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PIER Working Paper
18-029

The Curse of Long Horizons

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First Version: July 24, 2016
This Version: December 3, 2018

https://ssrn.com/abstract=3295838

The Curse of Long Horizons*

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December 3, 2018§

Abstract

We study dynamic moral hazard when the principal can only commit to spot contracts. Principal and agent are ex ante symmetrically uncertain about the difficulty of the job, and update their beliefs on observing output. Since the agent's effort is private, he has an additional incentive to shirk when the principal induces effort: shirking results in the principal having incorrect beliefs, giving rise to future informational rents. We show that the effort inducing contract must provide increasingly high powered incentives as the length of the relationship increases. Thus it is never optimal to always induce effort in very long relationships.

Keywords: principal-agency, moral hazard, differences in beliefs, high-powered incentives.

JEL Classification Codes: D01, D23, D86, J30.

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*We thank Yiman Sun for excellent RA support. Mailath thanks the National Science Foundation for research support (grants #SES-1260753 and #SES-1559369). Bhaskar thanks the National Science Foundation for its support via grant 201503942. We thank the referees for helpful comments.

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§First Version: July 24, 2016

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1 Introduction

We analyze the long-run implications of the ratchet effect, arising from the introduction of new technology, in a context where both firm and worker are learning about its efficacy. Milgrom and Roberts (1992, pages 232-236) provide a lucid statement of the problem: when a firm installs new equipment, firms and workers must learn the appropriate work standard. It is efficient to use information to adjust the standard, but this reduces work incentives today.[1] The ratchet effect arises from the combination of learning, moral hazard and lack of long term commitment by the employer.

Earlier work on the ratchet effect usually assumes ex ante differential information. The agent has private information on the nature of the job, and the principal is unable to make long term commitments. The problem is formulated as one of dynamic mechanism design without commitment in which the principal aims to induce the agent to reveal her private information.

We differ from this literature in formulating the ratchet effect as arising from a learning problem under symmetric incomplete information and moral hazard (since worker effort is not observed by the principal). The principal and the agent are symmetrically uncertain about the difficulty of the worker's job. We assume the principal cannot commit to long term contracts, and has all the bargaining power when choosing optimal spot (short-term) contracts. We also assume there is no limited liability, so the agent is indifferent between accepting the principal's optimal spot contract and taking her outside option. Furthermore, since uncertainty pertains to the nature of the job, the outside option does not depend too heavily upon what is learned. Finally, the principal cannot separately learn about job difficulty and agent behavior (in the sense that a signal that the state is good is also a signal of high effort, and conversely). Our assumption on the structure of the signals is natural, being satisfied by most parametric models.

The ratchet effect arises from the agent's possible manipulation of the principal's beliefs by shirking. In a pure strategy equilibrium in which high effort is chosen, the principal correctly anticipates the agent's effort choices, and the beliefs of the two parties about the nature of the job agree. However, when the agent deviates and shirks, the beliefs of the two parties differ, at least temporarily. Our analysis begins with a simple observation: In a two period world, such a deviation increases the expected continuation value of

[1] In the sociology literature, Mathewson (1931), Roy (1952), and Edwards (1979) are workplace studies that document the importance of output restriction in order to influence the firm's beliefs.

the agent. In consequence, any incentive compatible contract inducing high effort must be sufficiently high powered to offset this deviation gain. The ratchet effect gives rise to a dynamic incentive cost (which we term the *future information rent from shirking*, or *FIRS*), since the agent must be exposed to additional risk in order to overcome the incentive problem (Proposition 1). Since the principal must compensate the agent for increased risk, his wage costs increase. This finding generalizes Milgrom and Roberts's (1992) earlier demonstration of the need for high powered incentives in a two period model with linear technology and normal signals.

The bulk of our analysis concerns the behavior of dynamic incentive costs as the time horizon T increases. For most of the paper we focus is on the principal's cost minimization problem when she induces effort in every period—this is a prerequisite for analyzing overall profit maximization. Specifically, we consider *sequentially incentive efficient contracts*. These are contracts which induce high effort in every period at minimum cost. While it is intuitive that the future information rents from shirking in any period should increase with the time horizon, there is a subtlety. The dynamic incentive cost is essentially the opportunity cost of not shirking, and little is known about the comparative statics of the optimal effort contract with respect to costs (or benefits) of shirking. Nonetheless, it turns out that the intuitive increase with the time horizon does occur if either the agent has a specific form of CRRA preferences (Proposition A.1) or if the signal distribution satisfies one additional collinearity restriction (the collinearity assumption is always satisfied with binary signals).

However, a plausible conjecture is that this effect, when present, tapers off: since both the principal and agent learn the state of the world, there is very little uncertainty remaining towards the end of a long relationship. Our main result is that this conjecture is false. Under the collinearity restriction on the signal distribution the future information rents from shirking in any period are bounded below by a linear function of the remaining duration of the relationship (Proposition 3). The key insight is the following. Compare the problem of inducing effort in the initial period in two cases: a) a three-period relationship, and b) a two-period relationship. In both cases, the agent receives second period rents by shirking in the first period. However, these rents are larger in the three-period setting than in the two-period setting, because the principal has to provide more high-powered incentives in the penultimate period of the three-period relationship, as we have already seen from the analysis of the two-period model. We provide an example showing that in the absence of this positive feedback from one period's future information rents from shirking to earlier periods, the value from having different beliefs in all future periods is bounded (Proposition 4). We also derive a lower bound on the future information rents from shirking when the horizon is infinite and the agent discounts (Proposition C.1).[2]

Since the future information rents from shirking are bounded below by a linear function of remaining duration, t, the expected wage costs are similarly bounded below. Consequently, it is never optimal for the principal to always induce effort when t is sufficiently large. While characterizing the optimal optimal pattern of elicited effort is complex and beyond the scope of the current analysis, we do report some suggestive numerical calculations.

1.1 Related Literature

This paper is related to a growing literature on dynamic moral hazard with learning/experimentation. Holmström's (1982) career concerns model is a pioneering example.[3] Like us, Holmström (1982) assumes there is symmetric incomplete information, but critically, in that paper, the agent does not receive an incentive wage contract from a principal. Rather, the agent receives his market wage which is determined by the market beliefs about the unknown state (there interpreted as the the talent of the agent, and not the nature of the job). Our substantive conclusions are the opposite of Holmström's—dynamic incentives make shirking more attractive, whereas in Holmström, they provide incentives to work. To delineate the differences, we allow the agent's outside option to depend on the public belief about the difficulty of the job. If the outside option is not too sensitive to the public beiief, then our results apply, and incentivizing effort becomes more difficult in long term relationships. Conversely, if the outside option is sufficinently sensitive, then dynamic considerations provide an incentive for the agent to work.

Much of the work on the ratchet effect focuses on the asymmetric information case, where the principal wishes to elicit the private information of the agent. Lazear (1986) argues that high powered incentives are able to overcome the ratchet effect, without any efficiency loss, assuming that the worker is risk neutral. Gibbons (1987) shows that Lazear's result depends upon an implicit assumption of long term commitment; in its absence, one cannot induce efficient effort provision by the more productive type.[4] Laffont and Tirole (1988) prove that in general one cannot induce full separation

[2] The lower bound in this case goes to infinity as the discount factor goes to one.

[3] Gibbons and Murphy (1992) extend Holmström (1982) by having the market wage be a linear function of output.

[4] See also Freixas, Guesnerie, and Tirole (1985) and Carmichael and MacLeod (2000).

of a continuum of types.[5] Laffont and Tirole (1993) have a comprehensive discussion, and consider both the case of binary and a continuum of types. Gerardi and Maestri (2015) analyze an infinite horizon model with binary types.

More recently, there is an increased interest in agency models with learning, where the uncertainty also pertains to the nature of the project. Bergemann and Hege (1998, 2005), Manso (2011), Hörner and Samuelson (2015), and Kwon (2011) analyze agency models with binary effort, binary signals, and limited liability. Bhaskar (2014) studies a two-period model that makes the same informational and contracting assumptions as in the present paper, but allows for continuum effort choices (rather than binary). The main finding is that the principal cannot implement interior effort choices in the first period. Since the agent can increase his continuation value by shirking, this must be dissuaded by high powered incentives. However, this implies that the agent can deviate upwards, and increase his current payoff, without any loss in continuation value since he can always quit the job tomorrow.

There is also recent work on learning in agency models with private actions in continuous time and continuum action spaces including DeMarzo and Sannikov (2011), Cisternas (2018), and Prat and Jovanovic (2014), that examines the agent's incentives for belief manipulation. More generally, deviations can lead to persistent divergences in belief between the principal and agent in a variety of contexts. In Sannikov (2014), the agent's actions have persistent effect upon output, while Williams (2011) studies a model where the agent's type is persistent. All these papers have continuous choices, and employ a "first-order" approach to compute the agent's informational rent from a deviation. In our setting, choices are discrete, and we use the payoffs from a single deviation to provide a lower bound on informational rents. We discuss the connection further at the end of Section 5.

2 The model

A risk neutral principal (whom we treat as female) repeatedly hires a risk averse agent (whom we treat as male) to undertake some task. In each period, the principal offers a spot contract to the agent, who accepts or rejects. If the agent rejects the contract, the relationship is dissolved and the game ends. If the agent accepts the contract, the agent then decides

[5] Malcomson (2016) shows that the no full-separation result also obtains in a relational contracting setting, where the principal need not have all the bargaining power, as long as continuation play following full separation is efficient.

whether to exert effort e (incurring a disutility of $c > 0$) or shirk s (which is costless). As usual, there is moral hazard, with this choice not observed by the principal. Moreover, there is uncertainty about the "difficulty" of the task. Specifically, there are two states of the world $\omega \in \{B, G\}$, with the task being easy in G, and hard in B. The uncertainty concerns how difficult it is to succeed on *this* job.

The choice $a \in \{e, s\}$ by the agent determines, with the state of the world, the probability distribution over signals $y \in Y$, where $Y := \{y^1, y^2, \ldots, y^K\}$ is a finite set of signals. The spot contract specifies the wage payment as a function of the realized signal.

The agent updates his beliefs about the state knowing his own effort choice and the realized public signal. The principal updates her beliefs knowing only the signal, since the agent's effort is not public (i.e., it is not observed by the principal).

The agent's flow utility from a wage payment $w \in \mathbb{R}$ is $u(w)$, where u is strictly increasing and concave. To guarantee individual rationality binds, we assume unlimited liability, so that there are no constraints on the size and sign of utility payments.

We find it more convenient to work with utility schedules, so we write a spot contract as a utility schedule $u := (u^1, \ldots, u^K)$, where u^k is the utility the agent will receive after signal y^k. The wage cost of providing utility level u^k is written $w(u^k) := u^{-1}(u^k)$.

We do not specify how output signals translate into revenues for the principal. While solving for the equilibrium of the game does require specifying the principal's trade-off between revenues and wage costs, that is not our focus. Our focus, rather, is on the important preliminary step of characterizing the expected cost minimizing sequence of spot contracts that induce effort in every period. This step is independent of the revenue consequences of effort.[6]

The probability of signal y^k at action $a \in \{s, e\}$ and state $\omega \in \{B, G\}$ is denoted by $p^k_{a\omega}$. Our interest is in settings where a signal that the state is good is also a signal of high effort (and conversely), so that it is impossible to disentangle the two. Most parametric models in the learning/experimentation literature satisfy this assumption. For example, it is satisfied when the signal is the number of Poisson distributed successes, with an arrival rate increasing in both the ease of the job and effort. We capture

[6]This analysis also does not depend upon the principal's time preference. Also, it is possible to generalize the results to the case where the principal is risk-averse, as long as the agent's incentive constraint binds in the static contract.

this by the following assumption.

Assumption 1.

1. *There exists an* informative *signal, i.e., there exists* $y^k \in Y$ *such that either* $p^k_{eG} \neq p^k_{sG}$ *or* $p^k_{eB} \neq p^k_{sB}$.
2. *For any informative signal* $y^k \in Y$,
$$\min\left\{p^k_{sB}, p^k_{eG}\right\} < p^k_{sG}, p^k_{eB} < \max\left\{p^k_{sB}, p^k_{eG}\right\}.$$
3. *Signals have full support:* $p^k_{a\omega} > 0$ *for all* k, a, ω.

We partition the set of signals into a set of "high" signals Y^H, a set of "low" signals Y^L, and a set of neutral signals $Y \setminus (Y^H \cup Y^L)$ by setting

$$y^k \in Y^H \text{ if } p^k_{eG} > p^k_{sG}$$

and

$$y^k \in Y^L \text{ if } p^k_{eG} < p^k_{sG}.$$

A signal is high if it is indicative of effort in state G. Assumption 1 implies that a high signal is indicative of effort also in state B. Also, it is indicative of the state being G under either effort or shirking.

A player with belief μ that the task is easy ($\omega = G$) assigns a probability to signal y^k of $p^k_{a\mu} := \mu p^k_{aG} + (1-\mu) p^k_{aB}$. Assumption 1 immediately implies

$$y^k \in Y^H \iff p^k_{eG} > p^k_{eB}, p^k_{sG} > p^k_{sB} \iff p^k_{e\mu} > p^k_{s\mu}$$

and

$$y^k \in Y^L \iff p^k_{eG} < p^k_{eB}, p^k_{sG} < p^k_{sB} \iff p^k_{e\mu} < p^k_{s\mu}.$$

In other words, high signals arise with higher probability when either the agent exerts effort *or* the state is good.

We now derive two important implications of Assumption 1 that are key for the rest of the paper. First, if the principal believes that the agent is exerting effort, but the agent is in fact shirking, then on average, the principal will become more pessimistic than the agent after observing the signal realization. Second, if the agent is more optimistic than the principal, and the one-period contract induces effort, the agent earns an informational rent.

Let $\psi^k_a(\mu)$ denote the Bayesian update on the initial belief μ after observing signal y^k, given that action a has been taken. Suppose that the principal believes that the agent has taken action a, when the agent in fact takes action $\tilde{a}$. The average belief of the principal, under the distribution over signals induced by $\tilde{a}$ is

$$\mathbf{E}(\psi_a^k(\mu))|\tilde{a}) = \sum_k \psi_a^k(\mu) p_{\mu\tilde{a}}^k.$$

The martingale property of beliefs ensures that $\mathbf{E}(\psi_a^k(\mu))|a) = \mu$. The following lemma states that under Assumption 1, the average belief of the principal is always lower than that of the agent, when the agent shirks and the principal believes that she has exerted effort.

Lemma 1. *If the signals satisfy Assumption 1, then*

$$\mathbf{E}(\psi_e^k(\mu))|s) < \mathbf{E}(\psi_s^k(\mu))|s) = \mu.$$

Proof. Assumption 1 implies

$$y^k \in Y^H \iff p_{e\mu}^k > p_{s\mu}^k \iff p_{eG}^k > p_{e\mu}^k$$

and

$$y^k \in Y^L \iff p_{e\mu}^k < p_{s\mu}^k \iff p_{eG}^k < p_{e\mu}^k.$$

Thus,

$$\begin{aligned} \mu - \sum_k p_{s\mu}^k \frac{\mu p_{eG}^k}{p_{e\mu}^k} &= \mu \sum_k (p_{e\mu}^k - p_{s\mu}^k) \frac{p_{eG}^k}{p_{e\mu}^k} \\ &> \mu \sum_k (p_{e\mu}^k - p_{s\mu}^k) = 0. \end{aligned}$$

■

Suppose the principal and agent both assign probability μ to the task being easy. We allow for some state dependence in the value of the outside option. We normalize the value of the outside option to zero when the state is B and denote the value by $\Lambda \geq 0$ when the state is G. When the state is unknown, we assume the outside option depends on the public belief μ, and equals $\mu\Lambda$. This assumption is consistent with the interpretation that the outside option is provided by the competition from other potential employers, and the uncertainty pertains to the agent's ability, as in Holmström (1999). However, our analysis differs from Holmström: the agent's productivity in the current job is assumed to be significantly greater than his productivity at other employers. Consequently, the employer has monopoly power, and can extract all the surplus from the relationship.

Our analysis applies also to the case where the outside option depends on the agent's private belief π regarding his ability, and equals $\pi\Lambda$. In this case, the outside option comes from self-employment. Since the contract offered by the principal only depends on the public belief, the analysis that follows also applies in this case.[7]

A *statically optimal spot contract* offered by the principal is a contract $u \in \mathbb{R}^K$ minimizing its expected cost of provision

$$p_{e\mu} \cdot w(u),$$

where $p_{a\mu} := (p^1_{a\mu}, \ldots, p^K_{a\mu})$, subject to *incentive compatibility*

$$p_{e\mu} \cdot u - c \geq p_{s\mu} \cdot u \tag{IC}$$

and *individual rationality*

$$p_{e\mu} \cdot u - c \geq \mu\Lambda. \tag{IR}$$

Since the principal is risk neutral and the agent is risk averse, both (IC) and (IR) bind at the statically optimal contract, which is unique and denoted $\hat{u}_\mu$.

Suppose the principal assigns probability μ to G and offers a static contract $\hat{u}_\mu$ at which the (IR) binds (given $p_{e\mu}$). If the agent has belief π and exerts effort, the agent's payoff from exerting effort is

$$\begin{aligned} V^\dagger(\pi, \mu) : &= p_{e\pi} \cdot \hat{u}_\mu - c \\ &= p_{e\mu} \cdot \hat{u}_\mu - c + (\pi - \mu)(p_{eG} - p_{eB}) \cdot \hat{u}_\mu \\ &= (\pi - \mu)(p_{eG} - p_{eB}) \cdot \hat{u}_\mu + \mu\Lambda. \end{aligned} \tag{1}$$

We now show that $(p_{eG} - p_{eB}) \cdot \hat{u}_\mu$ is strictly positive. From (IC), we have

$$(p_{e\mu} - p_{s\mu}) \cdot \hat{u}_\mu > 0.$$

Observe that $\hat{u}^k_\mu \geq \hat{u}^{k'}_\mu$ if $p^k_{e\mu} > p^k_{s\mu}$ and $p^{k'}_{e\mu} \leq p^{k'}_{s\mu}$. [If not, there exists k and k' such that $\hat{u}^k_\mu < \hat{u}^{k'}_\mu$ with $p^k_{e\mu} > p^k_{s\mu}$ and $p^{k'}_{e\mu} \leq p^{k'}_{s\mu}$. The contract that equals the old contract except at signals y^k and $y^{k'}$, where the utility promises are replaced by the constant value $(p^k_{e\mu}\hat{u}^k_\mu + p^{k'}_{e\mu}\hat{u}^{k'}_\mu)/(p^k_{e\mu} + p^{k'}_{e\mu})$, satisfies (IC) and (IR), at lower cost.] Assumption 1 then implies that $\hat{u}^k_\mu \geq \hat{u}^{k'}_\mu$ for $y^k \in Y^H$ and $y^{k'} \in Y^L$. Since the agent's payoff is at least the payoff from exerting effort, we have therefore proved the following lemma.

[7] The difference between the two specifications of the outside option only matters for the agent's quitting decision when $\pi < \mu$. Since our bounds on informational rents are derived assuming that the agent does not quit, they apply equally to both cases

Lemma 2. *Suppose the signals satisfy Assumption 1 and $\Lambda = 0$. If the principal has belief μ and offers the statically optimal contract $\hat{u}_\mu$, then the informational rent of the agent when her belief π exceeds μ is no less than*

$$(\pi - \mu)(p_{eG} - p_{eB}) \cdot \hat{u}_\mu > 0.$$

To summarize: When $\Lambda = 0$, Assumption 1 ensures that the agent always has a dynamic incentive to shirk, since by doing so, she induces the principal to be more pessimistic than the agent, and this divergence in beliefs translates to an informational rent in a one-period contract. While this phenomenon has been noted before for specific informational assumptions, Assumption 1 suffices.

When Λ is strictly positive, as we will see in the next section, the outside option moderates the future information rent from shirking.

3 Two Time Periods

We begin with the two period case. The principal minimizes total wage costs and the agent maximizes total expected payoff. To minimize notation, we assume the agent does not discount in the finite horizon setting. Our results also hold under discounting, with obvious modifications; we discuss discounting in more detail in Remark 2 and when we analyze the infinite horizon setting in Appendix C.

The principal cannot commit in period 1 to period 2 wages, while the agent cannot commit to participate, and so each period's spot contract must satisfy incentive compatibility (IC) and individual rationality (IR) in that period.

We are interested in the most efficient sequence of spot contracts inducing e in every period. Since there is incomplete information, we require that both the principal and the agent's behavior be sequentially rational after every history, and that both update using Bayes' rule whenever possible. The common prior probability on G is denoted $\mu^\dagger$. Let $\mu_a^k := \psi_a^k(\mu^\dagger)$ be the posterior probability on G after y^k under action a. Though the principal does not observe effort, under the sequence of incentive efficient contracts, she assigns probability one to the agent choosing e.

Denote the first period spot contract by $u(\varnothing) := (u^1(\varnothing), \ldots, u^K(\varnothing))$, and the second period spot contract offered by the principal after signal y^k by $u(y^k) := (u^1(y^k), \ldots, u^K(y^k))$. Since the only intertemporal linkage between the periods is the posterior belief update and the first period contract induces

e deterministically, the most efficient first period spot contract minimizes first period expected wage cost.

Definition 1. *A two period sequence of contracts* $(u(\varnothing), (u(y^k))_{y^k \in Y})$ *is* sequentially effort incentive efficient *if*

1. *for every first period signal realization* $y^k \in Y$, $u(y^k)$ *minimizes*

$$p_{e\mu_e^k} \cdot w(u) = \sum_{k'} p_{e\mu_e^k}^{k'} \; w(u^{k'})$$

subject to the agent finding it optimal to participate and exert effort in the second period after exerting effort in the first period, and

2. $u(\varnothing)$ *minimizes* $\sum p_{e\mu^\dagger}^k w(u^k)$ *subject to the agent finding it optimal to participate and exert effort in the first period.*

Under a sequentially effort incentive efficient sequence of contracts, the agent exerts effort in every period, and the second period beliefs of the agent and principal agree. In particular, after y^k, the second period effort incentive efficient contract solves the static problem with public beliefs μ_e^k.

The first period is more complicated, since the agent's deviation to shirking in the first period results in the principal and agent having different beliefs. After signal y^k, the agent has update μ_s^k, which differs from the principal's update of μ_e^k. In addition, the principal is mistaken in her conviction that the agent also has the belief μ_e^k.

We saw at the end of the previous section that if $\mu_s^k > \mu_e^k$, then the agent receives a strictly positive payoff from the contract $\hat{u}_{\mu_e^k}$. If $\Lambda = 0$, the agent's second period expected payoff from the contract *strictly increases* from shirking in the first period:

1. Lemma 1 implies there is a signal y^k such that $\mu_s^k > \mu_e^k$, with a resulting second period gain from deviation.

2. For any signal y^k satisfying $\mu_s^k < \mu_e^k$, the IR constraint is violated, and the agent walks away, obtaining his reservation utility.

When $\Lambda > 0$, the effect is ambiguous. The agent's second period payoff equals his outside option $\mu_e^k \Lambda$ after signal k, the expectation of which equals $\mu^\dagger \Lambda$ when he exerts effort. However, when the agent shirks, he reduces his outside option, since $\mathbf{E}(\psi_e^k(\mu^\dagger)|s) < \mu^\dagger$. Thus, by shirking, the agent induces two effects:

1. For any second-period outside option $\mu\Lambda$, the agent gets an informational rent whenever his private belief π exceeds μ.

2. The average second-period outside option is lower.

The future information rents from shirking are positive, as long as Λ is not too large. We need to do a little bookkeeping before we can bound Λ.

The first period incentive compatibility constraint is

$$p_{e\mu^\dagger} \cdot u(\varnothing) - c + \sum_k p^k_{e\mu^\dagger} \Lambda \mu^k_e \geq p_{s\mu^\dagger} \cdot u(\varnothing) + \widetilde{W}(\mu^\dagger),$$

where $\widetilde{W}(\mu^\dagger)$ is the second period value to the agent from shirking in the first period (while the principal expected effort in the first period). This can be rewritten as

$$p_{e\mu^\dagger} \cdot u(\varnothing) - c \geq p_{s\mu^\dagger} \cdot u(\varnothing) + W(\mu^\dagger), \tag{2}$$

where

$$W(\mu^\dagger) := \widetilde{W}(\mu^\dagger) - \mu^\dagger \Lambda.$$

The expression $W(\mu^\dagger)$ is the one period (normalized) *future information rent from shirking*. Bounding $\widetilde{W}(\mu^\dagger)$ from below by assuming the agent exerts effort in the second period if he does not take the outside option,

$$\begin{aligned} W(\mu^\dagger) &\geq \sum_k p^k_{s\mu^\dagger} \max\{V^\dagger(\mu^k_s, \mu^k_e),\ \mu^k_e \Lambda\} - \mu^\dagger \Lambda \\ &= \sum_k p^k_{s\mu^\dagger} \max\{V^\dagger(\mu^k_s, \mu^k_e) - \mu^k_s \Lambda,\ (\mu^k_e - \mu^k_s)\Lambda\} \\ &= \sum_k p^k_{s\mu^\dagger} \max\{V^*(\mu^k_s, \mu^k_e),\ (\mu^k_e - \mu^k_s)\Lambda\}, \end{aligned}$$

where

$$\begin{aligned} V^*(\pi, \mu) :&= V^\dagger(\pi, \mu) - \pi\Lambda \\ &= (\pi - \mu)\big((p_{eG} - p_{eB}) \cdot \hat{u}_\mu - \Lambda\big). \end{aligned} \tag{3}$$

Observe that that V^* is strictly positive as long as $\pi > \mu$ and $\Lambda < \underline{\Lambda} := \inf_\mu (p_{eG} - p_{eB}) \cdot \hat{u}_\mu > 0$. Moreover, if $\Lambda < \underline{\Lambda}$, $W(\mu^\dagger) > 0$ and the statically optimal contract $\hat{u}_{\mu^\dagger}$ does not satisfy (2). The first period spot contract must be more high powered than the statically optimally contract in order to deter shirking.

We summarize this discussion in the following proposition.

Proposition 1. *Suppose Assumption 1 holds and the two period sequence of contracts* $(u(\varnothing), (u(y^k))_{y^k \in Y})$ *is sequentially effort incentive efficient. Then, for* $\Lambda < \underline{\Lambda}$*, the first period contract* $u(\varnothing)$ *is more high powered than the statically optimal contract* $\hat{u}_{\mu^\dagger}$*:*

$$(p_{e\mu^\dagger} - p_{s\mu^\dagger}) \cdot u(\varnothing) > c = (p_{e\mu^\dagger} - p_{s\mu^\dagger}) \cdot \hat{u}_{\mu^\dagger}$$

and

$$p_{e\mu^\dagger} \cdot u(\varnothing) = p_{e\mu^\dagger} \cdot \hat{u}_{\mu^\dagger} = c + \Lambda\mu^\dagger.$$

If Λ is large, then even a constant first period wage may be enough to induce effort. With a constant wage, the cost of shirking in the first period is a loss of second period outside option of

$$\Lambda \sum_k p^k_{e\mu^\dagger}\mu^k_e - \Lambda \sum_k p^k_{s\mu^\dagger}\mu^k_e = \Lambda\mu^\dagger - \Lambda \sum_k p^k_{s\mu^\dagger}\mu^k_e > 0,$$

and for Λ large enough, this will exceed c, the cost of effort.

4 Finite Horizon

We consider next the finite horizon setting, with T periods in the relationship. We index periods backwards, so in period t, there are $t-1$ periods remaining after the current one. In period $\tau = T, \ldots, 1$, the principal has observed the history $h^\tau \in Y^{T-\tau}$, and offers a spot contract $u(h^\tau) \in \mathbb{R}^K$. In the following definition, $\hat{h}^t$ is the common $T-t$ initial segment of each h^τ.

Definition 2. *A sequence of contracts* $((u(h^\tau))_{h^\tau \in Y^{T-\tau}})_{\tau=1,\ldots,T}$ *is* sequentially effort incentive efficient *(*SEIE*) if for every* $t \in \{T, \ldots, 2, 1\}$ *and every* $\hat{h}^t \in Y^{T-t}$*, the spot contract* $u(\hat{h}^t)$ *minimizes, over* $\tilde{u} \in \mathbb{R}^K$*,*

$$\mathbf{E}[w(\tilde{u}^k) \mid \hat{h}^t, a^t = e, a^T = \cdots = a^{t+1} = e] \\ + \sum_{\tau=1}^{t-1} \mathbf{E}[w(u^k(h^\tau)) \mid \hat{h}^t, a^\tau = e, a^T = \cdots = a^{\tau+1} = e]$$

subject to the agent finding it optimal to participate and exert effort in period t *and in every subsequent period after every public history, conditional on the agent having exerted effort in every previous period.*

Since the behavior of the principal in any period is completely determined by her beliefs about the state updated from the public history, we can solve for SEIE contracts recursively, beginning in the last period (period 1; recall we index periods backwards).

We need to consider situations in which the agent and principal have different beliefs. Let $\widetilde{V}(\pi,\mu,t)$ denote the agent's value function in period t when his belief is π and the principal's belief is μ (for our purposes, these beliefs are the result of updating using $h^t \in Y^{T-t}$, the period t public history). Denote the effort incentive efficient contract offered by the principal in period t by $u_\mu(t)$.

In the last period, period 1, the principal, given his updated beliefs μ, offers the contract $u_\mu(1) := \hat{u}_\mu$. The agent's value from this contract is

$$\widetilde{V}(\pi,\mu,1) = \max\left\{p_{e\pi}\cdot u_\mu(1) - c,\ p_{s\pi}\cdot u_\mu(1),\ \Lambda\mu\right\}.$$

If beliefs agree the value is the outside option, i.e., $\widetilde{V}(\mu,\mu,1) = \Lambda\mu$.

Proceeding recursively, in period t,

$$\begin{aligned}\widetilde{V}(\pi,\mu,t) = \max\big\{&p_{e\pi}\cdot u_\mu(t) - c + \textstyle\sum_k p^k_{e\pi}\widetilde{V}(\psi^k_e(\pi),\psi^k_e(\mu),t-1),\\ &p_{s\pi}\cdot u_\mu(t) + \textstyle\sum_k p^k_{s\pi}\widetilde{V}(\psi^k_s(\pi),\psi^k_e(\mu),t-1),\ t\Lambda\mu\big\},\end{aligned}$$

where $\psi^k_a(\beta)$ is the posterior probability on G after y^k under action a, given a prior β. We assume the agent receives $\Lambda\mu$ in each period after he takes his outside option.

On the equilibrium path, the agent always exerts effort, so that in period t, at belief μ, the contract $u_\mu(t)$ satisfies the incentive constraint

$$\begin{aligned}p_{e\mu}\cdot u_\mu(t) - c + \textstyle\sum_k p^k_{e\mu}&\widetilde{V}(\psi^k_e(\mu),\psi^k_e(\mu),t-1)\\ &\geq p_{s\mu}\cdot u_\mu(t) + \textstyle\sum_k p^k_{s\mu}\widetilde{V}(\psi^k_s(\mu),\psi^k_e(\mu),t-1)\end{aligned}$$

and the participation constraint binds, so that

$$\widetilde{V}(\mu,\mu,t) = t\Lambda\mu.$$

As we saw in the previous section, it is more convenient to work with the surplus net of the outside option prevailing if the principal has the same beliefs as the agent. So we define

$$V(\pi,\mu,t) := \widetilde{V}(\pi,\mu,t) - \Lambda\pi t. \tag{4}$$

Defining

$$W(\mu,t) := \textstyle\sum_k p^k_{s\mu} V(\psi^k_s(\mu),\psi^k_e(\mu),t-1) \tag{5}$$

as the *future information rent from shirking* (FIRS) in period t, the period-t incentive constraint can then be written as

$$p_{e\mu} \cdot u_\mu(t) - c \geq p_{s\mu} \cdot u_\mu(t) + W(\mu, t).$$

Summarizing this discussion, we have:

Proposition 2. *A sequence of contracts $((u(h^\tau))_{h^\tau \in Y^{T-\tau}})_{\tau=1,\dots,T}$ is sequentially effort incentive efficient if and only if $u = u(h^\tau) \in \mathbb{R}^K$ minimizes*

$$\sum\nolimits_k p^k_{e\mu(h^\tau)} w(u^k)$$

subject to

1. $p_{e\mu(h^\tau)} \cdot u - c \geq p_{s\mu(h^\tau)} \cdot u + W(\mu(h^\tau), t)$ *and*
2. $p_{e\mu(h^\tau)} \cdot u - c \geq \mu(h^\tau)\Lambda,$

where $\mu(h^\tau) = \Pr[G \mid h^\tau, a^T = \cdots a^{\tau-1} = e]$. Furthermore, the two inequalities hold as equalities in every SEIE contract.

From Section 3, we know for Λ close to zero, $W(\mu, 2) > 0$. Is $W(\mu, t)$ increasing in t, and if it is increasing, does it increase without bound?

Intuitively, $W(\mu, 3)$ should be larger than $W(\mu, 2)$, because the latter reflects the value of different beliefs induced by shirking under a statically optimal contract for a less demanding incentive compatibility constraint. This is essentially a question of comparative statics on static contracts with respect to the opportunity cost of shirking, which turns out to be a lot harder than comparative statics with respect to the disutility of effort. The next section outlines the problem and a resolution.

5 Comparative Statics of Optimal Contracts

The contract $u_\mu(t)$ described in Proposition 2 solves the static incentive problem:

$$\min_{\{u^k\}} \sum\nolimits_k p^k_{e\mu} w(u^k)$$

subject to

$$\sum\nolimits_k p^k_{e\mu} u^k - c \geq \sum\nolimits_k p^k_{s\mu} u^k + W \qquad \text{(IC}^*\text{)}$$

$$\text{and} \qquad \sum\nolimits_k p^k_{e\mu} u^k - c \geq \Lambda\mu^\dagger. \qquad \text{(IR}^*\text{)}$$

Suppose W' and W'' are two distinct future informational rents from shirking in period t, with $W' > W''$. We would like to show that in the preceding period, $t+1$, the informational rent from shirking is greater under W' than under W''. Let u' and u'' denote the vectors of utilities in the corresponding optimal period t contracts. Since incentive compatibility holds with equality we have

$$(p_{e\mu} - p_{s\mu}) \cdot u' = c + W'$$

and

$$(p_{e\mu} - p_{s\mu}) \cdot u'' = c + W''.$$

The informational rent from shirking in $t+1$ is directly related to the properties of the vector $u'' - u'$. In particular, recalling (1), if

$$W' > W'' \implies (p_{eG} - p_{eB}) \cdot (u' - u'') > 0, \tag{6}$$

then the informational rent is larger under W' than under W''.

While we know

$$(p_{e\mu} - p_{s\mu}) \cdot (u' - u'') = W' - W'', \tag{7}$$

without further assumptions, this does not imply (6).

If the agent has specific CRRA preferences, then it turns out that, even with general probabilities, the implication in (6) holds, and so $W(\mu, t)$ is monotonic in t (see Appendix A). But we do not know if there is a useful lower bound on $W(\mu, t))$ in this case.

We now pursue a direct path to link (6) and (7) by assuming the vectors $(p_{eG} - p_{eB})$ and $(p_{e\mu} - p_{s\mu})$ are collinear.

Consider the vectors $(p_{eG} - p_{sB})$, $(p_{eB} - p_{sB})$ and $(p_{sG} - p_{sB})$. Under Assumption 1, each component in $(p_{eB} - p_{sB})$ has the same sign as the corresponding component in $(p_{eG} - p_{sB})$, but its absolute magnitude is smaller; the same is true for each component in $(p_{sG} - p_{sB})$. The following assumption strengthens this, by requiring that these vectors are collinear.

Assumption 2. *There exists scalars $\alpha \in (0,1)$ and $\beta \in (0,1)$ such that*

$$p_{sG} = \alpha p_{eG} + (1-\alpha) p_{sB} \text{ and}$$
$$p_{eB} = \beta p_{eG} + (1-\beta) p_{sB}.$$

Under Assumption 1, the combination eG corresponds to the "best" state-action pair, while sB corresponds to the worst. The collinearity assumption states that the intermediate state-action pairs – eB and sG – each induce a distribution that is a convex combination of those arising from the best and the worst. This assumption is reminiscent of one made by Hart and Holmström (1987) in order to ensure the validity of the first-order approach to the moral hazard problem. They consider a model where effort must be chosen from the unit interval, and assume that the distribution on output signals induced by any interior effort level a is a convex combination of the distributions from the extremal efforts, with the weight on the distribution induced by effort 1 being an increasing concave function of a. Our assumption pertains to state-action pairs, rather than just effort, and essentially states that a better state is equivalent to a higher level of effort. Thus eG and sB are the highest and lowest, with sG and eB inducing a convex combination of the two extremes. Observe that Assumption 2 implies Assumption 1.

Assumption 2 does *not* require that productivity of effort is state independent, as in Holmström (1999). The parameter $1-\alpha$ is a measure of the productivity of effort in the good state, while β measures the productivity of effort in the bad state. Our formulation allows one of these parameters to be arbitrarily close to one, with the other being arbitrarily close to zero, so that effort would essentially affect the distribution of output only in one state.

Remark 1. With binary signals, the collinearity assumption is automatically satisfied: the space of probabilities is one-dimensional and α and β must belong to $(0,1)$, due to Assumption 1.

Assumption 3. *The outside option* Λ *satisfies*

$$K := \min_{\mu} \frac{(1-\beta)c}{[\mu(1-\alpha)+(1-\mu)\beta]} - \Lambda > 0.$$

When $\Lambda = 0$, so that the outside option does not vary with project quality, Assumption 2 implies $K > 0$, since $\beta < 1$. Consequently, Assumption 3 is satisfied for Λ small.

Our goal is to bound $W(\mu,t)$ as a function of t, since larger information rents require more high powered incentives. We bound $W(\mu,t)$ from below by bounding $V(\pi,\mu,t)$. Obtaining tight bounds for the value function is in general difficult. However, under the Collinearity Assumption 2, we are able to obtain useful bounds by considering a particular specification of continuation play of the agent, namely always exert effort. Denote by $V^*(\pi,\mu,t)$ the agent's value function in period t when his belief is π and the principal's belief is μ, and the agent always chooses effort. Since

$$V(\pi,\mu,t) \geq V^*(\pi,\mu,t), \tag{8}$$

it is enough to bound $V^*(\pi,\mu,t)$. The value recursion for V^* is

$$V^*(\pi,\mu,t) = \big(p_{e\pi}\cdot u_\mu(t) - c - \Lambda\pi\big) + \sum_k p^k_{e\pi} V^*(\psi^k_e(\pi), \psi^k_e(\mu), t-1). \tag{9}$$

As we saw from (1), if $\pi > \mu$, the first flow term is positive for small Λ, with subsequent flows reflecting additional rents from updated differences in beliefs. However, beliefs *merge* (Blackwell and Dubins, 1962): the difference between the agent's and the principal's posteriors vanishes. Consequently, in a long relationship, the impact of a difference in beliefs after a deviation in the initial period on the expected information rent in the last period is small.

Nonetheless, in the last period, any small information rent leads to an increase (albeit small) in the power of the required incentives in the penultimate period. This implies that the information rents in period 2 generated from a difference in beliefs are greater than they would have been in the last period. This in turn requires more high powered incentives in period 3, and so on. This cascading effect implies that the effect of an additional period upon period 1 incentives are non-negligible, no matter how long the time horizon T is.

Proposition 3. *Suppose that Assumptions 2 and 3 are satisfied. For any integer t,*

$$\pi \geq \mu \implies V(\pi,\mu,t) \geq V^*(\pi,\mu,t) \geq (\pi-\mu)Kt. \tag{10}$$

The future information rent from shirking is bounded below by a linear function of time: For all $\mu \in (0,1)$, there is a constant $\xi^*(\mu) \in (0,1)$ for which

$$W(\mu,t) \geq \xi^*(\mu)(t-1). \tag{11}$$

Proof. We first state some implications of the assumed structure on signals. Under the collinarity assumption,

$$p_{e\mu} - p_{s\mu} = [\mu(1-\alpha) + (1-\mu)\beta][p_{eG} - p_{sB}].$$

The optimal spot contract in period t satisfies

$$c + W(\mu,t) = (p_{e\mu} - p_{s\mu})\cdot u_\mu(t) = [\mu(1-\alpha)+(1-\mu)\beta][p_{eG} - p_{sB}]\cdot u_\mu(t), \tag{12}$$

where $W(\mu,1)=0$, and so, since (IR) binds on $u_\mu(t)$ at belief μ and recalling (1),

$$\begin{aligned}p_{e\pi}\cdot u_\mu(t)-c-\Lambda\pi &= (\pi-\mu)[(p_{eG}-p_{eB})\cdot u_\mu(t)-\Lambda],\\ &= (\pi-\mu)\left(\frac{(1-\beta)}{\mu(1-\alpha)+(1-\mu)}\big(c+W(\mu,t)\big)-\Lambda\right).\end{aligned} \tag{13}$$

The first inequality in (10) is simply (8).

From the value recursion for V^* given in (9), we have

$$\begin{aligned}V^*(\pi,\mu,t) &= p_{e\pi}\cdot u_\mu(t)-c-\Lambda\pi+\sum\nolimits_k p_{e\pi}^k V^*(\psi_e^k(\pi),\psi_e^k(\mu),t-1)\\ &= (\pi-\mu)\left(\frac{(1-\beta)}{\mu(1-\alpha)+(1-\mu)\beta}\big(c+W(\mu,t)\big)-\Lambda\right)\\ &\quad+\sum\nolimits_k p_{e\pi}^k V^*(\psi_e^k(\pi),\psi_e^k(\mu),t-1).\end{aligned} \tag{14}$$

A natural way to proceed is by induction. Suppose $t=1$. Using (12),

$$\begin{aligned}V(\pi,\mu,1) &\geq V^*(\pi,\mu,1)\\ &= (\pi-\mu)\big((p_{eG}-p_{eB})\cdot u_\mu(1)-\Lambda\big)\\ &= (\pi-\mu)\big((1-\beta)((p_{eG}-p_{sB})\cdot u_\mu(1)-\Lambda\big)\\ &\geq (\pi-\mu)K.\end{aligned}$$

The inductive hypothesis is

$$\pi\geq\mu \implies V^*(\pi,\mu,t-1)\geq(\pi-\mu)K(t-1).$$

If this implied

$$\sum\nolimits_k p_{e\pi}^k V^*(\psi_e^k(\pi),\psi_e^k(\mu),t-1)\geq(\pi-\mu)K(t-1), \tag{15}$$

then we would be done, since $W(\mu,t)\geq 0$ and so

$$(\pi-\mu)\left(K+\frac{(1-\beta)}{\mu(1-\alpha)+(1-\mu)\beta}W(\mu,t)\right)\geq(\pi-\mu)K.$$

Note that $\pi\geq\mu$ implies $\psi_e^k(\pi)\geq\psi_e^k(\mu)$. However, (15) fails because beliefs merge. From the inductive hypothesis we have

$$\sum\nolimits_k p_{e\pi}^k V^*(\psi_e^k(\pi),\psi_e^k(\mu),t-1)\geq K(t-1)\sum\nolimits_k p_{e\pi}^k(\psi_e^k(\pi)-\psi_e^k(\mu)).$$

Using the equality $p^k_{e\pi} = p^k_{e\mu} + (\pi - \mu)(p^k_{eG} - p^k_{eB})$, we have

$$\begin{aligned}\sum\nolimits_k p^k_{e\pi}(\psi^k_e(\pi) - \psi^k_e(\mu)) &= \pi - \sum\nolimits_k p^k_{e\pi}\frac{\mu p^k_{eG}}{p^k_{e\mu}} \\ &= \pi - \mu - (\pi-\mu)\mu\sum\nolimits_k (p^k_{eG} - p^k_{eB})\frac{p^k_{eG}}{p^k_{e\mu}} \\ &= (\pi-\mu)(1-\xi(\mu)), \end{aligned} \tag{16}$$

where

$$\xi(\mu) := \mu\sum\nolimits_k (p^k_{eG} - p^k_{eB})\frac{p^k_{eG}}{p^k_{e\mu}} > 0$$

is the *merging deficit*.[8] Therefore, all we can conclude from the inductive hypothesis with respect to the second term of (14) is

$$\sum\nolimits_k p^k_{e\pi}V^*(\psi^k_e(\pi), \psi^k_e(\mu), t-1) \geq (\pi-\mu)K(t-1)(1-\xi(\mu)). \tag{17}$$

For future reference, a straightforward calculation shows that under the collinear parameterization,

$$\xi(\mu) = \mu(1-\beta)\sum\nolimits_k (p^k_{eG} - p^k_{sB})\frac{p^k_{eG}}{p^k_{e\mu}}. \tag{18}$$

But the inductive hypothesis also bounds the future information rents from shirking,

$$\begin{aligned} W(\mu,t) &= \sum\nolimits_k p^k_{s\mu}V(\psi^k_s(\mu), \psi^k_e(\mu), t-1) \\ &\geq \sum\nolimits_{\{k:\psi^k_s(\mu)\geq\psi^k_e(\mu)\}} p^k_{s\mu}V(\psi^k_s(\mu), \psi^k_e(\mu), t-1) \\ &\geq K(t-1)\sum\nolimits_{\{k:\psi^k_s(\mu)\geq\psi^k_e(\mu)\}} p^k_{s\mu}(\psi^k_s(\mu) - \psi^k_e(\mu)) \\ &\geq K(t-1)\sum\nolimits_k p^k_{s\mu}(\psi^k_s(\mu) - \psi^k_e(\mu)). \end{aligned}$$

[8] The strict positivity of $\xi(\mu)$ for $\mu \notin \{0,1\}$ is an immediate implication of Assumption 1. Consistent with the interpretation of ξ as a measure of the merging deficit, $\xi(\mu) \to 0$ as $\mu \to 0$ or 1 (the summation equals 0 when $\mu = 1$, i.e.,when $p^k_{eG} = p^k_{e\mu} = 1$).

Now,

$$
\begin{aligned}
\sum\nolimits_k p^k_{s\mu}(\psi^k_s(\mu) - \psi^k_e(\mu)) &= \mu - \sum\nolimits_k p^k_{s\mu} \frac{\mu p^k_{eG}}{p^k_{e\mu}} \\
&= \mu \sum\nolimits_k \left\{ p^k_{e\mu} - p^k_{s\mu} \right\} \frac{p^k_{eG}}{p^k_{e\mu}} \\
&= \mu \sum\nolimits_k [\mu(1-\alpha) + (1-\mu)\beta](p^k_{eG} - p^k_{sB}) \frac{p^k_{eG}}{p^k_{e\mu}} \\
&= \frac{[\mu(1-\alpha) + (1-\mu)\beta]}{(1-\beta)} \xi(\mu). \qquad (19)
\end{aligned}
$$

Hence,

$$
\frac{(1-\beta)}{[\mu(1-\alpha) + (1-\mu)\beta]} W(\mu, t) \geq K(t-1)\xi(\mu). \qquad (20)
$$

Substituting (17) and (20) into (14) yields

$$
V^*(\pi, \mu, t) \geq (\pi - \mu)K[1 + (t-1)\xi(\mu) + (t-1)(1 - \xi(\mu))] = (\pi - \mu)Kt,
$$

completing the proof of (10). The inequality (11) is a rearrangement of (20), with

$$
\xi^*(\mu) = \frac{[\mu(1-\alpha) + (1-\mu)\beta]}{(1-\beta)} K\xi(\mu). \qquad (21)
$$

■

Remark 2. Suppose that the agent discounts the future at a rate $\delta < 1$. Define $S(t) = \sum_{\tau=0}^{t-1} \delta^\tau$, i.e., $S(t)$ is the discounted length of time corresponding to a horizon t. The above results hold without modification provided that we replace t by $S(t)$. In particular, $W(\mu, S(t))$ is bounded below a linear function of $S(t)$. Thus if $\delta \to 1$ and $t \to \infty$, $S(t) \to \infty$, and the future informational rent from shirking grows linearly in $S(t)$.

We analyze the infinite horizon case with discounting in Appendix C and show that a similar phenomenon arises in this case as well (Proposition C.1).

♦

The assumption on the structure of signals plays two roles in the analysis. The first is to provide a relationship between $p_{e\pi} \cdot u_\mu(t) - c$ and $W(\mu, t)$. The second is connect the merging deficit with the bound on $W(\mu, t)$. While it is possible to provide a relationship between $p_{e\pi} \cdot u_\mu(t) - c$ and $W(\mu, t)$ under weaker assumptions, the connection of the merging deficit with the bound on $W(\mu, t)$ is more subtle, and we have not found a more general condition.

We now relate our analysis to previous work on dynamic moral hazard and information rents, such as DeMarzo and Sannikov (2011), Sannikov (2014), Williams (2011) and Cisternas (2018), which use a "first-order approach" to study informational rents. The informational rents of the agent are computed using one-shot deviations, where the agent deviates from her equilibrium effort level incrementally.[9] Similarly, in order to bound informational rents, we have used a one-time deviation by the agent. Nonetheless, the foundations of the two analyses are quite different. When the agent's choice variable (effort) can be varied continuously, and has no discrete components, the envelope theorem implies that the agent's value function can be computed by using incremental one-time deviations. However, since our model has the agent making discrete choices, on effort as well as participation, one-time deviations only provide a lower bound on the agent's continuation value. Even with continuous effort choices, the first order approach does not apply, since the agent's participation decision may well be discrete (see Bhaskar, 2014).

Finally, actual future information rents from shirking need not be linear in t, since the lower bound may underestimate rents. In numerical examples, we are able to compute the agent's exact rents under his optimal continuation strategy following a deviation. This may involve leaving the job or shirking after some histories. We find that the lower bound can substantially underestimate the actual information rents from shirking.

6 Merging with Binary Signals

We have already seen in the two period case that the initial period contract must be more high powered than the one period contract in order to compensate for the one period FIRS. But this means that in the three period contract, the FIRS reflects the increased value of different beliefs in period 2 from the more high powered period 2 contract, in addition to the value of different beliefs in period 1.

How much of the lower bound on future information rents from shirking is due to the value from having different beliefs in all future periods, and how much is due to the positive feedback from one period's increase in the required power of the incentives to the previous period?

[9]In discrete time models, the increment is on the effort dimension and for one period, while in continuous time models, it pertains to both effort and time dimensions.

$p^H_{a\omega}$	$a = e$	$a = s$
$\omega = G$	r	$q + (2r - 1)$
$\omega = B$	$1 - r$	q

Figure 1: The probability of the high signal y^H as a function of the state ω and action a, with $0 < q < r < 1$ and $2r - 1 > 0$ and $q + r < 1$.

To shed light on this issue, we consider a symmetric environment with two signals y^H and y^L, the probability of y^H given in Figure 1, and $\Lambda = 0$.

By construction, beginning from a common prior, if the principal expects effort, but the agent shirks, then the agents is more optimistic than the principal after both y^H and y^L. We are interested in the value to the agent of shirking in the initial period (and so being more optimistic in every future period), when there are *no* expected information rents after the initial period.

Suppose that in each period (after the initial period), the principal offers the *statically optimal* contract $\hat{u}_\mu(t)$, where μ is the posterior update assuming the agent has exerted effort previously. The principal has belief $\psi_e(\mu, h^\tau) =: \mu^\tau$. This contract solves

$$u^H - u^L = \frac{c}{p^H_{e\mu^\tau} - p^H_{s\mu^\tau}}$$

and

$$p^H_{e\mu^\tau} u^H + p^L_{e\mu^\tau} u^L = 0.$$

The flow benefit to the agent from exerting effort is then, from (1),

$$[\psi_e(\pi, h^\tau) - \mu^\tau](p^H_{eG} - p^H_{eB})(u^H - u^L) = [\psi_e(\pi, h^\tau) - \mu^\tau]\frac{(2r-1)c}{p^H_{e\mu^\tau} - p^H_{s\mu^\tau}}.$$

The value to the agent of having belief $\pi > \mu$ at the end of the initial period with t periods remaining is

$$V^\dagger(\pi, \mu, t) = E_{e\pi} \sum_{\tau=1}^{t} [\psi_e(\pi, h^\tau) - \mu^\tau]\frac{(2r-1)c}{p^H_{e\mu^\tau} - p^H_{s\mu^\tau}},$$

where, as before, $h^\tau \in Y^{t-\tau}$. At the risk of emphasizing the obvious, observe that because $\pi > \mu$, for all h^τ we have that $\psi_e(\pi, h^\tau) - \mu^\tau = \psi_e(\pi, h^\tau) - \psi_e(\mu, h^\tau) > 0$.

We have the following proposition, proved in Appendix B.

Proposition 4. *Suppose there are two signals with distributions given in Figure 1, $16r^3(1-r) < 1$, and $\Lambda = 0$. There exists $\bar{V} \in \mathbb{R}$ such that for all t, and $\pi > \mu$,*

$$V^\dagger(\pi, \mu, t) < \bar{V}.$$

While we have not been able to bound $V^\dagger$ for other parameterizations (in particular, r must be close to one and q close to zero[10]), we conjecture the result holds more generally. This result supports our intuition that the incentive costs are unbounded in t due to the positive feedback from the power of the incentives.

7 The Cost of Inducing Effort

We have shown that the agent's opportunity cost of effort increases at least linearly in the length of the relationship under Assumption 2. We now examine how this translates to the principal's expected wage cost in any period, as a function of the length of the remaining relationship. The analysis of this section is based on the comparative statics of the optimal static contract when the agent's opportunity cost of shirking changes. As the time horizon t increases, $W(\mu, t)$ increases at least linearly. Let $\mathbf{w}(\mu, t)$ denote the expected wage cost of inducing effort in period t and belief μ, given that the principal induces effort in every period $s < t$. Our focus is on the behavior of $\mathbf{w}(\mu, t)$, as a function of t for a fixed μ.

It is an immediate consequence of Proposition A.1 that if the utility of the agent is CRRA with coefficient of relative risk aversion 2, then the wage cost is quadratic and increasing in $W(\mu, t)$, and therefore quadratic and increasing in t. We will prove that this generalizes: if the second derivative of $w(u)$ (the inverse of the worker's utility function) is bounded away from zero, then the wage cost is at least quadratic in the length of the relationship. Moreover, for any strictly concave utility function, the expected wage cost is at least linear in the length of the relationship.

Before we prove the above claims, we note that the wage cost is exponential in the length of the relationship when the agent's utility function is

[10] More precisely, the bound $16r^3(1-r) < 1$ requires approximately $r > 0.92$ and so $q < .08$.

log:

$$\begin{aligned}
\mathbf{w}(\mu,t) &= p_{e\mu}\cdot w(u_\mu(t)) \\
&= \sum\nolimits_k (p^k_{eB} + \mu(1-\beta)(p_{eG}-p_{sB})^k)\exp(u^k_\mu(t)) \\
&\geq \mu(1-\beta)\sum\nolimits_k (p_{eG}-p_{sB})^k \exp(u^k_\mu(t)) \\
&= \frac{\mu(1-\beta)}{\mu(1-\alpha)+(1-\mu)\beta}\sum\nolimits_k (p_{e\mu}-p_{s\mu})^k\exp(u^k_\mu(t)) \\
&\geq \frac{\mu(1-\beta)}{\mu(1-\alpha)+(1-\mu)\beta}\exp((p_{e\mu}-p_{s\mu})\cdot u_\mu(t)) \\
&\geq \frac{\mu(1-\beta)}{\mu(1-\alpha)+(1-\mu)\beta}\exp(c+W(\mu,t)).
\end{aligned}$$

Turning to more general agent utility functions, we first argue that we can restrict attention to binary signals without loss of generality.

Lemma 3. *Fix a general signal structure (Y,p). There exists a binary signal structure $(\{y^L,y^H\},p^H_a)$, such that the expected wage cost of inducing effort under (Y,p) is at least as large as the expected wage cost of inducing effort under the binary signal structure.*

Proof. Let y^K denote the signal with maximum likelihood ratio (p^k_e/p^k_s) and y^1 the signal with minimum likelihood ratio. Construct a new information structure from the original information structure by replacing each signal y^k, $k\neq 1,K$, with two signals $\bar{y}^k$ and $\underline{y}^k$ having probabilities $\theta^k_a p^k_{a\mu}$ and $(1-\theta^k_a)p^k_{a\mu}$, respectively, under the action $a\in\{s,e\}$. The numbers θ^k_a are chosen to satisfy

$$\frac{\theta^k_e p^k_{e\mu}}{\theta^k_s p^k_{s\mu}} = \frac{p^K_{e\mu}}{p^K_{s\mu}}$$

and

$$\frac{(1-\theta^k_e)p^k_{e\mu}}{(1-\theta^k_s)p^k_{s\mu}} = \frac{p^1_{e\mu}}{p^1_{s\mu}},$$

so that the likelihood ratio of $\bar{y}^k$ equals that of y^K, while the likelihood ratio of $\underline{y}^k$ equals that of y^1.

Since the optimal spot contract under the original information structure is feasible under the new information structure (by treating the pooled $\{\underline{y}^k,\bar{y}^k\}$ as y^k), the expected wage cost of inducing effort under the original information structure is at least as large as that from the optimal contract inducing effort under the new structure.

But since there are only two likelihood ratios under the new information structure, there are only two wages offered in the optimal spot contract. That is, the optimal spot contract partitions the signal space into two, $\{y^1\} \cup \{\underline{y}^k : k = 2, \ldots, K-1\}$ and $\{y^K\} \cup \{\bar{y}^k : k = 2, \ldots, K-1\}$. The desired binary signal structure treats each element of the partition as a signal. ∎

Consider now an arbitrary strictly concave and twice differentiable u. Under binary signals, the optimal contract in period t, $u_\mu(t) = (u^L_\mu(t), u^H_\mu(t))$, satisfies

$$\Delta u_\mu(t) := u^H_\mu(t) - u^L_\mu(t) = \frac{c + W(\mu, t)}{p^H_{e\mu} - p^H_{s\mu}}.$$

The optimal contract is the pair $(u^L_\mu(t), u^H_\mu(t))$ solving

$$u^H_\mu(t) = u^L_\mu(t) + \Delta u_\mu(t) \qquad \text{and}$$
$$c = p^H_{e\mu} u^H_\mu(t) + (1 - p^H_{e\mu}) u^L_\mu(t),$$

so that

$$u^L_\mu(t) = c - p^H_{e\mu} \Delta u_\mu(t) \text{ and } u^H_\mu(t) = c + (1 - p^H_{e\mu}) \Delta u_\mu(t). \tag{22}$$

Expected wages under the optimal contract are

$$\mathbf{w}(\mu, t) = (1 - p^H_{e\mu}) w(u^L_\mu(t)) + p^H_{e\mu} w(u^H_\mu(t)).$$

A second order Taylor-expansion of $w(u)$ around $u = 0$ yields

$$\mathbf{w}(\mu, t) = w(0) + \frac{1}{2}(1 - p^H_{e\mu}) w''(\hat{u}^L)(u^L_\mu(t))^2 + \frac{1}{2} p^H_{e\mu} w''(\hat{u}^H)(u^H_\mu(t))^2,$$

where $\hat{u}^L \in (u^L_\mu(t)), 0)$ and $\hat{u}^H \in (0, (u^H_\mu(t))$. If w'' is bounded below by $\rho > 0$, then the expected wage cost is a quadratic function of $u^L_\mu(t)$ and $u^H_\mu(t)$, and thus an increasing quadratic function of W, and so t.

Finally, we show that the expected wage cost increases at least linearly in t even if $w''(.)$ is not bounded away from zero. Let $u^H_\mu(1)$ and $u^L_\mu(1)$ denote the optimal contract in the static case, i.e., when $t = 1$, and let $w^H_\mu(1)$ and $w^L_\mu(1)$ denote the corresponding wages. Since $u^H_\mu(1) - u^L_\mu(1) = \frac{c}{(p^H_{e\mu} - p^H_{s\mu})}$ (due to the incentive constraint), and since u is strictly concave $u'(w^H_\mu(1)) < u'(w^L_\mu(1))$. Let $a := u'(w^H_\mu(1))$, and $b := u'(w^L_\mu(1))$. We approximate the function u by the piece-wise linear function $\tilde{u}$,

$$\tilde{u}(w) = \begin{cases} \tilde{u}_0 + a(w - \tilde{w}), & w \geq \tilde{w}, \\ \tilde{u}_0 - b(\tilde{w} - w)\ , & w < \tilde{w}, \end{cases}$$

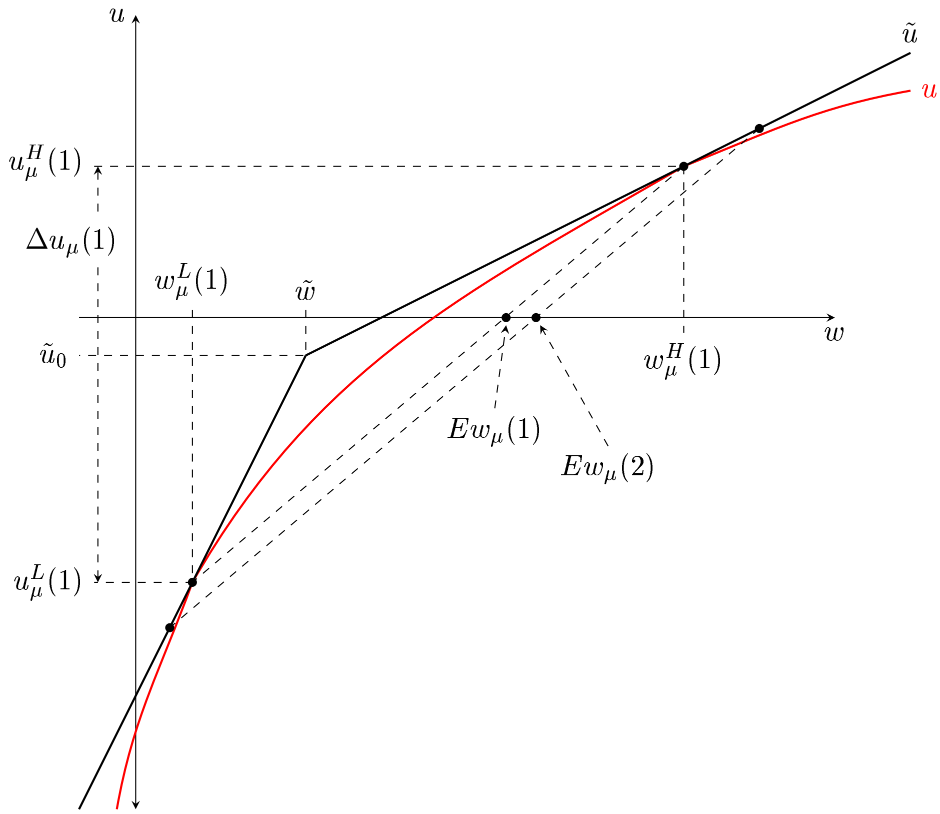

Figure 2: The original utility function u, the approximating piecewise linear utility function $\tilde{u}$, and the statically optimal contract $u_\mu(1)$ for binary signals. The contract $u_\mu(1)$ is determined by $\Delta u_\mu(1)$ and the requirement that expected utility (under $p_{e\mu}$) is zero. The expected cost of the contract is then the corresponding value on the w-axis.

where $(\tilde{w}, \tilde{u}_0)$ is defined so that $\tilde{u}$ is a continuous function, by the condition

$$\tilde{u}_0 := u^L_\mu(1) + b\left(\tilde{w} - w^L_\mu(1)\right) = u^H_\mu(1) - a\left(w^H_\mu(1) - \tilde{w}\right)$$

as depicted in Figure 2.

As before, the optimal contract satisfies (22). It is straightforward to verify that the expected cost of this contract is of the same order as $W(\mu, t)$, and so linear in t – see Figure 2. It is also straightforward to verify that the expected wage cost under the true utility function u is strictly greater than that under $\tilde{u}$. Thus the expected wage costs under the true utility function are bounded below by a linear function.[11]

To summarize: for any binary signal structure, we have established that wage costs are are bounded below by an increasing linear or quadratic function of t. Since Lemma 3 establishes that wage costs under an arbitrary information structure are greater than those under some binary signal structure, we have the following proposition.

Proposition 5. *Under the collinearity Assumption 2, $\mathbf{w}(\mu, t)$ is bounded below by a strictly increasing linear function of t. If additionally, there is a strictly positive lower bound on w'', then the bound on $\mathbf{w}(\mu, t)$ is increasing and quadratic in t. If agent utility is logarithmic, the bound is exponential in t.*

8 Endogenous Effort

We conclude with a few comments on endogenous effort. We maintain the assumptions of Proposition 3, and consider the case of finite horizons only.

First, we need to specify the effects of the the agent's action choices upon the principal's revenue. Suppose the signal is the level of revenue accruing to the principal, so that the expected revenue under action a and belief μ is $E_{a\mu}y$. Let $w_0 := w^{-1}(0)$ denote the constant wage that meets the agent's IR constraint when the agent shirks. We assume $E_{a\mu}y - w_0 > 0$ for all a and μ so that employing the agent is always optimal in the one-period problem. This implies that employment is also efficient in the dynamic case, since the principal can always hire the worker for a constant wage of w_0 (inducing shirking in every period). Let

$$R(\mu) := E_{e\mu}y - E_{s\mu}y$$

denote the principal's incremental revenue from effort over shirk at belief μ. Denote the expected wage cost of the contract $u_\mu(t)$ by $w_\mu(t)$.

In the one-period problem, the principal's optimal policy is to induce effort if

$$R(\mu) > w_\mu(1) - w_0,$$

and to induce shirking otherwise.[12] We assume inducing effort is optimal in the static setting for all beliefs μ.

[11] Since we can find a sequence of strictly concave utility functions that converge to $\tilde{u}$, one cannot in general do better than a linear bound, if we do not assume a bound on the second derivative.

[12] We assume that the principal induces shirking when she is indifferent, thereby focusing attention on the principal optimal equilibrium, since such a policy minimizes the deviation gain of the agent. Such an indifference does not arise generically.

Suppose first that, like the agent, the principal does not discount future payoffs. In addition to increasing expected revenues, effort may generate more informative signals than shirking. If effort is more informative, then the principal may find it profitable to induce effort even if the flow returns are negative. Nonetheless, we do have two straightforward observations about the profitability of always inducing effort. Since the value of information is less than linear in the horizon (because of merging), for a fixed prior, for a sufficiently long horizon, it cannot be optimal for the principal to induce effort in every period. On the other hand, for a fixed horizon, if the prior is sufficiently close the boundary, then always inducing effort can be optimal.

Proposition 6. *Suppose for all $\mu \in [0,1]$,*

$$R(\mu) > w_\mu(1) - w_0.$$

1. *For all $\mu_0 \in (0,1)$, there exists T_0 such that if $T \geq T_0$, then it is not optimal for the principal to induce effort in every period.*
2. *For all T_0, there exists $\varepsilon > 0$ such that if $\mu_0 \in (0,\varepsilon) \cup (1-\varepsilon,1)$, then it is optimal for the principal to induce effort in every period of a T-period relationship if $T \leq T_0$.*

Proof.

1. We prove this by contradiction. Suppose the principal induces effort in every period. Let v_τ be the τ-period expected benefit of e rather than s in period 1, assuming e is chosen in every subsequent period (v_τ will be negative if e generates less informative signals than s). Merging implies $v_\tau \to 0$ as $\tau \to 0$. We claim that the value of information is therefore less than linear in the horizon. More precisely, we claim that for all $\varepsilon > 0$, there exists $T_\varepsilon > 0$ such that for all $t \geq T_\varepsilon$,

$$\sum_{\tau=1}^{t} v_\tau < t\varepsilon.$$

   To prove this, fix $\varepsilon > 0$ and choose T' so that for all $t \geq T'$, we have $v_\tau < \varepsilon/2$. Define $\bar{v} := \max\{v_\tau : 1 \leq \tau \leq T'\}$. Then,

$$\sum_{\tau=1}^{T'} v_\tau + \sum_{\tau=T'+1}^{t} v_\tau \leq T'\bar{v} + (t - T' - 1)\varepsilon/2.$$

The last expression is less than $t\epsilon$ (as required) if

$$t \geq T_\varepsilon := \frac{2T'\bar{v}}{\varepsilon} - (T' + 1).$$

It is now immediate from Proposition 5 that not inducing effort in the initial period (and then inducing effort in every period) is more profitable than inducing effort in the initial period as well, when the time horizon is sufficiently long.

2. Fix T_0. Define

$$B(\mu, \mu') := \max_{t \leq T_0} |w_\mu(t) - w_{\mu'}(t)|.$$

Observe that $B(\mu, \mu')$ is continuous in μ and μ', and equals 0 when $\mu = \mu'$.

By assumption, inducing effort is statically strictly optimal, so that $\eta := \max_\mu R(\mu) - (w_\mu(1) - w_0) > 0$. Since the domain of B is compact, B is uniformly continuous, and so there exists $\varepsilon' > 0$ such that if the posterior belief μ after any history of signals and agent action choices is within ε' of the prior μ_0, then $B(\mu_0.\mu) < \eta$.

The proof is completed by observing that if the prior μ_0 is sufficiently close to the boundary (either 0 or 1) then every potential posterior must be within ε' of μ_0.

∎

8.1 The Time Path of Optimally Induced Efforts

In general, determining the principal's optimal sequence of induced actions is complicated, not least because it will also involve elements of active learning (experimentation) and possibly randomization over effort.

Consider first random effort. The principal will not induce randomized effort if the information structure is such that $\psi_s^k(\mu) \geq \psi_e^k(\mu)$ for every k, so that the agent never quits after shirking. Under this informational assumption, it is always optimal for the principal to induce a deterministic level of effort, i.e., it is strictly dominated for the principal to induce the agent to randomize between effort and shirking. If the principal induces random effort at date t, then at $t-1$, then after any signal realization y^k, the principal faces a screening problem, where the types of the agent correspond to the beliefs associated with the two different effort choices. If $\psi_s^k(\mu) \geq \psi_e^k(\mu)$ for every k, the agent never gets any informational rent after working, while he gets a rent from shirking, just as in the case the principal induces working for sure. This implies that the incentive constraint and participation constraint at date t for inducing random effort are identical to the constraints for inducing work for sure, thus ensuring that inducing random effort is dominated by one of the two pure effort levels. If the informational structure is such that there is a signal realization for which the agent is more pessimistic after shirking than after working, then inducing random effort allows the principal to commit to pay future rents to the agent who exerted effort. This relaxes both incentive and participation constraints, and in this case, inducing random effort can be optimal.

To illustrate how complicated the optimal sequence could be, we now focus on the case of a sequence of short-lived principals contracting with the long-lived agent. A short-lived principal will not induce effort for purposes of experimentation/learning. She will only induce effort if the expected wage cost of doing so is less than $R(\mu)$.

As for a long-lived principal and log utility agent, it is obviously never optimal for a short-lived principal to induce effort when all future principals induce effort and the horizon is long. A short run principal may induce random effort. As discussed above, a sufficient condition for not inducing random effort is that $\psi_s^k(\mu) \geq \psi_e^k(\mu)$ for every k.

A plausible conjecture is that for a sufficiently long lived agent, initially the short-lived principals induce shirking, followed by a second and final phase where they induce work. Intuitively, the initial phase reduces the time horizon and uncertainty regarding the state, both of which reduce the future informational rents from shirking, thereby permitting the principal to induce effort in the second phase. This conjecture is incorrect, due the following critical feature of our model: even if the agent is more optimistic than the principal, he only gets informational rents in a period if the principal induces effort in that period. As the time horizon T increases, the initial phase of shirking must also increase. But once the shirking phase becomes sufficiently long, and T increases further, it becomes profitable to induce effort in the first period of the relationship. In any subsequent period, the agent only receives an information rent if effort is induced in that period. Thus the future information rent from shirking in the first period is small when it is followed by a long phase of shirking, where not only does the agent not receive any rents, but her informational advantage is also eroded due to the merging of beliefs.

The above considerations suggest that even for the case of short-lived

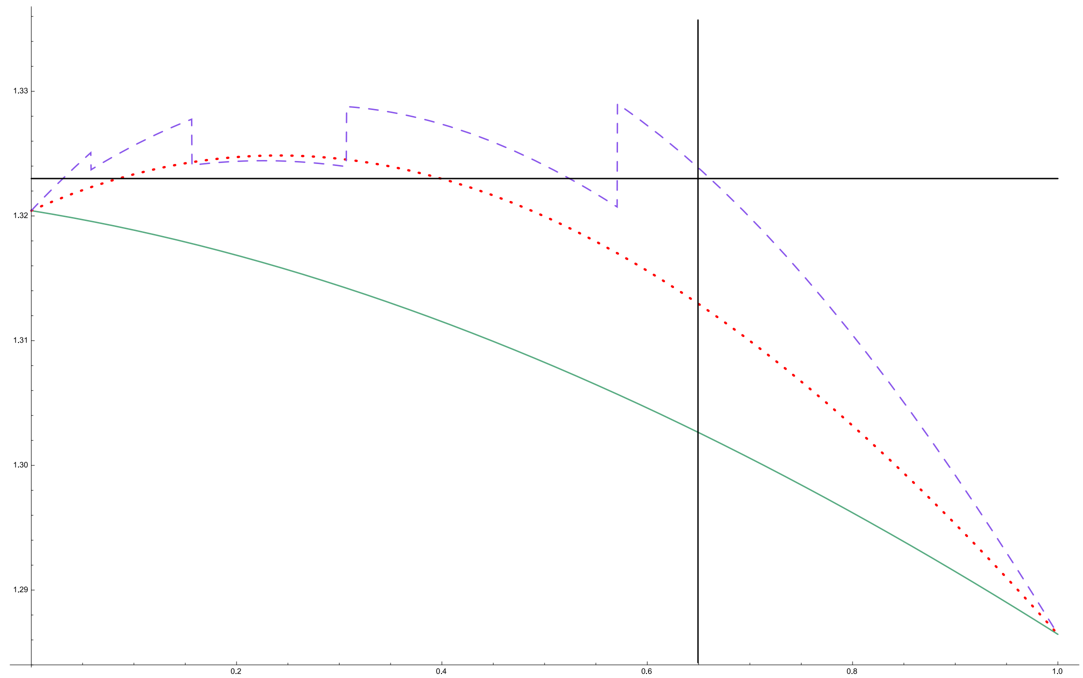

Figure 3: The expected wage cost of inducing effort in the initial period, as a function of the prior μ and length of the horizon. There are two signals, with $p^H_{eG} = \frac{3}{4}$, $p^H_{eB} = p^H_{sG} = \frac{1}{2}$, and $p^H_{sB} = \frac{1}{4}$. The cost of effort is $c = 2$ and the agent's utility functions if $u(w) = 10 \log w$. Finally, $y^H - y^L = 1.292$, so that $R(\mu) + w_0 = 1.323$. The horizontal line is $R(\mu) + w_0$, the vertical line is $\mu = 0.65$. The cyan solid line is for $T = 1$, the red dotted line is for $T = 2$, and the purple dashed line is for $T = 3$.

principals, the optimal sequence of induced agent behavior is complicated. This is illustrated in Figures 3 and 4, which report the expected wage cost of inducing effort given *optimal* future effort inducement (that is, effort in a period is only induced if that period's principal finds it optimal to do so). For example, for $T = 2$, the initial period principal does not induce effort for $\mu \in (.1, .2)$. Consequently, for $T = 3$, there is a discontinuity in the expected wage cost of inducing effort in the initial period at $\mu = \frac{4}{7}$: For $\mu > \frac{4}{7}$, effort is induced in the second periods after all signals, and so the FIRS is high. For μ just below $\frac{4}{7}$, the low signal leads to a posterior in $(.1, .2)$ and so a constant wage (no effort induced) in the second period,

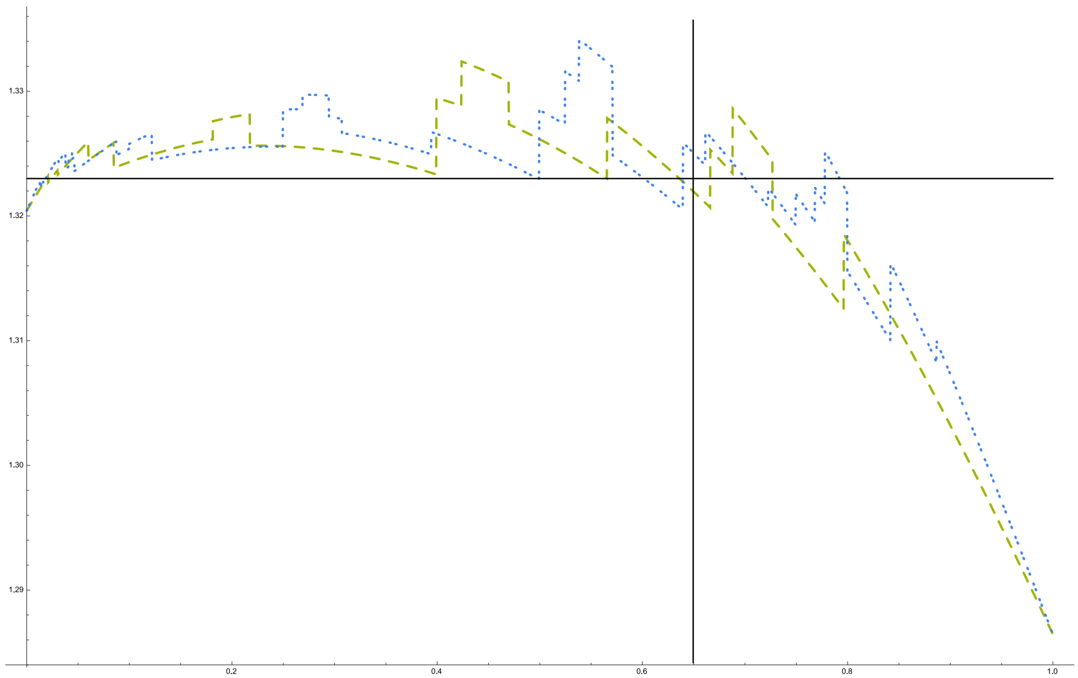

Figure 4: The expected wage cost of inducing effort in the initial period, as a function of the prior μ for $T = 4$ and $T = 5$. The green dashed line is for $T = 4$, and the blue dotted line is for $T = 5$. The parameter values are the same as in Figure 3.

with a consequently lower FIRS. Moreover, if slightly larger $y^H - y^L$, we see it is possible for some priors to induce effort in the initial period for $T = 3$ but not for $T = 2$. Finally, we see that is there is no monotonicity of the expected wage cost in either T or μ. At $\mu = .65$, for example, this results in effort being induced in the initial period for $T = 1, 2$, and 4 but not for $T = 3$ and 5.

9 Conclusions

We model the ratchet effect that arises when principal and agent face symmetric uncertainty about job difficulty and the principal does not observe the agent's effort choice. In order to overcome the ratchet effect in any period t but the last one, the principal needs to provide high-powered incentives in period t if she wants to induce effort. But such high powered incentives increase informational rents, and thereby aggravate the ratchet effect in every period prior to t. This makes always inducing effort unprofitable if the relationship is long enough.

Appendix

A CRRA Preferences

We assume the agent's utility function is unbounded below, but only to ensure individual rationality is always binding. While CRRA utility functions are not unbounded below, individual rationality will still be binding if $w(u^k) > -A$ for all y^k. In particular, it will be for many periods for the utility function in Proposition A.1 for sufficiently large A.

Proposition A.1. *Suppose the agent's utility function is given by*

$$u(w) = \sqrt{A + w},$$

where $A > 0$. If $w(u^k) > -A$ for all y^k under W and $\widetilde{W}$, then the implication (6) holds. There exists $\tilde{T}$ such that for all time horizons $T < \tilde{T}$, the agent's future informational rent from shirking, $W(T)$, is strictly increasing in T, and the principal's wage cost in the intial period is quadratic in $W(T)$.

Proof. Since $w'(u^k) = 2u^k$, the first order conditions for the principal's problem can be written as

$$2u^k = \lambda + \zeta \left(1 - \frac{p^k_{s\mu}}{p^k_{e\mu}}\right), \qquad k = 1, \ldots, K, \tag{A.1}$$

where λ is the multiplier on the IR constraint and ζ is the multiplier on the IC constraint. The incentive constraint $(p_{e\mu} - p_{s\mu}) \cdot u = c + W$ can then be rewritten as

$$\begin{aligned} c + W &= \sum\nolimits_k (p^k_{e\mu} - p^k_{s\mu}) \left[\frac{\lambda}{2} + \frac{\zeta}{2}\left(1 - \frac{p^k_{s\mu}}{p^k_{e\mu}}\right)\right] \\ &= \sum\nolimits_k \frac{\zeta}{2}(p^k_{e\mu} - p^k_{s\mu})\left(1 - \frac{p^k_{s\mu}}{p^k_{e\mu}}\right) \\ &=: \frac{\zeta X(\mu)}{2}, \end{aligned}$$

where $X(\mu) > 0$ since there exists an informative signal such that $p^k_{e\mu} \neq p^k_{s\mu}$. This implies

$$\zeta = \frac{2(c+W)}{X(\mu)},$$

and so

$$\begin{aligned}(p_{eG} - p_{eB}) \cdot (u - \tilde{u}) &= \sum\nolimits_k (p^k_{eG} - p^k_{eB}) \frac{(W - \widetilde{W})}{X(\mu)} \left(1 - \frac{p^k_{s\mu}}{p^k_{e\mu}}\right) \\ &=: X^*(\mu)(W - \widetilde{W}),\end{aligned}$$

where $X^*(\mu) > 0$ from Assumption 1.

Further, from (A.1),

$$2\mathbf{E}(u) = \lambda + \zeta \sum\nolimits_k (p^k_{e\mu} - p^k_{s\mu}) = \lambda,$$

and since $\mathbf{E}(u) - c = 0$ from the binding IR constraint, λ is independent of W, the opportunity cost of shirking.

Turning to expected wage costs, from (A.1),

$$\begin{aligned}4\mathbf{E}(w) &= \lambda^2 + \zeta^2 \sum\nolimits_k \left(p^k_{e\mu} - p^k_{s\mu}\right) \left(1 - \frac{p^k_{s\mu}}{p^k_{e\mu}}\right) \\ &= \lambda^2 + \frac{4(c+W)^2}{X(\mu)}.\end{aligned}$$

Since λ does not depend upon W, $\mathbf{E}(w)$ is quadratic in $c + W$. ■

B Proofs for Section 6

Lemma B.1. *Suppose μ, $\pi > \frac{1}{2}$. Then, there exists $\sigma \in (0, 1)$ such that for all $\mu, \pi \geq \frac{1}{2}$ and for all $y^k \in Y^H$,*

$$\left|\psi^k_e(\pi) - \psi^k_e(\mu)\right| \leq \sigma \left|\pi - \mu\right|.$$

Proof. From some straightforward calculations, we have

$$\begin{aligned}\psi^k_e(\pi) - \psi^k_e(\mu) &= \frac{\pi p^k_{eG}}{p^k_{e\pi}} - \frac{\mu p^k_{eG}}{p^k_{e\mu}} \\ &= \frac{(\pi - \mu) p^k_{eG} p^k_{eB}}{p^k_{e\pi} p^k_{e\mu}},\end{aligned}$$

and so it remains to bound the ratio of probabilities.

Now, consider

$$\begin{aligned} f^k(\pi,\mu) &:= p^k_{e\pi}p^k_{e\mu} - p^k_{eG}p^k_{eB} \\ &= \pi\mu(p^k_{eG})^2 + (1-\pi)(1-\mu)(p^k_{eB})^2 \\ &\qquad - [\pi\mu + (1-\pi)(1-\mu)]p^k_{eG}p^k_{eB}. \end{aligned}$$

This function is increasing in π and μ (since $y^k \in Y^H$), and so is minimized at $\pi = \mu = \frac{1}{2}$ over $\pi, \mu \geq \frac{1}{2}$. That is,

$$f^k(\pi,\mu) \geq \frac{1}{4}(p^k_{eG} - p^k_{eB})^2 \quad \forall \pi, \mu \geq \frac{1}{2}.$$

Define

$$X := \min_{y^k \in Y^H} \frac{(p^k_{eG} - p^k_{eB})^2}{4p^k_{eG}p^k_{eB}}$$

and set

$$\sigma = \frac{1}{1+X} \in (0,1). \tag{B.1}$$

Then,

$$\begin{aligned} p^k_{e\pi}p^k_{e\mu} - p^k_{eG}p^k_{eB} &= f^k(\pi,\mu) \\ &\geq X p^k_{eG}p^k_{eB} \\ &= \left(\frac{1}{\sigma} - 1\right) p^k_{eG}p^k_{eB}, \end{aligned}$$

and so

$$\frac{p^k_{eG}p^k_{eB}}{p^k_{e\pi}p^k_{e\mu}} \leq \sigma.$$

■

Proof of Proposition 4. For the purposes of this proof, it is more convenient to index periods forward rather than backward, so that h^τ is the τ length history leading to period τ, with $T - \tau$ periods remaining.

Given h^τ, let $n(h^\tau)$ denote the difference between the number of y^H and y^L realizations in h^τ. Then, since $p^H_{eB} = p^L_{eG}$, histories of different lengths lead to the same posterior as long as they agree in $n(h^\tau)$, i.e., for all h^τ and $h^{\tau'}$, with τ possibly different from τ',

$$n(h^\tau) = n(\hat{h}^{\tau'}) \Rightarrow \psi_e(\mu, h^\tau) = \psi_e(\mu, \hat{h}^{\tau'}).$$

We proceed by conditioning on G (the unconditional expectation is then the average of the conditioning on G and the symmetric term from B). Moreover, for large t, conditional on G, the probability that $n(h^t)$ is negative goes to zero sufficiently fast, that it is enough to show that

$$\Pr\{n(h^\tau) \geq 0 \text{ for } \tau = 0, \ldots, t-1\} \times \\ E\left\{ \sum_{\tau=0}^{t-1} [\psi_e(\pi, h^\tau) - \psi_e(\mu, h^\tau)] \,\middle|\, G, a^\tau = e, n(h^\tau) \geq 0 \right\} \tag{B.2}$$

is bounded. Moreover, we can also assume $\mu > 1/2$, since conditional on G, the probability that $n(h^\tau)$ is small becomes arbitrarily small as t becomes large.

From Lemma B.1 (using the value of σ from (B.1)), we have that for $\sigma := 4r(1-r) \in (0,1)$, if $\pi, \mu > \frac{1}{2}$, then

$$\psi_e(\pi, n(h^\tau)) - \psi_e(\mu, n(h^\tau)) < \sigma^{n(h^\tau)}(\pi - \mu).$$

Then the expression in (B.2) is bounded above by

$$\begin{aligned} &\sum_{\tau=0}^{t-1} \sum_{n=0}^{\tau} \Pr(n(h^\tau) = n)\sigma^n(\pi - \mu) \\ &= (\pi - \mu) \sum_{n=0}^{t-1} \sigma^n \sum_{\tau=n}^{t-1} \Pr(n(h^\tau) = n) \\ &\leq (\pi - \mu) \sum_{n=0}^{\infty} \sigma^n \sum_{\tau=n}^{\infty} \Pr(n(h^\tau) = n). \end{aligned} \tag{B.3}$$

We first bound

$$\Pr(n(h^\tau) = n) = b((\tau + n)/2; \tau, p) = \binom{\tau}{(\tau+n)/2} r^{(\tau+n)/2}(1-r)^{(\tau-n)/2}.$$

Using Stirling's formula[13]

$$\sqrt{2\pi}\; m^{m+1/2}e^{-m} \leq m! \leq e\; m^{m+1/2}e^{-m} \text{ for all positive integers } m,$$

[13]See, for example, Abramowitz and Stegun (1972, 6.1.38).

we bound the binomial coefficients as follows

$$
\begin{aligned}
\binom{\tau}{(\tau+n)/2} &= \frac{\tau!}{\frac{(\tau+n)}{2}!\frac{(\tau-n)}{2}!} \\
&\leq \frac{e\ \tau^{\tau+\frac{1}{2}}e^{-\tau}}{2\pi\left[\frac{(\tau+n)}{2}\right]^{\frac{(\tau+n)}{2}+\frac{1}{2}}e^{-\frac{(\tau+n)}{2}}\left[\frac{(\tau-n)}{2}\right]^{\frac{(\tau-n)}{2}+\frac{1}{2}}e^{-\frac{(\tau-n)}{2}}} \\
&\leq \frac{\tau^{\tau+\frac{1}{2}}}{\sqrt{2}\left[\frac{(\tau+n)}{2}\right]^{\frac{(\tau+n)}{2}+\frac{1}{2}}\left[\frac{(\tau-n)}{2}\right]^{\frac{(\tau-n)}{2}+\frac{1}{2}}} \\
&= \frac{(2\tau)^{\tau+\frac{1}{2}}}{(\tau^2-n^2)^{\frac{\tau}{2}+\frac{1}{2}}} \times \left(\frac{\tau-n}{\tau+n}\right)^{n/2} \\
&\leq \frac{(2\tau)^{\tau+\frac{1}{2}}}{(\tau^2-n^2)^{\frac{\tau}{2}+\frac{1}{2}}} \\
&\leq 2^{\tau+\frac{1}{2}}\left(\frac{\tau^2}{\tau^2-n^2}\right)^{\frac{\tau}{2}+\frac{1}{4}}.
\end{aligned}
$$

We also need the following calculation. Setting $k := \sqrt{(1+\sigma)/(1-\sigma)}$, gives for all $\tau > kn$,

$$
\begin{aligned}
\frac{\tau^2}{\tau^2-n^2}\sigma &< \frac{k^2n^2}{k^2n^2-n^2}\sigma \\
&= \frac{k^2}{k^2-1}\sigma \\
&= \frac{1+\sigma}{2\sigma}\sigma = \frac{1+\sigma}{2} =: y < 1,
\end{aligned}
$$

where the final inequality holds because $\sigma < 1$.

We are now in a position to bound (B.3), since

$$\begin{aligned}\sum_{n=0}^{\infty}\sigma^n\sum_{\tau=n}^{\infty}\Pr(n(h^\tau)=n) &= \sum_{n=0}^{\infty}\sigma^n\sum_{\tau=n}^{kn}\Pr(n(h^\tau)=n)\\ &\quad+\sum_{n=0}^{\infty}\left(\sigma^2\frac{r}{1-r}\right)^{n/2}\sum_{\tau=kn+1}^{\infty}\binom{\tau}{(\tau+n)/2}[r(1-r)]^{\tau/2}\\ &\le \sum_{n=0}^{\infty}\sigma^n(k-1)n\\ &\quad+\sqrt{2}\sum_{n=0}^{\infty}\left(\sigma^2\frac{r}{1-r}\right)^{n/2}\sum_{\tau=kn+1}^{\infty}\left(\frac{\tau^2}{\tau^2-n^2}\right)^{\frac{\tau}{2}+\frac{1}{4}}[4r(1-r)]^{\tau/2}\\ &\le \sum_{n=0}^{\infty}\sigma^n(k-1)n\\ &\quad+\sqrt{2}\sum_{n=0}^{\infty}\left(\sigma^2\frac{r}{1-r}\right)^{n/2}\sum_{\tau=kn+1}^{\infty}\left(\frac{\tau^2}{\tau^2-n^2}\right)^{\frac{1}{4}}y^{\tau/2}.\end{aligned}$$

Since $\sigma<1$ and $y<1$, this expression is bounded if

$$1>\sigma^2\frac{r}{1-r}=16r^2(1-r)^2\frac{r}{1-r}=16r^3(1-r).$$

C Infinite Horizon

In this section, we maintain the hypotheses on the probability distributions of Proposition 3 and show that a similar phenomenon arises with an infinite horizon. We assume both the principal and agent discount with possibly different discount factors δ_A and $\delta_P<1$. We focus on stationary high effort incentive efficient contracts.

Proposition C.1. *Suppose the probability distributions satisfy the conditions in Proposition 3. Suppose a stationary high effort incentive efficient contract exists and $V(\pi,\mu)$ is the agent's value when his belief is π and the principal's belief is μ. Then,*

$$\pi\ge\mu \implies V(\pi,\mu)\ge\frac{K(\pi-\mu)}{1-\delta_A}. \tag{C.1}$$

The future information rent from shirking becomes unbounded as the agent becomes arbitrarily patient (where $\xi^*(\mu)$ is given in (21)):

$$W(\mu;V):=\sum\nolimits_k p^k_{s\mu}V(\psi^k_s(\mu),\psi^k_e(\mu))\ge\frac{\xi^*(\mu)}{(1-\delta_A)}.$$

The denominator $1 - \delta_A$ replaces the horizon, and analogously to the finite horizon, as the agent becomes patient, future information rents from shirking become arbitrarily large.

We present the argument for $\Lambda = 0$; the case of $\Lambda > 0$ is handled mutatis mutandis (by working with the surplus net of the outside option).

Let $\mathcal{Y}$ be the set of all functions mapping $[0,1]^2$ to $\mathbb{R}$ equalling zero on the diagonal (i.e., $V(\mu,\mu) = 0$ for all $\mu \in [0,1]$ and all $V \in \mathcal{Y}$),[14] and let $\Psi : \mathcal{Y} \to \mathcal{Y}$ be the mapping defined by $V' = \Psi(V)$ given by

$$V'(\pi,\mu) := \max\Big\{p_{e\pi} \cdot u_\mu^V - c + \delta_A \sum\nolimits_k p_{e\pi}^k V(\psi_e^k(\pi), \psi_e^k(\mu)), \\ p_{s\pi} \cdot u_\mu^V + \delta_A \sum\nolimits_k p_{s\pi}^k V(\psi_s^k(\pi), \psi_e^k(\mu)), 0\Big\}, \quad \text{(C.2)}$$

where u_μ^V is the unique cost minimizing vector of utilities satisfying

$$p_{e\mu} \cdot u_\mu^V - c \geq p_{s\mu} \cdot u_\mu^V + \delta_A W(\mu; V) \quad \text{(C.3)}$$

$$\text{and} \quad p_{e\mu} \cdot u_\mu^V - c \geq 0. \quad \text{(C.4)}$$

For any stationary high effort incentive efficient contract, the value function V describing the agent's value when his belief is π and the principal's belief is μ is a fixed point of Ψ.

We proceed as in the finite horizon case, bounding V by the value function when the agent exerts effort. Consequently, as for the finite horizon case, we do not need to know the precise details of the spot contracts, here u_μ^V. It is enough to know that

$$p_{e\pi} \cdot u_\mu^V - c = (\pi - \mu)\frac{(1-\beta)}{[\mu(1-\alpha) + (1-\mu)\beta]}\left(c + \delta_A W(\mu; V)\right),$$

which follows from familiar arguments (see (12) and (13)).

Lemma C.1. *Denote by $\mathcal{V}$ the subset of $\mathcal{Y}$ satisfying the inequality in (C.1). The mapping $\Psi^e : \mathcal{Y} \to \mathcal{Y}$ defined by $V^* = \Psi^e(V)$, where*

$$V^*(\pi,\mu) := p_{e\pi} \cdot u_\mu^V - c + \delta_A \sum\nolimits_k p_{e\pi}^k V(\psi_e^k(\pi), \psi_e^k(\mu)) \quad \text{(C.5)}$$

is a self-map on $\mathcal{V}$, i.e.,

$$\Psi^e : \mathcal{V} \to \mathcal{V}.$$

[14] We have already seen in the finite horizon setting that this property holds, and it could be deduced here as well. Assuming it directly is without loss of generality and simplifies our analysis.

Proof. For $V \in \mathcal{V}$,

$$\begin{aligned}
W(\mu; V) &= \sum\nolimits_k p^k_{s\mu} V(\psi^k_s(\mu), \psi^k_e(\mu)) \\
&\geq \sum\nolimits_{\{k:\psi^k_s(\mu)\geq\psi^k_e(\mu)\}} p^k_{s\mu} V(\psi^k_s(\mu), \psi^k_e(\mu)) \\
&\geq \sum\nolimits_{\{k:\psi^k_s(\mu)\geq\psi^k_e(\mu)\}} p^k_{s\mu} \frac{K(\psi^k_s(\mu) - \psi^k_e(\mu))}{1-\delta_A} \\
&\geq \sum\nolimits_k p^k_{s\mu} \frac{K(\psi^k_s(\mu) - \psi^k_e(\mu))}{1-\delta_A} \\
&\geq \frac{[\mu(1-\alpha) + (1-\mu)\beta]}{(1-\beta)} \frac{K\xi(\mu)}{(1-\delta_A)}
\end{aligned}$$

(where the last inequality follows from (19)). This gives

$$V^*(\pi, \mu) \geq (\pi - \mu)K \left\{ 1 + \delta_A \frac{\xi(\mu)}{(1-\delta_A)} \right\} + \delta_A \sum\nolimits_k p^k_{e\pi} V(\psi^k_e(\pi), \psi^k_e(\mu)).$$

Turning to the second term, supposing $\pi \geq \mu$, and applying (16) to obtain the equality gives

$$\begin{aligned}
\sum\nolimits_k p^k_{e\pi} V(\psi^k_e(\pi), \psi^k_e(\mu)) &\geq \frac{K \sum_k p^k_{e\pi}(\psi^k_e(\pi) - \psi^k_e(\mu))}{1-\delta_A} \\
&= (\pi - \mu) \frac{K}{(1-\delta_A)} (1 - \xi(\mu)),
\end{aligned}$$

so that

$$\begin{aligned}
V^*(\pi, \mu) &\geq (\pi - \mu) \frac{K}{(1-\delta_A)} \{1 - \delta_A + \delta_A \xi(\mu) + \delta_A(1 - \xi(\mu))\} \\
&= (\pi - \mu) \frac{K}{(1-\delta_A)},
\end{aligned}$$

and so $V^* \in \mathcal{V}$. ■

Since

$$\Psi(V) \geq \Psi^e(V)$$

pointwise (i.e., for all (π, μ), $\Psi(V)(\pi, \mu) \geq \Psi^e(V)(\pi, \mu)$) and $\Psi^e : \mathcal{V} \to \mathcal{V}$, we have $\Psi : \mathcal{V} \to \mathcal{V}$.

We now argue that any fixed point of Ψ must lie in $\mathcal{V}$, which proves Proposition C.1. Since Ψ need not be a contraction, we argue indirectly.

Let $\mathcal{Y}^0 := \{V \in \mathcal{Y} \mid V(\pi, \mu) \geq 0 \ \forall (\pi, \mu)\}$. Clearly, $\Psi : \mathcal{Y} \to \mathcal{Y}^0$. For all $V \in \mathcal{Y}^0$,

$$\pi \geq \mu \implies \Psi(V)(\pi, \mu) \geq \Psi^e(V)(\pi, \mu) \geq (\pi - \mu)Kc.$$

Lemma C.2. *Defining*

$$\mathcal{Y}^\kappa := \{V \in \mathcal{Y}^{\kappa-1} \mid V(\pi,\mu) \geq (\pi-\mu)K(1-\delta_A^\kappa)/(1-\delta_A),\ \forall \pi \geq \mu\},$$

we have

$$\Psi : \mathcal{Y}^\kappa \to \mathcal{Y}^{\kappa+1}, \qquad \forall \kappa \geq 0.$$

Proof. For $V \in \mathcal{Y}^\kappa$, applying (19),

$$\begin{aligned} W(\mu;V) &\geq \sum\nolimits_{\{k:\psi_s^k(\mu)\geq\psi_e^k(\mu)\}} p_{s\mu}^k V(\psi_s^k(\mu),\psi_e^k(\mu)) \\ &\geq \sum\nolimits_{\{k:\psi_s^k(\mu)\geq\psi_e^k(\mu)\}} p_{s\mu}^k \frac{K(1-\delta_A^\kappa)(\psi_s^k(\mu)-\psi_e^k(\mu))}{1-\delta_A} \\ &\geq \sum\nolimits_k p_{s\mu}^k \frac{K(1-\delta_A^\kappa)(\psi_s^k(\mu)-\psi_e^k(\mu))}{(1-\delta_A)} \\ &\geq \frac{[\mu(1-\alpha)+(1-\mu)\beta]}{(1-\beta)}\frac{K(1-\delta_A^\kappa)\xi(\mu)}{(1-\delta_A)}. \end{aligned}$$

Then, as in the beginning of the proof of Lemma C.1,

$$\begin{aligned} \Psi^e(V)(\pi,\mu) \geq (\pi-\mu)K\left\{1+\delta_A\frac{(1-\delta_A^\kappa)\xi(\mu)}{(1-\delta_A)}\right\} \\ + \delta_A \sum\nolimits_k p_{e\pi}^k V(\psi_e^k(\pi),\psi_e^k(\mu)). \end{aligned}$$

But, for $\pi \geq \mu$,

$$\begin{aligned} \sum\nolimits_k p_{e\pi}^k V(\psi_e^k(\pi),\psi_e^k(\mu)) &\geq \frac{K(1-\delta_A^\kappa)\sum_k p_{e\pi}^k(\psi_e^k(\pi)-\psi_e^k(\mu))}{1-\delta_A} \\ &= (\pi-\mu)\frac{K(1-\delta_A^\kappa)}{(1-\delta_A)}(1-\xi(\mu)), \end{aligned}$$

so that

$$\begin{aligned} \Psi(V)(\pi,\mu) &\geq \Psi^e(V)(\pi,\mu) \\ &\geq (\pi-\mu)\frac{K}{(1-\delta_A)}\{1-\delta_A+\delta_A(1-\delta_A^\kappa)\xi(\mu)+\delta_A(1-\delta_A^\kappa)(1-\xi(\mu))\} \\ &= (\pi-\mu)\frac{K(1-\delta_A^{\kappa+1})}{(1-\delta_A)}, \end{aligned}$$

and so $V^* \in \mathcal{Y}^{\kappa+1}$. ■

Since $\mathcal{V} = \bigcap \mathcal{Y}^\kappa$, we have the desired result:

Lemma C.3. *Every fixed point of* Ψ *is in* $\mathcal{V}$.

Proof. Each fixed point of Ψ must be in every $\mathcal{Y}^\kappa$, so that

$$V(\pi,\mu) \geq \frac{(\pi-\mu)K(1-\delta_A^\kappa)}{(1-\delta_A)}, \qquad \forall \pi \geq \mu,$$

for all κ, implying $V \in \mathcal{V}$. ∎

C.1 Existence of stationary high effort incentive efficient contracts

A natural approach to obtaining existence of a well-defined value function is to find conditions under which Ψ is a contraction. Since Ψ is the point-wise maximum of Ψ^e (defined in (C.5)), Ψ^s (the analogous operator in which the agent shirks in the current period, corresponding to the second term in (C.2)), and the zero function, Ψ will be a contraction (under the sup norm) if Ψ^e and Ψ^s are (again, under the sup norm).

Suppose $V, \hat{V} \in \mathcal{V}$. Then,

$$\begin{aligned}
&|\Psi^e(V) - \Psi^e(\hat{V})| \\
&\qquad \leq \sup_{\pi,\mu}\left|\frac{(\pi-\mu)(1-\beta)}{\mu(1-\alpha)+(1-\mu)\beta}\right| \\
&\qquad\qquad \times \delta_A \left|\sum\nolimits_k p_{s\pi}^k [V(\psi_s^k(\pi), \psi_e^k(\mu)) - \hat{V}(\psi_s^k(\pi), \psi_e^k(\mu))]\right| \\
&\qquad\qquad\qquad + \delta_A \left|\sum\nolimits_k p_{e\pi}^k [V(\psi_e^k(\pi), \psi_e^k(\mu)) - \hat{V}(\psi_e^k(\pi), \psi_e^k(\mu))]\right| \\
&\qquad \leq |V-\hat{V}| \times \left\{\sup_{\pi,\mu}\left|\frac{(\pi-\mu)(1-\beta)}{\mu(1-\alpha)+(1-\mu)\beta}\right| + 1\right\}\delta_A.
\end{aligned}$$

This simple calculation shows that when Ψ^e is not a contraction, the failure arises from the future information rent from shirking (which contributes the sup term in the last expression. We also see that Ψ^e is a contraction if that sup term is sufficiently small (relative to $(1-\delta_A)/\delta_A$). A similar calculation shows that Ψ^s is also a contraction if a similar sup term is sufficiently small (also relative to $(1-\delta_A)/\delta_A$).[15]

A second approach to obtaining existence is to impose a parameter restriction that allows us to obtain a closed form expression for the value functions.

[15] The sup in Ψ^e is being taken over $|(p_{e\pi}^H - p_{e\mu}^H)/(p_{e\pi}^H - p_{s\mu}^H)|$, while the sup in Ψ^s is being taken over $|(p_{s\pi}^H - p_{e\mu}^H)/(p_{e\pi}^H - p_{s\mu}^H)|$.

Lemma C.4. *Suppose $\beta = 1 - \alpha$. The mapping Ψ has as a fixed point the function*

$$V(\pi, \mu) = \frac{\alpha c(\pi - \mu)}{(1-\alpha)(1-\delta_A)}, \tag{C.6}$$

and the associated stationary high effort incentive efficient contract is the unique cost minimizing vector of utilities satisfying (C.3) and (C.4).

Proof. We need only show that the function specified in (C.6) is a fixed point of Ψ. It is straightforward to verify that (C.6)) is a fixed point of Ψ^e. Some algebra verifies that $\Psi^e(V) - \Psi^s(V) = 0$ for V given by (C.6), and so (C.6) does indeed describe a fixed point of Ψ.[16] ■

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[16] Analogous calculations for the finite horizon can be found in the proof of Proposition 3 in an earlier version of this paper, Bhaskar and Mailath (2016).

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