Abstract

Why do organizations that want to produce information intervene in the allocation of workers to teams? Our answer is that decentralization may be inefficient. We posit a two-stage game: first, workers match one-to-one and second, produce information. Workers want to predict a Gaussian state and may acquire correlated Gaussian signals about it. While they form teams cooperatively, production is non-cooperative. Consequently, we define a new solution concept called Coalitional Sub-game Perfect Equilibrium. We prove a Pareto Efficient Equilibrium exists, though not every Equilibrium is Pareto Efficient. Further, the Equilibrium may not maximize utilitarian welfare or the production of information.

*We thank SangMok Lee, George Mailath, Steven Matthews, and Andy Postlewaite for support throughout the project. We also thank participants at the UPenn Micro Theory Lunch, the 28th Jerusalem School of Economic Theory, and the 2017 Summer School of the Econometric Society. In particular, we have benefited from the comments of Yeon-Koo Che, In-Koo Cho, Jan Eeckhout, Daniel Hauser, Eric Maskin, Stephen Morris, Leaat Yariv, and last, but not least, the support of our fellow graduate students.

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1 Introduction

Organizations that want to produce information must decide whether or not to intervene in its production. In a University, academic collaboration is unsupervised. But the University is the exception rather than the rule. In hierarchical business firms, managers assign workers to teams. In the United States House of Representatives, party leaders allocate congresspeople to Committees. We ask: why do organizations allocate workers to information-producing teams instead of decentralizing production?

Our answer is that the decentralized allocation of workers to teams may be inefficient. Organizations have experimented with decentralization in the past. Consider the case of the Denmark-based hearing aid manufacturer, Oticon. In the 1990s, Oticon replaced hierarchical production with self-enforced research teams. Despite the initial profitability of this change, today, management plays an active role in information production. For example, managers allocate workers to teams and appoint team leaders. Foss (2003) argues that incentive problems inherent to decentralization made Oticon revert back to hierarchical production.

The contribution of this paper is to posit a new model of cooperative team formation and non-cooperative production that rationalizes the widespread practice of management intervention in the allocation of workers to information-producing teams. In our model, workers want to predict a Gaussian state. They may acquire correlated, Gaussian signals about it at a cost. First, workers match one-to-one and second, acquire signals. After observing the signal realizations of their teammates, each worker takes an action to minimize a quadratic loss function.

In the second stage, which we call the Production Subgame, teammates acquire signals simultaneously and play a Payoff-Dominant Nash Equilibrium (PDN). We select Payoff-Dominant equilibria to account for pre-play communication. A fixed collection of PDN, one for every feasible team, and a partition of workers into teams is a Coalitional Subgame Perfect Equilibrium (CPSE) if there is no deviating team making each of its members better off. A CSPE differs from the Core because it limits the scope of re-negotiation; in the Core, after teams are formed, workers are free to coordinate on a PDN not pre-
scribed initially. CSPE are attractive from an applied point of view; in practice, managers may define roles for workers in teams even if they cannot dictate team composition.

Our main results are: (1) there are cutoff values on pairwise correlations which order equilibria of the Production Subgame by their symmetry; (2) a Pareto Efficient CSPE always exists; and (3) a CSPE may not be Pareto Efficient, maximize welfare, or maximize the production of information. We outline the logic behind each and provide a roadmap of the paper.

After formally defining the model and solution concepts in Section 2, we characterize the equilibria of the Production Subgame in Section 3. Existence of a PDN is shown by redefining the game as a Potential Game. Under a mild assumption on the cost of acquiring the first signal, we show that there is a cutoff value on pairwise correlations $\rho^*$ below which there is a unique symmetric PDN and a cutoff $\rho^{**}$ above which there is a unique asymmetric PDN in which one worker takes zero draws. To understand the intuition, consider a team of two workers with the lowest feasible pairwise correlation of -1. If both workers acquire a signal they learn the realized state perfectly; the two draws are located symmetrically around it. Hence, a symmetric PDN will be played. Now consider a team of two workers with the highest feasible pairwise correlation of 1. If one worker acquires a signal, the other has no incentive to acquire one because its realization will be identical. Hence, an asymmetric PDN in which one worker takes zero draws will be played. If the signal-to-prior variance ratio is greater than 1, an even stronger characterization holds: there is another cutoff $\rho^{***}$ below which there is a symmetric PDN and above which all PDN are asymmetric.

In Section 4, we consider existence in the two-stage game. First, we show that the Core may be empty. The reason is that preference cycles in the spirit of the Roomate Problem of [Gale and Shapley, 1962] cannot be broken. By eliminating them, we provide a sufficient condition for a non-empty Core. Finally, we prove a CSPE exists. The idea is to rank teams sequentially in decreasing order of utility attainable to a leader, defined as a teammate taking a weakly larger number of draws than her teammate. Exploiting the power of a manager to fix PDN off-path, each teammate can be selected to be the leader in any other feasible team. Hence, there are no profitable deviations from the proposed
teams.

In Section 5, we show that a CSPE outcome may be inefficient under a variety of optimality criterion. Our benchmark outcome is one that can be attained by a manager who can reassign workers to teams and choose which PDN is played inside each team. First, we show that a CSPE outcome may be Pareto Inefficient. It may be the case that selecting equilibria off-path may prevent mutually beneficial re-negotiation. Nonetheless, a Pareto Efficient CSPE always exists because any Pareto-improvement must be consistent with another CSPE. Next, we present and generalize two frictions that generate Welfare and Information Inefficiency. The first friction is exclusion. Exclusion happens when high complementary workers (those with low pairwise correlations) form teams and excluded low complementary workers are forced to form teams. A planner may better exploit the entire matrix of correlations. The second friction is free riding. Free riding causes non-complementary workers to form teams even though more learning occurs among complementary workers. A worker in a complementary team may be persuaded to form a non-complementary team if she can acquire fewer signals than her partner in the non-complementary team.

In Section 6, we explore several extensions to our model. First, we summarize the results of three robustness checks: (i) we assume the state and signal are binary instead of Gaussian; (ii) we assume workers acquire signals sequentially instead of simultaneously; and (iii) we assume a continuous action space instead of a discrete one. We show our model is qualitatively robust to these modifications. Second, to facilitate comparison to (Chade and Eeckhout 2017), we consider the case in which variances are heterogenous and pairwise correlations are constant. Third, we consider the case in which teams can have any number of members. We show that a CSPE may not exist and that the grand coalition may not be formed. Section 7 concludes and describes two applications of our model: one to the multinational firm and the other to congressional committees.
1.1 Related Literature

Team Formation. A general lesson of the team formation literature (e.g. [Page (2008), Prat (2002)]) is that forming teams comprised of heterogeneous agents is optimal. We show that there are cases in which such teams do not endogenously form. Teams may be comprised of similar individuals even when heterogeneous teams are available.

The most related paper in the literature is Chade and Eeckhout (2017). In their environment, agents have quadratic utility and form teams to share information. Agents can only obtain one draw from their signaling technology. Technologies have different pairwise precisions, but pairwise correlations are the same for all workers. The key difference in our environment is that we allow individuals to choose how many signals to acquire. In Section 6.2 we fix the signal structure of Chade and Eeckhout (2017), but allow individuals to choose any number of draws. It is no longer true that negative assortative matching is optimal. The reason is that when there is a large difference in the precision of two individual’s technologies, the individual obtaining less precise signals may not take any draws. Furthermore, we find that even if matching the most negatively correlated couple is optimal, these outcomes may not be reached endogenously.

Matching. Since teams have at most two individuals, we think of the allocation of workers to teams as a one-to-one matching problem. In contrast to existing approaches, the utility of matching with someone is not given exogenously, but depends on strategically-taken actions after the matched has been formed. From Gale and Shapley (1962), we know that in a one-sided matching problem the existence of a Stable matching is not guaranteed. But introducing an ex-post matching stage helps with existence; a planner can enforce stability by suggesting which equilibria to play off path.

As far as we know, only Kaya and Vereshchagina (2014) have considered a matching problem with ex-post concerns. In their environment, individuals match and decide how much effort to exert in the production of output. If effort is not observable there is a trade-off between providing incentives by destroying output or by bringing in a budget-breaker to suggest an action profile. Focusing on symmetric equilibria, they find
conditions under which the budget-breaker is optimal. While they focus on which organization is optimal ex-ante, we focus on the incentives to form teams inside a given organization.

*Strategic Experimentation.* The after-match game is analogous to a multi-agent multi-armed state-dependent bandit problem. As in Bolton and Harris (1999) and Keller, Rady, and Cripps (2005), individuals have incentives to free ride off of the information produced by others. In our model, there are two ways in which the free riding problem affects the number of draws taken by individuals. First, individuals do not take into account the positive externality of producing information. Second, for many correlation values the number of draws taken differs across individuals. This means that some players will provide larger quantities of the public good (i.e. the within group equilibrium may be asymmetric). We refer to the second effect as free riding.

## 2 Model

There is a finite set of workers \( N := \{1, ..., N\} \) who are uncertain about a state \( \theta \). They have a Gaussian common prior with mean \( \mu_\theta \) and variance \( \sigma_\theta^2 \). Each can obtain, at a cost, a Gaussian signal about the state from a worker-specific technology. Technologies are unbiased and have the same precision, but produce correlated signals; the joint distribution of signals is given by,\(^1\)

\[1,2\]

\[^1\]We justify the assumption that signals are drawn from a normal distribution by appealing to the Central Limit Theorem. If a signal represents the outcome of an experiment and each experiment involves a large number of repetitions, the normal distribution is a good approximation of the distribution of the signal. Even without a large sample justification, we show the qualitative analysis of our model is largely unchanged when modeling draws as outcomes of a binomial experiment. See Online Appendix D.1.

\[^2\]We fix variances and vary the correlation matrix because introducing heterogeneous variances complicates without illuminating. However, to facilitate direct comparison between our environment and that of Chade and Eeckhout (2017) directly, we consider the case in which correlations are fixed across players and variances are heterogeneous in Section 6.2.
We consider a two-stage game. In the first stage, workers form teams that have at most two members. The set of possible teams is given by $S := \mathcal{N} \times \mathcal{N}$ where $(i, j) \in S$ denotes a team with two members if $i \neq j$ and one member if $i = j$. A Partition is a function $\Pi : \mathcal{N} \to S$ such that $\Pi(i) = (i, j)$ implies $\Pi(j) = (i, j)$. Let $S_\Pi(i)$ denote the team in $\Pi$ which $i$ belongs to. Adding a teammate has a fixed cost; the cost of forming a team $S = (i, j) \in S$ is given by $K \ast I^S$, where $K > 0$ and $I^S$ is one if $i \neq j$ and zero otherwise.

In the second-stage, which we call the Production Subgame, each worker simultaneously chooses a positive integer of independent signals, $m_i$, from her technology. Fixing a team $(i, j)$ with integers $m_i$ and $m_j$, nature produces signals according to the following algorithm.

1. Set $n = 1$.

2. If $m_i \geq n$, $m_j \geq n$, and $i \neq j$, two signals are drawn from the distribution, $N \left( \begin{pmatrix} \theta \\ \theta \end{pmatrix}, \sigma^2 \begin{bmatrix} 1 & \rho_{12} & \cdots & \rho_{1N} \\ \rho_{21} & 1 & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{N1} & \cdots & \cdots & 1 \end{bmatrix} \right)$.

   If $i = j$, or if only $m_i \geq n$ or $m_j \geq n$, a single draw is taken from $N(\theta, 1)$.

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$^3$Online Appendix [B] extends the analysis to any group size. We summarize the main findings in Section [6.3].

$^4$Discreteness in the number of draws is not an important assumption as shown in Online Appendix [D.3]. Online Appendix [D.2], however, shows that the restrictions that draws are taken simultaneously may, indeed, be restrictive.

$^5$The independence assumption is not necessary to obtain our qualitative results. That inter-period persistence is constant across individuals is the relevant part of the assumption.
Draw Worker $i$ Worker $j$

First \[\rho_{ij}\]

-------- Independent --------

Second \[\rho_{ij}\]

-------- Independent --------

Third \[\rho_{ij}\]

Figure 1: Nature Draw Procedure.

3. If $m_i$ and $m_j$ are both less than or equal to $n$ the procedure stops. Else, repeat the previous step replacing $n$ with $n + 1$.

The procedure is depicted graphically in Figure 1 for a team $(i,j)$ in which $m_i = 3$ and $m_j = 2$.

Let $x_i$ denote the realized vector of length $m_i$ produced by $i$'s technology. Denote the concatenation of signals observed in team $S$ by $x^S$. After observing the signal realizations of every team member, each worker takes an action $a^* \in \mathbb{R}$ that maximizes the expected value of $u(a, \theta) = -(a - \theta)^2$,

$$a^* \in \arg \max_{a \in \mathbb{R}} -E_\theta \left[ (a - \theta)^2 \mid x^S \right].$$

We summarize the timing in Figure 2.

<table>
<thead>
<tr>
<th>Teams Form</th>
<th>Workers choose number of signals</th>
<th>Nature draws signals</th>
<th>Workers take action</th>
</tr>
</thead>
</table>

Figure 2: Timing.
2.1 Payoff-Dominant Nash Equilibrium

In a team \( S = (i, j) \), given a vector \((m_i, m_j)\), the utility of individual \( i \) conditional on membership in team \( S \) is,

\[
v_i^S(m_i, m_j) = -E_{x^S} \left[ \max_{a \in \mathbb{R}} E_{\theta} \left[ (a - \theta)^2 \mid x^S \right] \right] - c(m_i),
\]

(1)

where \( c : \mathbb{Z}_+ \to \mathbb{R} \) is such that the marginal costs are nondecreasing and positive, and \( c(0) = 0 \). We require that for each couple \( S = (i, j) \), the chosen vector \( m^*(S) \equiv (m_i^*(S), m_j^*(S)) \) is a pure strategy Payoff-Dominant Nash Equilibrium (PDN) of the normal form game \((\mathbb{Z}_+^2, \{v_i(S), v_j(S)\}_{j=1}^k)\). A PDN is a Nash Equilibrium that is not Pareto-dominated by any other Nash Equilibrium. In section 3.2, we show a Nash Equilibrium exists, and hence a Payoff-Dominant Nash Equilibrium exists, by redefining the normal form game as a Potential Game. We use this selection criterion to account for pre-play communication.\(^6\)

2.2 Core and Coalitional Subgame Perfection

We define two solution concepts for the two-stage game: Coalitional Subgame Perfect Equilibrium (CSPE) and the Core. A partition is a part of a CSPE if, fixing a PDN in every feasible team, there is no profitable deviating team \( S' \in S \).\(^7\) We prove a CSPE exists for any parameter values and feasible correlation matrix in Section 4.3.

**Definition 1** Let \( \Pi \) be a partition of \( N \) and \( M^* = \{m^*(S)\}_{S} \) be a collection of PDN for all \( S \in S \). The tuple \((\Pi, M^*)\) is a **Coalitional Subgame Perfect Equilibrium** if there does not exist a team \( S' \in S \) such that for all \( i \in S' \),

\[
v_i(m^*(S')) - K \cdot I_{S'} > v_i(m^*(S_{\Pi}(i))) - K \cdot I_{S_{\Pi}(i)}.
\]

\(^6\)In the general analysis in Online Appendix B, we use Coalition-Proof Nash Equilibrium (CPN) as our Production Subgame solution concept. When teams have at most two members, the set of CPN is equivalent to the set of Payoff-Dominant Nash Equilibria (Bernheim, Peleg, and Whinston (1987)).

\(^7\)From an applied perspective, it is plausible, revisiting the Oticon example, that a team leader may have the power to assign roles to individuals inside of a research team so that information is produced following a specific protocol.
A partition in the Core must be robust to another round of negotiation; after teams are formed, we require that workers cannot form a deviating team \( S' \in S \) and choose a PDN that makes each worker better off.\(^8\) Hence, both definitions coincide when there is a unique PDN within every feasible team. We show that the Core may be empty in Section 4.1 and provide sufficient conditions under which it is non-empty in Section 4.2.

**Definition 2** Let \( \Pi \) be a partition of \( N \) and \( \hat{M} = \{ \hat{m}(S) \}_{S \in \Pi} \) be a collection of PDN, one for each team in \( \Pi \). The tuple \( (\Pi, \hat{M}) \) is in the Core if there does not exist a team \( S' \in S \) and a PDN \( m^*(S') \) such that for all \( i \in S' \),

\[
v_i(m^*(S')) - K \ast I^{S'} > v_i(\hat{m}(S_{\Pi}(i))) - K \ast I^{S_{\Pi}(i)}.
\]

Notice, both definitions may be written using a matching function. We write them in terms of partitions so that they extend easily to the case in which there are no restrictions on team size. We discuss this extension in Section 6.3.

### 3 Production Subgame Analysis

Because the utility function is quadratic, the optimal action for a worker given any signal realization is the posterior mean. The expected utility of a worker when signals are costless is the negative posterior variance. In Lemma 1, we provide closed-form solutions for the posterior variance in any feasible team and re-state the preceding observations.\(^9\)

**Lemma 1** The optimal action for each agent in a team \( S \) is,

\[
a = E(\theta \mid x^S).
\]

If signals are costless, the expected utility of a teammate is the negative posterior variance. The posterior variance in a one-worker team acquiring \( n \) signals is,

\[
f(n) := \left( n\sigma^{-2} + \sigma_\theta^{-2} \right)^{-1}.
\]

---

\(^8\)The second definition may be thought of as the Core of a coalition game in which the valuation of a coalition is determined by the equilibrium correspondence of the Production Subgame.

\(^9\)The details of this proof, and all others that are not in the main text, can be found in Appendix A.
The posterior variance in a two-worker team acquiring \((m,n)\) signals with pairwise correlation \(\rho \in (-1,1)\) is,

\[
f(\rho, m, n) := \left( \frac{2}{1+\rho} + |m-n| \right) \sigma^{-2} + \sigma^{-2}_\theta \right)^{-1}.
\]

Holding \(m\) and \(n\) constant, the posterior variance in a two-worker team, \(f(\rho, m, n)\), is increasing in their pairwise correlation \(\rho\). For intuition, consider the case in which teammates have a pairwise correlation, \(\rho\), close to \(-1\). If each partner takes one draw, so that \(m = n = 1\), the team learns the state almost perfectly because the signals are located almost symmetrically around it. As \(\rho\) approaches \(-1\) information becomes perfectly complementary and the conditional variance, \(f(\rho, m, n)\), approaches zero. Now, consider a team with pairwise correlation \(1\). If only one teammate takes a draw, i.e. \(m = 1\) and \(n = 0\), the teammate taking zero draws has no incentive to take one. The information she produces is perfectly substitutable with her teammate since it is completely redundant. Note, however, that the marginal value of a draw by any worker when \(m = n = c\), for any positive integer \(c\), is not affected by the worker’s correlation with her teammate.

### 3.1 The Marginal Value of Information

We now study the marginal value of information for interior pairwise correlations \(\rho \in (-1,1)\). The marginal value of draw \(n_i\) by player \(i\) given that player \(j\) takes \(n_j\) draws is,

\[
MV(n_i; n_j, \rho) \equiv [-f(\rho, n_i, n_j)] - [-f(\rho, n_i - 1, n_j)] = f(\rho, n_i - 1, n_j) - f(\rho, n_i, n_j).
\]

We analyze \(MV(n_i; n_j, \rho)\) in Corollary [1] through Corollary [4]. In Corollary [1], we show that the marginal value of a draw by a leader, defined as a teammate taking weakly more draws than her partner, is increasing in \(\rho\). The reason is that the information left to learn, fixing the number of draws by each teammate, is increasing in \(\rho\).

**Corollary 1 (Leader Comparative Statics in \(\rho\))** For \(n_i > n_j\), \(MV(n_i; n_j, \rho)\) is increasing in \(\rho\).

\[\text{Recall, the expected utility, before the cost of acquiring information, of the agent is } -f(\rho, m, n) \text{ when one agent takes } m \text{ draws and the other takes } n \text{ draws.}\]
The same property does not hold for a follower, defined as a teammate taking strictly fewer draws than her partner. While the amount of information left to learn, fixing the number of draws by each teammate, increases in $\rho$, the value of matching a leader’s draw decreases in $\rho$ because the information produced is more redundant. Hence, the marginal benefit of a draw by a follower is non-monotonic. In Corollary 2 we prove it is strictly concave in the pairwise correlation $\rho$ and has a unique maximizer.

**Corollary 2 (Follower Comparative Statics in $\rho$)** For $n_i \leq n_j$ where $n_j \geq 1$, $MV(n_i; n_j, \rho)$ is strictly concave in $\rho$ with a unique maximizer,

$$\hat{\rho}(n_i, n_j, \gamma) = \frac{(n_j + \gamma - \sqrt{n_i(n_i + 1)})^2}{-(n_j + \gamma)^2 + n_i(n_i + 1)}.$$

We now make stepwise comparisons between the marginal value of a draw by a leader to the marginal value of a draw by a follower. Workers initially take $n - 1$ draws. The leader’s marginal value is the payoff of taking an $n$-th draw. The follower’s marginal value is the payoff of taking an $n$-th draw, given that the leader already took an $n$-th draw. In Corollary 3 we show that for any number $n \geq 1$ and signal-to-prior variance ratio $\gamma = \frac{\sigma^2}{\sigma_\theta^2}$, there is a unique correlation, $\hat{\rho}(n, \gamma)$, below which the marginal value of the leader is less than the marginal value of the follower, and above which the opposite holds.

**Corollary 3 (Leader-Follower MV Comparison 1)** Fix $n_i \geq 1$ and $\gamma$. Then,

$$\frac{MV(n_i; n_i - 1, \rho)}{MV(n_i; n_i, \rho)} < \begin{array}{ll}
\text{Marginal Value Leader} & \text{Marginal Value Follower}
\end{array}$$

if and only if,

$$\rho < \hat{\rho}(n_i, \gamma) = \frac{-(\gamma - 1 + 2n_i) + \sqrt{(\gamma - 1 + 2n_i)^2 - 4\gamma}}{2\gamma} < 0.$$

In Corollary 4 we show that if $\gamma$ is sufficiently large the pairwise correlation at which the marginal value of a follower is maximized, $\tilde{\rho}(n_i, n_j, \gamma)$, must be less than $\hat{\rho}(n_i, \gamma)$. This property is necessary to order equilibria in terms of their symmetry.
(a) Marginal value of second draw for leader and follower.
(b) Ex-post Variance given different strategies.

Figure 3: Ex-post variances and marginal values when $\sigma = \sigma_\theta = 1$.

Corollary 4 (Leader-Follower MV Comparison 2) Fix $n_i \geq 2$. Then, there exists $\gamma^*(n_i) \in \left[\frac{1}{2}, 1\right]$ such that $\gamma \geq \gamma^*(n_i)$ if and only if

$$\tilde{\rho}(n_i, n_i - 1, \gamma) \leq \hat{\rho}(n_i, \gamma).$$

Figure 3 illustrates Corollary 1 through Corollary 4 when $\gamma = 1$ and $n_i = 2$. By Corollary 1, the marginal value of the leader is increasing in $\rho$ because the information left to learn increases. Corollary 2 implies the follower’s marginal value has the hump-shape depicted in 3a. Figure 3b illustrates how diminishing marginal returns to information production generates it; when $\rho$ is lower the follower has less to learn, but produces less redundant information. For $\rho > \tilde{\rho}$, the former effect dominates the latter and the marginal value decreases. By Corollary 3, the marginal value of the leader is less than the marginal value of the follower below a negative cutoff $\hat{\rho}$. Since $\gamma = 1$, the condition in Corollary 4 is satisfied. Hence, the correlation that maximizes the follower’s marginal value, $\tilde{\rho}$, is less than $\hat{\rho}$.

\footnote{For $n_i = 1$, $\tilde{\rho}(n_i, n_i - 1, \gamma) = -1$, so the inequality is satisfied for any $\gamma$.}
3.2 Existence of Nash Equilibrium

Before characterizing the equilibrium correspondence, we need to prove an equilibrium exists.\footnote{Information is strategically complementary for correlations below $\hat{\rho}(n, \gamma)$ and strategically substitutable above it. Because the game does not exhibit strategic complementarities, we cannot invoke Tarski’s Fixed Point Theorem to prove existence.} The next Lemma states that we may bound the action space without loss of generality. The reason is that diminishing marginal returns to information production imply that, eventually, the marginal value of a draw must be less than the marginal cost regardless of the behavior of one’s partner.

Lemma 2 There is a positive integer $\bar{M}$ such that for each positive integer $m \geq \bar{M}$, $m$ is not a best response by player $i$ to any strategy by player $j$.

Since we can bound the action space, we may redefine the game as a finite potential game to show that there exists a pure strategy\footnote{In the rest of the paper, we focus on equilibrium in pure strategies. One reason we do not study mixed strategy equilibria is that they perform worse in terms of social welfare than at least one pure strategy equilibrium. Further, the results are not heavily dependent on the restriction to pure strategies. We will point out when the analysis changes.} Nash equilibrium.

Proposition 1 There exists a pure strategy Nash Equilibrium of the Production Subgame.

Proof Given that no individual will optimally choose a number of draws larger than $\bar{M}$, we can redefine, without loss of generality, the Production Game as the normal form game given by $(\{0, 1, \ldots, \bar{M}\}^2, \{v_i, v_j\})$. Define the function,

$$
\Phi(m, n, \rho) = -f(m, n, \rho) - dh(m) - dh(n)
$$

It is a potential function since

$$v_1(m, n, \rho) - v_1(m', n, \rho) = -f(m, n, \rho) - dh(m) + f(m', n, \rho) + dh(m') = \Phi(m, n, \rho) - \Phi(m', n, \rho)$$

$$v_2(m, n, \rho) - v_2(m, n', \rho) = -f(m, n, \rho) - dh(n) + f(m, n', \rho) + dh(n') = \Phi(m, n, \rho) - \Phi(m, n', \rho).$$

Hence, the redefined game is a finite potential game and is guaranteed to have a pure strategy Nash Equilibrium by Corollary 2.2 of Monderer and Shapley\textsuperscript{[1996].}
If there is a symmetric pure strategy Nash Equilibrium it must be a PDN. However, it is possible that there are no symmetric pure strategy Nash Equilibrium. Since the game is symmetric, there will always exist a symmetric mixed strategy equilibrium. We next show that no mixed strategy equilibrium Pareto Dominates all Nash equilibria in pure strategies. Hence, we can always find a pure strategy PDN of the Production Subgame.

**Corollary 5** There exists a pure strategy PDN of the Production Subgame.

### 3.3 Characterization

We now characterize the equilibrium correspondence. Assumption 1 bounds the cost function so that the following properties hold in equilibrium:

1. In any team, at least one worker takes at least one draw.
2. In a two-worker team with pairwise correlation \(-1\), both workers take one draw.

**Assumption 1** \(c(1) < \frac{\min\{\sigma^2, \theta^2\}}{1 + \gamma}\).

By Corollary 1 the marginal value of the leader is increasing monotonically in \(\rho\) and by Corollary 3 the marginal value of the follower is decreasing monotonically after \(\hat{\rho}\). Hence, a high enough pairwise correlation generates a unique asymmetric equilibrium. On the other hand, when the pairwise correlation is sufficiently small and information is complementary, the unique equilibrium is symmetric.

**Proposition 2** Suppose Assumption 1 is satisfied. There exist cutoff values \(\rho^*\) and \(\rho^{**}\) such that,

\[-1 < \rho^* \leq \rho^{**} < 1\]

and the following properties hold:

1. For all \(\rho \leq \rho^*\) there is a unique, symmetric PDN.
2. For all \(\rho > \rho^{**}\) there is a unique (up to identity) PDN in which one player takes a strictly positive number of draws and the other takes none.
The Proposition does not characterize equilibria for correlations between $\rho^*$ and $\rho^{**}$. Without additional restrictions, there are two complications. First, there may be multiple PDN as illustrated in Figure 4. For correlations around $\rho = 0$, there are two asymmetric equilibria. Second, there may be a correlation at which the unique equilibrium is asymmetric and for a higher correlation there is a symmetric equilibrium. In Figure 4, when $\rho = -0.29$ the unique equilibrium is asymmetric, but there is a unique symmetric equilibrium for a slightly higher correlation.

Figure 4: Equilibrium correspondence when $c(m) = 0.019m$, $\sigma^2 = \frac{1}{2}$ and $\sigma^2_0 = 1$, so that $\gamma = \frac{1}{2}$.

The second phenomenon deserves further attention. The key to the example in Figure 4 is that $\hat{\rho} < \tilde{\rho}$, in contrast to the situation depicted in Figure 3. For $\rho = -0.29 \in (\hat{\rho}, \tilde{\rho})$ and $n = 1$, the marginal value of a draw for a leader is greater than the marginal value of a draw for a follower because $\rho > \hat{\rho}$. Here, we fix the marginal cost of a second draw so that the asymmetric equilibrium $(2, 1)$ is played. If $\rho$ increases, however, the marginal value of the follower *increases* and may exceed the chosen marginal cost. The former phenomena occurs because the increase in the value of information left to learn for the follower offsets the declining value of matching the leader’s draw. If the follower takes a second draw, however, the leader has no incentive to take a third draw because the information left to learn decreases sufficiently. Hence, a symmetric equilibrium $(2, 2)$ is played.
The previous phenomena could not have happened if $\hat{\rho} < \tilde{\rho}$. When an asymmetric equilibria is played, the marginal value of the leader is greater than the marginal value of the follower at the leader’s last draw. Hence, $\rho > \hat{\rho}$. But increasing $\rho$ past $\hat{\rho}$ must decrease the marginal value of the follower because $\rho > \hat{\rho} > \tilde{\rho}$. Hence, the follower will not be induced to match the leader’s draw and play a symmetric equilibrium. In general, as we have seen in Corollary 4, the relationship between $\hat{\rho}$ and $\tilde{\rho}$ depends on the relationship between $\sigma$ and $\sigma_0$. There we proved that if $\gamma = \frac{\sigma^2}{\sigma_0^2} \geq 1$ and $n_i \geq 2$, then $\tilde{\rho}(n_i, n_i - 1, \gamma) \leq \hat{\rho}(n_i, \gamma)$. Hence, if $\gamma \geq 1$ we obtain a stronger characterization: in addition to the properties already established, there is a cutoff below which there exists a symmetric PDN and above which all PDN are asymmetric.\(^{14}\)

**Proposition 3** Suppose Assumption 1 is satisfied and $\gamma = \frac{\sigma^2}{\sigma_0^2} \geq 1$. There exist cutoff values $\rho^*$, $\rho^{**}$, and $\rho^{***}$ such that,

$$-1 < \rho^* \leq \rho^{***} \leq \rho^{**} < 1$$

and the following properties hold:

1. For all $\rho \leq \rho^*$ there is a unique, symmetric PDN.

2. For all $\rho \in (\rho^*, \rho^{***})$, there is at least one symmetric PDN and one asymmetric equilibrium.

3. For all $\rho > \rho^{***}$ all PDN are asymmetric.

4. For all $\rho > \rho^{**}$ there is a unique (up to identity) PDN in which one player takes a strictly positive number of draws and the other takes none.

### 4 Existence

Having analyzed the Production Subgame, we now prove existence of equilibrium in the two-stage game. We first show that the Core may be empty because preference cycles

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\(^{14}\)We refer the reader interested in further properties of the equilibrium correspondence to Appendix C, in which we analyze it using the tools of Monotonic Comparative Statics.
cannot be broken. Next, we provide a sufficient condition for a non-empty Core by eliminating such cycles. Finally, we prove the general existence of a Coalitional Subgame Perfect Equilibrium.

### 4.1 Empty Core Example

We fix parameters in Table 1. Because \( c(1) = 0.002 < \frac{1}{3} = \frac{\min\{\sigma^2, \sigma^2\}}{1 + \gamma} \) and \( \gamma = \frac{\sigma^2}{\sigma^2} = 2 \), Proposition 3 holds. The cutoff values are: \( \rho^* = \rho^{***} = -0.006 \) and \( \rho^{**} = 0.849 \).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Interpretation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma^2 )</td>
<td>Signal Variance</td>
<td>2</td>
</tr>
<tr>
<td>( \sigma^2_\theta )</td>
<td>Prior Variance</td>
<td>1</td>
</tr>
<tr>
<td>( c(m) )</td>
<td>Cost of ( m ) Draws</td>
<td>0.002 ( m^2 )</td>
</tr>
<tr>
<td>( K )</td>
<td>Cost of Teammate</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 1: Parameters for Example 4.1 and Example 5.1

Suppose there are three workers and their technologies are correlated according to the network depicted in Figure 5. The number inside each circle is the identity of the worker and the numbers next to the edges connecting the circles are pairwise correlations.

![Correlation Matrix for an Empty Core](image)

Figure 5: Correlation Matrix for an Empty Core.

The equilibria and payoffs of the Production Subgame for each feasible team are as follows. The pairwise correlation in couple (1, 2) is below the threshold \( \rho^* \) and there is a unique, symmetric equilibrium (4, 4). Workers obtain a payoff of –0.224. Pairwise correlations in the teams (1, 3) and (2, 3) exceed the threshold \( \rho^{***} \) after which all PDN are asymmetric. In both teams, there is a unique equilibrium, up to identity, (5, 4). In
the team \((1, 3)\), the leader obtains a payoff of \(-0.232\) and the follower obtains a payoff of \(-0.214\). In the team \((2, 3)\), the leader obtains a payoff of \(-0.238\) and the follower obtains a payoff of \(-0.220\). Finally, if any individual remains alone, she will take three draws and obtain a utility of \(-0.320\).

By exhibiting a preference cycle, we show the Core is empty. Suppose the team \((2, 1)\) is formed. Then, Worker 3 can make an offer to Worker 1 and form a mutually beneficial deviating team in which Worker 3 is the leader. Suppose the team \((1, 3)\) is formed. Then, Worker 2 can make an offer to the leader of the team and form a mutually beneficial deviating team in which Worker 2 is the leader. Suppose the team \((3, 2)\) is formed. Then, Worker 1 can make an offer to the leader of the team and form a mutually beneficial deviating team in which Worker 1 is the leader. Finally, if all teams are singletons, Worker 1 and Worker 2 can form a team and be made better off. Hence, there is no partition in the Core.

### 4.2 Sufficient Conditions for a Non-Empty Core

In light of the previous negative result, we find conditions under which the Core is non-empty. To accomplish this task, we define three sets of correlations using our characterization in Proposition 2. First, we define the set of correlations for which the only CPN is symmetric:

\[ P^1 = [-1, \rho^*]. \]

Second, we define a set that is the union of the elements in \(P^1\) for which a worker can obtain a higher utility in an asymmetric equilibrium and the set of correlations for which there may be multiple equilibria:

\[ P^2 = \left\{ \rho \in P^1 : \bar{v}_i(\rho) < \sup_{\tilde{\rho} \in (P^1)^c} (\bar{v}_i(\tilde{\rho})) \right\} \cup [\rho^*, \rho^{**}], \]

where \(\bar{v}_i(\rho)\) is the maximum equilibrium utility of an individual inside a team with correlation \(\rho\). Third, we define the set of correlations for which there is a unique asymmetric
equilibrium in which one worker takes zero draws:

\[ p^3 = (\rho^{**}, 1]. \]

**Theorem 1** Suppose Assumption 1 and one of the following two conditions holds:

1. All correlations are in \( P^1 \cup P^3 \).

2. If individual \( j \) has more than one pairwise correlation in \( P^2 \), then for all \( i \) such that \( \rho_{ij} \in P^2 \), \( \rho_{ij} \) is the only pairwise correlation in \( P^2 \) for individual \( i \).

Then, the Core is non-empty.

The logic of the first condition is that if all PDN are symmetric or include complete free riding, we can form teams with pairwise correlations in \( P^1 \) sequentially in decreasing order of utilities. Since those with correlations in \( P^1 \) will be matched with their preferred feasible choice and nobody wants to be in a team with a partner that takes zero draws, there are no profitable deviations. The second condition in Theorem 1 ensures there are no cycles of profitable deviations to non-trivial asymmetric equilibria, i.e. to teams with correlations outside of \( P^3 \). Hence, the example in Figure 5 is ruled out.

### 4.3 Existence of Coalitional Subgame Perfect Equilibrium

We now prove the existence of a CSPE. In the empty Core example, non-existence is driven by incentives to re-negotiate the equilibrium played in teams off the path-of-play. If a planner can select which PDN are played off path, incentives to deviate can be eliminated.\(^{15}\) Consider the example in Figure 5 and the partition \( \Pi = \{ (1, 2), (3) \} \). If the equilibrium in teams \( (1, 3) \) and \( (2, 3) \) is fixed to be \( (3, 1) \), there is no free-riding temptation because Worker 1 and Worker 2 will be leaders in a team with Worker 3. Allowing such enforcement is not too permissive; the suggested partition is the only one compatible with a CSPE.

\(^{15}\)We believe this weaker definition of equilibrium describes situations where there is a third party that can suggest which teams form by creating discussion groups or by assigning different tasks to individuals inside a team. For example, the manager of a firm can designate someone as the boss of some research team and make her more accountable for the joint output.
Theorem 2 For any parameter values and correlation matrix there exists a CSPE.

The key to the proof is that we may order teams sequentially in decreasing order of the utility attainable to a leader. Choosing PDN off-path, so that each teammate is the leader in any other feasible team, ensures there are no profitable deviations.\textsuperscript{16}

5 Inefficiency

We show that a CSPE outcome may be inefficient relative to a benchmark in which a planner can reassign workers to teams and choose which PDN is played inside each team. Formally, define an outcome as a partition, $\Pi^*$, and a PDN in each team in the partition, $(m^*(S))_{S \in \Pi^*}$. An outcome is \textbf{Pareto Efficient} if there does not exist a partition and a PDN in each team that weakly improves each worker’s payoff and strictly improves one worker’s payoff. An outcome is \textbf{Welfare Efficient} if it maximizes the sum of worker payoffs. Denote $P^{||\Pi^*||}$ as the set of partitions containing the same number of teams as $\Pi^*$. An outcome is \textbf{Information Efficient} if it minimizes the sum of posterior variances when partitions must be in $P^{||\Pi^*||}$:

$$\{\Pi^*, (m^*(S))_{S \in \Pi^*}\} \in \arg\min_{\Pi \in P^{||\Pi^*||}, (m(S))_{S \in \Pi}} \left(\sum_{S \in \Pi} f_\rho(S, m(S))^{-1}\right)^{-1}.$$

Though Welfare and Information Efficient outcomes are related they do not necessarily coincide. A planner maximizing information does not consider private costs of acquiring signals. Hence, the Information Efficient outcome may be Welfare dominated by an outcome in which workers produce less information, but some workers take fewer draws.

5.1 Pareto Inefficiency\textsuperscript{17}

First, we show that a CSPE may be Pareto Inefficient. Fix the parameters in Table 1. Suppose technologies are correlated according to the network in Figure 6. The correla-

\textsuperscript{16} We extensively exploit the symmetry of the game. In particular, in any couple any individual can be selected to be the team leader.

\textsuperscript{17} We thank Yeon-Koo Che for drawing our attention to this issue.
tions imply unique PDN in each size-two team. Equilibrium strategies and payoffs are reported below for convenience.

<table>
<thead>
<tr>
<th>Correlation</th>
<th>PDN</th>
<th>Payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>−0.05</td>
<td>(4, 4)</td>
<td>(−0.224, −0.224)</td>
</tr>
<tr>
<td>0</td>
<td>(4, 5)</td>
<td>(−0.2138, −0.2318)</td>
</tr>
<tr>
<td>0.4</td>
<td>(4, 5)</td>
<td>(−0.2334, −0.2874)</td>
</tr>
<tr>
<td>0.9</td>
<td>(0, 7)</td>
<td>(−0.2222, −0.3202)</td>
</tr>
</tbody>
</table>

Figure 6: Pareto Inefficient Correlation Matrix and PDN.

There is a CSPE with partition $\Pi^* = \{(1, 2), (3, 4)\}$ and on-path PDN $(m^*(S))_{S \in \Pi^*} = \{(4, 4), (4, 5)\}$. However, every worker is better off when teams $(1, 3)$ and $(2, 4)$ are formed and the PDN $(4, 5)$ is played in each team. The reason for Pareto Inefficiency is that fixing PDN off-path may prevent mutually beneficial re-negotiation. In fact, the deviating partition and collection of PDN used to demonstrate Pareto Inefficiency is the unique Core outcome. Furthermore, it is Pareto Efficient. Since every Core outcome is also a CSPE outcome, there exists a Pareto Efficient CSPE. It turns out this is a general property. Fix a CSPE outcome. If there is a Pareto-improving outcome, it has to be a CSPE outcome. As there are only a finite number of equilibria, we may iterate until there are no more Pareto improvements.

**Theorem 3 (Pareto Efficiency)** There exists at least one CSPE that is Pareto Efficient.

### 5.2 Welfare and Productive Inefficiency

Even if we select a Pareto Efficient CSPE, its outcome need not be Welfare or Information Efficient. We illustrate the two frictions, information exclusion and after-match free rid-
ing, that create both Welfare and Information Inefficiency. For the numerical examples, we choose the parameters presented in Table 2. Because \( c(1) = 0.01 < \frac{1}{2} = \frac{\min(\sigma^2_\theta, \sigma^2)}{1 + \gamma} \) and \( \gamma = \frac{\sigma^2_\theta}{\sigma^2} = 1 \), Proposition 3 holds. The cutoff values are: \( \rho^* = 0.1038 \), \( \rho^{**} = 0.3333 \), and \( \rho^{***} = 0.7143 \).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Interpretation</th>
<th>Value</th>
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<tbody>
<tr>
<td>( \sigma^2 )</td>
<td>Signal Variance</td>
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<tr>
<td>( \sigma^2_\theta )</td>
<td>Prior Variance</td>
<td>1</td>
</tr>
<tr>
<td>( c(m) )</td>
<td>Cost of ( m ) Draws</td>
<td>0.01( m^2 )</td>
</tr>
<tr>
<td>( K )</td>
<td>Cost of Teammate</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 2: Parameters for Example 5.2.1 and Example 5.2.2.

5.2.1 Exclusion

Suppose technologies are correlated according to the network in Figure 7. PDN in all size-two teams are unique, and are presented below the network. A symmetric equilibrium (2,2) is played in each team except when the pairwise correlation is 0.9. In the example, the unique Core outcome is also the unique CSPE outcome. Two teams of size two are formed: the most efficient learning team (3,4) and a relatively inefficient team (1,2). No teammate in team (3,4) has a profitable deviation because each obtains the highest attainable payoff. Worker 1 and Worker 2 are excluded. Each prefers forming the team (1,2) over remaining alone.

Even though the most efficient learning team is formed, the Equilibrium outcome is inefficient. The reason is that the excluded individuals, Worker 1 and Worker 2, produce positively correlated information (\( \rho_{12} = 0.1 \)). The planner can increase the production of information by forming teams (1,3) and (2,4). Worker 1’s information complements Worker 3’s information and Worker 2’s information complements Worker

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\(^{18}\) In a simulation study, available upon request, we provide a lower bound on the measure of inefficient correlation matrices. The economy has four workers, \( c(m) = 0.01m^2 \), and \( \sigma = \sigma_\theta = 1 \). We restrict matrices to those that have unique CSPE partitions; hence, inefficiencies are driven by the Exclusion problem discussed in detail in section 5.3. We find that 21.06% and 18.28% of the correlation matrices with a unique CSPE partition do not maximize welfare and information production, respectively.
4’s information. Even though the most efficient team (3, 4) is disrupted, the efficiency loss from pairing Worker 1 and Worker 2 is mitigated. The example illustrates that individual incentives clash with managerial objectives to exploit the entire correlation matrix.

We now generalize the example. Suppose there are an arbitrary number of individuals and two size-two teams \((i, j)\) and \((i', j')\) are formed in a CSPE. The general correlation structure among these workers is displayed in Figure 8.\(^{19}\) As in the numerical example, suppose teams \((i, j)\) and \((i', j')\) play the same equilibrium. Without loss of generality, we omit the cross-correlations between \(i\) and \(i'\) and \(j\) and \(j'\) because they are not necessary for our argument.

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\(^{19}\) We omit the cross-correlations between \(i\) and \(i'\) and \(j\) and \(j'\) because they are not necessary for our argument.
suppose $\rho_{i'j'}$ is larger than $\rho_{ij}$ so that $(i, j)$ produces more complementary information. Then, we can always choose $\rho_{i'j'}$ and $\rho_{ij}$ slightly larger than $\rho_{ij}$, but less than $\rho_{i'j'}$ such that more information is produced by matching $i$ with $j'$ and $j$ with $i'$. Further, the inefficiency increases in $\rho_{i'j'}$ because the excluded team produces more redundant information.\footnote{While Example 5.2.1 has symmetric equilibria played on and off-path, this need not happen to obtain exclusion efficiency.}

**Theorem 4 (Exclusion Inefficiency)** Suppose Assumption 1 holds. Fix a CSPE $\{\Pi^*, (m^*(S))_S\}$ containing two-worker teams $(i, j)$ and $(i', j')$. Without loss of generality, suppose $\rho_{ij} < \rho_{i'j'}$ so that $(i', j')$ is an excluded team. Suppose the following conditions hold:

1. $m^*((i, j)) = m^*((i', j'))$.
2. $\rho_{i'j'}, \rho_{ij} \in (\rho_{ij}, \rho_{ij} + \epsilon)$ for a small enough $\epsilon > 0$.

Then, the CSPE outcome is neither Welfare Efficient nor Information Efficient. The inefficiency is increasing in the pairwise correlation of the excluded team $\rho_{i'j'}$.

### 5.2.2 Free Riding

Now, suppose signaling technologies are correlated according to the network in Figure 9. The PDN in all size-two teams are unique up to identity, and are presented below the network. In the unique Core outcome, two teams of size two are formed: $(1, 3)$ and $(2, 4)$.\footnote{If we allow for equilibrium in mixed strategies this allocation is not in the Core anymore. However, it is a CSPE and the analysis of inefficiency goes through.} In contrast to the previous example, the most efficient learning team, $(1, 2)$, does not form endogenously even though it is a part of the optimal partition.

The team $(1, 2)$ is disrupted whenever Worker 1 can take fewer draws in a PDN with Worker 3. Despite learning less, Worker 1 obtains a higher payoff. Analogous reasoning holds if Worker 2 can take fewer draws in a PDN with Worker 4.

The optimal partition $\Pi^* = \{(1, 2), (3, 4)\}$ can be decentralized as a CSPE. If the manager is able to assign roles within teams, she can make Worker 1 (Worker 2) be the team leader when matched with Worker 3 (Worker 4). Worker 1 and Worker 2 will then
match endogenously. Notice, the unique Core is Welfare and Information Inefficient, while there exists a CSPE that is Welfare and Information Efficient. Fixing equilibria off-path can help a manager attain a better outcome when inefficiency is driven by after-match behavior.

We perform an analogous exercise to generalize the example. If Assumption 1 holds and $\gamma \geq 1$, by Proposition 3 there exists a cutoff, $\rho^*$, below which there is a unique symmetric equilibria and above which there is at least one asymmetric equilibrium, and a cutoff, $\rho^{**}$, above which there is a unique asymmetric equilibria in which one teammate takes zero draws. Suppose teams $(i', j)$ and $(i, j')$ are formed in a CSPE as portrayed in Figure 10. Two properties are important for free-riding inefficiency: first, a superior
team \((i, j)\) is not formed because \(i\) obtains a higher payoff as a follower in a team with \(j'\), and second \(j'\) obtains a higher payoff as a leader in team \((i, j')\) than as a leader in the manager’s preferred team \((i', j')\).

To satisfy the first property, there must exist a \(\rho_{ij'}\) in \([\rho^*, \rho^{**}]\) for which there is an asymmetric equilibrium in team \((i, j')\) where no teammate takes zero draws. Then, we can select \(\rho_{ij}\) such that team \((i, j)\) plays a unique symmetric PDN, but such that player \(i\) can obtain a higher payoff as a follower in a team with \(j'\). Recall, this means \(\rho_{ij} \in P^1 \cap P^2\), where \(P^1\) are correlations less than \(\rho^{**}\) and its intersection with \(P^2\) contains correlations for which a higher payoff is possible in an asymmetric equilibrium. To satisfy the second property, select \(\rho_{i'j'} \geq \rho_{ij'} \geq \rho^*\) and an asymmetric PDN. Then, if \(j'\) has to be a leader in a team she would rather be the leader in a team with \(i\).

We next ensure \(j\) and \(i'\) have the same incentives as \(i\) and \(j'\) described above. Hence, we add the condition that \(\rho_{i'j}\) is in \([\rho^*, \rho^{**}]\) and that \(\rho_{i'j'} \geq \rho_{i'j}\). Consequently, we obtain the following Theorem.

**Theorem 5 (Free Riding Inefficiency)** Suppose Assumption 1 holds and \(\gamma \geq 1\). Fix a CSPE \(\{\Pi^*, (m^*(S))_S\}\) containing teams \((i, j')\) and \((i', j)\) for which the following conditions hold:

1. \(\rho_{ij'}, \rho_{i'j} \in [\rho^*, \rho^{**}]\).

2. \(\rho_{i'j'} \geq \rho_{ij'} \geq \rho^*\) and \(\rho_{i'j'} \geq \rho_{i'j}\).

Then, there exists a \(\rho_{ij} \in P^1 \cap P^2\) such that the Equilibrium is Welfare Inefficient.

Notice, the Theorem does not prove that the outcome is Information Inefficient. Although in the numerical example above welfare maximizing teams maximize information production (we report the posterior variance, \(f(\rho, m^*(S))\), in Figure 9, so that this can be verified), this is not true in general. The reason is that there are two ways a welfare-optimal team playing a symmetric equilibrium can be broken. First, the deviating teammate may take fewer draws in another asymmetric equilibrium. In this case, the asymmetric equilibrium generates less information and Welfare Efficiency coincides with Information Efficiency. It is also possible, however, that a teammate takes the same number of draws in a welfare-inefficient asymmetric equilibrium and that the leader that
takes more draws than her initial partner. Then, better information may be produced in
the welfare-inefficient team than in the welfare-efficient one. In Online Appendix E we
present such an example.

6 Extensions

6.1 Summary of Robustness Checks

The environment we consider is stylized. We argue our results generalize to other similar
environments. For those interested, we present a detailed discussion in Online Appendix D.

We have assumed throughout the paper that the prior and signals are Gaussian. Another possible assumption is that the state and the signal are binary. In this case, the ex-post variance does not admit a simple closed-form solution. In Online Appendix D.1 we show that when the number of draws is small, however, the marginal value of a draw satisfies the same qualitative properties as the ones described in section 3. Hence, all subsequent results hold.

Another important assumption in our environment is that workers acquire signals simultaneously. We could have assumed that workers instead acquire signals sequentially. In Online Appendix D.2 we study a finite extensive game with sequential decisions. The main conclusion is that, for many correlations, there is a Subgame Perfect Equilibrium of the sequential game that coincides with the most symmetric equilibrium of the simultaneous game. The cases in which the Subgame Perfect Equilibrium do not coincide with any equilibrium of the simultaneous game occur when the pairwise correlation between teammates is in the region $(\bar{\rho}, \rho^{**})$ for a fixed $\bar{\rho} > 0$. In this region, the most symmetric equilibrium is such that the difference in the number of draws by each worker is at least two and neither worker takes zero draws.

Finally, in Online Appendix D.3 we consider a variation of the after-match game in which the action space is continuous. We show that the equilibrium is unique and symmetric for negative correlations and asymmetric, and not necessarily unique, for
positive correlations. This is consistent with our results in Proposition 3.

### 6.2 Heterogeneous Variances

We have assumed worker technologies have the same precision, but do not restrict their correlation. Chade and Eeckhout (2017) study the case in which precisions are different, but pairwise correlations are the same for any two individuals. The key difference in our environment, however, is that we allow individuals to play an ex-post game. At some cost, individuals may take as many signals as they want after matching while in Chade and Eeckhout (2017) each individual receives one signal exogenously. This distinction is important even if we consider the signal structure in Chade and Eeckhout (2017). In particular, we show that their main result that negative assortative matching is optimal need not hold.

Suppose we form a couple that has technologies with different variances that produce conditionally independent signals, but have constant pairwise correlations. If individuals have the same cost function and each individual takes the same number of draws, the individual with a higher precision always has a larger incentive to acquire a signal. In the example portrayed in Figure 11, the equilibrium in the ex-post game is symmetric only if technologies sufficiently similar. If precisions are different enough, the individual with the less precise signals takes zero draws.

Suppose $c = 0.001$, $h(m) = m^2$ and there are four individuals with variances 0.25, 0.5, 1 and 1.25. In the team whose members have the variances $(0.25, 1.25)$, the unique equilibrium has the individual with the highest precision take two draws while the other takes no draws. In this case, negative assortative matching is inefficient even without membership costs because one individual does not produce anything.\(^22\) Here the efficient match is $\{(0.25, 1), (0.5, 1.25)\}$ as opposed to the negative assortative match $\{(0.25, 1.25), (0.5, 1)\}$. The efficient match is the most negative assortative matching such that all individuals take a positive number of draws.\(^23\)

\(^{22}\)Notice that if there is an increasing membership cost, the player with variance 0.25 does not want to be part of this team either.

\(^{23}\)Another interesting observation is that a smaller variance is not always beneficial for an individual. If an individual reduces her variance, she may have to acquire more information or may reduce the number
Figure 11: Equilibrium correspondence when $\rho_{ij} = 0$, the variance of Player $i$ is $1$, $\sigma_0 = 1$, $c(m) = 0.01m^2$ and allow the variance for player $j$ to take different values.

### 6.3 Unrestricted Team Size

In this section, we briefly summarize how the analysis changes when we allow for larger teams. More detailed explanations are given in Online Appendix B. The existence of a pure strategy Nash Equilibrium inside the team is guaranteed because the best response of any player is bounded and the modified game is a finite Potential Game. The characterization of equilibrium is complicated since there are more parameters to keep track of.\(^{24}\) Take the case of a group of 3 individuals. In this case, there are three pairwise correlations. If the three individuals take a positive number of draws, Lemma 4 in Online Appendix B implies that the individuals’ utility depends on the three correlations through the inverse of the correlation matrix. This inverse is a non-linear function.

When we allow for larger teams, a natural question that arises is whether the character of signals acquired by her teammate. Hence, pre-match investments may not be profitable. For example, suppose that Player 1 knows that in equilibrium she will match with Player 2, who has variance equal to unity. Suppose Player 1 has a variance of 0.7 and can pay some fee to reduce her variance to 0.6. Maintaining the previous cost parameters, the only equilibrium of the couple is $(2, 2)$ which gives Player 1 a payoff of $-0.2107$. After the investment, the only equilibrium is $(2, 1)$ which gives Player 1 a payoff of $-0.2275$.

\(^{24}\)Characterizing the equilibrium in a group with $n$ individuals requires to understand how $\frac{n(n-1)}{2}$ parameters interact with each other.
teristic function implied by the equilibrium of the game inside the team is super-additive. That is, by adding a member to a team can we always increase the information it generates? Lemma 5 in the Online Appendix shows that if we fix the number of draws taken by each individual, the implied characteristic function is super-additive. Therefore, if we fix the number of draws by each player and the membership cost is independent of team size, the only partition in the Core is the grand coalition. However, there are many correlation matrices for which little or no information is added by an extra player. There are two reasons for this. First, in equilibrium a new team member may decide to take zero draws and then all other players obtain the same utility as before. Second, the cross correlations could be such that the information added by a new individual is small. In particular, the information could be completely redundant. Since we assumed that becoming part of a larger group increases the membership fee, we conclude that large teams may not be formed.

Another interesting aspect is that it is neither easier or more difficult to find a partition in the Core when we allow for larger teams. In section B.2 we presented an example in which the core is empty when allowing only for teams of size two. When allowing for larger teams, the Core is non-empty. However, in Online Appendix B we present an example in which the Core is empty when teams are unrestricted in size and non-empty when teams are restricted to have at most two members. Finally, in section B.3 we present an example for which there is no CSPE when we allow for large teams. The intuition for the failure of Theorem 2 is that the game inside each team is no longer symmetric when teams have more than two members. Hence, off the path-of-play, it may not be possible to force a teammate to be a leader.

7 Discussion

We have introduced a two-stage game to capture how workers form teams to produce information. In the second stage, workers acquire signals simultaneously and non-cooperatively. In the first stage, given common knowledge of the entire signal structure, workers cooperatively form teams. Our main conclusion is that decentralized team
formation generates inefficiency due to two frictions: exclusion and free riding. The first refers to each worker’s incentive to form the best team she can without accounting for her affect on other workers. The second refers to the possibility of exerting different effort levels in different teams; even when a more productive team is available, a worker may prefer to form a less productive team where she can exert less effort than her partner. The presence of such inefficiencies is a plausible explanation for why firms intervene in R&D team formation. We conclude by describing two other environments to which our model may be applied.

Lazear (1998) argues that multinational firms benefit from the complementarities of its workers across countries. A firm capable of expanding across countries can take advantage of complementarities in knowledge by facilitating information exchanges. Our model illustrates that such interactions may be profitable, but may not endogenously occur. Hence, we suggest an informational rationale for the multinational corporation.

Krehbiel (1992) portrays the committee system in the House of Representatives as a specialization-of-labor arrangement with committee composition to be determined by informational objectives. Our model suggests that endogenous sorting into committees may be inefficient due to information exclusion and after-match free riding. In practice, senior party leaders allocate legislators to committees and both major parties are well-represented in each committee. We find these observations unsurprising in light of our analysis.

References


In contrast to existing approaches, we abstract from strategic information revelation and focus on a sorting problem with moral hazard. This may be particularly relevant as the workload of congresspeople has increased dramatically over time (Wawro and Schickler (2013)).


A Proofs

Proof Lemma 1

For any function $g : X \rightarrow \mathbb{R}$, where $X$ is the set of possible realizations of the signals, we have,

$$-\mathbb{E}[(g(x) - \theta)^2] \leq -\mathbb{E}[\mathbb{E}(\theta | x) - \theta]^2 = -\mathbb{E}[\mathbb{E}(\theta | x) - \theta]^2 | x] = -\text{Var}(\theta | x),$$

where the first equality follows from the Law of Iterated Expectations and the second one from the definition of conditional variance.

To solve for a closed form we calculate the posterior distribution. The ex-post distribution of $\theta$ given $x$ is proportional to,

$$f(x|\theta)f(\theta) \propto \exp \left( -\frac{1}{2} \left[ (\theta - \mu_\theta)^2 \sigma_\theta^{-2} + (\theta \cdot 1_N - x)' \sigma^{-2} \Sigma^{-1} (\theta \cdot 1_N - x) \right] \right)$$

$$\propto \exp \left( -\frac{1}{2} \left[ \theta^2 (\sigma_\theta^{-2} + \sigma^{-2} 1_N' \Sigma^{-1} 1_N) - \theta (2 \mu_\theta \sigma_\theta^{-2} + \sigma^{-2} (x' \Sigma^{-1} 1_N + 1_N' \Sigma^{-1} x)) \right] \right)$$

$$\propto \exp \left( -\frac{1}{2} \left[ [\theta - A]' B [\theta - A] \right] \right)$$

where $B = (\sigma_\theta^{-2} + \sigma^{-2} 1_N' \Sigma^{-1} 1_N)$ and $A = B^{-1}(\mu_\theta \sigma_\theta^{-2} + \sigma^{-2} 1_N' \Sigma^{-1} x)$. Then, $\text{Var}(\theta | x) = B^{-1}$, since the derived expression is the kernel of a normal distribution.

Now consider the case where there are only two players. Suppose player 1 is drawing $m$ times and player 2 is drawing $n$ times. Assume without loss of generality that $m \geq n$. The covariance matrix is a diagonal block matrix with $n$ blocks

$$\Sigma_0 = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$$

and $m - n$ blocks consisting of the scalar 1. By Lemma 1 we have $\text{Var}(\theta | x) = (\sigma_\theta^{-2} 1_N' \Sigma^{-1} 1_N + \sigma^{-2})^{-1}$. We know that the inverse of a block diagonal matrix is equal to the block diagonal matrix formed by each of the inverses of the blocks. Each of the nontrivial blocks has inverse,

$$\Sigma_0^{-1} = \frac{1}{1 - \rho^2} \begin{pmatrix} 1 & -\rho \\ -\rho & 1 \end{pmatrix}$$
After some algebra, \(1_{2}\Sigma^{-1}_{0}1_{2} = \frac{2}{1+\rho}\). Then \(1_{N}'\Sigma^{-1}1_{N}\) is the sum of \(n\) of these blocks and \(m-n\) times 1. That is,

\[
Var(\theta | x) = \left( \min\{n,m\} \frac{2}{1+\rho} + |m-n| \right) \sigma^{-2} + \sigma^{-2}_{\theta} \right)^{-1}
\]

Define \(f(\rho, m, n)\) to be this conditional variance.

**Proof Corollary 1**

We have for \(n_i > n_j\),

\[
\frac{\partial f(\rho, n_i-1, n_j)}{\rho} \propto \left( \left( \frac{1-\rho}{1+\rho} + n_i \right) \sigma^{-2} + \sigma^{-2}_{\theta} \right) + \left( \left( \frac{1-\rho}{1+\rho} + n_i - 1 \right) \sigma^{-2} + \sigma^{-2}_{\theta} \right)
\]

what is clearly positive.

**Proof Corollary 2**

We have that the marginal benefit for player 2 of taking the draw \(n+1\), given \(t\) draws by player 1, is given by,

\[
f(\rho, t, n) - f(\rho, t, n+1) = \frac{\left( \frac{1-\rho}{1+\rho} \right) \sigma^{-2}}{\left( \frac{n_{\Sigma}^2 + 2t - n}{\sigma_{\theta}^2 + \sigma_{\theta}^2} \right)} \left( \left( \frac{1-\rho}{1+\rho} + n \right) \sigma^{-2} + \sigma^{-2}_{\theta} \right) + \left( \left( \frac{1-\rho}{1+\rho} + n - 1 \right) \sigma^{-2} + \sigma^{-2}_{\theta} \right)
\]

Therefore,

\[
\frac{\partial f(\rho, t, n) - f(\rho, t, n+1)}{\partial \rho} \propto -\frac{2\sigma^{-2}}{(1+\rho)^2} \left( \frac{t(1+\rho)(n-1)+\rho}{1+\rho} \sigma^{-2} + \sigma^{-2}_{\theta} \right) + \frac{2(n+1)\sigma^{-2}}{(1+\rho)^2} \left( \frac{t(1+\rho)(n-1)+\rho}{1+\rho} \sigma^{-2} + \sigma^{-2}_{\theta} \right)
\]

\[
\propto -t^2 + n(n+1)\left( \frac{1-\rho}{1+\rho} \right)^2 - 2t\gamma - \gamma^2
\]

By recovering the proportionality constants we obtain,

\[
\frac{\partial f(\rho, t, n) - f(\rho, t, n+1)}{\partial \rho} = \frac{2\sigma^{-2} (-t + \gamma)^2 (1+\rho)^2 + n(n-1)(1-\rho)^2)}{(2n + (t-n+\gamma)(1+\rho))^2 (2(n+1) + (t-n-1+\gamma)(1+\rho))^2}
\]

From this expression we can find the second derivative,
\[
\frac{\partial^2 f(\rho,t,n)}{\partial \rho^2} - \frac{\partial^2 f(\rho,t,n+1)}{\partial \rho^2} \propto \left( - (t + \gamma)^2 (1 + \rho) - n(n - 1)(1 - \rho) \right) \cdot \left( 2n + (t - n + \gamma)(1 + \rho) \right) \cdot \left( 2(n + 1) + (t - n - 1 + \gamma)(1 + \rho) \right) \\
- \left[ (2(n + 1) + (t - n - 1 + \gamma)(1 + \rho))(t - n + \gamma) + (2n + (t - n + \gamma)(1 + \rho))(t - n - 1 + \gamma) \right] \cdot \left( (t + \gamma)^2 (1 + \rho)^2 + n(n - 1)(1 - \rho)^2 \right) \\
\propto 4n(n + 1) \left( - (t + \gamma)^2 (1 + \rho) - n(n - 1)(1 - \rho) \right) \\
+n(n + 1) \left[ 2(n - n - 1 + \gamma) + 2(n + 1)(t - n + \gamma) + (t - n - 1 + \gamma)(t - n + \gamma) \right] (2\rho - 2) < 0
\]

Then the marginal value \( f(\rho,t,n) - f(\rho,t,n+1) \) is strictly concave.

The unique maximizer \( \hat{\rho}(t,n,\gamma) \) has to satisfy,

\[
(t + \gamma)^2 (1 + \hat{\rho}(t,n,\gamma))^2 = n(n + 1)(1 - \hat{\rho}(t,n,\gamma))^2
\]

a quadratic equation with respect to \( \rho \). The solutions to this equation are given by

\[
\rho = \frac{\left( t + \gamma \pm \sqrt{n(n + 1)} \right)^2}{-(t + \gamma)^2 + n(n + 1)}
\]

Notice that the denominator is negative. The solution with the sum is less than \(-1\). The solution with the minus corresponds to a negative number less than \(-1\), since \( n + 1 \leq t \). Define \( \tilde{\rho}(t,n,\gamma) \) to be the solution with minus.

**Proof Corollary**

The following claim implies the first part of the result.

**Claim 6** There is a unique \( \hat{\rho}(t,\gamma) < 0 \) such that

\[
f(\hat{\rho}(t,\gamma),t-1,t) - f(\hat{\rho}(t,\gamma),t,t) = f(\hat{\rho}(t,\gamma),t-1,t-1) - f(\hat{\rho}(t,\gamma),t-1,t)
\]

with

\[
\hat{\rho}(t,\gamma) = \frac{- (\gamma - 1 + 2t) + \sqrt{(\gamma - 1 + 2t)^2 - 4\gamma}}{2\gamma}
\]

**Proof** We have,

\[
f(\rho,t-1,t) - f(\rho,t,t) \geq f(\rho,t-1,t-1) - f(\rho,t-1,t)
\]

\[
\Leftrightarrow \left( 1 \right) \left( \frac{1}{(t-1)\frac{2}{1+\rho} + 1} \right) \sigma^{-2} + \frac{\rho^{-2}}{1+\rho} \leq \left( \frac{1}{(t-1)\frac{2}{1+\rho} + 1} \right) \sigma^{-2} + \frac{\rho^{-2}}{1+\rho} \leq \left( \frac{1}{(t-1)\frac{2}{1+\rho} + 1} \right) \sigma^{-2} + \frac{\rho^{-2}}{1+\rho}
\]
\[ \frac{1 - \rho}{1 + \rho} \left( (t - 1) \frac{2}{1 + \rho} \sigma^{-2} + \sigma^{-2} \right) \geq \left( t \frac{2}{1 + \rho} \sigma^{-2} + \sigma^{-2} \right) \]

\[ 0 \geq \gamma \rho^2 + (\gamma - 1 + 2t) \rho + 1 \]

Notice that the final expression is a quadratic concave function in \( \rho \). Solving we get

\[ \rho(t) = \frac{-(\gamma - 1 + 2t) \pm \sqrt{(\gamma - 1 + 2t)^2 - 4\gamma}}{2\gamma} \]

Call \( \rho^{-}(t) \) the negative root and \( \rho^{+} \) the positive root. We show now that \( \rho^{-}(t) < -1 \) and \( \rho^{+}(t) \in [-1, 0) \), so \( \hat{\rho}(t, \gamma) = \rho^{+}(t) \) as stated in the proposition.

First, we have,

\[ \rho^{-}(t) - \rho^{-}(t + 1) = \frac{2 + \sqrt{(\gamma - 1 + 2(t + 1))^2 - 4\gamma}}{2\gamma} - \frac{\sqrt{\gamma - 1 + 2(t + 1)^2 - 4\gamma}}{2\gamma} \]

Now, \( (\gamma - 1 + 2t)^2 - 4\gamma > 0 \) since it is increasing in \( t \) and when evaluated at \( t = 1 \) we get \( (\gamma - 1)^2 > 0 \). This means that the expression above is positive. Therefore, if \( \rho^{-}(1) \leq -1 \) it is true that for all \( t \geq 1 \), \( \rho^{-}(t) < -1 \).

Finally,

\[ \rho^{-}(1) = \frac{-(\gamma + 1) - \sqrt{(\gamma - 1)^2}}{2\gamma} = \begin{cases} \frac{-2\gamma}{2\gamma} = -1 & \text{if } \gamma \geq 1 \\ \frac{-\gamma}{2\gamma} < -1 & \text{if } \gamma < 1 \end{cases} \]

Second, it is clear that \( \rho^{+}(t) < 0 \) since \( \sqrt{(\gamma - 1 + 2t)^2} > \sqrt{(\gamma - 1 + 2t)^2 - 4\gamma} \). Besides we have,

\[ \rho^{+}(t + 1) - \rho^{+}(t) = \frac{-2 + \sqrt{(\gamma - 1 + 2(t + 1))^2 - 4\gamma} - \sqrt{(\gamma - 1 + 2t)^2 - 4\gamma}}{2\gamma} \]

that is positive since,

\[ -2 + \sqrt{(\gamma - 1 + 2(t + 1))^2 - 4\gamma} > \sqrt{(\gamma - 1 + 2t)^2 - 4\gamma} \]

\[ \Leftrightarrow (\gamma + 1 + 2t)^2 > (\gamma - 1 + 2(t + 1))^2 - 4\gamma \Leftrightarrow 4\gamma > 0 \]

This means that \( \rho^{+}(t) \) is increasing in \( t \) and we have,

\[ \rho^{+}(1) = \frac{-(\gamma + 1) + \sqrt{(\gamma - 1)^2}}{2\gamma} = \begin{cases} \frac{-\gamma}{2\gamma} > -1 & \text{if } \gamma \geq 1 \\ \frac{-\gamma}{2\gamma} = -1 & \text{if } \gamma < 1 \end{cases} \]
We conclude that $\rho^+(t) \in [-1, 0)$ and call $\hat{\rho}(t, \gamma) = \rho^+(t)$.

The next claim proves that for and any $t \geq 2$ there is $\gamma^*(t) \in [\frac{1}{2}, 1)$ such that for $\gamma \geq \gamma^*(t)$ we have $\hat{\rho}(t, t - 1, \gamma) \leq \hat{\rho}(t, \gamma)$.

**Claim 7** For $t \geq 2$, there exists $\gamma^*(t) \in [\frac{1}{2}, 1)$ such that $\hat{\rho}(t, t - 1, \gamma) \leq \hat{\rho}(t, \gamma)$ if and only if $\gamma \geq \gamma^*(t)$.

**Proof** Let $g(t, \gamma) = \hat{\rho}(t, t - 1, \gamma) - \hat{\rho}(t, \gamma)$. This function has a unique zero in $[\frac{1}{2}, 1)$. First,

$$g\left(t, \frac{1}{2}\right) = \frac{-8t^2 - 1 + 8t + \frac{1}{2}}{8t + 1} \sqrt{t(t - 1)} - \left(\frac{1}{2} - 2t + \sqrt{4t^2 - 2t - \frac{7}{4}}\right)$$

Therefore, $g\left(t, \frac{1}{2}\right) > 0$ if and only if

$$8t^2 - 2t - \frac{3}{2} + 4(2t + 1) \sqrt{t(t - 1)} > (8t + 1) \sqrt{4t^2 - 2t - \frac{7}{4}}$$

$$\Leftrightarrow 64t^3 - 48t^2 - 15t - 1 > 0$$

what is clearly true for any $t \geq 2$. Besides, we have that,

$$g(t, 1) = \frac{-(t + 1)(t - 1) - 2(t + 1) \sqrt{t(t - 1)} + (3t + 1) \sqrt{(t + 1)(t - 1)}}{-(3t + 1)} < 0$$

since $(t + 1)(t - 1 + 2 \sqrt{t(t - 1)}) < (3t + 1) \sqrt{(t + 1)(t - 1)}$ for $t \geq 2$. By the intermediate value theorem there is $\gamma^* \in [\frac{1}{2}, 1)$ such that $g(t, \gamma^*) = 0$.

Now we prove that $g(\cdot, \gamma)$ is strictly decreasing with respect to $\gamma$. First, $\hat{\rho}(t, \gamma)$ is strictly increasing in $\gamma$. We have,

$$\frac{\partial \hat{\rho}(t, \gamma)}{\partial \gamma} = \frac{(-1 + (\gamma - 1 + 2t)^2 - 4\gamma)^{-0.5} (\gamma - 3 + 2t) 2\gamma - 2 (-3 - 2t + \sqrt{(\gamma - 1 + 2t)^2 - 4\gamma})}{4\gamma^2}$$

$$\propto 3\gamma - 2t\gamma - 1 + 4t - 4t^2 + (2t - 1) \sqrt{(\gamma - 1 + 2t)^2 - 4\gamma}$$

Therefore $\frac{\partial \hat{\rho}(t, \gamma)}{\partial \gamma} > 0$ if and only if

$$(2t - 1) \sqrt{(\gamma - 1 + 2t)^2 - 4\gamma} > \gamma(2t - 3) + (2t - 1)^2$$

$$\Leftrightarrow \gamma^2(2t - 1)^2 > \gamma^2(2t - 3)^2$$
Therefore \( \frac{\partial \hat{p}(t, \gamma)}{\partial \gamma} > 0 \). Finally, \( \hat{p}(t, t-1, \gamma) \) is decreasing with respect to \( \gamma \) since,

\[
\frac{\partial \hat{p}(t, t-1, \gamma)}{\partial \gamma} = \frac{(2(t+\gamma)-2\sqrt{t(t-1)}((t+\gamma)^2+t(t-1))+2(t+\gamma)((t+\gamma)^2+t(t-1)-2(t+\gamma)\sqrt{t(t-1)})}{(-t+\gamma)^2+t(t-1))^2} \\
= \frac{4(t+\gamma)((t(t-1)-(t+\gamma)\sqrt{t(t-1)})-2\sqrt{t(t-1))^2}4(t+\gamma)^2 \sqrt{t(t-1)}}{(-t+\gamma)^2+t(t-1))^2} < 0
\]

since \( \gamma > 0 \) and \( t-1 < \sqrt{t(t-1)} \). This proves that \( g(\cdot, \gamma) \) is strictly decreasing with respect to \( \gamma \). Therefore, \( g(t, \gamma) = 0 \) only when \( \gamma = \gamma^* \). ■

**Proof Lemma 2**

We first show that if \( n_i \neq n_j \), the marginal value of an additional draw for \( i \) strictly decreases in \( n_i \),

\[ MV(n_i + 1; n_j, \rho) - MV(n_i; n_j, \rho) < 0. \]

For \( n_i \leq n_j \) we have

\[
f(\rho, n_i-1, n_j) - f(\rho, n_i, n_j) = \frac{1}{(n_i-1)\frac{2}{1+\rho}+(n_j-n_i+1)}\sigma^2+\sigma_\theta^2 - \frac{1}{n_i\frac{2}{1+\rho}+(n_j-n_i)}\sigma^2+\sigma_\theta^2 \\
= \frac{\left(\frac{1-\rho}{1+\rho}\right)\sigma^2}{\left(n_i\frac{2}{1+\rho}+n_i+1-\frac{2}{1+\rho}\right)\sigma^2+\sigma_\theta^2} \left(\frac{\left(\frac{1-\rho}{1+\rho}\right)^2}{\left(n_j\frac{2}{1+\rho}+n_j\right)\sigma^2+\sigma_\theta^2}\right)
\]

that is strictly decreasing in \( n_j \) and in \( n_i \) since \( \frac{1-\rho}{1+\rho} \geq 0 \) with equality only if \( \rho = 1 \).

Next, we show that If \( n_i \neq n_j + 1 \), the marginal value of an additional draw by \( i \) strictly decreases in \( n_j \),

\[ MV(n_i; n_j+1, \rho) - MV(n_i; n_j, \rho) < 0. \]

For \( n_i \geq n_j + 1 \) we have,

\[
f(\rho, n_i-1, n_j) - f(\rho, n_i, n_j) = \frac{1}{(n_i-1)\frac{2}{1+\rho}+(n_j-1-n_i)}\sigma^2+\sigma_\theta^2 - \frac{1}{n_i\frac{2}{1+\rho}+(n_j-n_i)}\sigma^2+\sigma_\theta^2 \\
= \frac{\sigma^2}{\left(n_i\frac{2}{1+\rho}+n_i-1\right)\sigma^2+\sigma_\theta^2} \left(\frac{\left(\frac{1-\rho}{1+\rho}\right)^2}{\left(n_j\frac{2}{1+\rho}+n_j\right)\sigma^2+\sigma_\theta^2}\right)
\]

that is clearly strictly decreasing in \( n_i \) and in \( n_j \) since \( \frac{1-\rho}{1+\rho} \geq 0 \), with equality only if \( \rho = 1 \).

The preceding observations imply that the best response by player \( j \) is decreasing in
$n_i$, since the marginal value is strictly decreasing. We only need to prove is that the best response by player $j$ to 0 draws by player $i$ is finite.

Since the marginal value of an extra draw is decreasing in $n_i$, to prove that the best response when player $i$ takes 0 draws is finite we only need that there is $n \in \mathbb{Z}_+$ such that $f(\rho, n - 1, 0) - f(\rho, n, 0)$ is smaller than $d$. But we have,

$$f(\rho, n - 1, 0) - f(\rho, n, 0) = \frac{1}{(n-1)\sigma^2+\sigma^2_0} - \frac{1}{n\sigma^2+\sigma^2_0}$$

$$< \frac{\sigma^2}{n(n-1)}$$

Therefore, for $n > \frac{\sigma^2}{d} + 1$ we get the required inequality. Call $\bar{M}$ the smallest value that satisfies this inequality.

**Proof Proposition 5**

Suppose there are multiple pure strategy equilibrium and all of them are asymmetric. Let $(m^*_i, m^*_j)$ the preferred equilibrium by player $i$. Take a mixed strategy equilibrium $(\sigma_i, \sigma_j)$ and suppose that is a Pareto improvement over $(m^*_i, m^*_j)$. Then there has to be a strategy $(i, j)$ such that $u_i(i, j) \geq u_i(m^*_i, m^*_j)$ and $u_j(i, j) > u_j(m^*_i, m^*_j)$ (or $u_j(i, j) \geq u_j(m^*_i, m^*_j)$ and $u_i(i, j) > u_i(m^*_i, m^*_j)$), and $\sigma(i) > 0, \sigma_j > 0$. Since the game is symmetric then $u_i(m^*_i, m^*_j) \geq u_i(i, m^*_j)$ and $u_i(m^*_j, m^*_i) \geq u_i(m^*_j, j)$ for all $i$ and $j$. Therefore, $i \neq m^*_i$ and $j \neq m^*_j$.

There are two cases:

- $i > m^*_i$: If $j < m^*_j$ the inequality $u_i(i, j) \geq u_i(s^*_i, s^*_j)$ can not hold. Then $j > m^*_j$. If $i < j$ this can not happen because, by the prove of Lemma 2, the marginal value of a draw by player $j$ decreases in the number of draws by player $i$, contradicting $u_j(i, j) > u_j(m^*_i, m^*_j)$. A similar argument holds if $i \geq j$.

- $i < m^*_i$: If $j > m^*_j$ the inequality $u_j(i, j) > u_j(m^*_i, m^*_j)$ can not hold. Then $j < m^*_j$. If $i < j$ this can not happen because, by the prove of Lemma 2, the marginal value of a draw by player $i$ decreases in the number of draws by player $j$, contradicting $u_i(i, j) \geq u_i(m^*_i, m^*_j)$. A similar argument holds if $i \geq j$. 

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Since none of the cases is possible, we conclude that the mixed strategy equilibrium $(\sigma_i, \sigma_j)$ is not a Pareto improvement over $(m_i^*, m_j^*)$.

**Proof Proposition 2**

By assumption we know that for $\rho = -1$ and correlations close to it, both players are taking at least one draw.

A sufficient condition for a symmetric equilibrium is:

$$f(\rho, n - 1, n) - f(\rho, n, n) \geq f(\rho, n - 1, n - 1) - f(\rho, n - 1, n)$$

From Corollary we know that for correlations smaller than $\hat{\rho}(t, \gamma)$ the condition is satisfied. Besides, from the proof from the same lemma we know that $\hat{\rho}(t, \gamma)$ is increasing in $t$ and it is increasing in $\gamma$. We have,

$$\lim_{\gamma \to 0} \hat{\rho}(2, \gamma) = \lim_{\gamma \to 0} -\frac{(\gamma+3) + \sqrt{(\gamma+3)^2 - 4\gamma}}{2\gamma}$$

\[ \text{L'Hôpital} \]

\[ \lim_{\gamma \to 0} -\frac{1 + \frac{1}{2}(\sqrt{(\gamma+3)^2 - 4\gamma})^{-1}(2(\gamma+3)-4)}{2} = -\frac{1}{3} \]

Now, we know that as $\rho \to 1$ the marginal benefit of completing the first draw by the follower, $f(\rho, 0, 1) - f(\rho, 1, 1) \to 0$ and the marginal value is continuous. Then by continuity and monotonicity with respect to $\rho$ there exists a unique $\rho^{**} < 1$ such that $f(\rho^{**}, 0, 1) - f(\rho^{**}, 1, 1) = d$. Since by the proof of Lemma 2, the marginal value is decreasing with respect to the number of draws by the leader, for any correlation higher than $\rho^{**}$ the follower does not want to take even the first draw.

**Proof Proposition 3**

Suppose that for correlation $\rho$ any equilibrium $(m_1, m_2)$ is asymmetric, that is, $m_1 > m_2$. This implies that $\rho > \hat{\rho}(m_1, \gamma)$, since the marginal value of the leader is decreasing with respect to $\rho$.

We need to argue that for correlation $\rho' > \rho$ there is not a symmetric equilibrium. As we want to prove that there is not a symmetric equilibrium we can use the Sequential Research Response Algorithm. Notice that by Corollary 2 and Corollary 3 at any iteration $s$, we can conclude that $m_s^2(\rho') \leq m_s^2(\rho)$ since the marginal value of a draw by the follower
is always smaller when facing correlation $\rho'$. To complete the argument, Corollary 1 implies that when the process reaches iteration $s$, if player 1 decides to take draw $s$ when the correlation is $\rho$ he would take it when the correlation is $\rho'$, as well. This implies that the procedure will continue for more iterations for higher correlation and the equilibrium is going to be more asymmetric for $\rho'$ than for $\rho$.

**Proof of Theorem 1**

Recall, $k(2) > k(1)$ since forming a team of size 2 is costlier than being alone. Hence, no team will form if its members have a pairwise correlation in $P_3$. If they did, the leader of the team would have a profitable deviation to form a singleton as she would obtain the same information at the same cost without paying $k(2)$.

The set $P_1$ satisfies the top coalition property (see Banerjee, Konishi, and Sönmez (2001)) when teams have at most two members. Hence, if all members have pairwise correlations in $P_1$ there is a Core partition. The reasoning mirrors the proof of the existence of a Subgame Stable Partition. As no couple with a pairwise correlation in $P_3$ will form a team, the result follows if all correlations are in $P_1 \cup P_3$.

Now, suppose the second condition holds. Form the team with pairwise correlation in $P_2$ that gives each member the highest utility. By definition no teammate wants to form a deviating team in which members have a pairwise correlation outside $P_2$. As the team has the highest pairwise utility in $P_2$ no teammate can do better. We may inductively construct teams with pairwise correlations in $P_2$ in this manner.

After we have finished creating all teams with pairwise correlations in $P_2$, set aside these individuals. We obtain a new correlation matrix in which some individuals may have multiple pairwise correlations in $P_2^c \setminus P_3$. Pick any such individual and allow her to choose a teammate from the subset of her possible partners that prefer being the leader in the group with correlation in $P_2^c \setminus P_3$ than being alone. Play an equilibrium in which the player with multiple pairwise correlations in $P_2^c \setminus P_3$ is the follower. As she chooses this teammate, she does not have an incentive to deviate and the player she picks does not have an incentive to deviate either since all her other pairwise correlations are in $P_3$; a team with a pairwise correlation in $P_3$ cannot be self-enforced as argued previously.
Iterate until all individuals with multiple pairwise correlations in $\rho_{ij} \in P^2 \setminus P^3$ have been dealt with. There may be an even number of remaining individuals with one correlation in $\rho_{ij} \in P^2 \setminus P^3$. Form these couples if they prefer this than being alone; no member wants to deviate since their outside option is to be alone.

**Proof Theorem 2**

First, note that only $\binom{N}{2}$ couples can be formed where $N$ is the number of individuals in the population. For each couple, each equilibrium played inside the couple yields a utility for each teammate. Order every such equilibrium and couple by the utility accruing to the leader. Choose the couple, say $(i,j)$, and equilibrium such that the leader, say $i$, obtains the highest utility compared to any leader in any couple playing any equilibrium. If there is more than one such couple, equilibrium, leader combination, choose one arbitrarily.

Fix both players to be the leaders in any other group. If both players $i$ and $j$ know that in any other couple they will be the leader neither will want to deviate; individual $i$ knows that in any other couple the leader gets at most what she is getting now. As both $i$ and $j$ acquire the same information, but $i$ is taking more draws we know that in $(i,j)$ the utility of $j$ is larger than the utility of $i$. Hence, player $j$ knows that she is getting even more than $i$.

Set $i$ and $j$ aside and repeat the process with the individuals that are left (i.e. in $N \setminus \{i,j\}$). The only difference is that the individuals picked in the second round will be the leaders in any couple they can form not including $i$ or $j$. By induction, we will find a partition and strategy profile comprising a Coalitional Subgame Perfect Equilibrium.

**Proof Theorem 3**

By Theorem 2 there exists at least one Coalitional Subgame Perfect Equilibrium and since the number of possible partitions and equilibria for each team are finite, there is only a finite number of Coalitional Subgame Perfect equilibria.

Find a Coalitional Subgame Perfect Equilibrium $(\Pi, \{m^*(S)\}_S)$. If it is Pareto Optimal we are done. Suppose it is not. Then there is another feasible partition $\hat{\Pi} = \{T_1, \ldots, T_m\}$
and on-path equilibria, \(m(T)_{T \in \bar{\Pi}}\) such that for each individual,

\[ v_i(m(T_{\bar{\Pi}(i)})) - K \ast \mathbf{I}^{T_{\bar{\Pi}(i)}} \geq v_i(m^*(S_{\bar{\Pi}(i)})) - K \ast \mathbf{I}^{S_{\bar{\Pi}(i)}} \]

and the inequality is strict for at least one of them. Consider the profile \((\bar{\Pi}, \{m(S)\}_S)\)
where \(m(S) = m^*(S)\) if \(S \notin \bar{\Pi}\) and \(m(S) = \hat{m}(S)\) if \(S \in \bar{\Pi}\). This profile is a Coalitional Subgame Perfect Equilibrium, since it gives higher payoff than the original profile and the possible deviations give to all individuals the same payoff.

If the new Equilibrium is Pareto Optimal then we are done. If not repeat the process. As there is a finite number Coalitional Subgame Perfect Equilibria and in every step we are weakly increasing the utility for all individuals and strictly for at least one of them, the process stops in finite time.

**Proof Theorem 4**

Let \((m, n)\) be the equilibrium in groups \((i, j)\) and \((i', j')\). WLOG assume \(m \geq n\). By Corollary 2 the marginal value, given \(m\), of a draw by the follower is strictly concave with respect to the correlation, so in groups \((i, j')\) and \((i', j)\) the marginal value of the follower has to be larger than the marginal value of the follower in at least one of the other two groups. At the same time, Corollary 1 implies that, given \(n\), the marginal value of a draw by the leader is increasing in \(\rho\).

Then if \(m = n\) the only equilibrium in groups \((i, j')\) and \((i', j)\) is \((m, n)\). If \(m > n\), the most symmetric equilibrium in groups \((i, j')\) and \((i', j)\) has to be weakly more symmetric. By Lemma 3, the only way to get a most symmetric equilibrium, in say group \((i, j')\), is if \(\hat{\rho}(n + 1) < \rho_{ij'} < \hat{\rho}(n + 1, n) < 0\). So if such more symmetric equilibrium exists, it has to be that \(m - n = 1\), that is the new equilibrium is \((m, m)\). This equilibrium produces more information than \((m, m - 1)\) and higher utility since player 2 optimally wants to take the extra draw and player 1 get the rewards of extra information at no cost.

Then the strategy \((m, n)\) give us a lower bound in the utility and information that would be generated in groups \((i, j')\) and \((i', j)\). Given this strategy profile, each player’s
utility is decreasing in the pairwise correlation $\rho$,

$$\frac{\partial v_i(m,n,\rho)}{\rho} = \frac{-2m\sigma^{-2}}{(2m + (m-n)(1+\rho))\sigma^{-2} + (1+\rho)\sigma_{\theta}^{-2}} < 0.$$ 

Therefore, when both correlations $\rho_{ij}'$ and $\rho_{i'j}$ are small enough the sum of utilities has to be larger than the one given by the Subgame Stable partition. The order in total information and utilities is the same since an equilibrium in which individuals take weakly more draws is played.

**Proof Theorem 5**

By the definition of $P_1$ and $P_2$, for $\rho_{ij}',\rho_{i'j}$ in $[\rho^*,\rho^{**}]$ close to $\rho^*$ the utility of the follower in the most asymmetric equilibrium for any of the two correlations has to be larger than the utility obtained in an open set contained in $P_1 \setminus P_2$.

Let $\delta_{ij}'$ and $\delta_{i'j}$ be the difference in the utilities, under the most asymmetric equilibrium, between the follower and the leader in groups $(ij')$ and $(i'j)$, respectively.

Pick $\rho_{ij} \in P_1 \setminus P_2$ such that the utility of the players (the only equilibrium is symmetric) is close to the maximum of utility a follower could get in either groups $(ij')$ and $(i'j)$. In particular, pick it such that the difference is less than $\frac{\min\{\delta_{ij}',\delta_{i'j}\}}{\alpha}$, for a constant $\alpha > 2$. Then the utility of the group $(i,j)$ is strictly larger than the one in either of the groups $(ij')$ and $(i'j)$. By picking $\rho_{i'j}'$ smaller but close to $\min\{\rho_{ij}',\rho_{i'j}\}$ we make sure that we do not lose all the gains of re-allocating the players. Notice that $\rho_{i'j}'$ can be strictly smaller than $\min\{\rho_{ij}',\rho_{i'j}\}$, except for the case where there is a discontinuity in the CPN equilibrium correspondence at $\min\{\rho_{ij}',\rho_{i'j}\}$.

We need to argue that we have not broken the equilibrium by picking the correlations the way we did. First, players $i$ and $j$ do not want to form a group together if they are the followers in groups $(i,j')$ and $(j,i')$, respectively. Second, either $j'$ or $i'$ has to be the leader in the group $(i',j')$. Since $\rho_{i'j}' \geq \min\{\rho_{ij}',\rho_{i'j}\}$ and in all of these correlations the same equilibrium exists, at least one of them would be worse off by deviating to the team $(i',j')$. 
