Markups: A Search-Theoretic Perspective*

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Abstract

I derive a formula for the equilibrium distribution of markups in the search-theoretic model of imperfect competition of Butters (1977), Varian (1980), and Burdett and Judd (1983). The level of markups and the sign of the relationship between a seller's markup and its size depend on the extent of search frictions, as well as on other deep parameters. Markups are efficient. Markups are positive even though the varieties produced by sellers are perfect substitutes. Markups are heterogeneous even when all sellers operate the same production technology. Markups depend on size, even though the substitutability between a variety and the others does not depend on how much of that variety is consumed. Interpreting these markups through the lens of the monopolistic competition model of Dixit and Stiglitz (1977) would lead one to recover incorrect and unstable buyers' preferences. Interpreting these markups through the lens of the Dixit-Stiglitz model would also lead to incorrect policy recommendations. These results are a cautionary note on recent work in macroeconomics.

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1 Introduction

Across macroeconomics, the monopolistic competition framework of Dixit and Stiglitz (1977) has become the standard approach to modelling market power. The framework is applied to such disparate topics as the study of monetary policy (see, e.g., Blanchard and Kyiotaki 1985, Dotsey, King and Wolman 1999, Christiano, Eichenbaum and Evans 2005, Golosov and Lucas 2007), the cost of misallocation and the determination of aggregate TFP (see, e.g., Hsieh and Klenow 2009), the effect of trade liberalization (see, e.g., Krugman 1979, Krugman 1980, Melitz 2003, Edmonds, Midrigan and Xu 2015). In the Dixit-Stiglitz framework, every seller is a monopolist of its own product variety, and buyers perceive the varieties carried by different sellers as imperfect substitutes. The extent of sellers' market power and the size of markups is determined by the substitutability of different varieties in the buyers' utility function.[1]

An alternative approach to modelling market power is the imperfect competition framework of Butters (1977), Varian (1980) and Burdett and Judd (1983).[2] In this framework, a seller has market power not because it carries a good that has no perfect substitutes, but because (some) buyers do not have every seller in their choice set due to informational frictions (i.e., buyers are not aware of all the sellers in the market) or physical frictions (i.e., buyers cannot purchase from some of the sellers because trading costs are too high). The extent of sellers' market power and the size of markups is determined by the distribution of the size of buyers' choice sets. The search-theoretic framework of imperfect competition has been traditionally used to study price dispersion. More recently, it has more used to analyze price stickiness (Head et al. 2012, Burdett and Menzio 2017, 2018, Wang, Wright and Liu 2020), differences between expenditures and consumption (Pytka 2018 and Nord 2023), markups in retail markets (Sangani 2023), endogenous product differentiation and growth (Menzio 2023), and business cycle fluctuations (Kaplan and Menzio 2016).

In some markets, such as the retail market, the assumptions of the Dixit-Stiglitz framework strain credulity. Consider a 36-ounce plastic bottle of Heinz ketchup—an

[1] Whenever I mention the Dixit-Stiglitz framework of monopolistic competition, I mean any model in which: (a) each seller is a monopolist of the good that it produces; (b) each seller chooses the price of its good; and (c) each seller is small, in the sense that its price has no impact on equilibrium. The definition includes models in which buyers have preferences with a constant elasticity of substitution (CES) across goods (such as Krugman 1980 or Melitz 2003), models in which buyers have preferences with a variable elasticity of substitution (VES) across goods (such as Krugman 1979 or Dhingra and Morrow 2019), translog preferences (such as Feenstra 2003), Kimball preferences (such as Kimball 1995 or Edmond, Midrigan and Xu 2023). In some of these models markups are variable—in the sense that a firm's markup depends on its size. Even in these models, a firm's markup is still entirely determined by preferences, since the size of a firm simply affects the point where the buyers' utility function is evaluated and, in turn, the elasticity of demand facing the firm.

[2] The popularity of the Dixit-Stiglitz framework is a relatively recent phenomenon. In the Industrial Organization textbook by Tirole (1988), the chapter on product differentiation contains an exposition of the model by Butters (1977) and of a spin-off by Grossman and Shapiro (1984). The Dixit-Stiglitz framework is relegated to a supplementary section.

example taken from Kaplan and Menzio (2015). The bottle is sold by a large number of retailers in a given geographical area, and each retailer charges a substantial markup over the wholesale cost. It is hard to believe that the retailers can charge large markups because buyers perceive the bottle of ketchup at one store as a poor substitute of the very same bottle of ketchup at any other store. It seems more natural to think that the retailers can charge markups because some buyers cannot purchase the bottle of ketchup from the store with the lowest price. Some skepticism about the relevance of the Dixit-Stiglitz framework is also warranted in non-retail markets. Does a consulting company charge markups to its clients because there is no other company that can provide them with comparable services? Or could it be that that the consulting company can charge markups because some of its clients have limited access to its competitors? Having said that, the origin of market power would largely be a matter of semantics if the two theories were observationally equivalent and had the same welfare implications.

In this paper, I characterize equilibrium markups in the search-theoretic model of imperfect competition of Butters (1977), Varian (1980) and Burdett and Judd (1983). I characterize the distribution of markups, the relationship between a seller's markup and its size, and the effect of structural parameters on the level and shape of markups. I ask whether and how the Dixit-Stiglitz model could rationalize the equilibrium markups generated by the search-theoretic model. I then ask whether, by interpreting the markups from the search-theoretic model through the lens of the Dixit-Stiglitz model, one would reach the correct conclusions about efficiency and optimal policy, and whether one would make the right counterfactual predictions. It turns out that, while the Dixit-Stiglitz model can reproduce the markups generated by the search-theoretic model, it would lead to incorrect conclusions about efficiency, policy, and counterfactuals. Therefore, the theory of market power does, in principle, matter. Moreover, since the search-theoretic model can rationalize any empirical pattern of markups given the appropriate choice of parameters, it follows that markup data alone is not sufficient to make claims about welfare, policy, and counterfactuals. One also needs evidence on the origin of market power. The theory of market power does also matter in practice.[3]

In the first part of the paper, I consider a version of the search-theoretic model of imperfect competition in which sellers operate the same technology and produce varieties of the good that are perfect substitutes. I derive a formula for the markup of a seller as a function of its rank (quantile) in the equilibrium price distribution. The formula reveals that markups are the product of two terms. The first term is the monopoly markup, which depends on the buyer's valuation of the good and on the seller's marginal cost. The second term is a discount factor that depends on the ranking of the seller in the price distribution, and on the extent of competition in the market—as measured by the average

[3]For some issues, it has already been established that the search-theoretic model of imperfect competition leads to different policy implications based on the same observables. Indeed, Head et al. (2012) show that, in a search-theoretic model, nominal price stickiness do not necessarily imply that monetary shocks have an effect on real outcomes.

size of the buyers' choice sets. The discount factor is 1 (no discounting) for the seller at the top of the price distribution. The discount factor declines as we move from the top to the bottom of the price distribution, and it does so at a speed that depends on the extent of competition in the market—measured as the average size of the buyers' choice set. The markup formula implies that the markup of a seller is increasing in its price and decreasing in its size.

The Dixit-Stiglitz model of monopolistic competition can reproduce the same markups, but through a different channel and with different welfare and policy implications. Since markups are positive, the Dixit-Stiglitz model requires buyers to perceive the varieties of the good carried by different sellers as imperfect substitutes. Since markups are decreasing in a seller's size, the Dixit-Stiglitz model requires buyers to perceive the variety of a particular seller to be a closer substitute to other varieties the more of that variety they consume. Since markups are endogenous, the Dixit-Stiglitz model requires the buyers' preferences to change whenever the environment changes. Therefore, any counterfactual exercise carried out while keeping the buyers' preferences unchanged would be invalid.

When interpreted through the lens of the Dixit-Stiglitz model, these markups are symptomatic of inefficiencies. The fact that markups are positive implies that sellers produce an inefficiently low quantity of the good (see, e.g., Edmond, Midrigan and Xu 2023). The fact that markups are heterogeneous implies that high-markup sellers produce too little compared to low-markup sellers (see, e.g., Dhingra and Morrow 2019 or Edmond, Midrigan and Xu 2023). Hence, when markups are interpreted through the lens of the Dixit-Stiglitz model, one would conclude that the government should introduce subsidies to increase consumption and production at all sellers, and design the subsidies so as to reallocate production from low to high-markup sellers. None of these welfare and policy implications are, however, correct, since the equilibrium of the search-theoretic model of imperfect competition is efficient.

If the search-theoretic model of imperfect competition could only generate markups that are decreasing in size, the findings above would be of little practical interest, since, empirically, larger firms tend to charge higher markups. For this reason, in the second part of the paper, I consider a version of the search-theoretic model in which sellers produce varieties of the good that are perfect substitutes, but are heterogeneous with respect to their marginal cost. I derive a formula for the markups of a seller as a function of its ranking in the equilibrium price distribution, which happens to be the same as its ranking in the marginal cost distribution. The formula for the markup of a seller contains an additional term in the version of the model with heterogeneous sellers. The additional term captures the fact that the firms ranked above the seller in the price distribution operate a less efficient technology and, for this reason, put less competitive pressure on the seller. The additional term is a weighted sum of the ratio between the marginal cost of higher-ranked firms and the marginal cost of the seller. The weights are largest for firms that are ranked just above the seller and they progressively become smaller for firms that are ranked further way from the seller.

In contrast to the version of the model with homogeneous sellers, equilibrium markups need not be decreasing in size. Markups can be decreasing, constant, or increasing in a seller's size. Indeed, I prove that any twice-differentiable markup function can be generated as an equilibrium of the search-theoretic model of imperfect competition given appropriate parameter choices. Therefore, the search-theoretic model of market power can rationalize any pattern of markups observed in the data. As in the case of homogeneous sellers, the Dixit-Stiglitz model can reproduce the same markups as in the search-theoretic model, but it would do so with reduced-form preferences that depend on deep parameters and, hence, are unstable. As in the case of homogeneous sellers, the Dixit-Stiglitz model would imply inefficiencies in the overall level of production and in the allocation of inputs across different sellers. Yet, the equilibrium of the search-theoretic model is efficient. These observations imply that markup data alone cannot be used to reach conclusions about efficiency, policy, and counterfactuals.

In the last part of the paper, I examine the determinants of markups when the distribution of marginal costs across sellers is log-uniform. I show that markups are decreasing in the extent of competition in the market. I show that the sign of the relationship between the markup of a seller and its size depends on the degree of competition in the market. If the degree of competition is below a critical threshold, larger sellers charge higher markups than smaller sellers. If the degree of competition is at the critical threshold, markups are constant across sellers of different sizes. If the degree of competition is above the critical threshold, larger sellers charge lower markups than smaller sellers. Therefore, changes in the extent of competition would require changes not only in the elasticity of substitution across varieties in the Dixit-Stiglitz model, but also changes in the way in which the elasticity of substitutions varies with quantities.

The paper does not claim that the search-theoretic model of imperfect competition is any closer to the truth that the model of monopolistic competition (although, I do have some views about it). The paper makes the more modest, but not unimportant claim that both models can make sense of the pattern of markups observed in the data, and they have very different implications about welfare, policy and counterfactuals. Therefore, it is not enough to examine markup data to study the macroeconomic consequences of market power. It is necessary to uncover some evidence on the origin of market power.

Related literature. The paper contributes to several strands of literature. First, it contributes to the development of the search-theoretic framework of imperfect competition by Butters (1977), Varian (1980), Burdett and Judd (1983) and the related labor-market version by Burdett and Mortensen (1988). The paper contributes a characterization of the equilibrium distribution of markups in versions of the framework where sellers are either homogeneous or heterogeneous with respect to their production technology. Moreover, the paper provides a characterization of the determinants of the relationship between the size of a seller and its markup. The paper adds to recent theoretical analyses of the model (see, e.g., Kaplan et al. 2019, Menzio 2023, Albrecht, Menzio and Vroman 2023, Menzio 2024a, Hugonnier, Lester and Weill 2024).

Second, the paper contributes to the literature on markup dispersion. Recent empirical studies have documented that markups are heterogeneous and tend to increase with the size of a firm (see. e.g., Edmond, Midrigan and Xu 2015). Theoretical studies cast in the Dixit-Stiglitz framework rationalize the relationship between markups and size through preferences (see, e.g., Mrazova and Neary 2017, Dhingra and Morrow 2019, Edmond, Midrigan and Xu 2023). These preferences have the property that, as a buyer consumes more of a seller's variety, the elasticity of substitution between that variety and others declines. Other theoretical studies rationalize the relationship between markups and size by assuming oligopolistic competition among sellers of differentiated varieties over which buyers have CES preferences (see, e.g., Atkeson and Burnstein 2010, Edmond, Midrigan and Xu 2015). In this paper, I show that the empirical relationship between markups and size can be rationalized in a model where the varieties of different sellers are perfect substitutes, but buyers do not have all of the varieties in their choice set because of information frictions. Moreover, I show that whether the relationship between markups and size is positive, negative or missing is an endogenous outcome that depends, among other things, on the extent of search frictions.

Third, the paper contributes a cautionary note to the macroeconomic literature that uses the Dixit-Stiglitz framework to model market power and markups. From the normative point of view, the Dixit-Stiglitz framework implies that markups are associated with an inefficiently low level of production, as long as inputs are supplied elastically (see, e.g., Edmond, Midrigan and Xu 2023). The framework also implies that markup heterogeneity is associated with an inefficient allocation of inputs across sellers (Dhingra and Morrow 2019, Edmond, Midrigan and Xu 2023, Boar and Midrigan 2024). The estimation of Dixit-Stiglitz models reveals that the inefficiencies associated with heterogeneous markups are quantitatively important contributors to the welfare cost of inflation in sticky price models (e.g, Gali' 1995), the welfare gains from opening up to trade (e.g., Dhingra and Morrow 2019), the cost of market power (e.g., Boar and Midrigan 2024). In this paper, I show that neither the level nor the dispersion of markups observed in the data are necessarily symptomatic of any inefficiency. In a follow-up paper (Menzio 2024b), I show that the efficiency of the equilibrium of the search-theoretic model of imperfect competition extends to a general equilibrium setting with endogenous entry of firms.

From the descriptive point of view, the Dixit-Stiglitz framework generates markups only because the varieties produced by different sellers are imperfect substitutes. According to the framework, markups are higher if varieties are less substitutable, and markups are increasing in a seller's size if a variety become less substitutable when a buyer consumes more of it. In the Dixit-Stiglitz framework, the structure of markups is baked into buyers' preferences. In this paper, I consider an alternative view on markups based on information frictions. I show that the structure of markups depends endogenously on deep parameters of the model. If one were to model market power as in Dixit and Stiglitz when the actual source of market power are informational frictions, one would recover the incorrect preferences for buyers. Moreover, these incorrect preferences would be a reduced-form representation of the actual preferences and the actual source of market power and, for this reason, they would not be stable in response to policy changes or changes to the environment. These preferences would, therefore, be subject to the Lucas' critique: Any policy and counterfactual experiments carried out under the maintained assumption of stable reduced-form preferences would not produce valid predictions.

2 Markups with homogeneous sellers

2.1 Environment and equilibrium

Consider the market for some consumer good. On one side of the market, there is a measure 1 of homogeneous seller. Each seller posts a price p for the good. Each seller produces the good at a constant marginal cost of c, with $c \geq 0$.[4] Each seller enjoys a payoff of $q(p-c)$, if it sells q units of the good at the price p. On one side of the market, there is a measure $b > 0$ of homogeneous buyers per seller. Each buyer demands one unit of the good. Each buyer enjoys a payoff of $u - p$ if he purchases a unit of the good at the price p, and 0 if he does not purchase the good, with $u > c$.

The market is frictional, in the sense that a buyer cannot purchase the good from any seller in the market, but only from the subset of sellers with whom he is in contact. Specifically, each buyer is in contact with a number n of randomly-selected sellers, where n is a random variable distributed as a Poisson with coefficient λ, with $\lambda > 0$.[5] The buyer observes the price charged by each of the n sellers with whom he is in contact and decides whether and where to purchase the good.

A market equilibrium is such that: (i) Each buyer purchases the good from the seller that posts the lowest price among their contacts, as long as such price is non-greater than u; (ii) Each price p on the support of the price distribution $F(p)$ maximizes the profits of a seller.

2.2 Existence, uniqueness and properties of equilibrium

The profit for a seller posting the price $p \in [0, u]$ is

$$V(p) = \left[\sum\nolimits_{k=0}^{\infty} b_k \pi_k(p)\right](p-c), \tag{2.1}$$

[4]Menzio (2024a) characterizes the equilibrium of the model when sellers operate a technology with decreasing returns to scale. In this paper, I stick to the standard assumption of constant returns to scale.

[5]This is the same sampling process as in Butters (1977), and, more recently, by Menzio (2023) and Albrecht, Menzio and Vroman (2023). The sampling process in Burdett and Judd (1983) assumes that a buyer contacts 1 seller with some probability and 2 sellers with complementary probability. The results under the Burdett-Judd sampling process are qualitatively similar, but the algebra is not quite as clean.

where b_k denotes the measure of buyers that are in contact with the seller and with k other firms, and $\pi_k(p)$ denotes the probability that one of the b_k buyers purchases the good from the seller.

The measure of buyers that are in contact with the seller and with k other firms is given by

$$b_k = b\frac{e^{-\lambda}\lambda^{k+1}}{(k+1)!}(k+1). \tag{2.2}$$

The measure b_k is equal to the measure of buyers per seller, b, multiplied by the probability that a buyer has $k+1$ contacts, $b\exp(-\lambda)\lambda^{k+1}/(k+1)!$, and by the number of contacts held by each one of these buyers, $k+1$.

The probability $\pi_k(p)$ that one of the b_k buyers purchases the good from the seller is given by

$$\pi_k(p) = (1-F(p))^k + \sum_{j=1}^{k}\binom{k}{j}\frac{\chi(p)^j(1-F(p))^{k-j}}{j+1}, \tag{2.3}$$

where $\chi(p)$ denotes the fraction of sellers posting the price p. The probability $\pi_k(p)$ is equal to the sum of two terms. The first term is the probability that all of the other k contacts of the buyer charge a price strictly greater than p. The second term is the probability that j of the other k contacts of the buyer charge a price equal to p, $k-j$ of the other k contacts of the buyer charge a price strictly greater than p, and the buyer chooses to purchase from the seller.

The following lemma states that, in any equilibrium, the price distribution $F(p)$ cannot have any mass points. The logic of the proof is the same as in Butters (1977), Varian (1980), and Burdett and Judd (1983).

Lemma 1: *In any equilibrium, the price distribution F does not have any mass points.*

Proof: As a preliminary step, notice that the support of F does not include any price $p > u$, nor any price $p \leq c$. To see why this is the case, notice that the profit for a seller posting the price u is such that

$$\begin{aligned} V(u) &= \left[\sum\nolimits_{k=0}^{\infty} b_k\pi_k(u)\right](u-c) \\ &\geq be^{-\lambda}\lambda(u-c) > 0. \end{aligned} \tag{2.4}$$

The above inequalities simply states that a seller posting a price equal to the buyer's valuation trades, at least, with the positive measure of buyers that are in contact with no other firm, and it enjoys a strictly positive profit on each unit that it trades. Hence, the maximized profit for a seller must be strictly positive. Since a seller makes a profit of 0 by posting any price p strictly greater than u, it follows that the support of F cannot include any $p > u$. Since a seller makes a non-positive profit by posting any price p smaller than c, it follows that the support of F cannot include any $p \leq c$.

Next, I establish that F cannot have any mass points. Clearly, F cannot have a mass point at any $p_0 > u$ nor at any $p_0 \leq c$, since the support of F cannot include these prices.

To show that F cannot have a mass point at a price $p_0 \in (c, u]$, notice that the profit for a seller posting the price $\hat{p} = p_0 - \epsilon$, with $\epsilon > 0$, is given by

$$\begin{aligned} V(\hat{p}) &\geq \left\{ \sum\nolimits_{k=0}^{\infty} b_k \left[(1 - F(p_0))^k + \sum\nolimits_{j=1}^{k} \binom{k}{j} \chi(p_0)^j (1 - F(p_0))^{k-j} \right] \right\} (\hat{p} - c) \\ &= \left[\sum\nolimits_{k=0}^{\infty} b_k \pi_k(p_0) \right] (p_0 - c) - \left[\sum\nolimits_{k=0}^{\infty} b_k \pi_k(p_0) \right] \epsilon \\ &+ \left\{ \sum\nolimits_{k=0}^{\infty} b_k \left[\sum\nolimits_{j=1}^{k} \binom{k}{j} \chi(p_0)^j (1 - F(p_0))^{k-j} \left(1 - \frac{1}{j+1} \right) \right] \right\} (p_0 - \epsilon - c). \end{aligned} \tag{2.5}$$

The first line in (2.5) makes use of the fact that a seller posting a price $\hat{p} < p_0$ trades with all the b_k buyers that are in contact with k other firms that are charging a price non-smaller than p_0. The second and third lines in (2.5) make use of the definitions of b_k, $\pi_k(p_0)$, and $\hat{p}$. For $\epsilon > 0$ and small enough, it is clear that the second and third lines in (2.5) are strictly greater than $V(p_0)$, which is the first term in the second line. Since $V(\hat{p}) > V(p_0)$, it follows that p_0 cannot be on the support of F. ■

In light of Lemma 1, I can rewrite (2.1) as

$$\begin{aligned} V(p) &= \left[\sum\nolimits_{k=0}^{\infty} b \frac{e^{-\lambda} \lambda^{k+1}}{(k+1)!} (k+1)(1 - F(p))^k \right] (p - c) \\ &= \left[b\lambda e^{-\lambda F(p)} \sum\nolimits_{k=0}^{\infty} \frac{e^{-\lambda(1-F(p))} \lambda^k (1 - F(p))^k}{k!} \right] (p - c) \\ &= b\lambda e^{-\lambda F(p)} (p - c), \end{aligned} \tag{2.6}$$

where the first line is obtained by substituting (2.2) and (2.3) into (2.1) and using the fact that $\chi(p) = 0$, the second line is obtained by rearranging terms, and the last line is obtained by noticing that the summation in the second line equals 1.

The next lemma states that, in any equilibrium, the support of the price distribution $F(p)$ is an interval $[p_\ell, p_h]$, with $p_h = u$. Again, the logic of the proof is the same as in Butters (1977), Varian (1980), and Burdett and Judd (1983).

Lemma 2: *In any equilibrium, the support of* $F(p)$ *is an interval* $[p_\ell, p_h]$*, with* $p_h = u$.

Proof: I first show that the support of $F(p)$ must be an interval. On the way to a contradiction, suppose that the support of F has a gap between p_0 and p_1, with $p_0 < p_1$ and p_0, p_1 on the support of F. The profit for a seller posting the price p_1 is given by

$$\begin{aligned} V(p_1) &= b\lambda e^{-\lambda F(p_1)} (p_1 - c) \\ &> b\lambda e^{-\lambda F(p_0)} (p_0 - c) = V(p_0), \end{aligned} \tag{2.7}$$

where the inequality in the second line makes use of the fact that $F(p_0) = F(p_1)$ and of the fact that $p_0 < p_1$. Since $V(p_0) < V(p_1)$, p_0 cannot be on the support of F, which gives me the desired contradiction.

Next, I show that $p_h = u$. On the way to a contradiction, suppose $p_h > u$. In this case, the profit for a seller posting the price p_h is equal to 0. Since the maximized profit of a seller is strictly positive, it follows that p_h cannot be on the support of F, which is a contradiction. Alternatively, suppose $p_h < u$. In this case, the profit for a seller posting the price p_h is strictly smaller than the profit for a seller posting the price u. Since $V(p_h) < V(u)$, p_h cannot be on the support of F, which is another contradiction. Combining these findings establishes that $p_h = u$. ■

Lemma 2 implies that the profit $V(u)$ for a seller posting the price u equals its maximum V^*. Moreover, since Lemma 2 states that u is the highest price on the support of the distribution F, $F(u) = 1$. Combining these observations yields

$$V^* = b\lambda e^{-\lambda}(u - c). \tag{2.8}$$

Lemma 2 also implies that the profit $V(p)$ for a seller posting a price $p \in [p_\ell, p_h]$ equals V^*. Therefore,

$$V^* = b\lambda e^{-\lambda F(p)}(p - c). \tag{2.9}$$

Equating the right-hand sides of (2.8) and (2.9) yields an equation for the price distribution $F(p)$. The solution to the equation is

$$F(p) = 1 - \frac{1}{\lambda}\log\left(\frac{u - c}{p - c}\right). \tag{2.10}$$

Since p_ℓ is the lowest price on the support of the distribution, $F(p_\ell) = 0$. Given the expression for $F(p)$ in (2.10), I can solve the equation $F(p_\ell) = 0$ with respect to p_ℓ and obtain

$$p_\ell = c + e^{-\lambda(u-c)}. \tag{2.11}$$

The price distribution in (2.10) and (2.11) describes a unique candidate market equilibrium. In order to verify that the candidate equilibrium is an actual equilibrium, I need to check that a seller attains the same profit V^* for every p in the interval $[p_\ell, p_h]$, and that a seller attains a profit non-greater than V^* for any $p < p_\ell$ and any $p > p_h$. By construction of F, the seller's profit is equal to V^* in (2.8) for any $p \in [p_\ell, p_h]$. For any $p > p_h = u$, the seller's profit is equal to zero and, hence, strictly smaller than V^*. For any $p < p_\ell$, the seller's profit $V(p)$ is equal to $b\lambda(p - c)$, which is strictly smaller than $b\lambda(p_\ell - c)$, which in turn is equal to V^*. Therefore, the price distribution in (2.10) and (2.11) describes the unique market equilibrium.

Note that the equilibrium is efficient—in the sense that it maximizes the sum of the payoffs to the buyers and the sellers. It is easy to see why this is the case. A social planner that wants to maximize the sum of payoffs to buyers and sellers instructs buyers to purchase one unit of the good whenever they are in contact with at least one seller, since the buyer's payoff u from consuming a unit of the good exceeds the seller's cost c from producing the good. Whenever a buyer is in contact with multiple sellers, the planner does not care where the buyer purchases the good, since the difference between the buyer's payoff from consuming one unit of the good and the seller's cost from producing the good is $u - c$ at every seller. Since, in equilibrium, a buyer that contacts at least one seller purchases the good, it follows that the equilibrium is efficient.

The following proposition summarizes the properties of equilibrium.

Proposition 1: *(i) The equilibrium exists and is unique. The equilibrium is described by the price distribution $F(p)$ given in (2.10), with support over the interval $[p_\ell, p_h]$, where p_ℓ is given by (2.11) and p_h is equal to u. (ii) The equilibrium is efficient.*

2.3 Markups

I am interested in the equilibrium distribution of markups across sellers, and in the relationship between a seller's markup and its size. I define the gross markup μ of a seller as the ratio between its posted price p and its marginal cost c. I define the net markup of a seller as the difference between the gross markup μ and 1. I define the size of a seller as the quantity q of the good that the seller trades.

In order to characterize the properties of the distribution of markups, it is useful to identify sellers by their ranking in the price distribution F. A seller at the x-th quantile of the price distribution F posts a price $F(p(x)) = x$. Using (2.10), I can solve for $p(x)$ and obtain

$$p(x) = c + (u - c)e^{-\lambda(1-x)}. \tag{2.12}$$

From (2.12), it follows that a seller at the x-th quantile of the price distribution F charges a gross markup $\mu(x)$ given by

$$\mu(x) = 1 + \left(\frac{u}{c} - 1\right) e^{-\lambda(1-x)}. \tag{2.13}$$

The formula in (2.13) has a simple interpretation. The term $u/c - 1$ is the net markup for a monopolist, and it is equal to the ratio between the buyers' valuation of the good u and the sellers' cost of production c. The term $\exp(-\lambda(1 - x))$ is a discount factor that depends on the seller's rank x in the price distribution. The discount factor is equal to 1 for the seller at the top of the price distribution. That is, for the seller at the top of the price distribution, there is no discounting of the monopoly markup. The discount factor becomes smaller for sellers that are at a lower rank of the price distribution. That is, for seller at lower ranks of the price distribution, there is stronger discounting of the monopoly markup. The speed at which the discount factor decreases as the seller's rank in the price distribution declines depends on λ. The parameter λ is the coefficient of the Poisson distribution of the number of sellers with which a buyer is in contact, it is equal to the average number of sellers with which a buyer is in contact, and, in this sense, it is a measure of the extent of competition in the market. The discount factor reaches its minimum $\exp(-\lambda)$ for a seller at the bottom of the price distribution.

I have thus established the following result.

Theorem 1: *Given the buyer's valuation u, the seller's marginal cost c, and the extent of competition in the market λ, the markup function $\mu(x)$ is given by (2.13).*

Several observations about (2.13) are worthwhile. First, note that net markups are positive, even though the sellers carry products that are perfect substitutes to the buyers. The reason why markups are positive for sellers of identical goods is that, due to search frictions, sellers meet a positive measure of buyers that are captive, in the sense that these buyers cannot purchase from any other seller. For this reason, sellers' equilibrium profits must be strictly positive, and prices must be strictly above marginal cost.

Second, note that net markups are heterogeneous, even though the sellers carry products that are perfect substitutes to the buyers, and the sellers operate the same production technology. The reason why markups are heterogeneous is the same reason why there is price dispersion in the search-theoretic model of imperfect competition of Butters (1977), Varian (1980) and Burdett and Judd (1983). Namely, the fact that sellers meet a positive measure of buyers that are not captive, in the sense that these buyers can purchase from multiple sellers, implies that the price distribution cannot have any mass points above marginal cost. The fact that sellers meet a positive measure of buyers that are captive implies that prices must be strictly above marginal cost. Taken together, these two observations imply that sellers must post different prices and, hence, charge different markups.

Obviously, the markup charged by every seller must be optimal. Every seller faces a demand curve $q(p)$ given by

$$q(p) = b\lambda e^{-\lambda F(p)}, \text{ for all } p \in [p_\ell, p_h]. \tag{2.14}$$

Hence, every seller faces an elasticity of demand $\epsilon_q(p)$ given by

$$\epsilon_q(p) = \lambda F'(p)p, \text{ for all } p \in [p_\ell, p_h]. \tag{2.15}$$

The optimality condition for the seller's price is such that the marginal benefit of increasing the price equals the marginal cost of increasing the price, i.e.

$$q'(p)(p-c) + q(p) = 0. \tag{2.16}$$

The optimality condition above can be rewritten as the familiar formula for the optimal markup

$$\mu = \frac{\epsilon_q(\mu c)}{\epsilon_q(\mu c) - 1}. \tag{2.17}$$

For the markup charged by a seller to be optimal, it has to be the case that the elasticity of demand is such that $\epsilon_q(\mu(x)c)$ equals $\mu(x)/(\mu(x)-1)$ for every $x \in [0,1]$. This particular elasticity of demand, which makes homogeneous sellers indifferent between choosing any markup in a range between $\mu(0)$ and $\mu(1)$ does not emerge because the exogenous preferences of buyers happen to have a particular knife-edge structure. The elasticity of demand emerges necessarily as an equilibrium outcome from the density $F'(p)$ of sellers posting different prices.

Third, note that markups are increasing in the seller's price, since $\mu(x)$ and $p(x)$ are both increasing in x. The property is a direct consequence of the fact that all sellers have the same marginal cost and face the same demand curve. The property implies that the elasticity of demand is lower at higher prices. This is the opposite of what people sometimes refer to as "Marshall's second law of demand," which posits that the elasticity of demand ought to be increasing in the price. Similarly, note that markups are decreasing in the seller's size, since $\mu(x)$ is increasing in x and $q(x) = b\lambda \exp(-\lambda x)$ is decreasing in x. This property is also a direct consequence of the environment. The property implies that larger sellers face a higher elasticity of demand.

Fourth, note that markups depend on the buyers' valuation u and on the seller's cost c, which determine the monopoly component of the markup in (2.13), but they also they depend on the extent of competition λ in the market, which determine the markup discount factor in (2.13). In particular, the more competitive is the market—in the sense that the average number λ of contacts per buyer is higher—the steeper is the decline in markups as we move down the seller's ranking x in the price distribution F. The intuition for this property is simple. The higher is λ, the higher is the probability that a seller meets a non-captive buyer and, hence, the lower are the equilibrium markups.

From the perspective of the monopolistic competition model of Dixit and Stiglitz (1977), the equilibrium properties of markups are surprising. Markups are positive, even though the sellers carry varieties of the good that are perfect substitutes. Markups are heterogeneous, even though the sellers operate technologies that have the same marginal cost. Markups are increasing in a seller's price and decreasing in a seller's size, even though the substitutability of a seller's variety is independent of the amount of that variety consumed by buyers. Moreover, markups may change over time, even though buyers' preferences and technology remain constant.

If the distribution of markups generated by the search-theoretic model of imperfect competition was given to an economist bent on seeing the world through the lens of the monopolistic competition model of Dixit and Stiglitz (1977), they would reach a number of incorrect conclusions about preferences, technology, and shocks. From net markups being positive, they would conclude that the varieties of the product carried by different sellers are imperfect substitutes in the utility function of the buyers. From net markups being decreasing in the seller's size, they would conclude that the seller's variety becomes more substitutable the more of that variety is consumed by the buyers. From a decline in the markup caused by an increase in the extent of competition, they would conclude that the varieties carried by different sellers have become closer substitutes.

More importantly, the economist reading the markups through the lens of Dixit and Stiglitz (1977) would also reach incorrect conclusions about welfare and, in turn, make incorrect policy recommendations. They would conclude that the market is inefficient, since net markups are positive. Indeed, in Dixit and Stiglitz (1977), a positive net markup implies that sellers produce an inefficiently small quantity of the good, as long as the supply of the inputs of production is elastic (see, e.g., Edmond, Midrigan and Xu 2023). They would then recommend the introduction of consumption subsidies in order to increase buyers' consumption and sellers' production. Since markups are heterogeneous across sellers, they would conclude that inefficiencies are larger at high-markup sellers than at low-markup sellers. Indeed, in Dixit and Stiglitz (1977), a larger markup implies a bigger efficiency loss. They might then recommend finely-tuned production subsidies that reallocate inputs and consumption from low to high-markup sellers (see, e.g., Edmond, Midrigan and Xu 2023 or Boar and Midrigan 2024). These conclusions about welfare and, in turn, these policy recommendations would be wrong, since the equilibrium is efficient.

The root of these economists' mistake would lie in the interpretation of the demand curve $q(p)$. The gap between $q(p)$ and $q(c)$ is the quantity of the seller's variety that is not consumed by the buyers when the seller's price exceeds its marginal cost. In the Dixit and Stiglitz (1977) model of monopolistic competition, the quantity $q(p) - q(c)$ of the variety that is not consumed by the buyers represents a lost opportunity to exploit gains from trade and, for this reason, it is associated with an inefficiency. In the search-theoretic model of imperfect competition, the quantity of the variety that is not consumed by the buyers represents equally valuable trades that the buyers make with other sellers. Hence, the gap between $q(p) - q(c)$ is not associated with an inefficiency.

Even though the arguments above are straightforward, it is worth illustrating them in the context of a simple Dixit-Stiglitz model in the spirit of Krugman (1979). Consider a market for a good populated by a continuum of buyers and a continuum of sellers, both with measure 1. A buyer has preferences described by the utility function $\int_i v(q_i)di + z$, where z denotes consumption of a numeraire good, q_i denotes consumption of the variety of the good produced by seller i, and $v(q)$ is an increasing and concave function. Seller i produces its variety of the good at a constant marginal cost c_i, where c_i are units of the numeraire good. The market is frictionless, in the sense that buyers have access to all of the sellers. The market is monopolistic, in the sense that each seller is the sole producer of its variety of the good.

The optimality condition for the buyer's consumption of the variety produced by seller i is

$$v'(q_i) = p_i, \tag{2.18}$$

where p_i denotes the price posted by the seller. The optimality condition in (2.18) states that the buyer equates the marginal utility of the consumption of the variety produced by seller i to its price. The optimality condition in (2.18) generates an inverse demand $p(q)$ faced by each seller, where $p(q)$ is such that $p(q) = v'(q)$ and $p'(q) = v''(q)$.

The optimality condition for the seller's problem is

$$p'(q_i)q_i + p(q_i) = c_i. \tag{2.19}$$

The optimality condition in (2.19) states that the seller equates the marginal revenue to the marginal cost. The optimality condition in (2.19) can be written as

$$\frac{\mu_i - 1}{\mu_i} = -\frac{p'(q_i)q_i}{p(q_i)}, \tag{2.20}$$

where μ_i denotes the markup charged by the seller and it is defined as

$$\mu_i = p(q_i)/c_i. \tag{2.21}$$

I want to find the utility function $v(q)$ such that that the equilibrium of the Dixit-Stiglitz model generates the same combinations of markups $\mu(x)$ and quantities $q(x)$ as in the Burdett-Judd model, where $\mu(x)$ is given by (2.13) and $q(x)$ is given by $b\lambda \exp(-\lambda x)$. From (2.21) and (2.18), it follows that, in the Dixit-Stiglitz model, a seller produces $q(x)$ and charges the markup $\mu(x)$ only if

$$-\frac{v''(q(x))q(x)}{v'(q(x))} = \frac{\mu(x) - 1}{\mu(x)}. \tag{2.22}$$

Using the expressions for $q(x)$ and $\mu(x)$ to write the markup as a function of the quantity sold by the seller, I can reformulate (2.22) as

$$-\frac{v''(q)q}{v(q)} = 1 + \left(\frac{u}{c} - 1\right)\frac{b\lambda}{q}e^{-\lambda}. \tag{2.23}$$

The expression in (2.23) is a differential equation for the buyer's marginal utility $v'(q)$. The solution of the differential equation is

$$v'(q) = A\left[ce^{\lambda} + b\lambda\frac{u-c}{q}\right], \tag{2.24}$$

where $A > 0$ is a constant of integration. By construction, the buyer's utility function in (2.24) is such that the demand curve facing an individual seller is the same, up to the scaling factor A, in the Dixit-Stiglitz model as in the Burdett-Judd model, and sellers find it optimal to charge the same markups and produce the same quantities.

I can now point out the properties of the reduced-form utility function that rationalizes in a Dixit-Stiglitz model the equilibrium of the Burdett-Judd model. First, notice that the reduced-form utility function is such that $v'(q)$ is strictly decreasing in q. This property implies that buyers perceive the varieties produced by different sellers as imperfect substitutes and, hence, it is why sellers can charge positive markups. Second, notice that the reduced-form utility function is such that the elasticity of demand $v'(q)/(-v''(q)q)$ is strictly increasing in q. This property implies that the elasticity of demand is increasing in q, meaning that the elasticity of substitution is increasing in q. This is why larger sellers charge lower markups. Third, notice that the reduced-form utility function depends on the parameters of the search-theoretic model λ, u, c and b. Therefore, the reduced-form utility function is unstable, in the sense that changes in the environment and counterfactual experiments lead to changes in the reduced-form utility function.

The equilibrium of the Dixit-Stiglitz model is inefficient, in the sense that it does not maximize the sum of the payoffs to buyers and sellers. This is so, even though the equilibrium of the Burdett-Judd model is efficient. First, the quantity of the good produced by every seller is inefficiently low because net markups are strictly positive. Consider a seller that produces q_0 and charges the markup μ_0. Increasing the quantity of the good produced by the seller by dq_0 leads to a change in welfare equal to

$$\begin{aligned} dW &= (v'(q_0) - c_0)\, dq_0 \\ &= (\mu_0 - 1)\, c_0 dq_0 > 0. \end{aligned} \tag{2.25}$$

Second, inputs are misallocated because markups are heterogeneous. Consider a seller that produces q_0 and charges the markup μ_0, and a seller that produces q_1 and charges the markup μ_1, with $\mu_1 > \mu_0$. Increasing the quantity of the good produced by seller 1 by dq_1 and the quantity of the good produced by seller 0 by $dq_0 = -(c_1/c_0)dq_1$, so that the amount of inputs used in production is unchanged, leads to a change in welfare equal to

$$\begin{aligned} dW &= (v'(q_1) - c_1)\, dq_1 + (v'(q_0) - c_0)\, dq_0 \\ &= (\mu_1 - 1)\, c_1 dq_1 + (\mu_0 - 1)\, c_0 dq_0. \\ &= (\mu_1 - 1)\, c_1 dq_1 - (\mu_0 - 1)\, c_1 dq_1 > 0. \end{aligned} \tag{2.26}$$

3 Markups with heterogeneous sellers

3.1 Environment and equilibrium

I now consider a version of the model in which sellers are heterogeneous with respect to their marginal cost of production. Specifically, one side of the market is populated by a measure 1 of sellers. The distribution of sellers across marginal costs is given by a twice-differentiable cumulative distribution function $\Phi(c)$ with support over the interval $[c_\ell, c_h]$, where $c_h > c_\ell > 0$. Each seller posts a price p for the good. A seller with marginal cost c enjoys a payoff of $q(p - c)$, if it sells q units of the good at the price p.

The other side of the market is populated by a measure $b > 0$ of buyers per seller. Each buyer comes into contact with n randomly-selected sellers, where n is distributed as a Poisson with coefficient λ and is drawn independently for each buyer. Each buyer observes the price posted by the sellers with which he comes into contact, and decides whether and where to purchase a unit of the good. The buyer enjoys a payoff of $u - p$ if he purchases a unit of the good at the price p, and 0 if he does not purchase the good, with $u > c_h$.

An equilibrium is such that: (i) Each buyer purchases the good from the seller that posts the lowest price among their contacts, as long as such price is non-greater than u;

(ii) Each price p on the support of the price distribution $F(p)$ maximizes the profits of a seller.

3.2 Existence, uniqueness and properties of equilibrium

The profit for a seller with marginal cost c that posts the price $p \in [0, u]$ is

$$V(p, c) = \left[\sum_{k=0}^{\infty} b_k \pi_k(p)\right](p - c), \tag{3.1}$$

where b_k is given by

$$b_k = b\frac{e^{-\lambda}\lambda^{k+1}}{(k+1)!}(k+1), \tag{3.2}$$

and $\pi_k(p)$ is given by

$$\pi_k(p) = (1 - F(p))^k + \sum_{j=1}^{k} \binom{k}{j} \frac{\chi(p)^j (1 - F(p))^{k-j}}{j+1}. \tag{3.3}$$

It is straightforward to verify that Lemma 1 and Lemma 2 also apply to a version of the model in which sellers are heterogeneous with respect to their marginal cost. Lemma 1 then guarantees that the maximized profit for a seller is strictly positive and that the price distribution F does not have any mass points. Lemma 2 guarantees that the support of the price distribution F is some interval $[p_\ell, p_h]$, with $p_h = u$. In light of Lemma 1, I can rewrite (3.1) as

$$V(p, c) = b\lambda e^{-\lambda F(p)}(p - c). \tag{3.4}$$

The next lemma shows that the price posted by a seller is a strictly increasing function of the seller's marginal cost. The lemma involves three steps. In the first step, I show that the price posted by a seller is weakly increasing in the seller's marginal cost. In the second step, I use the fact that the price distribution F does not have any mass points to show that the price posted by a seller is strictly increasing in the seller's marginal cost. In the third and last step, I use the fact that the support of the price distribution does not have any gaps to show that the price posted by a seller is a function, not a correspondence, of the seller's marginal cost.

Lemma 3: *The price posted by a seller is a strictly increasing function $p(c)$ of the seller's cost c.*

Proof: First, I establish that the price posted by a seller is weakly increasing in the seller's cost c. Let p_0 denote the price posted by some seller with cost c_0 and let p_1 denote the price posted by some seller with cost c_1, with $c_1 > c_0$. Since a seller with cost c_0 finds it optimal to post the price p_0, it must be that

$$b\lambda e^{-\lambda F(p_0)}(p_0 - c_0) \geq b\lambda e^{-\lambda F(p_1)}(p_1 - c_0). \tag{3.5}$$

Since a seller with cost c_1 finds it optimal to post the price p_1, it must be that

$$b\lambda e^{-\lambda F(p_1)}(p_1 - c_1) \geq b\lambda e^{-\lambda F(p_0)}(p_0 - c_1). \tag{3.6}$$

Combining the above inequality yields

$$\left(e^{-\lambda F(p_0)} - e^{-\lambda F(p_1)}\right)(c_1 - c_0) \geq 0. \tag{3.7}$$

Since $\exp(-\lambda F(p))$ is strictly decreasing in p, (3.7) implies that $p_1 \geq p_0$.

Second, I establish that the price posted by a seller is strictly increasing in the seller's cost c. On the way to a contradiction, suppose that a seller with cost c_0 and a seller with cost $c_1 > c_0$ both post the price p. Since the price posted by a seller is weakly increasing in the seller's cost, the fact that a seller with cost c_0 and a seller with cost c_1 both post the price p implies that any seller with a cost $c \in (c_0, c_1)$ must post the price p as well. In turn, this implies that the distribution F must have a mass point at p, which contradicts Lemma 1.

Lastly, I establish that every seller with the same cost c posts the same price p and, hence, that the price posted by a seller is a function of the seller's cost. On the way to a contradiction, suppose that a seller with cost c posts the price p_0 and another seller with cost c posts the price p_1, with $p_1 > p_0$. Since a seller's price is strictly increasing in its cost, it follows that any seller with a cost $\hat{c} < c$ posts a price strictly smaller than p_0. Similarly, any seller with a cost $\hat{c} > c$ posts a price strictly greater than p_1. Taken together, these observations imply that $F(p_0) = F(p_1)$, which contradicts Lemma 2. ■

I now turn to the derivation of the price function $p(c)$. To this aim, first notice that the necessary condition for the optimality of $p(c)$ is

$$b\lambda e^{-\lambda F(p(c))} - b\lambda e^{-\lambda F(p(c))}\lambda F'(p(c))(p(c) - c) = 0. \tag{3.8}$$

The first term in (3.8) is the seller's marginal benefit from increasing the price, which is equal to the quantity of the good that the seller trades. The second term in (3.8) is the negative of the seller's marginal cost from increasing the price, which is equal to the decline in the quantity that the seller trades because of the price increase multiplied by the seller's profit margin. Condition (3.8) states that the seller's price $p(c)$ must equate marginal benefit and marginal cost.

Next, notice that the fraction of sellers that post a price non-greater than $p(c)$ must be equal to the fraction of sellers with a cost non-greater than c, since Lemma 3 states that a seller's price is a strictly increasing function of its cost. Formally, we have

$$F(p(c)) = \Phi(c). \tag{3.9}$$

Differentiating the above equation with respect to c yields

$$F'(p(c))p'(c) = \Phi'(c). \tag{3.10}$$

Lastly, notice that I can use (3.10) to substitute $F'(p(c))$ with $\Phi'(c)/p'(c)$ in the optimality condition (3.8) and obtain

$$p'(c) = \lambda\Phi'(c)(p(c) - c). \tag{3.11}$$

Since Lemma 2 states that the highest price p_h in the distribution $F(p)$ is the buyer's valuation u and since Lemma 3 states that the price posted by a seller is a strictly increasing function of the seller's cost c, it follows that

$$p(c_h) = u. \tag{3.12}$$

The expressions in (3.11) and (3.12) are, respectively, an ordinary differential equation for the price function $p(c)$, and a boundary condition. Clearly, the solution to (3.11) and (3.12) exists, it is unique, and it is strictly increasing.

The solution to (3.11) and (3.12) identifies a unique candidate equilibrium. To make sure that the candidate equilibrium is indeed an equilibrium, I need to verify that the necessary condition for the optimality of $p(c)$ in (3.8) identifies a global maximum for the profit of the seller. To this aim, consider a seller with cost c_0 posting the price $p_0 = p(c_0)$. By construction, the derivative of the seller's profit with respect to p on the left-hand side of (3.8) is equal to 0 at p_0. For any $p \in [p(c_\ell), p_0)$, the left-hand side of (3.8) is equal to 0 for a seller with cost $c < c_0$ and, hence, it is strictly positive for the seller with cost c_0. For $p < p(c_\ell)$, $F'(p) = 0$ and, hence, the left-hand side of (3.8) is strictly positive for the seller with cost c_0. For any $p \in (p_0, p(c_h)]$, the left-hand side of (3.8) is equal to 0 for a seller with cost $c > c_0$ and, hence, it is strictly negative for the seller with cost c_0. For any $p > p(c_h) = u$, the seller's profit is equal to 0, while it is strictly positive for p_0. These observations imply that the profit for a seller with cost c_0 attains its global maximum at p_0. The unique candidate equilibrium is indeed an equilibrium.

I now want to examine the welfare properties of equilibrium. To this aim, consider a social planner that wants to maximize the sum of payoffs to buyers and sellers. When a buyer is in contact with at least one seller, the planner instructs the buyer to purchase one unit of the good, since the buyer's payoff u from consuming one unit of the good is greater than the seller's cost of producing the good c. When a buyer is in contact with multiple sellers, the planner instructs the buyer to purchase the good from the seller with the lowest cost, since doing so maximizes the sum of the payoffs to the buyer and the seller. In equilibrium, whenever a buyer is in contact with at least one seller, he purchases one unit of the good, since every seller posts a price p non-greater than the buyer's valuation u. Moreover, in equilibrium, whenever a buyer is in contact with multiple sellers, he purchases the good from the seller with the lowest cost, since the seller with the lowest cost posts the lowest price. These observations imply that the equilibrium is efficient.

The following proposition summarizes the properties of equilibrium.

Proposition 2. *(i) The equilibrium exists and is unique. The equilibrium is described* *by the price function $p(c)$ that solves the differential equation (3.11) together with the boundary condition (3.12). (ii) The equilibrium is efficient.*

3.3 Markups

I want to characterize the distribution of markups across sellers, and the relationship between a seller's markup, its price, and its size. As in Section 2, it is useful to categorize sellers by their rank in the price distribution. Since the price of a seller is a strictly increasing function of its marginal cost, a seller's rank in the price distribution F is the same as a seller's rank in the cost distribution Φ.

Let $c(x)$ denote the marginal cost of a seller at the x-th quantile of the price distribution F and of the cost distribution Φ. The cost $c(x)$ is such that $\Phi(c(x)) = x$. Differentiating $\Phi(c(x)) = x$ with respect to x yields

$$\Phi'(c(x))c'(x) = 1. \tag{3.13}$$

Let $\hat{p}(x)$ denote the price posted by a seller at the x-th quantile of the price distribution F and of the cost distribution Φ. The price $\hat{p}(x)$ is such that $\hat{p}(x) = p(c(x))$. Differentiating $\hat{p}(x) = p(c(x))$ with respect to x yields

$$\hat{p}'(x) = p'(c(x))c'(x). \tag{3.14}$$

I use (3.13) and (3.14) to transform the differential equation (3.11) for the price of a seller as a function of its cost, $p(c)$, into a differential equation for the price of a seller as a function of its ranking in the cost distribution, $\hat{p}(x)$. Specifically, I evaluate the differential equation (3.11) at $c = c(x)$ and multiply both sides by $c'(x)$ to obtain

$$p'(c(x))c'(x) = \lambda\Phi'(c(x))c'(x)(p(c(x)) - c(x)). \tag{3.15}$$

I then use (3.13), (3.14) and the definition of $\hat{p}(x)$ to rewrite (3.15) as

$$\hat{p}'(x) = \lambda(\hat{p}(x) - c(x)). \tag{3.16}$$

The expression in (3.16) is a differential equation for $\hat{p}(x)$. I derive the boundary condition for (3.16) from the boundary condition for (3.11), rewritten using the fact that $p(c_h) = \hat{p}(1)$. Specifically, the boundary condition for (3.16) is

$$\hat{p}(1) = u. \tag{3.17}$$

Next, let $\mu(x)$ denote the gross markup charged by a seller at the x-th quantile of the price distribution F and of the cost distribution Φ. The markup $\mu(x)$ is such that

$\mu(x) = \hat{p}(x)/c(x)$. Differentiating $\mu(x) = \hat{p}(x)/c(x)$ yields

$$\mu'(x) = \frac{\hat{p}'(x)c(x) - \hat{p}(x)c'(x)}{c(x)^2}. \tag{3.18}$$

I now use (3.18) to transform (3.16) into a differential equation for the markup $\mu(x)$ charged by a seller as a function of its ranking in the cost distribution. Specifically, I divide both sides of (3.16) by $c(x)$ and subtract $\hat{p}(x)c'(x)/c(x)^2$ from both sides of (3.16) to obtain

$$\frac{\hat{p}'(x)}{c(x)} - \frac{\hat{p}(x)c'(x)}{c(x)^2} = \lambda\left(\frac{\hat{p}(x)}{c(x)} - 1\right) - \frac{\hat{p}(x)c'(x)}{c(x)^2}. \tag{3.19}$$

Then I use (3.18) and the definition $\mu(x) = \hat{p}(x)/c(x)$ to rewrite (3.19) as

$$\mu'(x) = \lambda\left(\mu(x) - 1\right) - \mu(x)\frac{c'(x)}{c(x)}. \tag{3.20}$$

The expression in (3.20) is a differential equation for $\mu(x)$. I derive the boundary condition for (3.20) from the boundary condition (3.17), rewritten using the fact that $\hat{p}(1)/c(1) = \mu(1)$. Specifically, the boundary condition for (3.20) is

$$\mu(1) = \frac{u}{c(1)}. \tag{3.21}$$

The solution to the differential equation (3.20) with the boundary condition (3.21) is

$$\mu(x) = 1 + \left(\frac{u}{c(x)} - 1\right)e^{-\lambda(1-x)} + \lambda\int_x^1 \left(\frac{c(\hat{x})}{c(x)} - 1\right)e^{-\lambda(\hat{x}-x)}d\hat{x}. \tag{3.22}$$

The formula above generalizes (2.13) to an environment in which sellers are heterogeneous. The term $u/c(x)-1$ is the net markup for a monopolist, and it is equal to the ratio between the buyers' valuation u and the seller's cost of production $c(x)$. The term $\exp(-\lambda(1-x))$ is a discount factor on the monopoly markup that depends on the seller's rank in the price distribution. The last term captures the additional markup that the seller can charge because the firms ranked above it in the price distribution produce at higher marginal cost. Indeed, the last term is zero if all the firms ranked above the seller have a marginal cost of $c(x)$. Otherwise, the last term is strictly positive. The excess marginal cost of firms ranked above the seller is weighted according to $\exp(-\lambda(\hat{x} - x))$, where $\hat{x} - x$ is the ranking differential between the firm and the seller. Therefore, the excess marginal cost of firms that are closer to the seller has a stronger impact on the seller's markup than the excess marginal cost of firms that are further away from the seller. The marginal cost of firms that are ranked below the seller does not affect the seller's markup at all.

I have thus established the following result.

Theorem 2. *Given the valuation for the good u by buyers, the quantile function $c(x)$ of marginal costs across sellers, and the degree of competition λ in the market, the markup function $\mu(x)$ is given by (3.22).*

In order to understand the properties of the markup function $\mu(x)$, it is useful to work with the phase diagram associated with the differential equation (3.20)-(3.21). To this aim, let me define the nullcline $\mu_n(x)$. If $\lambda > c'(x)/c(x)$, the nullcline $\mu_n(x)$ is given by

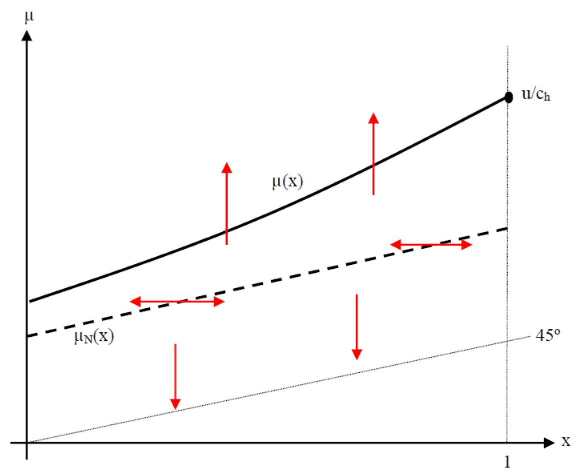
$$\mu_n(x) = \frac{\lambda}{\lambda - c'(x)/c(x)}. \tag{3.23}$$

If $\lambda \leq c'(x)/c(x)$, let $\mu_n(x) = +\infty$. Then, for any $\mu(x) > \mu_n(x)$, (3.20) implies that $\mu'(x) > 0$. For any $\mu(x) < \mu_n(x)$, (3.20) implies that $\mu'(x) < 0$. For any $\mu(x) = \mu_n(x)$, (3.20) implies that $\mu'(x) = 0$.

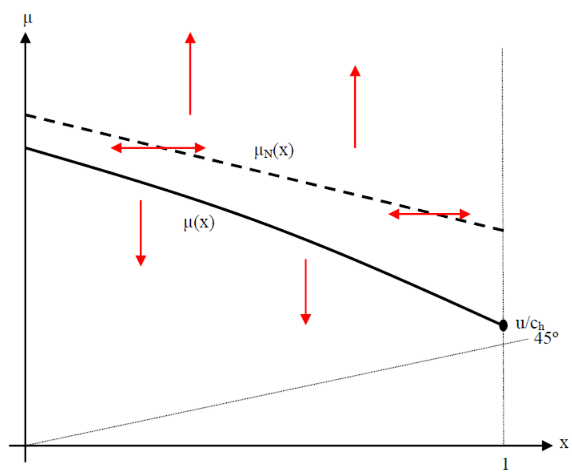
The location of the nullcline—which depends on the degree of competitiveness of the market, λ, and on the distribution of marginal costs across sellers, $c(x)$—together with the location of the boundary condition (3.21)—which depends on the buyers' valuation for the good u and on the sellers' highest marginal cost $c(1)$—determine the shape of the markup function $\mu(x)$.

Figure 1(a) illustrates a case in which the nullcline $\mu_n(x)$ is upward sloping and the boundary condition $u/c(1)$ lies above the nullcline at $x = 1$. In this case, the phase diagram implies that the solution to the differential equation (3.20)-(3.21) is a markup function $\mu(x)$ that is strictly increasing in x. In this case, a seller's markup is strictly increasing in its price and, hence, strictly decreasing in its size. These are the same properties of markups as in the version of the model with homogeneous sellers. Figure 1(b) illustrates a case in which the nullcline $\mu_n(x)$ is downward sloping and the boundary condition lies below the nullcline at $x = 1$. In this case, the phase diagram implies that the solution to the differential equation (3.20)-(3.21) is a markup function $\mu(x)$ that is strictly decreasing in x. In this case, a seller's markup is strictly decreasing in its price and, hence, strictly increasing in its size. These properties are opposite to those obtained in the version of the model with homogeneous sellers and they satisfy "Marshall's second law of demand." In Figure 1(c), the nullcline is upward sloping and the boundary condition $u/c(1)$ lies below the nullcline at $x = 1$ and above the nullcline at $x = 0$. In this case, the solution to (3.20)-(3.21) is a markup function $\mu(x)$ that is hump-shaped in x. Here, the markups are lowest for the sellers with the highest and lowest prices, and highest for the sellers in the middle of the distribution.

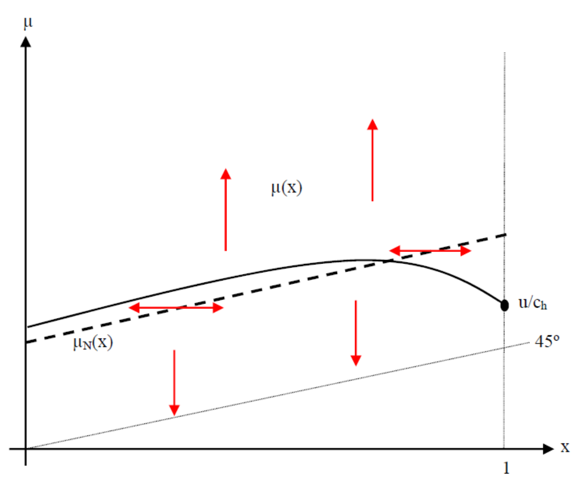
Figure 1 illustrates some of the shapes of the markup function that may emerge as an equilibrium outcome of the search-theoretic model of imperfect competition. Yet, any markup function can be generated by the model given the appropriate distribution of marginal costs across sellers and the appropriate Poisson distribution of contacts among sellers. To see why this is the case, let $\mu^*(x)$ denote an arbitrary twice-continuously differentiable function with $\mu^*(x) > 1$ for all $x \in [0, 1]$. The markup function $\mu^*(x)$ is an equilibrium outcome of the model given a parameter λ such that $\lambda(\mu^*(x) - 1) > \mu^{*\prime}(x)$



(a) Increasing markups



(b) Decreasing markups



(c) Hump-shaped markups

Figure 1: Phase diagram of $\mu(x)$

for all $x \in [0,1]$, and a cost distribution $c(x)$ that solves the differential equation

$$\frac{c'(x)}{c(x)} = \lambda \frac{\mu^*(x)-1}{\mu^*(x)} - \frac{\mu^{*\prime}(x)}{\mu^*(x)}, \tag{3.24}$$

together with the boundary condition

$$c(1) = \frac{u}{\mu^*(1)}. \tag{3.25}$$

The solution to the above differential equation is

$$c(x) = C\int_0^x \exp\left(\lambda \frac{\mu^*(x)-1}{\mu^*(x)} - \frac{\mu^{*\prime}(x)}{\mu^*(x)}\right) d\hat{x}, \tag{3.26}$$

where the constant of integration C is such that $c(1) = u/\mu^*(1)$. Clearly, the cost function $c(x)$ in (3.26) is a proper quantile function, since $c'(x)$ is guaranteed to be strictly positive by the choice of λ. By construction, the cost function $c(x)$ together with λ and u generates the desired markup function $\mu^*(x)$.

I have thus established the following "anything goes" result:

Theorem 3. *Any twice-continuously differentiable markup function $\mu^*(x)$ with $\mu^*(x) > 1$ for all $x \in [0,1]$ can be generated as an equilibrium outcome of the search-theoretic model of imperfect competition given some λ such that $\lambda(\mu^*(x)-1) > \mu^{*\prime}(x)$ for all $x \in [0,1]$ and a quantile function $c(x)$ of marginal costs across sellers given by*

$$c(x) = \frac{\mu^*(1)}{u} \frac{\int_0^x \exp\left(\lambda \frac{\mu^*(x)-1}{\mu^*(x)} - \frac{\mu^{*\prime}(x)}{\mu^*(x)}\right) d\hat{x}}{\int_0^1 \exp\left(\lambda \frac{\mu^*(x)-1}{\mu^*(x)} - \frac{\mu^{*\prime}(x)}{\mu^*(x)}\right) d\hat{x}}. \tag{3.27}$$

Following the same arguments as in Section 2, it is easy to show that the markups generated by the search-theoretic model can be rationalized by the Dixit-Stiglitz model. If markups are decreasing in size, buyers must have an elasticity of substitution that is increasing in the consumption of a particular variety. If markups are increasing in size, buyers must have an elasticity of substitution that is decreasing in the consumption of a particular variety. If markups are non-monotonic in size, the buyers' elasticity of substitution must be non-monotonic in consumption of a particular variety. The reduced-form preferences that rationalize the markups in the Dixit-Stiglitz model depend on the parameters of the Burdett-Judd model and, in this sense, are unstable. Following the same arguments as in Section 2, it is also easy to show that, when interpreted through the lens of the Dixit-Stiglitz model, markups imply an inefficiently low level production and an inefficient allocation of inputs across sellers. These conclusions about welfare are incorrect, since the equilibrium of the search-theoretic model is efficient.

Theorem 3 shows that the search-theoretic model can generate any pattern of markups that one might observe in the data. For this reason, markup data cannot be used to reject the search-theoretic model. As argued above, the monopolistic competition model can reproduce the same markups that are generated by the search-theoretic model. Therefore, both theories are consistent with empirical evidence on markups. Since the two theories have different implications about welfare, policy, and counterfactuals, markup data alone cannot be used to reach any definitive conclusions about welfare and optimal policy. Similarly, markup data alone cannot be used to reach any definitive conclusions about counterfactuals, such as the equilibrium and welfare effects of opening up to international trade. In order to make any predictions, one needs evidence on the source of market power in product markets.

Obviously, I could modify the search-theoretic model to make the equilibrium inefficient. For instance, I could remove the assumption of unit demand, or I could introduce consumption or production externalities. I could even write an entirely different model of markups where the equilibrium is inefficient. These observations do not diminish the main point of the paper: there exist two models that are based on sensible assumptions, that are well-established in the literature, that can explain the pattern of markups observed in the data, but have very different implications about welfare, policy and counterfactuals. Noting that there might be other models that can generate the empirical pattern of markups and might have yet different implications about welfare and policy only strengthens my point. Markup data alone is not sufficient to reach any firm conclusions about the macroeconomic implications of markups.

3.4 Determinants of the markup distribution

Theorem 3 states that any markup function can be rationalized by the appropriate choice of parameters of the model. I now want to understand how changes in the parameters of the model affect the markup function. To this aim, let me restrict attention to the log-uniform family of distributions for the sellers' marginal costs. That is, let me restrict attention to the family of distributions $\Phi(c)$ given by

$$\Phi(c) = 1 - \frac{\log c_h - \log c}{\log c_h - \log c_\ell}. \tag{3.28}$$

The marginal cost $c(x)$ for a seller at the x-th quantile of the distribution $\Phi(c)$ in (3.28) is

$$c(x) = c_h e^{-\kappa(1-x)}, \tag{3.29}$$

where κ is defined as

$$\kappa = \log c_h - \log c_\ell. \tag{3.30}$$

Expressed as in (3.29) the distribution $\Phi(c)$ of marginal costs across sellers depends on the parameters c_h and κ. The parameter c_h describes the marginal cost of the least efficient seller in the market. The parameter κ describes how quickly the seller's marginal cost declines as one moves from the top to the bottom of the distribution.

Given the cost distribution (3.29), the differential equation (3.20) simplifies to

$$\mu'(x) = \lambda(\mu(x) - 1) - \mu(x)\kappa. \tag{3.31}$$

The solution to the differential equation (3.31) that satisfies the boundary condition (3.21) is

$$\mu(x) = \left(\frac{u}{c_h} - \frac{\lambda}{\lambda - \kappa}\right) e^{-(\lambda-\kappa)(1-x)} + \frac{\lambda}{\lambda - \kappa}. \tag{3.32}$$

Let me begin by examining how the extent of competition, captured by the search parameter λ, affects the distribution of markups across sellers. First, notice that the markup function $\mu(x)$ is strictly decreasing in λ. To see why this is the case, consider λ_0 and λ_1, with $\lambda_0 < \lambda_1$. Let $\mu_0(x)$ denote the markup function associated with λ_0, and $\mu_1(x)$ the markup function associated with λ_1. Suppose that $\mu_0(x_0) = \mu_1(x_0)$ for some $x_0 \in [0, 1]$. From (3.31) it follows that $\mu_1'(x_0) > \mu_0'(x_0)$. In other words, if the markup functions $\mu_0(x)$ and $\mu_1(x)$ ever cross, $\mu_1(x)$ crosses $\mu_0(x)$ from below. Since the markup functions are continuous, this property implies that they can cross at most at one x_0. Moreover, $\mu_1(x) < \mu_0(x)$ for any $x \in [0, x_0)$ and $\mu_1(x) > \mu_0(x)$ for any $x \in (x_0, 1]$. Since $\mu_1(1) = \mu_0(1) = u/c_h$, it follows that $\mu_1(x) < \mu_0(x)$ for all $x \in [0, 1)$.

Second, notice that the sign of the slope of the markup function $\mu(x)$ depends on λ. In particular, there is a cutoff λ^* defined as

$$\lambda^* = \frac{u/c_h}{u/c_h - 1}\kappa. \tag{3.33}$$

For any $\lambda \in (0, \kappa]$, the nullcline $\mu_n(x)$ is infinite and, hence, the markup function $\mu(x)$ is strictly decreasing in x. For any $\lambda \in (\kappa, \lambda^*)$, the nullcline $\mu_n(x)$ is a finite constant that lies above the boundary condition u/c_h. Also in this case, the markup function $\mu(x)$ is strictly decreasing in x. For $\lambda = \lambda^*$, the nullcline is a finite constant that is equal to the boundary condition u/c_h. In this case, the markup function $\mu(x)$ is equal to the nullcline and independent of x. For $\lambda > \lambda^*$, the nullcline $\mu_n(x)$ is a finite constant that lies above the boundary condition u/c_h. In this case, the markup function $\mu(x)$ is strictly increasing in x. These properties are illustrated in Figure 2.

Overall, when competition is weak, in the sense that $\lambda < \lambda^*$, markups are high and decreasing in the seller's rank in the cost distribution Φ and in the price distribution F, which implies that markups are decreasing in a seller's price and increasing in a seller's size. When competition is strong, in the sense that $\lambda > \lambda^*$, markups are low and increasing in the seller's rank, which implies that markups are increasing in a seller's price and decreasing in a seller's size. When $\lambda = \lambda^*$, markups are intermediate and independent of the seller's rank, which implies that markups are independent of a seller's price and size.

There is a simple intuition for these findings. When λ is low, sellers are unlikely to compete for the same buyers and, for this reason, they can charge high markups. Moreover, when λ is low, low-cost sellers do not face much competitive pressure from

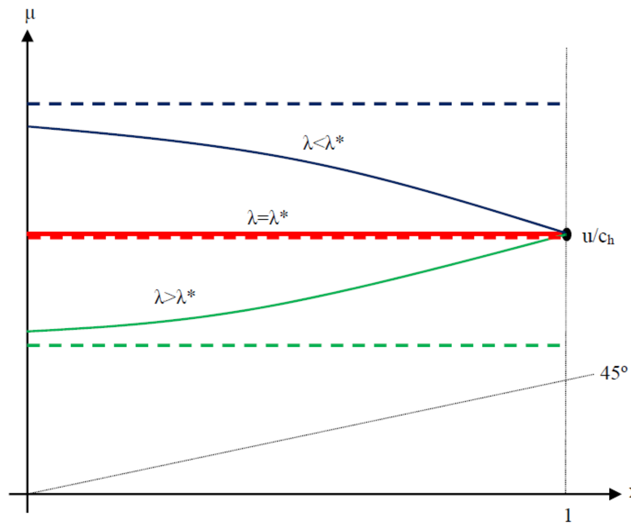


Figure 2: Markup function $\mu(x)$ and λ

high-cost sellers and, for this reason, they can charge higher markups than high-cost sellers. For instance, when almost all of the seller's potential customers are captive, every seller can charge a price close to the monopoly price u and, hence, markups are high, and they are higher for low-cost sellers. When λ is high, sellers are likely to compete for the same buyers and, for this reason, they have to charge low markups. Moreover, when λ is high, low-cost sellers are pushed by less efficient competitors to post prices that are so low as to make their markups lower. For instance, when buyers have a large number of contacts, low-cost sellers are pushed by less efficient competitors to charge prices close to marginal costs and, hence, they have negligible markups. Seller with the highest marginal costs, however, do not face any competitive pressure from above and, hence, they can charge markups close to u/c_h.

Next, let me examine the effect of the buyer's valuation u on the distribution of markups across sellers. First, notice that the markup function $\mu(x)$ is strictly increasing in u. To see why this is the case, consider u_0 and u_1, with $c_h < u_0 < u_1$. Let $\mu_0(x)$ denote the markup function associated with u_0, and $\mu_1(x)$ the markup function associated with u_1. The markup functions $\mu_0(x)$ and $\mu_1(x)$ are both solutions to the differential equation (3.31) but satisfy different boundary conditions. The boundary condition for $\mu_0(x)$ is $\mu_0(1) = u_0/c_h$, while the boundary condition for $\mu_1(x)$ is $\mu_1(1) = u_1/c_h$. Since $\mu_0(x)$ and $\mu_1(x)$ are solutions to the same differential equation, they cannot cross. Since $\mu_0(1) < \mu_1(1)$, $\mu_1(x)$ must be greater than $\mu_0(x)$ for all $x \in [0, 1]$.

Second, notice that the sign of the slope of the markup function $\mu(x)$ depends on u. For $\lambda \leq \kappa$, the nullcline $\mu_n(x)$ is infinite and, hence, the markup function $\mu(x)$ is strictly decreasing in x for any $u > c_h$. For $\lambda > \kappa$, the sign of the slope of the markup function

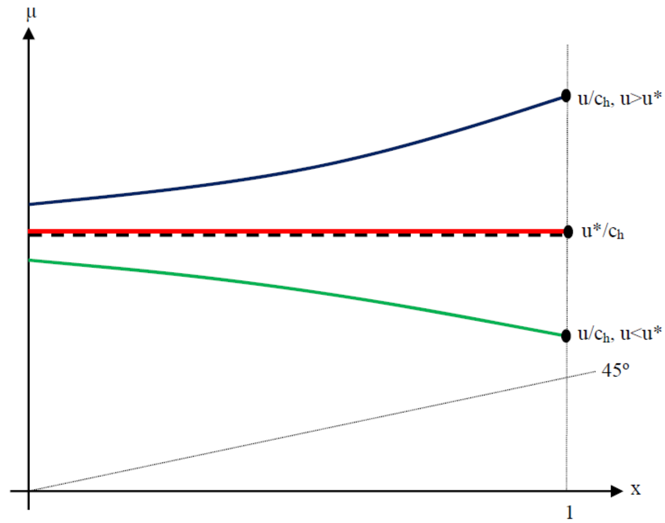


Figure 3: Markup function $\mu(x)$ and u

depends on u. In particular, there is a cutoff u^* defined as

$$u^* = \frac{\lambda/\kappa}{\lambda/\kappa - 1} c_h. \tag{3.34}$$

For $u \in (c_h, u^*)$, the nullcline $\mu_n(x)$ is a finite constant μ_n, with $\mu_n > u/c_h$. In this case, the markup function $\mu(x)$ is strictly decreasing in x. For $u = u^*$, the nullcline $\mu_n(x)$ is a finite constant μ_n, with $\mu_n = u/c_h$. In this case, the markup function $\mu(x)$ is independent of x. For $u > u^*$, the nullcline $\mu_n(x)$ is a finite constant μ_n, with $\mu_n < u/c_h$. In this case, the markup function $\mu(x)$ is strictly increasing in x. These properties are illustrated in Figure 3.

Let me explain the findings above. Markups are increasing in the buyer's valuation u. Intuitively, the higher is the buyer's valuation for the good, the higher is the markup charged by the seller with the highest marginal cost. In turn, if the seller with the highest marginal cost charges a higher markup, sellers with a lower marginal cost can also charge a higher markup. If the buyer's valuation u is below u^*, markups are decreasing in the seller's rank in the cost distribution Φ and in the price distribution F and, hence, they are decreasing in the seller's price and increasing in the seller's size. If the buyer's valuation u is above u^* (and $\lambda > \kappa$), markups are increasing in the seller's rank and, hence, they are increasing in the seller's prices and decreasing in the seller's size. Intuitively, for low-cost sellers the markup approaches $\lambda/(\lambda - \kappa)$, which is independent of u. For high-cost sellers the markup approaches u/c_h. Therefore, if u is low enough, markups are decreasing in the seller's cost. If u is high enough, markups are increasing in the seller's cost.

Given the effect of the buyer's valuation u on the distribution of markups across sellers, it is immediate to derive the effect of the seller's highest marginal cost c_h. Indeed, the solution (3.32) to the differential equation (3.31) depends on u and c_h only though their ratio u/c_h. Therefore, the effect of c_h on the distribution of markups is the opposite of the effect of u on the distribution of markups. Namely, the markup function $\mu(x)$ is strictly decreasing in c_h. For $\lambda \leq \kappa$, the markup function $\mu(x)$ is strictly decreasing in x for all $c_h \in (0, u)$. For $\lambda > \kappa$, the markup function $\mu(x)$ is strictly increasing in x for all $c_h \in (0, c_h^*)$, it is independent of x for $c_h = c_h^*$, and it is strictly decreasing in x for all $c_h \in (c_h^*, u)$, where the cutoff c_h^* is given by

$$c_h^* = \frac{\lambda/\kappa - 1}{\lambda/\kappa} u. \tag{3.35}$$

Lastly, I want to consider the effect of the parameter κ on the distribution of markups across sellers. Taking c_h as given, the parameter κ controls how steeply marginal costs decline as we move from the top to the bottom quantile of the cost distribution Φ. For $\kappa \to 0$, the marginal costs are approximately constant as we move from the top to the bottom quantile of Φ. In other words, for $\kappa \to 0$, all sellers have approximately a marginal cost equal to c_h. The higher is κ, the faster marginal costs decline as we move from the top to the bottom quantile of Φ. For $\kappa \to \infty$, almost all sellers have a marginal cost approximately equal to 0.

First, notice that the markup function $\mu(x)$ is strictly increasing in κ. To see why this is the case, consider κ_0 and κ_1, with $0 < \kappa_0 < \kappa_1$. Let $\mu_0(x)$ denote the markup function associated with κ_0, and $\mu_1(x)$ the markup function associated with κ_1. Suppose that $\mu_0(x_0) = \mu_1(x_0)$ for some $x_0 \in [0, 1]$. From (3.31) it follows that $\mu_1'(x_0) < \mu_0'(x_0)$. In other words, if the markup functions $\mu_0(x)$ and $\mu_1(x)$ ever cross, $\mu_1(x)$ crosses $\mu_0(x)$ from above. Since the markup functions are continuous, this property implies that they can cross at most at one x_0. Moreover, $\mu_1(x) > \mu_0(x)$ for any $x \in [0, x_0)$ and $\mu_1(x) < \mu_0(x)$ for any $x \in (x_0, 1]$. Since $\mu_1(1) = \mu_0(1) = c_h/u$, it follows that $\mu_1(x) > \mu_0(x)$ for all $x \in [0, 1)$.

Second, notice that the sign of the slope of the markup function $\mu(x)$ depends on κ. Specifically, there is a cutoff κ^* given by

$$\kappa^* = \frac{u/c_h - 1}{u/c_h} \lambda. \tag{3.36}$$

For any $\kappa \in (0, \kappa^*)$, the nullcline $\mu_n(x)$ is a finite constant that lies below the boundary condition u/c_h. In this case, the markup function $\mu(x)$ is strictly increasing in x. For $\kappa = \kappa^*$, the nullcline $\mu_n(x)$ is a finite constant that is equal to the boundary condition u/c_h. In this case, the markup function $\mu(x)$ is independent of x. For any $\kappa \in (\kappa^*, \lambda)$, the nullcline $\mu_n(x)$ is a finite constant that lies above the boundary condition u/c_h. In this case, the markup function $\mu(x)$ is strictly decreasing in x. For any $\kappa \geq \lambda$, the nullcline is infinite and the markup function $\mu(x)$ is also strictly decreasing in x. These findings are illustrated in Figure 4.

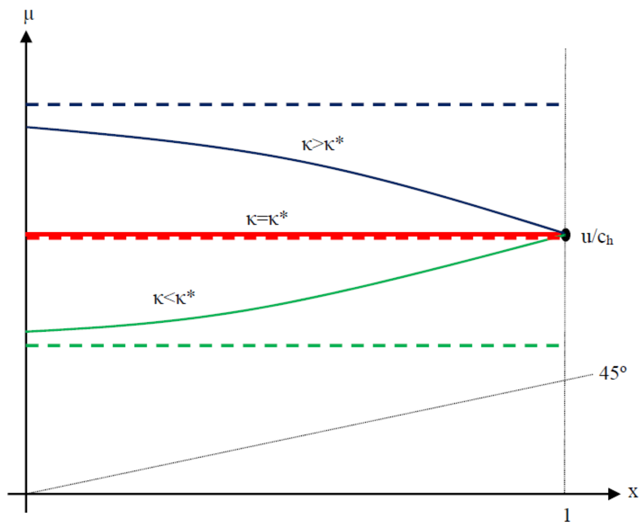


Figure 4: Markup function $\mu(x)$ and κ

The finding that markups are increasing in κ is intuitive. The higher is κ, the lower are the sellers' marginal costs and, for this reason, the higher are the markups that they charge. There is also a simple intuition for the finding that markups are increasing in x (and, hence, increasing in a seller's price and decreasing in a seller's size) for low values of κ, and that they are decreasing in x (and, hence, decreasing in a seller's prices and increasing in a seller's size) for high values of κ. For low values of κ, all sellers have marginal costs close to c_h. Therefore, as in the version of the model with homogeneous sellers, sellers that charge a higher price have a higher markup than sellers that charge a lower price. For high values of κ, low-cost sellers have a marginal cost close to zero and, hence, a very large markup. High-cost sellers, on the other hand, have a marginal cost close to c_h and a markup close to u/c_h.

Let me summarize the above analysis in the following theorem.

Theorem 4. *Let the distribution $\Phi(c)$ of marginal costs across sellers be log-uniform over the interval $[c_h \exp(-\kappa), c_h]$, with $\kappa > 0$.*

(i) *The markup function $\mu(x)$ is strictly decreasing in λ. The markup function is strictly decreasing in x for $\lambda \in (0, \lambda^*)$, independent of x for $\lambda = \lambda^*$, and strictly increasing in x for $\lambda > \lambda^*$, where λ^* is given by (3.33)*

(ii) *The markup function $\mu(x)$ is strictly increasing in u. For $\lambda \leq \kappa$, the markup function is strictly decreasing in x. For $\lambda > \kappa$, the markup function is strictly decreasing in x for $u \in (c_h, u^*)$, independent of x for $u = u^*$, and strictly increasing in x for $u > u^*$, where u^* is given by (3.34).*

(iii) *The markup function $\mu(x)$ is strictly decreasing in c_h. For $\lambda \leq \kappa$, the markup function is strictly decreasing in x. For $\lambda > \kappa$, the markup function is strictly* *increasing in x for $c_h \in (0, c_h^*)$, independent of x for $c_h = c_h^*$, and strictly increasing in x for $c_h \in (c_h^*, u)$, where c_h^* is given by (3.35).*

(iv) *The markup function $\mu(x)$ is strictly increasing in κ. The markup function is strictly increasing in x for $\kappa \in (0, \kappa^*)$, independent of x for $\kappa = \kappa^*$, and strictly decreasing in x for $\kappa > \kappa^*$, where κ^* is given by (3.36).*

Theorem 4 identifies the forces that determine the level and the shape of markups in the search-theoretic model of imperfect competition. Markups decrease with the extent of competition in the market λ, increase with the buyers' valuation for the good u, and with the rate κ at which the sellers' marginal costs decline as one goes from the top to the bottom of the cost distribution Φ. Markups are increasing in prices and decreasing in quantities when the extent of competition in the market is sufficiently strong, when the buyers' valuation for the good is sufficiently high, and when the sellers' marginal costs decline slowly enough. In contrast, markups are decreasing in prices and increasing in quantities when the extent of competition in the market is sufficiently weak, when the buyers' valuation for the good is sufficiently low, and when the sellers' marginal costs decline quickly enough. Between the region where markups are increasing in prices and the region where markups are decreasing in prices lies a knife-edge where markups are constant.

Theorem 4 applies only to the family of log-uniform cost distributions. Some of the results in Theorem 4, however, generalize to arbitrary cost distributions. For instance, it is immediate to see that the proof that the markup function $\mu(x)$ is strictly decreasing in λ, strictly increasing in u, strictly decreasing in c_h, and strictly increasing in $\kappa(x) = c'(x)/c(x)$ generalizes to any arbitrary cost distribution Φ. Partial analogues of the effect of λ, u, c_h and $\kappa(x)$ on the slope of the markup function $\mu(x)$ can also be derived for arbitrary cost functions. I am not going to report these results, as they tediously depend on the shape of the nullcline.

4 Conclusions

I characterized the equilibrium distribution of markups in the search-theoretic model of imperfect competition of Butters (1977), Varian (1980), and Burdett and Judd (1983). Markups are positive, even though sellers produce varieties that buyers perceive as perfect substitutes. Markups are heterogeneous, even when sellers produce varieties at the same marginal cost. Markups may be increasing, decreasing, or constant in a seller's size, even though the degree of substitutability between varieties is invariant to consumption. Moreover, markups are efficient. If these markups were interpreted through the lens of the model of monopolistic competition model of Dixit and Stiglitz (1977), one would reach incorrect conclusions about welfare and policy. If these markups were interpreted through the lens of Dixit and Stiglitz, one would infer buyers' preferences that are incorrect and, more importantly, unstable to changes in the environment. These findings suggest using some caution when interpreting the empirical evidence on markups.

As a rhetorical tool, I assumed that the data-generating process was the search-theoretic model of imperfect competition, and I asked whether one would reach some incorrect conclusions by interpreting the data through the lens of the Dixit-Stiglitz model. In reality, both theories are likely to be overly simplified descriptions of the world. Yet, and this is the point of the paper, the two theories build on two very different sources of market power, they provide two very different interpretations of markups, and they have very different implications for welfare and policy. The stark difference between the two theories suggests that it is critical to identify the relative importance of information frictions and product differentiation in the creation of market power. In other words, the question to be answered is "How much of the downward sloping demand curve facing a seller is due to the heterogeneity in buyer's outside options and how much is it due to preferences?"

The analysis contained in this paper does not only apply to product markets, but also to the labor market. It is straightforward to derive a closed-form formula for equilibrium markdowns in the search-theoretic model of the labor market of Burdett and Mortensen (1998), which is essentially a dynamic spin-off of Burdett and Judd (1983). The formula reveals that markdowns are positive, even though employers are perfect substitutes from the perspective of workers. The formula reveals that markdowns are heterogeneous, even when firms operate the same production technology. And that markdowns may be increasing, decreasing, or constant in the size of a firm. As in Burdett and Judd (1983), the equilibrium is efficient in Burdett and Mortensen (1998). Therefore, the same caution that I recommend using when interpreting markups should be applied to the interpretation of markdown data.

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