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Time Lotteries and Stochastic Impatience*

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Abstract

We study preferences over lotteries that pay a fixed prize at an uncertain future date: what we call *time lotteries*. The standard model of risk and time preferences, Expected Discounted Utility, implies that individuals must be risk *seeking* towards such lotteries (RSTL). In contrast, we show experimentally that almost all subjects violate this property. Our main contributions are theoretical. First, we show that risk aversion over time lotteries can be captured by a generalization of Expected Discounted Utility that is obtained by keeping the behavioral postulates of Discounted Utility and Expected Utility. Second, we introduce a new property termed Stochastic Impatience, a risky counterpart of standard Impatience, and show that not only the model above, but also substantial generalizations that allow for non-Expected Utility and non-exponential discounting, cannot jointly accommodate it and even a *single* instance of risk aversion over time lotteries (or, equivalently, any violation of RSTL), showing a fundamental tension between the two.

Key words: Expected Discounted Utility, Separation of Risk and Time preferences, Time Lotteries, Stochastic Impatience.

JEL: C91, D81, D90.

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1 Introduction

Consider a choice between *(i)* receiving a prize in period $\bar{t}$ for sure, or *(ii)* receiving the same prize in a random period t with mean $\bar{t}$. For example, the choice may involve receiving a desirable outcome (such as $100 or a dinner at a fancy restaurant) in 10 weeks for sure versus receiving it in either 5 or 15 weeks with equal probability. Both options deliver the same prize and have the same expected delivery date, but in one of them the date is uncertain. What would, or should, the individual choose? This paper studies these decisions, which we call *time lotteries*.[1]

Many real life decisions involve uncertainty about timing. For example, is an investment that will start paying dividends in five years *for sure* better than another that will start paying in five years *on average*? Is it worthwhile to pay an extra fee to ensure that a package scheduled to arrive in a random time is delivered on a guaranteed (average) date instead? Or, when picking a mode of transport, is it better to take a train that follows a deterministic timetable or a bus which has random arrival times (due to traffic)?

According to the standard model in economics, Expected Discounted Utility (EDU), subjects should always pick the option with an uncertain payment date in any possible instance – they must be *Risk Seeking over Time Lotteries* (RSTL). To see why, note that if u is the (positive) utility function over prizes and β is the discount factor, then the value of receiving x at time t is $\beta^t u(x)$, while that of the lottery with random date is $\mathbb{E}[\beta^t]u(x)$. Since β^t is convex in t, Jensen's inequality implies that the latter option must be preferred.

The main contribution of our paper is theoretical. However, we start with an incentivized experiment on time lotteries to motivate our theoretical analysis. We find that only a minuscule fraction of subjects are consistently RSTL; most subjects are Risk *Averse* over Time Lotteries (RATL) in the majority of the questions asked.[2] We also find that risk attitudes towards time lotteries are highly correlated with attitudes towards risk in standard lotteries (i.e., atemporal lotteries over money). While perhaps intuitive, this connection is missing from EDU, where the curvature of the utility function over prizes u plays no role for time lotteries.

We present two theoretical results on modeling risk attitudes toward time lotteries, focusing on preferences defined over lotteries on dated rewards, that is, pairs of the form (x, t), where x is a monetary prize and t is the time in which it is received.[3] First, we show that one can accommodate more flexible attitudes toward time lotteries, including RATL, while maintaining the main properties used to motivate EDU.

[1] Choices of this type are analyzed in Chesson and Viscusi (2003) and Onay and Öncüler (2007). We discuss them below.

[2] While there exist surveys about attitudes towards time lotteries on humans (Chesson and Viscusi, 2003; Onay and Öncüler, 2007) and other animals (Kacelnik and Bateson, 1996), to our knowledge ours is the first incentivized experiment to investigate it. See below for a discussion.

[3] We focus on this setup for simplicity. In Appendix B we show that our main results extend to preferences over lotteries of consumption streams.

EDU can be seen as merging the *functional form* of Discounted Utility without risk with the one of Expected Utility. Instead of merging their functional forms, we instead impose the axioms that are known to guarantee Discounted Utility without risk (Outcome Monotonicity, Impatience, and Stationarity) with those of Expected Utility (Independence and Continuity). We show that these axioms do *not* characterize EDU: rather, they lead to a more general model, which we call Generalized EDU (GEDU), in which there is a strictly increasing utility function over prizes u, a discount factor β, and a strictly increasing function ϕ, such that preferences are represented by

$$\mathbb{E}\big[\phi\big(\beta^t u(x)\big)\big].$$

GEDU is similar to models that have been adopted in the literature (in particular, Kihlstrom and Mirman 1974 applied to time). Importantly, this shows how merging the functional forms adds implicit assumptions and in particular eliminates the additional curvature given by ϕ. In GEDU, intertemporal substitution is regulated by u and β, while risk preferences are regulated by a combination of ϕ, u, and β.

Unlike EDU, GEDU can accommodate different attitudes towards time lotteries. We show that the individual is RATL if and only if ϕ is more concave than the log function. This result implies that one can easily accommodate violations of RSTL without having to drop the key motivations behind EDU. In fact, we show that, together with a strengthening of Stationarity to risk, RSTL is the additional property needed to obtain EDU from GEDU. Thus, RSTL is the implicit assumption that is made when one merges the functional forms from Discounted Utility and Expected Utility.

The second part of our analysis considers a new property, called Stochastic Impatience, and presents a number of impossibility results that highlight a fundamental incompatibility between this property and even a single instance of RATL (or, equivalently, any violation of RSTL). Intuitively, Stochastic Impatience states that, when facing lotteries that pay in different periods with the same probability, the individual prefers receiving higher payments earlier. Consider, for example, two prizes, say \$100 and \$20, and two time periods, say a day and a month. Stochastic Impatience requires the individual to prefer the 50/50 lottery that pays either \$100 in a day or \$20 in a month, over the 50/50 lottery that pays either \$100 in a month or \$20 in a day. More generally, the individual is Stochastically Impatient if, for any uniform distribution over dated rewards, he prefers to pair the i^{th} highest outcome with the i^{th} earliest time. This property may be seen as a counterpart of the standard Impatience assumption under risk. Indeed, the two notions coincide under EDU.

We then examine the relationship between violations of RSTL and Stochastic Impatience. We first show that, within GEDU, Stochastic Impatience holds if and only if the individual is R*S*TL. Intuitively, RATL and Stochastic Impatience push in opposite directions: in terms of *discounted utils*, the distribution of the 50/50 lottery between \$100 in a day and \$20 in a month has a higher mean but is also more spread out than the distribution of the 50/50 lottery between \$100 in a month or \$20 in a day; in particular, it contains in its support the worst possible outcome, \$20 in a month, meaning that the individual cannot be too risk averse — again, over discounted utils— to prefer it. The theorem shows that, under GEDU, the additional concavity imposed by the function ϕ to accommodate RATL is exactly the one that tilts preferences against Stochastic Impatience, leading to an equivalence result.

This relationship, in fact, holds much more generally. We show that within a very broad class of models, observing a single violation of RSTL is enough to allow us to construct a violation of Stochastic Impatience. This class of models further generalizes GEDU by allowing general forms of non-Expected Utility (local bi-linearity, which includes many popular models such as Rank-Dependent Utility, Cumulative Prospect Theory, and Disappointment Aversion) and general forms of discounting. Thus, the fundamental tension between even an instance of RATL and Stochastic Impatience cannot be resolved within a very broad class of models. In Appendix C we show that a similar impossibility holds also for the model of Epstein and Zin (1989), which can accommodate local versions of RATL, but in those cases must also violate Stochastic Impatience.[4]

To test the relationship between RSTL and Stochastic Impatience, we conducted one further experiment on Amazon Mechanical Turk. Most subjects exhibit RATL, replicating our results in the lab. Crucially, an even larger majority (85%) did *not* violate Stochastic Impatience, and there is no relationship between RATL and the tendency to satisfy Stochastic Impatience. While this evidence is only suggestive – as we tested only specific instances of Stochastic Impatience – it indicates the joint presence of two behaviors, RATL and Stochastic Impatience, that should not coexist under very general assumptions.

To summarize, our incentivized experiment indicates that violations of RSTL are widespread, and we investigate models that can accommodate them. If one is not concerned with Stochastic Impatience, we propose a model that allows for this pattern while maintaining the key properties at the base of EDU – except that it is built starting from axioms instead of merging functional forms. However, if one also wishes to maintain Stochastic Impatience, we show that it is impossible to accommodate any instance of RATL within a very broad class of models. The take-home message is therefore that if one believes that preferences may exhibit even a single instance of RATL-type behavior – as almost universally true in our experimental data – then either Stochastic Impatience *must* be violated, or one needs to consider models beyond commonly used ones.

We conclude the introduction with a discussion of the literature. To our knowledge, the first study of time lotteries appears in Chesson and Viscusi (2003), who show that EDU implies a preference for uncertain timing. They hypothesize that risk aversion over time may be due to high risk aversion or hyperbolic discounting. The latter is proven impossible by Onay and Öncüler (2007), who generalize their

[4]A companion paper, Dillenberger et al. (2018) discusses the broader relation between the Epstein-Zin model and Stochastic Impatience.

theoretical results, pointing out that (what we call) RSTL holds for any convex discount function. They link this to probability distortions. Ebert (2017) extends the analysis to higher-order risk preferences (prudence and temperance).[5] We show that RATL can be accommodated within Expected Utility, but if one wants to preserve Stochastic Impatience, then not even allowing for probability distortions is sufficient.

For experimental evidence, to our knowledge there are no incentivized experiments testing the attitude towards time lotteries. Chesson and Viscusi (2003) conduct a hypothetical survey with business owners and find that about a third of them are RATL. Onay and Öncüler (2007) run a non-incentivized survey with large hypothetical payments and find that most subjects are RATL. By contrast, Kacelnik and Bateson (1996) show evidence that animals in foraging decisions tend to be RSTL. Eliaz and Ortoleva (2016) show that subjects are ambiguity averse when timing is ambiguous.

The remainder of the paper is organized as follows. Section 2 formalizes risk attitudes towards time lotteries and briefly describes the results of the experiment. Section 3 includes the analysis of GEDU and how it accommodates RATL. Section 4 defines Stochastic Impatience and presents the impossibility results. Section 5 concludes. The Appendices include the proofs of the results in the text, the extension to consumption streams and to the model of Epstein and Zin (1989), and further experimental analysis and details. The Supplementary Appendix contains a discussion of local notions of RATL, the proofs of the results in the Appendix, and screenshots of the experiment.

2 Risk Aversion over Time Lotteries

Consider an interval of monetary prizes $[w, b] \subset \mathbb{R}_{++}$ and a set of dates T that is either a set of non-negative integers ("discrete time") or an interval of non-negative numbers ("continuous time"), with $0 \in T$ in both cases. We interpret each element of $[w, b] \times T$ as a dated reward, where (x, t) indicates receiving the monetary prize x in t periods (or at time t; the distinction is irrelevant here). Let Δ be the set of simple lotteries over $[w, b] \times T$ endowed with the topology of weak convergence, and $\delta_{(x,t)}$ denote the degenerate lottery that gives (x, t) with certainty.[6] We study a complete and transitive preference relation $\succsim$ over Δ, where $\sim$ and $\succ$ denote its symmetric and asymmetric parts, respectively.

Throughout the paper, both in the theoretical part and in the experiment, we focus on lotteries in which the resolution of uncertainty is immediate: once the individual chooses a lottery, it will be immediately acted out and the prize (and time) will be known. Therefore, preferences in our setting will not stem from any planning considerations.

A *time lottery* is a lottery that pays a fixed monetary prize x at a random date. For example, a lottery that pays \$10 in either ten or twenty periods with equal probability

[5] Chen (2013) also shows that EDU implies RSTL.

[6] We focus on monetary prizes for simplicity. It is easy to generalize the results to arbitrary sets.

is a time lottery. For any $x \in [w,b]$, we say that $p_x \in \Delta$ is a *time lottery with prize* x if $y = x$ for any (y,t) in its support.

We start by defining attitudes towards time lotteries:

Definition 1. $\succsim$ is Risk Averse over Time Lotteries (RATL) if for all $x \in [w,b]$ and all time lotteries p_x with prize x, if $\bar{t} = \sum_\tau p_x(x,\tau) \times \tau$ then

$$\delta_{(x,\bar{t})} \succsim p_x.$$

Analogously, $\succsim$ is Risk Seeking over Time Lotteries (RSTL) or Risk Neutral over Time Lotteries (RNTL) if the above holds with $\preceq$ or $\sim$, respectively.

In words, the individual is RATL if he always prefers to receive a certain amount in a sure time to receiving the same amount on a random time with the same mean. RSTL and RNTL are defined analogously.[7]

The standard model to study risk and time is the Expected Discounted Utility model (EDU), according to which lotteries are evaluated by

$$V(p) = \mathbb{E}_p\big[\beta^t u(x)\big], \tag{1}$$

where u is a positive valued utility function over monetary outcomes and $\beta \in (0,1)$ is a discount factor. Note that for any fixed x, EDU evaluates a time lottery with prize x as $\mathbb{E}_p\big[\beta^t\big]u(x)$. Since β^t is a convex function of t, it follows from Jensen's inequality that, independently of u, any preference relation in EDU must be RSTL. In fact, since this argument only relies on the convexity of the discount function, it holds more generally than for exponential discounting. Suppose preferences are represented by

$$V(p) = \mathbb{E}_p\big[D(t)u(x)\big], \tag{2}$$

where D is a strictly positive and strictly decreasing function with $D(0) = 1$. Then, preferences are RSTL if and only if D is convex.[8] Notice that all discount functions used in practice – including exponential, hyperbolic, and quasi-hyperbolic – are convex.[9] Moreover, when T is unbounded, no strictly decreasing function $D : T \to (0,1]$ can be concave. Thus, in this case no preference relation represented by (2) can be RATL.[10]

[7] More generally, one can define attitudes towards time lotteries in terms of preferences for mean-preserving spreads in time. In the GEDU model described in Section 3, RATL coincides with an aversion to mean-preserving spreads, so these definitions are equivalent.

[8] See Supplementary Appendix A for the definition of convexity when T is discrete.

[9] We also test convexity in our experiment (below) and find that the vast majority of subjects satisfy it. Note that convex discounting is implied by requiring Diminishing Impatience (or no future bias), a property that holds if for any $\tau > 0$, the ratio $\frac{D(t)}{D(t+\tau)}$ is not increasing in t.

[10] The risk attitudes towards time lotteries in Definition 1 are defined for arbitrary periods and prizes. In Supplementary Appendix A, we introduce their local counterparts, relate it to the local convexity/concavity of the discount function, and show that preferences represented by (2) must be locally-RSTL in all but a finite number of periods.

The impossibility of EDU to accommodate different attitudes towards time lotteries can be understood with an analogy to the classic work of Yaari (1987). Within the (atemporal) Expected Utility framework, diminishing marginal utility of income and risk aversion are bound together via the curvature of the utility function over prizes. But, as Yaari argues, these two properties are "horses of different colors" and hence, as a fundamental principle, a theory that keeps them separate may be desirable. In our setting, convex discounting, which is a property of deterministic settings, implies RSTL, a property of stochastic settings. There is no fundamental reason why the two should be related. Moreover, in Yaari's analysis even though diminishing marginal utility of income and risk aversion relate to two different phenomena, they are both reasonable and documented properties. In our case, however, while convex discounting is a plausible and documented behavioral property, we now provide evidence that most people violate RSTL.

2.1 Experimental evidence of RATL

We now describe the results of an experiment that measures attitudes towards time lotteries using incentivized questions. Because the purpose of this section is primarily to motivate the theoretical results that will be presented later, we postpone many of the details and additional analyses to Appendix E.

A total of 197 subjects took part in an experiment run at the Wharton Behavioral Lab. The experiment has three parts. Part I asks subjects to choose between different time lotteries: they were offered two options that paid the same prize at different dates, where the distribution of payment dates of one option was a mean preserving spread of that of the other. For example, the first question asked them to choose between i) \$15 in 2 weeks or ii) \$15 in 1 week with probability .75 and in 5 weeks with probability .25. Subjects answered five questions of this kind. In three of them, one of the options had a known date; in the others, both options had random payment dates. Table 3 in Appendix E lists the questions.[11]

Parts II and III use the multiple price list (MPL) method to measure time and risk preferences separately. Part II measures standard time preferences as well as attitudes towards time lotteries. Part III measures atemporal risk preferences, with payments taking place immediately at the end of the session. These include measures of regular risk aversion, as well as Allais' common-ratio-type questions to test and quantify violations of Expected Utility. Procedures in this part follow standard practice.

At the end of the experiment, one question was randomly selected for payment.

[11] A recent literature has discussed concerns with using monetary rewards to study intertemporal choice in experiments (Augenblick et al., 2015). Here, however, typical issues do not apply (e.g., the curvature of the utility function is inconsequential for time lotteries as the prize is fixed). Moreover, we are interested in studying the relation between attitude towards time lotteries and atemporal risk aversion. Since the latter is defined for monetary lotteries, we focus on monetary prizes and leave for future research an investigation of time lotteries involving different objects, noting that the same intuition applies more in general.

Table 1: Attitude Towards Time Lotteries in Part I

% of RATL choices	Long	Short
Question 1	65.7	56.0
All questions	60.6	47.9
Both dates random	69.0	45.6

# of RATL choices	Long		Short	
	Percent	Cum.	Percent	Cum.
0	2.9	2.9	9.9	9.9
1	9.5	12.4	16.4	26.3
2	22.9	35.2	24.3	50.6
3	23.8	59.0	26.4	76.9
4	28.6	87.6	19.8	96.7
5	12.4	100.0	3.3	100.0

The order of parts and of questions was partly randomized, except that all subjects received Part I first, and all subjects received the same first question in a separate sheet of paper. The answer to this question is a key indication of the subjects' preferences, as it captures their immediate reaction to this choice, uncontaminated by other questions.

We ran two treatments: a long delay treatment ('Long') with 105 subjects and a short delay treatment ('Short') with 91 subjects. The only difference between them is the length of the delay in payments. In the Long treatment, some payments were delayed by up to 12 weeks; in the Short, the maximum delay was 5 weeks.

Our main results pertain to the attitude towards time lotteries in Part I. Table 1 presents the percentage of RATL choices in (i) Question 1 of Part I, (ii) all questions, and (iii) only questions in which both lotteries pay at random dates. It also shows the distribution of the number of RATL answers that each subject gave (where 0 means never RATL, 5 means always RATL).

Only a minuscule (2.86% or 9.89%) fraction of subjects are consistently RSTL, whereas the majority of subjects choose according to RATL in the majority of questions. This pattern holds also in the first question and when both options are risky. Moreover, in most questions RATL is stronger in the Long rather than in the Short treatment.[12] This difference makes intuitive sense since, when the time horizon is relatively short, the two options are closer in value and thus the choices are closer to an even split. As the time horizon increases, as in the Long treatment, the options become less similar, and subjects move away from an even split and become *more* RATL. This contrasts with EDU, which not only predicts that individuals must al-

[12]For example, the proportion of subjects who exhibit a majority of RATL choices is about 65% in the Long vs. about 50% in the Short (test of proportion, p-value 0.015). Overall, the proportion of RATL answers are 60.6 vs. 47.9 (p-value <0.001).

ways pick the RSTL option, but also that the premium that they would be willing to pay to avoid the RATL option would increase in the time horizon.

We then analyze the relationship between RATL, convexity of discounting (measured using the questions on time preferences), atemporal risk aversion, and violations of Expected Utility (see Appendix E.2 for details and statistical analysis). In line with previous findings, 82% of our subjects exhibit convex discounting. We also find that 39.89% choose approximately according to Expected Utility throughout. Even focusing on subjects in either of these two groups, RATL is still prevalent, with almost identical proportions as in the entire sample. Regression analysis confirms that certainty bias or convexity of discounting are generally uncorrelated with RATL.[13]

Lastly, we test the relation between the tendency to exhibit RATL and atemporal risk aversion. Here we find a significant correlation: subjects who are more (atemporally) risk averse, also tend to be more RATL (see Table 9 in Appendix E.2). While this is intuitive since they are both forms of risk aversion, it is hard to reconcile this connection within EDU.

3 Modeling RATL under Expected Utility

We have seen that even a single violation of RSTL cannot be accommodated by EDU; we now explore whether it is possible to do so with a model that maintains the two core motivations behind EDU: 1) coincide with exponentially discounted utility when there is no risk; and 2) evaluate risk by calculating 'expected utility'. EDU is obtained by merging the two functional forms – simply taking the expectation of the discounted utility. Here, instead, we will characterize the model that satisfies the (known and simple) *axioms* underlying discounted utility without risk and expected utility with risk. Combining these axioms leads to a strictly more general model than EDU. Importantly, this will allow for more flexibility with respect to attitude towards time lotteries.

To posit exponentially discounted utility without risk, consider the following conditions over degenerate lotteries (sure outcomes):

Axiom 1 (Outcome Monotonicity)**.** *For all* $x, y \in [w, b]$*, and* $s \in T$*, if* $x > y$ *then* $\delta_{(x,s)} \succ \delta_{(y,s)}$.

Axiom 2 (Impatience)**.** *For all* $x \in [w, b]$ *and* $s, t \in T$*, if* $s < t$ *then* $\delta_{(x,s)} \succ \delta_{(x,t)}$.

Axiom 3 (Stationarity)**.** *For all* $x, y \in [w, b]$*,* $s, t \in T$*,* $\tau \in \mathbb{R}$ *with* $s + \tau, t + \tau \in T$*, if* $\delta_{(x,t)} \sim \delta_{(y,t+\tau)}$ *then* $\delta_{(x,s)} \sim \delta_{(y,s+\tau)}$.

[13]As shown in Section 2, this is not compatible with EDU, where RSTL is connected with the convexity of discounting. These results also indicate that RATL may not be due to violations of Expected Utility, as opposed to what has been suggested by Chesson and Viscusi (2003) and Onay and Öncüler (2007).

Outcome Monotonicity states that, holding the time fixed, higher prizes are better. Impatience states that earlier payments of the same prize are better. Stationarity posits that the preference between x in t periods and y in $t+\tau$ periods does not depend on t, a standard condition known to lead to exponential discounting. Fishburn and Rubinstein (1982) show that these axioms, together with continuity (below), are necessary and sufficient to guarantee that the restriction of $\succsim$ to degenerate lotteries is represented by Discounted Utility: $V(\delta_{(x,t)}) = \beta^t u(x)$.

Next, we posit the key postulate for Expected Utility, Independence,[14] as well as a continuity assumption:

Axiom 4 (Independence)**.** *For all $p, q, r \in \Delta$ and $\lambda \in (0,1)$,*

$$p \succsim q \;\Leftrightarrow \lambda p + (1-\lambda) r \succsim \lambda q + (1-\lambda) r.$$

Axiom 5 (Continuity)**.** *For all $(y,s) \in [w,b] \times T$ the sets $\{(x,t) \in [w,b] \times T : \delta_{(x,t)} \succsim \delta_{(y,s)}\}$ and $\{(x,t) \in [w,b] \times T : \delta_{(y,s)} \succsim \delta_{(x,t)}\}$ are closed. Moreover, for all $p, q, r \in \Delta$, if $p \succ q \succ r$, than there exist $a, b \in (0,1)$ such that $ap + (1-a)r \succ q$ and $q \succ bp + (1-b)r$.*

By standard arguments, these last two postulates ensure that the individual evaluates lotteries according to the Expected Utility criterion.

The following theorem characterizes the model that satisfy the postulates above.

Proposition 1. *The following are equivalent:*

1. *$\succsim$ satisfies Outcome Monotonicity, Impatience, Stationarity, Independence, and Continuity;*
2. *There exist $\beta \in (0,1)$, $u : [w,b] \to \mathbb{R}_{++}$ strictly increasing and continuous, and $\phi : \mathrm{Im}\big((\beta^{(\cdot)} u(\cdot)\big) \to \mathbb{R}$ strictly increasing,[15] such that $\succsim$ is represented by*

$$V(p) = \mathbb{E}_p(\phi(\beta^t u(x))),$$

We call this representation a *Generalized Expected Discounted Utility* model (GEDU) and identify it with the triple (u, β, ϕ). GEDU is similar to existing models in the literature. In particular, it can be seen as an application of the multi-attribute function of Kihlstrom and Mirman (1974) to the context of time.[16]

[14] Note that Independence is imposed on all lotteries, and not only on lotteries with a fixed payment date. While weaker versions have been proposed, we focus on this complete version since it is satisfied by EDU, and is often necessary for desired results (e.g., to guarantee the existence of Nash equilibria).

[15] $\mathrm{Im}((\beta^{(\cdot)} u(\cdot))$ is the image of $\beta^t u(x)$ over $[w,b] \times T$.

[16] A similar functional form was used, but not derived, by Andersen et al. (2017), to study intertemporal utility and correlation aversion, by Abdellaoui et al. (2017), to study different questions on time and risk, as well as by Edmans and Gabaix (2011) and Garrett and Pavan (2011).

Proposition 1 shows that combining the axioms that lead to exponential discounting without risk with the axioms that lead to Expected Utility does not generate EDU. Rather, it leads to a model that includes one additional curvature, captured by the function ϕ, applied *after* discounting has taken place. The model only coincides with EDU when ϕ is affine. It is important to highlight that the Proposition follows immediately from entirely standard arguments: it is a consequence of the fact that one cannot assume that the Bernoulli utility used in the Expected Utility form is, cardinally, the discounted utility.[17]

Under EDU, time and risk preferences are both governed solely by the curvature of u. This is no longer the case for GEDU. Intertemporal substitution is governed by u and β: without risk, the individual evaluates a prize x payed at time t by $\beta^t u(x)$.[18] Atemporal risk preferences, for lotteries with only immediate payments, are instead governed by $\phi \circ u$: a lottery p that pays only at time 0 is evaluated by $\mathbb{E}_p(\phi(u(x)))$. Thus, under GEDU, intertemporal substitution and risk aversion differ – the difference captured by the curvature of ϕ. One possible interpretation is that u represents the individual's utility function over deterministic payments, while ϕ represents risk attitude towards variations in 'discounted utils.'

The significance of this model is that, unlike EDU, GEDU does not constrain preferences to be RSTL.

Proposition 2. *Consider $\succsim$ that admits a GEDU representation (u, β, ϕ). Then:*

1. *$\succsim$ is RSTL if and only if ϕ is a convex transformation of* $\ln$*;*
2. *$\succsim$ is RNTL if and only if ϕ is an affine transformation of* $\ln$*;*
3. *$\succsim$ is RATL if and only if ϕ is a concave transformation of* $\ln$*.*

Proposition 2 follows from noticing that if $\phi = \ln$, then $\phi(\beta^t u(x)) = t \ln[\beta] + \ln[u(x)]$, an affine function of t, implying risk neutrality over time lotteries. If ϕ is "more concave than the log," preferences are RATL; if it is "more convex than the log," preferences are RSTL. Note also that the curvature of ϕ affects both risk attitudes towards time lotteries and atemporal risk aversion. As discussed in Section 2, the connection between these two forms of risk aversion is supported by our experimental results.

[17] From Fishburn and Rubinstein (1982) we know that Axioms 1-3 and Continuity guarantee that there exist a utility function u and a discount factor β such that degenerate lotteries are ranked according to Discounted Utility. Independence and Continuity guarantee that there exists a utility function over prize-dates pairs, $v : [w, b] \times T \to \mathbb{R}$, such that preferences over lotteries follow Expected Utility using Bernoulli utility v. The key observation is that $v(x, t)$ need not coincide with $\beta^t u(x)$: they must be *ordinally*, but not necessarily *cardinally*, equivalent. Thus, there must exist a strictly increasing function ϕ such that $v(x, t) = \phi(\beta^t u(x))$. Put differently, the function ϕ is needed because the curvature emerging from Discounted Utility, $\beta^t u(x)$, may not be the correct one to capture the risk preferences.

[18] According to the model, he evaluates (x, t) by $\phi(\beta^t u(x))$, which is a strictly increasing transformation of $\beta^t u(x)$.

It is worth mentioning that RSTL is not only a property implied by EDU. In Appendix A we show that, in addition to Risk Stationarity – a stationarity assumption on risky prospects – RSTL is *the* characterizing feature of EDU among the class of preferences that admit a GEDU representation. In Appendix A we also show that, adding Risk Stationarity, RATL characterizes preferences that admit a "Negative EDU" representation: EDU except that the discount factor β exceeds 1 and the utility function over outcomes u takes negative values (such as, for example, CRRA functions more concave than the log). Since u is negative, earlier payments are better despite the fact that $\beta > 1$ (so this model also satisfies Impatience). This is a simple modification of the EDU model that can easily accommodate RATL and satisfies all properties above.[19]

Since RSTL is implicitly assumed whenever one uses EDU, it is natural to ask if this is a reasonable property. From a positive point of view, we saw that most subjects violate RSTL. From a normative point of view, we contend that this assumption does *not* have the same appeal of the postulates imposed above: while Outcome Monotonicity, Impatience, and Independence have well-known normative justifications, and while both Stationarity and Risk Stationarity can be justified based on dynamic consistency, we find no equivalent arguments to justify why individuals should necessarily be risk seeking over time lotteries.

4 RATL and Stochastic Impatience: Impossibility Results

Thus far we have analyzed the extent to which we can relax RSTL by properly combining the postulates of Discounted Utility over sure outcomes with the ones of Expected Utility under risk. In this section, we show that relaxing RSTL generally comes at the 'cost' of violating a plausible novel property we term Stochastic Impatience. We establish this result first within the context of GEDU; then, we show that it holds in a much more general class of models, highlighting a fundamental connection between RSTL and Stochastic Impatience.

4.1 Stochastic Impatience

Suppose that an individual is asked to choose between the following two alternatives:

A Receive either \$100 today or \$20 in a month, with probability 0.5 each;

B Receive either \$20 today or \$100 in a month, with probability 0.5 each.

Both options involve the same prizes, probabilities, and dates, but in the first one the higher prize is paid earlier, keeping the same odds. One could imagine that, to the

[19]Indeed this is a special case of GEDU. It is easy to see that whenever $\phi(x) = -\frac{1}{x^\alpha}$ for $\alpha > 1$, then GEDU can be re-written as a Negative EDU model.

extent that the individual prefers higher payments sooner, this latter option should be preferred. A related argument could be made by decomposing each alternative into two parts. Observe that both A and B offer the exact same basic lottery in which the agent receives \$20 either today or in a month. The difference between them is when an increment of \$80 is received: option A yields it today, while option B yields it in a month. Insofar as the agent prefers obtaining it sooner, then option A should be preferred. Therefore, Stochastic Impatience can be seen as an analogue of Impatience for risky environments.

We formalize this in the following axiom, which we call *Stochastic Impatience*. A companion paper, Dillenberger et al. (2018), discusses its relation with models that separate risk and time preferences. To our knowledge, this is a novel condition.

Axiom 6 (Stochastic Impatience)**.** *Let* $\mathbf{t} = (t_1, t_2, ..., t_n)$ *and* $\mathbf{x} = (x_1, x_2, ..., x_n)$ *be finite collections of* n *time periods and outcomes, with* $t_1 < t_2 <, ... < t_n$ *and* $x_1 > x_2 >, ... > x_n$*. Let* $\iota(\mathbf{t})$ *be a permutation of* $\mathbf{t}$ *and* $\iota'(\mathbf{x})$ *be a permutation of* $\mathbf{x}$*. Preferences satisfy Stochastic Impatience if for any* n*-ordered sequences* $\mathbf{t}$*,* $\mathbf{x}$ *with corresponding perturbations* $\iota(\mathbf{t})$ *and* $\iota'(\mathbf{x})$*,*

$$\sum_{i=1}^{n} \frac{1}{n} \delta_{(x_i, t_i)} \succsim \sum_{i=1}^{n} \frac{1}{n} \delta_{(x_{\iota'(i)}, t_{\iota(i)})}.$$

In words, the individual is Stochastically Impatient if and only if for any uniform distribution over dated rewards, he always prefers to pair the i^{th} highest outcome with the i^{th} earliest time.

Impatience and Stochastic Impatience are equivalent when lotteries are evaluated by $\mathbb{E}[D(t)u(x)]$ for some weakly decreasing (but not constant) D.[20] Thus, in the particular case of EDU, these are also equivalent conditions.

4.2 RATL and Stochastic Impatience under GEDU

Our next result shows that in the context of GEDU, Stochastic Impatience is incompatible with RATL; in fact, it is *equivalent* to RSTL.

Theorem 1. *Suppose that* $\succsim$ *admits a GEDU representation* (u, ϕ, β)*. The following are equivalent:*

1. *The relation* $\succsim$ *satisfies Stochastic Impatience;*
2. *The relation* $\succsim$ *satisfies Risk Seeking over Time Lotteries.*

[20] Let $A := \sum_{i=1}^{n} D(t_i)$, and set $u(x_{n+1}) = 0$ and $\sum_{j=n+1}^{n} D(t_j) = 0$. Note that $\sum_{i=1}^{n} u(x_i) D(t_i) = \sum_{i=1}^{n} \left[A - \sum_{j=i+1}^{n} D(t_j)\right] [u(x_i) - u(x_{i+1})] \geq \sum_{i=1}^{n} \left[A - \sum_{j=i+1}^{n} D(t_{\iota(j)})\right] [u(x_i) - u(x_{i+1})] = \sum_{i=1}^{n} u(x_{\iota'(i)}) D(t_{\iota(i)})$, with equality if and only if $D(t) = 1$ for all t.

This result shows a fundamental incompatibility between any violation of RSTL and Stochastic Impatience. To get intuition, recall Options A and B in the previous example, and observe that the two prizes offered by Option B (\$20 today or \$100 in a month) are, in terms of desirability, strictly in between the two prizes offered by Option A (\$100 today or \$20 in a month). There is a sense in which Option A is 'more spread out,' in discounted utility terms, although it has a higher mean. Under EDU, this spread does not matter, and Option A is preferred. Under GEDU the additional curvature through ϕ adds a layer of intertemporal risk aversion – precisely what allowed it to capture RATL. But this is also what makes the individual sensitive to the spread in utilities of Option A. In particular, if the curvature of ϕ is very high, then the value of each option will be close to that of its worst possible outcome, in which case Option B will be preferred, violating Stochastic Impatience. Put differently, the same additional curvature that allowed us to capture RATL also pushes against satisfying Stochastic Impatience.

In fact, Theorem 1 shows not only that RATL and Stochastic Impatience push in opposite directions, but that the latter is *equivalent* to the opposite of the former under GEDU. To see why, recall that we have already seen how, when $\phi = \ln$, GEDU becomes linear with respect to time; higher concavity leads to RATL, lower to RSTL. Similarly, if $\phi = \ln$, the individual is indifferent between Options A and B above, since $0.5 \ln [u(100)] + 0.5 \ln [\beta u(20)] = 0.5 \ln [u(20)] + 0.5 \ln [\beta u(100)]$. But the distribution of the 50/50 lottery between $\ln [\beta u(20)]$ and $\ln [u(100)]$ is a mean-preserving spread of that between $\ln [u(20)]$ and $\ln [\beta u(100)]$. Thus, any expected utility maximizer with utility function $\phi = g \circ \ln$ for some convex (resp., concave) function g will prefer Option A over Option B (resp., B over A). Thus Stochastic Impatience goes hand in hand with RSTL, whereas the opposite property corresponds to RATL.

The equivalence in Theorem 1 can also be understood through the lens of matching theory; in fact, our general proof is an application of a familiar result about supermodular functions.[21] Consider a general two-sided matching model with equal number of participants in each side, sorted according to a one-dimensional characteristic that represents their type. Denote by $f(a, b)$ the match output between type a and type b, where f is increasing. It is well-known (e.g., Becker 1973) that positive assortative matching — where the i^{th}-highest type a pairs with the i^{th}-highest type b — maximizes the sum of match outputs when f is supermodular, i.e., $\frac{\partial^2 f}{\partial a \partial b}(a, b) \geq 0$. In our context, let $a = u(x)$, $b = \beta^t$, and $f(u(x), \beta^t) = \phi(u(x)\beta^t) = g(\ln(u(x)\beta^t))$, and note that $\frac{\partial^2 f}{\partial u(x) \partial \beta^t}(u(x), \beta^t) = \frac{g''(\ln(u(x)\beta^t))}{u(x)\beta^t}$, which is positive if and only if g is convex – thus when ϕ is more convex than ln, which is equivalent to preferences being RSTL. Put differently, both Stochastic Impatience and RSTL imply, and are implied by, supermodularity between the size of the prize (as captured by the utility function) and the time it is received (as captured by the discount factor).

[21] We thank Rakesh Vohra for pointing out this connection.

4.3 Beyond Expected Utility: still an impossibility

Having seen that any violation of RSTL is incompatible with Stochastic Impatience under GEDU, it is natural to ask whether they can be simultaneously accommodated in a more general model. If possible, this result would suggest that one should move beyond GEDU to allow for both properties. As we have mentioned, previous papers have suggested that RATL is related to non-expected utility. We show below, however, that accommodating both remains impossible in a much larger class of models.

To prove our general result, we consider a model that we call *Generalized Local Bilinear Discounted Utility* (GL-BDU). This model generalizes GEDU in two ways. First, it posits that dated prizes are evaluated by $\phi\big(D(t)u(x)\big)$, where ϕ is as in GEDU, and D is a strictly decreasing discount function (with $D(0) = 1$) that satisfies *Diminishing Impatience*, i.e., $\frac{D(t)}{D(t+\tau)} \geq \frac{D(t')}{D(t'+\tau)}$ if $t' > t$ and $\tau > 0$. This version of the discount function includes as special cases not only exponential discounting, but also many well-known alternative specifications, such as hyperbolic and quasi-hyperbolic discounting.

Second, it replaces Expected Utility with the assumption that 50/50 lotteries between (x, t) and (x', t'), where (x, t) is better than (x', t'), are evaluated by weighting the utility of (x, t) by $\pi(0.5)$ and that of (x', t') by $(1 - \pi(0.5))$. When $\pi(0.5) = 0.5$, the model coincides with Expected Utility (for 50/50 lotteries); but if $\pi(0.5) < 0.5$, the agent underweights the better option. This very general model includes as special cases popular ones such as those of probability weighting (Rank-Dependent Utility, Quiggin 1982, and Cumulative Prospect Theory, Tversky and Kahneman 1992) and Disappointment Aversion (Gul, 1991), and restricts preferences only for 50/50 lotteries.[22]

Definition 2. We say that $\succsim$ admits a *Generalized Local Bilinear Discounted Utility* (GL-BDU) representation if there are continuous, increasing functions $u : X \to \mathbb{R}$ and $\phi : \mathrm{Im}\big(D(\cdot)u(\cdot)\big) \to \mathbb{R}$, and a strictly decreasing function $D : T \to (0, 1]$ satisfying Diminishing Impatience, such that for $\pi(0.5) \in (0, 1)$, $p = 0.5\delta_{(x,t)} + 0.5\delta_{(x',t')}$ with $D(t)u(x) \geq D(t')u(x')$ is evaluated according to:

$$V(p) = \pi(0.5)\phi\Big(D(t)u(x)\Big) + (1 - \pi(0.5))\phi\Big(D(t')u(x')\Big).$$

To reiterate, a GL-BDU representation is a very general class that subsumes the vast majority of commonly used models. For risk preferences, it restricts only how the individual evaluates 50/50 prospects, generalizing both Cumulative Prospect Theory and Disappointment Aversion. For time preferences, it allows for any discount function with Diminishing Impatience, allowing, for example, for hyperbolic and quasi-

[22]This specification also allows for generalizations of Rank-Dependent Expected Utility, e.g., the minimum from a set of probability distortions (Dean and Ortoleva, 2017). On the other hand, it does not encompass all known models of risk preferences (e.g., it does not encompass Cautious Expected Utility, Cerreia-Vioglio et al. 2015). Formally, this model is a local specification (at 0.5) of the bilinear (or biseparable) model of Ghirardato and Marinacci (2001).

hyperbolic discounting. And, for the interaction between time and risk, it includes the distortion ϕ, which we have seen under GEDU.[23]

The main result of this section is that even in this very general class of models, violations of RSTL are still incompatible with Stochastic Impatience. As GL-BDU only restricts preferences over binary and equal-chance lotteries, we will focus on this type of lotteries.

Theorem 2. *Suppose $\succsim$ admits a GL-BDU representation. If $\delta_{(x,t_2)} \succ 0.5\delta_{(x,t_1)} + 0.5\delta_{(x,t_3)}$ for some x and $t_2 = \frac{t_1+t_3}{2}$, and $D(t_3)u(b) > D(t_1)u(w)$, then $\succsim$ violates Stochastic Impatience.*

The intuition for this result is the same as the one for Proposition 1: the additional 'risk aversion' that leads to RATL – through the additional curvature ϕ, the probability distortion π, or a combination of both – must lead to a violation of Stochastic Impatience. Unlike Theorem 1, in this case RSTL and Stochastic Impatience are not equivalent: there exist instances of the model above that violate Stochastic Impatience and still satisfy RSTL. Nevertheless, the most interesting part of Theorem 1 (the fact that Stochastic Impatience implies RSTL) still holds.

We should also highlight that the proof of Theorem 2 is constructive and only requires a minimal richness condition on the space of prizes.[24] If we observe an instance of RATL, we can design a choice problem involving binary lotteries, with similar outcomes and delivery times, in which the individual violates Stochastic Impatience.

In light of the results above, a natural question is whether it is possible to reconcile violations of RSTL with Stochastic Impatience by considering more complex setups. In Appendix B we show that the same results hold when preferences are defined over lotteries over streams. One may then wish to consider the space of temporal lotteries, as in Kreps and Porteus (1978) or Epstein and Zin (1989). It is well-known that this richer setup allows more freedom to accommodate otherwise conflicting properties. The following remark partly addresses this question.

Remark 1. *In Appendix C we show that a similar impossibility holds also for the common specification of Epstein and Zin (1989) — with CRRA Expected Utility preferences and a CES aggregator. While this model can accommodate local versions of RATL, in those cases it must also violate Stochastic Impatience. That is, we have the same impossibility as in Theorem 2.*[25]

Similar to the discussion at the end of Section 2, the results in this section emphasize that the seemingly obvious link between Impatience and Stochastic Impatience

[23]The only similar generalization we are aware of appears in Abdellaoui et al. (2017) that study preferences over risk and time using a model that generalizes GEDU (for streams) allowing for cumulative probability weighting (following Rank-Dependent Expected Utility).

[24]This is the condition $D(t_3)u(b) > D(t_1)u(w)$ in the theorem, which guarantees that, at least for the t_1 and t_3 considered, the utility of the worst and best prizes are distant enough. This would be always satisfied were we to include a zero element ($u(w) = 0$).

[25]We refer to Dillenberger et al. (2018) for a discussion of the broader relation between Epstein and Zin (1989) and Stochastic Impatience.

Table 2: Attitude Towards Time Lotteries and Stochastic Impatience, MTurk

	SI=0	SI=1	Total
RATL=0	12 (7.6%)	55 (35.0%)	67 (42.7%)
RATL=1	14 (8.9%)	76 (48.4%)	90 (57.3%)
Total	26 (16.6%)	131 (83.4%)	157

could be more subtle, due to the presence of risk. Recall the intuition given right after the statement of Theorem 1. In the context of risk and time, Stochastic Impatience not only sorts the time-payoff pairs in a way that is most consistent with Impatience, but it also maximizes the spread, in terms of discounted utils, among all other possible combinations. This additional spread is inconsequential in EDU because of its additively separable structure, but it may matter if there is complementary between outcomes.[26] To the extent that the individual cares about such spread, the descriptive validity of Stochastic Impatience may not be obvious.

4.4 Additional Experimental Evidence

The results above show that, within a very general class of models, it is impossible to allow for any violation of RSTL while keeping Stochastic Impatience. We have documented that RATL is widespread. Do preferences satisfy Stochastic Impatience, in general and in conjunction with RATL?

As an initial attempt to shed some light on this issue, we ran an incentivized experiment on Amazon's Mechanical Turk (March 2018, 157 subjects from the US). The experiment consisted of two simple questions. The first question concerned attitudes towards time lotteries. Subjects were offered a choice between receiving \$5 in 2 weeks for sure (RATL=1), and receiving \$5 in either 1 or 3 weeks with 50% chance each (RATL=0). The second question concerned Stochastic Impatience. In this question, subjects had to choose between receiving either \$5 in 1 week or \$1 in 3 weeks with 50% chance each (SI=1), and receiving either \$1 in 1 week or \$5 in 3 weeks with 50% chance each (SI=0).

Table 2 presents the main findings of this experiment. In line with our lab experiment, a majority of subjects chose the RATL option (57.3%). More importantly, we found that the vast majority of subjects do not violate Stochastic Impatience (only 16.5% do); and that the few subjects who violate Stochastic Impatience are not more likely to choose the RATL option.[27]

[26] This resembles the idea in Machina (2009) of seeing ambiguity aversion as a manifestation of event-nonseparability.

[27] Stochastic Impatience is violated by 12 out of the 90 subjects who choose the RATL option and by 14 out of the 67 subjects who choose the RSTL option (correlation of 0.031 with a p-value of 0.697).

These results are naturally only suggestive, as they test only one instance of Stochastic Impatience. It does not rule out the possibility that the same subjects would violate Stochastic Impatience if we choose different amounts. (As usual, this axiom cannot be tested in finite time but only falsified.) Nevertheless they support the possibility that a sizable fraction of subjects satisfies Stochastic Impatience while at the same time being RATL – a combination that, as we have seen, is ruled out by a large class of models.

5 Conclusion

This paper studies time lotteries and makes three contributions. First, as a motivation, it shows in an incentivized experiment that subjects typically violate RSTL, a behavior that is not compatible with the standard Expected Discounted Utility model (EDU).

Second, it shows that this behavior can be accommodated by a model that preserves many of the motivating features of EDU, reducing to exponentially discounted utility without risk and using Expected Utility for risk, but adding an additional curvature. This model, which we refer to the Generalized Expected Discounted Utility model (GEDU), is characterized by the axioms of Discounted and Expected Utility, with no other assumptions.

Third, it shows a fundamental tension between any instance of RATL and a new property called Stochastic Impatience. Within GEDU, individuals satisfy one if and only if they violate the other. But this incompatibility extends well beyond GEDU. We provide a similar impossibility result for a much larger class of preferences that allows for a broad class of non-Expected Utility behavior and non-exponential discounting.

Overall, the message of the paper can be summarized as follows. Despite what is predicted by EDU, most subjects exhibit violations of RSTL. If maintaining Stochastic Impatience is not a concern, then one can preserve the main features of EDU in a model that allows for this pattern of behavior. If, however, maintaining Stochastic Impatience is important, then even if we consider large generalizations, it is not possible to allow for any instance of RATL, unless one is willing to abandon even more fundamental properties that underline Theorem 2, such as no future bias or monotonicity of the utility function over monetary outcomes.

Appendices

These appendices contain some extensions, as well as the proofs of the results in the main text. The proofs of results from the appendices are in the Supplementary Appendix.[28]

A Obtaining EDU via RSTL

In the characterization of GEDU in Section 3 we imposed stationarity only on trade-offs involving deterministic payments (Axiom 3). One may consider a stronger notion that includes risky prospects, so that the ranking between two lotteries would not change if we move all payments in the support of the lotteries by the same number of periods. Formally, for any $p \in \Delta$, let $p_{+\tau}$ denote the lottery in which each prize is shifted by τ periods: $p_{+\tau}((x, t+\tau)) = p((x,t))$ for all $(x,t) \in [w,b] \times T$.

Axiom 7 (Risk Stationarity). *For every $p, q \in \Delta$ and τ such that $p_{+\tau}, q_{+\tau} \in T$,*

$$p \succsim q \;\Leftrightarrow\; p_{+\tau} \succsim q_{+\tau}.$$

It is not obvious that Risk Stationarity is a desirable property: its normative appeal, linked to dynamic consistency, is to be contrasted with robust evidence of its violations.[29]

It is easy to see that Risk Stationarity is satisfied by EDU. As the next proposition shows, however, EDU is not the only case of GEDU preferences that satisfies Risk Stationarity. Rather, starting from GEDU, EDU is characterized by imposing **both** Risk Stationarity and RSTL.

Proposition 3. *Suppose T is an interval and consider $\succsim$ that admits a GEDU representation and satisfies Risk Stationarity. Then:*

1. *$\succsim$ is RSTL (and not RNTL) if and only if it admits an EDU representation.*

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[28] The Supplementary Appendix is available online here.

[29] Typical findings are that subjects are more risk tolerant for delayed payments (Shelley, 1994; Sagristano et al., 2002; Noussair and Wu, 2006; Baucells and Heukamp, 2010; Coble and Lusk, 2010; Abdellaoui et al., 2011), a pattern that violates Risk Stationarity but is compatible with GEDU: if ϕ exhibits (strictly) increasing relative risk aversion, then $\succcurlyeq$ exhibits (strictly) higher risk tolerance for delayed rewards, i.e., if $p \sim (x,t)$ then for all τ we have $p_{+\tau} (\succ) \succsim (x, t+\tau)$. Intuitively, under GEDU pushing all rewards to the future is akin to "shrinking them" by the discount factor β; and if the individual has a increasing relative risk aversion he should have higher risk tolerance for these smaller amounts. A natural functional form for this is $\phi(x) = 1 - e^{-\gamma x}$ for some $\gamma \in \mathbb{R}_{++}$. With this functional form, the individual would be averse to time lotteries with large enough prizes (since, in this case, the exponential function is more risk averse than the log). Note that if u is CRRA and ϕ is CARA, then their combination $\phi \circ u$ exhibits increasing relative risk aversion, which, under GEDU, must be the case for atemporal risk preferences. This prediction appears to be supported by recent experimental evidence (Binswanger, 1981; Kachelmeier and Shehata, 1992; Bosch-Domènech and Silvestre, 1999; Holt and Laury, 2002; Fehr-Duda et al., 2010).

2. $\succsim$ *is RATL (and not RNTL) if and only if there exists* $\beta > 1$ *and a strictly increasing* $u : [w, b] \to \mathbb{R}_{--}$ *such that* $\succsim$ *is represented by* $V(p) = \mathbb{E}_p(\beta^t u(x))$. *We call this a Negative-EDU representation.*

The proposition above shows that RSTL is not only implied by EDU: it is the characterizing feature of EDU in the class of preferences that satisfy the axioms of Expected Utility, Discounted Utility, and Risk Stationarity.

The proposition also shows that there are other GEDU preferences that allow for RATL while maintaining Risk Stationarity: the Negative-EDU model, which takes $\beta > 1$ and a negative utility (such as, for example, any CRRA function more concave than the *log*) – a minor twist over EDU and probably just as tractable. It is easy to see that earlier payments are still preferred, despite the fact that $\beta > 1$, because u is negative (thus, the model still satisfies Impatience).

B Extension to consumption streams

We now extend our results to lotteries over consumption streams.

Consider the interval $[w, b]$ of prizes and a set of dates $T = \mathbb{N}$.[30] A consumption program $x = (x_0, x_1, \dots)$ yields consumption $x_t \in [w, b]$ in period $t \in T$. A program x is *ultimately constant* if there exist $a \in [w, b]$ and $\underline{\mathrm{t}} \in \mathbb{N}$ such that $x_t = a$ for all $t > \underline{\mathrm{t}}$. Let $X_{\underline{\mathrm{t}}}$ denote the set of all ultimately constant program that are constant starting from period $\underline{\mathrm{t}} + 1$, and let $\mathcal{X}$ denote the set of all ultimately constant programs.[31] Let $\Delta(\mathcal{X})$ be the set of all simple probability measures over $\mathcal{X}$. Our primitive is a complete and transitive preference relation $\succsim'$ over $\Delta(\mathcal{X})$.

We focus on the following representation, that extends GEDU to streams: there exist a continuous and strictly increasing $u : [w, b] \to \mathbb{R}$, $\beta \in (0, 1)$, and a strictly increasing $\phi : \mathbb{R}_+ \to \mathbb{R}$ such that $\succsim'$ is represented by

$$V(P) = \mathbb{E}_p \left[\phi \left(\sum_{t=0}^{\infty} \beta^t u(x_t) \right) \right].$$

This model is precisely the model in Kihlstrom and Mirman (1974) applied to time. It can be easily characterized like we did for GEDU, by imposing the axioms for Discounted Utility and Expected Utility. The only difference is that, to obtain Discounted Utility without risk, instead of the postulates of Fishburn and Rubinstein (1982), one should posit only on degenerate lotteries over streams those in Bleichrodt et al. (2008) (based on Koopmans 1960). See also Dillenberger et al. (2018).

[30] For brevity, we focus on discrete time and infinite horizon here. The case of continuous time is identical with the adapted formalism. For the finite horizon case, we would need to adapt to this framework the axioms in Fishburn (1970, Th. 7.5), which is an easy exercise.

[31] The restriction to ultimately constant programs guarantees that the discounted utility is well-defined.

We now consider our other results in this setup of streams: first, we show that this model can accommodate RATL; then, we show that even if we generalize it by allowing for non-Expected Utility, we have an incompatibility between Stochastic Impatience and RATL.

To discuss these results, we need to extend our notions of RATL and Stochastic Impatience to consumption streams. To do so, let $c \in [w,b]$ denote the consumption in the absence of any prizes ("background consumption"). In this context, a binary lottery $\alpha \times \delta(x_1,t_1) + (1-\alpha) \times \delta(x_2,t_2)$ gives with probability α the stream of consumption $\{c, c, ..., \underbrace{c+x_1}_{t_1}, c, ...\}$, and with probability $1-\alpha$ the stream $\{c, c, ..., \underbrace{c+x_2}_{t_2}, c, ...\}$, where $c + x_i \in [w,b]$. (We will omit the requirement that all $c + x_i \in [w,b]$ in the definitions below). A time lottery p_x with prize x is to be understood as a lottery over such objects. Our preference relation over lotteries over consumption streams $\succsim'$ would then induce a preference over such lotteries, one for any given background consumption c. We denote this induced preference relation by $\succsim'_c$. Our main properties extend to this domain as follows:

Definition 3. We say that $\succsim'$ is Risk Averse over Time Lotteries' (RATL') if for all $x, c \in [w,b]$ and all time lotteries p_x with prize x, if $\bar{t} = \sum_\tau p_x(x,\tau) \times \tau$ then

$$\delta_{(x,\bar{t})} \succsim'_c p_x.$$

Definition 4. Let $\mathbf{t} = (t_1, t_2, ..., t_n)$ be any finite collection of n time periods, with $t_1 < t_2 <, ... < t_n$, and $\mathbf{x} = (x_1, x_2, ..., x_n)$ be any collection of n outcomes such that $x_1 > x_2 >, ... > x_n$. Let $\iota(\mathbf{t})$ be a permutation of $\mathbf{t}$ and $\iota'(\mathbf{x})$ be a permutation of $\mathbf{x}$. We say that $\succsim'$ satisfies Stochastic Impatience if for any n-ordered sequences $\mathbf{t}$, $\mathbf{x}$ with corresponding perturbations $\iota(\mathbf{t})$ and $\iota'(\mathbf{x})$ and for any $c \in [w,b]$,

$$\sum_{i=1}^{n} \frac{1}{n} \delta_{(x_i,t_i)} \succsim'_c \sum_{i=1}^{n} \frac{1}{n} \delta_{(x_{\iota'(i)}, t_{\iota(i)})}.$$

It is easy to see that the model above can accommodate RATL. In particular, if ϕ is concave enough then the value of the non-degenerate lottery gets arbitrary close to the value of the worst consumption stream in the support (that is, the one in which the prize x is received in the latest possible time), compared to which the individual would prefer to receive x instead in the average time $\bar{t}$. In the case of lotteries over streams, however, risk attitude towards time lotteries may — due to the additive terms — depend on the magnitude of x and c (and not only on the curvature of ϕ). Therefore, there is no counterpart to Theorem 1. Theorem 2, on the other hand, is readily extended to this domain. To see this, first note that the GL-BDU representation becomes:

Definition 5. We say that $\succsim'$ admits a *Generalized Local Bilinear Discounted Utility* (GL-BDU) representation, if for any $c \in [w,b]$ there are continuous increasing functions $u : X \to \mathbb{R}$, $\phi : \mathbb{R}_+ \to \mathbb{R}$, and a strictly decreasing $D : T \to (0,1]$ which exhibits Diminishing Impatience, such that for $\pi(0.5) \in (0,1)$, $0.5\delta_{(x,t)} + 0.5\delta_{(x',t')}$ with $D(t)u(x+c) + \sum_{\tau \neq t} D(t)u(c) \geq D(t')u(x'+c) + \sum_{\tau \neq t'} D(t)u(c)$ is evaluated by:

$$\pi(0.5)\phi\Big(D(t)u(x+c)+\textstyle\sum_{\tau\neq t}D(\tau)u(c)\Big)+(1-\pi(0.5))\phi\Big(D(t')u(x'+c)+\textstyle\sum_{\tau\neq t'}D(\tau)u(c)\Big).$$

Theorem 3. *Suppose $\succsim'$ admits a GL-BDU representation. If $\delta_{(x,t_2)} \succ'_c 0.5\delta_{(x,t_1)} + 0.5\delta_{(x,t_3)}$ for some $x, c \in [w, b]$ and $t_2 = \frac{t_1+t_3}{2}$, and if either*

$$D(t_1)(u(w+c) - u(c)) < D(t_2)(u(x+c) - u(c))$$

or

$$D(t_3)(u(b) - u(c)) > D(t_2)(u(x+c) - u(c)),$$

then $\succsim_c$ violates Stochastic Impatience.

The constructive proof follows the exact same steps as in the case for lotteries over dated rewards, and the richness condition (that is now written in a weaker version as two separate inequalities) plays the exact same role in it.

C Epstein-Zin 1989 preferences

In this appendix we show that the model of Epstein and Zin (1989) (henceforth, EZ) with CRRA Expected Utility preferences and CES aggregator cannot accommodate violations of RSTL without also violating Stochastic Impatience. We adopt the same formal recursive setup of their paper, which we do not discuss here for brevity. Consider a preference relation $\hat{\succsim}$ over the recursive framework that admits a recursive representation of the form:

$$V_t = \left\{(1-\beta)\, C_t^{1-\rho} + \beta \left[E_t\left(V_{t+1}^{1-\alpha}\right)\right]^{\frac{1-\rho}{1-\alpha}}\right\}^{\frac{1}{1-\rho}}, \tag{3}$$

where C_t denotes consumption at time t, $\alpha \geq 0$ is the coefficient of relative risk aversion, and $\rho \geq 0$ is the inverse of the elasticity of intertemporal substitution (with $\alpha \neq 1$ and $\rho \neq 1$, so that the formula is well-defined). EZ coincides with EDU when $\alpha = \rho$.

We now turn to discuss whether EZ can simultaneously accommodate Stochastic Impatience and RATL. Since this model is defined over consumption streams, we use the definitions of Stochastic Impatience and RATL for streams as introduced in Appendix B. In this setup we also need to specify when the uncertainty is resolved: for all the lotteries in question, we assume that the uncertainty is resolved immediately after the current period.

We first show that EZ allows for violations of RSTL although it cannot accommodate (global) RATL:

Proposition 4. *Under EZ, for any β, ρ, and x, there exists $\bar{\alpha}_{\rho,\beta,x} > \max\{\rho, 1\}$ such that $\delta_{(x,t)} \hat{\succ} 0.5\delta_{(x,t-1)} + 0.5\delta_{(x,t+1)}$ if and only if $\alpha > \bar{\alpha}_{\rho,\beta,x}$. Moreover, $\lim_{x\searrow 0} \bar{\alpha}_{\rho,\beta,x} = +\infty$.*

Proposition 4 shows that, controlling for discounting β, elasticity of intertemporal substitution $1/\rho$, and the size of the prize x, more risk averse individuals are more likely to prefer the safe lottery.[32] That is, as with GEDU, there is also a connection between risk aversion over time lotteries and risk aversion over temporal lotteries in EZ. Moreover, the risky lottery is always preferred if the utility function is less concave than a logarithmic function ($\alpha < 1$) and if $\alpha \leq \rho$,[33]. Proposition 4 also shows that $\lim_{x\searrow 0} \bar{\alpha}_{\rho,\beta,x} = +\infty$, which means that, when the prize x is small enough, the risky time lottery is always preferred. That is, EZ preferences cannot be (globally) RATL.

Our main result is that EZ cannot accommodate violations of RSTL without also violating Stochastic Impatience:

Proposition 5. *Suppose that $\hat{\succsim}$ admits an EZ representation. Then, if $\hat{\succsim}$ satisfies Stochastic Impatience, it also satisfies RSTL.*

This result shows that even in this richer setting, the impossibility result established with Theorem 2 continues to hold. The intuition is very similar to the one previously given: the extra "intertemporal" risk aversion needed to accommodate RATL is going to generate a violation of Stochastic Impatience. We refer to Dillenberger et al. (2018) for an in-depth discussion of the implications of Stochastic Impatience for EZ.

D Proofs of the Results in the text

D.1 Proof of Proposition 1

Necessity is immediate. To show sufficiency, note that by Continuity, for all $(x,t) \in [w,b] \times T$ the sets $\{(x,t) \in [w,b] \times T : \delta_{(x,t)} \succsim \delta_{(y,s)}\}$ and $\{(x,t) \in [w,b] \times T : \delta_{(y,s)} \succsim \delta_{(x,t)}\}$ are closed in the product topology on $[w,b] \times T$. Define $\succsim'$ on $[w,b] \times T$ by $(x,s) \succsim' (y,t)$ if and only if $\delta_{(x,s)} \succsim \delta_{(y,t)}$, and note that $\succsim'$ satisfies Axioms A0-A5 in Fishburn and Rubinstein (1982). Then, by Theorem 2 in that paper, there exist $\beta \in (0,1)$ and a strictly increasing and continuous $u : [w,b] \to \mathbb{R}_{++}$ such that

$$\delta_{(x,s)} \succsim \delta_{(y,t)} \;\Leftrightarrow\; (x,s) \succsim' (y,t) \;\Leftrightarrow\; \beta^s u(y) \geq \beta^t u(x).^{34}$$

[32] That is, $\alpha' > \alpha$ implies that if the decision maker with coefficient of risk aversion α prefers the safe lottery over the risky one, so does the decision maker with α' (holding other parameters fixed).

[33] Starting with Kreps and Porteus (1978), a large literature has studied preferences over the timing of resolution of uncertainty. With EZ, early resolution of uncertainty is preferred if and only if $\alpha > \rho$ (Epstein et al., 2014). Proposition 4 then implies that this condition is also needed for the safe time lottery to be preferred.

[34] Recall that our domain includes only strictly positive prizes, so that we do not add the requirements for u on weakly negative outcomes.

By Independence and Continuity there exists $U : [w, b] \times T \to \mathbb{R}$ such that

$$p \succsim q \;\Leftrightarrow\; \mathbb{E}_p(U) \geq \mathbb{E}_p(U).$$

By Continuity, U is also continuous.

It follows that for all $(x, s), (y, t) \in [w, b] \times T$, $\beta^s u(x) \geq \beta^t u(y)$ if and only if $U(x, s) \geq U(y, t)$. Let $F(x, t) = \beta^t u(x)$. The existence of $\phi : F([w, b] \times T) \to \mathbb{R}$ such that $U(x, t) = \phi(F(x, t))$ follows from standard arguments, as both U and F represent the same preferences. The continuity of ϕ is also immediate. We are left with showing that such ϕ is strictly increasing. If not, then there exist $a, b \in F([w, b] \times T)$ such that $a > b$ but $\phi(a) = \phi(b)$. Since $a, b \in F([w, b] \times T)$, there exist $(x, t), (y, s) \in [w, b] \times T$ such that $F(x, t) = a > b = F(y, s)$, thus $\delta_{(x,t)} \succ \delta_{(y,s)}$. But since $\phi(a) = \phi(b)$ we have $U(x, t) = U(y, s)$, thus $\delta_{(x,t)} \sim \delta_{(y,s)}$, a contradiction. ■

D.2 Proof of Proposition 2

Let r_x be a time lottery which yields x in a random time t with $E_r(t) = \bar{t}$. Then

$$V(\delta_{(x,\bar{t})}) = \phi\left(\beta^{\bar{t}} u(x)\right)$$

and

$$V(r_x) = E_r \phi\left(\beta^t u(x)\right).$$

Note that if $\phi = \ln$, then $V(\delta_{(x,\bar{t})}) = V(r_x) = \bar{t} \ln \beta + \ln u(x)$. Since the distribution of t is a mean-preserving spread of the distribution of $\bar{t}$, Jensen inequality implies that $V(\delta_{(x,\bar{t})}) >$ (resp., $<$) $V(r_x)$ whenever there is a concave (resp., convex) function h such that $\phi = h \circ \log$.

Since r_x was arbitrary, the concavity (resp., convexity) of g should be global to ensure no violation of RATL (resp., RSTL). ■

D.3 Proof of Theorem 1

That $\succsim$ is RSTL if an only if ϕ is a convex transformation of ln has been established in Proposition 2 of Section 3. We now show that $\succsim$ displays Stochastic Impatience if an only if ϕ is a convex transformation of ln. To see this, let $\mathbf{t}' = (\beta^{t_1}, \beta^{t_2}, ..., \beta^{t_n})$ for $\beta \in (0, 1)$ and $t_1 < t_2 < ... < t_n$, and $\mathbf{x}' = (u(x_1), u(x_2), ..., u(x_n))$ for strictly increasing and positive-valued u over X and $x_1 > x_2 >, ... > x_n$. Let $\iota_t(\mathbf{t}')$ be a permutation of $\mathbf{t}'$ and $\iota_x(\mathbf{x}')$ be a permutation of $\mathbf{x}'$. If $\phi = \ln$ then, and only then, $\sum_{i=1}^n \frac{1}{n} \delta_{(x_i, t_i)} \sim \sum_{i=1}^n \frac{1}{n} \delta_{(x_{\iota'(i)}, t_{\iota(i)})}$. But note that $f(u(x), \beta^t) := g(\ln(u(x), \beta^t))$ is supermodular if and only if g is convex. By Becker (1973), supermodularity of f is both a necessary and sufficient condition for the positive assortative pairing — that is, a positive correlation in sorting between the values of $u(x)$ and β^t —- to maximize the sum of the $f(u(x), \beta^t)$ terms across all possible permutations. ■

D.4 Proof of Theorem 2

Let $t_1 < t_2 < t_3$ with $t_2 = \frac{t_1+t_3}{2}$ and for some prize x consider the two time lotteries $\delta_{(x,t_2)}$ and $r_x = 0.5\delta_{(x,t_1)} + 0.5\delta_{(x,t_3)}$. According to the GL-BDU representation we have

$$V(\delta_{(x,t_2)}) = \phi(D(t_2)u(x))$$

and

$$V(r_x) = \pi(0.5)\phi(D(t_1)u(x)) + (1-\pi(0.5))\phi(D(t_3)u(x)).$$

Let $\overline{\pi(0.5)}$ be the value such that $V(\delta_{(x,t_2)}) = V(r_x)$, or

$$\overline{\pi(0.5)} = \frac{\phi(D(t_2)u(x)) - \phi(D(t_3)u(x))}{\phi(D(t_1)u(x)) - \phi(D(t_3)u(x))} \in (0,1).$$

The richness condition $D(t_1)u(w) < D(t_3)u(b)$ guarantees that either (i) there is $x' < x$ such that $D(t_1)u(x') = D(t_2)u(x)$; or (ii) there is $x' > x$ such that $D(t_3)u(x') = D(t_2)u(x)$; or both.

Consider first case *(1)*. Take $x' < x$ such that $D(t_1)u(x') = D(t_2)u(x)$. Let $p = 0.5\delta_{(x,t_1)} + 0.5\delta_{(x',t_2)}$ and $q = 0.5\delta_{(x',t_1)} + 0.5\delta_{(x,t_2)}$. We have

$$V(p) = \pi(0.5)\phi(D(t_1)u(x)) + (1-\pi(0.5))\phi(D(t_2)u(x'))$$

and

$$V(q) = \phi(D(t_2)u(x)).$$

Let $\widehat{\pi(0.5)}$ be the value such that $V(p) = V(q)$, or

$$\widehat{\pi(0.5)} = \frac{\phi(D(t_2)u(x)) - \phi(D(t_2)u(x'))}{\phi(D(t_1)u(x)) - \phi(D(t_2)u(x'))} \in (0,1).$$

Note that $\pi(0.5) > \widehat{\pi(0.5)}$ implies $V(p) > V(q)$ while $\pi(0.5) < \overline{\pi(0.5)}$ implies $V(\delta_{(x,t_2)}) > V(r)$. Since ϕ is strictly increasing, we will be done if we show that $D(t_2)u(x') \leq D(t_3)u(x)$, since this implies that $\widehat{\pi(0.5)} \geq \overline{\pi(0.5)}$, so that Stochastic Impatience and RATL contradict one another. But by Diminishing Impatience and the definition of x' we have

$$\frac{D(t_2)}{D(t_3)} \leqq \frac{D(t_1)}{D(t_2)} = \frac{u(x)}{u(x')}$$

or $D(t_2)u(x') \leq D(t_3)u(x)$.

If case (i) is not satisfied, then consider now case (ii). Take $x' > x$ such that $D(t_3)u(x') = D(t_2)u(x)$. Let $p = 0.5\delta_{(x',t_2)} + 0.5\delta_{(x,t_3)}$ and $q = 0.5\delta_{(x,t_2)} + 0.5\delta_{(x',t_3)}$. We have

$$V(p) = \pi(0.5)\phi(D(t_2)u(x')) + (1-\pi(0.5))\phi(D(t_3)u(x))$$

and

$$V(q) = \phi(D(t_2)u(x)).$$

Let $\widehat{\pi(0.5)}$ be the value such that $V(p) = V(q)$, or

$$\widehat{\pi(0.5)} = \frac{\phi(D(t_2)u(x)) - \phi(D(t_3)u(x))}{\phi(D(t_2)u(x')) - \phi(D(t_3)u(x))} \in (0,1).$$

If $D(t_2)u(x') \leq D(t_1)u(x)$ then we have $\widehat{\pi(0.5)} \geq \overline{\pi(0.5)}$. By Diminishing Impatience and the definition of x' we have

$$\frac{D(t_1)}{D(t_2)} \geq \frac{D(t_2)}{D(t_3)} = \frac{u(x')}{u(x)}$$

or $D(t_2)u(x') \leq D(t_1)u(x)$. This completes the proof. ■

E Experiment: additional information

A total of 197 subjects took part in an experiment run at the Wharton Behavioral Lab at the Wharton School of the University of Pennsylvania. We used a paper-and-pencil questionnaire. Some questions involved immediate payments, that were made at the end of each session. Others involved payments to be made in the future; for these, subjects were told that their payment would be available to pick up from the lab starting from the date indicated.[35]

We ran two treatments: 'long delay' and 'short delay,' labeled Long and Short in what follows. A total of 105 and 91 subjects participated in each, respectively. The only difference was the length of delays in some of the questions: in the Long treatment, some payments were delayed by up to 12 weeks, while in the Short treatment the maximum delay was 5 weeks.[36]

[35] All payment dates were expressed in weeks, with the goal of reducing heterogeneity in transaction costs between the dates, under the assumption that students have a regular schedule each week during the semester. An email was then sent to remind them of the approaching date (they were told they would receive it). Subjects were also given the contact details of one of the authors, at the time a full-time faculty at Wharton. Returning to the lab to collect the payment involve transaction costs, a typical concern. However, in our experiment *all* payments related to time lotteries were designed to take place in future dates, thus holding constant the transaction cost. We have already mentioned that a second concern may relate to the use of monetary prizes, which could be seen as problematic to study intertemporal choice (Augenblick et al., 2015). As we have seen, typical issues do not apply here (for example, the curvature of the utility function is inconsequential on ranking of time lotteries). Moreover, since are interested in studying the relation between risk aversion over time lotteries and atemporal risk aversion, and the latter is defined for monetary lotteries, we focus our experiment on monetary prizes. Our broad intuition would naturally apply more in general; we leave its experimental investigations to future research.

[36] Testing both treatments allows us to study long times spans, where differences between time lotteries become more pronounced; as well as shorter ones, where students' schedules are more stable, reducing heterogeneous sources of variation. In the Short version all payments were scheduled before the end of the semester; no payment was scheduled during exam week.

Table 3: Questions in Part I

	Long Delay				Short Delay			
Q.	$$	Option 1	*vs.*	Option 2	$$	Option 1	*vs.*	Option 2
1	$20	2 wk		75% 1 wk, 25% 5 wk	$20	2 wk		75% 1 wk, 25% 5 wk
2	$15	3 wk		90% 2 wk, 10% 12 wk	$15	3 wk		50% 1 wk, 50% 5 wk
3	$10	2 wk		50% 1 wk, 50% 3 wk	$10	2 wk		50% 1 wk, 50% 3 wk
4	$20	50% 2 wk, 50% 3 wk		50% 1 wk, 50% 4 wk	$20	50% 2 wk, 50% 3 wk		75% 2 wk, 25% 4 wk
5	$15	50% 2 wk, 50% 5 wk		75% 1 wk, 25% 11 wk	$10	50% 2 wk, 50% 5 wk		75% 3 wk, 25% 5 wk

Notes. Each lottery pays the same prize with different delays (in weeks). Subjects in the long delay treatment chose between 'Option 1' and 'Option 2, Long Delay.' Those in the short delay treatment chose between 'Option 1' and 'Option 2, Short Delay.'

The experiment has three parts.[37] Part I asks subjects to choose between different time lotteries and it is the main part of our experiment. For example, the first question asked them to choose between $15 in 2 weeks or $15 in 1 week with probability .75 and in 5 weeks with probability .25. Subjects answered five questions of this kind. Table 3 lists the questions asked in each treatment. All questions offered two options that paid the same prize at different dates, where the distribution of payment dates of one option was a mean preserving spread of that of the other. In three questions, one of the options had a known date; in the others, both options had random payment dates. All subjects received the same first question (Question 1 in Table 3) in a separate sheet of paper. The answer to this question is a key indication of the subjects' preferences, as it captures their immediate reaction to this choice, uncontaminated by other questions.[38]

Parts II and III use the multiple price list (MPL) method of Holt and Laury (2002) to measure time and risk preferences separately.[39] Part II measures standard time

[37]Subjects received general instructions and specific instructions about the first part when they entered the room. Separate instructions were distributed before each of the following parts. The order of parts and of questions was partly randomized, as we discuss below.

[38]One potential concern with offering a list of similar questions is that subjects may 'try' different answers even if they have a mild preference in one direction with some hedging concern in mind (Agranov and Ortoleva, 2017).

[39]In a MPL, each question has a table with two columns and multiple rows. The subject is asked to make a choice in each row. One of the options is always the same, while the other gets better and better as we proceed down the rows. These questions are typically interpreted as follows: if a subject chooses the option on the left for all rows above a point, and the option on the right below that point, then the indifference point should be where the switching takes place. Subjects who understand the procedure should not switch more than once. This is indeed the case for the large majority of answers: 13% of subjects gave a non-monotone answer in at least one of the 12 MPL questions, and only 4.6% gave non-monotone answers in more than one. These are substantially lower numbers (i.e., fewer violations) than what previous studies have found (Holt and Laury, 2002). The non-monotone behavior did not concentrate in any specific question. Following the typical approach, these answers are disregarded. Alternatively, we could have dropped any subject that exhibits a non-monotone

Table 4: Questions in Part II

	Long Delay			Short Delay		
Q.	Option 1	*vs.*	Option 2	Option 1	*vs.*	Option 2
6	$10 today		x in 2 wk	$10 today		x in 2 wk
7	$10 in 1 wk		x in 2 wk	$10 in 1 wk		x in 2 wk
8	$10 in 1 wk		x in 5 wk	$10 in 1 wk		x in 3 wk
9	$10 in 1 wk		x in 12 wk	$10 in 1 wk		x in 4 wk
10	$20 in 4 wk		$20, x% in 2wk, (1-x)% in 12wk	$25 in 3 wk		$25, x% in 2wk, (1-x)% in 5wk
11	$25 in 2 wk		$25, x% in 1wk, (1-x)% in 5wk	$25 in 2 wk		$25, x% in 1wk, (1-x)% in 5wk

Notes. Questions 6-9 ask the amount $\$x$ that would make subjects indifferent between each option. Questions 10-11 ask the probability $x\%$ that would make subjects indifferent between each option. These amounts were determined using MPL.

Table 5: Questions in Part III

Q.	Option 1	*vs.*	Option 2
12	$15		x% of $20, (1-x)% of $8
13	50% of $15, 50% of $8		x% of $20, (1-x)% of $8
14	20% of $15, 80% of $8		x% of $20, (1-x)% of $8
15	$20		x% of $30, (1-x)% of $5
16	50% of $20, 50% of $5		x% of $30, (1-x)% of $3
17	10% of $20, 90% of $5		x% of $30, (1-x)% of $3

Notes. Questions ask the probability $x\%$ that would make subjects indifferent between each option, determined using MPL. All payments were scheduled for the day of the experiment.

preferences as well as attitudes towards time lotteries (Question 10 and 11). Part III measures atemporal risk preferences, with payments taking place immediately at the end of the session. These include questions to measure regular risk aversion, as well as Allais' common-ratio-type questions, that allow us to test and quantify violations of Expected Utility theory. Tables 4 and 5 include the list of questions asked in these two parts.

At the end of the experiment one question was randomly selected from Parts I, II, and III for payment. The randomization of the question selected for payment, as well as the outcome of any lottery, was resolved with dice.[40] Crucially, all uncertainty was resolved at the end of the experiment, including the one regarding payment dates. The instructions explicitly stated that subjects would know all payment dates before

behavior at least once; this leave our results essentially unchanged.

[40] Specifically, one participant was selected as 'the assistant,' using the roll of a die. This subject was then in charge of rolling the die and checking the outcomes. This was done to reduce the fear that the experimenter could manipulate the outcome. All was clearly explained beforehand.

leaving the room.

The order of parts and of questions within parts was partly randomized at a session level. Because Part I is the key one, all subjects saw it first to avoid contamination. For the same reason, within Part I, Question 1 was always the same. All other elements were randomized.[41] We find no significant effects of ordering.[42]

We conclude by noting that our incentive scheme, the random payment mechanism, as well as the multiple price list method, are incentive compatible for Expected Utility maximizers, but not necessarily for more general preferences over risk.[43] Since this is the procedure used by most studies, a significant methodological work has been done to examine whether this creates relevant differences, with some reassuring results.[44]

E.1 Results

We start from the main variable of interest: risk attitude towards time lotteries. This can be measured in three different ways. First, we can measure it using Question 1 of Part I, the first question that subjects see. Second, we can look at the answers to all five questions in Part I and ask whether subjects exhibited RATL in the majority of them (for the purpose of this section, we say that subjects are RATL in a given question if, in that question, they chose the option with the smallest variance of the payment date). A third way is to look at the answers given in Questions 10 and 11

[41] Specifically: for questions in Part I other than the first, half of the subjects answered questions in one specific order (the one used above), while the other half used a randomized order. In each of them, which option appears on the left and which on the right was also determined randomly. The order of Parts II and III was randomized. For both parts, it was determined randomly whether in the MPL the constant option would appear on the left or on the right. This was done (independently) for each part, but not for each question within a part: in Part II or III the constant option of the MPL was either on the left or on the right for all questions of that part. This is typical for experiments that use the MPL method, as it makes the procedure easier to explain.

[42] The only exception is that out of the five questions in the first part, subjects have a significant (moderate) preference for the option on the right in the second question. While this is most likely a spurious significance (due to the large number of tests run), the order was randomized for all sessions and thus this should have no impact on our analysis.

[43] Holt (1986) points out that a subject who obeys the Reduction of compound lotteries but violates the Independence axiom may make different choices under a randomly incentivized elicitation procedure than he would make in each choice in isolation. Conversely, if the decision maker treats compound lotteries by first assessing the certainty equivalents of all first stage lotteries and then plugging these numbers into a second stage lottery (as in Segal, 1990), then this procedure is incentive compatible. Karni and Safra (1987) prove the non-existence of an incentive compatible mechanism for general non-Expected Utility preferences.

[44] Beattie and Loomes (1997), Cubitt et al. (1998) and Hey and Lee (2005) all compare the behavior of subjects in randomly incentivized treatments to those that answer just one choice, and find little difference. Also encouragingly, Kurata et al. (2009) compare the behavior of subjects that do and do not violate Expected Utility in the Becker-DeGroot-Marschak procedure (which is strategically equivalent to MPL) and find no difference. On the other hand, Freeman et al. (2015) find that subjects tend to choose the riskier lottery more often in choices from lists than in pairwise choices.

of Part II, that compute RATL using MPL.

Table 6 presents the percentage of RATL answers for each of these measures. The results are consistent: in most questions, especially in the Long treatment, the majority of subjects are RATL. Note that most subjects are still RATL when *both* options are risky but one of the options is a mean preserving spread of the other (Questions 4 and 5). Thus, the data suggest an aversion to mean preserving spreads, not simply an attraction towards certainty.

Table 6: Percentage of RATL in each question

Question	Long	Short
1	65.71	56.04
2	50.48	54.95
3	48.57	37.36
4	64.76	38.46
5	73.33	52.75
Majority in 1-5	64.76	49.45
10	44.23	54.44
11	57.28	41.11
Either in 10 or 11	64.07	66.66

Table 7: Frequency of RATL answers in Part I

Frequency of RATL	Long Delay		Short Delay	
	Percent	Cum.	Percent	Cum.
0	2.86	2.86	9.89	9.89
1	9.52	12.38	16.48	26.37
2	22.86	35.24	24.28	50.55
3	23.81	59.05	26.37	76.92
4	28.57	87.62	19.78	96.70
5	12.38	100.00	3.30	100

In most questions, RATL is stronger in the Long rather than in the Short treatment.[45] This is intuitive: when the time horizon is relatively short, the difference between the options decreases and subjects should become closer to being indifferent – and their choices closer to an even split. In the Long treatment the difference in time horizon increases, and so does the differences between the options. While the standard model suggests that this should push more strongly towards RSTL, the *opposite* holds in our data.

While most answers are consistent with RATL, it could be that a non-trivial fraction of our subjects still consistently chooses the risky option, as predicted by EDU. Table 7 shows that this is not the case: the fraction of subjects who does so is minuscule in the Long treatment (2.86%) and very small in the Short one (9.89 %). By contrast, in the Long treatment almost 41% give risk averse answers at least 4 out of 5 times, and 59% at least three times. (These numbers are about 23% and 48.45% in the Short treatment.)

Overall, these finding are not compatible with RSTL and thus with EDU: only a minuscule fraction of subjects is consistently risk seeking over time lotteries, while the majority is tends to be risk *averse* over time lotteries. Thus, the assumption of

[45] The proportion of subjects who exhibit a majority of RATL choices is about 65% in the Long vs. about 50% in the Short (test of proportion, p-value 0.015). Overall, the proportion of RATL answers are 60.6 vs. 47.9 (p-value <0.001).

risk seeking overt time lotteries, implicitly present when using EDU, does not seem to be have a positive appeal.

E.2 RATL, Convexity, Expected Utility, and Risk Aversion

We now turn to analyze the relationship between RATL and convex discounting, violations of Expected Utility, and atemporal risk aversion. Under EDU, all subjects with convex discounting should be RSTL; in turn, this means that such tendency should be negatively related to convexity of the discount function. Under GEDU, RATL should be positively correlated with atemporal risk aversion. Finally, if RATL were due to violations of Expected Utility, as suggested by Chesson and Viscusi (2003) and Onay and Öncüler (2007), then it should be linked to certainty bias and violations of Expected Utility.

We quantify convexity of the discount function, violations of Expected Utility, and atemporal risk aversion using the MPL measures collected in Parts II and III. We determine which subjects have convex discounting based on their answers in Part II (see Questions 7, 8, and 9 in Table 4). Unsurprisingly, we find that 82% of our subjects exhibit it (this is an established finding). From the questions in Part III we can construct two related measures of violations of Expected Utility. First, we can determine if subjects exhibit certainty bias (Kahneman and Tversky, 1979), which is implied by pessimistic probability weighting.[46] We find that a small number of subjects exhibit it (15.71%).[47] Second, we can use the same three questions to determine whether the subjects give answers that are jointly consistent with Expected Utility. Since this is a very demanding requirement – it is well-known that these measures are very noisy –, we consider as "approximately Expected Utility" those subjects who would abide by Independence across all three questions if we changed their answer in at most one of the lines. These are 39.89% of the pool.

Table 8 shows that, based on the four different measures, subjects are still RATL in each of the subsamples above. The table also shows the results of Chi-squared tests on whether subjects in each of subsample are statistically different from those

[46] This could be done using Questions 12 and 13, or 12 and 14 (see Table 5). Suppose that in Question 12 the subject switches at x_{12}, while in Question 13 he switches at x_{13}. If the subject follows Expected Utility, we should have $2x_{13} = x_{12}$. A certainty-biased subject would instead have $x_{12} > 2x_{13}$; because he is attracted by the certainty of Option 1 in Question 12, he demands a high probability of receiving the high prize in Option 2 to be indifferent. Thus, the answers to Question 12 and 13 allow us to identify subjects who are certainty biased and to quantify it. In what follows, when we need to identify subjects who are certainty biased, we use this measure. A similar measure can be obtained from the answers to Questions 12 and 14: the results using it are essentially identical and are reported in Section E.3. When we need to quantify certainty bias (in the regression analysis), we use instead the principal component of the two measures, which should reduce the observation error (essentially identical results hold using either of the two measures or their average).

[47] These small numbers are not surprising: it is a stylized fact that certainty bias is less frequent when stakes are small, as in this part of our experiment (Conlisk, 1989; Camerer, 1989; Burke et al., 1996; Fan, 2002; Huck and Müller, 2012). See the discussion in Cerreia-Vioglio et al. (2015).

Table 8: Proportion of RATL subjects

Sample	Convex Discounting		Approximately Exp. Ut.		No Certainty Bias	
Treatment	Long	Short	Long	Short	Long	Short
Question 1	67.78	50.70*	66.67	60.61	67.50	55.00
Majority in Q1-5	65.56	43.66**	64.29	60.61*	68.75*	50.00
Question 10	46.07	52.86	54.76*	68.75**	47.50	51.90
Question 11	57.95	44.29**	64.29	56.25	54.43	48.10
Observations	90	71	42	33	80	80

Notes. The first row measures RATL using Question 1. The second row identifies as RATL subjects who chose the safe option in the majority of Questions 1-5. The third and fourth rows use answers to MPL Questions 10 and 11. Columns present the proportion of RATL subjects in the subsamples of subjects with convex discounting, approximately Expected Utility, and those with no Certainty Bias as measured using Questions 12 and 13. * and ** denote significance at the 10% and 5% level in a Chi-squared test of whether each subset is different from its complement.

outside of it. We find a majority of RATL among subjects who either have convex discounting, or who have no certainty bias, or who are "approximately Expected Utility." In most cases there is no significant difference in the proportions of RATL between these groups and their complement. These results are in direct contrast with the predictions of EDU, and with the explanation of RATL suggested by Chesson and Viscusi (2003) and Onay and Öncüler (2007) based on probability weighting: according to the former, there should be no RATL with convex discounting; according to the latter, there should be no RATL without certainty bias, or for subjects that (approximately) follow Expected Utility. In Section E.3 we present regression analysis to confirm these results, where we show how certainty bias or convexity of discounting is generally not related, or poorly related, to the tendency to exhibit RATL: see Table 11.

All our findings thus far are compatible with the GEDU model. However, as we pointed out, GEDU makes one additional prediction, which allows us to test it in our data: RATL should be related to standard atemporal risk aversion. Table 9 presents the coefficients from a Probit regression with our four RATL measures as dependent variables and the degree of risk aversion (as measured in Question 12) as the independent variable. Consistently with the model, the coefficients are positive and, with the exception of the Short treatment in Question 1, they are all statistically significant at the 5% level. (Similar results hold constructing risk aversion from other questions, e.g., Question 15, or using a linear probability model.)

To summarize, we find that subjects who are either (i) approximately Expected Utility maximizers, or (ii) satisfy either convex discounting, or (iii) satisfy no certainty bias, also have a tendency to be RATL. In fact, the proportions in these groups are almost identical to the one in the overall population. Regression analysis shows

Table 9: Probit Regressions: RATL and Atemporal Risk Aversion

Dep. Variable	RATL Q.1		RATL Majority Q.1-5		RATL Q.10		RATL Q.11	
Treatment (Probit)	Long (1)	Short (2)	Long (3)	Short (4)	Long (5)	Short (6)	Long (7)	Short (8)
Risk Aversion, Atemporal	.336** (2.41)	.175 (1.30)	.308** (2.27)	.341** (2.40)	.459*** (3.27)	.435*** (2.91)	.571*** (3.81)	.239*** (1.75)
Constant	.07 (0.38)	-.01 (-0.07)	.07 (0.38)	-.34* (-1.79)	-.60*** (-3.01)	-.27 (-1.47)	-.37*** (-1.87)	-.19 (-1.08)
Pseudo-R^2	0.047	0.014	0.040	0.049	0.083	0.076	0.121	0.025
Observations	101	90	101	90	101	89	100	89

Notes. Dependent variables are indicated in the first row. Atemporal risk aversion measure is obtained from Question 12. RATL measures were obtained from Question 1 (Regressions 1 and 2), having chosen the safe option in the majority of Questions 1-5 (Regressions 3 and 4), and MPL Questions 10 and 11 (Regressions 5-8). Coefficients in brackets are z-statistics. *, **, and *** denote significance at the 10%, 5% and 1% level.

that RATL is unrelated to violations of Expected Utility and generally unrelated to convexity. It is, however, related to (atemporal) risk aversion. These findings are not compatible with RSTL, EDU, or to explanations based on probability weighting, but they are compatible with GEDU.

E.3 Additional Analysis

Table 10: Proportion of RATL subjects

Sample	No Cert. Bias (12-13)		No Cert. Bias (12-14)	
Treatment	Long	Short	Long	Short
Question 1	67.50	55.00	62.50	54.17
Majority in Q1-5	68.75*	50.00	63.89	47.22
MPL in Q10	47.50	51.90	45.83	50.00
MPL in Q11	54.43	48.10	56.94	48.61
Observations	80	80	72	72

Notes. Same as Table 8, including certainty bias measure from Questions 12 and 14 (see footnote 46).

Table 11: Probit Regressions: RATL and Convexity and Certainty Bias

Dep. Variable	RATL Q1		RATL Majority Q1-5		RATL Q10		RATL Q11	
Treatment	Long	Short	Long	Short	Long	Short	Long	Short
(Probit)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Certainty Bias	-.25*	.18	-.20	.18	-.03	.44***	.16	.23
	(-1.94)	(1.17)	(-1.60)	(1.20)	(-.25)	(2.77)	(1.42)	(1.52)
Convexity	4.27*	-4.45	.06	-11.10***	3.47	-1.77	.99	-7.73**
	(1.82)	(-1.29)	(.03)	(-2.86)	(1.64)	(-1.29)	(.46)	(-2.16)
Constant	.19	.28*	.39**	.19	-.26	.16	.19	.12
	(1.07)	(1.82)	(2.15)	(1.20)	(-1.52)	(1.03)	(1.08)	(.79)
Pseudo-R^2	.06	.02	.02	.01	.02	.07	.02	.06
Obs.	92	86	92	86	92	85	92	85

Notes. Dependent variables are indicated in the first row. Coefficients in brackets are z-statistics. *, **, and *** denote significance at the 10%, 5% and 1% level.

Table 12: Probit Regressions: RATL and Atemporal Risk Aversion

Dep. Var.	RATL Q.1				RATL Majority Q.1-5			
Treatment	Long		Short		Long		Short	
(Probit)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Cert. Bias		-.19		.18		-.21		.12
		(-1.56)		(1.20)		(-1.71)		(0.82)
Convexity	3.73*		-4.17		-.39		-10.60***	
	(1.68)		(-1.25)		(-0.19)		(-2.83)	
Constant	.22	.40***	.25*	.19	.41**	.37***	.15	.01
	(1.35)	(2.93)	(1.67)	(1.41)	(2.50)	(2.75)	(1.02)	(.09)
Pseudo-R^2	.02	.02	.01	.01	.01	.03	.07	.01
Obs.	101	95	88	88	101	95	88	88

Notes. Same as Table 11. Each regression excludes one dependent variable.

Table 13: Probit Regressions: RATL and Convexity and Certainty Bias

Dep. Var.	RATL Q.10				RATL Q.11			
Treatment	Long		Short		Long		Short	
(Probit)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Convexity	3.63*		-1.59		1.13		-7.59**	
	(1.73)		(-2.83)		(-0.53)		(-2.20)	
Cert. Bias		-.06		.45***		.19*	.18	
		(-0.52)		(2.84)		(1.71)		(1.23)
Constant	-.29*	-.11	.13	.13	.12	.24*	.12	.18
	(-1.79)	(-.88)	(.88)	(.92)	(0.75)	(1.80)	(.81)	(1.23)
Pseudo-R^2	.02	.01	.01	.07	.01	.02	.04	.01
Obs.	101	95	87	87	101	95	87	87

Notes. Same as Table 11. Each regression excludes one dependent variable.

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