Choosing the Right Pond: Status vs. Prestige in School Choice & Party Formation

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1 Introduction

Is it better to be a big fish in a small pond or a small fish in a big pond? The small pond is better according to both a Chinese proverb ("it is better to be the head of a chicken than the tail of a phoenix") and its Spanish counterpart ("it is better to be the head of a mouse than the tail of a lion"). The big pond is better according to both President Javier Milei of Argentina ("it is better to be the tail of a lion than the head of a mouse") and American singer-songwriter Taylor Swift ("if you are the smartest person in the room, you are in the wrong room"). We do not set out to resolve the dilemma. Instead, we acknowledge the trade-off it represents and focus on examining the trade-off's equilibrium implications.

The trade-off between being ranked higher within the group to which one belongs and belonging to a group that ranks higher relative to other groups is present in a myriad of applications. We focus on two: a school-choice problem with peer effects and a party-formation problem with office-motivated candidates.

In the school-choice problem, each student prefers being ranked higher toward the top of his class—to bask in teachers' attention, to boost his self-esteem, or (in California and Texas) to gain priority admission to a state university. At the same time, the student prefers attending a school

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whose rank, induced by the students' mean ability, is higher, and that, as a result, promises better career and social prospects. In this setting, students do not care about schools per se; they only care about peers. While different from (and complementary to) theoretical work on school choice in the tradition of Abdulkadiroğlu and Sönmez (2003), our pure peer-effects formulation agrees with latest empirical findings: parents value peers but not school effectiveness (Abdulkadiroglu, Pathak, Schellenberg and Walters, 2020; Rothstein, 2006); peers matter because they affect school prestige, not because students learn from them (MacLeod and Urquiola, 2015); and the worst student at a (prestigious) selective school would be better off as the best student at a (nonprestigious) nonselective school (Abdulkadiroglu, Angrist and Pathak, 2014).

In the party-formation problem, ambitious office-seekers join political parties as candidates in anticipation of an election under the open-list representative electoral system. Candidates are distinguished by their valence, which is a nonideological characteristic (e.g., competence, integrity, or charisma) that all voters view as desirable (Stokes, 1963). Each candidate prefers being ranked higher toward the top of his party in valence, to outshine his peers. At the same time, the candidate prefers belonging to a party whose rank, induced by its candidates' mean valence, is higher, and that, as a result, commands greater resources and affords greater visibility. Candidates' lack of concern for party ideology agrees with the theory of political ambition of Aldrich (2011, Chapter 2) and Aldrich and Bianco (1992) and is descriptive of politics in Brazil (Iaryczower, Kim and Montero, 2024; Samuels, 2003).

Both the school-choice and the party-formation problems are special cases of a model that we study in the abstract. This model has a continuum of agents differentiated by a number in $[0, 1]$, which we interpret as ability. The goal is to partition agents into two or more groups. Each agent cares about his rank—or **status**—within the group to which he belongs. Each agent also cares about his group's rank—or **prestige**—induced by the average ability of the group members. Agents' preferences are identical: all agents trade off status against prestige in the same way. The solution concept, (myopic) stability, assumes coalitions of agents to be capable of seeking improvements in welfare by switching groups. A **stable** partition is immune to (myopic) blocking by improvement-seeking agents. Blocking is myopic because no coalition conceives of the possibility that the outcome of its blocking could itself be blocked by another coalition. Which partitions are stable depends on which blocking coalitions are allowed. We focus on two scenarios, which have our two applications as special cases. In both scenarios, motivated by coordination frictions, we require blocking coalitions to be of "arbitrarily small" but positive measure.

Our first scenario assumes that all that agents in a blocking coalition can do is to exchange their group assignments among themselves. This **exchange scenario** encompasses the school-choice problem, in which stability is a notion of fairness: a coalition of students may condemn a partition as unfair by pointing out that all of them would have been happier if only they had they been allowed to exchange their assignments among themselves.

For the exchange scenario, we show that a stable partition always exists. Indeed, any partition in which each group is a "representative sample" of the entire population—and, as a result, all groups have the same prestige—can be shown to be stable. Other partitions—including those with no ties in prestige—can also be stable. Our main result for the exchange scenario is a characterization of all stable partitions. We show that, in order to verify stability, it is both necessary and sufficient to rule out blocking pairs and blocking triplets. Our blocking pair is a convenient figure of speech that does not comply with the standard definition encountered in models of two-sided matching. We say that a **blocking pair** exists if one can find a blocking coalition (of positive measure) whose members' abilities are all "concentrated" around just two values in $[0, 1]$. Similarly, a **blocking triplet** exists if one can find a blocking coalition (of positive measure) whose members' abilities are all concentrated around exactly three values in $[0, 1]$. The no-blocking-pair and the no-blocking-triplet conditions can be expressed as restrictions on the probability distributions (CDFs) of abilities within each group: no two CDFs should be too far apart, and any two CDFs must cross at most once.

Viewed through the prism of the school-choice problem, the exchange scenario admits a stable partition of students into schools for any number of schools of any capacities. In one such stable partition, each school is a representative sample of the entire population, which is what the Educational Option admission method in New York City aims to accomplish (presumably motivated by the considerations of diversity, equity, and inclusion). Another stable partition equates prestige across schools (achieving equity) but does not equate school composition (sacrificing diversity and inclusion) in order to achieve equal treatment of equals: no students of the same ability are scattered across multiple schools. Yet another stable partition is a prototype of elite education: it maximizes the inequality in school quality. It turns out that insistence on stability as one pursues inequality amounts to introducing an admission lottery for a subpopulation of students.

Our second scenario enlarges the set of potential blocking coalitions by allowing each member of a blocking coalition to discard his existing group assignment and fabricate a new one to any group he likes, an existing or a newly founded one. This **farication scenario** is exemplified by the party formation problem, in which stability is a descriptive property: each candidate is free to leave his party and to join another (existing or newly founded) one.

Under the fabrication scenario, nothing is stable. Every partition can be blocked by a coalition whose members fabricate assignments to a new group that they jointly found. This coalition draws on agents with diverse abilities from a highly ranked group and over-represents higher-ability agents from that group. Every partition can also be blocked by a coalition whose low-ability members all come from the same lowly-ranked group and jointly move to a highly-ranked existing group. While other blocking coalitions exist, each of them contains a blocking coalition of one of the two types described above. The structure of blocking coalitions hints at the nature of the turmoil one should expect if agents form blocking coalitions myopically, as exemplified by the perpetual churning of political candidates and the emergence of new political parties.

In both scenarios, all unstable partitions can be blocked by a coalition that is not too "complex." We define the complexity of a blocking coalition as the number (possibly infinite) of distinct values in $[0, 1]$ around which the abilities of the coalition's members are concentrated. For instance, under the exchange scenario, all unstable partitions can be blocked by a coalition of complexity two or three because the absence of blocking pairs and blocking triplets is necessary and sufficient for stability. Under the fabrication scenario, all partitions can be blocked by a coalition of complexity one, which is the archetypical coalition whose members all leave one group to join another. The other archetypical coalition, one that founds a new group, may have to be arbitrarily complex in order to succeed at blocking, depending on model parameters.

We probe our model's boundaries by asking whether the assumptions that coalitions be small and that utility be nontransferable are restrictive. They are. Large blocking coalitions may not only break the ties in prestige (just as small coalitions sometimes do) but also reverse strict rankings (which small coalitions cannot do). The possibility of this reversal renders some (not all) otherwise stable partitions unstable. The introduction of transfers into the coalitional calculus of deviations shrinks the set of stable partitions, as well.

Our analysis starts out with, and lingers on, the special case of two-group partitions. The focus on this case does not impoverish the economics of the model much. It turns out that, whatever the number of groups in a partition, if the partition can be blocked, it can be blocked by a coalition whose members are recruited from at most two groups. As a result, the no-blocking-pair and the no-blocking-triplet conditions that secure stability in the two-group case continue to secure stability in the general case, in which these conditions are applied to all pairs of groups.