Efficient Imperfect Competition with an Application to International Trade*

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Abstract

I study the equilibrium and the welfare effects of international trade when product markets are imperfectly competitive due of search frictions—as in Burdett and Judd (1983)—rather than product differentiation—as in Dixit and Stiglitz (1977). Markups are positive, even though there are multiple firms producing identical goods. Markups depend negatively on the number of firms producing identical goods, which, in turn, determines the extent of competition in the market. Markups may be increasing, constant, decreasing or non-monotonic in firm's size, depending on the extent of competition and on the distribution of marginal costs. The entry of firms and the quantity of output produced by each firm are efficient, even though the market is imperfectly competitive. International trade increases the measure of firms in the market, intensifies competition, lowers markups, and unambiguously increases welfare. These "natural" effects of trade emerge generically in the Burdett-Judd model of imperfect competition. In the Dixit-Stiglitz model of imperfect competition, these effects are an artifact of particular specifications of preferences.

JEL Codes: D43, D83, F12, L16.

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1 Introduction

I study the equilibrium and welfare effects of international trade when product markets are imperfectly competitive because of search frictions—as in Butters (1977), Varian (1980), and Burdett and Judd (1983)—rather than product differentiation—as in Dixit and Stiglitz (1977). I show that markups are positive, even though there are multiple

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firms producing the same good. In the Dixit-Stiglitz framework markups are positive only to the extent that firms produce differentiated goods. I show that markups depend on the extent of competition in the market, which is determined by the number of active firms. In the Dixit-Stiglitz framework, markups depend exclusively on the elasticity of substitution between varieties in the buyers' utility function. The number of firms and the quantity of output produced by each firm are efficient, even though the product market is imperfectly competitive. In the Dixit-Stiglitz framework, equilibrium entry and output are generally inefficient. Opening up to international trade increases the measure of active firms and, hence, lowers markups. In the Dixit-Stiglitz framework, international trade does not generically lower markups. International trade unambiguously improves welfare, even though product markets are imperfectly competitive. In the Dixit-Stiglitz framework, international trade need not increase welfare.

The analysis is centered around a general equilibrium version of the search-theoretic model of imperfect competition by Butters (1977), Varian (1980) and Burdett and Judd (1983). Households demand a unit of a continuum of goods. Firms enter the market for a particular good by paying a fixed cost. After paying the fixed cost, firms realize an idiosyncratic marginal cost of production and post prices. Informational or physical frictions limit the extent of competition in the market for a particular good. Specifically, households can only purchase the good from a discrete subset of firms of size n, where n is a random variable with a mean that depends linearly in the number of firms in the market. The input of production is a numeraire good that is supplied by households and demanded by firms in a perfectly competitive market.

In the first part of the paper, I study a closed-economy version of the model. Equilibrium exists and is unique. Equilibrium is characterized by a free-entry condition and an optimal pricing condition. The free-entry condition pins down the equilibrium measure of firms, and depends on the entry cost and on the population of households. The optimal pricing condition maps the rank of a firm in the marginal cost distribution to its price. The optimal pricing condition is such that the markup charged by a particular firm is strictly decreasing in the measure of firms in the market, markups converge to zero when the measure of firms in the market goes to infinity, and markups converge to their monopoly values when the measure of firms in the market goes to zero. Markups depend negatively on the measure of firms in the market because, quite naturally, the measure of firms in the market affects the size of the households' choice sets and, in turn, the extent of competition. Markups may be increasing, decreasing, constant or non-monotonic in the size of a firm, depending on the measure of firms in the market and the distribution of marginal costs across firms. In the Dixit-Stiglitz framework, markups may increase, decrease or be constant in the measure of firms, depending on the structure of buyers' preferences. Similarly, the structure of buyers' preferences determines whether markups are decreasing, increasing or constant in the size of a firm.

Equilibrium is efficient, in the sense that it decentralizes the solution of a utilitarian social planner. The measure of firms entering the market for a particular good is efficient. The quantity of output produced by each firm and the quantity of output consumed by each household are efficient. Equilibrium entry is efficient because the profits enjoyed by a firm exactly reflect the social value of the meetings between the firm and the households. Equilibrium quantities are efficient because households purchase from the firm with the lowest price among those that they contact, and the firm with the lowest price is the firm with the lowest marginal cost. In the Dixit-Stiglitz framework, in contrast, the entry of firms and the quantity of output produced by firms are generally inefficient.

In the second part of the paper, I study an open-economy version of the model. Specifically, I consider a world economy comprised of two identical countries. The measure of firms that operate in the market for a particular good is higher when international trade is allowed than in autarky. The markup charged by a firm with a given marginal cost is lower when international trade is allowed than in autarky. The equilibrium effects of international trade are the result of natural economic forces. When international trade is allowed, the market for a particular good is larger and, for this reason, more firms find it optimal to enter. When more firms enter, the market becomes more competitive, and firms find it optimal to lower their markups. In the Dixit-Stiglitz framework, these natural effects of international trade can be mimicked only when buyers' preferences are such that the elasticity of substitution between the variety produced by a firm and the other varieties is decreasing in the quantity of that firm's variety that is consumed by buyers.

International trade always increases welfare. I derive a formula for the welfare effects of trade. International trade has a shopping effect—the change in welfare caused by the increase in the measure of firms, keeping prices constant—and a competition effect—the change in welfare caused by the decline in markups, keeping the measure of firms constant. Both the shopping effect and the competition effects are strictly positive and, hence, international trade unambiguously increases welfare. In the Dixit-Stiglitz framework, international trade need not increase welfare. Intuitively, in the Dixit-Stiglitz framework, equilibrium is inefficient and, hence, a positive technological change (international trade) need not increase welfare. In my framework, equilibrium is efficient and, hence, any positive technological change is necessarily welfare improving.

In the third part of the paper, I generalize the model of international trade in several directions. First, I consider a version of the model in which households are more likely to contact local than foreign sellers. I refer to the gap between the likelihood that a household contacts a local firm and a foreign firm as the informational distance between the two countries. I show that the equilibrium and welfare effects of international trade are monotonically decreasing in the informational distance between the two countries. I show that, whenever there is some informational distance, the household's consumption features some home bias. Second, I consider a version of the model in which the two countries are asymmetric in size. I show that, when the two countries are asymmetric in size, a firm finds it optimal to price to market. Specifically, a firm charges a higer price and a higher markup in the smaller country, where prices are higher, than in the larger country, where prices are lower. Third, I consider a version of the model in which firms have to pay a fixed cost to become exporters. I show that only the more efficient firms choose to become exporters, while the less efficient firms only sell in the local market. For all of these extensions, I show that the equilibrium and welfare effects of international trade are the same, qualitatively, as in the baseline model.

In the last part of the paper, I consider a version of the model in which the input of production is labor rather than a numeraire good, and the household's supply of labor is imperfectly elastic. I establish the existence and uniqueness of equilibrium. I establish the efficiency of equilibrium. I establish that international trade unambiguously increases welfare.

Related Literature. The paper is an application of the framework of imperfect competition by Butters (1977), Varian (1980), and Burdett and Judd (1983). One appealing feature of the framework is that it generates price dispersion. This feature of the framework has motivated a number of empirical applications to price dispersion (Sorensen 2000, Hong and Shum 2006, Kaplan and Menzio 2015, Menzio 2023) and price stickiness (Head et al. 2012 and Burdett and Menzio 2018). Another appealing feature of the framework is that the extent of competition spans the spectrum from pure monopoly to perfect competition as one changes the distribution of the size of the buyers' choice sets. This feature of the framework has motivated a strand of empirical applications focused on the effect of buyers' heterogeneity with respect to shopping activities (Kaplan and Menzio 2016, Pytka 2018, Nord 2023 and Sangani 2023). In Menzio (2024b), I derive a formula for the distribution of markups in a version of the framework with heterogeneous firms, and characterize the effect of search frictions on the level of markups and on the relationship between markups and firm's size. I also show that production and consumption are efficient, even though markups are positive and generally heterogeneous. In this paper, I expand on Menzio (2024b) by endogenizing the entry of firms and showing that production, consumption, and entry are all efficient. I apply the model to study the equilibrium and welfare effects of international trade.

The efficiency of entry, production and consumption established in this paper stands in sharp contrast with the welfare properties of the monopolistic competition framework of Dixit and Stiglitz (1977). In the Dixit-Stiglitz framework, positive markups imply that production and consumption are inefficiently low, as long as the supply of inputs is not perfectly inelastic (Edmond, Midrigan and Xu 2023). Heterogeneous markups imply that production and consumption are misallocated, so that high-markup firms produce too little compared to low-markup firms (Baqaee, Farhi and Sangani 2022, Edmond, Midrigan and Xu 2023). The entry of firms is generally inefficient, and may be too high or too low depending on the relative strength of a negative business stealing externality and a positive surplus externality (Mankiw and Whinston 1985). Only when preferences are CES, the entry of firms is efficient (Dhingra and Morrow 2019). Overall, in the Dixit-Stiglitz framework, equilibrium is efficient only when inputs are fixed and preferences are CES. This paper shows that observing that firms have market power does not imply that equilibrium is inefficient. The source of the market power matters. Since there is not much evidence on the source of the firms' market power, some caution must be used when making policy recommendations based on the measurement of markups alone.

The paper also contributes to the literature on international trade. Starting with Krugman (1980), a large strand of the literature has studied the equilibrium and welfare effects of trade in imperfectly competitive markets modelled as in Dixit and Stiglitz (1977). In these models, international trade leads to an increase in the measure of varieties that are available to buyers. The increase in available varieties, however, does not have an effect on the extent of competition in the market, because the firms' market power is dictated by the structure of buyers' preferences, not by the interactions between firms. Indeed, in Krugman (1980) and Melitz (2003), international trade does not affect markups at all. Given that the idea that the number of firms of firms should increase competition is so natural, there have been several attempts at modifying the structure of preferences so that international trade would lead to lower markups. In Krugman (1979) and many subsequent papers (e.g., Baqaee, Sangani and Farhi 2022, Edmond, Midrigan and Xu 2023), the elasticity of substitution between varieties is assumed to be decreasing in the quantity of a particular variety that is consumed by a buyer. International trade increases the number of varieties and lowers the quantity of each variety, which, under such preferences, causes markups to fall. International trade lowers markups, but it does so by rigging preferences. In the search-theoretic framework of imperfect competition presented here, international trade lowers markups because it intensifies competition between firms.

The welfare effects of international trade are also different than in the trade literature that builds on the Dixit-Stiglitz framework. Since equilibrium is not efficient in Dixit-Stiglitz, there is no guarantee that a positive technological shock such as opening a country to trade would increase welfare. Helpman and Krugman (1985) and Dhingra and Morrow (2019) derive sufficient conditions under which international trade is welfare-improving. In the search-theoretic model of imperfect competition presented here, equilibrium is efficient. Therefore, any positive technological shock, including opening a country to international trade, increases welfare.

There is an earlier literature that uses the search-theoretic framework of imperfect competition of Burdett and Judd (1983) to study issues in international trade. Alessandria (2004, 2009) uses the framework to study international deviation from the law of one price. Alessandria and Kaboski (2011) use the framework to study pricing-to-market. Herrenbrueck (2015, 2017) uses the framework to study the effect of monetary policy on international terms of trade. Even though these papers use the Burdett-Judd framework as I do, the models and the focus are quite different.

2 Closed Economy

In this section, I propose a general equilibrium version of the search-theoretic model of imperfect competition by Butters (1977), Varian (1980) and Burdett and Judd (1988), in which the measure of firms producing and selling a particular good is endogenously determined by free entry. In Section 2.1, I establish the existence and uniqueness of equilibrium, as well as its properties. In Section 2.2, I formulate and solve the problem of a utilitarian social planner, and use the solution to establish the efficiency of equilibrium. Throughout the section, I compare the properties of the search-theoretic model of imperfect competition with the properties of the monopolistic model of monopolistic competition by Dixit and Stigliz (1977). In contrast to Dixit and Stiglitz (1977), markups are endogenous and determined by the extent of competition in the market, which, in turn, is determined by the measure of active firms. In contrast to Dixit and Stiglitz (1977), the equilibrium is efficient with respect to both the measure of active firms, and the quantity of output produced by each firm.

2.1 Environment

The economy is populated by ex-ante identical households (buyers) and ex-ante identical firms (sellers). Households supply a numeraire good that is used by firms as an input of production. Households and firms trade the numeraire good in a perfectly competitive market. Households demand a unit of a continuum of search goods indexed by $i \in [0,1]$ that are produced by firms. Households and firms trade the search goods in frictional markets.

Let me describe households in more detail. The economy is populated by a continuum of households with measure $b > 0$ per market i. The preferences of a household are described by

$$U(y,z) = \int_i y_i u di + z, \tag{2.1}$$

where $z \in \mathbb{R}$ denotes consumption of the numeraire good, $y_i \in \{0,1\}$ is an indicator function that takes the value 1 if the household consumes one unit of good i, and $u > 0$ is a parameter that describes the household's utility from consuming one unit of good i expressed in terms of the utility of the numeraire good. Households are endowed with $h > 0$ units of the numeraire good and with the ownership of the firms.[1]

Next, let me describe firms in more detail. The economy is populated by a positive measure of firms per market $i \in [0,1]$. The measure of firms in the market for search good i is endogenous. In order to enter the market for search good i, a firm has to pay a quantity $\zeta > 0$ of the numeraire good. After entering the market for search good i, a firm draws

[1]The utility function in (2.1) is linear in the consumption z of the numeraire, which is allowed to be negative. For the readers that are uncomfortable with the notion of negative consumption, it is worth poiting out that $z \geq 0$ for a sufficiently large endowment h of the numeraire.

its idiosyncratic type c from the distribution $\Phi(c)$, where $\Phi(c)$ is a twice-continuously differentiable cumulative distribution with support $[c_\ell, c_h]$, $0 < c_\ell < c_h < u$. A firm of type c operates a constant return to scale technology such that it requires a quantity c of the numeraire good to produce a measure 1 of units of good i.[2] After observing its type, a firm chooses the price p_i for its good, where p_i is measured in units of the numeraire good.

The market for good $i \in [0, 1]$ is frictional, in the sense that a household cannot purchase good i from any firm in the market but only from the subset of firms with whom he comes into contact. In particular, a household in market i comes into contact with n_i randomly-selected firms, where n_i is a draw from a Poisson distribution with coefficient λs_i, $\lambda > 0$ is a coefficient and s_i is the measure of firms in market i.[3] The buyer observes the price charged by each of the n_i firms and decides whether and where to purchase a unit of the good.

The environment described above is a general equilibrium version of the imperfect competition framework of Burdett and Judd (1983). As in these models, buyers cannot purchase from any seller but only from a subset of them. A buyer cannot purchase from any seller because of information frictions (he is not aware of all the sellers, he does not understand the specifics of the variety of the good carried by all the sellers, etc...) or physical frictions (he cannot reach the seller). Instead, the buyer can purchase from a number of sellers that is randomly drawn from a Poisson distribution. Hence, the buyer may not be able to purchase the good, he may be able to purchase the good from a single seller, or he may be able to purchase the good from multiple sellers. Since buyers cannot purchase from any seller, the market need not be perfectly competitive. The competitiveness of the market and, in turn, prices and markups depend on the size of the buyers' choice sets. The size of the buyers' choice set is, on average, λs_i, where s_i is the measure of sellers in the market, which, in contrast to Burdett and Judd (1983), is endogenous and determined by free entry.

2.2 Existence, uniqueness and properties of equilibrium

In order to characterize an equilibrium of the economy, let me start by formulating the problem of a household. Consider a household that contacts n_i firms in the market for the search good i. The distribution of prices posted by firms in the market i is given by some distribution $F_i(p)$. Let $\tilde{p}_i$ denote the lowest price posted by one the n_i firms that the household contacts in market i. If $n_i = 0$, in the sense that the household does not

[2] Menzio (2024a) characterizes the equilibrium of the model when sellers operate a technology with decreasing returns to scale. In this paper, I stick to the standard assumption of constant returns to scale.

[3] The average number of contacts λs for an individual household is linear in the measure s of firms in the market. For this reason, the entry of an additional firm does not affect the probability that a households meets the other firms. This property seems the most natural in a consumer good context.

contact any firms in market i, let $\tilde{p}_i = +\infty$. The problem for the household is

$$\begin{aligned} &\max_{y_i, z} \textstyle\int_i y_i u di + z, \text{ s.t.} \\ &\textstyle\int_i y_i \tilde{p}_i + z = h + \Pi. \end{aligned} \tag{2.2}$$

The household's problem in (2.2) is easy to understand. The household chooses how much of the numeraire good to consume, $z \in \mathbb{R}$, and whether or not to consume one unit of good i at the price $\tilde{p}_i$, $y_i \in \{0, 1\}$, so as to maximize its utility subject to its budget constraint. The left-hand side of the budget constraint is the cost of the household's consumption given the prices that it faces. The right-hand side of the budget constraint is the value of the household's endowment, which is equal to the value h of its endowment of the numeraire good and the profit Π that it receives as owner of the firms. Solving the budget constraint with respect to z and substituting the solution into the objective function reveals that the household finds it optimal to consume a unit of good i if and only if $\tilde{p}_i \leq u$.

I now turn to the pricing problem of a firm in the market for good i. To keep notation light, I will omit the dependence of variables from i. Consider a firm of type c posting the price $p \in [0, u]$.[4] The firm's profit $V(p, c)$ is given by

$$V(p, c) = \left[\sum\nolimits_{n=0}^{\infty} b_n \pi_n(p)\right](p - c), \tag{2.3}$$

where b_n is given by

$$b_n = \frac{b}{s} \frac{e^{-\lambda s}(\lambda s)^{n+1}}{(n+1)!}(n+1), \tag{2.4}$$

and $\omega_n(p)$ is given by

$$\omega_n(p) = (1 - F(p))^n + \sum_{j=1}^{n} \binom{n}{j} \frac{\chi(p)^j (1 - F(p))^{n-j}}{j+1}, \tag{2.5}$$

where $\chi(p)$ denotes the fraction of firms posting the price p.

The firm's profit function in (2.3) is easy to understand. The firm meets a measure b_n of households that are in contact with n other sellers in the same market. The measure b_n is equal to the measure of households per seller, b/s, multiplied by the fraction of households $\exp(-\lambda s)(\lambda s)^{n+1}/(n+1)!$ that come into contact with $n+1$ sellers (including the firm), multiplied by the number of contacts $n+1$ that each one of these households have. The probability $\omega_n(p)$ that one of the b_n households purchases the good from the firm is the sum of the probability of two events. The first event is one in which all of the household's n other contacts post a price strictly greater than p. The second event is one in which j of the household's n other contacts post a price equal to p, the remaining $n - j$ contacts post a price strictly greater than p, and the household chooses to purchase

[4]Obviously, if a firm posts a price p strictly greater than u, it does not sell any good and it enjoys a profit of 0.

the good from the firm rather than from one of the other j sellers posting p. The sum of $b_n\omega_n(p)$ for $n = 1, 2, \ldots$ is the quantity of the good that is sold by the firm. The firm's profit for every unit sold is $p - c$.

The expression for the firm's profit in (2.3) can be used to establish some properties of the equilibrium price distribution F. Lemma 1 establishes that, in any equilibrium, the price distribution F does not have a mass point. The proof of the lemma is in Menzio (2024b, Lemma 1). The intuition for the lemma is the same as in Burdett and Judd (1983). If the price distribution F had a mass point at some price p_0, the demand curve faced by an individual firm would have a discontinuity at p_0. At the price p_0, the firm would sell the good to a fraction $1/(j + 1)$ of the positive measure of households that are in contact with j sellers posting the price p_0 and with $n - j$ sellers positing a price strictly greater than p_0. At any price $p_0 - \epsilon$, for any arbitrarily small $\epsilon > 0$, the firm would sell the good to all of the positive measure of households that are in contact with j sellers posting the price p_0 and with $n - j$ sellers posting a price greater than p_0. Therefore, no firm of type $c < p_0$ finds it optimal to post the price p_0. No firm of type $c \geq p_0$ finds it optimal to post the price p_0 because the firm can guarantee itself strictly a positive profit by positing the price u and selling the good to the households that are not in contact with any other seller. Since no firm finds it optimal to post p_0, F cannot have a mass point at that price.

Lemma 1: *In any equilibrium, the price distribution F does not have any mass points.*

In light of Lemma 1, I can rewrite the firm's profit $V(p, c)$ as

$$\begin{aligned} V(p,c) &= \left[\sum\nolimits_{n=0}^{\infty} \frac{b}{s} \frac{e^{-\lambda s}(\lambda s)^{n+1}}{(n+1)!}(n+1)(1-F(p))^n\right](p-c) \\ &= \left[b\lambda e^{-\lambda s F(p)} \sum\nolimits_{n=0}^{\infty} \frac{e^{-\lambda s(1-F(p))}(\lambda s)^n (1-F(p))^n}{n!}\right](p-c) \\ &= b\lambda e^{-\lambda s F(p)}(p-c). \end{aligned} \tag{2.6}$$

The first line in (2.6) is obtained by substituting the expressions for b_n and $\omega_n(p)$ into (2.3) and by making use of the fact that F does not have any mass points. The second line is obtained by collecting terms in the first line. The last line is obtained by noting that the summation in the second line equals 1.

Lemma 2 establishes that the support of the price distribution F is an interval $[p_\ell, p_h]$, with $p_h = u$. The proof of the lemma is in Menzio (2024b, Lemma 2). The intuition for the lemma is the same as in Burdett and Judd (1983). If the price distribution had a gap between any two prices p_0 and p_1, the demand curve faced by an individual firm would be flat. At the price p_0, the firm would sell the good to a households with n other contacts if and only if all of these other contacts post a price greater than p_0. At the price p_1, the firm would sell the good to a household with n other contacts if and only if all of these n other contacts post a price greater than p_1. Since there are no sellers posting prices between p_0 and p_1, the firm would sell the same quantity at p_0 and p_1, but it would enjoy a strictly greater profit margin at p_1. Therefore, no firm would find it optimal to post the price p_0. A similar argument lies behind the fact that the highest price on the support of F must be the buyers' valuation u.

Lemma 2: *In any equilibrium, the support of* $F(p)$ *is an interval* $[p_\ell, p_h]$, *with* $p_h = u$.

Lemma 3 establishes that the price posted by a firm is a strictly increasing function $p(c)$ of the firm's marginal cost c. The proof of the lemma is in Menzio (2024b, Lemma 3). There is a simple intuition for the lemma. First, consider a firm of type c_0 posting a price p_0, and a firm of type c_1 posting the price p_1, with $c_0 < c_1$. Revealed preferences imply that $p_1 \geq p_0$. That is, the price of a firm is weakly increasing in the firm's marginal cost. Second, suppose that p_1 was equal to p_0. Every firm of type $c \in (c_0, c_1)$ would have to post the price p_0 and, hence, the price distribution F would have a mass point. Since this would contradict Lemma 1, $p_1 > p_0$. That is, the price of a firm is strictly increasing in the firm's marginal cost. Third, suppose that a firm of type c_0 posted the price p_0 and another firm of type c_0 posted the price p_1, with $p_0 < p_1$. Every firm of type $c > c_0$ would have to post a price strictly greater than p_1. Every firm of type $c < c_0$ would have to post a price strictly smaller than p_1. Hence, the price distribution F would have a gap on its support. Since this would contradict Lemma 2, every firm with the same marginal cost must be posting the same price. That is, the firm's price is a function of its marginal cost.

Lemma 3: *The price posted by a seller is a strictly increasing function* $p(c)$ *of the seller's cost* c.

The price $p(c)$ posted by a firm of type c must satisfy the following optimality condition

$$b\lambda e^{-\lambda s F(p(c))} = b\lambda e^{-\lambda s F(p(c))} \lambda s F'(p(c))(p(c) - c). \tag{2.7}$$

The left-hand side of (2.7) is the marginal benefit of a price increase, which is given by the quantity of the good sold by the firm. The right-hand side of (2.7) is the marginal cost of a price increase, which is given by the decline in the quantity sold by the firm multiplied by the firm's profit margin. Condition (2.7) states that marginal cost and marginal benefit must be equated at the price $p(c)$.

The fraction of firms posting a price smaller than $p(c)$ must be equal to the fraction of firms with a marginal cost smaller than c, since $p(c)$ is a strictly increasing function. That is, $F(p(c))$ must be equal to $\Phi(c)$. Differentiating $F(p(c)) = \Phi(c)$ with respect to c yields

$$F'(p(c))p'(c) = \Phi'(c). \tag{2.8}$$

Using (2.8) to substitute out $F'(p(c))$, I can rewrite the optimality condition (2.7) as

$$p'(c) = \lambda s \Phi'(c)(p(c) - c). \tag{2.9}$$

Using the fact that $p(c)$ is strictly increasing in c and the fact that $p_h = u$, I obtain

$$p(c_h) = u. \tag{2.10}$$

The expressions in (2.9) and (2.10) represent a differential equation for $p(c)$ together with a boundary condition that must be satisfied in any equilibrium. The solution to (2.9)-(2.10) exists and is unique for any measure of firms s. As established in Menzio (2024b), the solution to (2.9)-(2.10) not only satisfies the firm's first-order condition (2.7), but it also satisfies the firm's second-order condition. Thus, $p(c)$ is the price at which the profit of a firm of type c attains its global maximum.

It is also useful to derive the markups charged by firms. To this aim, let $c(x)$ denote the marginal cost for a firm at the x-th quantile of the cost distribution Φ, i.e. $\Phi(c(x)) = x$, and let $\mu(x)$ denote the markup charged by a firm at the x-th quantile of Φ, i.e. $\mu(x) = p(c(x))/c(x)$. Using the definition of $\mu(x)$, I can rewrite (2.9) as

$$\mu'(x) = \lambda s \left(\mu(x) - 1\right) - \mu(x)\frac{c'(x)}{c(x)}, \tag{2.11}$$

and the boundary condition (2.10) as

$$\mu(1) = \frac{u}{c(1)}. \tag{2.12}$$

The solution to the differential equation (2.11)-(2.12) is given by

$$\mu(x) = 1 + \left(\frac{u}{c(x)} - 1\right) e^{-\lambda s(1-x)} + \lambda s \int_x^1 \left(\frac{c(\hat{x})}{c(x)} - 1\right) e^{-\lambda s(\hat{x}-x)} d\hat{x}. \tag{2.13}$$

The formula in (2.13) is easy to understand. The markup charged by a firm at the x-th quantile of Φ is given by three terms. The first term, $u/c(x) - 1$, is the markup that the firm could charge if it were a monopolist. The second term, $\exp(-\lambda s(1-x))$, is a discount factor that is applied to the monopoly markup. The discount factor is equal to 1, i.e. no discounting, for a firm at the top of the cost distribution Φ. The discount factor becomes smaller and smaller, i.e. stronger and stronger discounting, the lower is the firm's quantile x in the cost distribution Φ. The last term in (2.13) is an additional markup that a firm can charge because sellers at higher quantiles of the Φ distribution are not as efficient at producing the good and, hence, they put less competitive pressure on the firm.[5] It is easy to show that the markup $\mu(x)$ charged by a firm at the x-th quantile of Φ is strictly decreasing in the measure s of firms in the market.[6] For $s \to 0$, the markup $\mu(x)$ converges to the monopoly markup. For $s \to \infty$, the markup $\mu(x)$ converges to the competitive markup 1.

Next, I turn to the firm's entry problem. The firm's cost from entering the market for search good i is given by ζ. The firm's benefit from entering the market is given by

[5] Menzio (2024b) shows that the markup function in a model with homogeneous sellers is simply given by the first two terms in (2.13).

[6] Formally, $\mu(x)$ is strictly decreasing in x for every $x \in [0, 1)$. For $x = 1$, $\mu(x) = u/c(1)$ independently of s. Throughout the paper, I will abuse language and say that $\mu(x)$ is strictly decreasing when it is so for every $x \in [0, 1)$.

the profit $V^*(c) = V(p(c), c)$ that the firm enjoys by producing the good at the cost c and selling it at the price $p(c)$, averaged across the distribution $\Phi(c)$. That is, the firm's benefit from entering the market is given by

$$\int_{c_\ell}^{c_h} V^*(c)\Phi'(c)dc. \tag{2.14}$$

If the firm draws the highest marginal cost c_h, its profit is

$$\begin{aligned} V^*(c_h) &= b\lambda e^{-\lambda s F(p(c_h))}(p(c_h) - c_h) \\ &= b\lambda e^{-\lambda s}(u - c_h), \end{aligned} \tag{2.15}$$

where the second line makes use of the fact that $p(c_h) = u$ and $F(u) = 1$.

If the firm draws the marginal cost c, its profit is

$$V^*(c) = b\lambda e^{-\lambda s F(p(c))}(p(c) - c). \tag{2.16}$$

Differentiating the above expression with respect to c yields

$$\begin{aligned} V^{*\prime}(c) &= -b\lambda e^{-\lambda s F(p(c))} + b\lambda e^{-\lambda s F(p(c))}\left[1 - \lambda s F'(p(c))(p(c) - c)\right] \\ &= -b\lambda e^{-\lambda s F(p(c))}, \end{aligned} \tag{2.17}$$

where the second equality follows from (2.7). Using (2.17), I can write the firm's profit $V^*(c)$ as

$$\begin{aligned} V^*(c) &= V^*(c_h) - \int_c^{c_h} V^{*\prime}(\hat{c})d\hat{c} \\ &= V^*(c_h) + \int_c^{c_h} b\lambda e^{-\lambda s \Phi(\hat{c})}d\hat{c}. \end{aligned} \tag{2.18}$$

Using (2.18), I can rewrite the firm's expected profit as

$$\begin{aligned} &\int_{c_\ell}^{c_h} V^*(c)\Phi'(c)dc \\ =\;& V^*(c_h) + \int_{c_\ell}^{c_h}\left[\int_c^{c_h} b\lambda e^{-\lambda s \Phi(\hat{c})}d\hat{c}\right]\Phi'(c)dc \\ =\;& V^*(c_h) + \left|\left(\int_c^{c_h} b\lambda e^{-\lambda s \Phi(\hat{c})}d\hat{c}\right)\Phi(c)\right|_{c_\ell}^{c_h} + \int_{c_\ell}^{c_h} b\lambda e^{-\lambda s \Phi(c)}\Phi(c)dc \\ =\;& b\lambda e^{-\lambda s}(u - c_h) + \int_{c_\ell}^{c_h} b\lambda e^{-\lambda s \Phi(c)}\Phi(c)dc, \end{aligned} \tag{2.19}$$

where the second equality makes use of integration by parts, and the third equality is obtained by noting that the first term in the second equality is equal to zero and that $V^*(c_h)$ is given by (2.15). The expression in the last line of (2.19) is easy to understand. The firm's expected profit is the sum of two terms. The first term is the profit for a firm with the highest cost c_h. The firm with the highest cost posts a price equal to the households' valuation for the good, u, and sells only to the households that do not contact any other seller in the market, of which it meets a measure $b\lambda \exp(-\lambda s)$. The second term is the extra profit that a firm enjoys if its cost c is drawn from the distribution Φ, rather than being c_h. It is immediate to see that the firm's expected profit from entering a market is strictly decreasing in the measure s of firms in the market. For $s = 0$, the firm's expected profit converges to the monopoly profit $b\lambda(u - \mathbb{E}(c))$. For $s \to \infty$, the firm's expected profit converges to the competitive profit 0.

If the firm's cost ζ of entering the market for a search good is strictly greater than the firm's benefit in (2.19), the equilibrium measure of sellers in the market must be equal to zero. If the firm's cost of entering the market for a search good is equal to the firm's benefit, the equilibrium measure of sellers in the market may be strictly positive. Overall, the equilibrium measure s of sellers in the market must be such that

$$\zeta \geq b\lambda e^{-\lambda s}(u - c_h) + \int_{c_\ell}^{c_h} b\lambda e^{-\lambda s \Phi(c)}\Phi(c)dc, \tag{2.20}$$

and $s \geq 0$, where the two inequalities hold with complementary slackness. Since the firm's benefit of entering the market for a search good in (2.19) is strictly decreasing in s and converges to zero for s going to infinity, there is one and only one s that satisfies (2.20).

The following theorem summarizes my findings.

Theorem 1: *An equilibrium exists and is unique. The equilibrium is such that in the market for good $i \in [0, 1]$:*

(a) *There is a measure s of firms where s is the unique solution to*

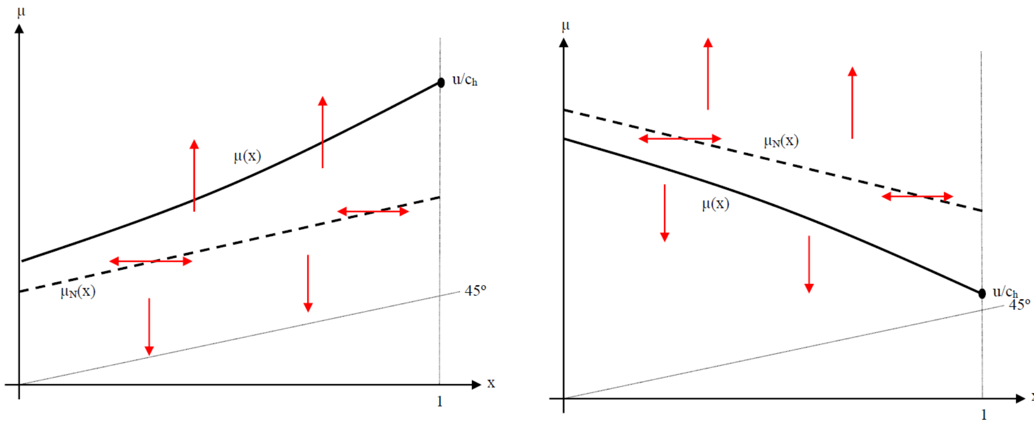
$$\zeta \geq b\lambda e^{-\lambda s}(u - c_h) + \int_{c_\ell}^{c_h} b\lambda e^{-\lambda s \Phi(c)}\Phi(c)dc, \tag{2.21}$$

and $s \geq 0$, with complementary slackness.

(b) *A firm at the x-th quantile of the cost distribution Φ has a cost $c(x)$ and charges a markup*

$$\mu(x) = 1 + \left(\frac{u}{c(x)} - 1\right) e^{-\lambda s(1-x)} + \lambda s \int_x^1 \left(\frac{c(\hat{x})}{c(x)} - 1\right) e^{-\lambda s(\hat{x}-x)} d\hat{x}. \tag{2.22}$$

It is worth discussing some of the properties of equilibrium, as they are very different than in a model of monopolistic competition in the style of Dixit and Stiglitz (1977) where market power originates from the fact that the varieties of the goods carried by different firms are inperfect substitutes in the household's utility function. The distribution of markups is described by function $\mu(x)$ in (2.22). Net markups are positive, even though all the firms in the market carry varieties of good i that are perfect substitutes. Net markups are positive because, due to search frictions, some households can only purchase the good from a single firm and, for this reason, firms' profits must be strictly positive and prices must exceed marginal costs. Net markups are heterogeneous, even when all the



(a) Increasing markups

(b) Decreasing markups

Figure 1: Equilibrium markups $\mu(x)$

firms in the market have the same marginal cost and the last term in (2.22) is zero. Net markups are always heterogeneous because equilibrium requires price dispersion. Since some households are in contact with multiple firms, a firm would have an incentive to undercut the competition if all competitors posted the same price. Net markups depend negatively on s, even though the households' elasticity of substitution between the variety of a firm and the other varieties does not depend on s. Net markups depend negatively on s because, when there are more firms, the average choice set of households becomes larger, and competition between firms intensifies.

To further understand the properties of markups, it is useful to examine the differential equation (2.11). The behavior of the solution of the differential equation depends on whether μ is above or below $\mu_N(x)$, where $\mu_N(x)$ is the nullcline and it is given by $\lambda s/\left(\lambda s - c'(x)/c(x)\right)$ if $\lambda s > c'(x)/c(x)$, and by $+\infty$ if $\lambda s \leq c'(x)/c(x)$. The solid lines in Figures 1(a) and 1(b) plot the nullcline $\mu_N(x)$. If $\mu > \mu_N(x)$, the solution of the differential equation is increasing. If $\mu = \mu_N(x)$, the solution of the differential equation is constant. If $\mu < \mu_N(x)$, the solution of the differential equation is decreasing. The relevant solution of the differential equation is the one that passes through the boundary condition $\mu(1) = u/c(1)$. In the left panel of Figure 1, I illustrate a case in which the nullcline is increasing and the boundary condition lies above the nullcline. In this case, the relevant solution of the differential equation is a markup function $\mu(x)$ that is strictly increasing in x. In the right panel of Figure 1, I illustrate a case in which the nullcline is decreasing and the boundary condition lies below the nullcline. In this case, the relevant solution of the differential equation is a markup function that is strictly decreasing in x. Firms at a higher quantile x of the cost distribution Φ charge higher prices and, hence, they are smaller. Therefore, the left panel illustrates a case in which markups are decreasing in the firm's size, and the right panel illustrates a case in which markups

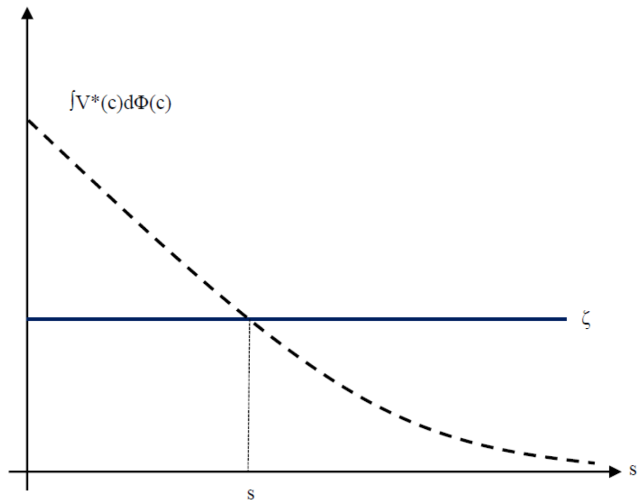


Figure 2: Equilibrium measure of firms

are increasing in the firm's size. More generally, markups can be increasing, decreasing, constant or non-monotonic in the firm's size depending on the specification of the cost distribution and on other parameters of the model (see, Theorem 3 in Menzio 2024b). Markups depend on size, even though the elasticity of substitution between the variety of a seller and the other varieties does not depend on the quantity of the good consumed by a household.

The firm's entry decision is characterized by the free-entry condition (2.21), which is illustrated in Figure 2. The dashed line is the firm's benefit of entering the market for a search good as a function of the measure s of firms in the market. The firm's benefit of entering the market is strictly decreasing with respect to s, since the markup that a firm can charge is strictly decreasing with respect to s. The solid line is the firm's cost of entering the market for a search good. The firm's cost of entering the market is ζ, and does not depend on s. The intersection between the dashed line and the solid line identifies the equilibrium measure of firms in the market.[7]

Using Figures 1 and 2, it is easy to understand the effect of changes in fundamentals on the equilibrium. Consider, for example, an increase in the entry cost ζ. From Figure 2, it follows that an increase in ζ leads to a decline in the measure s of firms in the market. From Figure 1, it follows that a decline in s leads to an increase in the markup function $\mu(x)$. Overall, when the entry cost increases, fewer firms enter the market, competition declines, and firms can charge higher markups. Consider, as another example, an increase in the measure of households b. From Figure 2, it follows that an increase in b leads to an increase in the measure s of firms in the market. From Figure 1, it follows that an

[7] If the firm's cost of entry exceeds the firm's benefit for $s = 0$, the solid line is everywhere above the dashed line and the equilibrium does not feature any entry.

increase in s leads to a decline in the markup function $\mu(x)$. Overall, when the market becomes bigger, more firms enter, competition intensifies, and firms have to lower their markups.

2.3 Welfare properties of equilibrium

In order to assess the welfare properties of equilibrium, let me start by computing equilibrium welfare, defined as the sum of the utilities enjoyed by all households. Equilibrium welfare is given by

$$W = \left[\sum_{n=1}^{\infty} b \frac{e^{-\lambda s}(\lambda s)^n}{n!} u \right] + bz, \tag{2.23}$$

where

$$bz = bh - \left[\sum_{n=1}^{\infty} b \frac{e^{-\lambda s}(\lambda s)^n}{n!} \int_{p_\ell}^{p_h} pn(1 - F(p))^{n-1} F'(p) dp \right]. \tag{2.24}$$

The expressions above are easy to understand. In the market for search good i, a measure $b \exp(-\lambda s)(\lambda s)^n/n!$ of households contact n firms, $n = 1, 2, \ldots$. These households purchase one unit of the good and enjoy the utility u. Every household enjoys a utility z from consuming a quantity z of the numeraire good. The total quantity bz of the numeraire good consumed by the households is derived by aggregating the households' budget constraints. In aggregate, households are endowed with a quantity bh of the numeraire good. In the market for search good i, a measure $b \exp(-\lambda s)(\lambda s)^n/n!$ of households contact n firms. These households purchase the good from the firm that has the lowest price p among the n contacted firms. The lowest price p charged by n firms is distributed according to the cumulative function $1 - (1 - F(p))^n$, which has a density $n(1 - F(p))^{n-1} F'(p)$. Every household also receives profits from the firms, but the free-entry condition (2.21) guarantees that profits are zero.

Substituting (2.24) into (2.23) yields

$$\begin{aligned} W &= \left[\sum_{n=1}^{\infty} b \frac{e^{-\lambda s}(\lambda s)^n}{n!} \int_{p_\ell}^{p_h} (u - c^{-1}(p)) n(1 - F(p))^{n-1} F'(p) dp \right] \\ &- \left[\sum_{n=1}^{\infty} b \frac{e^{-\lambda s}(\lambda s)^n}{n!} \int_{p_\ell}^{p_h} (p - c^{-1}(p)) n(1 - F(p))^{n-1} F'(p) dp \right] + bh. \end{aligned} \tag{2.25}$$

Because the household's preferences are quasi-linear and because of the firms' free-entry condition, welfare can be written as in (2.25). The first term denotes the gains from trade between households and firms in the market for search goods, measured in units of the numeraire good. The second term denotes the negative of the profits enjoyed by firms in the market for search goods, also measured in units of the numeraire good. Therefore, the sum of the first two terms denotes the surplus captured by households in the market for search goods. The last term is the households' endowment of the numeraire good.

The first term in (2.25) is such that

$$\begin{aligned} &\sum_{n=1}^{\infty} b\frac{e^{-\lambda s}(\lambda s)^n}{n!}\int_{p_\ell}^{p_h}(u-c^{-1}(p))n(1-F(p))^{n-1}F'(p)dp \\ = &\sum_{n=1}^{\infty} b\frac{e^{-\lambda s}(\lambda s)^n}{n!}\int_{c_\ell}^{c_h}(u-c)n(1-\Phi(c))^{n-1}\Phi'(c)dc. \end{aligned} \tag{2.26}$$

The second line in (2.26) is obtained by changing the variable of integration from p to c, and then noting that $F(p(c)) = \Phi(c)$ and $F'(p(c))p'(c) = \Phi'(c)$. The second term in (2.25) is such that

$$\sum_{n=1}^{\infty} b\frac{e^{-\lambda s}(\lambda s)^n}{n!}\int_{p_\ell}^{p_h}(p-c)n(1-F(p))^{n-1}F'(p)dp = \zeta s. \tag{2.27}$$

Indeed, the free-entry condition (2.21) guarantees that the firms' profits from selling the search goods, the left-hand side of (2.27), must be equal to the firms' costs of entering the markets for search goods, the right-hand side of (2.27).

Using (2.26) and (2.27), I can write equilibrium welfare as

$$W = \left[\sum_{n=1}^{\infty} b\frac{e^{-\lambda s}(\lambda s)^n}{n!}\int_{c_\ell}^{c_h}(u-c)n(1-\Phi(c))^{n-1}\Phi'(c)dc\right] + bh - \zeta s. \tag{2.28}$$

Next, let me consider the problem of a utilitarian social planner

$$W^* = \max_{y(c),z,s}\left[\sum_{n=1}^{\infty} b\frac{e^{-\lambda s}(\lambda s)^n}{n!}\int_{c_\ell}^{c_h}y(c)un(1-\Phi(c))^{n-1}\Phi'(c)dc\right] + bz \tag{2.29}$$

subject to

$$bh = \left[\sum_{n=1}^{\infty} b\frac{e^{-\lambda s}(\lambda s)^n}{n!}\int_{c_\ell}^{c_h}y(c)cn(1-\Phi(c))^{n-1}\Phi'(c)dc\right] + bz + \zeta s. \tag{2.30}$$

The objective of the planner is the sum of the utility of every household. In the market for search good i, a measure $b\exp(-\lambda s)(\lambda s)^n/n!$ of households contact n randomly-selected firms. The lowest marginal cost c among n randomly-selected firms is distributed according to the cumulative distribution function $1-(1-\Phi(c))^n$, which has a density $n(1-\Phi(c))^{n-1}\Phi'(c)$. If the household purchases the good from the lowest-cost firm, it enjoys a utility of u. Every household enjoys a utility z from consuming z units of the numeraire good. The constraint of the planner is the aggregate feasibility constraint. The planner has access to a quantity bh of the numeraire good. The planner allocates bz of the numeraire good to households' consumption. In the market for search good i, the planner allocates a quantity ζs of the numeraire good to let s firms into the market. In the market for search good i, a measure $b\exp(-\lambda s)(\lambda s)^n/n!$ of households contact n firms. If the lowest-cost firm contacted by the household has a cost c and the household purchases a unit of the good from that firm, the planner allocates a quantity c of the numeraire good to production. The planner chooses the measure of firms in each market, s, and whether a household should purchase the good from the most efficient of the firms that the household contacts, $y(c)$. Obviously, the planner never finds it optimal to instruct a household to purchase a good from anyone but the most efficient firm.

Substituting the planner's constraint into the planner's objective yields

$$W^* = \max_{y(c),s} \left[\sum_{n=1}^{\infty} b \frac{e^{-\lambda s}(\lambda s)^n}{n!} \int_{c_\ell}^{c_h} y(c)(u-c)n(1-\Phi(c))^{n-1}\Phi'(c)dc \right] + bh - \zeta s. \quad (2.31)$$

The planner's optimal choice for $y(c)$ is 1 for all $c \in [c_\ell, c_h]$, since any firm's cost c of producing the good is smaller than the household's utility u from consuming the good. Since $y(c) = 1$, the planner's optimal choice for s is such that

$$\zeta \geq b \left[\sum_{n=1}^{\infty} \left(\frac{e^{-\lambda s}\lambda^n s^{n-1}}{n!} n - \frac{\lambda e^{-\lambda s}(\lambda s)^n}{n!} \right) \int_{c_\ell}^{c_h} (u-c)n(1-\Phi(c))^{n-1}\Phi'(c)dc \right] \quad (2.32)$$

and $s \geq 0$, with complementary slackness. The left-hand side of (2.32) is the marginal cost to the planner of increasing the measure of firms in the market for a search good. The marginal cost is equal to ζ. The right-hand side of (2.32) is the marginal benefit to the planner of increasing the measure of firms in the market for a search good. The marginal benefit is given by the change in the measure of buyers that contact n firms multiplied by the expected utility of a trade between a household and the most efficient of n firm.

I want to simplify the integral in the right-hand side of (2.32). To this aim, notice that

$$\begin{aligned} & \int_{c_\ell}^{c_h} (u-c)n(1-\Phi(c))^{n-1}\Phi'(c)dc \\ = & \ -|(u-c)(1-\Phi(c))^n|_{c_\ell}^{c_h} - \textstyle\int_{c_\ell}^{c_h} (1-\Phi(c))^n dc \\ = & \ u - c_\ell - \textstyle\int_{c_\ell}^{c_h} (1-\Phi(c))^n dc, \end{aligned} \quad (2.33)$$

where the second line follows from integration by parts.

Using (2.33), I can rewrite the right-hand side of (2.32) as

$$\begin{aligned} & b \left[\sum_{n=1}^{\infty} \left(\frac{e^{-\lambda s}\lambda^n s^{n-1}}{n!} n - \frac{\lambda e^{-\lambda s}(\lambda s)^n}{n!} \right) (u - c_\ell) \right] \\ - & \ b \int_{c_\ell}^{c_h} \left[\sum_{n=1}^{\infty} \left(\frac{e^{-\lambda s}\lambda^n s^{n-1}}{n!} n - \frac{\lambda e^{-\lambda s}(\lambda s)^n}{n!} \right) (1-\Phi(c))^n \right] dc. \end{aligned} \quad (2.34)$$

To further simplify (2.34), notice that

$$\sum_{n=1}^{\infty} \frac{e^{-\lambda s}\lambda^n s^{n-1}}{n!} n = \lambda \sum_{n=1}^{\infty} \frac{e^{-\lambda s}(\lambda s)^{n-1}}{(n-1)!} = \lambda, \quad (2.35)$$

and that

$$\begin{aligned}
&\sum\nolimits_{n=1}^{\infty} \frac{\lambda e^{-\lambda s}(\lambda s)^n}{n!} \\
= \; & \lambda \left[\sum\nolimits_{n=0}^{\infty} \frac{\lambda e^{-\lambda s}(\lambda s)^n}{n!} - e^{-\lambda s}\right] = \lambda \left(1 - e^{-\lambda s}\right).
\end{aligned} \tag{2.36}$$

Similarly, notice that

$$\begin{aligned}
&\sum\nolimits_{n=1}^{\infty} \frac{e^{-\lambda s}\lambda^n s^{n-1} \left(1-\Phi(c)\right)^n}{n!} n \\
= \; & \lambda e^{-\lambda s \Phi(c)}(1-\Phi(c)) \sum\nolimits_{n=1}^{\infty} \frac{e^{-\lambda s(1-\Phi(c))}(\lambda s)^{n-1} \left(1-\Phi(c)\right)^{n-1}}{(n-1)!} \\
= \; & \lambda e^{-\lambda s \Phi(c)}(1-\Phi(c)),
\end{aligned} \tag{2.37}$$

and that

$$\begin{aligned}
&\sum\nolimits_{n=1}^{\infty} \frac{\lambda e^{-\lambda s}(\lambda s)^n \left(1-\Phi(c)\right)^n}{n!} \\
= \; & \sum\nolimits_{n=0}^{\infty} \frac{\lambda e^{-\lambda s}(\lambda s)^n \left(1-\Phi(c)\right)^n}{n!} - \lambda e^{-\lambda s} \\
= \; & \lambda e^{-\lambda s \Phi(c)} \sum\nolimits_{n=0}^{\infty} \frac{\lambda e^{-\lambda s(1-\Phi(c))}(\lambda s)^n \left(1-\Phi(c)\right)^n}{n!} - \lambda e^{-\lambda s} \\
= \; & \lambda \left(e^{-\lambda s \Phi(c)} - e^{-\lambda s}\right).
\end{aligned} \tag{2.38}$$

Using (2.35)-(2.38), I can reduce (2.34) to

$$\begin{aligned}
& b\lambda e^{-\lambda s}(u - c_\ell) + \int_{c_\ell}^{c_h} b\lambda e^{-\lambda s \Phi(c)} \Phi(c) dc - \int_{c_\ell}^{c_h} b\lambda e^{-\lambda s} dc \\
= \; & b\lambda e^{-\lambda s}(u - c_h) + \int_{c_\ell}^{c_h} b\lambda e^{-\lambda s \Phi(c)} \Phi(c) dc,
\end{aligned} \tag{2.39}$$

where the second line is obtained by solving the last integral in the first line.

From (2.39), it follows that the optimality condition (2.32) can be written as

$$\zeta \geq b\lambda e^{-\lambda s}(u - c_h) + \int_{c_\ell}^{c_h} b\lambda e^{-\lambda s \Phi(c)} \Phi(c) dc, \tag{2.40}$$

and $s \geq 0$, with complementary slackness. Note that the optimality condition (2.40) for the planner's choice of s is identical to the equilibrium free-entry condition (2.21). Hence, the measure of firms that the planner chooses to let into the market for a search good is the same as the measure of firms that enter the market in equilibrium. Also note that, if the measure of firms that the planner chooses to let into the market for a search good is the same as the measure of firms that enter the market in equilibrium, the planner's maximized welfare (2.31) is identical to equilibrium welfare (2.28). From these observations, it follows that the equilibrium is efficient.

Theorem 2: *The equilibrium is efficient, in the sense that the equilibrium allocation* *maximizes the sum of households' utilities.*

There are two parts to the efficiency result in Theorem 2. First, taking as given the measure of firms in the market, the allocation of production and consumption is efficient. Second, the measure of firms that enters the market is efficient.

First, let me explain why consumption and production are efficient, taking as given the measure of firms in the market. Consider a household that meets a single firm. The planner wants the household to purchase the good because $u > c_h$ implies that the social value of the trade between the household and the firm is positive. In equilibrium, the household purchases the good from the firm because $u = p_h$ implies that the value of the trade to the household is positive. Consider a household that meets n firms, with $n \geq 2$. The planner wants the household to purchase the good from the firm that has the lowest cost, since that trade has the highest social value. In equilibrium, the household purchases the good from the firm that has the lowest cost, since the firm with the lowest cost posts the lowest price and, hence, it offers to the household the highest value. These observations imply that, taking as given the measure of firms in the market, consumption and production are efficient.

Next, let me explain why the measure of firms that enter the market is efficient. By entering the market, a firm generates some meetings with households. The firm generates some meetings with households that are not in contact with any other seller. The social value of these meetings is $u-c$, where c denotes the firm's realization of the idiosyncratic cost of production. The firm generates some meetings with households that are in contact with other sellers, all of which have cost of production higher than c. The social value of these meetings is $c - \hat{c}$, where $\hat{c}$ denotes the lowest cost of production among the other sellers with which the household is in contact. Lastly, the firm generates some meetings with households that are in contact with other sellers, some of which have cost of production lower than c. The social value of these meetings is zero. Formally, the social value of a firm with cost c is given by

$$v(c) = \frac{b}{s} e^{-\lambda s} \lambda s (u - c) + \sum_{n=1}^{\in} \left[\frac{b}{s} \frac{e^{-\lambda s} (\lambda s)^{n+1}}{(n+1)!} (n+1) \int_{c}^{c_h} (c - \hat{c}) \Phi'(\hat{c}) d\hat{c} \right]. \tag{2.41}$$

When integrated over Φ, (2.41) returns the social value of an additional firm on the right-hand side of (2.40).

Now, suppose that the price paid by a household was determined by a procurement auction between all the firms that are in contact with that household. When a firm with production cost c meets a household that is not in contact with any other seller, the outcome of the auction would be a price of u, and the firm would enjoy a profit of $u - c$. When the firm meets a household that is in contact with other sellers, all of which have cost of production higher than c, the outcome of the procurement auction would be a price of $\hat{c}$, where $\hat{c}$ denotes the lowest cost among the other sellers with which the household is in contact, and the firm would enjoy a profit of $\hat{c} - c$. When a firm meets a household that is on contact with other sellers, some of which have a cost lower than c, the firm would lose the auction and its profit would be 0. Hence, if prices were determined by a procurement auction, the firm's profit would coincide with its social value $v(c)$, and the entry of firms would be efficient.

Firms, however, do not participate in a procurement auction to sell a unit of the good to each particular household that they meet. Firms simply post prices. Then why is the entry of firm efficient? The answer is that there is a revenue equivalence at work. Specifically, the profit of a firm when prices are set by a procurement auction are the same as the profit of a firm when prices are posted. Indeed, after integrating by parts and solving some infinite sums, the profit $v(c)$ of a firm when prices are set by an auction can be written as

$$v(c) = b\lambda e^{-\lambda s}(u - c_h) + \int_c^{c_h} b\lambda e^{-\lambda s \Phi(\hat{c})} d\hat{c}, \tag{2.42}$$

which is exactly the profit $V^*(c)$ of a firm when prices are posted.

The efficiency properties of equilibrium are different than in the model of monopolistic competition of Dixit and Stiglitz (1977). First, taking as given the measure of firms, consumption and production are generally inefficient (see, e.g., Edmond, Midrigan and Xu 2023 or Baqaee, Sangani and Farhi 2022). Since firms charge a strictly positive markup, there is a gap between the household's marginal utility of consumption (which is equated to the price) and the firm's marginal cost of production. Therefore, as long as the supply of inputs is not perfectly inelastic, production and consumption are inefficiently low. Moreover, if firms charge different markups, the inputs of production are misallocated, in the sense that welfare would increase if inputs, production and consumption were reallocated from low to high-markup firms. Second, even when the supply of inputs is perfectly inelastic, the measure of firms in the market is generally inefficient (see, e.g., Dixit and Stiglitz 1977, Mankiw and Whinston 1986, Dhingra and Morrow 2019). Intuitively, the entry of firms generates a negative business stealing externality that an entering firm imposes on other firms, and a positive surplus externality that an entering firm has on households. The two externalities cancel each other out only in the special case where household's preferences are CES.

The Dixit-Stiglitz logic is so ingrained that markups are now synonymous with inefficiency, and measures of markups are sometimes presented as sufficient statistics to carry out welfare analysis and to issue policy recommendations. Theorem 2 shows that observations on markups are not sufficient to reach conclusions about welfare and optimal policy. The origin of market power and markups matters too.

It is also useful to interpret Theorem 2 from the perspective of search theory.[8] First,

[8] The efficiency results in Menzio (2023) and Albrecht, Menzio and Vroman (2024) are different, even though they are about applications of the Burdett-Judd framework. Menzio (2023) establishes the efficiency of the extent of horizontal differentiation of a firm's variety. Albrecht, Menzio and Vroman (2024) establish the efficiency of the extent of vertical differentiation of a firm's variety. These models assume that the measure of firms is fixed.

notice that the measure of meetings between firms and households, blams, is linear in the measure s of firms in the market. This means that all the meetings between an additional firm and households are additional meetings, and do not come at the expense of meetings of other firms. Second, notice that firms set prices and, in this sense, they have all the bargaining power. Taken together, these two observations imply, heuristically, that the entry of firms does not have a negative congestion externality on other firm, nor a positive surplus externality on households. In other words, the Mortensen rule (Mortensen 1982), or equivalently the Hosios condition (Hosios 1990), is satisfied.

The above observations imply that the efficiency of equilibrium depends on the assumption that entering firms do not reduce the number of meetings between other firms, and that firms post prices. Both assumptions seem natural in the context of the product market. Indeed, it seems uncontroversial to assume that firms post prices in the market for consumer goods. It also seems quite natural to assume that when household can buy from a new firm, they would not become unable to buy from firms with which they were previously in contact. If, however, firms do congest each other because, say, they compete for the limited attention of households, entry of new firms would generate a negative externality on other firms and the equilibrium would be inefficient.

3 Open Economy

In this section, I examine the equilibrium and welfare effects of international trade. I consider a world economy comprised of two identical countries. In Section 3.1, I compare the properties of equilibrium when international trade is allowed and when it is not (autarky). I show that the measure of local and foreign firms that enter the market for a particular good is greater when international trade is allowed than in autarky. A direct consequence of this finding is that firms charge lower markups when international trade is allowed than in autarky. In Section 3.2, I derive a formula for the welfare gains of international trade. I show that the welfare effect of international trade is the sum of a shopping effect—i.e., the increase in the household's choice set keeping prices constant—and a competition effect—i.e., the decline in the firm's prices keeping the household's choice set constant. Both effects are positive and, hence, international trade increases welfare. In contrast with models of international trade based on the Dixit-Stiglitz framework, international trade unambiguously lowers markups and unambiguously increases welfare.

3.1 Equilibrium effects of international trade

Consider a world comprised of two countries, 1 and 2. The two countries are identical. In country $j \in \{1, 2\}$, there is a measure b of ex-ante identical households per search good $i \in [0, 1]$. Every household has preferences over consumption of the search goods and of the numeraire good that are described by the utility function (2.1). Every household is endowed with a quantity h of the numeraire good, and with an equal share of the local firms. In the market for each search good, there is a measure of potential firms. If a firm enters the market for a search good, it pays a cost ζ and draws an idiosyncratic marginal cost of production c from the cumulative function $\Phi(c)$. After observing its marginal cost c, a firm posts a price p for its variety of the search good. If trade between the two countries is not allowed, i.e. in autarky, a household in the market for a search good can only come into contact with local firms. In particular, a household contacts n randomly-selected local firms, where n is distributed as a Poisson with coefficient λs_a, and s_a denotes the measure of local firms in the market. If trade between the two countries is allowed, i.e. when international trade is open, a household may contact both local and foreign firms. In particular, a household contacts n randomly-selected local or foreign firms, where n is distributed as a Poisson with coefficient λs_t, where s_t denotes the measure of local and foreign firms in the market.[9]

If trade between the two countries is not allowed, the equilibrium in country j is described by Theorem 1. Therefore, in the market for good i, the measure s_a of firms is such that

$$\zeta \geq b\lambda e^{-\lambda s_a}(u - c_h) + \int_{c_\ell}^{c_h} b\lambda e^{-\lambda s_a \Phi(c)}\Phi(c)dc, \tag{3.1}$$

and $s_a \geq 0$, where the two inequalities hold with complementary slackness. In the market for good i, the markup $\mu_a(x)$ charged by a firm at the x-th quantile of the cost distribution Φ is such that

$$\mu_a(x) = 1 + \left(\frac{u}{c(x)} - 1\right) e^{-\lambda s_a(1-x)} + \lambda s_a \int_x^1 \left(\frac{c(\hat{x})}{c(x)} - 1\right) e^{-\lambda s_a(\hat{x}-x)}d\hat{x}, \tag{3.2}$$

where $c(x)$ is defined as $\Phi(c(x)) = x$.

Suppose now that trade between the two countries is allowed. In the market for good i, the measure s_t of local and foreign firms is such that

$$\zeta \geq 2b\lambda e^{-\lambda s_t}(u - c_h) + \int_{c_\ell}^{c_h} 2b\lambda e^{-\lambda s_t \Phi(c)}\Phi(c)dc, \tag{3.3}$$

and $s_t \geq 0$, where the two inequalities hold with complementary slackness. The left-hand side of (3.3) is the firm's cost of entering the market. The right-hand side is the firm's benefit of entering the market, which is given by the sum of two terms. The first term is the operating profit for a firm that draws the highest cost c_h, posts the price u, and trades only with those households that are not in contact with any other seller. The second term is the average with respect to Φ of the additional operating profit enjoyed by a firm that draws a cost c rather than c_h. Condition (3.3) is the same as condition (3.1), except that the firm's benefit from entering the market is proportional to $2b$ rather than b. Indeed, if the two countries are allowed to trade, a firm comes into contact with both local and

[9] Since a household is equally likely to meet local and foreing firms, the market for good i in country j and the market for good i in country $-j$ are perfectly integrated, and I will refer to them as the market for good i.

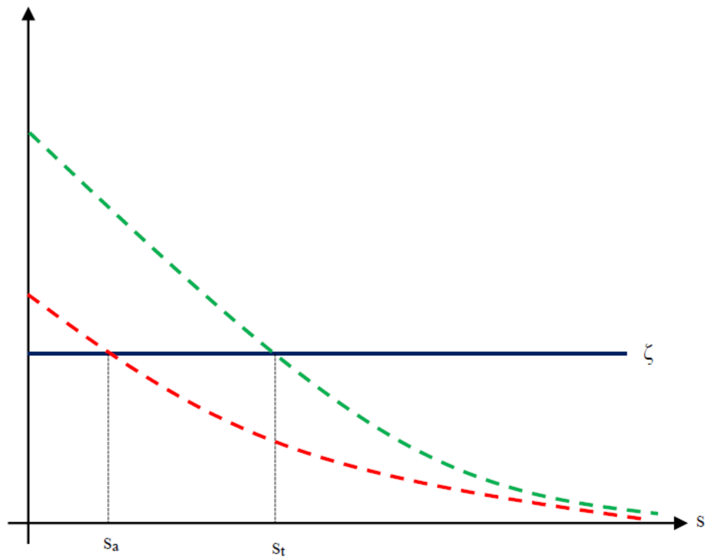


Figure 3: Equilibrium measure of firms

foreign households, of which there is a measure $2b$. If the two countries are not allowed to trade, a firm only comes into contact with local households, of which there is a measure b.

The solid line in Figure 3 is the firm's cost of entering the market for a search good. The dashed line is the firm's benefit of entering the market if international trade is closed. The dotted line is the firm's benefit of entering the market if international trade is open. The dotted line lies above the dashed line because the right-hand side of (3.3) is strictly greater than the right-hand side of (3.1) for any measure of firms in the market s. Both the dotted line and the dashed line are downward sloping since both the right-hand side of (3.3) and the right-hand side of (3.1) are strictly decreasing in s. As it is clear from the figure, $s_t > s_a$.[10] In words, the measure of local and foreign firms entering the market for good i when international trade is open exceeds the measure of local firms entering the market for good i of country j when international trade is closed.

If trade between the two countries is allowed, a firms at the x-th quantile of the cost distribution Φ charges the markup

$$\mu_t(x) = 1 + \left(\frac{u}{c(x)} - 1\right) e^{-\lambda s_t(1-x)} + \lambda s_t \int_x^1 \left(\frac{c(\hat{x})}{c(x)} - 1\right) e^{-\lambda s_t(\hat{x}-x)} d\hat{x}. \tag{3.4}$$

The markup is given by three terms. The first term is the monopoly markup. The second term is a discount on the monopoly markup, which depends on the firm's rank x and on

[10] When comparing equilibrium with and without international trade, I assume that $s_t > 0$. I shall ignore the uninteresting case in which the entry cost ζ is so high that $s_t = 0$ and, hence, $s_a = 0$. In this uninteresting case, international trade has neither equilibrium nor welfare effects.

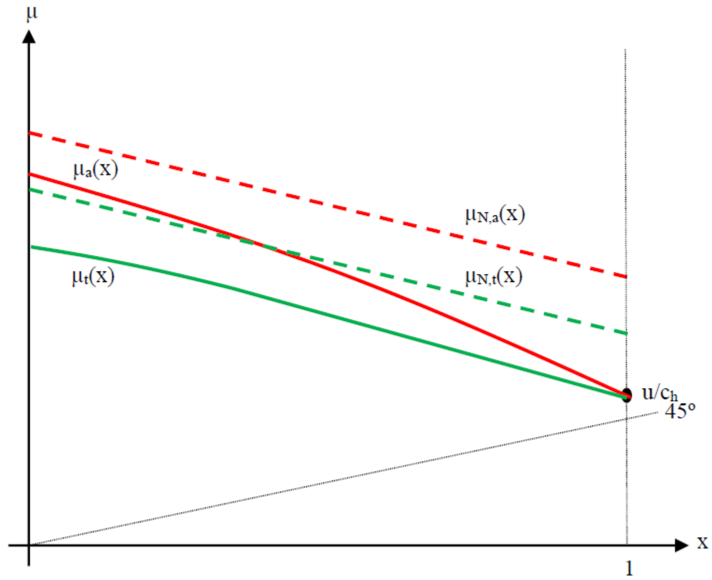


Figure 4: Equilibrium markups

the average size of the households' choice set λs_t. The last term is an extra markup that the firm can charge because higher-ranked sellers produce the good less efficiently, and it depends on the quantile function $c(x)$ and on λs_t. The markup function (3.4) is the same as (3.2), except that λs_t replaces λs_a. Indeed, if the two countries are allowed to trade, the households' choice set includes an average of λs_t firms. If the two countries are not allowed to trade, the households' choice set contains an average of λs_a firms. Since the expression in (3.4) is strictly decreasing in s and $s_t > s_a$, $\mu_t(x) < \mu_a(x)$. In words, a firm with cost c posts a lower price and charges a lower markup when international trade is open than when international trade is closed. This finding is illustrated in Figure 4.

Next, let me compute the size of firms and the extent of market concentration. I measure the size of a firm as the quantity of the good that it sells. If countries are allowed to trade, the quantity sold by a firm at the x-th quantile of the cost distribution Φ is

$$q_t(x) = 2b\lambda e^{-\lambda s_t x}. \tag{3.5}$$

If countries are not allowed to trade, the quantity sold by a firm at the x-th quantile of the cost distribution Φ is

$$q_a(x) = b\lambda e^{-\lambda s_a x}. \tag{3.6}$$

Clearly, $q_t(x)$ and $q_t(x)$ are both strictly decreasing in x. Indeed, firms that are ranked higher in the cost distribution have a higher marginal cost, they post a higher price, and they sell a smaller quantity of the good. Moreover, $q_t(x) > q_a(x)$ for all $x \leq x_c$ and $q_t(x) < q_a(x)$ for all $x \geq x_c$, where $x_c > 0$. In words, when the two countries are allowed to trade, firms with a relatively low cost of production become larger and firms with a relatively high cost of production become smaller. To see why this is the case, it is sufficient to notice that $q_t(0) > q_a(0)$ and that $q_t(x) = q_a(x)$ implies that $q_t'(x) - q_a'(x) < 0$.

I measure market concentration as the fraction of the good that is sold by the $x\%$ of largest firms, which are the firms below the x-th quantile of the cost distribution. If countries are allowed to trade, the fraction of sales made by the $x\%$ of largest firms is

$$Q_t(x) = \frac{\int_0^x q_t(\hat{x})d\hat{x}}{\int_0^1 q_t(\hat{x})d\hat{x}} = \frac{1 - e^{-\lambda s_t x}}{1 - e^{-\lambda s_t}}. \tag{3.7}$$

If countries are not allowed to trade, the fraction of sales made by the $x\%$ of largest firms is

$$Q_a(x) = \frac{\int_0^x q_a(\hat{x})d\hat{x}}{\int_0^1 q_a(\hat{x})d\hat{x}} = \frac{1 - e^{-\lambda s_a x}}{1 - e^{-\lambda s_a}}. \tag{3.8}$$

Since $s_t > s_a$, it is easy to see that $Q_t(x) > Q_a(x)$ for any x. In words, when the two countries are allowed to trade, market concentration increases.

The following theorem summarizes the equilibrium effects of international trade.

Theorem 3: *Consider two identical countries.*

(a) *Suppose that the two countries are not allowed to trade. The equilibrium is such that in country $j \in \{1,2\}$ and in the market for good $i \in [0,1]$* (i) *the measure s_a of firms is given by (3.1);* (ii) *the markup $\mu_a(x)$ charged by a firm at the x-th quantile of the cost distribution Φ is given by (3.2);* (iii) *the fraction $Q_a(x)$ of sales made by the $x\%$ of largest firms is given by (3.8).*

(b) *Suppose that the two countries are allowed to trade. The equilibrium is such that in the market for good $i \in [0,1]$:* (i) *the measure s_t of firms is given by (3.3) and it is such that $s_t > s_a$;* (ii) *the markup $\mu_t(x)$ charged by a firm a the x-th quantile of the cost distribution Φ is given by (3.4), and it is such that $\mu_t(x) < \mu_a(x)$;* (iii) *the fraction $Q_t(x)$ of sales made by the $x\%$ of largest firms is given by (3.7), and it is such that $Q_t(x) > Q_a(x)$.*

The logic behind the findings in Theorem 3 is the manifestation of simple and natural economic forces. When trade between the two countries opens up, a firm that enters the market has access to a larger pool of potential buyers. Therefore, when trade between the two countries opens up, more firms find it optimal to enter the market. The increase in the measure of firms implies that, on average, households can purchase the same good from a larger number of sellers. Therefore, the market becomes more competitive, and firms find it optimal to lower their prices and their markups. The increase in the measure of firms also implies that households are more likely to come into contact with firms with a lower marginal cost and, since firms with a lower marginal cost post lower prices, households are more likely to purchase from them. Therefore, the market becomes more concentrated.

The economic forces behind the findings in Theorem 3 are simple and natural, but they are not the same as in models based on the monopolistic competition framework of

Dixit and Stiglitz (1977). In models of monopolistic competition, households have access to all the firms, but firms sell goods that are imperfect substitutes. The markups charged by a particular firm depends on the elasticity of substitution between its variety and the varieties sold by other firms. If the elasticity of substitution between one variety and others is constant, as is the case when households have CES preferences, international trade has no effect whatsoever on markups (see, e.g., Krugman 1980, Melitz 2003). If the elasticity of substitution between one variety and others is decreasing in the quantity of that variety consumed by a household, international trade lowers markups (see, e.g., Krugman 1979). If the elasticity of substitution between one variety and others is increasing in the quantity of that variety consumed by a household, international trade increases markups. In any case, the effect of international trade on markups has nothing to do with an increase of competition between firms—which is always perfect in a frictionless market—but with the shape of the households' preferences. In contrast, in the search-theoretic model presented here, international trade always lowers markups, and it does so because it allows household to purchase the same good from a larger number of firms.

Another difference between the search-theoretic model of international trade presented here and the monopolistic competition models is the motive for trade. In the monopolistic competition models, a household purchases both local and foreign varieties because they perceive them as different goods. In contrast, in the search-theoretic model presented here, a household may purchase the same good from a local or a foreign firm depending on which one charges a lower price. Due to search frictions, firms find it optimal to charge different prices for the same good. Due to search frictions, an individual household is in contact with only n firms selling the same good and, among those firms, the one posting the lowest price may sometimes be local and sometimes be foreign.

3.2 Welfare effects of international trade

I now compute the welfare gains associated with opening up international trade. First, suppose that the two countries are not allowed to trade. In this case, welfare in country $j \in \{1, 2\}$ is given by

$$\begin{aligned} W_a &= bh + \sum_{n=1}^{\infty} b\frac{e^{-\lambda s_a}(\lambda s_a)^n}{n!} \int_{p_\ell}^{p_h} (u-p)n(1-F_a(p))^{n-1}F_a'(p)dp \\ &= bh + \sum_{n=1}^{\infty} b\frac{e^{-\lambda s_a}(\lambda s_a)^n}{n!} \int_{c_\ell}^{c_h} (u-p_a(c))n(1-F_a(p_a(c)))^{n-1}F_a'(p_a(c))p_a'(c)dp \\ &= bh + \sum_{n=1}^{\infty} b\frac{e^{-\lambda s_a}(\lambda s_a)^n}{n!} \int_{c_\ell}^{c_h} (u-p_a(c))n(1-\Phi(c))^{n-1}\Phi'(c)dc, \end{aligned} \tag{3.9}$$

where s_a denotes the measure of firms in the market for a search good when international trade is not allowed, F_a denotes the distribution of prices posted by firms when international trade is not allowed, and $p_a(c)$ denotes the price posted by a firm with marginal cost c when international trade is not allowed. The expression in the first line of (3.9) is the expression for welfare in (2.25) and is obtained by substituting the household's budget constraint into the households' objective function. The expression in the second line of (3.9) is obtained by changing the variable of integration from p to c. The expression in the third line is obtained by noting that $F_a(p_a(c)) = \Phi(c)$ and, hence, $F_a'(p_a(c))p_a'(c) = \Phi'(c)$.

I can further simplify (3.9) as

$$\begin{aligned} W_a &= bh + \sum_{n=1}^{\infty} b\frac{e^{-\lambda s_a}(\lambda s_a)^n}{n!}\left[u - \int_{c_\ell}^{c_h} p_a(c)n(1-\Phi(c))^{n-1}\Phi'(c)dc\right] \\ &= bh + \sum_{n=0}^{\infty} b\frac{e^{-\lambda s_a}(\lambda s_a)^n}{n!}(u - \bar{p}_{a,n}), \end{aligned} \tag{3.10}$$

The first line in (3.10) is obtained by noting that $n(1-\Phi(c))^{n-1}\Phi'(c)$ is the density associated with the cumulative distribution function $1-(1-\Phi(c))^n$, which describes the distribution of the lowest cost among n firms randomly drawn from Φ. Therefore, $n(1-\Phi(c))^{n-1}\Phi'(c)$ integrates up to 1. The second line in (3.10) is obtained by defining $\bar{p}_{a,0}$ as u and $\bar{p}_{a,n}$ as the average of the price posted by the lowest-cost firm among n firms randomly drawn from Φ and by letting , i.e.

$$\bar{p}_{a,0} = u, \quad \bar{p}_{a,n} = \int_{c_\ell}^{c_h} p_a(c)n(1-\Phi(c))^{n-1}\Phi'(c)dc \text{ for } n = 1, 2, ... \tag{3.11}$$

Notice that $1-(1-\Phi(c))^n$ is strictly smaller than $1-(1-\Phi(c))^{n+1}$ for $n = 1, 2, ...$ In words, the distribution of the lowest cost among n firms randomly drawn from Φ first-order stochastically dominates the distribution of the lowest cost among $n+1$ firms randomly drawn from Φ. Since $p_a(c)$ is a strictly decreasing function of Φ, it follows that $\bar{p}_{a,n} > \bar{p}_{a,n+1}$. Moreover, since $p(c_h) = u$ and $p(c)$ is strictly decreasing function of c, it follows that $\bar{p}_{a,1} < u$. Taken together, these observations imply

$$u = \bar{p}_{a,0} > \bar{p}_{a,1} > \bar{p}_{a,2} > ... \tag{3.12}$$

Next, suppose that trade between the two countries is allowed. In this case, welfare in country j is given by

$$\begin{aligned} W_t &= bh + \sum_{n=1}^{\infty} b\frac{e^{-\lambda s_t}(\lambda s_t)^n}{n!}\int_{p_\ell}^{p_h}(u-p)n(1-F_t(p))^{n-1}F_t'(p)dp \\ &= bh + \sum_{n=1}^{\infty} b\frac{e^{-\lambda s_t}(\lambda s_t)^n}{n!}\int_{c_\ell}^{c_h}(u-p_t(c))n(1-\Phi(c))^{n-1}\Phi'(c)dc, \end{aligned} \tag{3.13}$$

where s_t denotes the measure of firms in the market for a search good when international trade is allowed, F_t denotes the distribution of prices posted by firms when international trade is allowed, and $p_t(c)$ denotes the price posted by a firm with marginal cost c when international trade is allowed. The expression in the first line of (3.13) is obtained by substituting the household's budget constraint into the households' objective function. The expression in the second line of (3.13) is obtained by changing the variable of integration from p to c, and by noting that $F_t(p_t(c)) = \Phi(c)$ and, hence, $F_t'(p_t(c))p_t'(c) = \Phi'(c)$.

I can further simplify (3.13) as

$$\begin{aligned} W_t &= bh + \sum_{n=1}^{\infty} b\frac{e^{-\lambda s_t}(\lambda s_t)^n}{n!}\left[u - \int_{c_\ell}^{c_h} p_t(c)n(1-\Phi(c))^{n-1}\Phi'(c)dc\right] \\ &= bh + \sum_{n=0}^{\infty} b\frac{e^{-\lambda s_a}(\lambda s_a)^n}{n!}\left(u - \bar{p}_{t,n}\right). \end{aligned} \tag{3.14}$$

The first line in (3.14) is obtained by noting that $n(1-\Phi(c))^{n-1}\Phi'(c)$ integrates up to 1. The second line in (3.14) is obtained by defining $\bar{p}_{t,n}$ as

$$\bar{p}_{t,0} = u, \quad \bar{p}_{t,n} = \int_{c_\ell}^{c_h} p_t(c)n(1-\Phi(c))^{n-1}\Phi'(c)dc \text{ for } n = 1, 2, ... \tag{3.15}$$

As before, it is immediate to see that $u = \bar{p}_{t,0} > \bar{p}_{t,1} > \bar{p}_{t,2} > ...$

Using (3.10) and (3.14), I can write the difference between W_t and W_a as

$$\begin{aligned} & W_t - W_a \\ = \; & \underbrace{b\sum_{n=0}^{\infty}\left(\frac{e^{-\lambda s_t}(\lambda s_t)^n}{n!} - \frac{e^{-\lambda s_a}(\lambda s_a)^n}{n!}\right)(u - \bar{p}_{t,n})}_{\text{shopping effect}} + \underbrace{b\sum_{n=0}^{\infty}\frac{e^{-\lambda s_a}(\lambda s_a)^n}{n!}(\bar{p}_{a,n} - \bar{p}_{t,n})}_{\text{competition effect}} \end{aligned} \tag{3.16}$$

International trade has two effects. First, international trade affects the measure of firms in the market and, hence, the probability that a household comes into contact with n firms. Second, international trade affects the prices posted by firms and, hence, the average price paid by a household with n contacts. The first term on the right-hand side of (3.16) captures the welfare effect of the change in the probability that a household contacts n firms, keeping the average price paid by such as a household unchanged. I refer to this as the *shopping effect* of international trade. The second term on the right-hand side of (3.16) captures the welfare effect of the change in the average price paid by a household with n contacts, keeping the probability that a household contacts n firms unchanged. I refer to this as the *competition effect* of international trade.

Let me first focus on the shopping effect of international trade. To this aim, suppose that a household comes into contact with n firms, where n is a random variable distributed as a Poisson with coefficient λs, and s is the measure of firms in the market. The probability that a household comes into contact with n firms is given by

$$\Pr(n|s) = \frac{e^{-\lambda s}(\lambda s)^n}{n!}. \tag{3.17}$$

Differentiating (3.17) with respect to s yields

$$\frac{d\Pr(n|s)}{ds} = \frac{e^{-\lambda s}\lambda^n s^{n-1}\left(-\lambda s + n\right)}{n!}, \tag{3.18}$$

which is strictly negative for all $n < \lambda s$, and strictly positive for all $n > \lambda s$. In words, an increase in the measure s of firms in the market reduces the probability that a household comes into contact with a relatively small number of firms, and increases the probability that a household comes into contact with a relatively large number of firms.

The derivative with respect to s of the probability that a household contacts no more than n firms is given by

$$\begin{aligned} d\left(\sum\nolimits_{k=0}^{n} \Pr(k|s)\right) \Big/ ds &= \sum\nolimits_{k=0}^{n} \frac{d\Pr(k|s)}{ds} \\ &= -\sum\nolimits_{k=n+1}^{\infty} \frac{d\Pr(k|s)}{ds}, \end{aligned} \tag{3.19}$$

where the second line makes use of the fact that $\sum_{k=0}^{\infty} \Pr(k|s) = 1$ for all s. For any $n < \lambda s$, the derivative in (3.19) is strictly negative. Indeed, for any $n < \lambda s$, the first line in (3.19) is the sum of the strictly negative terms $d\Pr(k|s)/ds$. For any $n > \lambda s$, the derivative in (3.19) is also strictly negative. Indeed, for any $n > \lambda s$, the second line in (3.19) is the negative of the sum of the strictly positive terms $d\Pr(k|s)/ds$. Overall, the derivative with respect to s of the probability that a household contacts no more than n firms is strictly negative. Therefore, the distribution of households' contacts is strictly increasing (in the sense of first-order stochastic dominance) with respect to s.

The measure of firms in the market is strictly greater under international trade, i.e. $s_t > s_a$. It then follows from (3.19) that the distribution of households' contacts under international trade first-order stochastically dominates the distribution of households' contacts under autarky. Also recall that the average price paid $\bar{p}_{t,n}$ by a household with n contacts is strictly decreasing in n and, hence, the average surplus $u - \bar{p}_{t,n}$ enjoyed by a household with n contacts is strictly increasing in n. These observations imply that the shopping effect of international trade is positive, i.e.

$$b\sum_{n=0}^{\infty}\left(\frac{e^{-\lambda s_t}(\lambda s_t)^n}{n!} - \frac{e^{-\lambda s_a}(\lambda s_a)^n}{n!}\right)(u - \bar{p}_{t,n}) > 0. \tag{3.20}$$

Next, let me focus on the competition effect of international trade. Since $\mu_t(x) < \mu_a(x)$ for all $x \in [0, 1)$, $p_t(c) < p_a(c)$ for all $c \in [c_\ell, c_h)$. Hence, for $n = 1, 2, 3, \ldots$, the average price $\bar{p}_{t,n}$ paid by a household with n contacts when the two countries are allowed to trade is strictly smaller than the average price $\bar{p}_{a,n}$ paid by a household with n contacts when the two countries are in autarky. For $n = 0$, $\bar{p}_{t,0} = \bar{p}_{a,0} = u$. From these observations it follows that the competition effect of international trade is positive, i.e.

$$b \sum_{n=0}^{\infty} \frac{e^{-\lambda s_a} (\lambda s_a)^n}{n!} (\bar{p}_{a,n} - \bar{p}_{t,n}) > 0. \tag{3.21}$$

The following theorem summarizes the welfare effects of international trade.

Theorem 4: *Consider two identical countries. The welfare gain of international trade is*

$$\begin{aligned} W_t - W_a \quad &= b \sum_{n=0}^{\infty} \left(\frac{e^{-\lambda s_t} (\lambda s_t)^n}{n!} - \frac{e^{-\lambda s_a} (\lambda s_a)^n}{n!} \right) (u - \bar{p}_{t,n}) \\ &+ b \sum_{n=0}^{\infty} \frac{e^{-\lambda s_a} (\lambda s_a)^n}{n!} (\bar{p}_{a,n} - \bar{p}_{t,n}). \end{aligned} \tag{3.22}$$

The first term on the right-hand side of (3.22) is the shopping effect of international trade, and it is strictly positive. The second term on the right-hand side of (3.22) is the competition effect of international trade, and it is strictly positive. The welfare gains from international trade are strictly positive.

The intuition behind Theorem 4 is simple. The welfare of a country is equal to the surplus that the local households capture in the markets for search goods. When trade between the two countries opens up, the measure of firms in the market for any search good increases, and local households come into contact with a larger number of firms. Since local households come into contact with a larger number of firms and firms post different prices, local households end up paying lower prices and capturing more surplus, even if firms do not change their prices. Firms, however, do change their prices. In fact, when trade between the two countries opens up, firms understand that their potential customers are in contact with more competitors and, for this reason, all firms lower their prices. Therefore, local households end up paying lower prices and capturing more surplus, even keeping the distribution of the households' contacts unchanged. Overall, international trade increases the local household's surplus by increasing the number of draws that households take from the price distribution (the shopping effect) and by shifting the price distribution to the left (the competition effect).

In the Dixit-Stiglitz model of monopolistic competition, international trade need not increase welfare. Indeed, since equilibrium is generally inefficient, a technological improvement, such as the integration of two markets, need not increase welfare.[11] In the Burdett-Judd model of imperfect competition, international trade always increases welfare. Indeed, since equilibrium is efficient, any technological improvement, including the integration of two markets, is guaranteed to increase welfare. Therefore, the observation that markets are imperfectly competitive and that firms charge positive markups is not

[11] Helpman and Krugman (1985) and Dhingra and Morrow (2019) derive sufficient conditions under which market integration leads to an increase in welfare.

evidence of inefficiencies, and it does not justify policies aimed at limiting international trade

4 Generalizations

In this section, I generalize the model of international trade and, in the process, highlight some additional features of equilibrium. In Section 4.1, I generalize the model to allow households to have a lower likelihood of contacting a foreign firm than a local firm. I refer to difference between the likelihoods as the informational distance between the two countries. I show that the effective measure of firms operating in a country is decreasing, and that the markups charged by firms are increasing in the informational distance between the countries. I show that households' consumption features home bias whenever there is some informational distance between the countries. In Section 4.2, I further generalize the model of the previous section to allow countries to differ in size. I show that, when countries are asymmetric in size, firms price to market—in the sense that the same firm posts a higher price and charges a higher markup in the smaller country, where prices are endogenously higher, than in the larger country, where prices are endogenously lower. In Section 4.3, I follow Melitz (2003) and generalize the model by introducing a fixed cost that a firm has to incur to become an exporter access the foreign market. I show that, in the presence of a fixed export cost, only the most efficient firms choose to export, while the rest sell only in the local market.

4.1 Informational proximity and home bias

Consider a world consisting of two identical countries. In country $j \in \{1, 2\}$, there is a measure b of households per search good $i \in [0, 1]$. Each household has preferences described by the utility function (2.1), defined over consumption of the search goods and consumption of the numeraire good. Each household is endowed with a quantity h of the numeraire good and with an equal share of the local firms. In country j, a firm pays a cost ζ to enter the market for a search good and, after paying the cost, it draws a marginal cost c of production from the distribution Φ. Each firm posts a price for its good for local households (the local market) and a possibly different price for its good for foreign households (the foreign market). In contrast to the version of the model presented in Section 3, I allow firms to price discriminate households based on their country of origin. I do so because, as it will be clear momentarily, the local and the foreign markets are not perfectly integrated and, hence, the demand curve faced by a firm might be different in the local and in the foreign markets.

A household in country j comes into contact with n_j local firms and n_{-j} foreign firms in the market for good i, where n_j is distributed as a Poisson with coefficient λs_j, n_{-j} is distributed as a Poisson with coefficient $\lambda \gamma s_{-j}$, s_j is the measure of local firms, s_{-j} is the measure of foreign firms, and γ is a parameter in the interval $(0, 1)$. For $\gamma = 0$, a household in country j does not contact any foreign firms. In this case, the equilibrium is the same as in the closed-economy model of Section 2. For $\gamma \in (0,1)$, a household in country j may contact some foreign firms, but the household is less likely to contact foreign firms than local firms. For $\gamma = 1$, a household in country j is equally likely to learn about foreign and local firms. In this case, the equilibrium is the same as in the open economy model of Section 3. The parameter γ measures the informational proximity between the two countries—the likelihood that a buyer learns about foreign relative to local sellers—and it captures the common-sense view that buyers are more likely to know about local than foreign sellers.

Consider the market for good $i \in [0,1]$ in country $j \in \{1,2\}$. A local firm with a marginal cost of c posting a price of p enjoys a profit $V_{j,j}(p,c)$, where

$$\begin{aligned} & V_{j,j}(p,c) \\ = & \left\{ \sum_{n=0}^{\infty} \frac{b}{s_j} \frac{e^{-\lambda s_j}(\lambda s_j)^{n+1}}{n!} \left[\sum_{k=0}^{\infty} \frac{e^{-\lambda\gamma s_{-j}}(\lambda\gamma s_{-j})^k}{k!} (1-F_{j,j}(p))^n (1-F_{-j,j}(p))^k \right] \right\} (p-c) \end{aligned} \tag{4.1}$$

The firm meets a measure $(b/s_j)\exp(-\lambda s_j)(\lambda s_j)^{n+1}/n!$ of households that are in contact with n additional local sellers, where b/s_j is the measure of households per local seller, $\exp(-\lambda s_j)(\lambda s_j)^{n+1}/(n+1)!$ is the probability that a household contacts $n+1$ local sellers (including the firm), and $n+1$ is the number of contacts that these households have with local sellers. The fraction of households that are in contact with k foreign sellers is $\exp(-\lambda\gamma s_{-j})(\lambda\gamma s_{-j})^k/k!$. The probability that a household that is in contact with n additional local sellers and with k foreign sellers is $(1-F_{j,j}(p))^n \cdot (1-F_{-j,j}(p))^k$, where $F_{j,j}(p)$ is the distribution of prices posted by local firms and $F_{-j,j}(p)$ is the distribution of prices posted by foreign firms. Since Lemma 1 obviously applies, (4.1) already makes use of the fact that $F_{j,j}(p)$ and $F_{-j,j}(p)$ do not have mass points.

The firm's profit can be rewritten as

$$\begin{aligned} V_{j,j}(p,c) = & \left\{ \begin{array}{l} \sum_{n=0}^{\infty} \frac{b}{s_j} \frac{e^{-\lambda s_j}(\lambda s_j)^{n+1}(1-F_{j,j}(p))^n}{n!} \\ \cdot e^{-\lambda\gamma s_{-j}F_{-j,j}(p)} \left[\sum_{k=0}^{\infty} \frac{e^{-\lambda\gamma s_{-j}(1-F_{-j,j}(p))}(\lambda\gamma s_{-j})^k}{k!} (1-F_{-j,j}(p))^k \right] \end{array} \right\} (p-c) \\ = & \; b\lambda \left[\sum_{n=0}^{\infty} \frac{e^{-\lambda s_j}(\lambda s_j)^n (1-F_{j,j}(p))^n}{n!} e^{-\lambda\gamma s_{-j}F_{j-,j}(p)} \right] (p-c) \\ = & \; b\lambda e^{-\lambda s_j F_{j,j}(p)} e^{-\lambda\gamma s_{-j}F_{j-,j}(p)} \left[\sum_{n=0}^{\infty} \frac{e^{-\lambda s_j(1-F_{j,j}(p))}(\lambda s_j)^n (1-F_{j,j}(p))^n}{n!} \right] (p-c) \\ = & \; b\lambda e^{-\lambda s_j F_{j,j}(p)} e^{-\lambda\gamma s_{-j}F_{j-,j}(p)} (p-c). \end{aligned} \tag{4.2}$$

The first line is obtained by collecting terms in (4.1). The second line is obtained by recognizing the summation with respect to k in the first line equals 1. The third line is obtained by collecting terms in the second line. The fourth line is obtained by recognizing that the summation with respect to n in the third line equals 1.

Next, consider a foreign firm with marginal cost c posting the price p. The firm's profit is given by

$$\begin{aligned} & V_{-j,j}(p,c) \\ = & \left\{ \sum_{n=0}^{\infty} \frac{b}{s_{-j}} \frac{e^{-\lambda\gamma s_{-j}}(\lambda\gamma s_{-j})^{n+1}}{n!} \left[\sum_{k=0}^{\infty} \frac{e^{-\lambda s_j}(\lambda s_j)^k}{k!} (1-F_{-j,j}(p))^n (1-F_{j,j}(p))^k \right] \right\} (p-c) \end{aligned} \tag{4.3}$$

The firm meets a measure $(b/s_{-j})\exp(-\lambda\gamma s_{-j})(\lambda\gamma s_{-j})^{n+1}/n!$ of households that are in contact with n additional foreign sellers, where b/s_{-j} is the measure of households per foreign seller, $\exp(-\lambda\gamma s_{-j})(\lambda\gamma s_{-j})^{n+1}/(n+1)!$ is the probability that a household contacts $n+1$ foreign sellers (including the firm), and $n+1$ is the number of contacts that these households have with foreign sellers. The fraction of households that are in contact with k local sellers is $\exp(-\lambda s_j)(\lambda s_j)^k/k!$. The probability that a household that is in contact with n additional foreign sellers and with k local sellers is $(1-F_{-j,j}(p))^n \cdot (1-F_{j,j}(p))^k$.

I can rewrite the firm's profit as

$$V_{-j,j}(p,c) = b\gamma\lambda e^{-\lambda\gamma s_{-j}F_{-j,j}(p)} e^{-\lambda s_j F_{j,j}(p)}(p-c). \tag{4.4}$$

The profit function (4.4) for a foreign firm is equal to γ times the profit function (4.2) for a local firm. Therefore, a foreign firm has the same preferences over prices as a local firm. Using this observation and following the same steps as in Lemma 3, it is easy to show that the price posted by a firm—be it local or foreign—is a strictly increasing function $p_j(c)$ of its marginal cost c. Since the cost distribution for local and foreign firms is the same, the distribution of prices posted by foreign firms is the same as the distribution of prices posted by local firms, i.e. $F_{-j,j}(p) = F_{j,j}(p) = F_j(p)$.

The price function $p_j(c)$ must equate the marginal cost and the marginal benefit of posting a higher price for a firm of type c, i.e.

$$\lambda(s_j + \gamma s_{-j})F_j'(p_j(c))\,(p_j(c) - c) = 1. \tag{4.5}$$

Since the pricing function $p_j(c)$ is strictly increasing in c, the fraction of firms positing a price smaller than $p_j(c)$ is equal to the fraction of firms with a marginal cost smaller than c, i.e.

$$F_j(p_j(c)) = \Phi(c) \Longrightarrow F_j'(p_j(c))p_j'(c) = \Phi'(c). \tag{4.6}$$

Combining the above observations yields

$$p_j'(c) = \lambda\left(s_j + \gamma s_{-j}\right)\Phi'(c)\left(p_j(c) - c\right). \tag{4.7}$$

The pricing function $p_j(c)$ must satisfy the differential equation (4.7). The pricing function must satisfy the boundary condition $p_j(c_h) = u$, since $p_j(c)$ is a strictly increasing function of c and the highest price on the support of the distribution F_j is equal to u.

From the differential equation (4.7) for $p_j(c)$, it is immediate to derive a differential equation for the markup $\mu_j(x)$ charged by a firm at the x-th quantile of the cost distribution Φ in the market for a search good in country j. The differential equation for $\mu_j(x)$ is

$$\mu_j'(x) = \lambda\left(s_j + \gamma s_{-j}\right)\left(\mu_j(x) - 1\right) - \mu_j(x)\frac{c'(x)}{c(x)}, \tag{4.8}$$

together with the boundary condition $\mu_j(1) = u/c(1)$.

The benefit that a firm from country j expects from entering the market for good i is given by

$$\int_{c_\ell}^{c_h} \left[V_{j,j}(p_j(c), c) + V_{j,-j}(p_{-j}(c), c)\right] d\Phi(c). \tag{4.9}$$

The first term in (4.9) is the firm's expected profit from selling the good in country j. The second term in (4.9) is the firm's expected profit from selling the good in country $-j$, where $V_{j,-j}(p, c)$ is given as in (4.4) with j and $-j$ swapped, and $p_{-j}(c)$ is given as in (4.7) with j and $-j$ swapped. Using the same steps as in Section 2, it is easy to show that

$$\int_{c_\ell}^{c_h} V_{j,j}(p_j(c), c) d\Phi(c) = b\lambda\left[e^{-\lambda(s_j+\gamma s_{-j})}(u - c_h) + \int_{c_\ell}^{c_h} e^{-\lambda(s_j+\gamma s_{-j})\Phi(c)}\Phi(c)dc\right] \tag{4.10}$$

and

$$\int_{c_\ell}^{c_h} V_{j,-j}(p_{-j}(c), c) d\Phi(c) = b\lambda\gamma\left[e^{-\lambda(s_{-j}+\gamma s_j)}(u - c_h) + \int_{c_\ell}^{c_h} e^{-\lambda(s_{-j}+\gamma s_j)\Phi(c)}\Phi(c)dc\right]. \tag{4.11}$$

If the measure s_j of firms from country j that enter the market for a search good is strictly positive, a firm's benefit from entering the market must equal the cost. If the measure s_j of firms from country j that enter the market for a search good is zero, a firm's benefit must be non-greater than the cost. Overall, the measure s_j is determined by the free-entry condition

$$\begin{aligned} \zeta \quad &\geq b\lambda\left[e^{-\lambda(s_j+\gamma s_{-j})}(u - c_h) + \int_{c_\ell}^{c_h} e^{-\lambda(s_j+\gamma s_{-j})\Phi(c)}\Phi(c)dc\right] \\ &+b\lambda\gamma\left[e^{-\lambda(s_{-j}+\gamma s_j)}(u - c_h) + \int_{c_\ell}^{c_h} e^{-\lambda(s_{-j}+\gamma s_j)\Phi(c)}\Phi(c)dc\right], \end{aligned} \tag{4.12}$$

and $s_j \geq 0$, where the two inequalities hold with complementary slackness. Notice that the right-hand side of (4.12) is strictly decreasing in s_j and, hence, there exists at most one s_j that satisfies the free-entry condition. Similarly, there exists at most one s_{-j} that satisfies the version of the free-entry condition (4.12) for country $-j$. Since the free-entry conditions for the two countries are identical except for the roles of s_j and s_{-j} being reversed, s_{-j} equals s_j in any equilibrium.

Let $s_t = s_j + \gamma s_{-j}$ denote the effective measure of firms in the market for search good

i in country j, which is the same as $s_{-j} + \gamma s_j$ since $s_j = s_{-j}$. Using the definition of s_t, I can rewrite the firm's free-entry condition (4.12) as

$$\zeta \geq b(1+\gamma)\lambda e^{-\lambda s_t}(u - c_h) + \int_{c_\ell}^{c_h} b(1+\gamma)\lambda e^{-\lambda s_t \Phi(c)}\Phi(c)dc, \tag{4.13}$$

and $s_t \geq 0$, where the two inequalities hold with complementary slackness. Since the right-hand side of (4.13) is strictly decreasing in s_t and converges to zero for s_t going to infinity, there is one and only one s_t that solves (4.13). Since the right-hand side of (4.12) is strictly increasing in γ for any s_t, the unique solution to (4.13) is strictly increasing in γ. In words, the effective firms in the market for good i in country j is strictly increasing in the informational proximity between the two countries. Since the right-hand side of (3.1) is the same as the right-hand side of (4.12) for $\gamma = 0$, it follows that $s_t > s_a$. In words, the effective measure of firms in the market for good i in country j is strictly higher when the two countries are allowed to trade than when they are in autarky.

Using the definition of s_t, I can rewrite the firm's optimal markup condition (4.8) as

$$\mu_t'(x) = \lambda s_t \left(\mu_t(x) - 1\right) - \mu_t(x)\frac{c'(x)}{c(x)} \tag{4.14}$$

together with the boundary condition $\mu_t(1) = u/c(1)$. Since s_t is strictly increasing in γ, the markup function $\mu_t(x)$ is strictly decreasing in γ. In words, the markups charged by firms are strictly decreasing in the informational proximity between the two countries. Since (4.14) coincides with (3.2) for $\gamma = 0$, it follows that $\mu_t(x) < \mu_a(x)$. In words, the markup charged by firms are strictly smaller when the two countries are allowed to trade than when they are in autarky.

A measure international trade is the quantity T of a search good that is sold to local households by foreign firms. The quantity T is given by

$$T = b\left(1 - e^{-\lambda s_t}\right) \cdot \frac{\gamma}{1+\gamma}. \tag{4.15}$$

The first term on the right-hand side of (4.15) is the quantity of the search good that is consumed by local households, which is equal to the measure $b\exp(-\lambda s_t)$ of local households that contact at least one firm. The second term on the right-hand side of (4.15) is the probability that the cheapest firm contacted by a local household is foreign, which is given by $\gamma/(1+\gamma)$. Since both $b\exp(-\lambda s_t)$ and $\gamma/(1+\gamma)$ are strictly increasing in γ, T is strictly increasing in γ. For $\gamma = 0$, there is no international trade. For $\gamma = 1$, international trade accounts for half of the consumption of local households. For $\gamma \in (0,1)$, international trade accounts for a fraction of the consumption of local households that is strictly positive, but strictly smaller than half. In this sense, as long as $\gamma \in (0,1)$, there is international trade but consumption displays some home bias. In contrast to standard models of international trade, home bias does not originate from the fact that households have different preferences for local and foreign varieties, nor from the fact that foreign firms face transportation costs. Home bias originates from the fact that households are more likely to learn about local than foreign firms.

Lastly, let me turn to the welfare effects of international trade. The difference between the welfare W_t of a country when international trade is allowed and the welfare W_a of the same country in autarky is

$$\begin{aligned} W_t - W_a \quad &= b \sum_{n=0}^{\infty} \left(\frac{e^{-\lambda s_t} (\lambda s_t)^n}{n!} - \frac{e^{-\lambda s_a} (\lambda s_a)^n}{n!} \right) (u - \bar{p}_{t,n}) \\ &+ b \sum_{n=0}^{\infty} \frac{e^{-\lambda s_a} (\lambda s_a)^n}{n!} (\bar{p}_{a,n} - \bar{p}_{t,n}). \end{aligned} \tag{4.16}$$

The first term on the right-hand side of (4.16) is the shopping effect of international trade. This term is strictly positive, since s_t is strictly increasing in γ and equals s_a for $\gamma = 0$. This term is strictly increasing in γ since s_t is strictly increasing in γ, and $\bar{p}_{t,n}$ is strictly decreasing in γ. The second term on the right-hand side of (4.16) is the competition effect of international trade. This term is positive, since $\bar{p}_{t,n}$ is strictly decreasing in γ and equals $\bar{p}_{a,n}$ for $\gamma = 0$. This term is strictly increasing in γ, since $\bar{p}_{t,n}$ is strictly decreasing in γ. From these observations, it follows that the welfare gains from international trade are strictly positive, and strictly increasing in the degree of informational proximity γ between the two countries.

4.2 Asymmetry and pricing to market

Consider a world consisting of two countries that differ in size. Country 1 is populated by a measure b_1 of households per search good $i \in [0, 1]$. Country 2 is populated by a measure b_2 of households per search good $i \in [0, 1]$. Country 1 is larger than country 2, in the sense that $b_1 > b_2$. Every household has preferences described by the utility function (2.1). Every household is endowed with a quantity h of the numeraire good, and with a share of the local firms. Country 1 and country 2 are also populated by an endogenous measure of firms. Firms pay a cost ζ to enter the market for a search good and, after paying the cost, they draw a marginal cost c from the distribution $\Phi(c)$. After observing its marginal cost, a firm posts a price for its good for local households (the local market), and a possibly different price for foreign households (the foreign market). Households in country $j \in \{1, 2\}$ comes into contact with n_j local firms and n_{-j} foreign firms in the market for a search good, where n_j is distributed as a Poisson with coefficient λs_j, n_{-j} is distributed as a Poisson with coefficient $\lambda \gamma s_{-j}$, s_j is the measure of local firms, s_{-j} is the measure of foreign firms, and γ is a parameter in the interval $(0, 1)$.

Consider the market for good i in country j. A local firm with marginal cost c posting the price p enjoys a profit

$$V_{j,j}(p, c) = b_j \lambda e^{-\lambda s_j F_{j,j}(p)} e^{-\lambda \gamma s_{-j} F_{-j,j}(p)} (p - c). \tag{4.17}$$

A foreign firm with marginal cost c posting the price p enjoys a profit

$$V_{-j,j}(p,c) = b_j \gamma \lambda e^{-\lambda s_j F_{j,j}(p)} e^{-\lambda \gamma s_{-j} F_{-j,j}(p)} (p-c). \tag{4.18}$$

In (4.17) and (4.18), $F_{j,j}(p)$ denotes the distribution of prices posted by local firms in country j, and $F_{-j,j}(p)$ denotes the distribution of prices posted by foreign firms in country j. Lemma 1 implies that $F_{j,j}$ and $F_{-j,j}$ have no mass points. Lemma 2 implies that the support of the distribution $(s_j F_{j,j} + \gamma s_{-j} F_{-j,j})/(s_j + \gamma s_{-j})$ is an interval and that the highest price on the interval is u. Since the profit function (4.18) for a foreign firm is equal to γ times the profit function (4.17) for a local firm, a foreign firm has the same preferences over prices as a local firm. Combining this observation with Lemma 3, I can show that the price posted in country j by any firm, local or foreign, is a strictly increasing function $p_j(c)$ of the firm's marginal cost c. Since the distribution of marginal costs among local and foreign firms is the same, it follows that $F_{j,j}(p) = F_{-j,j}(p) = F_j(p)$.

Using the necessary condition for the optimality of $p_j(c)$ and the strict monotonicity of $p_j(c)$, I can show that the pricing function $p_j(c)$ must satisfy the differential equation

$$p_j'(c) = \lambda(s_j + \gamma s_{-j})\Phi'(c)(p_j(c) - c). \tag{4.19}$$

Using the strict monotonicity of $p_j(c)$ and the fact that the highest price on the support of F_j is u, I can show that the pricing function $p_j(c)$ must satisfy the boundary condition $p_j(c_h) = u$.

The differential equation for $p_j(c)$ can be transformed into a differential equation for the markup $\mu_j(x)$ charged by a firm at the x-th quantile of the cost distribution Φ. Specifically, $\mu_j(x)$ solves the differential equation

$$\mu_j'(x) = \lambda\left(s_j + \gamma s_{-j}\right)\left(\mu_j(x) - 1\right) - \mu_j(x)\frac{c(x)}{c(x)} \tag{4.20}$$

together with the boundary condition $\mu_j(1) = u/c(1)$.

The free-entry condition for firms from country j is given by

$$\begin{aligned} \zeta \quad &\geq b_j \lambda \left[e^{-\lambda(s_j + \gamma s_{-j})}(u - c_h) + \int_{c_\ell}^{c_h} e^{-\lambda(s_j+\gamma s_{-j})\Phi(c)} \Phi(c) dc \right] \\ &+ b_{-j}\lambda\gamma \left[e^{-\lambda(s_{-j}+\gamma s_j)}(u - c_h) + \int_{c_\ell}^{c_h} e^{-\lambda(s_{-j}+\gamma s_j)\Phi(c)} \Phi(c) dc \right], \end{aligned} \tag{4.21}$$

and $s_j \geq 0$, where the two inequalities hold with complementary slackness. The left-hand side of (4.21) is the cost that a firm from country j has to pay to enter the market for a search good. The right-hand side of (4.21) is the benefit that a firm from country j enjoys by entering the market, and it is derived making use of the optimality condition for prices (4.19). The first line on the right-hand side of (4.21) is the expected profit that the firm from country j enjoys in the local market. The second line is the expected profit that the firm from country j enjoys in the foreign market.

For the sake of exposition, let me assume that ζ is low enough to guarantee that $s_1 > 0$ and $s_2 > 0$.[12] Then, equating the right-hand side of (4.21) for $j = 1$ to the right-hand side of (4.21) for $j = 2$ yields

$$\begin{aligned} & b_1(1-\gamma)\lambda \left[e^{-\lambda(s_1+\gamma s_2)}(u - c_h) + \int_{c_\ell}^{c_h} e^{-\lambda(s_1+\gamma s_2)\Phi(c)}\Phi(c)dc \right] \\ = \quad & b_2(1-\gamma)\lambda \left[e^{-\lambda(s_2+\gamma s_1)}(u - c_h) + \int_{c_\ell}^{c_h} e^{-\lambda(s_2+\gamma s_1)\Phi(c)}\Phi(c)dc \right]. \end{aligned} \tag{4.22}$$

If $s_1 = s_2$, the left-hand side of (4.22) is strictly greater than the right-hand side because $b_1 > b_2$, $\gamma \in (0,1)$ and $s_1+\gamma s_2 < s_2+\gamma s_1$. If $s_1 < s_2$, the left-hand side of (4.22) is strictly greater than the right-hand side because $b_1 > b_2$, $\gamma \in (0,1)$, and $s_1+\gamma s_2 < s_2+\gamma s_1$. From these observations, it follows that $s_1 > s_2$. In words, the equilibrium is such that more firms enter the market from the larger country than from the smaller country. There is a simple intuition for this finding. Households are more likely to come into contact with local than with foreign firms. Since there are more households in country 1 than in country 2, firms in country 1 are contacted by more buyers and, hence, they have a stronger incentive to enter.

The markups charged by firms in country 1 and country 2 are, respectively, given by

$$\mu_1'(x) = \lambda\left(s_1 + \gamma s_2\right)\left(\mu_1(x) - 1\right) - \mu_1(x)\frac{c'(x)}{c(x)}, \tag{4.23}$$

and

$$\mu_2'(x) = \lambda\left(s_2 + \gamma s_1\right)\left(\mu_2(x) - 1\right) - \mu_2(x)\frac{c'(x)}{c(x)}, \tag{4.24}$$

with the boundary conditions $\mu_1(1) = u/c(1)$ and $\mu_2(1) = u/c(1)$. Since $s_1 > s_2$ and $\gamma \in (0,1)$, $s_1 + \gamma s_2 > s_2 + \gamma s_1$. In turn $s_1 + \gamma s_2 > s_2 + \gamma s_1$ implies that $\mu_1(x) < \mu_2(x)$ for all $x \in [0,1)$. In words, the markups charged by firms in the larger country are lower than the markups charged by firms in the smaller country. The intuition for this result is simple. As there are more firms from country 1 than from country 2 and households are more likely to come into contact with local than with foreign firms, the choice set of households in country 1 (the larger country) is larger than the choice set of households in country 2 (the smaller country). For this reason, competition is more intense, and markups are lower in the larger country than in the smaller country.

Now, consider a firm at the x-th quantile of the cost distribution Φ. The firm finds it optimal to post the price $c(x)\mu_1(x)$ in country 1, and the price $c(x)\mu_2(x)$ in country 2. Since $\mu_1(x) \neq \mu_2(x)$, the firm finds it optimal to price discriminate customers based on their country of origin. Since $\mu_1(x) < \mu_2(x)$, the firm finds it optimal to charge a lower price to customers from country 1, where prices are lower, than to customers from country

[12] I make the assumption to keep the exposition light. The results are general.

2, where prices are higher. In this sense, the firm prices to market. The firm prices to market not because firms are more efficient in country 1 than in country 2, but because the market is endogenously more competitive in country 1 than in country 2.

The comparison of the equilibrium outcomes with and without international trade is the same as in Section 4.1. Namely, the effective measure $s_j + \gamma s_{-j}$ of firms operating in country j when international trade is allowed is strictly greater than the measure $s_{a,j}$ of firms in country j when international trade is not allowed. The markups $\mu_j(x)$ charged by firms in country j when international trade is allowed are strictly smaller than the markups $\mu_{a,j}(x)$ charged by firms in country j when international trade is not allowed. Since international trade increases the size of the households' choice sets and it lowers markups, it unambiguously increases welfare.

4.3 Fixed cost and selection into export

Consider a world consisting of two identical countries. Country $j \in \{0, 1\}$ is populated by a measure b of households per search good $i \in [0, 1]$. Every household has preferences described by the utility function (2.1), defined over the consumption of the search goods and the consumption of the numeraire good. Every household is endowed with a quantity h of the numeraire good, and a share of local firms. Country j is also populated by an endogenous measure of firms. A firm pays a cost ζ to enter the market for a search good and, after paying the cost, it draws a marginal cost c from the distribution $\Phi(c)$. As in Melitz (2003), I assume that, after observing c, the firm chooses whether to become an exporter by paying a cost $\eta > 0$. The firm then chooses a price for households of country j (the local market) and, if it does become an exporter, a price for households of country $-j$ (the foreign market). Households in country j come into contact with n_j firms, where n_j is distributed as a Poisson with coefficient $\lambda(s_j + e_j)$, s_j is the measure of local firms, and e_j is the measure of foreign firms exporting to country j.

Consider the market for good i in country j. A firm with marginal cost c posting the price p enjoys the profit

$$V_j(p, c) = b\lambda e^{-\lambda(s_j+e_j)} F_j(p)(p - c), \tag{4.25}$$

where F_j denotes the price distribution in country j. Lemma 1 implies that F_j has no mass points. Lemma 2 implies that the support of the distribution F_j is an interval, and that the highest price on such interval is u. Lemma 3 implies that the price posted by a firm is a strictly increasing function $p_j(c)$ of the firm's marginal cost c.

The profit function $V_j(p, c)$ is strictly decreasing in c for any p. For this reason, the maximum $V_j^*(c)$ of the profit function $V_j(p, c)$ with respect to p is strictly decreasing in c. Since $V_j^*(c)$ is strictly decreasing in c, there exists a cutoff c_j^e such that $V_j^*(c_j^e) = \eta$, $V_j^*(c) > \eta$ if $c < c_j^e$, and $V_j^*(c) < \eta$ if $c > c_j^e$. Therefore, there exists a cutoff c_j^e such that a firm from country $-j$ exports to country j if and only if $c \leq c_j^e$. From this observation, it follows that the measure e_j of firms from country $-j$ that export to country j is given by $s_{-j}\Phi(c_j^e)$, and the distribution of marginal costs among firms from country $-j$ that export to country j is given by $\Phi(c)/\Phi(c_j^e)$.

The first-order condition for the optimality of $p_j(c)$ is

$$\lambda(s_j + e_j)F_j'(p_j(c))\,(p_j(c) - c) = 1. \tag{4.26}$$

Since $p_j(c)$ is strictly increasing in c, the fraction $F_j(p_j(c))$ of firms that post a price smaller than $p_j(c)$ must be equal to the fraction $\Phi_j(c)$ of firms with a cost smaller than c, where

$$\Phi_j(c) = \begin{cases} \dfrac{s_j\Phi(c) + e_j\Phi(c)}{s_j + e_j}, & \text{if } c \in [c_\ell, c_j^e], \\ \dfrac{s_j\Phi(c) + e_j}{s_j + e_j}, & \text{if } c \in [c_j^e, c_h]. \end{cases} \tag{4.27}$$

Differentiating $F_j(p_j(c)) = \Phi_j(c)$ with respect to c yields $F_j'(p_j(c))p_j'(c) = \Phi_j'(c)$, where

$$\Phi_j'(c) = \begin{cases} \Phi'(c), & \text{if } c \in [c_\ell, c_j^e], \\ \dfrac{s_j}{s_j + e_j}\Phi'(c), & \text{if } c \in [c_j^e, c_h]. \end{cases} \tag{4.28}$$

Using the fact that $F_j'(p_j(c))p_j'(c) = \Phi_j'(c)$, I can rewrite the first-order condition for the optimality of $p_j(c)$ as

$$p_j'(c) = \lambda(s_j + e_j)\Phi_j'(c)\,(p_j(c) - c)\,. \tag{4.29}$$

The expression in (4.29) is a differential equation for $p_j(c)$. This differential equation must satisfy the boundary condition $p_j(c_h) = u$, since the price function $p_j(c)$ is strictly increasing in c and the highest price on the support of F_j is u.

Using the fact that $F_j(p_j(c)) = \Phi_j(c)$, I can write the profit that a firm from country j expects in the local market as

$$\begin{aligned} \int_{c_\ell}^{c_h} V_j^*(c)d\Phi(c) &= V_j^*(c_h) - \int_{c_\ell}^{c_h}\left[\int_c^{c_h} V_j^{*\prime}(\hat{c})d\hat{c}\right]\Phi'(c)dc \\ &= b\lambda e^{-\lambda(s_j+e_j)}(u - c_h) + b\lambda\int_{c_\ell}^{c_h} e^{-\lambda(s_j+e_j)\Phi_j(c)}\Phi(c)dc. \end{aligned} \tag{4.30}$$

The first term on the right-hand side of (4.30) is the profit that a firm from country enjoys in the local market if its marginal cost is c_h. The second term on the right-hand side of (4.30) is the expectation of the extra profit that the firm enjoys in the local market because its marginal cost is drawn from Φ rather than being equal to c_h. The expression in the second line is derived by computing $V_j^*(c)$ and then integrating by parts.

Using the fact that $F_j(p_j(c)) = \Phi_j(c)$, I can write the profit that a firm from country

$-j$ expects in the foreign market as

$$\begin{aligned}\int_{c_\ell}^{c_h} \max\{V_j^*(c)-\eta,0\}d\Phi(c) &= V_j^*(c_j^e)-\eta-\int_{c_\ell}^{c_j^e}\left[\int_c^{c_j^e} V_j^{*\prime}(\hat{c})d\hat{c}\right]\Phi'(c)dc \\ &= b\lambda\int_{c_\ell}^{c_j^e} e^{-\lambda(s_j+e_j)\Phi_j(c)}\Phi(c)dc.\end{aligned} \tag{4.31}$$

The first line on the right-hand side of (4.31) makes use of the fact that $V_j^*(c)\geq\eta$ if and only if $c\leq c_j^e$. The second line on the right-hand side of (4.31) makes use of $V_j^*(c_j^e)=\eta$, the expression for $V_j^{*\prime}(c)$, and integration by parts.[13]

The export threshold c_j^e is such that $V_j^*(c_j^e)=\eta$. I can rewrite this condition as

$$\begin{aligned}\eta &= V_j^*(c_h)-\int_{c_{-j}^e}^{c_h} V_j^{*\prime}(\hat{c})d\hat{c} \\ &= b\lambda e^{-\lambda(s_j+e_j)}(u-c_h)+b\lambda\int_{c_j^e}^{c_h} e^{-\lambda(s_j+e_j)\Phi_j(\hat{c})}d\hat{c}.\end{aligned} \tag{4.32}$$

The first line on the right-hand side of (4.32) expresses $V_j^*(c_j^e)$ as $V_j^*(c_h)$ plus the negative of the integral of the derivative $V_j^{*\prime}(\hat{c})$ between c_j^e and c_h. The second line is obtained by computing $V_j^{*\prime}(\hat{c})$.

The free-entry condition for firms from country j is

$$\begin{aligned}\zeta &\geq b\lambda e^{-\lambda(s_j+e_j)}(u-c_h)+\int_{c_\ell}^{c_h} e^{-\lambda(s_j+e_j)\Phi_j(c)}\Phi(c)dc \\ &+\int_{c_\ell}^{c_{-j}^e} e^{-\lambda(s_{-j}+e_{-j})\Phi_{-j}(c)}\Phi(c)dc.\end{aligned} \tag{4.33}$$

and $s_j\geq 0$, with complementary slackness. The left-hand side of (4.33) is the cost of entry for a firm from country j, which is given by ζ. The right-hand side of (4.33) is the benefit of entry for a firm from country j, which is given by the expected profit from sales in the local market and from sales in the foreign market (net of the export cost η).

The characterization of equilibrium is now complete. The free-entry condition (4.33) for firms from country j pins down s_j, and the analogous free-entry condition for firms from country $-j$ pins down s_{-j}. The exporting indifference condition (4.32) for firms from country $-j$ pins down c_j^e and, in turn, the measure of exporters e_j, and the distribution $\Phi_j(c)$ of marginal costs across firms operating in country j. The analogous indifference condition for firms from country j pins down c_{-j}^e, e_{-j}, and $\Phi_{-j}(c)$. The optimality condition (4.29) for firms in country j pins down the price function $p_j(c)$. The analogous optimality condition for firms in country $-j$ pins down $p_{-j}(c)$.

In contrast to the baseline model of Section 3, the measure s_j+e_j of firms operating

[13] Notice that $V_j^*(c_j^e)=\eta$ only when $c_j^e\in(c_h,c_\ell)$. I will maintain this implicit assumption though the rest of the analysis.

in country j when international trade is allowed need not be larger than the measure s_a of firms when country j is in autarky. For this reason, the prices charged by firms in country j need not be lower when international trade is allowed. Nevertheless, international trade increases welfare in country j. Indeed, it is easy to verify that the equilibrium decentralizes the solution of the problem of a utilitarian social planner and, hence, it maximizes the sum of the welfare of households in country j and $-j$. Since the two countries are identical, the equilibrium maximizes the welfare of each individual country j. Since the social planner has always the option of setting c_j^e and c_{-j}^e to c_ℓ and, hence, to reproduce the equilibrium outcomes when international trade is not allowed, welfare is higher with international trade than in autarky.

5 Imperfectly Elastic Supply of Inputs

In the previous sections, I maintained the assumptions that the household's utility function is linear in the numeraire good and that the numeraire good is the only input of production. Hence, the results in the previous sections are established under the assumption that the input supply is perfectly elastic. In this section, I show that the results in Sections 2 and 3 generalize to an environment in which the household's utility function is concave in a consumption aggregator of the search good and a numeraire good and in leisure, and that labor is the only input of production. Hence, this section shows that the results in Sections 2 and 3 generalize to an environment in which the input supply is imperfectly elastic.

5.1 Closed economy

The economy is populated by a continuum of households with measure b per search good $i \in [0, 1]$. The preferences of a household are described by

$$U\left(z + \int_0^1 y_i u di, 1 - \ell\right), \tag{5.1}$$

where $1 - \ell$ denotes leisure, z denotes consumption of the numeraire good, y_i is an indicator function that takes the value 1 if the household consumes one unit of good i, and $u > 0$ is a parameter that describes the household's utility from consuming one unit of good i expressed in terms of the numeraire good. The utility function U is strictly increasing and strictly concave in consumption and leisure. The utility function U is separable in consumption and leisure, in the sense that the marginal utility of leisure is independent of consumption, and the marginal utility of consumption is independent of leisure. Households are endowed with a quantity h of the numeraire good, a quantity 1 of time, and the ownership of the firms.

The economy is populated by a positive measure of firms in the market for search good $i \in [0, 1]$. The measure of firms in the market for i is endogenous. In order to enter the market for i, a firm has to pay a quantity ζ of labor. After entering market i, a firm draws its idiosyncratic type c from the distribution $\Phi(c)$, where $\Phi(c)$ is a twice-continuously differentiable cumulative distribution with support $[c_\ell, c_h]$, $0 < c_\ell < c_h$. A firm of type c operates a constant return to scale technology such that it requires a quantity c of labor to produce a measure 1 of units of good i. After observing its type, a firm chooses the price p for its good, where p is measured in units of the numeraire good.

The market for good $i \in [0, 1]$ is frictional, in the sense that a household cannot purchase good i from any firm in the market but only from the subset of firms with whom he comes into contact. In particular, a household in market i comes into contact with n randomly-selected firms, where n is a draw from a Poisson distribution with coefficient λs, λ is a parameter, and s is the measure of firms in the market. The buyer observes the price charged by each of the n firms and decides whether and where to purchase a unit of the good.

I charcaterize the equilibrium under several assumptions. First, I assume that the household's valuation u for a search good exceeds the firm's marginal cost wc for all $c \in [c_\ell, c_h]$, i.e. $u > wc_h$. Second, I assume that the household's choice of labor ℓ is interior, i.e. $\ell \in (0, 1)$. Third, I assume that a positive measure of firms find it optimal to enter the market for a search good, i.e. $s > 0$. These assumptions are not critical for the results. The assumptions simplify the exposition of the results, by sparing me and the readers with corner solutions..

The first-order condition for the optimality of the households' choice of labor ℓ is

$$U_2\left(h + (1 - e^{-\lambda s})u, 1 - \ell\right) = U_1\left(h + (1 - e^{-\lambda s})u, 1 - \ell\right) w, \tag{5.2}$$

where U_1 denotes the derivative of the utility function (5.1) with respect to its first argument (the aggregator of the search goods and the numeraire good), and U_2 denotes the derivative of the utility function (5.1) with respect to its second argument (leisure). The left-hand side of (5.2) is the household's marginal utility of leisure. The right-hand side of (5.2) is the household's marginal utility of consumption multiplied by the wage w expressed in units of the numeraire good. The household's marginal utilities in (5.2) are evaluated at the equilibrium levels of consumption and leisure. The equilibrium level of consumption is $h + (1 - \exp(-\lambda s))u$. In fact, firms post prices for the search goods that are non-greater than u. Hence, a household purchases a unit of the search good as long as it comes into contact with at least one firm, an event that occurs with probability $1 - \exp(-\lambda s)$. A household purchases a unit of the search good by giving some of the numeraire good to the seller. Firms, however, rebate these revenues back to the households as profits. Hence, a household ends up consuming its endowment of the numeraire good.

The first-order condition for the optimality of the firm's price $p(c)$ is

$$p'(c) = \lambda s \Phi'(c)\left(p(c) - wc\right), \tag{5.3}$$

with the boundary condition $p(c_h) = u$. Condition (5.3) is derived by noting that Lemma 1, Lemma 2 and Lemma 3 immediately apply to this version of the model. Lemma 1 guarantees that the price distribution F is atomless. Lemma 2 guarantees that the support of the price distribution F is an interval $[p_\ell, p_h]$, with $p_h = u$. Lemma 3 guarantees that the price posted by a firm is a strictly increasing function $p(c)$ of the firm's marginal cost c. Therefore, the fraction of firms with a price smaller than $p(c)$ is equal to the fraction of firms with a marginal cost smaller than c and, hence, $F(p(c)) = \Phi(c)$ and $F'(p(c))p'(c) = \Phi'(c)$. The differential equation in (5.3) is the necessary condition for the optimality of $p(c)$ for a firm with cost c, rewritten using the fact that $F'(p(c))p'(c) = \Phi'(c)$. The boundary condition follows from Lemma 2 and Lemma 3.

The free-entry condition for firms is given by

$$\zeta = b\lambda e^{-\lambda s}\left(\frac{u}{w} - c_h\right) + b\lambda \int_{c_\ell}^{c_h} e^{-\lambda s \Phi(c)} \Phi(c) dc. \tag{5.4}$$

The left-hand side of (5.4) is a firm's cost of entering the market for a search good, measured in units of labor. The right-hand side of (5.4) is the firm's benefit of entering the market for a search good, also measured in units of labor. The right-hand side of (5.4) is obtained following the same steps as in Section 2 and it has the same interpretation as in Section 2. Namely, the first term is the firm's benefit of entering when its marginal cost of production is c_h. The second term is the firm's additional benefit of entering when its marginal cost c is a random draw from the distribution Φ rather than c_h. Condition (5.4) states that the firm's cost and benefit of entry must be equal.

The market-clearing condition for labor is

$$b\ell = \zeta s + \sum_{n=1}^{\infty} b \frac{e^{-\lambda s}(\lambda s)^n}{n!} \int_{c_\ell}^{c_h} cn \left(1 - \Phi(c)\right)^{n-1} \Phi'(c) dc. \tag{5.5}$$

The left-hand side of (5.5) is the aggregate supply of labor, which is given by the measure of households multiplied by the supply of labor per household. The right-hand side of (5.5) is the aggregate demand of labor, and it is the sum of two terms. The first term is the quantity of labor demanded by firms to enter the market for search goods, which is given by the measure of firms multiplied by the labor cost of entry per firm. The second term is the quantity of labor demanded by firms to produce the search goods. There is a measure $b \exp(-\lambda s)(\lambda s)^n/n!$ of households that come into contact with n firms. The households purchase the good from the firm with the lowest price, which is also the firm with the lowest marginal cost, among the n contacted firms. The distribution of the lowest marginal cost among n firms is given by the cumulative function $1 - (1 - \Phi(c))^n$, which has a density $n\left(1 - \Phi(c)\right)^{n-1} \Phi'(c)$. Therefore, the firm's expected labor cost of production for households with n contacts is given by c integrated with respect to the density $n\left(1 - \Phi(c)\right)^{n-1} \Phi'(c)$.

The first-order condition for the optimality of the households' choice of labor ℓ pins down the wage. In fact, (5.2) can be written as

$$w(s,\ell) = \frac{U_2\left(h + (1-\exp(-\lambda s))u, 1-\ell\right)}{U_1\left(h + (1-\exp(-\lambda s))u, 1-\ell\right)}. \tag{5.6}$$

The utility function U is strictly increasing and strictly concave in consumption and leisure, i.e. $U_{1,1} < 0$ and $U_{2,2} < 0$. The utility function is separable in consumption and leisure, i.e. $U_{1,2} = U_{2,1} = 0$. From these observations, it follows that the wage w is strictly increasing in the amount of labor ℓ supplied by the household. Similarly, the wage w is strictly increasing in the measure s of firms in the market.

The free-entry condition for firms pins down the measure of firms in the market for search goods. Using (5.6), condition (5.4) can be written as

$$\zeta = b\lambda e^{-\lambda s}\left(\frac{u}{w(s,\ell)} - c_h\right) + b\lambda \int_{c_\ell}^{c_h} e^{-\lambda s \Phi(c)}\Phi(c)dc. \tag{5.7}$$

The right-hand side of (5.7) is strictly decreasing in s and it converges to zero for s going to infinity. Hence, there exists a unique measure s of firms that solves the free-entry condition (5.7). I denote the solution as $s(\ell)$. The right-hand side of (5.7) is strictly decreasing in ℓ, since $w(s,\ell)$ is strictly increasing in ℓ. Hence, the measure of firms $s(\ell)$ that solves (5.7) is strictly decreasing in the amount of labor ℓ supplied by households.

The market-clearing condition pins down the amount of labor supplied by households. Using (5.2) and integration by parts, condition (5.5) be written as

$$b\ell = \zeta s(\ell) + b\left[c_\ell - e^{-\lambda s(\ell)}c_h + \int_{c_\ell}^{c_h} e^{-\lambda s(\ell)\Phi(c)}dc\right]. \tag{5.8}$$

The left-hand side of (5.8) takes the value 0 for $\ell = 0$, and it is strictly increasing in ℓ. The right-hand side of (5.8) strictly positive for $\ell = 0$, and it is strictly decreasing in ℓ. In fact, the derivative of the right-hand side of (5.8) with respect to ℓ is

$$\begin{aligned} &\left\{\zeta + b\left[\lambda e^{-\lambda s(\ell)}c_h - \lambda\int_{c_\ell}^{c_h} e^{-\lambda s(\ell)\Phi(c)}\Phi(c)dc\right]\right\} s'(\ell) \\ = &\left\{b\lambda e^{-\lambda s(\ell)}\frac{u}{w(s(\ell),\ell)}\right\} s'(\ell) < 0, \end{aligned} \tag{5.9}$$

where the second line in (5.9) is obtained by using (5.7) to replace ζ. From these observations, it follows that there exists a unique ℓ that satisfies condition (5.8). In turn, this implies that there is a unique $s = s(\ell)$ that satisfies condition (5.7), and there exists a unique wage $w = w(s(\ell),\ell)$ that satisfies condition (5.6). Therefore, the equilibrium exists and it is unique.

5.2 Welfare properties of equilibrium

The problem of a utilitarian social planner is

$$\max_{\ell,s} bU\left(h+(1-e^{-\lambda s})u, 1-\ell\right), \text{ s.t.}$$
$$b\ell = \zeta s + \sum\nolimits_{n=1}^{\infty} b\frac{e^{-\lambda s}(\lambda s)^n}{n!}\int_{c_\ell}^{c_h} cn\left(1-\Phi(c)\right)^{n-1}\Phi'(c)dc. \tag{5.10}$$

The planner's objective is the maximize the sum of the households' utilities. The planner chooses how much labor ℓ the households should supply, and how many firms s should enter the market. The planner's choices of ℓ and s are constrained by the fact that the amount of labor supplied by the households must be equal to the amount of labor used by the firms to enter the market and to produce the search goods. Note that, in formulating the planner's problem, I make use of the fact that the planner finds it optimal to instruct a household to purchase a unit of search good i whenever the household meets a firm, and to purchase the good from the firm with the lowest cost whenever the household meets multiple firms.

Using integration by parts, I can rewrite the planner's resource constraint as

$$b\ell = \zeta s + b\left[c_\ell - e^{-\lambda s}c_h + \int_{c_\ell}^{c_h} e^{-\lambda s\Phi(c)}dc\right]. \tag{5.11}$$

The first-order condition for the optimality of ℓ is

$$U_2\left(h+(1-e^{-\lambda s})u, 1-\ell\right) = \mu. \tag{5.12}$$

The left-hand side of (5.12) is the planner's marginal cost of increasing the amount of labor supplied by the household, which is the household's marginal utility of leisure. The right-hand side of (5.12) is the planner's marginal benefit of increasing the amount of labor supplied by the household, which is given by the Lagrange multiplier μ on the resource constraint.

The first-order condition for the optimality of s is

$$bU_1\left(h+(1-e^{-\lambda s})u, 1-\ell\right)\lambda e^{-\lambda s}u = \mu\left[\zeta + b\lambda e^{-\lambda s}c_h - b\lambda\int e^{-\lambda s\Phi(c)}\Phi(c)dc\right]. \tag{5.13}$$

The left-hand side of (5.13) is the planner's marginal benefit of increasing the measure s of firms in the market for search goods, which is given by the household's marginal utility of consumption multiplied by the increase in consumption generated by an increase in s. The right-hand side of (5.13) is the planner's marginal cost of increasing the measure s of firms in the market for search goods, which is given by the amount of labor needed by an additional firm scaled by the Lagrange multiplier on the resource constraint.

Using (5.12) to replace μ, I can rewrite the optimality condition (5.13) as

$$b\lambda e^{-\lambda s} u \frac{U_1\left(h+(1-e^{-\lambda s})u, 1-\ell\right)}{U_2\left(h+(1-e^{-\lambda s})u, 1-\ell\right)} = \zeta + b\lambda e^{-\lambda s} c_h - b\lambda \int e^{-\lambda s \Phi(c)} \Phi(c) dc. \tag{5.14}$$

Rearranging terms, I can rewrite (5.14) as

$$\zeta = b\lambda e^{-\lambda s} \left(u \frac{U_2}{U_1} - c_h \right) + b\lambda \int e^{-\lambda s \Phi(c)} \Phi(c) dc. \tag{5.15}$$

The expressions in (5.11) and (5.15) are necessary conditions for the optimality of the planner's choices of ℓ and s. The expressions in (5.11) and (5.15) are the same as the clearing condition for the labor market (5.8) and the free-entry condition for firms (5.7). Since there exists a unique solution to (5.8) and (5.7), (5.11) and (5.15) uniquely pin down the ℓ and s that solve the planner's problem. Moreover, the equilibrium ℓ and s coincide with the solution to the social planner's problem. That is, the equilibrium is efficient.

5.3 Open economy

I consider a world economy comprised of two identical countries. In country $j \in \{1, 2\}$, households have the same preferences and the same endowment as in Section 5.1. In country j, firms operate the same technology as in Section 5.1. If international trade is not allowed, a household in country j contacts n firms in the market for a search good, where n is distributed as a Poisson with coefficient λs_a, and s_a is the measure of local firms producing the good. If international trade is allowed, a household in country j comes into contact with n firms, where n is distributed as a Poisson with coefficient λs_t, and s_t is the measure of local and foreign firms producing the good.

If international trade is not allowed, the equilibrium in each country is given by a measure s_a of firms, a quantity ℓ_a of labor supplied by each household, and by a wage w_a such that $\{s_a, \ell_a, w_a\}$ satisfy the firm's free-entry condition

$$\frac{\zeta}{b} = \lambda e^{-\lambda s_a} \left(\frac{u}{w_a} - c_h \right) + \lambda \int_{c_\ell}^{c_h} e^{-\lambda s_a \Phi(c)} \Phi(c) dc, \tag{5.16}$$

the household's optimality condition

$$w_a = \frac{U_2\left(h+(1-e^{-\lambda s_a})u, 1-\ell_a\right)}{U_1\left(h+(1-e^{-\lambda s_a})u, 1-\ell_a\right)}, \tag{5.17}$$

and the market-clearing condition

$$\ell_a = \left(\frac{\zeta}{b}\right) s_a + \left[c_\ell - e^{-\lambda s_a} c_h + \int_{c_\ell}^{c_h} e^{-\lambda s_a \Phi(c)} dc \right]. \tag{5.18}$$

If international trade is allowed, the equilibrium in each country is given by a measure s_t of local and foreign firms, a quantity ℓ_t of labor supplied by each household, and by a wage w_t such that $\{s_t, \ell_t, w_t\}$ satisfy the firm's free-entry condition

$$\frac{\zeta}{2b} = \lambda e^{-\lambda s_t}\left(\frac{u}{w_t} - c_h\right) + \lambda \int_{c_\ell}^{c_h} e^{-\lambda s_t \Phi(c)} \Phi(c) dc, \tag{5.19}$$

the household's optimality condition

$$w_t = \frac{U_2\left(h + (1 - e^{-\lambda s_t})u, 1 - \ell_t\right)}{U_1\left(h + (1 - e^{-\lambda s_t})u, 1 - \ell_t\right)}, \tag{5.20}$$

and the market-clearing condition

$$\ell_t = \left(\frac{\zeta}{b}\right)\frac{s_t}{2} + \left[c_\ell - e^{-\lambda s_t} c_h + \int_{c_\ell}^{c_h} e^{-\lambda s_t \Phi(c)} dc\right]. \tag{5.21}$$

Notice that the system of equations (5.19)-(5.21) is the same as the system of equations (5.16)-(5.18), except that $\zeta/2b$ takes the place of ζ/b. Therefore, to characterize the difference between $\{s_a, \ell_a, w_a\}$ and $\{s_t, \ell_t, w_t\}$, it is useful to analyze the effect of changes in $\hat{\zeta} \equiv \zeta/b$ on the solution to the system of equations (5.16)-(5.18). To this aim, let me rewrite the free-entry condition (5.16) as

$$f(s, \ell, \hat{\zeta}) = \lambda e^{-\lambda s}\left(u\frac{U_1}{U_2} - c_h\right) + \lambda \int_{c_\ell}^{c_h} e^{-\lambda s \Phi(c)} \Phi(c) dc - \hat{\zeta} = 0, \tag{5.22}$$

where I substituted in the household's optimality condition (5.17). The derivative of f with respect to s is

$$f_1(s, \ell, \hat{\zeta}) = -\lambda^2 e^{-\lambda s}\left(u\frac{U_1}{U_2} - c_h\right) + \lambda^2 \int_{c_\ell}^{c_h} e^{-\lambda s \Phi(c)} (\Phi(c))^2 dc + \lambda^2 e^{-2\lambda s} u^2 \frac{U_{1,1}}{U_2}. \tag{5.23}$$

The derivative of f with respect to ℓ is

$$f_2(s, \ell, \hat{\zeta}) = \lambda e^{-\lambda s} u \frac{U_{2,2} U_1}{U_2^2}. \tag{5.24}$$

The derivative of f with respect to $\hat{\zeta}$ is

$$f_3(s, \ell, \hat{\zeta}) = -1. \tag{5.25}$$

Let me rewrite the market-clearing condition (5.18) as

$$g(s, \ell, \hat{\zeta}) = \hat{\zeta} s + c_\ell - e^{-\lambda s} c_h + \int_{c_\ell}^{c_h} e^{-\lambda s \Phi(c)} dc - \ell = 0. \tag{5.26}$$

The derivative of g with respect to s is

$$g_1(s, \ell, \hat{\zeta}) = \hat{\zeta} + \lambda e^{-\lambda s} c_h - \lambda \int_{c_\ell}^{c_h} e^{-\lambda s \Phi(c)} \Phi(c) dc = \lambda e^{-\lambda s} u \frac{U_1}{U_2}. \tag{5.27}$$

The derivative of g with respect to ℓ is

$$g_2(s, \ell, \hat{\zeta}) = -1. \tag{5.28}$$

The derivative of g with respect to $\hat{\zeta}$ is

$$g_3(s, \ell, \hat{\zeta}) = s. \tag{5.29}$$

Once linearized, the system of equations (5.16) and (5.18) becomes

$$\begin{bmatrix} f_1 & f_2 \\ g_1 & g_2 \end{bmatrix} \cdot \begin{bmatrix} ds \\ d\ell \end{bmatrix} = \begin{bmatrix} -f_3 \\ -g_3 \end{bmatrix} d\hat{\zeta}. \tag{5.30}$$

Applying Cramer's rule to solve (5.30) yields

$$\frac{ds}{d\hat{\zeta}} = \frac{-f_3 g_2 + f_2 g_3}{f_1 g_2 - f_2 g_1}, \tag{5.31}$$

and

$$\frac{d\ell}{d\hat{\zeta}} = \frac{-f_1 g_3 + f_3 g_1}{f_1 g_2 - f_2 g_1}. \tag{5.32}$$

The denominator in (5.31) and (5.32) is strictly positive, since $f_1 < 0$, $f_2 < 0$, $g_1 > 0$ and $g_2 < 0$. The numerator in (5.31) is strictly negative, since $f_2 < 0$, $f_3 < 0$, $g_2 < 0$ and $g_3 > 0$. Therefore, $ds/d\hat{\zeta} < 0$. The numerator in (5.32) cannot be signed, since $f_1 < 0$, $f_3 < 0$, $g_1 > 0$ and $g_3 > 0$. Therefore, $d\ell/d\hat{\zeta}$ may be positive or negative.

I am now in the position to characterize the equilibrium effects of international trade. Since $\zeta/2b < \zeta/b$ and $ds/d\hat{\zeta} < 0$, the measure of local and foreign firms selling a search good when international trade is allowed is strictly greater than the measure of firms selling a search good in autraky, i.e. $s_t > s_a$. Since $s_t > s_a$, (5.3) implies that firms post lower prices and charge lower markups when international trade is allowed than in autarky, i.e. $p_t(c) < p_a(c)$ and $\mu_t(x) < \mu_a(x)$. Since $d\ell/d\hat{\zeta}$ may be positive or negative, the quantity of labor supplied by households may be larger or smaller when international trade is allowed than in autarky, i.e. ℓ_t may be larger or smaller than ℓ_a. Overall, international trade has the same equilibrium effects in this version of the model, where the supply of inputs is imperfectly elastic, as in the baseline model, where the supply of inputs is perfectly elastic. Namely, international trade increases the measure of firms entering the market for search goods and, for this reason, it increases competition and leads to lower markups. Moreover, international trade may or may not lead to a more intense use of production inputs, depending on the parameters of the model.

The welfare effect of international trade in this version of the model, where the supply of inputs is imperfectly elastic, is the same as in the baseline model, where the supply of inputs is perfectly elastic. When international trade is not allowed, the equilibrium coincides with the solution to the social planner's problem. Hence, when international trade is not allowed, welfare W_a in a given country is given by

$$\begin{aligned} W_a = \quad & \max_{\ell,s} bU\left(h + (1-e^{-\lambda s})u, 1-\ell\right), \text{ s.t.} \\ & \ell = \frac{\zeta}{b}s + \sum\nolimits_{n=1}^{\infty} \frac{e^{-\lambda s}(\lambda s)^n}{n!} \int_{c_\ell}^{c_h} cn\left(1-\Phi(c)\right)^{n-1} \Phi'(c)dc. \end{aligned} \tag{5.33}$$

When international trade is allowed, the equilibrium coincides with the solution to the problem of a social planner that aims at maximizing the utility of households in both countries. Hence, when international trade is allowed, welfare W_t in a given country is given by

$$\begin{aligned} 2W_t = \quad & \max_{\ell,s} 2bU\left(h + (1-e^{-\lambda s})u, 1-\ell\right), \text{ s.t.} \\ & \ell = \frac{\zeta}{2b}s + \sum\nolimits_{n=1}^{\infty} \frac{e^{-\lambda s}(\lambda s)^n}{n!} \int_{c_\ell}^{c_h} cn\left(1-\Phi(c)\right)^{n-1} \Phi'(c)dc. \end{aligned} \tag{5.34}$$

The problem in (5.34) is the same as the problem in (5.33), except that $\zeta/2b$ takes the place of ζ/b in the resource constraint. Since $\zeta/2b < \zeta/b$, welfare when international trade is allowed is strictly greater than welfare in autarky, i.e. $W_t > W_a$.

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