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Marriage market dynamics, gender, and the age gap

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Abstract

We present a general discrete choice framework for analysing household formation and dissolution decisions in an equilibrium limited-commitment collective framework that allows for marriage both within and across birth cohorts. Using Panel Study of Income Dynamics and American Community Survey data, we apply our framework to empirically implement a time allocation model with labour market earnings risk, human capital accumulation, home production activities, fertility, and both within- and across-cohort marital matching. Our model replicates the bivariate marriage distribution by age, and explains some of the most salient life-cycle patterns of marriage, divorce, remarriage, and time allocation behaviour. We use our estimated model to quantify the impact of the significant reduction in the gender wage gap since the 1980s on marriage outcomes.

Keywords: Marriage, divorce, collective household models, life-cycle, search and matching, intrahousehold allocation, structural estimation.

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1 Introduction

For many Americans, marriage, and increasingly divorce and remarriage, are important life-course events. There is considerable gender asymmetry in the timing and incidence of these events. In the United States, as indeed is true throughout the world, women marry at a younger age, with marriages in which the husband is older than his wife being more common than both same-age and women-older marriages. Gender differences are even more pronounced in later-age marriages and remarriage. Not only are remarried men more likely than those in a first marriage to have a spouse who is younger, in many cases she is much younger.[1]

These well-known patterns suggest that age, for reasons that we later describe, is an important marriage matching characteristic. As a consequence, age is significant within marriage not only through the usual life-cycle channels, but because spouses of different ages have differential, and potentially gender-asymmetric, desirability in the marriage market. This mechanism has implications for behaviour within the household, including patterns of specialisation and the likelihood of divorce, with both of these varying in economically significant ways with the marital age gap. The primary objective of this paper is to develop a quantitative framework that can account for these empirical patterns, in an environment where the economic value in both singlehood and marriage is micro-founded, and where opportunities in a dynamically evolving marriage market and behaviour within the household are intimately linked.

The methodological framework that we introduce is an *equilibrium* intertemporal limited commitment collective model that allows for marriage both within and across birth cohorts. Intertemporal collective models extend the collective approach to household decision making introduced by Apps and Rees (1988) and Chiappori (1988, 1992) to dynamic settings. In an environment with limited commitment, as considered in Mazzocco (2007) and Voena (2015) among others, married couples cooperate when making decisions but are unable to commit to future allocations of resources. Household decisions are therefore made efficiently, subject to the constraints that both spouses are able to dissolve the relationship and receive their value from outside of the relationship. These outside options, which determine the bargaining weight of each household member,

[1] Lundberg and Pollak (2007) and Stevenson and Wolfers (2007) document how marriage patterns in the United States (including the marriage age gap) have changed over time. For U.S. evidence on age differences in remarriage and over the life-cycle, see, for example, Vera, Berardo and Berardo (1985) and England and McClintock (2009). International evidence on the average age difference in marriage, and how it has evolved over time, is presented United Nations (1990, 2017).

depend on future prospects in the marriage market. They are therefore governed by the entire distribution of potential future spouses from all marriageable cohorts, and in this paper we make explicit that these distributions are endogenously determined in a marriage market equilibrium.[2]

We present a general discrete choice framework for analysing equilibrium intertemporal collective models with limited commitment. We consider an overlapping generations economy where marriage matching is subject to informational frictions: in each period, single individuals meet at most one potential spouse from all marriageable cohorts, observe a marital match quality, and decide whether or not to marry.[3] When married, the marital match quality evolves stochastically, and households make decisions that affect the evolution of state variables and their value both inside and outside of the relationship. The bargaining weight within marriage also evolves as a function of these outside options, adjusting whenever is necessary for the continuation of the relationship. If the household dissolves, then individuals may remarry in the future. In this framework we adopt a convenient within-period timing structure, which together with our persistent-transitory parametrisation of the marital match component, jointly yield an empirically tractable model. Within this general class of model, we characterise theoretical properties of the model and provide a proof of equilibrium existence. We describe methods for computing the model equilibrium, and exploit our explicit equilibrium characterisation in the subsequent estimation procedure.

We apply our equilibrium intertemporal limited commitment framework to explore the age structure of marriages as an equilibrium marriage market phenomenon. While age patterns of marriage are somewhat less studied in the economics literature,[4] the sociology and demography literature (e.g., England and McClintock, 2009) has documented important facts, such as the phenomenon of *age hypergamy* (men marrying women younger than themselves) becoming much more extreme the older men are when they marry. Building on this evidence, we also show important differences in the time allocation behaviour depending on the marital age gap. In particular, the labour supply

[2] While the importance of extending household model to equilibrium environments is well recognised (e.g., Chiappori and Mazzocco, 2017), in the context of life-cycle models this has previously been considered extremely difficult or "infeasible" (Eckstein, Keane and Lifshitz, 2019). An alternative framework which also incorporates a life-cycle in a dynamic marriage market model is presented in Ciscato (2019).

[3] To the best of our knowledge, all existing applications of limited commitment household models studies restrict marriages to be within cohort (or equivalently at a fixed age difference).

[4] Exceptions include Bergstrom and Bagnoli (1993), Siow (1998), Choo and Siow (2006), Coles and Francesconi (2011), Díaz-Giménez and Giolito (2013), Choo (2015), Ríos-Rull, Seitz and Tanaka (2016), Low (2017), and Gershoni and Low (2018).

of married women is lower the older is her husband relative to her, even conditional on characteristics including husband's income. Motivated by these patterns, we present an empirical model with both within- and across-cohort marital matching, and incorporate marital age preferences, labour market earnings risk, human capital accumulation, home production activities, and fertility. Individual characteristics are therefore both directly and indirectly related to age. We structurally estimate our model using data from the American Community Survey (ACS) and Panel Study of Income Dynamics (PSID), and demonstrate that our parsimoniously parametrised model can simultaneously explain some of the most salient facts regarding life-cycle patterns of marriage, divorce, remarriage, and time allocation behaviour.

In our framework, marriage matching patterns and behaviour within the household are intimately linked. One of the most important ways in which the age distribution of marriages has changed over time, is the gradual narrowing of the marriage age gap. This change has been accompanied by a contemporaneous reduction in the gender *wage* gap. Using our estimated equilibrium model we then explore the quantitative relationship between gender wage disparities and both household behaviour and marriage outcomes. We show that the significant increase in women's relative earnings since the 1980s, simultaneously results in increased female employment, reduced male employment, an increase in the age-of-first marriage for women, and a reduction in the marital age gap. Overall, we attribute a third of the reduction in the marital age gap to the decline of the gender wage gap.

Related Literature. Our analysis firstly relates to the existing literature that has developed intertemporal household models with limited commitment. These models, cast in non-equilibrium settings, have emerged as a leading paradigm in the intertemporal analysis of household decisions, and have been used to study a range of different problems. These include the shift from mutual consent to unilateral divorce laws (Voena, 2015), the gender gap in college graduation (Bronson, 2015), the difference between cohabitation and marriage (Gemici and Laufer, 2014), an evaluation of the U.S. Earned Income Tax Credit programme (Mazzocco, Ruiz and Yamaguchi, 2013), the impact of time limits in welfare receipt (Low et al., 2018), and a comparison of alternative systems of family income taxation (Bronson and Mazzocco, 2018).

Second, our analysis relates to quantitative equilibrium marriage matching models that have been developed using alternative frameworks. Choo and Siow (2006) present a tractable frictionless marriage market model with transferable utility. Their empirical matching framework has subsequently been extended to incorporate static collective time allocation models in, for example, Choo and Seitz (2013) and Gayle and Shephard (2019). While Chiappori, Costa Dias and Meghir (2018) and Reynoso (2018) also consider life-cycle collective models (with full and limited commitment respectively), marriage matching occurs at an initial stage with the market clearing at a single point in time. A fully dynamic overlapping-generations version of Choo and Siow (2006) with full commitment, transferable utility, and exogenous divorce is developed in Choo (2015), which estimates the gains from marriage by age. In contrast, ours is a model where utility is imperfectly transferable, divorce is endogenous, and in which the marriage market is subject to search frictions. As such, it also relates to the two-sided search-and-matching model in Díaz-Giménez and Giolito (2013) which emphasises the role of differential fecundity in explaining marriage age patterns, the stationary marital search model in Goussé, Jacquemet and Robin (2017),[5] and the quantitative macro-economic literature that includes Aiyagari, Greenwood and Guner (2000), Caucutt, Guner and Knowles (2002), Chade and Ventura (2002), Greenwood, Guner and Knowles (2003), Guvenen and Rendall (2015), and Greenwood et al. (2016).

Most related is the recent marital search model developed in parallel work by Ciscato (2019), which presents a tractable extension of the Goussé, Jacquemet and Robin (2017) framework to a life-cycle setting, and examines how changes in the wage structure relate to the decline of marriage in the United States. As in this paper, Ciscato (2019) incorporates across-cohort marriage matching, but in contrast considers an environment with transferable utility and no commitment (rather than limited commitment).

The remainder of the paper proceeds as follows. In Section 2 we present a general discrete choice framework for our equilibrium intertemporal limited commitment collective model. Here we detail the behaviour of both single and married households, characterise the stationary equilibrium of the economy, and present our main theoretical results. In Section 3 we describe the application of our general model and present our empirical specification, while Section 4 describes the associated parameter estimates and model fit. Section 5 then studies the impact that reductions in the gender wage gap have on outcomes including the age structure of marriages. Finally, Section 6 concludes. Computational details and theoretical proofs are presented in the paper appendix.

[5]Other recent microeconometric studies that incorporate marital search in equilibrium frameworks include Wong (2003), Seitz (2009), Flabbi and Flinn (2015), and Beauchamp et al. (2018).

2 An equilibrium limited-commitment model

2.1 Environment and timing

We consider an overlapping generations economy, in which time is discrete and the time horizon is infinite. Every period a new generation (comprising an exogenous measure of women and men) is born, with each generation living for $A < \infty$ periods.[6] Women (men) are characterised by their age a_f (a_m) and their current state vector $\boldsymbol{\omega}_f$ ($\boldsymbol{\omega}_m$), whose support is taken to be discrete and finite. As we restrict our attention to stationary equilibria, we do not index any quantity by calendar time.

In what follows it is convenient to adopt a within period timing structure. The *start-of-period* is defined prior to the opening of the marriage market. All surviving individuals enter a new period with an updated state vector (which evolves according to some law-of-motion described below) and are either single or married, with marriage pairings occurring both within ($a_f = a_m$) and across birth cohorts ($a_f \neq a_m$). All newly born enter the pool of single individuals, as do individuals with non-surviving spouses. At an interim stage, spousal search, matching, renegotiation, and divorce (under a unilateral divorce regime) take place.[7] Single women and men meet each other according to some endogenous meeting probabilities that depend upon the *equilibrium* measure of single individuals. The decision to marry then depends on how the value of marriage (including any match-specific component that evolves throughout marriage) compares to the outside options of both individuals. Within marriage household decisions are made efficiently, as in Apps and Rees (1988) and Chiappori (1988, 1992), with the household Pareto weight (which is a continuous state variable) determining the chosen allocation.

If marriage takes place, it follows that the Pareto weight must be such that the marriage participation constraints are satisfied for both spouses. That is, the value within marriage for both husband and wife must exceed their respective values from singlehood. Importantly, while married couples cooperate when making decisions, we assume that they are unable to commit to future allocations of resources. As in Mazzocco (2007), Mazzocco, Ruiz and Yamaguchi (2013), Gemici and Laufer (2014), Voena (2015), Bron-

[6] The framework generalises to incorporate exogenous mortality risk, and we include this is our empirical application in Section 3. We abstract from this (and other) considerations in our presentation of the model as they add little to the formal analysis, but require additional notation.

[7] A number of studies have examined the impact of the shift from mutual consent to unilateral divorce laws in the United States. These include Chiappori, Fortin and Lacroix (2002), Friedberg (1998), Wolfers (2006), Stevenson (2007), Voena (2015), Fernández and Wong (2017), and Reynoso (2018).

son (2015), Low et al. (2018), among others, we therefore consider a limited-commitment intertemporal collective model.[8] Amongst continuing marriages, the Pareto weight remains unchanged if the marriage participation constraints for both the wife and her husband continue to be satisfied. Otherwise, there is renegotiation, with the Pareto weight adjusting by the smallest amount such they are both satisfied. If no Pareto weight exists such that both participation constraints can be simultaneously satisfied, then the couple divorces. Divorced individuals may remarry in future periods.

The *end-of-period* is then defined following spousal search, matching, renegotiation, and divorce. At this point, further uncertainty may be realised,[9] and household allocation decisions are made. These household decisions influence the future evolution of the state vectors. All individuals have the common discount factor $\beta \in [0, 1]$.

A central feature of the environment that we consider is that the value both within and outside of marriage depends upon future prospects in the marriage market. These prospects are governed by the entire distribution of potential future spouses from all marriageable cohorts. The equilibrium limited-commitment intertemporal collective framework that we develop here makes explicit that these distributions are determined in equilibrium. Equilibrium consistency requires that all individuals and households behave optimally at the end-of-period allocation stage, and in their marriage formation/dissolution decisions, given the marriage market meeting probabilities. Moreover, this behaviour then induces stationary distributions of single and matched individuals that are consistent with these meeting probabilities.

2.2 End-of-period decision problem

Following marriage, divorce, and renegotiation, a household decision problem is solved. We consider a general discrete choice formulation where the decision problem is represented as the choice over a finite set of alternatives, and where each choice is associated

[8] Chiappori and Mazzocco (2017) provide a recent survey of this literature. Using U.S. data, Mazzocco (2007) tests the consistency of intertemporal household allocations with alternative models of commitment. While the full-commitment intertemporal model (which assumes that the couple can commit ex-ante to future allocations, with the Pareto weight fixed from the time of marriage) is rejected, the limited-commitment intertemporal model (where such commitment is not possible and the Pareto weight evolves given outside options) is not rejected. See, also, the recent contribution in Lise and Yamada (Forthcoming), whose estimates also favour limited commitment within the household.

[9] We do not allow further renegotiation of the Pareto weight at this stage. This implies that the none of the threshold values that we later derive when characterising marriage/divorce decisions, and the evolution of the Pareto weight, depend upon the *realisation* of this end-of-period uncertainty.

with some additive alternative-specific error that are only realised at the end-of-period.[10]

2.2.1 Single women

Consider a single woman and let $\mathcal{T}_f = \{1, \ldots, T\}$ be the index representation of her choice set. Associated with each alternative $t_f \in \mathcal{T}_f$ is the period indirect utility function $v_f^S(t_f; a_f, \boldsymbol{\omega}_f)$, which is bounded, and an additively separable utility shock ε_{t_f} with $\boldsymbol{\varepsilon}_{t_f} \in \mathbb{R}^T$. Preferences are intertemporally separable, with the woman's *alternative-specific value function* consisting of two terms: the per-period utility flow and her discounted continuation pay-off.[11] It obeys the Bellman (Bellman, 1957) equation

$$
\begin{aligned}
V_f^S(t_f; a_f, \boldsymbol{\omega}_f) + \varepsilon_{t_f} \equiv\ & v_f^S(t_f; a_f, \boldsymbol{\omega}_f) + \varepsilon_{t_f} \\
& + \beta \sum_{\boldsymbol{\omega}'_f} E\widetilde{V}_f^S(a_f + 1, \boldsymbol{\omega}'_f)\pi_f(\boldsymbol{\omega}'_f | a_f + 1, \boldsymbol{\omega}_f, t_f),
\end{aligned} \tag{1}
$$

where $E\widetilde{V}_f^S(a_f + 1, \boldsymbol{\omega}'_f)$ corresponds to the *start-of-period* expected value from being single at age $a_f + 1$ and with state vector $\boldsymbol{\omega}'_f$. (As a matter of convention, we use a tilde to denote start-of-period objects.) Recall that start-of-period objects are defined prior to marital search and matching, with this expected value therefore reflecting expected marriage market prospects. The evolution of her state vector is described by the Markov state transition matrix $\pi_f(\boldsymbol{\omega}'_f | a_f + 1, \boldsymbol{\omega}_f, t_f)$. The solution to the allocation problem is given by

$$
t_f^*(a_f, \boldsymbol{\omega}_f, \boldsymbol{\varepsilon}_{t_f}) = \arg\max_{t_f} \{V_f^S(t_f; a_f, \boldsymbol{\omega}_f) + \varepsilon_{t_f}\}. \tag{2}
$$

We define the *end-of-period* expected value function after marital search and matching, but prior to the realisation of the additive utility shocks. Under the maintained assumption that these random utility shocks are independent and identically distributed (i.i.d.) Type-I extreme value errors, and with the state transition matrix exhibiting conditional independence, it follows from well known results (e.g., McFadden, 1978) that the

[10] Note that our framework accommodates continuous choices that have been optimised over conditional on each discrete alternative, provided that such continuous choice variables do not enter the state variable transition matrix.

[11] The assumption of additively separable utility and a choice-specific scalar unobservable component, as in Rust (1987), is a very convenient and common assumption in the dynamic discrete choice literature. Alternatives to additively separability are discussed in Keane, Todd and Wolpin (2011).

end-of-period expected value function is given by

$$EV_f^S(a_f, \boldsymbol{\omega}_f) \equiv \mathbb{E}[V_f^S(t_f^*(a_f, \boldsymbol{\omega}_f, \boldsymbol{\varepsilon}_{t_f}); a_f, \boldsymbol{\omega}_f) + \varepsilon_{t_f} | a_f, \boldsymbol{\omega}_f]$$
$$= \sigma_\varepsilon \gamma + \sigma_\varepsilon \log \left[\sum\nolimits_{t_f} \exp\left(V_f^S(t_f; a_f, \boldsymbol{\omega}_f)/\sigma_\varepsilon \right) \right], \tag{3}$$

where $\sigma_\varepsilon > 0$ is the Type-I extreme value scale parameter, $\gamma \approx 0.5772$ is the Euler–Mascheroni constant, and where the expectation is taken over the realisations of the alternative-specific utility shocks $\boldsymbol{\varepsilon}_{t_f}$. Finally, we denote the conditional choice probability for alternative t_f being chosen by a single $(a_f, \boldsymbol{\omega}_f)$-woman as $\mathbb{P}_f^S(t_f; a_f, \boldsymbol{\omega}_f) = \exp(V_f^S(t_f; a_f, \boldsymbol{\omega}_f)/\sigma_\varepsilon) / \sum_{t'_f} \exp(V_f^S(t'_f; a_f, \boldsymbol{\omega}_f)/\sigma_\varepsilon)$. The end-of-period allocation problem for single men (and the associated value functions and conditional choice probabilities) are described symmetrically.

2.2.2 Married couples

In addition to being characterised by their ages $\mathbf{a} = [a_f, a_m]$ and discrete states $\boldsymbol{\omega} = [\boldsymbol{\omega}_f, \boldsymbol{\omega}_m]$, married couples are also characterised by their continuous household Pareto weight and marital match quality. The Pareto weight, denoted $\lambda \in [0, 1]$, is fixed at the time of the end-of-period decision process, and determines how much weight is given to the woman when the household collectively determines the allocation. The marital match component consists of a persistent distributional parameter ξ (which has discrete and finite support and evolves throughout the duration of the marriage), and a continuously distributed distributed idiosyncratic component denoted θ. We make the following assumption:

Assumption 1. *The period utility function is additively separable in the idiosynctatic marital match component θ, which is common to both spouses. It is continuously distributed, with full support on the real line, and with the cumulative distribution function H_ξ.*

We refer to θ as the current period match quality. As will soon become clear, this persistent-transitory characterisation of the marital match quality is convenient as it will imply the existence of various θ-threshold values that are useful when characterising both value functions and the equilibrium of the marriage market. Moreover, we rely upon this parametrisation in establishing identification.

The discrete choice set for a married couple is given by $\mathcal{T} = \mathcal{T}_f \times \mathcal{T}_m$. Conditional on each joint alternative $\mathbf{t} \in \mathcal{T}$, we assume that couples are able to transfer current period utility, albeit imperfectly,[12] such that the indirect utility functions within marriage, $v_f(\mathbf{t};\mathbf{a},\boldsymbol{\omega},\lambda)$ and $v_m(\mathbf{t};\mathbf{a},\boldsymbol{\omega},\lambda)$, also depend upon the household Pareto weight. We assume that these indirect utility functions satisfy the following properties:

Assumption 2. *The indirect utility functions $v_f(\mathbf{t};\mathbf{a},\boldsymbol{\omega},\lambda)$ and $v_m(\mathbf{t};\mathbf{a},\boldsymbol{\omega},\lambda)$ are continuously differentiable on $\lambda \in (0,1)$, with $\partial v_f(\mathbf{t};\mathbf{a},\boldsymbol{\omega},\lambda)/\partial\lambda > 0$ and $\partial v_m(\mathbf{t};\mathbf{a},\boldsymbol{\omega},\lambda)/\partial\lambda < 0$.*

Assumption 3. *Utility transfers are unbounded from below and bounded from above. That is, $\lim_{\lambda\to 0} v_f(\mathbf{t};\mathbf{a},\boldsymbol{\omega},\lambda) = \lim_{\lambda\to 1} v_m(\mathbf{t};\mathbf{a},\boldsymbol{\omega},\lambda) = -\infty$ and $(v_f, v_m)\circ(\mathbf{t};\mathbf{a},\boldsymbol{\omega},\lambda) < \infty$ for all λ.*

These assumptions will hold under suitable conditions on the household utility possibility frontier.[13] For reasons of tractability that will become clear below, we additionally assume that associated with each joint alternative are additive utility shocks $\varepsilon_{\mathbf{t}}$ that are public in the household, with $\boldsymbol{\varepsilon}_{\mathbf{t}} \in \mathbb{R}^{T\times T}$. The choice-specific value function for a married woman is defined as

$$\begin{aligned} V_f(\mathbf{t};\mathbf{a},\boldsymbol{\omega},\xi,\lambda) + \theta + \varepsilon_{\mathbf{t}} = {} & v_f(\mathbf{t};\mathbf{a},\boldsymbol{\omega},\lambda) + \theta + \varepsilon_{\mathbf{t}} \\ & + \beta\sum_{\boldsymbol{\omega}'}\sum_{\xi'} E\widetilde{V}_f(\mathbf{a}+\mathbf{1},\boldsymbol{\omega}',\xi',\lambda)b(\xi'|\xi)\pi(\boldsymbol{\omega}'|\mathbf{a}+1,\boldsymbol{\omega},\mathbf{t}), \end{aligned} \tag{4}$$

where $E\widetilde{V}_f(\mathbf{a}+\mathbf{1},\boldsymbol{\omega}',\xi',\lambda)$ is the start-of-period expected value function for a married woman.[14] As this start-of-period expected value is defined prior to the opening of the marriage market, it reflects uncertainty in the idiosyncratic match quality realisations, and therefore the possibility of divorce or renegotiation of the household Pareto weight. The evolution of household states is described by the state transition matrix $\pi(\boldsymbol{\omega}'|\mathbf{a}+$

[12] See Galichon, Kominers and Weber (Forthcoming) for a general framework for analysing static frictionless matching models with imperfectly transferable utility.

[13] The possibility frontier is defined over period utilities: if the female gets period utility U_f, then the male gets flow utility $U_m(U_f;\mathbf{t},\mathbf{a},\boldsymbol{\omega})$. It is sufficient to assume that U_m is twice continuously differentiable, with $U_m'' < 0$ so that utility is imperfectly transferable across spouses. Moreover, utility is bounded from above, unbounded from below and becomes arbitrarily hard to transfer: $\lim_{U_f\to-\infty} U_m' = 0$ and $\lim_{U_m\to-\infty} U_m' = -\infty$. The optimization for a couple with Pareto weight λ is then

$$\left\{v_f(\mathbf{t};\mathbf{a},\boldsymbol{\omega},\lambda), v_m(\mathbf{t};\mathbf{a},\boldsymbol{\omega},\lambda)\right\} = \underset{U_f,U_m\in\mathbb{R}}{\arg\max}\, \lambda U_f + (1-\lambda)U_m(U_f;\mathbf{t},\mathbf{a},\omega).$$

Our assumptions here imply that the first order condition with respect to U_f is both necessary and sufficient, with $v_f(\mathbf{t};\mathbf{a},\omega,\lambda) = U_m'^{-1}(-\lambda/(1-\lambda);\mathbf{t},\mathbf{a},\boldsymbol{\omega})$ and $v_m(\mathbf{t};\mathbf{a},\boldsymbol{\omega},\lambda) = U_m(v_f(\mathbf{t};\mathbf{a},\boldsymbol{\omega},\lambda);\mathbf{t},\mathbf{a},\boldsymbol{\omega})$. There exists a solution to this for all $\lambda \in (0,1)$ since the range of U_m' is $(0,-\infty)$.

[14] For women with a non-surviving spouse we define $E\widetilde{V}_f([a_f+1, A+1],\boldsymbol{\omega}',\xi',\lambda) = E\widetilde{V}_f^S(a_f+1,\boldsymbol{\omega}_f')$ for all $(\boldsymbol{\omega}_m',\xi',\lambda)$.

$1, \boldsymbol{\omega}, \mathbf{t})$ which depends on household choices, while the evolution of the persistent marital quality component is similarly described by $b(\xi'|\xi)$. The choice-specific value function for a married man is defined symmetrically.

The *household choice-specific value function* is defined as the Pareto-weighted sum of the wife's and husband's choice specific value functions

$$V_{fm}(\mathbf{t}; \mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) + \theta + \varepsilon_{\mathbf{t}} = \lambda V_f(\mathbf{t}; \mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) + (1-\lambda) V_m(\mathbf{t}; \mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) + \theta + \varepsilon_{\mathbf{t}},$$

which when maximised over the set of alternatives yields the solution to the household allocation problem

$$\mathbf{t}^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda, \boldsymbol{\varepsilon}_{\mathbf{t}}) = \arg\max_{\mathbf{t}} \{V_{fm}(\mathbf{t}; \mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) + \varepsilon_{\mathbf{t}}\}. \tag{5}$$

If the household's alternative-specific utility shocks are Type-I extreme value with scale σ_ε then the *end-of-period* expected value for the wife can be shown to be given by

$$\begin{aligned} EV_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) &\equiv \mathbb{E}[V_f(\mathbf{t}^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda, \boldsymbol{\varepsilon}_{\mathbf{t}}); \mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) | \mathbf{a}, \boldsymbol{\omega}, \xi, \lambda] \\ &= \sigma_\varepsilon \gamma + \textstyle\sum_{\mathbf{t}} \mathbb{P}(\mathbf{t}; \mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) \cdot \left[V_f(\mathbf{t}; \mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) - \sigma_\varepsilon \log[\mathbb{P}(\mathbf{t}; \mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)]\right], \end{aligned} \tag{6}$$

where $\mathbb{P}(\mathbf{t}; \mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) = \exp(V_{fm}(\mathbf{t}; \mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)/\sigma_\varepsilon) / \sum_{\mathbf{t}'} \exp(V_{fm}(\mathbf{t}; \mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)/\sigma_\varepsilon)$ defines the conditional choice probability for a type-$(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$ married couple.[15] The end-of-period expected value function for the husband is defined symmetrically, while important properties of these end-of-period expected value functions are provided in Lemma 1.

Lemma 1. *The wife's end-of-period value function $EV_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$ is continuously differentiable with respect to the Pareto weight $\lambda \in (0,1)$, with $\partial EV_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)/\partial\lambda > 0$. The husband's end-of-period value function $EV_m(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$ is continuously differentiable with respect to the Pareto weight $\lambda \in (0,1)$, with $\partial EV_m(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)/\partial\lambda < 0$.*

Proof of Lemma 1. See Appendix A.1. □

[15] This follows from the result that the distribution of Type-I extreme value errors conditional on a particular alternative being optimal is also Type-I extreme value, with a common scale σ_ε parameter and the shifted location parameter, $-\sigma_\varepsilon \log \mathbb{P}(\mathbf{t}|\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$. An alternative representation is given by

$$\begin{aligned} EV_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) &= \sigma_\varepsilon \gamma + \sigma_\varepsilon \log\left[\textstyle\sum_{\mathbf{t}} \exp\left(V_{fm}(\mathbf{t}; \mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)/\sigma_\varepsilon\right)\right] \\ &\quad + (1-\lambda) \textstyle\sum_{\mathbf{t}} \mathbb{P}(\mathbf{t}; \mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) \cdot \left[V_f(\mathbf{t}; \mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) - V_m(\mathbf{t}; \mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)\right], \end{aligned}$$

such that the end-of-period expected value function is equal to the sum of the expected household value function plus an individual expectation adjustment term.

2.3 Marriage and the start-of-period decision problem

Individuals enter every period with a given marital status. At an interim stage, spousal search, matching, renegotiation, and divorce take place. We now describe this stage. First, we characterise marriage and divorce decisions. Second, we show how the Pareto weight evolves within a marriage. Third, we define a marriage matching function and construct meeting probabilities. Fourth, we use the behaviour at this interim stage to derive expressions for the start-of-period expected value functions.

2.3.1 Reservation match values

A $(a_f, \boldsymbol{\omega}_f)$-woman and $(a_m, \boldsymbol{\omega}_m)$-man get married whenever the current period match quality θ exceeds the reservation match value $\underline{\theta}(\mathbf{a}, \boldsymbol{\omega}, \xi)$, which we define as

$$\begin{aligned}\underline{\theta}(\mathbf{a}, \boldsymbol{\omega}, \xi) = \min\{\theta : \exists \lambda \in [0,1] \text{ s.t. } & EV_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) + \theta \geq EV_f^S(a_f, \boldsymbol{\omega}_f) \\ \wedge \;\; & EV_m(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) + \theta \geq EV_m^S(a_m, \boldsymbol{\omega}_m)\}. \end{aligned} \tag{7}$$

That is, the reservation match value $\underline{\theta}(\mathbf{a}, \boldsymbol{\omega}, \xi)$ defines the lowest value of θ for which there exists a household Pareto weight λ such that both spouses prefer to be married over being single. By the same token, and in the absence of any divorce costs, when $\theta < \underline{\theta}(\mathbf{a}, \boldsymbol{\omega}, \xi)$ an existing type-$(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$ marriage does not provide any marital surplus and will therefore dissolve.[16] Under Assumption 3, the end-of-period expected value for any individual can be made arbitrarily low through suitable choice of Pareto weight, i.e., $\lim_{\lambda \to 0} EV_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) = \lim_{\lambda \to 1} EV_m(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) = -\infty$. This implies that the participation constraints of both spouses will simultaneously bind at the reservation match value $\underline{\theta}(\mathbf{a}, \boldsymbol{\omega}, \xi)$ and that we must have $\lambda \in (0,1)$ in any marriage.[17]

2.3.2 Evolution of the Pareto weights

The household Pareto weight determines an intra-household allocation among the set of allocations on the Pareto frontier. Given an initial start-of-period weight λ, we follow the limited commitment literature by presenting a theory that describes how the Pareto

[16] We omit divorce costs from the main presentation to avoid introducing more cumbersome notation. See Section 3 for a discussion of this extension. While considering divorce as the outside option is common in intertemporal collective models, other papers have considered alternative outside option definitions, such as non-cooperative behaviour (e.g., Lundberg and Pollak, 1993 and Del Boca and Flinn, 2012).

[17] In Appendix A.3 we present a stronger result and show that all Pareto weights must lie in a closed interval.

weight evolves given outside options. To proceed, we define $\theta_f^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$ as the value of θ such that the participation constraint of a (a_f, ω_f)-woman just binds in a type-$(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$ marriage. That is

$$\theta_f^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) = EV_f^S(a_f, \omega_f) - EV_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda),$$

and we similarly define $\theta_m^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$ as the value of θ such that the participation constraint of the man binds in a type-$(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$ marriage. Before we proceed, we provide the following Lemma.

Lemma 2. *If $\underline{\theta}(\mathbf{a}, \boldsymbol{\omega}, \xi) < \theta_f^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$ then $\theta_m^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) \leq \underline{\theta}(\mathbf{a}, \boldsymbol{\omega}, \xi)$. Conversely, if $\underline{\theta}(\mathbf{a}, \boldsymbol{\omega}, \xi) < \theta_m^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$ then $\theta_f^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) \leq \underline{\theta}(\mathbf{a}, \boldsymbol{\omega}, \xi)$.*

Proof of Lemma 2. See Appendix A.2. □

We now describe the evolution of the Pareto weight for different realisations of the current period match quality. Suppose first that $\theta \geq \max\{\theta_f^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda), \theta_m^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)\}$. This means that the match quality is sufficiently high such that the participation constraint for each spouse is satisfied at λ. In this event, the Pareto weight is assumed to remain unchanged. Next, suppose that $\underline{\theta}(\mathbf{a}, \boldsymbol{\omega}, \xi) \leq \theta < \theta_f^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$. In this case the woman triggers the renegotiation of the Pareto weight. Following the limited commitment literature, e.g., Kocherlakota (1996) and Ligon, Thomas and Worrall (2002), we assume that the Pareto weight will adjust just enough to make the woman indifferent between being married at the renegotiated Pareto weight, which we denote $\lambda_f^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \theta)$, and being single.[18] Conversely, suppose that the current period match quality satisfies $\underline{\theta}(\mathbf{a}, \boldsymbol{\omega}, \xi) \leq \theta < \theta_m^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$, meaning that the man's participation constraint is violated at λ. In this case the Pareto weight will be renegotiated downwards to a new weight $\lambda_m^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \theta)$ such that man's participation constraint now binds. Note that our assumption of limited commitment within the household implies that while risk-sharing is present, it is imperfect.

The Pareto weight transition function, which we note is Markovian, can therefore be

[18]For $\underline{\theta}(\mathbf{a}, \boldsymbol{\omega}, \xi) \leq \theta \leq \theta_f^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$ it follows from Lemma 1 that the renegotiated weight $\lambda_f^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \theta)$ is uniquely determined by $EV_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda_f^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \theta)) + \theta = EV_f^S(a_f, \omega_f)$. Note that we assume that the process of renegotiation itself is costless. As this adjustment procedure only moves the Pareto weight by the minimal amount to maintain marriage, the deviation from the ex-ante efficient allocation is minimised.

summarised by the function $\lambda^*(\mathbf{a},\boldsymbol{\omega},\xi,\theta,\lambda)$ which we define as

$$\lambda^*(\mathbf{a},\boldsymbol{\omega},\xi,\theta,\lambda)=\begin{cases}\lambda & \text{if } \theta \geq \max\{\theta_f^*(\mathbf{a},\boldsymbol{\omega},\xi,\lambda),\theta_m^*(\mathbf{a},\boldsymbol{\omega},\xi,\lambda)\},\\ \lambda_f^*(\mathbf{a},\boldsymbol{\omega},\xi,\theta) & \text{if } \underline{\theta}(\mathbf{a},\boldsymbol{\omega},\xi)\leq\theta<\theta_f^*(\mathbf{a},\boldsymbol{\omega},\xi,\lambda),\\ \lambda_m^*(\mathbf{a},\boldsymbol{\omega},\xi,\theta) & \text{if } \underline{\theta}(\mathbf{a},\boldsymbol{\omega},\xi)\leq\theta<\theta_m^*(\mathbf{a},\boldsymbol{\omega},\xi,\lambda),\\ \emptyset & \text{if } \theta<\underline{\theta}(\mathbf{a},\boldsymbol{\omega},\xi).\end{cases} \tag{8}$$

2.3.3 Meeting probabilities

The marriage market is characterised by search frictions. We denote the respective probabilities that a $(a_f,\boldsymbol{\omega}_f)$-woman meets a $(a_m,\boldsymbol{\omega}_m)$-man and vice versa by $\eta_f(\mathbf{a},\boldsymbol{\omega})$ and $\eta_m(\mathbf{a},\boldsymbol{\omega})$. These meeting probabilities are endogenous objects that depend both upon the availability of single individuals and an efficiency parameter that determines the extent to which certain types of meetings may be more or less likely. In parametrizing the technology we use $\overline{\boldsymbol{\omega}}_f$ and $\overline{\boldsymbol{\omega}}_m$ to denote the respective subset of state variables that are fixed over the life-cycle. Letting $\gamma(\mathbf{a},\overline{\boldsymbol{\omega}})\geq 0$ we then define[19]

$$\eta_f(\mathbf{a},\boldsymbol{\omega})=\frac{\gamma(\mathbf{a},\overline{\boldsymbol{\omega}})\widetilde{g}_m^S(a_m,\boldsymbol{\omega}_m)}{\sum_{a'_m}\sum_{\boldsymbol{\omega}'_m}\gamma([a_f,a'_m],[\overline{\boldsymbol{\omega}}_f,\overline{\boldsymbol{\omega}}'_m])\mu_m(a'_m,\boldsymbol{\omega}'_m)} \tag{9a}$$

$$\eta_m(\mathbf{a},\boldsymbol{\omega})=\frac{\gamma(\mathbf{a},\overline{\boldsymbol{\omega}})\widetilde{g}_f^S(a_f,\boldsymbol{\omega}_f)}{\sum_{a'_f}\sum_{\boldsymbol{\omega}'_f}\gamma([a'_f,a_m],[\overline{\boldsymbol{\omega}}'_f,\overline{\boldsymbol{\omega}}_m])\mu_f(a'_f,\boldsymbol{\omega}'_f)}, \tag{9b}$$

where $\widetilde{g}_m^S(a_m,\boldsymbol{\omega}_m)$ is the *start-of-period* measure of single $(a_m,\boldsymbol{\omega}_m)$-men that we characterise below, and $\mu_m(a_m,\boldsymbol{\omega}_m)$ is the total measure (single and married) of such men. Similarly, $\widetilde{g}_f^S(a_f,\boldsymbol{\omega}_f)$ is the start-of-period measure of single $(a_f,\boldsymbol{\omega}_f)$-women and $\mu_f(a_f,\boldsymbol{\omega}_f)$ is the total measure of such women. For consistency we require that

$$\sum_{a'_f}\sum_{\boldsymbol{\omega}'_f}\gamma([a'_f,a_m],[\overline{\boldsymbol{\omega}}'_f,\overline{\boldsymbol{\omega}}_m])\mu_f(a'_f,\boldsymbol{\omega}'_f)=\sum_{a'_m}\sum_{\boldsymbol{\omega}'_m}\gamma([a_f,a'_m],[\overline{\boldsymbol{\omega}}_f,\overline{\boldsymbol{\omega}}'_m])\mu_m(a'_m,\boldsymbol{\omega}'_m)$$

for all a_f, a_m, $\boldsymbol{\omega}_f$, and $\boldsymbol{\omega}_m$, so that $\widetilde{g}_m^S(a_m,\boldsymbol{\omega}_m)\eta_m(\mathbf{a},\boldsymbol{\omega})=\widetilde{g}_f^S(a_f,\boldsymbol{\omega}_f)\eta_f(\mathbf{a},\boldsymbol{\omega})$. That is, the measure of single $(a_m,\boldsymbol{\omega}_m)$-men who meet single $(a_f,\boldsymbol{\omega}_f)$-women is equal to the measure

[19] For consistency we require that men meet women at the same rate as women meet men. With a general $\gamma(\mathbf{a},\boldsymbol{\omega})$ specification this requirement is difficult to enforce out of steady state as the measure of $(\mathbf{a},\boldsymbol{\omega})$ types is endogenously determined by equilibrium choices. Restricting the efficiency parameter to depend only on exogenous individual characteristics circumvents this complication.

of single $(a_f, \boldsymbol{\omega}_f)$-women who meet single $(a_m, \boldsymbol{\omega}_m)$-men.

2.3.4 Start-of-period expected value functions

The *start-of-period* expected value functions for married women and men are defined after the state vectors $\boldsymbol{\omega}$ are updated, and the new persistent marital quality parameter ξ is drawn, but before the current period marriage quality θ is realised. The expectation taken over θ therefore reflects any marriage formation/dissolution decisions, and any renegotiation of the start-of-period Pareto weight.

Consider the start-of-period expected value function for a married woman. If $\theta < \underline{\theta}(\mathbf{a}, \boldsymbol{\omega}, \xi)$ then the marriage can not be formed or continued as the surplus is negative for all Pareto weights. In this event, the woman must wait a period before re-entering the marriage market and therefore receives her value as a single, $EV_f^S(a_f, \boldsymbol{\omega}_f)$. Conversely, if $\theta \geq \underline{\theta}(\mathbf{a}, \boldsymbol{\omega}, \xi)$ the marriage is formed with the Pareto weight $\lambda^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \theta, \lambda)$, with this function reflecting any possible renegotiation of the weight given the outside opportunities of both the woman and the man. It therefore follows that the start-of-period expected value function is given by

$$
\begin{aligned}
E\widetilde{V}_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) &= H_\xi(\underline{\theta}(\mathbf{a}, \boldsymbol{\omega}, \xi)) EV_f^S(a_f, \boldsymbol{\omega}_f) \\
&\quad + \int_{\underline{\theta}(\mathbf{a}, \boldsymbol{\omega}, \xi)} \left[EV_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \theta, \lambda)) + \theta \right] \mathrm{d}H_\xi(\theta).
\end{aligned} \tag{10}
$$

Recall that our definition of $\lambda^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \theta, \lambda)$ reflects cases when both participation constraints are satisfied at λ (in which case λ^* reduces to the identity map), and also when the weight is renogotiated. Properties of the start-of-period expected value functions within marriage are described in Lemma 3.

Lemma 3. *The wife's start-of-period value function $E\widetilde{V}_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$ is continuously differentiable with respect to the Pareto weight $\lambda \in (0, 1)$, with $\partial E\widetilde{V}_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)/\partial\lambda > 0$. The husbands's end-of-period value function $E\widetilde{V}_m(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$ is continuously differentiable with respect to the Pareto weight $\lambda \in (0, 1)$, with $\partial E\widetilde{V}_m(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)/\partial\lambda < 0$.*

Proof of Lemma 3. See Appendix A.1. $\square$

Now consider a woman's expected value from being single before search and matching in the marriage market occurs. For new matches, the persistent marital component ξ has the probability mass function b_0, and we additionally assume the existence of an initial Pareto weight $\lambda_0 \in [0,1]$ at which potential marriages are first evaluated.[20] The start-of-period expected value for a single $(a_f, \boldsymbol{\omega}_f)$-woman is given by

$$\begin{aligned} E\widetilde{V}_f^S(a_f, \boldsymbol{\omega}_f) &= \sum_{a_m}\sum_{\boldsymbol{\omega}_m}\sum_{\xi} \eta_f(\mathbf{a}, \boldsymbol{\omega}) E\widetilde{V}_f(\mathbf{a}, \boldsymbol{\omega}, \lambda_0, \xi) b_0(\xi) \\ &\quad + \left(1 - \sum_{a_m}\sum_{\boldsymbol{\omega}_m} \eta_f(\mathbf{a}, \boldsymbol{\omega})\right) EV_f^S(a_f, \boldsymbol{\omega}_f), \end{aligned} \tag{11}$$

and where we recall that $\eta_f(\mathbf{a}, \boldsymbol{\omega})$ is the probability that a $(a_f, \boldsymbol{\omega}_f)$-woman meets a $(a_m, \boldsymbol{\omega}_m)$-man. The first line of equation (11) reflects the expected value associated with the different types of men that a given woman may meet. This expectation is defined prior to the realisation of θ and therefore reflects any renegotiation of the Pareto weight from λ_0, and that meetings may not result in marriage. The second line of the equation corresponds to the case when the woman does not meet any single man in the marriage market and therefore receives her end-of-period expected value from singlehood.

Finally, before proceeding to characterise the steady-state equilibrium of our economy, we first summarise the within-period timing structure and the associated policy functions. These are presented in Figure 1.

2.4 Steady state distributions

The start-of-period expected value functions for single women and men depend upon the probability of meeting a potential spouse of a given type. As made explicit in equations (9a) and (9b), these depend upon the measure of available potential spouses and so are equilibrium objects. In this section we present a theoretical characterization of these objects in the steady state, together with the joint measure of marriage matches. As in our presentation of value functions, it is useful to distinguish between *(i)* the *start-of-period* measures of marriage matches $\widetilde{g}^M(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$, single women $\widetilde{g}_f^S(a_f, \boldsymbol{\omega}_f)$, and single men $\widetilde{g}_m^S(a_m, \boldsymbol{\omega}_m)$; and *(ii)* the *end-of-period* measures of marriage matches $g^M(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$, single women $g_f^S(a_f, \boldsymbol{\omega}_f)$, and single men $g_m^S(a_m, \boldsymbol{\omega}_m)$.[21]

At the beginning of each period, a new generation of single women and men are born,

[20] The assumption of an initial weight λ_0 (which can be renegotiated) is convenient when calculating the equilibrium of our model. See Appendix B for details. An alternative assumption used in the limited-commitment intertemporal collective literature is that the initial weight is the result of a symmetric Nash bargaining problem, which for $\theta \geq \underline{\theta}(\mathbf{a}, \boldsymbol{\omega}, \xi)$, equates the surplus from marriage between spouses. That is, the initial Pareto weight λ_0 would solve $E\widetilde{V}_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda_0) - E\widetilde{V}_f^S(a_f, \boldsymbol{\omega}_f) = E\widetilde{V}_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda_0) - E\widetilde{V}_m^S(a_m, \boldsymbol{\omega}_m)$.

[21] It is not necessary to keep track of θ in the measure of matches, since θ is i.i.d. conditional on ξ.

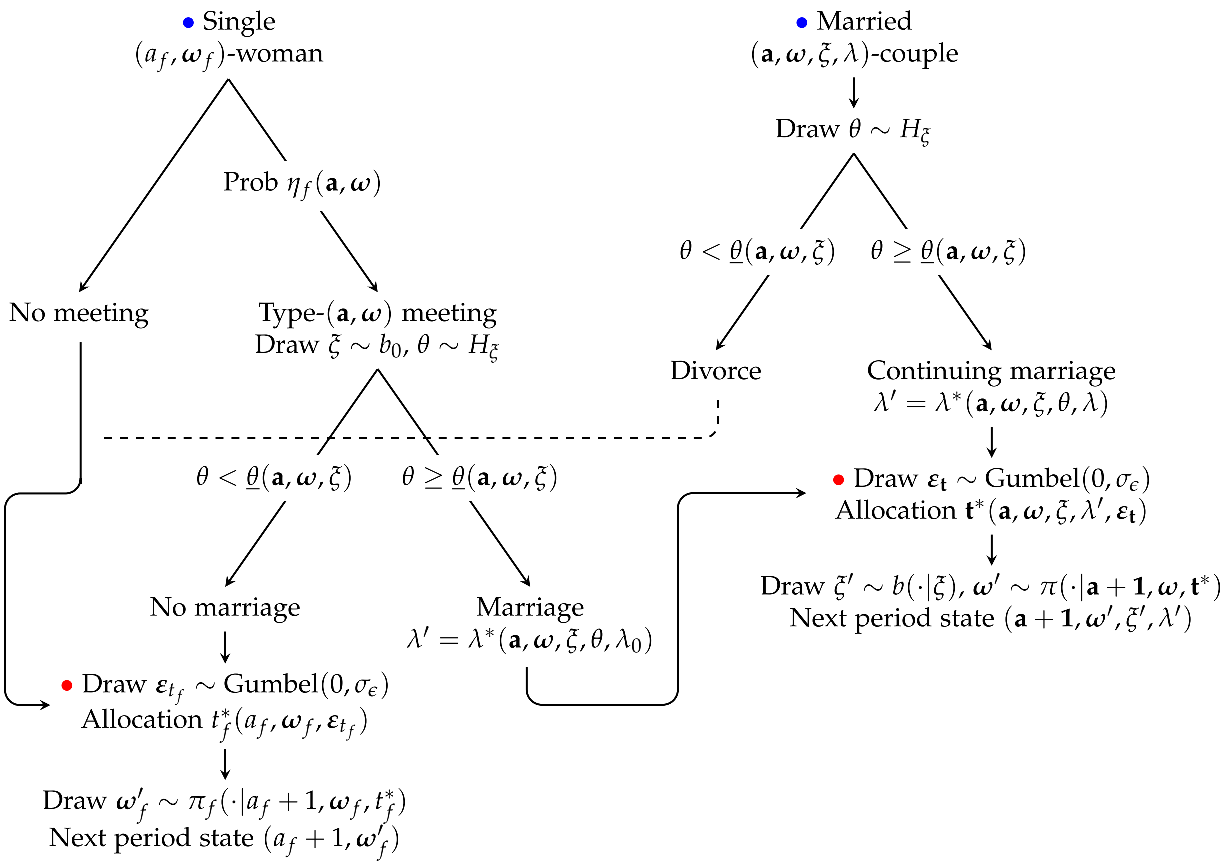


Figure 1: Timing structure. Diagram shows the within-period model timing structure and policy functions assuming that $a_f < A$ and $a_m < A$. The timing structure for single-men has been omitted for clarity of presentation, but proceeds as in the case for single-women. Blue (red) dots indicate the point at which *start-of-period* (*end-of-period*) objects and expectations are defined.

with initial measures over the states as given by $\pi_f^0(\boldsymbol{\omega}_f)$ and $\pi_m^0(\boldsymbol{\omega}_m)$. These define the age-1 start-of-period measures

$$\widetilde{g}_f^S(1, \boldsymbol{\omega}_f) = \pi_f^0(\boldsymbol{\omega}_f), \tag{12a}$$

$$\widetilde{g}_m^S(1, \boldsymbol{\omega}_m) = \pi_m^0(\boldsymbol{\omega}_m), \tag{12b}$$

$$\widetilde{g}^M([1, a], \boldsymbol{\omega}, \xi, \lambda) = \widetilde{g}^M([a, 1], \boldsymbol{\omega}, \xi, \lambda) = 0 \quad \forall a. \tag{12c}$$

The characterization of the start-of-period matching measures in equation (12c) follows as individuals are initially unmatched. Now consider the start-of-period measure of single females $\widetilde{g}_f^S(a_f, \boldsymbol{\omega}_f)$ for age $1 < a_f \leq A$. This comprises the measure of both single women from the previous period and women who became widows, whose state vector changed to $\boldsymbol{\omega}_f$. That is

$$\begin{aligned}\widetilde{g}_f^S(a_f, \boldsymbol{\omega}_f) &= \sum_{t'_f} \sum_{\boldsymbol{\omega}'_f} g_f^S(a_f - 1, \boldsymbol{\omega}'_f) \pi_f^S(\boldsymbol{\omega}_f | a_f, \boldsymbol{\omega}'_f, t'_f) \mathbb{P}_f^S(t'_f | a_f - 1, \boldsymbol{\omega}'_f) \\ &+ \sum_{\mathbf{t}'} \sum_{\boldsymbol{\omega}'} \sum_{\xi} \int_{\lambda} g^M([a_f - 1, A], \boldsymbol{\omega}', \xi, \lambda) \pi_f^S(\boldsymbol{\omega}_f | a_f, \boldsymbol{\omega}'_f, t'_f) \mathbb{P}(\mathbf{t}' | \boldsymbol{\omega}', [a_f - 1, A], \xi, \lambda) \, \mathrm{d}\lambda. \end{aligned} \tag{13}$$

The start-of-period measure of single males $\widetilde{g}_m^S(a_m, \boldsymbol{\omega}_m)$ for age $1 < a_m \leq A$ is defined symmetrically.

We similarly construct the start-of-period measure of matches for ages $1 < \mathbf{a} \leq A$. These correspond to the previous period matches, following the realizations of the joint state vectors $\boldsymbol{\omega}$ and the persistent marital state ξ, but before the idiosyncratic marital shock (and hence marriage continuation decisions). That is

$$\begin{aligned}&\widetilde{g}^M(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) = \\ &\quad \sum_{\mathbf{t}'} \sum_{\boldsymbol{\omega}'} \sum_{\xi'} g^M(\mathbf{a} - \mathbf{1}, \boldsymbol{\omega}', \xi', \lambda) \pi(\boldsymbol{\omega} | \mathbf{a}, \boldsymbol{\omega}', \mathbf{t}') \mathbb{P}(\mathbf{t}' | \mathbf{a} - \mathbf{1}, \boldsymbol{\omega}', \xi', \lambda) b(\xi | \xi'). \end{aligned} \tag{14}$$

To complete our characterization we need to define the end-of-period measures, after search, matching, and renegotiation has taken place in the marriage market. Firstly, consider the end-of-period measure of single women aged $1 \leq a_f \leq A$. This consists of the start-of-period measure of single $(a_f, \boldsymbol{\omega}_f)$-women who do not find a spouse and women of the same type who get divorced. The end-of-period measure of single females is therefore given by

$$
\begin{aligned}
g_f^S(a_f, \boldsymbol{\omega}_f) = \widetilde{g}_f^S(a_f, \boldsymbol{\omega}_f) \left[1 - \sum_{a_m} \sum_{\boldsymbol{\omega}_m} \sum_{\xi} \eta_f(\mathbf{a}, \boldsymbol{\omega}) \overline{H}_{\xi}(\underline{\theta}(\mathbf{a}, \boldsymbol{\omega}, \xi)) b_0(\xi) \right] \\
+ \sum_{a_m} \sum_{\boldsymbol{\omega}_m} \sum_{\xi} \int_{\lambda} \widetilde{g}^M(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) H_{\xi}(\underline{\theta}(\mathbf{a}, \boldsymbol{\omega}, \xi)) \, \mathrm{d}\lambda,
\end{aligned} \tag{15}
$$

where $\overline{H}_{\xi}(\theta) \equiv 1 - H_{\xi}(\theta)$. The end-of-period measure of single men $g_m^S(a_m, \boldsymbol{\omega}_m)$ is defined symmetrically.

The characterization of the end-of-period measure of marriage matches is complicated by the dynamics of the Pareto weight. Recall that this adjusts by the minimal amount if one spouse's participation constraint is not satisfied. To proceed, define $h_{\xi}(\theta) \equiv \mathrm{d}H_{\xi}(\theta) / \mathrm{d}\theta$ and denote by $\psi_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$ the density of θ that makes the woman indifferent between being married and not being married while exceeding the couple's reservation match value. Formally we have

$$
\psi_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) = \begin{cases} h_{\xi}(\theta_f^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)) \times \left| \frac{\partial}{\partial \lambda} \theta_f^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) \right| & \text{if } \theta_f^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) \geq \underline{\theta}(\mathbf{a}, \boldsymbol{\omega}, \xi) \\ 0 & \text{otherwise.} \end{cases}
$$

This density describes the distribution of draws of θ that result in an adjustment of the Pareto weight in the woman's favour to exactly λ. Symmetrically, define $\psi_m(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$ for men. The measure of type-$(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$ marriage matches then satisfies

$$
\begin{aligned}
g^M(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) = \widetilde{g}_f^S(a_f, \boldsymbol{\omega}_f) \eta_f(\mathbf{a}, \boldsymbol{\omega}) \psi_{fm}(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) b_0(\xi) \\
+ \overline{H}_{\xi}(\max\{\theta_f^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda), \theta_m^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)\}) \widetilde{g}^M(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) \\
+ \psi_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) \int_0^{\lambda} \widetilde{g}^M(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda^{-1}) \, \mathrm{d}\lambda^{-1} \\
+ \psi_m(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) \int_{\lambda}^{1} \widetilde{g}^M(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda^{-1}) \, \mathrm{d}\lambda^{-1},
\end{aligned} \tag{16}
$$

and where $\psi_{fm}(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$ reflects the density of θ depending upon whether the Pareto weight λ is less than, equal to, or greater than the initial weight λ_0.[22] The first term in equation (16) accounts for newly formed matches. The second term reflects the measure of matches that entered the period with Pareto weight λ and were not renegotiated. The third term corresponds to the measure of matches that entered the period with Pareto weight less than λ and were renegotiated to satisfy the female's participation constraint at exactly λ. Finally, the fourth term corresponds to the measure of matches that entered the period with Pareto weight less than λ and were renegotiated to satisfy the male's participation constraint at exactly λ.

2.5 Equilibrium

We restrict attention to stationary equilibria. In equilibrium, all individuals behave optimally when choosing from the end-of-period set of alternatives, and in their marriage formation/dissolution decisions, given the marriage market meeting probabilities. Equilibrium consistency requires that this behaviour induces stationary distributions of single and matched individuals that are consistent with these meeting probabilities. In Definition 1 we provide a formal definition of equilibrium.

Definition 1 (Equilibrium)**.** *A stationary equilibrium consists of* (i) *allocation choices for single women,* $t_f^*(a_f, \boldsymbol{\omega}_f, \boldsymbol{\varepsilon}_{t_f})$*, single men,* $t_m^*(a_m, \boldsymbol{\omega}_m, \boldsymbol{\varepsilon}_{t_m})$*, and for married couples,* $\mathbf{t}^*(\mathbf{a}, \boldsymbol{\omega}, \boldsymbol{\varepsilon}_{\mathbf{t}})$*;* (ii) *threshold reservation values for marriage and divorce decisions,* $\underline{\theta}(\mathbf{a}, \boldsymbol{\omega}, \xi)$*, and a transition rule for the Pareto weight,* $\lambda^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \theta, \lambda)$*;* (iii) *start-of-period and end-of-period value functions for single women* $(E\widetilde{V}_f^S, EV_f^S) \circ (a_f, \boldsymbol{\omega}_f)$ *and single men* $(E\widetilde{V}_m^S, EV_m^S) \circ (a_m, \boldsymbol{\omega}_m)$*, and for married women and men* $(E\widetilde{V}_f, EV_f^S, E\widetilde{V}_m, EV_m^S) \circ (\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$*;* (iv) *meeting probabilities* $(\eta_f, \eta_m) \circ (\mathbf{a}, \boldsymbol{\omega})$*. Such that*

1. *Household end-of-period allocation decisions solve equations* (2) *and* (5)*.*
2. *Marriage and divorce decisions are governed by a reservation threshold value given in equation* (7)*, and the Pareto weight evolves according to the transition function in equation* (8)*,*

[22] That is

$$\psi_{fm}(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) = \begin{cases} \psi_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) & \text{if } \lambda > \lambda_0 \\ \overline{H}_\xi(\max\{\theta_f^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda), \theta_m^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)\}) & \text{if } \lambda = \lambda_0 \\ \psi_m(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) & \text{if } \lambda < \lambda_0. \end{cases}$$

3. *Value functions for single women satisfy equations* (1), (3), *and* (11), *while value functions for married women satisfy equations* (4), (6), *and* (10) *(and similarly for men),*

4. *Meeting probabilities are consistent with the equilibrium measures of women and men as described by equations* (9a), (9b), (12a), (12b), (12c), (13), (14), (15), *and* (16).

We now state our formal existence proposition.

Proposition 1 (Existence)**.** *Under regularity conditions a stationary equilibrium exists.*

Proof of Proposition 1. See Appendix A.3. □

We use Brouwer's fixed-point theorem to prove existence. The main idea of our approach, which is also reflected in our numerical solution, is that the start-of-period expected value functions when single, $E\widetilde{V}_f^S(a_f, \omega_f)$ and $E\widetilde{V}_m^S(a_m, \omega_m)$, fully capture the value from marriage opportunities to spouses from all possible cohorts. Accordingly, we show how to construct a continuous update function that maps these value functions and start-of-period single measures to itself, such that a fixed-point (stationary equilibrium) exists.[23] In Appendix B we describe the numerical implementation of our fixed-point operator, together with practical numerical issues when calculating expected value functions and steady state measures. Note that we do not have any theoretical result that ensures uniqueness of the equilibrium, and as such, our framework is open to the possibility of multiple equilibria. While theoretical work is required to establish conditions under which uniqueness is guaranteed, in practice, we have always found that our fixed-point operator converges to the same equilibrium distribution.

3 Application: the age structure of marriages

In the United States there is important variation in the age structure of marriages, both cross-sectionally, and over the life-cycle. Firstly, while there exists considerable dispersion in the cross-section, there is the well-known tendency for men to be married to women younger than themselves (see Figure 2a), a phenomenon referred to as age hypergamy. The husband's age exceeds his wife's age by 2.3 years on average, with women older than men in only 20% of marriages. Secondly, and as documented in, e.g., England and McClintock (2009), while age hypergamy becomes much more extreme the

[23]The requirement to define an update mapping over the start-of-period measures (in addition to the expected value functions) only arises due to across-cohort marriage matching. See Appendix B.

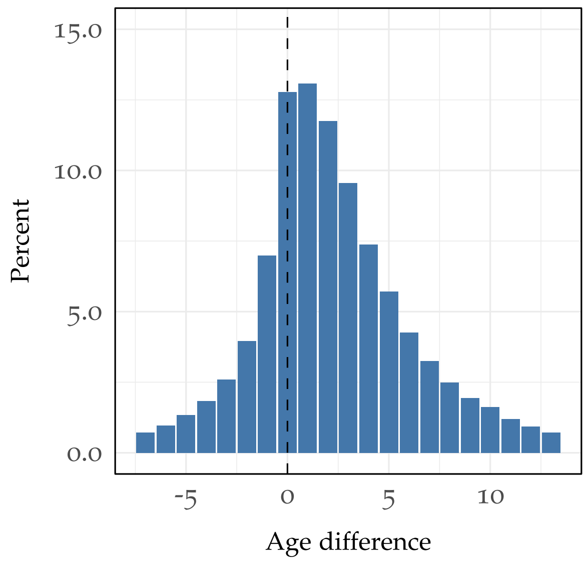
older men are when they marry, it is much less strongly related to the woman's age at marriage (see Figure 2b). Thirdly, as first marriages for women occur at younger ages compared to men, and both their marriage and remarriage rates are lower at older ages, there are significant imbalances in the relative number of single men compared to single women by age. For example, there are approximately 20% more single women in their fifties compared to single men in the same age group (see Figure 2c). Marital age differences also exhibit an important influence on patterns of specialisation within the household. In Figure 2d we examine the relationship between a woman's employment and the age difference in marriage. The employment rate of women is lower the older is her husband relative to her, with this negative relationship most pronounced for younger women. This negative relationship continues to hold even conditioning upon a rich set of controls including children, education levels, and her husband's income. In contrast, there is a much weaker relationship between the labour supply of men and the marital age gap (not illustrated here, but see Section 4 later).[24]

3.1 Empirical parametrisation

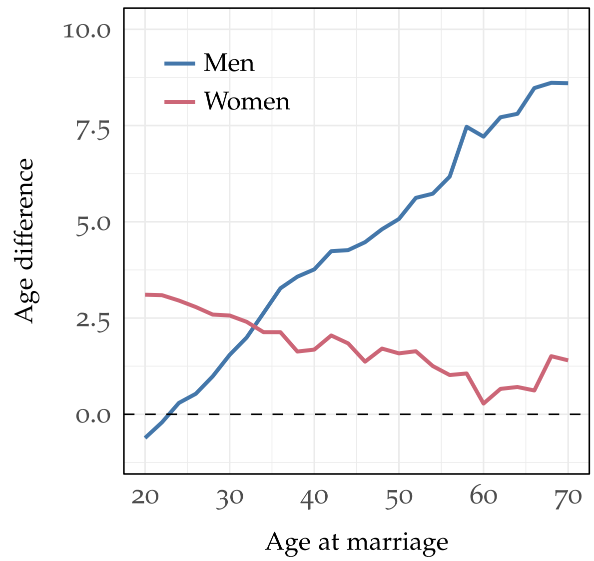
As an application of our equilibrium limited-commitment framework we empirically implement a model with labour market earnings risk, human capital accumulation, home production activities, fertility, and both within- and across-cohort marital matching.

Relative to the framework presented in Section 2, our application considers a slightly generalised environment, with these extensions omitted from the earlier presentation as they require further notation but do not fundamentally change the analysis. Firstly, we incorporate gender- and age-specific *mortality risk* by introducing an exogenous probability that an individual will survive to the next period. These survival probabilities change the discounting of the continuation value, and for individuals in couples, the continuation value also reflects that an individual with a non-surviving spouse is single next period. And while the start-of-period measures are suitably modified, our timing structure implies that the definition of the end-of-period measures is unaffected. Second, we introduce *divorce costs* as a one-time utility cost κ_{div} in the event of divorce. This introduces a wedge in the threshold values for marriage and divorce decisions, and the Markovian Pareto weight transition function. Third, in addition to the state-specific

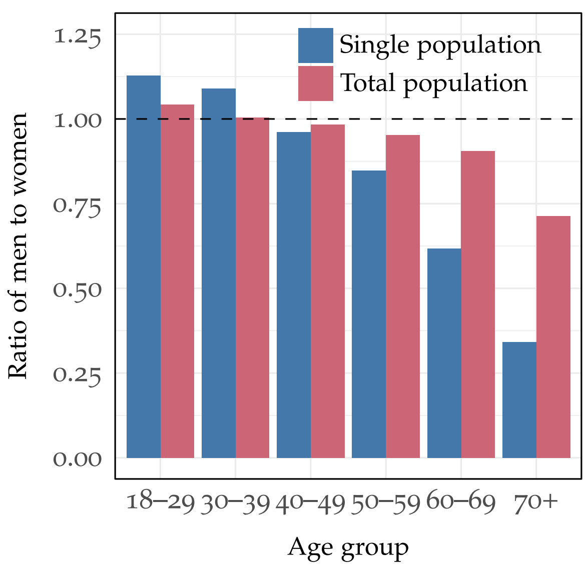
[24]Many of these patterns are true across a range of countries. For example, positive age gaps (defined as the husband's age less then wife's age) are found in all countries (see United Nations, 2017). Using a sample of Israeli Jewish women with a high school education or less, Grossbard-Shechtman and Neuman (1988) found that the employment rate of women was decreasing in the marital age gap.



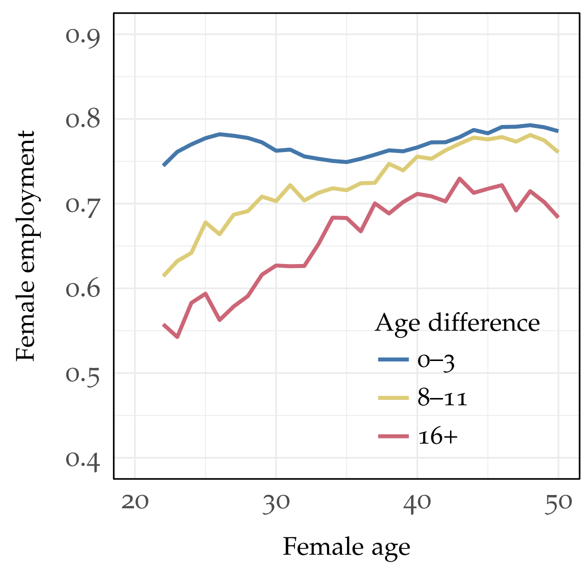
(a) Age gap distribution, all marriages



(b) Average age gap, new marriages



(c) Sex ratios



(d) Employment by age and age gap

Figure 2: Panel a shows the cross-sectional distribution of the age gap for married couples (defined as the husband's age less then wife's age, $a_m - a_f$). Panel b shows the average age gap as a function of the age of the husband or wife for new marriages. Panel c presents the ratio of men to women in the whole population and in the population of singles by age group. Panel d shows the employment rate of married women as a function of female age and the age gap. Source: Author's calculations with pooled 2008–2015 American Community Survey data.

errors we allow for additional sources of *end-of-period uncertainty*. The end-of-period expected value function are then calculated by integrating over the respective distributions.

3.1.1 Preferences, endowments, and choices

Risk averse individuals enter the economy at age 18 as singles with no children and are endowed with an education level $s \in \{s_L, s_H\}$, which respectively corresponds to high school graduate and below, and college and above. Individuals live until (at most) age 80, with a model period corresponding to two years. At the *end-of-period* decision stage, individual's and household's choose consumption and time allocations given their current state. For a single woman, this will depend upon: her age a_f, number of children n_c, age of youngest child y_c, human capital level k_f, education s_f, transitory wage realisation ϵ_{wf}, and vector of state-specific preference shocks $\boldsymbol{\varepsilon}_{t_f}$. Conditional upon these, she chooses how to allocate her time between leisure ℓ_f, market work time h_{qf}, and home production activities h_{Qf}.[25] Her within-period preferences are described by a direct utility function that is defined over her leisure ℓ_f, consumption of a private market good q_f, and consumption of a non-marketable good Q_f that is produced with home time. We adopt the parametrisation

$$u_f(\ell_f, q_f, Q_f; \boldsymbol{\omega}_f) = \frac{q_f^{1-\sigma_q} \cdot \exp[(1-\sigma_q)(\nu_f(\ell_f) + \nu_Q(Q_f))]}{1-\sigma_q}, \tag{17}$$

which exhibits curvature in the utility function over consumption of the private market good, with this subutility interacted with both leisure (as in Attanasio, Low and Sánchez-Marcos, 2008, and Blundell et al., 2016, among others) and consumption of the non-marketable home produced good. The preferences and decisions of a single man are defined symmetrically.[26]

The consumption and time allocation choice of married individuals will depend upon the characteristics of all household members, $(\mathbf{a}, n_c, y_c, \mathbf{k}, \mathbf{s}, \boldsymbol{\epsilon}, \boldsymbol{\varepsilon}_{\mathbf{t}})$, together with the persistent marital quality component ξ and the Pareto weight λ. The within-period prefer-

[25]We do not consider any retirement decision in our application, for which spousal age differences are likely important. It is well-documented that spouses often retire within a short time from each other (see for example, Hurd, 1990 and Blau, 1998). Casanova (2010) presents a dynamic model of joint retirement, but does not consider marriage formation or divorce.

[26]We have $\nu_Q(Q) = \beta_Q \times Q^{1-\sigma_Q}/(1-\sigma_Q)$. The function $\nu_j(\ell_j)$ comprises (leisure) alternative-specific constants, with $\nu_j(\underline{\ell}) = 0$. For married individuals, an additive term $\nu_{jj'}$ is present when their spouse works.

ences for each spouse take the same form as for single individuals, but are additionally interacted with a term that reflects direct spousal age preference. For a gender-j individual we define $\widetilde{\Delta}_j(\mathbf{a}) = [a_m^{\gamma_{\eta_j}} \times (1 - a_f/a_m) - \mu_{\eta_j}]/\sigma_{\eta_j}$ and specify the subutility function (which interacts with equation (17) multiplicatively) as

$$\eta_j(\mathbf{a}) = \exp\left((1-\sigma_q) \times \beta_{\eta_j} \times \left\{\text{normalPdf}[\widetilde{\Delta}_j(\mathbf{a})] \times \text{normalCdf}[\alpha_{\eta_j}\widetilde{\Delta}_j(\mathbf{a})] - (8\pi)^{-1/2}\right\}\right),$$

where normalPdf[$\cdot$] and normalCdf[$\cdot$] are respectively the standard normal density and cumulative distribution function. This flexible specification provides a low-dimensional (five parameter) way of capturing different marital age preferences. Consistent with existing stated-preference evidence over spousal age (see Section 4), it allows preferences to vary with an individuals age. It can accommodate preferences for individuals being similar in age and also somewhat younger/older than themselves. Moreover, it allows asymmetry in the preference for younger and older spouses relative to the most preferred age. Note that by construction we have $\eta_j(\mathbf{a}) = 1$ whenever $\widetilde{\Delta}_j(\mathbf{a}) = 0$.[27]

Given these preferences, and the constraints and technology of the household, we next proceed to characterise the period indirect utility functions for single and married women and men.

3.1.2 Singles: End-of-period time allocation problem

Consider a single $(a_f, \boldsymbol{\omega}_f)$-woman. From a finite and discrete set of alternatives she chooses how to allocate her time between leisure ℓ_f, market work time h_{qf}, and home production time h_{Qf}.[28] Her consumption of the private market good depends on her work hours h_{qf} through the static budget constraint

$$q_f = F_f(h_{qf}, \boldsymbol{\omega}_f, \epsilon_f) \equiv w_f \cdot h_{qf} - T_S(w_f h_{qf}; n_c, y_c) - C_S(h_{qf}; n_c, y_c),$$

[27] There may exist combinations of the spousal preference parameters that imply very similar values for $\eta_j(\mathbf{a})$. Based upon an initial estimation, the elements of the male preference parameter vector over female age were not well-identified, and in the results presented we restrict the skew parameter α_{η_m} to be zero.

[28] We allow for 8 alternatives for each *individual* with the equivalent of 115 hours per week of non-discretionary time. Expressed in hours per week, and suppressing the indexing by gender, the index representation of an individual choice set is given by $(h_q^t)_{t\in\mathcal{T}} = [0, 20, 20, 40, 40, 40, 60, 60]$, $(h_Q^t)_{t\in\mathcal{T}} = [45, 25, 45, 5, 25, 45, 5, 25]$, and with leisure then defined as the residual time, $\ell^t = 115 - h_q^t - h_Q^t$.

where $w_f = w_f(\boldsymbol{\omega}_f, \boldsymbol{\epsilon}_f)$ is her hourly wage (which also depends on the realisation of *end-of-period* uncertainty in the form of a transitory wage shock), $T_S(\cdot)$ is the tax schedule for a single individual,[29] and $C_S(\cdot)$ are childcare expenditures that depend on her labour supply and both the number and age of any children. Similarly, consumption of the non-marketable home good depends upon the woman's time input h_{Qf} through the production function $Q_f = Q_f(h_{Q_f}; \boldsymbol{\omega}_f) \equiv \zeta(s_f, y_c, n_c) \cdot h_{Q_f}$. The home efficiency parameter depends upon the woman's education, and both the number and age of her children. Substituting the budget constraint and home production technology in her utility function we obtain the indirect utility function

$$v_f^S(t_f; a_f, \boldsymbol{\omega}_f, \boldsymbol{\epsilon}_f) \equiv u_f(\ell_f, F_f(h_{qf}, \boldsymbol{\omega}_f, \boldsymbol{\epsilon}_f), Q_f(h_{Qf}; \boldsymbol{\omega}_f); a_f, \boldsymbol{\omega}_f),$$

where $t_f = t_f(\ell_f, h_{qf}, h_{Qf})$ is the bijective function that defines the index representation of the time alternatives. We obtain $v_m^S(t_m; a_m, \boldsymbol{\omega}_m, \boldsymbol{\epsilon}_m)$ symmetrically.

3.1.3 Married couples: End-of-period time allocation problem

Consider now a married $(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$-couple. The household time allocation determines the total consumption of the private good, together with the consumption of the non-marketable home produced good. The latter is produced by combining the home time of the husband and wife, and is public within the household. The production technology is parametrised as $Q = Q_{fm}(\mathbf{h}_Q; \boldsymbol{\omega}) \equiv \zeta_{fm}(\mathbf{s}, n_c, y_c) \cdot h_{Qf}^{\alpha} \cdot h_{Qm}^{1-\alpha}$, with the efficiency parameter depending upon education and the number and age of any children.[30] Given labour supplies, the *household* consumption of the private good is then uniquely determined by the household budget constraint

$$q = F_{fm}(\mathbf{h}_q; \boldsymbol{\omega}, \boldsymbol{\epsilon}) = \mathbf{w}'\mathbf{h}_q - T(\mathbf{w}'\mathbf{h}_q; n_c, y_c) - C(h_{qf}; n_c, y_c).$$

With a static budget constraint, the private good resource division problem conditional on $\mathbf{t} \in \mathcal{T}$ reduces to a a static optimisation problem which determines utility transfers

$$\max_{0 \le q_f \le q} \lambda u_f(\ell_f, q_f, Q_{fm}(\mathbf{h}_Q; \boldsymbol{\omega}); \mathbf{a}, \boldsymbol{\omega}_f) + (1-\lambda) u_m(\ell_m, q - q_f, Q_{fm}(\mathbf{h}_Q; \boldsymbol{\omega}); \mathbf{a}, \boldsymbol{\omega}_m).$$

[29]Our calculation of the net-tax schedule uses the institutional features of the (2015) U.S. tax system and major transfer programmes, and closely follows that described in Online Appendix C of Gayle and Shephard (2019), although here we do not allow for state variation.

[30]We specify the home efficiency parameters as a log-linear index function of the state variables. As a scale normalisation, we omit an intercept term in the efficiency parameter for married couples.

The solution to this constrained maximization problem defines private consumption for both the wife $q_f(\mathbf{t}; \mathbf{a}, \boldsymbol{\omega}, \boldsymbol{\epsilon}, \lambda)$ and her husband $q_m(\mathbf{t}; \mathbf{a}, \boldsymbol{\omega}, \boldsymbol{\epsilon}, \lambda)$, satisfying $q_f + q_m = q$. The period indirect utility function can then be obtained as

$$v_f(\mathbf{t}; \mathbf{a}, \boldsymbol{\omega}, \boldsymbol{\epsilon}, \lambda) = u_f(\ell_f, q_f(\mathbf{t}; \mathbf{a}, \boldsymbol{\omega}, \boldsymbol{\epsilon}, \lambda), Q_{fm}(\mathbf{h}_Q; \boldsymbol{\omega}); \mathbf{a}, \boldsymbol{\omega}_f)$$
$$v_m(\mathbf{t}; \mathbf{a}, \boldsymbol{\omega}, \boldsymbol{\epsilon}, \lambda) = u_m(\ell_m, q_m(\mathbf{t}; \mathbf{a}, \boldsymbol{\omega}, \boldsymbol{\epsilon}, \lambda), Q_{fm}(\mathbf{h}_Q; \boldsymbol{\omega}); \mathbf{a}, \boldsymbol{\omega}_m).$$

A low decision weight for an individual is therefore reflected both in the patterns of time allocation, and through less access to private consumption goods.[31] Note that given our specification of the period utility function (equation (17)), we require that the $\sigma_q > 1$ for Assumption 3 to hold and impose this restriction in our subsequent estimation.

3.1.4 Wages and human capital

Individuals accumulate skills while working through a learning by doing process.[32] The log hourly wage offer for individual-i of gender $j \in \{f, m\}$, schooling s, and age a is given by

$$\ln w_{ia} = r_{js} + \alpha_{js} \ln(1 + k_{ia}) + \epsilon_{wia}, \quad \epsilon_{wia} \sim \mathcal{N}(0, \sigma^2_{js}), \tag{18}$$

and where we note that the parameters of the wage process, including the distribution of shocks, are both education- and gender-specific. The variable k_{ia} measures acquired human capital, which is restricted to take pre-specified values on a grid, $k_{ia} \in [0 = k_1, \ldots, k_K]$.[33] In our empirical application we set $K = 3$ with an exogenously specified and uniform-spaced grid. All workers enter the model with $k_{i1} = k_1 = 0$ which then evolves according to a discrete state Markov chain.

We choose a specification that closely links future returns in the labour market to current labour supply, and which allows career interruptions to be costly. To this end, we write the human capital transition matrix as $\pi_k(k', k, h_q) = \Pr[k_{i,a+1} = k' | k_{ia} = k, h_q]$, which depends on current labour supply h_q. We consider a low-dimensional parametri-

[31] Through their impact on outside options, our specification implies a relationship between the distribution of wages within the household and consumption inequality. Using detailed expenditure data from the United Kingdom, Lise and Seitz (2011) present empirical evidence that relates differences in the wages between husband and wife to differences in consumption allocations.

[32] Other studies which incorporate human capital accumulation in a life-cycle labour supply model include Shaw (1989), Eckstein and Wolpin (1989), Keane and Wolpin (2001), Imai and Keane (2004), and Blundell et al. (2016).

[33] Note that variables including the number and age of children, together with spousal characteristics, affect the decision to work but not wage offers. These therefore provide important exclusion restrictions.

sation of the transition matrix by defining $\pi_k(k', k, 0)$ to be a lower-triangular matrix which, for $k > 1$, defines a constant probability δ_0 of an incremental reduction in their human capital level. Similarly, let $\pi_k(k', k, \overline{h}_q)$ be a upper-triangular matrix, which for $k < K$, defines a constant probability δ_k of an incremental improvement in human capital when working maximal hours ($h_q = \overline{h}_q$). For general h_q we construct a weighted average of these transition matrices. Finally, the residual component in the log-wage equation comprises an i.i.d. transitory component ϵ_{wia}.[34]

3.1.5 Fertility and children

As in Siow (1998) and Díaz-Giménez and Giolito (2013) we introduce a role for differentiable fecundity. We do not explicitly model the fertility decision, but rather assume that children arrive according to some stochastic process. To this end, we estimate non-parametric regression models that describe the probability that a child is born as a function of the woman's age.[35] Separate regressions are performed depending upon marital status, the education level of the woman, and whether there are any other children in the household. These imply non-parametric estimates for the probabilities $\Pr[y_{c,a_f} = 0 | s, a_f - 1, n_{c,a_f - 1}, m_{a_f - 1}]$.

Children enter the model in the following ways. First, they enter the budget constraint, with children affecting both taxes and costs of work (through childcare costs). Second, they are considered public goods in the household, with children affecting the productivity of home time. Note that given our preference specification in equation (17), changes in the household decision weight can have an important impact on the allocation of time and the quantity of the home good that is produced. In the event of divorce, any children are assumed to remain with the mother and no longer enter the (now) ex-husband's state space.[36] When a single women with children marries, her children

[34] When forming our end-of-period expected value functions we numerically integrate over the distribution of these transitory wage realisations using Gaussian quadrature. See Meghir and Pistaferri (2011) for a survey of the literature that characterises and estimates models of earnings dynamics. For computational reasons the earnings dynamics process adopted here is relatively simple, with the incorporation of richer family income dynamics an important empirical extension for future work.

[35] Estimation is performed using kernel-weighted local polynomial regression. An alternative approach would be to model fertility as a choice variable. Recent papers that estimate non-equilibrium life-cycle models with endogenous fertility decisions include Adda, Dustmann and Stevens (2017) and Eckstein, Keane and Lifshitz (2019). Both approaches allow younger women to have greater fertility capital.

[36] This is a simplifying assumption which implies that there is no interaction between divorcees. An alternative approach that has been followed in the literature is that children remain a public good in divorce (Weiss and Willis, 1985), with both divorcees then contributing to this public good. This is a considerably more complicated problem in an environment with remarriage, as it is both necessary to keep

(regardless of whether they were born in a previous marriage or when single) enter the combined state space and the new household treats the children as its own. To help rationalise the observation that single women with children have lower marriage and remarriage rates than those without children, we follow Bronson (2015) by incorporating a one-time utility cost κ_{mar} when marriages with existing children are formed.[37]

3.1.6 Marriage quality and matching

All initial marriage meetings are evaluated at the Pareto weight $\lambda_0 = 1/2$, with the weight then renegotiated to $\lambda^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \theta, \lambda_0)$ if necessary for the formation of the marriage. The marital match component consists of a persistent distributional parameter ξ, and a continuously distributed distributed idiosyncratic component $\theta \sim H_\xi$. We allow the distributional parameter to take two values, $\xi \in \{\xi_L, \xi_H\}$, with $b(\xi'|\xi) = \Pr[\xi_{\mathbf{a+1}} = \xi'|\xi_{\mathbf{a}} = \xi]$ defining the respective Markovian transition matrix. The idiosyncratic component (current period match quality) is parametrised as a Logistic distribution, with mean μ_{θ_ξ} and common scale parameter σ_θ. We impose $\mu_{\theta_L} < \mu_{\theta_H}$ and therefore interpret ξ_L and ξ_H as respectively representing lower and higher quality marriages. While our parametrisation differs from, e.g., Voena (2015) and Greenwood et al. (2016), our persistent-transitory parametrisation also implies autocorrelation in the marital match-quality over time, and therefore has implications for the degree of duration dependence in the divorce hazard.

While our theoretical model does not restrict the degree of across-cohort marital matching, in our application we restrict the maximal absolute age gap $|a_m - a_f| \leq \Delta a_{\max}$, which we parametrise as an absolute age difference no greater than 16 years. In our data, this is true for almost 99% of couples. We then allow the marriage matching efficiency parameter to depend upon age $\mathbf{a}$ and education $\mathbf{s}$. We set

$$\gamma(\mathbf{a}, \overline{\boldsymbol{\omega}}) = \begin{cases} \gamma_s(\mathbf{s}) \times \left(1 - \left[\frac{a_m - a_f}{\Delta a_{\max} + 1}\right]^2\right)^{\gamma_a} & \text{if } |a_m - a_f| \leq \Delta a_{\max} \\ 0 & \text{otherwise.} \end{cases}$$

track of children from all previous marriages, and to solve for a decision problem involving (potentially multiple) children outside of the household.

[37]To limit the size of the state space, we represent children in the household by two state variables: the age of the youngest child y_c, and the number of children n_c. Assuming children exit the household at some fixed age, it is not possible to update n_c exactly without knowing the age of all children. We proceed by approximating this law-of-motion by assuming that all children leave the household when the youngest child does so (at age 18). The difficulty with incorporating the full age structure of children in dynamic programming models is discussed in Keane, Todd and Wolpin (2011).

The parameter $\gamma_a \geq 0$ characterises the degree of age homophily in meetings, i.e., how likely are individuals to meet potential spouses who are similar in age. As this parameter gets large, these meetings are much more likely to take place at similar ages. Conversely, as this parameter approaches zero, such meetings become more uniform across ages.

Finally, we note that we have age entering both preferences and the meeting technology. To understand identification suppose first that there is no persistence in the marriage quality component (i.e. ξ is not a state variable), and for expositional simplicity that there are no state variables other than age. In this simplified model we then have that the probability of divorce conditional on the household state is given by $H(\underline{\theta}(\mathbf{a}))$. As this same probability enters the observed marriage probabilities, we are then able infer the meeting efficiency parameter $\gamma(\mathbf{a})$ using equations (9a) and (9b) as single measures are also observed. Thus, we would infer that an infrequent marriage-pairing, which is long-lasting when it does take place, to be high marital surplus and that the lack of marriage pairings is due to infrequent meetings. This is essentially the argument in Goussé, Jacquemet and Robin (2017).

This same identification argument does not follow when we have auto-correlation in the marriage quality, as we have here, as the distribution of the unobserved persistent component differs in new-marriage meetings versus continuing marriages. Nonetheless, we can still establish identification in this case by relating the divorce probabilities to marriage duration. This follows as marriages of different durations have different mixing distributions that we may characterise. In new marriages, the mixing distribution over ξ depends upon both $b_0(\xi)$ and the conditional marriage formation probability $H_\xi(\underline{\theta}(\mathbf{a}, \xi))$. In marriages that were formed one period ago, this depends on $\{b_0(\xi)$, $H_\xi(\underline{\theta}(\mathbf{a}, \xi))$, $H_\xi(\underline{\theta}(\mathbf{a} - \mathbf{1}, \xi))$, and $b(\xi'|\xi)\}$, and so on. Thus, we may construct a system of equations that relates the observed divorce probabilities (for marriages of different ages and durations) to these probabilities, which is identified provided sufficient divorce probabilities are observed.[38]

3.2 Data

We use two data sources for our estimation. First, we use pooled data from the 2008–2015 American Community Survey (ACS) which provides us with information on education,

[38]As an example consider same age marriages when the persistent marriage component takes two values. Let τ_d denote marriage duration. We observe $DP(a, \tau_d)$ for $a = 2, \ldots, A, \tau_d = 1, \ldots a-1$. This provides $A \times (A-1)/2$ known quantities. We wish to identify $b_0(\xi_L)$, $b(\xi_L|\xi_L)$, $b(\xi_H|\xi_H)$, $H_{\xi_L}(\underline{\theta}(\mathbf{a}, \xi_L))$, and $H_{\xi_H}(\underline{\theta}(\mathbf{a}, \xi_H))$. This comprises $3 + 2 \times A$ unknown parameters. For $A \geq 6$ we have identification.

marital patterns, marriage events, demographics, incomes, and labour supply.[39] Both the size of the sample and the information collected in the ACS, make it particularly well suited for analysing the age distribution of marriages.

We additionally use data from the Panel Study of Income Dynamics (PSID), a longitudinal panel survey of a representative sample of U.S. individuals and families.[40] While the sample size is significantly smaller than the ACS, it provides us with measures of labour market experience and broad home production time that includes both housework and time spent with children.[41] Moreover, as a true panel data set, it allows rich labour market and marriage market histories to be constructed. As in, for example, Choo (2015), we assume that the data is obtained from a stationary data generating process that is in steady state.[42,43]

[39]The ACS, available from Ruggles et al. (2017), is the U.S. Census Bureau's replacement for the long form of the decennial census. The full implementation of the ACS, which began in 2005, provides timely information on a range of economic, demographic, and social outcomes. Beginning in 2008, the ACS asks questions about marital events that have occurred in the previous 12 months, the number of times a person has been married, and the year of the most recent marriage. These questions facilitate the analysis of marriage and divorce rates.

[40]The PSID began in 1968 with a sample of 4,800 U.S. families (including a low-income oversample). These original families, and the split-off families formed by children and other family unit members as they established their own households, have been re-interviewed on an annual basis from 1968–1997, and biennially since then. The survey collects information on a range of demographic, economic, and social outcomes over the life course of these families.

[41]Data on the time that parents spend with children is derived from the PSID Child Development Supplement (CDS). The CDS provides detailed information on a subset of children from the PSID main interview sample, starting in 1997. We use data from both the initial wave and subsequent waves (2003, 2007, 2014). We construct our childcare measure using the CDS child time diaries, which contain information including the type and duration of activities performed by the child, as well as information on who else was present or participation in each activity, over two 24-hour periods (a randomly sampled weekday and weekend day). For each of these 24 hour periods, we equate a parent's childcare time to the total time that the parent was participating in activities with the child, and impute a weekly measure by multiplying the weekday totals by 5 and the weekend day totals by 2. Given the diaries are at the child (not parent) level, to avoid double counting parental time in multi-child families, we exclude activity time for additional children when both the parent and a sibling were participating in that activity.

[42]Stationarity is a strong assumption. Extending the theoretical framework to allow for differences across birth cohorts would allow the dynamics of secular changes (such as changing educational attainment, life-expectancy, and social attitudes) to be analysed. This is an important and challenging extension for future work.

[43]We obtain gender and age-specific mortality risk from life tables produced by the United States National Center for Health Statistics (see Elizabeth Arias, Melonie Heron and Xu, 2017). These are used to construct gender and age-specific population sizes for a synthetic cohort. In calculating data moments, we apply a set of constructed weights. These weights are calculated to ensure consistency with the constructed population counts, while also being close to the empirical (joint age) marriage matching function.

3.3 Estimation procedure

Existing empirical applications of limited commitment models all use a simulation-based indirect inference estimation procedure (Gourieroux, Monfort and Renault, 1993). In this approach, the dynamic programming problem is first solved given a candidate parameter vector, and an artificial dataset is then generated using the model data generating process. The objective of the estimation concerns the choice of parameter vector that minimises the distance between the auxiliary parameters estimated on the actual data and those estimated on the simulated data. One of the main practical difficulties with simulation based estimation is that the objective function is typically non-smooth which precludes the use of gradient-based numerical optimization.[44]

We impose all equilibrium restrictions in our estimation procedure, and by virtue of characterising the equilibrium, we do not require simulation. Conditional on the model parameter vector $\mathbf{\Theta}$ we first solve the joint dynamic programming and marriage market equilibrium problem as described in Appendix B. Note that the solution to this problem yields equilibrium joint distributions and the associated policy functions. Thus, model moments/auxiliary parameters that condition on any subset of the dynamic programming state variables (such as marriage market matching patterns, marriage transitions, time allocation decisions, etc.) may be calculated directly. For example, the marriage rate for single women conditional on age would be given by

$$\sum_{\boldsymbol{\omega}_f} \widetilde{g}_f^S(a_f, \boldsymbol{\omega}_f) \times \sum_{a_m} \sum_{\boldsymbol{\omega}_m} \sum_{\xi} \eta_f(\mathbf{a}, \boldsymbol{\omega}) \overline{H}_{\xi}(\underline{\theta}(\mathbf{a}, \boldsymbol{\omega}, \xi)) b_0(\xi) \times \left[\sum_{\boldsymbol{\omega}_f} \widetilde{g}_f^S(a_f, \boldsymbol{\omega}_f) \right]^{-1}.$$

Moments that condition on variables that are not state variables of the dynamic programming problem are also of interest and may be calculated by constructing the respective match distributions. Importantly, this may be done following computation of the equilibrium in a *non-iterative* step. As a simple example, consider a moment that conditions on marriage duration $\tau_d = 1, \ldots, A$, and (with slight abuse of notation) let $g^M(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda, \tau_d)$ denote the end-of-period measure of $(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$-matches of duration τ_d. Then for new marriages $g^M(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda, \tau_d = 1) = \widetilde{g}_f^S(a_f, \boldsymbol{\omega}_f) \eta_f(\mathbf{a}, \boldsymbol{\omega}) \phi(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) b_0(\xi)$, while for contin-

[44] Non-smoothness naturally arises when there are discrete choices since a marginal change in the parameter vector may induce zero or discontinuous changes in behaviour and therefore the estimation criterion function. Sauer and Taber (2017) discuss the use of importance sampling to circumvent non-differentiability in indirect inference.

uing marriages ($\tau_d > 1$) we have

$$g^M(\mathbf{a},\boldsymbol{\omega},\xi,\lambda,\tau_d) = \overline{H}_\xi(\max\{\theta_f^*(\mathbf{a},\boldsymbol{\omega},\xi,\lambda),\theta_m^*(\mathbf{a},\boldsymbol{\omega},\xi,\lambda)\})\widetilde{g}^M(\mathbf{a},\boldsymbol{\omega},\xi,\lambda,\tau_d-1)$$
$$+ \psi_f(\mathbf{a},\boldsymbol{\omega},\xi,\lambda)\int_0^\lambda \widetilde{g}^M(\mathbf{a},\boldsymbol{\omega},\xi,\lambda^{-1},\tau_d-1)\,\mathrm{d}\lambda^{-1}$$
$$+ \psi_m(\mathbf{a},\boldsymbol{\omega},\xi,\lambda)\int_\lambda^1 \widetilde{g}^M(\mathbf{a},\boldsymbol{\omega},\xi,\lambda^{-1},\tau_d-1)\,\mathrm{d}\lambda^{-1},$$

where $\widetilde{g}^M(\mathbf{a},\boldsymbol{\omega},\xi,\lambda,\tau_d)$ naturally represents that start-of-period measure of matches of duration τ_d. Note that in the above $\widetilde{g}_f^S(a_f,\boldsymbol{\omega}_f)$, $\eta_f(\mathbf{a},\boldsymbol{\omega})$, $\phi(\mathbf{a},\boldsymbol{\omega},\xi,\lambda)$, $\theta_f^*(\mathbf{a},\boldsymbol{\omega},\xi,\lambda)$, $\theta_m^*(\mathbf{a},\boldsymbol{\omega},\xi,\lambda)$, $\psi_f(\mathbf{a},\boldsymbol{\omega},\xi,\lambda)$, and $\psi_m(\mathbf{a},\boldsymbol{\omega},\xi,\lambda)$ are all independent of τ_d and have been calculated as part of the initial equilibrium computation. Similar arguments can be used to calculate distributions with lagged employment, lagged wages, marital histories, and so on.[45]

From these equilibrium distributions and policy functions we construct a vector of moments $\mathbf{m}(\boldsymbol{\Theta})$ that summarise both the static and dynamic implications of our model, and that can be matched to moments $\mathbf{m}_{\text{data}}$ calculated from the observed data. Given a positive definite weighting matrix $\mathbf{W}$ the objective of the estimation procedure is to choose the parameter vector $\widehat{\boldsymbol{\Theta}}$ that minimises the weighted distance between model and empirical moments. Formally

$$\widehat{\boldsymbol{\Theta}} = \arg\min_{\boldsymbol{\Theta}} \left[\mathbf{m}(\boldsymbol{\Theta}) - \mathbf{m}_{\text{data}}\right]^\intercal \mathbf{W} \left[\mathbf{m}(\boldsymbol{\Theta}) - \mathbf{m}_{\text{data}}\right].$$

[45]Depending upon the auxiliary parameter of interest, the characterisation and calculation may be somewhat more complicated compared to a simulation based estimation approach. In the context of within and across cohort marital matching, a simulation based procedure proceeds as follows. At the start of each period t, a fixed number of N_{sim} women and N_{sim} men are born in the single state, and all surviving individuals age one period. The new born ("generation t") individuals draw a state vector from the initial exogenous state distribution; individuals from older generations draw an update to their state vector depending on both their period $t-1$ state vector and household allocation decision. Individuals are first randomly matched to another individual (uniquely characterised by a birth year, a gender, and an individual identifier $i = 1, \ldots, N_{\text{sim}}$) according to the meeting probabilities. If they are both single, persistent and idiosyncratic marital shock component must be drawn and given these it is then determined whether the match will be consummated. Similarly, for individuals who were married at the start of the period, new persistent and idiosyncratic components are drawn and it is then evaluated whether that match will continue. For both new and surviving couples, the Pareto weight is adjusted if necessary. A vector of idiosyncratic preference and wage shocks are obtained, and using the equilibrium values functions obtained from the initial dynamic programming problem, the allocation problem for both singles and couples may be solved. With marriage matching across birth cohorts, it is necessary to forward simulate the economy for a large number of generations until a stationary distribution is obtained. This procedure yields a simulated panel dataset with A active generations, and with each generation characterised by a partial life-cycle history.

Given the well-known problems associated with the use of the optimal weighting matrix (Altonji and Segal, 1996), we choose $\mathbf{W}$ to be a diagonal matrix.[46]

The full list of moments used to identify the model is provided in Appendix C. For moments that may be calculated both with the ACS and PSID, we use the ACS because of the much larger sample size that this offers.

4 Model estimates and fit

We present parameter estimates, together with accompanying standard errors, in Appendix D. Here, we comment on some of the main features, together with the implications that they have for life-cycle marriage and time-allocation outcomes. We first note that there are important differences in the wage process by both gender and education. By education, the initial wage is higher for college graduates and so too are the returns to labour market experience. Both the return to human capital and the initial wages levels are also estimated to be higher for men compared to women. This, together with strong human capital depreciation when not working, partial accumulation from part-time work, and the well-documented career interruptions of women, allows us to explain both the divergent life-cycle wage profiles by education and gender, together with differential wage growth by work hours. In Table 1 we present fit to the (two-year) wage growth rate conditional on lagged labour supply, together with the coefficients from a linear regression model of log hourly wages on a quadratic in actual work experience. Table 2 reports the fit to the life-cycle profile of wages by gender and marital status. While the model does generate wage differences by marital status, these differences are less pronounced relative to what we see in the data for married compared to single men.[47] This same table also show that the model can replicate some of the key patterns of life-cycle labour supply at both the extensive and intensive margins, although the

[46]The covariance matrix of our estimator is

$$\left[\mathbf{D}_{\mathbf{m}}^{\intercal}\mathbf{W}\mathbf{D}_{\mathbf{m}}\right]^{-1}\mathbf{D}_{\mathbf{m}}^{\intercal}\mathbf{W}\mathbf{\Sigma}\mathbf{W}^{\intercal}\mathbf{D}_{\mathbf{m}}\left[\mathbf{D}_{\mathbf{m}}^{\intercal}\mathbf{W}\mathbf{D}_{\mathbf{m}}\right]^{-1},$$

where $\mathbf{\Sigma}$ is the covariance matrix of the empirical moments, and $\mathbf{D}_{\mathbf{m}} = \partial \mathbf{m}_{\mathbf{sim}}(\mathbf{\Theta})/\partial\mathbf{\Theta}|_{\mathbf{\Theta}=\widehat{\mathbf{\Theta}}}$ is the derivative matrix of the moment conditions with respect to the model parameters evaluated at $\mathbf{\Theta} = \widehat{\mathbf{\Theta}}$. As the ACS and PSID data sets have very different sample sizes, and because the conditioning sets for the various moments often differ substantially in size, we choose to compute the default inverse weights using the estimated asymptotic variance of the sample moment, and not their finite-sample variance. Relative to this default, extra weight is placed on a small number of moments (see Appendix C).

[47]See Eckstein, Keane and Lifshitz (2019) for evidence on how the marriage wage premium has changed over time and the factors responsible for this.

Table 1: Wage growth and wage regression

	Women				Men			
	No college		College		No college		College	
	Data	Model	Data	Model	Data	Model	Data	Model
Log-wage changes								
Part-time	-0.01	0.01	0.03	0.01	-0.02	0.00	0.02	0.00
Full-time	0.04	0.07	0.07	0.08	0.04	0.06	0.06	0.07
Log-wage regression								
Constant	2.12	2.14	2.62	2.65	2.22	2.20	2.64	2.73
Experience	0.04	0.04	0.05	0.03	0.06	0.06	0.07	0.05
Experience squared / 100	-0.06	-0.06	-0.09	-0.04	-0.10	-0.11	-0.15	-0.08
Residual s.d.	0.52	0.49	0.54	0.58	0.52	0.53	0.53	0.59

Notes: Table shows empirical and simulated wage growth and wage regression coefficients. *Log-wage changes* measures the change in log-wages over a period of two-years, conditional on part-time or full-time employment status. *Log-wage regression* reports the coefficients from a linear regression model of log hourly wages on a quadratic in *Experience* (measured as the number of years of actual labour market experience). Empirical moments calculated with PSID data.

model does generate too high employment for single individuals over the life-cycle.

The public good property of home production activities provides an important economic benefit of marriage, with the estimated efficiency of home time strongly linked to both the number and age of children. This, together with the age-related decline in fertility for women, increases the desirability of younger women and is also reflected in the marriage matching patterns (discussed below). Within marriage, female home time is estimated to be a much more important input in the home production technology than is male home time. In Appendix D we also show the model is able to successfully explain the differences in the time allocations patterns (of both market work and home activities) for both men and women across different family structures.

The marital match quality parameters have important implications both for the type of marriages that are formed, and how marriages and outcomes within marriage evolve. In terms of the stochastic component, recall that the persistent state of the marriage quality distribution is parametrised to take two values that are associated with different mean values ("higher" and "lower" quality marriages). While the match quality from the majority of initial meetings is estimated to be in the lower state, those meetings where it is higher are much more likely to be consummated. Moreover, the estimated Markovian

Table 2: Life-cycle labour market outcomes

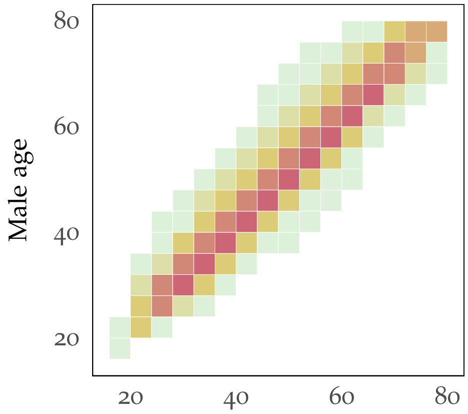
	Women						Men					
	Employment		Work hours		Log wage		Employment		Work hours		Log wage	
	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model
Single individuals												
20–29	0.85	0.94	36.01	38.86	2.53	2.54	0.89	0.99	38.24	40.23	2.59	2.65
30–39	0.86	0.92	38.11	38.27	2.79	2.79	0.94	0.99	40.42	40.38	2.89	2.99
40–49	0.88	0.97	38.58	38.82	2.89	2.85	0.94	0.99	40.66	40.19	3.01	3.08
50–59	0.87	0.99	38.75	39.71	2.92	2.90	0.93	0.99	40.56	40.07	3.04	3.10
Married individuals												
20–29	0.74	0.76	36.30	37.18	2.70	2.59	0.97	0.98	41.82	42.96	2.81	2.76
30–39	0.74	0.73	36.42	37.17	2.96	2.79	0.98	0.97	42.35	42.68	3.14	3.05
40–49	0.78	0.81	36.40	37.92	2.95	2.83	0.98	0.97	42.45	42.35	3.26	3.17
50–59	0.80	0.87	36.82	38.76	2.94	2.86	0.97	0.98	42.33	42.24	3.27	3.22

Notes: Table shows the empirical and simulated employment rates, conditional work hours, and log-wages, for both single and married women and men by aggregated age groups. Incomes expressed in average 2013 prices. Empirical moments calculated with ACS data.

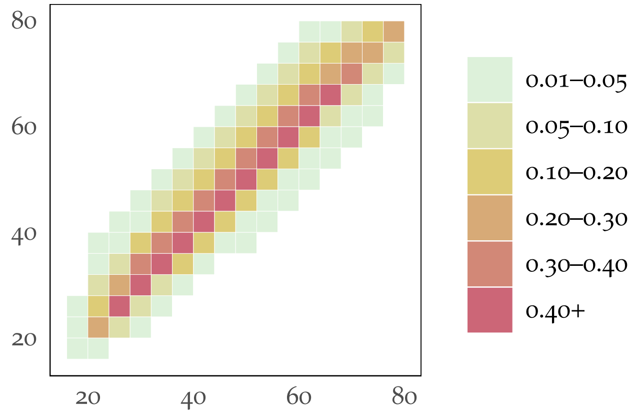
Table 3: Divorce hazard by age difference

	Age difference, $a_m - a_f$						
	(5+)	(1–4)	0–3	4–7	8–11	12–15	16+
Data	0.07	0.05	0.03	0.04	0.05	0.05	0.07
Model	0.08	0.04	0.02	0.03	0.05	0.08	0.10

Notes: Table shows the empirical and simulated divorce hazard rates (defined over a period of two-years) as a function of the age difference within marriage (defined as the husband's age less the wife's age, $a_m - a_f$). Age differences presented in parentheses correspond to negative age gaps. Empirical moments calculated with PSID data.



(a) Empirical



(b) Simulated

Figure 3: Marriage matching function. Figure shows the empirical and simulated marriage matching function by age amongst married, with age aggregated into age groups of 2 (equivalent to 4 years). Population size at age 18 is equal to normalised to one for men and women. Measures less than 0.01 are unfilled. Empirical moments calculated with ACS data.

transition matrix suggests that once a marriage enters the higher state it is very unlikely to revert to the lower state. This simple stochastic structure allows the model to generate the declining divorce rate with marriage duration. Similarly, given the preferences over spousal age that we describe below, in Table 3 we show that the model is able to replicate the empirical relationship between the martial age-difference and divorce rates, with the divorce hazard rate higher in more age-dissimilar unions.

The fit to the stationary distribution of marriages by male and female ages is presented as a heatmap in Figure 3, where age has been aggregated into four year bins, and where the warmer colours represent a greater probability mass. The model is remarkably successful in terms of replicating the cross-sectional bivariate distribution of marriages by age. We obtain the largest probability mass along and slightly above the diagonal, with the dispersion in matches increasing in both male and female age. A more detailed presentation of these facts is also provided Appendix D.

In Figure 4 we present the life-cycle profile of marital histories by gender (partitioned into single never married, first marriage, remarriage, and divorced). The model does well in explaining the broad patterns of marriage over the life-cycle, including important gender differences in the age at first marriage, although it does under-predict the incidence of remarriage in the middle of the life-cycle for both men and women.[48] In Table 4 we

[48]In Figure 4 we use "divorced" to refer to single individuals who were previously married and include

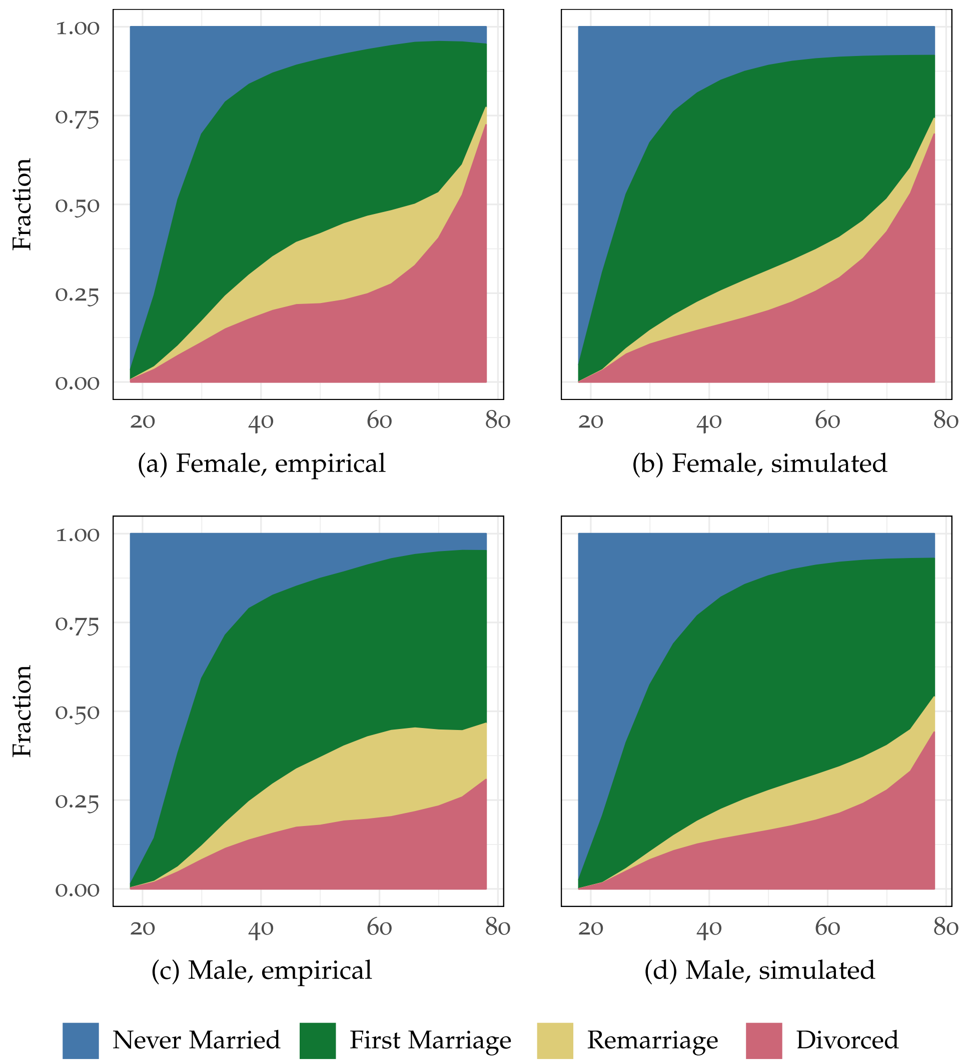
additionally present life-cycle marriage and divorce hazards. Consistent with the data, we obtain marriage hazards that are higher at younger ages for women than men, and with this pattern reversing at older ages. The same table also presents the age-difference in new marriages (defined as the husband's age less the wife's age, $a_m - a_f$) by age. The model generates that age hypergamy (men marrying women younger than themselves) becomes more extreme the older men are when they marry, although this relationship is less pronounced at more advanced ages (60+) compared with what we see in the data. As in the data, we also obtain a much flatter relationship between female age at marriage and the marital age gap.

The age curvature parameter of meeting technology is low, which implies that marriage meetings are relatively uniformly distributed for absolute age differences within $\Delta a_{\max}$. And while interpreting the individual parameters in the age preference function $\eta_j(\mathbf{a})$ can be difficult, the implied patterns, which we illustrate in Figure 5 as consumption equivalents relative to a spouse of the same age, are clear: men have a preference for women younger than themselves, while women most prefer men who are either the same age or a little older than themselves. These estimated spousal age preferences are economically very significant which suggests a limited role for policies such as tax reforms to have a large impact on the age distribution of marriages. For example, relative to having a wife of the same age, the increase in utility for a man whose wife is around five years younger is equivalent to his private consumption being 50% higher. Similarly, for women aged around 40 and younger there is a decrease in the wife's utility (equivalent to her private consumption being around 50% lower) when her husband is two years younger than her. While we are agnostic regarding the source of these age preferences, the qualitative patterns are consistent with statements of preferences that have been obtained from alternative sources, such as direct survey questions (e.g., Bozon, 1991), analysis of newspaper advertisements (e.g., Kenrick and Keefe, 1992), and the stated preferences from internet dating users.[49]

An important feature of our framework is that these spousal preferences not only have implications for the patterns of marital matching in the cross-section and over the

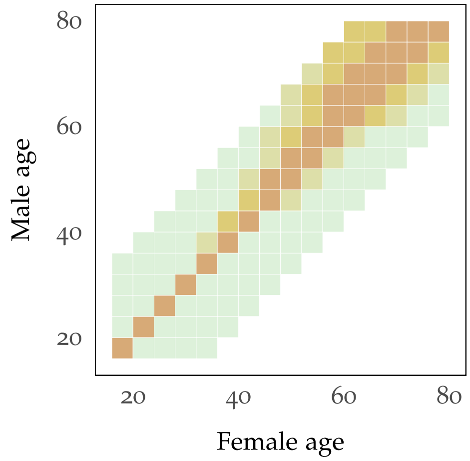
both widows and widowers. Recall that married individuals must re-enter the single pool, followed by search and matching, before they may remarry. Incorporating "on-the-marriage" search may help the model better explain the incidence of remarriage. See Burdett, Imai and Wright (2004) for a theoretical model where matched agents may undertake costly search for a different partner.

[49]Different arguments have been made for such preferences. For example, theories and evidence in the evolutionary psychology literature (e.g., Buss, 1989, Kenrick and Keefe, 1992, and Kenrick et al., 1996), argue that these spousal age preferences result from selective processes in our evolutionary past.

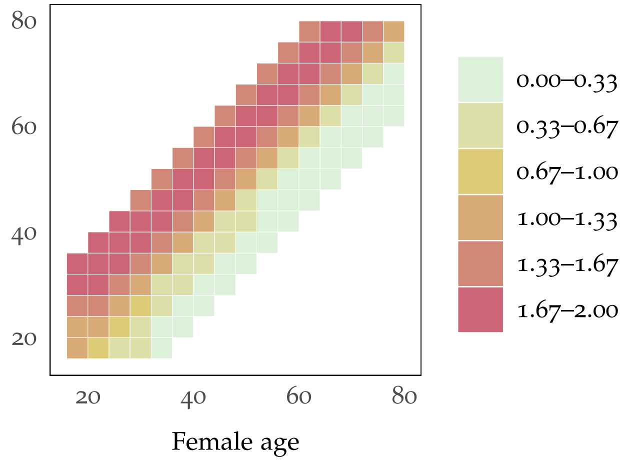


(a) Female, empirical
(b) Female, simulated
(c) Male, empirical
(d) Male, simulated

Figure 4: Life-cycle marital histories. Figure shows the empirical and simulated life-cycle marital state of women and men, categorised as *Never Married*, *First Marriage*, *Remarriage*, and *Divorced*. Divorced refers to single individuals who were previously married and includes widows and widowers. Horizontal axis measures age. Simulated moments correspond to the end-of-period state. Empirical moments calculated with ACS data.



(a) Female preference



(b) Female preference

Figure 5: Static age preference. Figure shows the estimated direct spousal age preference component for men and women, $\eta_j(\mathbf{a})$, with age aggregated into age groups of 2 (equivalent to 4 years). Preferences are expressed as consumption equivalents and are measured relative to a spouse of the same age.

life-cycle, but are also reflected in the Pareto weights within marriage and by consequence, the time allocation patterns within the household.[50] In Table 5 we show the ability of the model to generate these empirical patterns. As in the data, we obtain that male employment is relatively flat with the age-gap, whereas for married women, we obtain that female employment is lower (and her Pareto weight is higher) the older is her husband relative to her. To better understand the impact that the Pareto weights has upon these patterns of household specialisation (versus compositional differences), we simulate the model with the same reservation match values $\underline{\theta}(\mathbf{a}, \boldsymbol{\omega}, \xi)$ from the estimated model, but instead use $\lambda = \lambda_0$ when solving the time allocation problem. This results in a much flatter relationship between female employment and the marital age gap. For example, for married women aged 20–29, the female employment rate in age-similar unions ($0 \leq a_m - a_f \leq 3$) is 0.76 (compared to 0.81 from the estimated model), while in highly age-discrepant unions ($a_m - a_f \geq 12$) the female employment rate is 0.72 (significantly higher than 0.63 from the estimated model).

[50]In Figure 6 from the following section, we present the distribution of current period Pareto weights in alternative marriage-age pairings.

Table 4: Life-cycle marriage outcomes

	Women						Men					
	Div. hazard		Mar. hazard		Age diff.		Div. hazard		Mar. hazard		Age diff.	
	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model
20–29	0.07	0.10	0.13	0.15	2.78	2.89	0.06	0.09	0.10	0.11	0.40	0.30
30–39	0.05	0.04	0.14	0.14	2.14	2.62	0.05	0.05	0.15	0.15	2.43	3.49
40–49	0.05	0.03	0.07	0.09	1.82	2.67	0.04	0.03	0.08	0.11	4.42	4.45
50–59	0.02	0.02	0.04	0.05	1.67	1.88	0.03	0.02	0.05	0.08	5.86	4.61
60+	0.01	0.01	0.02	0.02	0.89	0.13	0.02	0.01	0.03	0.04	7.49	5.16

Notes: Table shows the empirical and simulated marriage and divorce hazard rates for women and men by aggregated age groups, and measures the probability that a single (married) individual will marry (divorce) over a period of two-years. *Age diff.* refers to the age gap in new marriages (defined as the husband's age less the wife's age, $a_m - a_f$). Empirical moments calculated with ACS data and adjusted to model period.

Table 5: Employment outcomes by age difference

	Female employment						Male employment					
	20–29		30–39		40–49		20–29		30–39		40–49	
	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model
(1+)	0.77	0.80	0.76	0.76	0.78	0.82	0.96	0.98	0.98	0.97	0.98	0.98
0–3	0.76	0.81	0.75	0.78	0.79	0.85	0.97	0.97	0.98	0.96	0.98	0.97
4–7	0.70	0.69	0.74	0.68	0.79	0.79	0.96	0.99	0.98	0.99	0.98	0.98
8–11	0.66	0.64	0.72	0.61	0.79	0.74	0.95	0.99	0.97	0.99	0.97	0.99
12+	0.61	0.63	0.70	0.58	0.77	0.71	–	–	0.97	0.99	0.96	0.99

Notes: Table shows employment by age, gender, and the age difference within marriage (defined as the husband's age less the wife's age, $a_m - a_f$). Age differences presented in parentheses correspond to negative age gaps. Empirical moments calculated with ACS data.

5 Age, marriage, and the gender wage gap

One of the most important ways in which the age distribution of marriages has changed over time, is the gradual narrowing of the marriage age gap. In 1960 the average marital age gap in the United States was 3.3 years. By 1980 it had fallen to 2.8 years, and it is currently 2.3 years.[51] These long-term trends in the United States are also mirrored in many other industrialised countries (United Nations, 1990).

In parallel, the gender *wage* gap has also declined, particularly since the 1980s (Blau and Kahn, 2017). Using our estimated equilibrium intertemporal limited commitment model, we provide a quantitative assessment that explores how gender wage differentials, which change the relative importance of age as a matching characteristic, affect the timing of marriage, the age structure of marriages, household specialisation patterns, and the relative bargaining weight within marriages. To this end, our exercise proceeds by changing parameters of the female wage offer function (the intercept and the return to human capital) such that gender differences in accepted (average) log wages over the life-cycle correspond to those observed in 1980.[52]

We present the impact that these wage differences have on life-cycle labour supply in Table 6. Here, and in what follows, we report changes with the 1980 gender wage differentials taken as the baseline. There are very important changes in specialisation patterns. First, we see that these changes have very pronounced effects on female labour supply. For married women, the reduction in the gender wage gap results in increased employment of between around 7 and 9 percentage points during the working life, while conditional work hours increase by the equivalent of around 3 hours per week. The same qualitative patterns are true for single women, although the magnitudes are smaller. Opposite patterns are observed for men, with conditional work hours decreased by around 1.5 hours per week. These changes are broadly consistent with the actual well-documented labour supply trends for men and women since 1980.

Any change in wages and household specialisation patterns has implications for the economic value in both singlehood and in alternative marriage pairings. This therefore changes the equilibrium of the marriage market, including the distribution of Pareto

[51] Author's calculations using 1960/1980 Census Public Use Microdata Sample data (Ruggles et al., 2017).

[52] Regalia, Ríos-Rull and Short (2019) and Ciscato (2019) also investigate how changes in wages has affected marriage outcomes, although neither relate these changes to the age structure of marriages. Here, we implement this change by modifying the wage equation from equation (18) by reducing r_{fs} by 0.09 and reducing α_{fs} by 0.15. We allow for both selection and endogenous changes to the stock of human capital when generating this gender wage gap.

Table 6: Reduction in gender wage gap: labour market outcomes

	Women		Men	
	Δ Employment	Δ Work hours	Δ Employment	Δ Work hours
Single individuals				
20–29	3.66	1.43	-0.01	-0.18
30–39	6.73	1.82	-0.01	-0.16
40–49	3.54	1.94	-0.00	-0.10
50–59	0.98	1.67	-0.00	-0.06
Married individuals				
20–29	6.85	2.54	-1.03	-1.33
30–39	8.80	2.83	-1.40	-1.54
40–49	7.74	3.15	-1.36	-1.59
50–59	6.68	3.25	-1.26	-1.62

Notes: Table shows the change in employment rates and conditional work hours as the gender wage gap is reduced. Employment changes are measured in percentage points. Hours changes are measured in hours per week.

Table 7: Reduction in gender wage gap: marriage outcomes

	Women		Men	
	Δ Marriage	Δ Age diff.	Δ Marriage	Δ Age diff.
20–29	-0.90	-0.17	-0.19	-0.01
30–39	-0.22	-0.11	0.12	-0.25
40–49	0.62	-0.06	0.52	-0.31
50–59	0.87	-0.05	0.74	-0.22
60+	0.95	-0.06	0.75	-0.12

Notes: Table shows the change in marriage rates and the average age difference in new marriages as the gender wage gap is reduced. Marriage rate changes are measured in percentage points. Age difference changes are measured in years.

weights within marriage. First we note that the average female Pareto weight increases in every marriage-age pairing. This is consistent with the evidence presented in Lise and Seitz (2011), which shows that the contemporaneous narrowing of the gender wage gap in the United Kingdom has reduced within household consumption inequality. In Figure 6 we illustrate the impact of this change on the stationary distribution of Pareto weights (approximated with a probability mass distribution) in continuing marriages. Here we show how, for alternative values of the martial age gap, the reduction in the gender wage gap (from *Base* to *Reform*) results in an improvement in the female weight. The changes are largest in relatively age-homogeneous marriages, where there is a clear shift of the distribution towards weights favouring the wife, and with more modest changes in the weight in age-discrepant marriages.

In Table 7 we then show how wage differentials matter for family formation decisions. We note the following important features. First, as the gender wage gap is reduced there is an accompanying reduction in the number of women who are married at younger ages, while the reverse is true at older ages. For women aged 20–29, there is a decrease in the marriage rate of around 1 percentage point, which represents around 20% of the actual decline over this period. Second, there are important reductions in the marital age gap over the life-cycle. Overall, the average age gap in the cross section declines by 0.15 years, which corresponds to around a third of the overall decline since the beginning of the 1980s. Taken together, these results show that the narrowing of the gender wage gap has been important not only in terms of labour market decisions, but also in terms of its impact on inequality within the household, and family formation patterns (including the marital age gap).

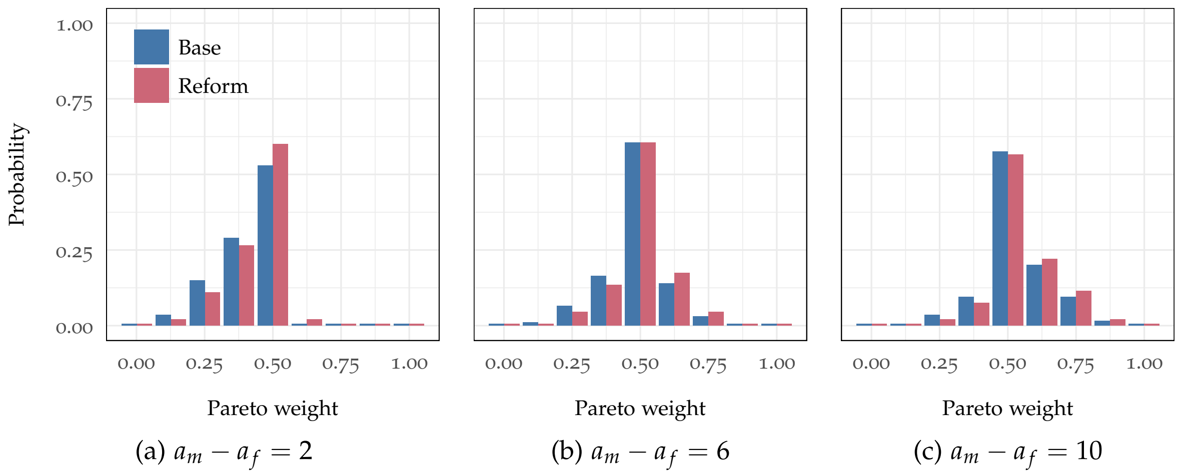


Figure 6: Pareto weight distribution. Figure shows the distribution of Pareto weights in marriages with alternative marital age differences, $a_m - a_f$, and where the Pareto weight distribution is approximated with a 9 point probability mass distribution. *Reform* corresponds to the stationary weight distribution from the estimated model. *Base* corresponds to the stationary weight distribution when the female wage process is modified to mimic the 1980 gender wage gap as described in Footnote 52.

6 Conclusion

We have presented an empirical search and matching framework for analysing intertemporal time allocation and household formation and dissolution decisions in an equilibrium limited-commitment collective framework with imperfectly transferable utility. The discrete choice framework we develop is very general: it allows for features including marriage within and across birth cohorts, persistence in the marital match component, and endogenous evolution of the state variables. In this general model we describe a series of assumptions that jointly yield a tractable model, and describe sufficient conditions to obtain existence of a stationary equilibrium.

A model with labour supply, endogenous human capital accumulation, fertility, private consumption, and public home production, is then empirically implemented using American Community Survey and Panel Study of Income Dynamics data. We impose all equilibrium conditions in estimation and show how, by virtue of characterising the equilibrium of the model, the estimation problem remains tractable. We show that the model can explain marriage patterns in the cross-section, together with the life-cycle dynamics of marriage, divorce, and remarriage. We replicate the bivariate age distribution of marriages and important gender asymmetries, including the phenomenon of age hypergamy becoming more extreme the older men are when they marry.

We use our estimated model to explore the relationship between gender wage disparities and both household behaviour and marriage outcomes. We find that the significant increase in women's relative earnings since the 1980s, simultaneously results in increased female employment, reduced male employment, an increase in the age-of-first marriage for women, and a reduction in the marital age gap. Overall, we attribute a third of the reduction in the marital age gap to the decline of the gender wage gap.

We believe that this paper represents an important step in the development of equilibrium models of life-cycle marital matching and household behaviour. While there are many potential applications of such a model, there are also important departures from this model environment that should be considered. As in the dynamic marriage matching model presented in Goussé, Jacquemet and Robin (2017), we posit a model with informational frictions and use this as a framework to understand the dynamics of marital search and matching. An important and unexplored question is the extent to which a *frictionless* marriage matching model, the leading paragim in the static matching literature, may also be able to generate similar dynamics. Similarly, we have maintained the assumption of limited commitment in the household. While the existing empirical evidence rejects full-commitment (e.g., Mazzocco, 2007), the implications of alternative household commitment assumptions should be assessed. The exploration of these and other issues is left for future research.

Appendices

A Theoretical properties and proofs

A.1 Proof of Lemma 1 and Lemma 3

In this Appendix we characterise properties of the expected value functions and provide a proof of Lemma 1 and Lemma 3. To proceed, we note that marriages end either through divorce or when one spouse dies. Therefore, when one spouse is at the terminal age $A < \infty$, continuation payoffs do not depend on the Pareto weight. The expression for a married woman's end-of-period expected value function $EV_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$, as given by equation (6), is then a closed form function of λ. Notably, it is continuously differentiable with respect to λ, and with $\partial EV_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)/\partial\lambda > 0$ following from Assumption 2. These properties hold for all female value functions by the following backward induction argument.

Claim 1. *Fixing type $(\mathbf{a}, \boldsymbol{\omega}, \xi)$, if the end-of-period expected value function $EV_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$ is continuously differentiable in λ, then so too is the start-of-period expected value function $E\widetilde{V}_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$. If in addition $\partial EV_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)/\partial\lambda > 0$ for all $\lambda \in (0, 1)$, then we also have that $\partial E\widetilde{V}_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)/\partial\lambda > 0$.*

Proof of Claim 1. Define $\theta^*_{fm}(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) = \max\{\theta^*_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda), \theta^*_m(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)\}$. The derivative of the Pareto weight transition function $\lambda^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \theta, \lambda)$ with respect to λ is

$$\frac{\partial\lambda^*}{\partial\lambda}(\mathbf{a}, \boldsymbol{\omega}, \xi, \theta, \lambda) = \begin{cases} 1 & \text{if } \theta > \theta^*_{fm}(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) \\ 0 & \text{otherwise,} \end{cases}$$

since the transition function is independent of the start-of-period Pareto weight whenever there is renegotiation. Note that by definition of $\theta^*_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$ we have $\partial\theta^*_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)/\partial\lambda = -\partial E\widetilde{V}_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)/\partial\lambda$ (and similarly for $\theta^*_m(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$). Thus, $\theta^*_{fm}(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$ is continuously differentiable in λ.

It therefore follows that the derivative of $E\widetilde{V}_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$ with respect to λ is

$$\begin{aligned} \frac{\partial E\widetilde{V}_f}{\partial\lambda}(\mathbf{a}, \boldsymbol{\omega}, \lambda, \xi) &= \int_{\theta^*_{fm}(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)} \frac{\partial EV_f}{\partial\lambda}(\mathbf{a}, \boldsymbol{\omega}, \lambda, \xi)\, \mathrm{d}H_\xi(\theta) \\ &= \frac{\partial EV_f}{\partial\lambda}(\mathbf{a}, \boldsymbol{\omega}, \lambda, \xi) \cdot \left(1 - H_\xi\left(\theta^*_{fm}(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)\right)\right), \end{aligned}$$

which is continuous in λ. Further, it is strictly positive if $\partial EV_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)/\partial\lambda > 0$ because θ has full support on the real line by Assumption 1. $\square$

Claim 2. *Fixing age $\mathbf{a} < A$, if the start-of-period expected value function $E\widetilde{V}_f(\mathbf{a}+\mathbf{1}, \boldsymbol{\omega}, \lambda, \xi)$ is continuously differentiable in λ for all types $(\boldsymbol{\omega}, \xi)$, then so too is the end-of-period expected value function $E\widetilde{V}_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$. If in addition $\partial E\widetilde{V}_f(\mathbf{a}+\mathbf{1}, \boldsymbol{\omega}, \lambda, \xi)/\partial\lambda > 0$ for all $\lambda \in (0, 1)$, then we also have that $\partial EV_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)/\partial\lambda > 0$.*

Proof of Claim 2. From equations (4) and (6), we can see that $EV_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$ is the finite sum of continuously differentiable functions, and is therefore itself continuously differentiable. If in addition $\partial E\widetilde{V}_f(\mathbf{a}+\mathbf{1}, \boldsymbol{\omega}, \lambda, \xi)/\partial\lambda > 0$ for all $\lambda \in (0, 1)$, then differentiating equation (6) immediately gives that $\partial EV_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)/\partial\lambda > 0$ as well. $\square$

Result 1 (Value function differentiability). *All end- and start-of-period value functions are continuously differentiable with respect to the Pareto weight* $\lambda \in (0,1)$*. In addition,* $(\partial EV_f/\partial\lambda, \partial E\widetilde{V}_f/\partial\lambda) \circ (\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) > 0$ *and* $(\partial EV_m/\partial\lambda, \partial E\widetilde{V}_m/\partial\lambda) \circ (\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) < 0$*.*

Proof of Result 1. This result follows from backward induction by combining Claim 1 and Claim 2 (and analogous claims for male value functions) with the fact that these properties hold for the end-of-period value functions when one at least one spouse is aged A. This result therefore establishes the proof of Lemma 1 and Lemma 3. □

A.2 Proof of Lemma 2

We prove Lemma 2 by contradiction and suppose that it does not hold. Then there exists an interval of non-zero measure on $[\underline{\theta}(\mathbf{a}, \boldsymbol{\omega}, \xi), \min\{\theta_f^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda), \theta_m^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)\}]$ where the participation constraints of both spouses are violated at λ. From Lemma 1 no change in λ can simultaneously improve both spouses value within marriage, which contradicts the definition of $\underline{\theta}(\mathbf{a}, \boldsymbol{\omega}, \xi)$.

A.3 Proof of Proposition 1

In our numerical implementation we construct an *update function* Ψ that has a fixed point if and only if there exists a stationary equilibrium. This function takes in an initial guess for $E\widetilde{V}^S$ and $\widetilde{g}^S$ and computes everything else in the model using the equilibrium equations presented in Section 2. The function then outputs an update of this guess.[53]

Definition 2. *Let* $\mathcal{V}$ *denote the space of start-of-period expected value functions for singles* $E\widetilde{V}^S$*, and let* $\mathcal{G}$ *denote the space of start-of-period measures for singles. The "update function"* $\Psi : \mathcal{V} \times \mathcal{G} \to \mathcal{V} \times \mathcal{G}$ *constructs an update according to Step 3 of Appendix B.1. Let the components of* Ψ *be denoted by the subscripts* $\Psi_V : \mathcal{V} \times \mathcal{G} \to \mathcal{V}$ *and* $\Psi_G : \mathcal{V} \times \mathcal{G} \to \mathcal{G}$*.*

The update $\Psi_V(E\widetilde{V}^S, \widetilde{g}^S)$ is the maximised start-of-period expected utility for singles if the continuation value of singlehood is characterised by $E\widetilde{V}^S$ and the spousal match probabilities η correspond with $\widetilde{g}^S$. Similarly, the update $\Psi_G(E\widetilde{V}^S, \widetilde{g}^S)$ is the measure of singles that results. We prove that Ψ has a fixed point by constructing a compact convex set $\widehat{\mathcal{V}} \times \widehat{\mathcal{G}}$ such that

[53] In this Appendix, whenever we omit gender-specific subscripts, the objects should be understood to be obtained by stacking the respective female and male objects.

1. Ψ maps $\widehat{\mathcal{V}} \times \widehat{\mathcal{G}}$ into itself, and

2. Ψ is continuous on $\widehat{\mathcal{V}} \times \widehat{\mathcal{G}}$.

Then, by Brouwer's fixed-point theorem, a stationary equilibrium exists.

A.3.1 Constructing $\widehat{\mathcal{V}}$

To bound the space of start-of-period expected value functions for singles we introduce the following definitions.

Definition 3. *Let W denote the value function in a world without marriage.*

Definition 4. *Let B_f denote the value function for a woman in a world where (1) females can choose any spouse and persistent marriage quality ξ at the start of each period and (2) the Pareto weight is always $\lambda = 1$. (Define B_m analogously with $\lambda = 0$.)*

Definition 5. *Define the set $\widehat{V}$ to be*

$$\widehat{\mathcal{V}} \equiv \left\{ E\widetilde{V}^S \in \mathcal{V} \middle| E\widetilde{W}^S \leq E\widetilde{V}^S \leq E\widetilde{B}^S \right\}.$$

Claim 3. $E\widetilde{W}^S \leq \Psi_V(E\widetilde{V}^S, \widetilde{g}^S) \leq E\widetilde{B}^S$ *for all* $(E\widetilde{V}^S, \widetilde{g}^S) \in \widehat{\mathcal{V}} \times \mathcal{G}$.

Proof of Claim 3. By definition $\Psi_V(E\widetilde{V}^S, \widetilde{g}^S)$ is the maximised start-of-period expected utility for singles if the continuation value of singlehood is characterised by $E\widetilde{V}^S$. Because $E\widetilde{W}^S \leq E\widetilde{V}^S$, singles can receive at least $E\widetilde{W}^S$ utility by remaining single another period, so their maximised utility is at least $E\widetilde{W}^S$. And, $E\widetilde{B}^S$ is the maximised utility in a relaxed problem with higher continuation payoffs ($E\widetilde{V}^S \leq E\widetilde{B}^S$) and is therefore an upper bound. □

A.3.2 Constructing $\widehat{\mathcal{G}}$

The meeting probabilities η are continuous in the measure of singles $\widetilde{g}^S$ by construction if the measure is bounded strictly away from zero for all *feasible* types.

Definition 6. *A type $(a, \boldsymbol{\omega})$ is "feasible" if there exists an initial type $(1, \boldsymbol{\omega}')$ in the support of π^0 that for some series of marriage and discrete choices can transition to $(a, \boldsymbol{\omega})$ with strictly positive probability.*

Assumption 4. *For every feasible type $(a, \boldsymbol{\omega})$ there exists a type $(1, \boldsymbol{\omega}')$ in the support of π^0 that for some series of discrete choices can transition to $(a, \boldsymbol{\omega})$ with strictly positive probability while remaining single.*

The conditional probability of remaining single $H_{\xi}(\underline{\theta}(\cdot))$ is bounded away from zero because the start-of-period expected value functions are bounded between $E\widetilde{W}^S$ and $E\widetilde{B}^S$, and the current period match quality θ spans the entire real line (Assumption 1). Similarly, the probability of any discrete choice $\mathbb{P}^S(t|\cdot)$ is also bounded away from zero given the choice-specific value functions are bounded and the associated state-specific errors $\boldsymbol{\varepsilon}_t \in \mathbb{R}^T$ have full support. Therefore, there is a lower bound on the probability any particular type remains single and chooses any series of discrete choices. Then, by Assumption 4, there is a lower bound on the measure of feasible single types that does not depend on the meeting probabilities.

Claim 4. *There exists a measure $\underline{\widetilde{g}}^S$ such that for all feasible types $(a, \boldsymbol{\omega})$*

$$\Psi_G\left(E\widetilde{V}^S, \widetilde{g}^S\right) \circ (a, \boldsymbol{\omega}) \geq \underline{\widetilde{g}}^S(a, \boldsymbol{\omega}) > 0, \quad \text{for all } (E\widetilde{V}^S, \widetilde{g}^S) \in \widehat{\mathcal{V}} \times \mathcal{G}.$$

Definition 7. *Let $\underline{\widetilde{g}}^S$ be as in Claim 4, we then define $\widehat{\mathcal{G}}$ to be the following subset of $\mathcal{G}$*

$$\widehat{\mathcal{G}} \equiv \left\{ \widetilde{g}^S \in \mathcal{G} \middle| \widetilde{g}^S(a, \boldsymbol{\omega}) \geq \underline{\widetilde{g}}^S(a, \boldsymbol{\omega}) \text{ for all feasible types } (a, \boldsymbol{\omega}) \right\}.$$

It then immediately follows by Claim 3 and Claim 4 that

Result 2 (Self-map)**.** *Ψ maps $\widehat{\mathcal{V}} \times \widehat{\mathcal{G}}$ into itself.*

The following additional assumption ensures that equations (9a) and (9b) for the meeting probabilities η are well-defined and continuous on $\widehat{\mathcal{G}}$ by bounding the denominator strictly away from zero.

Assumption 5. *For every feasible type $(a, \boldsymbol{\omega})$ there exists a match $(\mathbf{a}, \boldsymbol{\omega})$ with another feasible type such that $\gamma(\mathbf{a}, \boldsymbol{\omega}) > 0$.*

Henceforth, the domain of the start-of-period single measures $\widetilde{g}^S$ is treated as $\widehat{\mathcal{G}}$.

A.3.3 Constructing Λ

The objective of this section is to show that the Pareto weights arising from value functions in $\widehat{\mathcal{V}}$ all lie in a closed interval $\Lambda = [\underline{\lambda}, \overline{\lambda}] \subset (0, 1)$. This compactification greatly simplifies concepts of continuity in the next section.

Definition 8. *Model objects are determined by $E\widetilde{V}^S$ and $\widetilde{g}^S$ through a backward induction procedure as described in Appendix B.1. We denote these objects as $EV(\cdot|E\widetilde{V}^S)$, $\lambda^*(\cdot|E\widetilde{V}^S)$, and $\eta(\cdot|\widetilde{g}^S)$, etc. For example, $EV(\cdot|E\widetilde{V}^S)$ refers to the function EV over the arguments $(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$ conditional on a fixed $E\widetilde{V}^S$.*

Claim 5. *There exists $\Lambda = [\underline{\lambda}, \overline{\lambda}] \subset (0, 1)$ with $\lambda_0 \in \Lambda$ such that the Pareto weight transition function $\lambda^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \theta, \lambda|E\widetilde{V}^S) \in \Lambda \cup \emptyset$ for all types $(\mathbf{a}, \boldsymbol{\omega}, \xi, \theta)$, $\lambda \in \Lambda$, and $E\widetilde{V}^S \in \widehat{\mathcal{V}}$.*

Proof of Claim 5. Choose $\underline{\lambda}(\mathbf{a}, \boldsymbol{\omega}, \xi) > 0$ to be sufficiently small such that

$$
\begin{aligned}
v_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \underline{\lambda}(\mathbf{a}, \boldsymbol{\omega}, \xi)) + \beta \max\nolimits_{\{\omega'_f\}} E\widetilde{B}_f(a_f + 1, \boldsymbol{\omega}'_f)& \\
< v_m(\mathbf{a}, \boldsymbol{\omega}, \xi, \underline{\lambda}(\mathbf{a}, \boldsymbol{\omega}, \xi)) + \beta \min\nolimits_{\{\omega'_m\}} E\widetilde{W}_m(a_m + 1, \boldsymbol{\omega}'_m)&
\end{aligned}
$$

There exists such a $\underline{\lambda}(\mathbf{a}, \boldsymbol{\omega}, \xi) > 0$ because we have $\lim_{\lambda \to 0} v_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) = -\infty$ while $v_m(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$ remains bounded. By construction, $\theta^*_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda|E\widetilde{V}^S) > \theta^*_m(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda|E\widetilde{V}^S)$ for any $\lambda \leq \underline{\lambda}(\mathbf{a}, \boldsymbol{\omega}, \xi)$ and $E\widetilde{V}^S \in \widehat{\mathcal{V}}$. Therefore, $\lambda^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \theta, \lambda|E\widetilde{V}^S) \geq \lambda$ for any $\lambda \leq \underline{\lambda}(\mathbf{a}, \boldsymbol{\omega}, \xi)$, $E\widetilde{V}^S \in \widehat{\mathcal{V}}$, and $\theta \in \mathbb{R}$. In addition, $\lambda^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \theta, \lambda|E\widetilde{V}^S) \geq \underline{\lambda}(\mathbf{a}, \boldsymbol{\omega}, \xi)$ because λ^* is weakly increasing in λ. Setting $\underline{\lambda} \equiv \max_{\{\mathbf{a}, \boldsymbol{\omega}, \xi\}} \underline{\lambda}(\mathbf{a}, \boldsymbol{\omega}, \xi)$ gives the desired lower bound. An upper bound $\overline{\lambda}$ is constructed symmetrically. □

In what follows the domain of Pareto weights λ is taken to be Λ.

A.3.4 Continuity of Ψ

The proof that Ψ is continuous follows the same backward induction argument as in Appendix A.1. In what follows, Claim 6 proves that $EV(\cdot|\mathbf{a}, E\widetilde{V}^S)$ is continuous in $E\widetilde{V}^S$ if $EV(\cdot|\mathbf{a} + \mathbf{1}, E\widetilde{V}^S)$ is continuous in $E\widetilde{V}^S$, and Claim 9 proves that $E\widetilde{V}(\cdot|\mathbf{a}, E\widetilde{V}^S)$ is continuous in $E\widetilde{V}^S$ if $EV(\cdot|\mathbf{a}, E\widetilde{V}^S)$ is continuous in $E\widetilde{V}^S$. Together these imply $E\widetilde{V}(\cdot|\mathbf{a}, E\widetilde{V}^S)$ is continuous in $E\widetilde{V}^S$ for all ages and therefore Ψ is continuous.[54]

In the following we use the discrete metric for types $(\mathbf{a}, \boldsymbol{\omega}, \xi)$, and the max Euclidean

[54] A complication arises because $E\widetilde{V}(\cdot|\mathbf{a}, E\widetilde{V}^S)$ with respect to $E\widetilde{V}^S$ is a function over functions for which continuity is defined with respect to the sup-norm. Fortunately, since all the functions have been restricted to compact domains, pointwise continuity implies uniform continuity, which in turn implies continuity of functions in $E\widetilde{V}^S$ under the sup-norm. The proof therefore naively proceeds by arguing only that the equilibrium equations for $E\widetilde{V}$ and EV are pointwise continuous.

metric for λ and $E\widetilde{V}^S$. For example, the distance metric d over the entire space is

$$d\left((\mathbf{a},\boldsymbol{\omega},\xi,\lambda,E\widetilde{V}^S),(\mathbf{a}',\boldsymbol{\omega}',\xi',\lambda',E\widetilde{V}^{S\prime})\right) \\ = \max\left\{\mathbb{1}\{(\mathbf{a},\boldsymbol{\omega},\xi)\neq(\mathbf{a}',\boldsymbol{\omega}',\xi')\},|\lambda-\lambda'|,|E\widetilde{V}^S-E\widetilde{V}^{S\prime}|\right\}.$$

And where we note that we restrict the domain of $E\widetilde{V}^S$ to $\widehat{\mathcal{V}}$, $\widetilde{g}^S$ to $\widehat{\mathcal{G}}$, and λ to Λ. All equilibrium objects now explicitly depend on $E\widetilde{V}^S$ and/or $\widetilde{g}^S$. Continuity of a function $EV(\cdot|\mathbf{a})$ refers to pointwise continuity of the arguments $(\boldsymbol{\omega},\xi,\lambda,E\widetilde{V}^S)$ for a fixed $\mathbf{a}$ unless otherwise stated.

Claim 6. *$EV(\cdot|\mathbf{a})$ is continuous if $E\widetilde{V}(\cdot|\mathbf{a}+\mathbf{1})$ is continuous.*

Proof of Claim 6. Given that $E\widetilde{V}(\cdot|\mathbf{a}+\mathbf{1})$ is continuous, the choice-specific value function $V(\cdot|\mathbf{a})$ in equation (4) is a finite sum of continuous functions and is therefore itself continuous. This in turn implies that $EV(\cdot|\mathbf{a})$ in equation (6) is continuous since it is a continuous function of the finitely many values $\{V(\mathbf{t};\boldsymbol{\omega},\xi,\lambda,E\widetilde{V}^S|\mathbf{a})\}_{\mathbf{t}\in\mathcal{T}}$. □

To proceed in the other direction, we first note that equation (10) for $E\widetilde{V}(\cdot|\mathbf{a})$ is continuous only if the reservation match value $\underline{\theta}(\cdot|\mathbf{a})$ is continuous and the Pareto weight transition function $\lambda^*(\cdot|\mathbf{a})$ is uniformly continuous. (Note that λ^* depends on $\theta\in\mathbb{R}$, so pointwise continuity does not necessarily imply uniform continuity.)

Definition 9. *Define $\underline{\theta}^*_{fm}$ and $\overline{\theta}^*_{fm}$ as*

$$\underline{\theta}^*_{fm}(\mathbf{a},\boldsymbol{\omega},\xi,\lambda,E\widetilde{V}^S)=\min\left\{\theta^*_f(\mathbf{a},\boldsymbol{\omega},\xi,\lambda,E\widetilde{V}^S),\ \theta^*_m(\mathbf{a},\boldsymbol{\omega},\xi,\lambda,E\widetilde{V}^S)\right\},$$
$$\overline{\theta}^*_{fm}(\mathbf{a},\boldsymbol{\omega},\xi,\lambda,E\widetilde{V}^S)=\max\left\{\theta^*_f(\mathbf{a},\boldsymbol{\omega},\xi,\lambda,E\widetilde{V}^S),\ \theta^*_m(\mathbf{a},\boldsymbol{\omega},\xi,\lambda,E\widetilde{V}^S)\right\}.$$

The functions θ^*, $\underline{\theta}^*_{fm}$, and $\overline{\theta}^*_{fm}$ are all continuous if $EV(\mathbf{a},\boldsymbol{\omega},\xi,\lambda,E\widetilde{V}^S)$ is continuous.

Claim 7. *$\underline{\theta}(\cdot|\mathbf{a})$ is continuous if $EV(\cdot|\mathbf{a})$ is continuous.*

Proof of Claim 7. Fix $(\boldsymbol{\omega},\xi,E\widetilde{V}^S)$, by equation (7) it follows that for any λ we have

$$\underline{\theta}^*_{fm}(\mathbf{a},\boldsymbol{\omega},\xi,\lambda,E\widetilde{V}^S)\leq\underline{\theta}(\mathbf{a},\boldsymbol{\omega},\xi,E\widetilde{V}^S)\leq\overline{\theta}^*_{fm}(\mathbf{a},\boldsymbol{\omega},\xi,\lambda,E\widetilde{V}^S).$$

The Intermediate Value Theorem implies that for each $(\mathbf{a},\boldsymbol{\omega},\xi,E\widetilde{V}^S)$ there exists a unique $\hat{\lambda}\in\Lambda$ such that θ^*_f and θ^*_m are equal. That is, $\underline{\theta}^*_{fm}(\mathbf{a},\boldsymbol{\omega},\xi,\hat{\lambda},E\widetilde{V}^S)=\underline{\theta}(\mathbf{a},\boldsymbol{\omega},\xi,E\widetilde{V}^S)=$

$\overline{\theta}^*_{fm}(\mathbf{a},\boldsymbol{\omega},\xi,\hat{\lambda},E\widetilde{V}^S)$. The above inequality then implies for any $(\boldsymbol{\omega}',\xi',E\widetilde{V}^{S\prime})$ that

$$\left|\underline{\theta}(\mathbf{a},\boldsymbol{\omega}',\xi',E\widetilde{V}^{S\prime})-\underline{\theta}(\mathbf{a},\boldsymbol{\omega},\xi,E\widetilde{V}^S)\right| \leq \max\Big\{\left|\underline{\theta}^*_{fm}(\mathbf{a},\boldsymbol{\omega}',\xi',\hat{\lambda},E\widetilde{V}^{S\prime})-\underline{\theta}^*_{fm}(\mathbf{a},\boldsymbol{\omega},\xi,\hat{\lambda},E\widetilde{V}^S)\right|, \\ \left|\overline{\theta}^*_{fm}(\mathbf{a},\boldsymbol{\omega}',\xi',\hat{\lambda},E\widetilde{V}^{S\prime})-\overline{\theta}^*_{fm}(\mathbf{a},\boldsymbol{\omega},\xi,\hat{\lambda},E\widetilde{V}^S)\right|\Big\}$$

Therefore, $\underline{\theta}(\cdot|\mathbf{a})$ is continuous at $(\boldsymbol{\omega},\xi,E\widetilde{V}^S)$ since both $\underline{\theta}^*_{fm}$ and $\overline{\theta}^*_{fm}$ are continuous. □

Claim 8. *$\lambda^*(\cdot|\mathbf{a})$ is uniformly continuous if $EV(\cdot|\mathbf{a})$ is continuous.*

Proof of Claim 8. Whenever $\theta \geq \overline{\theta}^*_{fm}(\mathbf{a},\boldsymbol{\omega},\xi,\lambda,E\widetilde{V}^S)$, the Pareto weight transition function reduces to the identity map, i.e. $\lambda^*(\mathbf{a},\boldsymbol{\omega},\xi,\lambda,E\widetilde{V}^S) = \lambda$. Alternatively, the set of points where $\underline{\theta}(\mathbf{a},\boldsymbol{\omega},\xi,\lambda,E\widetilde{V}^S) \leq \theta \leq \overline{\theta}^*_{fm}(\boldsymbol{\omega},\xi,\lambda,E\widetilde{V}^S)$ is compact since $\underline{\theta}$ and $\overline{\theta}^*_{fm}$ are continuous over their compact domain. Pointwise continuity of λ^* follows from its definition in equation (8) since both θ^*_f and θ^*_m are continuous and strictly monotonic in λ. Pointwise continuity implies uniform continuity over the compact set, which then implies uniform continuity over all points where $\theta \geq \underline{\theta}(\mathbf{a},\boldsymbol{\omega},\xi,\lambda,E\widetilde{V}^S)$. (Note that λ^* is not defined for points below $\underline{\theta}$ as such marriages are not formed.) □

Claim 9. *$E\widetilde{V}(\cdot|\mathbf{a})$ is continuous if $EV(\cdot|\mathbf{a})$ is continuous.*

Proof of Claim 9. Claims 7 and 8 together with θ being a continuous random variable imply that equation (10), which defines $E\widetilde{V}(\cdot|\mathbf{a})$, is continuous if $EV(\cdot|\mathbf{a})$ itself is continuous. □

Result 3 (Continuity of Ψ)**.** *$E\widetilde{V}$, EV, $\underline{\theta}$, and λ^* are all uniformly continuous in their respective $\mathbf{a}$, $\boldsymbol{\omega}$, ξ, λ, θ, and $E\widetilde{V}^S$. Therefore, Ψ is continuous on $\widehat{\mathcal{V}} \times \widehat{\mathcal{G}}$.*

Proof of Result 3. $E\widetilde{V}$, EV, $\underline{\theta}$, and λ^* are continuous by a backward induction argument using Claims 7–9. Equation (11) which defines $E\widetilde{V}^S$ is then a finite sum of continuous functions and therefore Ψ_V is continuous.

We can now argue by forward induction that the transitions of all measures g are (weakly) continuous and therefore Ψ_G is continuous. (1) The probability of a given match η is continuous, and the conditional probability of marriage is also continuous (because θ is continuously distributed by assumption and because $\underline{\theta}$ is continuous). (2) Uniform continuity of λ^* implies weak continuity of married measures during renegotiation. And (3), continuity of $E\widetilde{V}$ implies continuity in the conditional choice proabilities $\mathbb{P}$. Therefore, the new measures Ψ_G generated by forward induction on the initial cohort are continuous. □

A.3.5 Existence of a fixed point

Under Assumptions 1–5 we have that the function Ψ maps $\widehat{\mathcal{V}} \times \widehat{\mathcal{G}}$ into itself (by Result 2) and is continuous (by Result 3). Since $\widehat{\mathcal{V}} \times \widehat{\mathcal{G}}$ is a compact convex set, Ψ has a fixed point by Brouwer's theorem, which implies that a stationary equilibrium exists. This therefore establishes a proof of Proposition 1.

B Numerical implementation

B.1 Model Solution

Step 1: Compute known objects. Given a known terminal value, end-of-period value functions for single individuals aged A, and for couples where both spouses are aged A can be calculated outside of the iterative loop. Similarly, age-1 start-of-period measures of single women and single men are known by assumption.

Step 2: Initialization. Provide initial guesses for *(i)* the start-of-period measures of single women and men, $\widetilde{g}^S_f(a_f, \boldsymbol{\omega}_f)$ and $\widetilde{g}^S_m(a_m, \boldsymbol{\omega}_m)$, that are not known from Step 1; and *(ii)* the expected start-of-period value functions when single $E\widetilde{V}^S_f(a_f, \boldsymbol{\omega}_f)$ and $E\widetilde{V}^S_m(a_m, \boldsymbol{\omega}_m)$.

Step 3: Iteration. The iteration step mirrors the *update mapping* that we describe in Appendix A.3. Iterate over the start-of-period expected value functions for single individuals, and the start-of-period measures of singles, using the following sequence:

a. **Single value functions**. Calculate end-of-period single expected value functions $EV^S_f(a_f, \boldsymbol{\omega}_f)$ and $EV^S_m(a_m, \boldsymbol{\omega}_m)$ for $a_f, a_m < A$ using the current guess for $E\widetilde{V}^S_f(a_f + 1, \boldsymbol{\omega}'_f)$ and $E\widetilde{V}^S_m(a_m + 1, \boldsymbol{\omega}'_m)$, and the state transition functions. These calculations also imply the conditional choice probabilities $\mathbb{P}^S_f(t_f; a_f, \boldsymbol{\omega}_f)$ and $\mathbb{P}^S_m(t_m; a_m, \boldsymbol{\omega}_m)$.

b. **Couples value functions, main diagonal**. Backward induct along the main diagonal where $a_f = a_m = a$. For $a = A-1, \ldots, 1$, compute end-of-period expected value functions $(EV_f, EV_m) \circ (\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$ given $(E\widetilde{V}_f, E\widetilde{V}_m) \circ (\mathbf{a} + \mathbf{1}, \boldsymbol{\omega}', \xi', \lambda)$, and the state transition functions. From this calculate the threshold values, $\underline{\theta}(\mathbf{a}, \boldsymbol{\omega}, \xi)$, $\theta^*_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$, and $\theta^*_m(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$, as well as the transition function $\lambda^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \theta, \lambda)$, and the couples' start-of-period expected value functions $(E\widetilde{V}_f, E\widetilde{V}_m) \circ (\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$. These imply the conditional choice probabilities, $\mathbb{P}(\mathbf{t}; \mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$.

c. **Couples value functions, off-diagonal**. For the age difference $\Delta a = 1, 2, \ldots, A-1$ compute the end-of-period expected value functions $EV_f([A-\Delta a, A], \boldsymbol{\omega}, \xi, \lambda)$ and $EV_m([A, A-\Delta a], \boldsymbol{\omega}, \xi, \lambda)$, exploiting that someone who is married to a spouse aged A today will be single next period. Then calculate the associated threshold match values as in Step 3b and the expected start-of-period expected value functions $(E\widetilde{V}_f, E\widetilde{V}_m) \circ ([A-\Delta a, A], \boldsymbol{\omega}, \xi, \lambda)$, and $(E\widetilde{V}_f, E\widetilde{V}_m) \circ ([A, A-\Delta a], \boldsymbol{\omega}, \xi, \lambda)$. Conditional on the age difference Δa iterate backwards with $a = A-1, \ldots, \Delta a + 1$, and calculate objects for both $(a - \Delta a, a)$ and $(a, a - \Delta a)$ marriage pairings.

d. **Update single measures**. Calculate the end-of-period measure of marriage matches where at least one spouse is aged 1 using equation (16) and the current guess of the start-of-period single measures. (Note that the start-of-period measure of such couples is known and is identically zero.) This gives $g^M([1, a], \boldsymbol{\omega}, \xi, \lambda)$ and $g^M([a, 1], \boldsymbol{\omega}, \xi, \lambda)$ for all $a \leq A$. Then, calculate $\widetilde{g}^M([2, a+1], \boldsymbol{\omega}, \xi, \lambda)$ and $\widetilde{g}^M([a+1, 2], \boldsymbol{\omega}, \xi, \lambda)$ for $a \leq A-1$ using equation (14) together with the state transition functions and the conditional choice probabilities. Repeated forward induction yields the complete start-of-period and end-of-period measure of matches. From these, updates of the start-of-period single measures can be obtained using equations (13) and (15).[55]

e. **Update single expected value function**. The current start-of-period single measures allows the meeting probabilities $\eta_f(\mathbf{a}, \boldsymbol{\omega})$ and $\eta_m(\mathbf{a}, \boldsymbol{\omega})$ to be calculated. These, together with the *end-of-period* expected value functions for single women and men (from Step 3a), $EV^S_f(a_f, \boldsymbol{\omega}_f)$ and $EV^S_m(a_m, \boldsymbol{\omega}_m)$, and the *start-of-period* expected values in marriage (from Step 3b and 3c), provides updated start-of-period expected values for single women and men, $E\widetilde{V}^S_f(a_f, \boldsymbol{\omega}_f)$ and $E\widetilde{V}^S_m(a_m, \boldsymbol{\omega}_m)$.

The distance between the updated and previous expected value functions and single measures is evaluated. If it is less than the specified tolerance δ_{tol} then terminate the iteration loop. Otherwise, return to Step 3a. Calculating expected value functions, and the measure of both single and matched individuals is central to our procedure. In Appendix B.2 we describe the numerical calculation of the start-of-period expected value functions, while in Appendix B.3 we describe the calculation of the match distribution.

[55]The requirement to iterate over both the start-of-period expected value functions and measures in Step 3 arises due to across-cohort marriage matching. Absent this feature, we are able to forward induct only using the initial (exogenous) start-or-period cohort measures as specified in equations (12a)–(12c). We can then proceed to update the start-of-period single expected value function as described in Step 3e.

Remark. An important feature of our model is that we allow household choices to influence the evolution of the state variables. This implies that the joint allocation conditional choice probabilities $\mathbb{P}(\mathbf{t}; \mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$ are required when updating the single measures in Step 3d. Given limitations on the availability of computer memory, this object may be prohibitively large in some applications as it contains T^2 more elements than the couples' expected value functions and measures. To avoid storing this full object, we can instead iterate on the *end-of-period* measures of single individuals *and* married couples. This then allows the *start-of-period* measures to be calculated during the backward induction phase.

B.2 Calculating start-of-period expected values

With the exception of the Pareto weight, all state variables are discrete. We implement the Pareto weight by constructing an ordered λ-grid, which takes the values $\boldsymbol{\lambda}_{\text{grid}} = [\lambda^1, \ldots, \lambda^L]$, with $\lambda^1 \gtrapprox 0$ and $\lambda^L \lessapprox 1$. Consider the calculation of the female start-of-period expected value $E\widetilde{V}_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$ with $\lambda \in \boldsymbol{\lambda}^{\text{grid}}$. In the case where $\theta_f^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) \geq \theta_m^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$, as we increase θ the man's participation constraint is satisfied before the woman's and so the expected value function from equation (10) simplifies to

$$\begin{aligned}E\widetilde{V}_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) = {} & H_\xi(\theta_f^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda))EV_f^S(a_f, \boldsymbol{\omega}_f) \\ & + \overline{H}_\xi(\theta_f^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)) \left[EV_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) + \mathbb{E}[\theta | \theta \geq \theta_f^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)\right].\end{aligned}$$

As the above involves known value functions, and (for a known distribution of marital shocks) the evaluation of a cumulative distribution function and a partial expectation, the calculation in this case is straightforward.

The more complicated case is when the woman's participation constraint is satisfied first, i.e., $\theta_f^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) < \theta_m^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$, as for $\theta \in [\underline{\theta}(\mathbf{a}, \boldsymbol{\omega}, \xi), \theta_f^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)]$ it becomes necessary to calculate the female expected value function with the Pareto weight adjusting to $\lambda^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \theta, \lambda) = \lambda_m^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \theta)$. Equation (10) in this case becomes

$$\begin{aligned}E\widetilde{V}_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) = {} & H_\xi(\underline{\theta}(\mathbf{a}, \boldsymbol{\omega}, \xi))EV_f^S(a_f, \boldsymbol{\omega}_f) \\ & + \int\limits_{\underline{\theta}(\mathbf{a}, \boldsymbol{\omega}, \xi)}^{\theta_m^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)} \left[EV_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda_m^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \theta)) + \theta\right] \mathrm{d}H_\xi(\theta) \\ & + \overline{H}_\xi(\theta_m^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)) \left[EV_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda) + \mathbb{E}\left[\theta | \theta \geq \theta_m^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)\right]\right].\end{aligned}$$

In practice we calculate the second term in this equation by first obtaining the Pareto weight $\underset{\sim}{\lambda}(\mathbf{a}, \boldsymbol{\omega}, \xi)$ that is associated with the reservation match value $\underline{\theta}(\mathbf{a}, \boldsymbol{\omega}, \xi)$. Under Assumption 3 both participation constraints simultaneously bind at the reservation match value, such that $\underset{\sim}{\lambda}(\mathbf{a}, \boldsymbol{\omega}, \xi)$ can therefore be obtained as the unique solution to $EV_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \underset{\sim}{\lambda}(\mathbf{a}, \boldsymbol{\omega}, \xi)) - EV_f^S(a_f, \omega_f) = EV_m(\mathbf{a}, \boldsymbol{\omega}, \xi, \underset{\sim}{\lambda}(\mathbf{a}, \boldsymbol{\omega}, \xi)) - EV_m^S(a_m, \omega_m)$. The reservation match value $\underline{\theta}(\mathbf{a}, \boldsymbol{\omega}, \xi)$ can then be obtained using the participation constraint of either spouse. Conditional on $(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$, we then construct an ordered θ-subgrid, which takes values $\boldsymbol{\theta}_{\text{subgrid}} = [\theta_s^1, \dots, \theta_s^{L_s}]$, with $\theta_s^1 = \underline{\theta}(\mathbf{a}, \boldsymbol{\omega}, \xi)$ and $\theta_s^{L_s} = \theta_m^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$. For each $\theta_s^l \in \boldsymbol{\theta}_{\text{subgrid}}$ we construct an interpolating function to obtain $EV_f(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda_s^l)$, where $\lambda_s^l = \lambda_m^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \theta_s^l)$ is such that the male's participation constraint binds at θ_s^l. The integral is then evaluated using Newton-Cotes quadrature rules. The calculation of the male start-of-period expected value function $E\widetilde{V}_m(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda)$ proceeds similarly.

B.3 Calculating the measure of matches

It is not possible to calculate the measure of marriage matches exactly as the Pareto weight is a continuous state variable. Instead, in characterising these measures we construct a discrete probability distribution over $\boldsymbol{\lambda}_{\text{grid}}$. Consider a couple with an initial Pareto weight λ^j. If $\theta \geq \max\{\theta_f^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda^j), \theta_m^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda^j)\}$ then both participation constraints are satisfied and the Pareto weight remains unchanged.

Suppose instead that $\underline{\theta}(\mathbf{a}, \boldsymbol{\omega}, \xi) \leq \theta < \max\{\theta_f^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda^j), \theta_m^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda^j)\}$. In this case the couple will remain married, but the Pareto weight will adjust. In practice we adjust the weight in the woman's favour from λ^j to $\lambda^i > \lambda^j$ for all values of θ such that $\theta \geq \underline{\theta}(\mathbf{a}, \boldsymbol{\omega}, \xi)$ and $\theta_f^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda^i) < \theta \leq \theta_f^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda^{i-1})$. Similarly, we adjust the weight in the man's favour from λ^j to $\lambda^i < \lambda^j$ for all values of θ such that $\theta \geq \underline{\theta}(\mathbf{a}, \boldsymbol{\omega}, \xi)$ and $\theta_m^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda^{i+1}) < \theta \leq \theta_m^*(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda^i)$. Suppressing the explicit conditioning on the other state variables, $(\mathbf{a}, \boldsymbol{\omega}, \xi)$, we therefore have the following law-of-motion

$$\mathbb{P}_\lambda[\lambda^i|\lambda^j] = \begin{cases} \overline{H}_\xi(\max\{\theta_f^*(\lambda^i), \theta_m^*(\lambda^i)\}) & \text{if } \lambda^i = \lambda^j \\ H_\xi(\max\{\theta_f^*(\lambda^{i-1}), \underline{\theta}\}) - H_\xi(\max\{\theta_f^*(\lambda^i), \underline{\theta}\}) & \text{if } \lambda^i > \lambda^j \\ H_\xi(\max\{\theta_m^*(\lambda^{i+1}), \underline{\theta}\}) - H_\xi(\max\{\theta_m^*(\lambda^i), \underline{\theta}\}) & \text{if } \lambda^i < \lambda^j. \end{cases}$$

Note that by construction we have $\sum_i \mathbb{P}_\lambda[\lambda^i|\lambda^j; \mathbf{a}, \boldsymbol{\omega}, \xi] = \overline{H}_\xi(\underline{\theta}(\mathbf{a}, \boldsymbol{\omega}, \xi))$ for all $j = 1, \dots, L$. Using $\mathbb{P}_\lambda[\lambda^i|\lambda^j; \mathbf{a}, \boldsymbol{\omega}, \xi]$ we then calculate the end-of-period measure of matches for all

$(\mathbf{a}, \boldsymbol{\omega}, \xi)$ and $\lambda^i \in \boldsymbol{\lambda}_{\text{grid}}$ in equation (16) as

$$g^M(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda^i) = \widetilde{g}^S_f(a_f, \boldsymbol{\omega}_f) \cdot \eta_f(\mathbf{a}, \boldsymbol{\omega}) \cdot \mathbb{P}_\lambda[\lambda^i | \lambda^{i_0}; \mathbf{a}, \boldsymbol{\omega}, \xi] \cdot b_0(\xi) \\ + \sum_j \mathbb{P}_\lambda[\lambda^i | \lambda^j; \mathbf{a}, \boldsymbol{\omega}, \xi] \cdot \widetilde{g}^M(\mathbf{a}, \boldsymbol{\omega}, \xi, \lambda^j),$$

where i_0 is the index of $\boldsymbol{\lambda}_{\text{grid}}$ corresponding to λ_0.

C Estimation moments

In this appendix we describe the set of estimation moments used to identify the model. First, we define the following conditioning sets: *(a). Children*: no children; and at least one child. *(b). Youngest child*: no children; one child aged 0–5; one child aged 6–11; one child aged 12+; two or more children, youngest child aged 0–5; two or more children, youngest child aged 6–11; and two or more children, youngest child aged 12+. *(c). Education*: less than college; and college and above. *(d). Marital status*: single; and married. *(e). Marriage duration*: four year duration bins starting with 2–5 years and ending with 34+ years duration. *(f). Age*: four year age bins starting with ages 18–21 and ending with ages 78–81; *(g). Age group*: ten year age bins starting with ages 20–29 and ending with ages 50–59, followed by ages 60 and above. *(h). Working age group*: ten year age bins starting with ages 20–29 and ending with ages 50–59. *(i). Age difference*: the difference $a_m - a_f$ starting no greater than (negative) 5 years, and proceeding in four year age bin starting from (negative) 4–1 years to (positive) 11–15 years, followed by (positive) 16+ years. *(j). Hours*: part-time (weekly hours no greater than 30); full-time (weekly hours exceeding 30); *(k). Sex*: female; and male.

C.1 List of moments

(1). Cross-sectional marriage matching patterns: marriage matching function by female *education* and male *education*; marriage matching function by female *age* and male *age*. **(2). Marriage and divorce dynamics and history**: divorce hazard rate by *children*; divorce hazard rate by *marriage duration*; divorce hazard rate by *age difference*; marriage hazard rate by *age group*, *education*, and *sex*; female marriage hazard rate by *working age group* and *children*; divorce hazard rate by *age group*, *education*, and *sex*; never married rate by *age* and *sex*; first marriage rate by *age* and *sex*; remarriage rate by *age* and *sex*; divorced rate (including widows and widowers) by *age* and *sex*; new marriage age gap (mean) by *age group* and *sex*; new marriage age gap (standard deviation) by *age group* and *sex*. **(3). Labour supply:** employment by *youngest child*, *marital status*, *education* and *sex*; mean conditional work hours by *youngest child*, *marital status*, *education*, and *sex*; employment by *working age group*, *marital status*, *education*, and *sex*; conditional work hours (mean) by *working age group*, *marital status*, *education* and *sex*; conditional work hours (standard deviation) by *working-age group*, *marital status*, *education*, and *sex*; employment by *age difference*, *working-age group* and *sex*. **(4). Labour supply dynamics:** non-employment to employment transition rates by *education* and *sex*; employment to non-employment transition rates by *education* and *sex*. **(5). Home time:** home production hours (mean) by *youngest child*, *education*, and *sex*; home production hours (standard deviation) by *youngest child*, *education* and *sex*. **(6). Wages**: log wages (mean) by *hours*, *education*, and *sex*; log wages (standard deviation) by *hours*, *education*, and *sex*; wages (mean) by *working-age group*, *marital status*, *education*, and *sex*; wages (standard deviation) by *working-age group*, *marital status*, *education*, and *sex*. **(7). Wage dynamics**: one-period log-wage changes (mean) by *hours*, *education* and *sex*; one-period log-wage changes (standard deviation) by *hours*, *education*, and *sex*; log-wages from non-employment (mean) by *education* and *sex*; log-wages from non-employment (standard deviation) by *education* and *sex*; log-wage linear regression model coefficients (constant, experience, experience squared, standard deviation of residual) by *education* and *sex*.[56]

D Additional tables and results

In Table D.1 we present the parameter estimates and accompanying standard errors. Table D.2 presents model fit to the marriage matching function by age. Table D.3 shows model fit to the cross-sectional labour supply patterns for different demographic groups, while in Table D.4 we similarly report cross-sectional home-time patterns.

[56]There are a small number of additional restrictions on the conditioning sets. For example, as education is a fixed characteristic, and the model begins at age 18, when constructing our theoretical moments we only include college-educated individuals age 22 and above. In Footnote 46 we describe the default weighting that we apply to this set of moments when evaluating our estimation criterion function. To help emphasise certain moments of interests, we increase the weight (relative to the default) on employment by *age difference*, *working-age group* and *sex*, and new marriage age gap (mean/standard deviation) by *age group* and *sex*, by a factor of around four.

Table D.1: Parameter estimates

		Estimate	Standard Error
Preference parameters			
σ_q	Consumption curvature	1.010	0.003
σ_Q	Home good curvature	0.182	0.032
β_Q	Home good scale	1.574	0.091
$\nu_f(\ell_2)$	Female leisure, *medium*	0.028	0.021
$\nu_f(\ell_3)$	Female lesiure, *high*	1.439	0.008
ν_{fm}	Female leisure, *spouse works*	0.440	0.017
$\nu_m(\ell_2)$	Male leisure, *medium*	0.001	0.032
$\nu_m(\ell_3)$	Male lesiure, *high*	1.468	0.011
ν_{mf}	Male leisure, *spouse works*	0.284	0.011
σ_ε	State specific error s.d.	0.254	0.005
Wages and human capital			
$r_{f,\text{nc}}$	Female intercept, *no college*	2.068	0.007
$r_{f,\text{col}}$	Female intercept, *college*	2.405	0.009
$\alpha_{f,\text{nc}}$	Female human capital slope, *no college*	0.295	0.003
$\alpha_{f,\text{col}}$	Female human capital slope, *college*	0.361	0.003
$\sigma_{f,\text{nc}}$	Female residual s.d., *no college*	0.225	0.005
$\sigma_{f,\text{col}}$	Female residual s.d., *college*	0.100	0.012
$r_{m,\text{nc}}$	Male intercept, *no college*	2.138	0.010
$r_{m,\text{col}}$	Male intercept, *college*	2.534	0.021
$\alpha_{m,\text{nc}}$	Male human capital slope, *no college*	0.345	0.004
$\alpha_{m,\text{col}}$	Male human capital slope, *college*	0.378	0.006
$\sigma_{m,\text{nc}}$	Male residual s.d., *no college*	0.138	0.011
$\sigma_{m,\text{col}}$	Male residual s.d., *college*	0.112	0.025
δ_0	Human capital depreciation	0.328	0.017
δ_1	Human capital appreciation, *low* to *medium*	0.257	0.005
δ_2	Human capital appreciation, *medium* to *high*	0.219	0.034
Home technology			
ζ_0^S	Single productivity, *intercept*	-0.549	0.063
ζ_1^S	Single productivity, *pre-school*	0.245	0.038
ζ_2^S	Single productivity, *primary school*	0.219	0.032
ζ_3^S	Single productivity, *one child*	1.278	0.097
ζ_4^S	Single productivity, *more than one child*	1.198	0.091
ζ_1	Household productivity, *pre-school*	0.543	0.041
ζ_2	Household productivity, *primary school*	0.212	0.033
ζ_3	Household productivity, *one child*	0.608	0.039
ζ_4	Household productivity, *more than one child*	0.563	0.044

Continued...

Table D.1: (continued)

		Estimate	Standard Error
Marriage quality and preferences			
$b_0(\xi_L)$	Initial match probability, *lower*	0.990	0.001
$b_L(\xi_H)$	Match transition probability, *lower* to *higher*	0.326	0.007
$b_H(\xi_L)$	Match transition probability, *higher* to *lower*	0.001	0.000
μ_{θ_L}	Mean match quality, *lower*	-13.675	0.518
μ_{θ_H}	Mean match quality, *higher*	5.341	0.281
σ_θ	Match quality, *scale*	2.634	0.108
γ_s	Meeting education homophily	0.659	0.001
γ_a	Meeting age homophily	0.025	0.024
μ_{η_m}	Male spousal age preference, *location*	10.002	2.169
σ_{η_m}	Male spousal age preference, *spread*	27.664	2.689
β_{η_m}	Male spousal age preference, *scale*	49.466	5.188
γ_{η_m}	Male spousal age preference, *curvature*	1.226	0.078
μ_{η_m}	Female spousal age preference, *location*	0.033	0.001
μ_{η_f}	Female spousal age preference, *spread*	0.050	0.002
β_{η_f}	Female spousal age preference, *scale*	20.030	0.626
α_{η_f}	Female spousal age preference, *skew*	-2.001	0.135
γ_{η_f}	Female spousal age preference, *curvature*	-0.753	0.019
κ_{mar}	Marriage cost with children	3.434	0.206
κ_{div}	Divorce cost	0.500	0.176

Notes: All parameters estimated simultaneously using a moment based estimation procedure as detailed in Section 3.3.

Table D.2: Empirical and simulated marital sorting patterns by age

Age of female		Age of male 18–21	22–25	26–29	30–33	34–37	38–41	42–45	46–49	50–53	54–57	58–61	62–65	66–69	70–73	74–77	78+
		1.97 [1.95]	1.74 [1.61]	1.32 [1.26]	0.97 [1.00]	0.78 [0.82]	0.68 [0.69]	0.64 [0.61]	0.61 [0.56]	0.57 [0.53]	0.55 [0.51]	0.50 [0.50]	0.46 [0.49]	0.44 [0.50]	0.42 [0.51]	0.40 [0.52]	0.39 [0.56]
18–21	1.94 [1.90]	0.02 [0.03]	0.03 [0.04]	0.01 [0.02]	0.00 [0.01]	0.00 [0.00]											
22–25	1.58 [1.45]	0.01 [0.01]	0.17 [0.27]	0.16 [0.17]	0.05 [0.06]	0.02 [0.03]	0.01 [0.01]	0.00	0.00								
26–29	1.12 [1.09]	[0.00]	0.04 [0.05]	0.38 [0.43]	0.30 [0.27]	0.10 [0.09]	0.03 [0.04]	0.01 [0.02]	0.01	0.00	0.00						
30–33	0.82 [0.86]	[0.00]	0.01 [0.01]	0.08 [0.08]	0.49 [0.51]	0.37 [0.33]	0.13 [0.12]	0.05 [0.05]	0.02 [0.02]	0.01	0.00	0.00					
34–37	0.72 [0.72]		0.00 [0.00]	0.02 [0.01]	0.11 [0.09]	0.51 [0.55]	0.39 [0.37]	0.14 [0.14]	0.05 [0.06]	0.02 [0.02]	0.01	0.00	0.00				
38–41	0.67 [0.65]		[0.00]	0.01 [0.01]	0.03 [0.02]	0.13 [0.10]	0.51 [0.56]	0.39 [0.39]	0.15 [0.16]	0.06 [0.06]	0.02 [0.02]	0.01	0.00	0.00			
42–45	0.65 [0.61]			0.00 [0.00]	0.01 [0.01]	0.03 [0.02]	0.14 [0.11]	0.50 [0.56]	0.38 [0.39]	0.15 [0.16]	0.06 [0.07]	0.02 [0.03]	0.01	0.00	0.00		
46–49	0.64 [0.60]				0.00 [0.00]	0.01 [0.01]	0.04 [0.02]	0.14 [0.11]	0.49 [0.55]	0.39 [0.39]	0.15 [0.16]	0.06 [0.07]	0.02 [0.03]	0.01	0.00	0.00	
50–53	0.60 [0.60]				0.00	0.00 [0.00]	0.01 [0.01]	0.04 [0.02]	0.14 [0.11]	0.49 [0.54]	0.39 [0.38]	0.15 [0.16]	0.06 [0.07]	0.03 [0.03]	0.01	0.00	0.00
54–57	0.59 [0.61]					0.00	0.00 [0.01]	0.01 [0.01]	0.04 [0.03]	0.13 [0.11]	0.49 [0.52]	0.39 [0.37]	0.15 [0.15]	0.06 [0.06]	0.03 [0.02]	0.01	0.00
58–61	0.58 [0.64]						0.00	0.00 [0.01]	0.01 [0.01]	0.04 [0.03]	0.12 [0.11]	0.49 [0.50]	0.39 [0.34]	0.14 [0.14]	0.06 [0.06]	0.02 [0.02]	0.01
62–65	0.60 [0.68]							0.00	0.00 [0.01]	0.01 [0.01]	0.03 [0.03]	0.10 [0.10]	0.47 [0.46]	0.39 [0.31]	0.13 [0.13]	0.05 [0.05]	0.02 [0.02]
66–69	0.65 [0.75]								0.00	0.00 [0.01]	0.01 [0.01]	0.03 [0.02]	0.08 [0.10]	0.42 [0.42]	0.37 [0.28]	0.12 [0.11]	0.04 [0.04]
70–73	0.74 [0.83]									0.00	0.00 [0.01]	0.01 [0.01]	0.02 [0.02]	0.07 [0.09]	0.37 [0.37]	0.33 [0.24]	0.11 [0.09]
74–77	0.87 [0.93]											0.00 [0.01]	0.01 [0.01]	0.02 [0.02]	0.05 [0.07]	0.30 [0.30]	0.27 [0.18]
78+	1.05 [1.05]												0.00 [0.00]	0.00 [0.01]	0.01 [0.02]	0.05 [0.06]	0.24 [0.21]

Notes: Table shows empirical and simulated marriage matching function by age, with age aggregated into age groups of 2 (equivalent to 4 years). Simulated values from the model are presented in brackets. Population size at age 18 is equal to normalised to one for men and women. Measures less than 0.001 are omitted. Empirical moments calculated using ACS data.

Table D.3: Cross-sectional labour supply patterns

			1 child						2+ children					
	No children		Pre-school		Primary		Secondary		Pre-school		Primary		Secondary	
	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model
Female, employment														
Single, no college	0.88	0.99	0.80	0.79	0.86	0.89	0.88	0.95	0.72	0.66	0.82	0.81	0.85	0.90
Single, with college	0.98	0.99	0.94	0.94	0.96	0.97	0.96	0.99	0.90	0.89	0.94	0.94	0.94	0.97
Married, no college	0.82	0.87	0.70	0.63	0.76	0.73	0.80	0.78	0.56	0.61	0.71	0.71	0.77	0.76
Married, with college	0.92	0.91	0.82	0.74	0.84	0.81	0.86	0.84	0.70	0.71	0.78	0.78	0.82	0.82
Female, conditional hours														
Single, no college	38.09	39.58	35.06	36.09	36.84	35.19	37.85	35.76	35.10	36.24	36.32	35.04	37.30	35.64
Single, with college	40.82	40.24	38.71	37.84	39.23	37.53	39.64	38.02	38.12	37.76	38.56	36.80	38.96	37.48
Married, no college	37.28	38.24	35.15	35.59	35.89	35.95	36.22	36.51	34.12	35.28	34.44	35.33	35.11	35.86
Married, with college	39.68	40.56	37.48	37.89	37.51	38.64	37.55	39.17	35.50	37.68	35.24	37.95	35.81	38.44
Male, employment														
Single, no college	0.95	0.99	–	–	–	–	–	–	–	–	–	–	–	–
Single, with college	0.98	0.99	–	–	–	–	–	–	–	–	–	–	–	–
Married, no college	0.97	0.98	0.97	0.96	0.97	0.97	0.97	0.97	0.97	0.96	0.98	0.97	0.98	0.97
Married, with college	0.99	0.99	0.99	0.97	0.98	0.98	0.99	0.98	0.99	0.97	0.99	0.98	0.99	0.98
Male, conditional hours														
Single, no college	40.55	40.12	–	–	–	–	–	–	–	–	–	–	–	–
Single, with college	42.02	40.67	–	–	–	–	–	–	–	–	–	–	–	–
Married, no college	41.87	42.10	41.88	42.57	41.75	41.67	42.04	41.27	41.92	42.15	42.02	41.56	42.13	41.33
Married, with college	42.76	43.60	42.61	44.32	42.60	43.32	43.32	42.86	43.30	44.10	43.54	43.51	43.65	43.11

Notes: Table shows empirical and simulated labour supply (employment, and conditional work hours), by gender, marital status, education, and the number and age of any children. *Pre-school*, *Primary*, and *Secondary*, respectively refer to the school age of the youngest child. Work hours are measured in weekly terms. Empirical moments calculated using ACS data.

Table D.4: Cross-sectional home time patterns

	No children		1 child						2+ children					
			Pre-school		Primary		Secondary		Pre-school		Primary		Secondary	
	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model
Female, home hours														
Single	10.89	9.71	30.35	32.27	28.56	30.16	23.78	24.50	31.01	32.90	28.16	29.95	24.04	23.83
Married	15.33	26.01	39.97	39.69	32.81	35.91	28.93	33.26	38.55	39.65	35.06	35.80	31.01	33.09
Male, home hours														
Single	9.89	9.39	–	–	–	–	–	–	–	–	–	–	–	–
Married	8.10	14.88	27.59	22.20	21.08	19.07	17.89	17.52	24.55	21.78	21.58	18.99	19.45	17.51

Notes: Table shows empirical and simulated home production time, by gender, marital status, education, and the number and age of any children. *Pre-school*, *Primary*, and *Secondary*, respectively refer to the school age of the youngest child. Home hours are measured in weekly terms. Empirical moments calculated using PSID data.

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