\delta > \frac{c_2}{\hat{\mu}(q)c_1}$, then

$$b_{1L}(x; DC, \delta) = \begin{cases} 1, & \text{if } x = 0, \\ 0, & \text{otherwise,} \end{cases}$$

$$b_{1H}(x;DC,\delta)=\begin{cases}1-\frac{c_2}{\delta c_1\hat{\mu}(q)}, & \textit{if } x=0,\\ \frac{c_2}{\delta\hat{\mu}(q)\hat{v}_H(q)}, & \textit{if } 0<x\le\frac{\hat{v}_H(q)}{c_1},\\ 0, & \textit{otherwise,}\end{cases}$$

$$b_2(x;DC,\delta)=\begin{cases}\frac{\delta c_1}{\hat{v}_H(q)}, & \textit{if } 0<x\le\frac{\hat{v}_H(q)}{\delta c_1},\\ 0, & \textit{otherwise.}\end{cases}$$

The equilibrium under (CD,δ) can be obtained similarly.

A.2 Expected Total Effort and the Expected Winner's Effort

Propositions 1 and 2 lead to the following.

Lemma 2 ***(Expected Total Effort under Different Contest Schemes)*** *Fixing a contest scheme (δ,γ) and a profile of marginal effort costs (c_1,c_2), the contest generates an equilibrium expected total effort*

$$TE(CC,\delta;c_1,c_2)=TE(DD,\delta;c_1,c_2)=\begin{cases}\frac{\delta\bar{v}(c_1+c_2)}{2c_2^2}, & \textit{if } \delta<\frac{c_2}{c_1},\\ \frac{\bar{v}(c_1+c_2)}{2\delta c_1^2}, & \textit{if } \delta\ge\frac{c_2}{c_1}\end{cases}\tag{4}$$

for symmetric disclosure schemes. Under asymmetric disclosure schemes, the equilibrium expected total effort of the contest can be obtained as

$$\begin{aligned}TE(DC,\delta;c_1,c_2)&=TE(CD,1/\delta;c_2,c_1)\\ &=\begin{cases}\frac{\delta(c_1+c_2)\left(\hat{v}_L(q)+\hat{\mu}(q)^2\hat{v}_H(q)-\hat{\mu}(q)^2\hat{v}_L(q)\right)}{2{c_2}^2}, & \textit{if } \delta<\frac{c_2}{c_1},\\ \frac{c_1+c_2}{2c_1c_2}\left[\frac{c_2}{\delta c_1}\hat{v}_L(q)+\frac{\delta c_1}{c_2}\hat{\mu}(q)^2(\hat{v}_H(q)-\hat{v}_L(q))\right], & \textit{if } \frac{c_2}{c_1}\le\delta\le\frac{c_2}{\hat{\mu}(q)c_1},\\ \frac{(c_1+c_2)\hat{v}_H(q)}{2\delta c_1^2}, & \textit{if } \delta>\frac{c_2}{\hat{\mu}(q)c_1}.\end{cases}\end{aligned}\tag{5}$$

Further, we derive the equilibrium expected winner's efforts. The following ensues.

Lemma 3 ***(Expected Winner's Effort under Different Contest Schemes)*** *Fixing a contest scheme (δ,γ) and a profile of marginal effort costs (c_1,c_2), the equilibrium expected winner's effort from the contest game is*

$$WE(CC,\delta;c_1,c_2)=WE(DD,\delta;c_1,c_2)=\begin{cases}\frac{\delta\bar{v}(2c_1+3c_2-c_1\delta)}{6{c_2}^2}, & \textit{if } \delta<\frac{c_2}{c_1},\\ \frac{\bar{v}(3c_1\delta-c_2+2c_2\delta)}{6{c_1}^2\delta^2}, & \textit{if } \delta\ge\frac{c_2}{c_1},\end{cases}\tag{6}$$

and

$$WE(DC,\delta;c_1,c_2)=\begin{cases}\frac{\hat{v}_L(q)}{6c_1c_2}\mathcal{W}_1\left(\hat{\mu}(q),\frac{\hat{v}_H(q)-\hat{v}_L(q)}{\hat{v}_L(q)},\frac{\delta c_1}{c_2};c_1,c_2\right), & \textit{if } \delta<\frac{c_2}{c_1},\\ \frac{\hat{v}_L(q)}{6c_1c_2}\mathcal{W}_2\left(\hat{\mu}(q),\frac{\hat{v}_H(q)-\hat{v}_L(q)}{\hat{v}_L(q)},\frac{\delta c_1}{c_2};c_1,c_2\right), & \textit{if } \frac{c_2}{c_1}\le\delta\le\frac{c_2}{\hat{\mu}(q)c_1},\\ \frac{\hat{v}_H(q)}{6c_1c_2}\mathcal{W}_3\left(\frac{\delta c_1}{c_2};c_1,c_2\right), & \textit{if } \delta>\frac{c_2}{\hat{\mu}(q)c_1},\end{cases} \tag{7}$$

where $\mathcal{W}_1(\cdot,\cdot,\cdot)$*,* $\mathcal{W}_2(\cdot,\cdot,\cdot)$*, and* $\mathcal{W}_3(\cdot)$ *are defined as follows:*

$$\mathcal{W}_1(u,z,d;c_1,c_2):=-c_2\left(u^3z+1\right)d^2+\left\{u^2z\left[3(c_1+c_2)-c_1u\right]+2c_1+3c_2\right\}d,$$

$$\mathcal{W}_2(u,z,d;c_1,c_2):=\frac{d^3\left(-u^2\right)z\left[u(c_1+c_2d)-3(c_1+c_2)\right]+3c_1d-c_1+2c_2d}{d^2},$$

$$\mathcal{W}_3(d;c_1,c_2):=\frac{c_1(3d-1)+2c_2d}{d^2}.$$

Moreover, we have that $WE(CD,\delta;c_1,c_2)=WE(DC,1/\delta;c_2,c_1)$*.*

Lemmas 2 and 3 pave the way for our analysis of the optimal contest design.

A.3 Proofs of Propositions 1 and 2 and Lemmas 2 and 3

Proof. It can be verified that the strategy profiles provided in Propositions 1 and 2 constitute an equilibrium under $\gamma\in\{CC,DD\}$ and $\gamma\in\{DC,CD\}$, respectively. The equilibrium uniqueness in Proposition 1 follows from Hillman and Riley (1989) and Baye, Kovenock, and De Vries (1996), and that in Proposition 2 follows from Siegel (2014). Lemmas 2 and 3 follow immediately from the equilibrium characterizations in Propositions 1 and 2. ■

Appendix B Proofs

Proof of Theorem 1

Proof. See the proof of Theorem 2. ■

Proof of Theorem 2

Proof. We first prove part (i) of the theorem. From (4), it is straightforward to verify that $\delta = \frac{c_2}{c_1}$ maximizes $TE(CC, \delta; c_1, c_2)$ and $TE(DD, \delta; c_1, c_2)$, and the maximum expected total effort is $\frac{(c_1+c_2)\bar{v}}{2c_1c_2}$. Similarly, from (5), it can be verified that either $\delta = \frac{c_2}{c_1}$ or $\delta = \frac{c_2}{\hat{\mu}(q)c_1}$ maximizes $TE(DC, \delta; c_1, c_2)$. Moreover, we have that

$$\begin{aligned} TE\left(DC, \frac{c_2}{c_1}; c_1, c_2\right) &= \frac{(c_1+c_2)\left\{\hat{\mu}^2(q)\hat{v}_H(q) + \left[1-\hat{\mu}^2(q)\right]\hat{v}_L(q)\right\}}{2c_1c_2} \\ &< \frac{(c_1+c_2)\left\{\hat{\mu}(q)\hat{v}_H(q) + \left[1-\hat{\mu}(q)\right]\hat{v}_L(q)\right\}}{2c_1c_2} \\ &= \frac{(c_1+c_2)\bar{v}}{2c_1c_2} = TE\left(CC, \frac{c_2}{c_1}; c_1, c_2\right), \end{aligned}$$

and

$$\begin{aligned} TE\left(DC, \frac{c_2}{\hat{\mu}(q)c_1}; c_1, c_2\right) &= \frac{(c_1+c_2)\hat{\mu}(q)\hat{v}_H(q)}{2c_1c_2} \\ &< \frac{(c_1+c_2)\left\{\hat{\mu}(q)\hat{v}_H(q) + \left[1-\hat{\mu}(q)\right]\hat{v}_L(q)\right\}}{2c_1c_2} \\ &= \frac{(c_1+c_2)\bar{v}}{2c_1c_2} = TE\left(CC, \frac{c_2}{c_1}; c_1, c_2\right). \end{aligned}$$

Therefore, choosing $\gamma \in \{CC, DD\}$ with $\delta = \frac{c_2}{c_1}$ generates strictly more expected total effort to the designer than choosing $\gamma = DC$ with any $\delta > 0$. Recall $TE(DC, \delta; c_1, c_2) = TE(CD, 1/\delta; c_2, c_1)$ from Lemma 2. This immediately implies that choosing $\gamma \in \{CC, DD\}$ with $\delta = \frac{c_2}{c_1}$ generates strictly more expected total effort for the designer than choosing $\gamma = CD$ with any $\delta > 0$.

Next, we prove part (ii). It is useful to prove an intermediate result.

Lemma 4 *Fix $q \in (1/2, 1]$. $WE(DC, \delta; c_1, c_2)$ is maximized at $\delta = \frac{c_2}{c_1}$ or $\delta = \frac{c_2}{\hat{\mu}(q)c_1}$.*

Proof. Fix $u \in (0, 1)$ and $z \in \mathbb{R}_{++}$. First, for $d \in (0, 1)$, we have that

$$\frac{\partial \mathcal{W}_1(u, z, d; c_1, c_2)}{\partial d} = u^2 z\left[(3-u)c_1 + (3-2u)c_2\right] + (2c_1 + c_2) + 2\left(c_2 u^3 z + c_2\right)(1-d) > 0.$$

Therefore, $\mathcal{W}_1(u, z, d; c_1, c_2)$ is increasing in d for $d \in (0, 1)$.

Next, we show that $\mathcal{W}_2(u, z, d; c_1, c_2)$, with $d \in [1, 1/\mu]$, is maximized at $d = 1$ or $d = 1/u$. Simple algebra would verify that

$$\frac{\partial \mathcal{W}_2(u, z, d; c_1, c_2)}{\partial d} = \frac{\left[zu^2 \mathcal{W}_4(u, d; c_1, c_2) - 1\right](3c_1 d + 2c_2 d - 2c_1)}{d^3},$$

where $\mathcal{W}_4(u, d; c_1, c_2) := \frac{3(c_1+c_2)-u(c_1+2c_2 d)}{3c_1 d+2c_2 d-2c_1} d^3$. Note that

$$\frac{\partial \mathcal{W}_4(u, d; c_1, c_2)}{\partial d} = \frac{6d^2 \mathcal{W}_5(u, d; c_1, c_2)}{\left[c_1(3d-2) + 2c_2 d\right]^2},$$

where $\mathcal{W}_5(u, d; c_1, c_2) := -c_2 u(3c_1+2c_2)d^2 + \left[c_1^2(3-u) + c_1 c_2(2u+5) + 2c_2^2\right] d + c_1\left[c_1 u - 3(c_1 + c_2)\right]$. Note that $\mathcal{W}_5(u, d; c_1, c_2)$ is concave in d, which implies that

$$\mathcal{W}_5(u, d; c_1, c_2) \geq \min\left\{\mathcal{W}_5(u, 1; c_1, c_2), \mathcal{W}_5(u, 1/u; c_1, c_2)\right\}, \text{ for } d \in [1, 1/\mu];$$

together with $\mathcal{W}_5(u, 1; c_1, c_2) = 2c_2(c_1 + c_2) - c_2 u(c_1 + 2c_2) > 0$ and $\mathcal{W}_5(u, 1/u; c_1, c_2) = \frac{c_1(c_1(3-u)(1-u)+c_2(2-u))}{u} > 0$, we can conclude that $\mathcal{W}_5(u, d; c_1, c_2) > 0$. As a result, $\frac{\partial \mathcal{W}_4(u,d;c_1,c_2)}{\partial d} > 0$ and thus $\mathcal{W}_4(u, d; c_1, c_2)$ is increasing in d for $d \in [1, 1/u]$, which in turn implies that

$$\frac{\partial \mathcal{W}_2(u, z, d; c_1, c_2)}{\partial d} \gtreqless 0 \Leftrightarrow zu^2 \mathcal{W}_4(u, d; c_1, c_2) \gtreqless 1.$$

Therefore, $\mathcal{W}_2(u, z, d; c_1, c_2)$ is either monotonic or U-shaped in $d \in [1, 1/u]$. This implies that $\mathcal{W}_2(u, z, d; c_1, c_2)$ is maximized at $d = 1$ or $d = 1/u$.

Finally, for $d > 1$, we have that

$$\frac{\partial \mathcal{W}_3(d; c_1, c_2)}{\partial d} = -\frac{3c_1 d - 2c_1 + 2c_2 d}{d^3} < 0,$$

which implies that $\mathcal{W}_3(d; c_1, c_2)$ is decreasing in d for $d > 1$.

In summary, (i) $\mathcal{W}_1(u, z, d; c_1, c_2)$ is increasing in d for $d \in (0, 1)$; (ii) $\mathcal{W}_2(u, z, d; c_1, c_2)$ is maximized at $d = 1$ or $d = 1/u$; and (iii) $\mathcal{W}_3(d; c_1, c_2)$ is decreasing in d for $d > 1$. All together, these facts imply that $WE(DC, \delta; c_1, c_2)$ is maximized at $\delta = \frac{c_2}{c_1}$ or $\delta = \frac{c_2}{\hat{\mu}(q)c_1}$, which concludes the proof. ■

For $\gamma \in \{CC, DD\}$, we have that

$$\frac{\partial WE(CC, \delta; c_1, c_2)}{\partial \delta} = \frac{\partial WE(DD, \delta; c_1, c_2)}{\partial \delta} = \begin{cases} \frac{(3c_2 - 2c_1\delta + 2c_1)\bar{v}}{6c_2^2} > 0, & \text{if } \delta < \frac{c_2}{c_1}; \\ -\frac{(3c_1\delta - 2c_2 + 2c_2\delta)\bar{v}}{6c_1^2\delta^3} < 0, & \text{if } \delta \geq \frac{c_2}{c_1}. \end{cases}$$

Therefore, $WE(CC, \delta; c_1, c_2)$ and $WE(DD, \delta; c_1, c_2)$ are both maximized at $\delta = \frac{c_2}{c_1}$. The maximum expected winner's effort is $\frac{(c_1+c_2)\bar{v}}{3c_1c_2}$.

Further, fixing $q \in (1/2, 1]$, it follows from Lemma 4 that $WE(DC, \delta; c_1, c_2)$ is maximized at $\delta = \frac{c_2}{c_1}$ or $\delta = \frac{c_2}{\hat{\mu}(q)c_1}$. Carrying out the algebra, we can obtain that

$$\begin{aligned} WE\left(DC, \frac{c_2}{c_1}; c_1, c_2\right) &= \frac{(c_1+c_2)\left\{2\bar{v} - \left[2-\hat{\mu}(q)\right]\left[1-\hat{\mu}(q)\right]\hat{\mu}(q)\left[\hat{v}_H(q) - \hat{v}_L(q)\right]\right\}}{6c_1c_2} \\ &< \frac{(c_1+c_2)\bar{v}}{3c_1c_2} = WE\left(CC, \frac{c_2}{c_1}; c_1, c_2\right), \end{aligned}$$

and

$$WE\left(DC, \frac{c_2}{\hat{\mu}(q)c_1}; c_1, c_2\right) = \frac{\hat{\mu}(q)\hat{v}_H(q)\left\{2c_2 + c_1\left[3-\hat{\mu}(q)\right]\right\}}{6c_1c_2}.$$

Further, by Lemma 3, we have that $WE(CD, \delta; c_1, c_2) = WE(DC, 1/\delta; c_2, c_1)$; together with the above analysis, we can conclude that $WE(CD, \delta; c_1, c_2)$ is maximized at $\delta = \frac{c_2}{c_1}$ or $\delta = \frac{\hat{\mu}(q)c_2}{c_1}$. Moreover, we have that

$$WE\left(CD, \frac{c_2}{c_1}; c_1, c_2\right) = WE\left(DC, \frac{c_1}{c_2}; c_2, c_1\right) = WE\left(DC, \frac{c_2}{c_1}; c_1, c_2\right) < WE\left(CC, \frac{c_2}{c_1}; c_1, c_2\right),$$

and

$$\begin{aligned} WE\left(CD, \frac{\hat{\mu}(q)c_2}{c_1}; c_1, c_2\right) &= \frac{\hat{\mu}(q)\hat{v}_H(q)\left\{2c_1 + c_2\left[3-\hat{\mu}(q)\right]\right\}}{6c_1c_2} \\ &< \frac{\hat{\mu}(q)\hat{v}_H(q)\left\{2c_2 + c_1\left[3-\hat{\mu}(q)\right]\right\}}{6c_1c_2} = WE\left(DC, \frac{c_2}{\hat{\mu}(q)c_1}; c_1, c_2\right), \end{aligned}$$

where the strict inequality follows from $c_1 \geq c_2$ and $3 - \hat{\mu}(q) > 2$. As a result, $\gamma = CD$ would not arise in the optimum.

In summary, fixing $q \in (1/2, 1]$, the expected winner's effort from the contest is maximized by either $(\gamma, \delta) = \left(CC \text{ or } DD, \frac{c_2}{c_1}\right)$ or $(\gamma, \delta) = \left(DC, \frac{c_2}{\hat{\mu}(q)c_1}\right)$. Carrying out the algebra, we have that

$$WE\left(CC, \frac{c_2}{c_1}; c_1, c_2\right) - WE\left(DC, \frac{c_2}{\hat{\mu}(q)c_1}; c_1, c_2\right) = \frac{\left[1-\hat{\mu}(q)\right] \times \left[2(c_1+c_2)\hat{v}_L(q) - c_1\hat{\mu}(q)\hat{v}_H(q)\right]}{6c_1c_2}.$$

It can be verified that $WE\left(CC, \frac{c_2}{c_1}; c_1, c_2\right) > WE\left(DC, \frac{c_2}{\hat{\mu}(q)c_1}; c_1, c_2\right)$ is equivalent to $\hat{\mu}(q)\hat{v}_H(q) <$

$\left(2\frac{c_2}{c_1}+2\right)\hat{v}_L(q)$, which concludes the proof. ■

Proof of Theorem 3

Proof. The proof of part (i) of the theorem closely follows that of Theorem 2(i), and it remains to prove part (ii). It is useful to prove an intermediate result.

Lemma 5 *Suppose that $\gamma = DC$. Fix an arbitrary tuple $(v_H^\pi, v_L^\pi, \mu^\pi)$ that satisfies (2) and let the designer set the scoring bias $\delta > 0$. Then the expected winner's effort from the contest is maximized at $\delta = \frac{c_2}{c_1}$ or $\delta = \frac{c_2}{\mu^\pi c_1}$.*

Proof. The proof closely follows that of Lemma 4 and is omitted for brevity. ■

Following the same steps in the proof of Theorem 2, we can show that for an arbitrary tuple $(v_H^\pi, v_L^\pi, \mu^\pi)$ that satisfies (2), the expected winner's effort from the contest is maximized by $(\delta,\gamma) = (c_2/c_1, CC)$, $(\delta,\gamma) = (c_2/c_1, DD)$, or $(\delta,\gamma) = \left(\frac{c_2}{\mu^\pi c_1}, DC\right)$. The first two contest schemes generate an expected winner's effort of $\frac{c_1+c_2}{3c_1c_2}\bar{v}$, while the third one generates an expected winner's effort of $\frac{\mu^\pi v_H^\pi(2c_2+3c_1-c_1\mu^\pi)}{6c_1c_2}$. The optimization problem thus boils down to

$$\max_{\{v_L^\pi, v_H^\pi, \mu^\pi\}} WE^\pi := \max\left\{\frac{c_1+c_2}{3c_1c_2}\bar{v},\ \frac{\mu^\pi v_H^\pi(2c_2+3c_1-c_1\mu^\pi)}{6c_1c_2}\right\}\ s.t.\ (2).$$

It is straightforward to verify that $\frac{\mu^\pi v_H^\pi(2c_2+3c_1-c_1\mu^\pi)}{6c_1c_2}$ is increasing in $\mu^\pi \in (0,1)$. By (2), we can obtain that

$$\mu^\pi = \frac{\bar{v} - v_L^\pi}{v_H^\pi - v_L^\pi} = 1 - \frac{v_H^\pi - \bar{v}}{v_H^\pi - v_L^\pi},$$

which implies that μ^π is decreasing in v_L^π. Therefore, $v_L^\pi = v_L$ in the optimum. Plugging $v_L^\pi = v_L$ into (2) yields that $v_H^\pi = v_L + \frac{\bar{v}-v_L}{\mu^\pi}$. Replacing (v_L^π, v_H^π) with $(v_L, v_L + \frac{\bar{v}-v_L}{\mu^\pi})$ in WE^π, the above maximization problem can be further simplified as

$$\max_{\mu^\pi\in[\mu,1)} \max\left\{\frac{c_1+c_2}{3c_1c_2}\bar{v}, \mathcal{V}(\mu^\pi)\right\},$$

where

$$\mathcal{V}(\mu^\pi) := \frac{-c_1v_L(\mu^\pi)^2 + \left[(4c_1+2c_2)v_L - c_1\bar{v}\right]\mu^\pi + (3c_1+2c_2)(\bar{v}-v_L)}{6c_1c_2},$$

and the constraint $\mu^\pi \geq \mu$ is due to the constraint $v_H^\pi = v_L + \frac{\bar{v}-v_L}{\mu^\pi} \leq v_H$ imposed in (2).

Note that $\mathcal{V}(1) = \frac{c_1+c_2}{3c_1c_2}\bar{v}$ and $\mathcal{V}(\mu^\pi)$ is quadratic and inverted U-shaped in μ^π. Therefore, $\max_{\mu^\pi\in[\mu,1)}\mathcal{V}(\mu^\pi) > \frac{c_1+c_2}{3c_1c_2}\bar{v}$ if and only if

$$\frac{(4c_1+2c_2)v_L - c_1\bar{v}}{2c_1v_L} < 1 \iff \frac{\bar{v}}{v_L} > 2 + 2\frac{c_2}{c_1}.$$

In this case, $\mu^\pi = \max\left\{\mu, \frac{(4c_1+2c_2)v_L - c_1\bar{v}}{2c_1 v_L}\right\}$ in the optimum.

In summary, if $\frac{\bar{v}}{v_L} > 2 + 2\frac{c_2}{c_1}$, the expected winner's effort is maximized by a contest scheme with $(\delta, \gamma) = \left(\frac{c_2}{\mu^\pi c_1}, DC\right)$ and

$$\pi\big(H|v_H\big) = 1, \pi\big(H|v_L\big) = \begin{cases} 0, & \text{if } \frac{\bar{v}}{v_L} \geq 4 - 2\mu + 2\frac{c_2}{c_1}, \\ \frac{4-2\mu+2\frac{c_2}{c_1}-\frac{\bar{v}}{v_L}}{2(1-\mu)}, & \text{if } 2 + 2\frac{c_2}{c_1} < \frac{\bar{v}}{v_L} < 4 - 2\mu + 2\frac{c_2}{c_1}; \end{cases}$$

otherwise, it is maximized by $(\delta, \gamma) = (c_2/c_1, CC)$ or $(\delta, \gamma) = (c_2/c_1, DD)$ with an arbitrary information structure $\big\{\pi(\cdot|v_H), \pi(\cdot|v_L)\big\}$. This concludes the proof. ■

Proof of Theorem 4

Proof. From the equilibrium characterization established in Propositions 1 and 2, we can obtain the following:

$$ME(CC; c_1, c_2) = \frac{(3c_1^2 + c_2^2)\bar{v}}{6c_1^2 c_2}, \text{ and}$$
$$ME\left(CD, \frac{\hat{\mu}(q)c_2}{c_1}; c_1, c_2\right) = \frac{\left\{3c_1^2 + 3[1-\hat{\mu}(q)]c_1c_2 + [\hat{\mu}(q)]^2 c_2^2\right\}\hat{\mu}(q)\hat{v}_H(q)}{6c_1^2 c_2}.$$

It can be verified that $ME(CC; c_1, c_2) > ME\left(CD, \hat{\mu}(q)c_2/c_1; c_1, c_2\right)$ is equivalent to

$$\frac{\hat{v}_L(q)}{\hat{v}_H(q)} > \frac{\frac{c_2}{c_1}\hat{\mu}(q)\left\{3 - [1+\hat{\mu}(q)]\frac{c_2}{c_1}\right\}}{3 + (\frac{c_2}{c_1})^2},$$

which concludes the proof. ■