Shaping Inequality and Intergenerational Persistence of Poverty: Free College or Better Schools*

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Abstract

We evaluate the aggregate, distributional and welfare consequences of alternative government education policies to encourage college completion, such as making college free and improving funding for public schooling. To do so, we construct a general equilibrium overlapping generations model with intergenerational linkages, a higher education choice as well as a multi-stage human capital production process during childhood and adolescence with parental and government schooling investments. The model features rich cross-sectional heterogeneity, distinguishes between single and married parents, and is disciplined by US household survey data on income, wealth, education and time use. Studying the transitions induced by unexpected policy reforms we show that the "free college" and the "better schools" reform generate significant welfare gains, which take time to materialize and are lower in general than in partial equilibrium. It is optimal to combine both reforms: tuition subsidies make college affordable even for children from poorer parental backgrounds and better schools increase human capital thereby reducing dropout risk.

Keywords: education spending, public transfers, welfare benefits, inequality, poverty, intergenerational persistence

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1 Introduction

In the U.S., 16% of children grow up in poverty.[1] Educational achievement gaps between children of different socio-economic backgrounds appear early in life and persist into adulthood, and are associated with a strong parental income gradient of college attendance (Chetty, Hendren, Kline, and Saez 2014). These pre-labor market entry gaps have been identified as a key factor determining differences in later life economic outcomes (see, e.g., Keane and Wolpin 1997; Carneiro and Heckman 2002; Huggett, Ventura, and Yaron 2011). This has sparked an active academic and policy debate concerning the question whether public education policies can lessen these disparities and lift children out of poverty, most recently, the proposal by the Biden administration of free community college, as well as further initiatives that would make higher education less costly.

This paper seeks to provide a joint analysis of education policies targeted at the pre-college human capital accumulation stage by secondary school financing on the one hand, and policies designed to make college education more financially affordable, on the other hand. Suppose the main problem holding poor children back from successfully pursuing a college education are not potentially binding credit constraints, but rather that these children arrive at college age ill-equipped to successfully apply and ultimately graduate from college. Then policies tackling this latter problem are the appropriate tool to increase the average and reduce the cross-sectional dispersion and intergenerational persistence of college attendance, earnings and welfare.

To positively and normatively evaluate these policies we develop a general equilibrium overlapping generations framework with intergenerational links through altruistically-motivated education and wealth transfers, in the spirit of Barro and Becker (1988), and with rich cross-sectional heterogeneity of labor productivity (and thus earnings), human capital, wealth and marital status. Human capital is accumulated at different stages of a child's development, depending on parental resource- and time investments as well as public education funding. Crucially, human capital acquired at earlier stages of child development determines the productivity of all future human capital investments. The human capital acquired prior to the higher education (college) stage determines both the chances to succeed in college and the expected returns from a college education. Altruistically motivated and rationally forward-looking parents respond to policies affecting the labor market stage of their children's life cycle. At the same time, parents react to education (financing) reforms by adjusting both their own education choices and investments in their children. These interactions between education subsidies targeted at different stages of child and adolescent development and progressive taxation suggest that these policy reforms must be

[1] Data from the 2022 U.S. Census, see: https://www.census.gov/library/stories/2023/12/poverty-rate-varies-by-age-groups.html.

studied jointly. The dynastic modeling framework with intergenerational linkages allows us to evaluate the implications of policy reforms not only for cross-sectional inequality but also for intergenerational earnings- and education mobility.

Our policy analysis starts from an initial stationary equilibrium calibrated to the status quo of the US economy in the 2010's. We then investigate the impact of policy reforms along the transition of the economy towards a final steady state. Along this transition, the government may issue new government debt to finance education policies that in the long run raise human capital and thus the tax base (a fiscal externality), but take time to materialize their full impact. By issuing debt the government may thus smooth out the transitional costs of the policy reforms.

We evaluate a set of once-and-for-all policy reforms that are motivated by the current political discussion and that are comparable in the present discounted value of their government expenditure requirements. The first reform is making college education free (motivated by President Biden's proposal to provide for universal free community college) which in our model is implemented by a 100% tertiary education subsidy by the government. This reform is used as benchmark to determine the size of the other reforms to make those fiscally comparable. The alternative policy reform focuses on human capital accumulation of younger children by increasing public spending for primary or secondary schools. Finally, we study the optimal mix of "better schools" and college tuition subsidies, holding the total fiscal cost of the policy reform constant in present discounted value terms across all interventions considered.

We find that, in terms of aggregates, both education reforms increase the share of a cohort having a college degree by roughly one-fourth (and roughly to the same degree in the long run), but the overall expansion of human capital is much more pronounced —and thus the college dropout share is substantially lower— under the pre-college school spending reform. Both reforms are more than self financing, in the sense that (as long as the government can issue new debt in the short run) the required permanent increase in the labor income tax rate to balance the intertemporal government budget is actually negative. Because of the stronger increase in human capital in the long run, the net present discounted value of government revenues rises more substantially under the school expenditure expansion than under the free college reform, and thus the labor income tax can be reduced by more. Both reforms generate significant long-run welfare gains, in the order of 11-15% of permanent consumption, when measured as consumption equivalent variation of newborn agents. Given the assumed altruism, these high welfare gains include the welfare benefits of children and all subsequent generations. Consistent with the more favorable human capital and tax revenue expansions, the welfare gain is larger by about 4 percentage points in the "better schools" reform than in the "free college" reform in the long run.

Second, the two policy reforms have vastly different distributional consequences. Most crucially, the pre-college expenditure reform also benefits children from households who will not go to college even if they do not have to pay tuition, as is the case under the "free college" reform.[2] This in turn has profound consequences for the intergenerational persistence of earnings and educational attainment. Perhaps the most striking contrast between the reforms is the differential impact on the educational attainment of the poorest children, which tend to be children growing up in a household with a single parent and low (less than high school) educational attainment. Although the "free college" reform is fairly successful in drawing these disadvantaged children into college, it does so at the cost of disproportionally high dropout rates and relatively low college wage premia.[3] In contrast, the additional human capital accumulation these children obtain with the "better schools" reform, although insufficient in most cases to push them above college enrollment and completion thresholds, strongly increases the chances of these children to at least complete high school, therefore strongly reducing the intergenerational persistence of dropping out of high school.

For the general equilibrium effects, the most important mechanism is that both reforms lead to an expansion of labor supply, especially of college labor, and thus to a fall of the capital-labor ratio, which increases the interest rate and suppresses the average wage level in the economy. Additionally, since college labor becomes abundant relative to non-college labor, the college wage premium declines. This second effect dominates for non-college workers so that their wages increase slightly, whereas those of college workers fall and so the college wage premium falls.

To quantify the importance of these general equilibrium adjustments for the overall welfare findings, we conduct a sequence of partial equilibrium exercises in which we hold prices constant. It turns out that for the size of the general welfare gains, the reduction in wages (and especially, the wages of college graduates) is quantitatively most important. Consequently, the welfare gains of the reforms in partial equilibrium, where this mechanism is absent by construction, are higher than in general equilibrium. Since the policies lead to wage compression within the population, the distributional consequences for ex-ante lifetime utility are positive, partially compensating for the welfare losses from the decline in the absolute wage level. Overall, and relative to a world where all factor prices are constant, the long-run welfare gains for the "free college" reform are 2.2 percentage points lower in general than in partial equilibrium, and they are 2.7 percentage points lower for the "better schools" reform. Thus, endogenous factor price movements not only reduce the welfare gains from the reforms, but also reduce the gap in the welfare consequences across the two reforms.

[2] At the same time, more generous high school financing additionally also benefits children even from most affluent backgrounds via increasing their post-graduation earnings. This is an important force driving the more beneficial fiscal implications of the "better schools" reform while the "free college" reform is completely ineffective for changing college decisions at the very top and simply provides a pure consumption transfer to these households.

[3] In the model, the college wage premium is not one parameter but follows a distribution that endogenously depends on the pre-college acquired human capital distribution.

Finally, we study the optimal combination of both better school financing and college subsidies. The two policy instruments can be relative welfare substitutes —they both decrease the human capital threshold it takes to attend college— or relative welfare complements —increasing human capital through increased school spending increases relative wages of college youngsters and reduces dropout probabilities. It is therefore a quantitative question whether our analysis of joint policies delivers a corner solution such that the "better schools" policy reform is the preferred policy choice, or an interior solution with an optimal mix of both instruments. We indeed find an interior optimum such that a policy that devotes ca. $1/3$ of its budget to better schools and $2/3$ to college subsidies maximizes aggregate welfare. This joint policy is almost as effective as the pure "free college" reform in generating a higher share of college graduates, but the higher average human capital upon entering college due to better schools limits the increase in dropout rates that the pure "free college" reform displays. The optimal policy is characterized by a higher weight on "free college" because it takes a sufficiently large college reform to draw a relevant mass of youngsters into college.

After briefly reviewing the extant literature, we describe the quantitative model and its equilibrium in Sections 2 and 3, respectively. Section 4 and 5 discusses calibration and validation of the model. In Section 6 we present the results of two policy reforms, and Section 7 discusses the optimal combination of both reforms. Section 8 concludes, and further details about the theory and quantification of the model, as well as additional results are contained in the appendix.

1.1 Related Literature

Our paper builds on a voluminous literature that has documented a prominent role of family income and family composition both in determining pre-college academic achievement (Dahl and Lochner 2012; Caucutt, Lochner, and Park 2017; Blandin and Herrington 2022; Ebrahimian 2023), and college entry decisions and graduation outcomes (e.g. Belley and Lochner 2007), even after controlling for initial conditions at high school graduation (see, e.g., Leukhina 2023). Specifically, we connect two broad literatures in macroeconomics and public finance for the study of currently proposed education finance and fiscal policy reforms. The first is concerned with (optimal) redistributive tax-transfer and education policies, see Benabou (2002), Hanushek, Leung, and Yilmaz (2003) and Bovenberg and Jacobs (2005) for foundational papers. Recent papers in this genre focusing on education (financing) reform include Abbott, Gallipoli, Meghir, and Violante (2019), Caucutt and Lochner (2020), Stantcheva (2017), Athreya, Ionescu, Neelakantan, and Vidangos (2019), Fogli and Guerrieri (2019), Capelle (2020), Fu, Ishimaru, and Kennan (2024) as well as our own work, Krueger and Ludwig (2016). An important part of this literature studies the impact of tax- and education policy on intergenerational mobility,[4] see, e.g., Restuccia and Urrutia (2004), Holter (2015), Lee and Seshadri (2019), Koeniger and Prat (2018) and Koeniger and Zanella (2022), and a complementary and equally relevant literature studies (optimal) tax-transfer and poverty alleviation policy (transitions), see, e.g., Boar and Midrigan (2022), Dyrda and Pedroni (2023), Daruich and Fernández (2024), Floden (2001), Ortigueira and Siassi (2023), Guner, Kaygusuz, and Ventura (2020) and Guner, Ventura, and Kaygusuz (2023). In contrast to most of this existing literature, we take as central tenet that the heterogeneity in initial conditions at labor market entry with respect to human capital and wealth is an endogenous object that can be affected by education and fiscal policies. Thus, it considers education policies as additional means of redistribution, by reducing education and achievement gaps of children from different socio-economic backgrounds and at different stages of the skill formation process. In addition, we contribute to this literature by distinguishing between the incidence of pre-college versus college subsidies while explicitly modeling the complementarity between ability and educational attainment for wages (see Jacobs and Bovenberg 2011 or Stantcheva 2017) and the dynamic complementarities in child human capital accumulation recently stressed by Cunha, Heckman, and Schennach (2010).

Therefore, into the above literature we seek to integrate an explicit modeling of life cycle choices with a production function for human capital at different stages of child development. In this regard, we build on a second recent literature in empirical microeconomics and quantitative macroeconomics that models child skill formation and human capital accumulation endogenously, see, e.g., Cunha, Heckman, and Lochner (2006), Cunha and Heckman (2007), Cunha, Heckman, and Schennach (2010), Caucutt, Lochner, Mullins, and Park (2020), Eckstein, Keane, and Lifshitz (2019), Daruich (2022), Blandin and Herrington (2022), Bolt, French, Maccuish, and O'Dea (2023), Yum (2023), Adamopoulou, Hannusch, Kopecky, and Obermeier (2024), Bellue and Mahler (2024), and our own work, Fuchs-Schündeln, Krueger, Ludwig, and Popova (2022) and Fuchs-Schündeln, Krueger, Kurmann, Lalé, Ludwig, and Popova (2023), to study the dynamic interactions between parental borrowing constraints and public education spending. On the modeling side we extend this literature by considering the endogenous time allocation choice for both parents between work, leisure and spending time with children of different ages. We also emphasize the importance of general equilibrium effects induced by the policy interventions. On the applied policy side, our main focus lies on the impact of (optimal) policy transitions (permitting government debt) on cross-sectional inequality and intergenerational persistence of economic outcomes, especially those at the lower end of the income and wealth distribution. This in turn

[4]Since intergenerational persistence in outcomes is impacted by intergenerational transfers, the empirical literature on these transfers in, e.g., Gale and Scholz (1994), Altonji, Hayashi, and Kotlikoff (1997) and especially Yang and Ripoll (2023) provides important references for the calibration of our the model.

requires the explicit model with intergenerational linkages and rich household heterogeneity especially with respect to family marital structure that we provide in this paper. It also highlights the importance of distinguishing the impact of policy reforms in the short run (early in the transition) and in the long run (in the final steady state).

We further complement an empirical literature on the interaction of welfare programs and the education and human capital accumulation of children, see, e.g., Del Boca, Flinn, and Wiswall (2014), Del Boca, Flinn, and Wiswall (2016), and Bailey, Hoynes, Rossin-Slater, and Walker (2023), and on the impact of day care- and education spending and financing on education and economic outcomes, see, e.g., Havnes and Mogstad (2011), Abramitzky and Lavy (2014), Jackson, Johnson, and Persico (2016), Deming and Walters (2017), Johnson and Jackson (2019), Black, Denning, Dettling, Goodman, and Turner (2020), García, Heckman, Leaf, and Prados (2020), Duncan, Kalil, Mogstad, and Rege (2022), Bastian and Lochner (2022), and Flood, McMurry, Sojourner, and Wiswall (2022) (as well as the survey by Handel and Hanushek 2023). Importantly, we show that our model-implied short-run partial equilibrium responses of college enrollment and high school graduation rates to small-scale expansions of both pre-college and college funding are consistent with the estimates reviewed in the meta-studies by Deming and Dynarski (2010) and Jackson and Mackevicius (2023). Relative to these studies, our model-based approach allows for an explicit analysis of the long-run impact of such policies, accounting for changes in the distribution of assets, human capital, and parental education in the population, as well as general equilibrium effects on wages, rates of return, taxes and government debt.[5]

2 The Quantitative Model

2.1 Overview

We employ a general equilibrium overlapping generations (OLG) model in which generations are linked through the intergenerational transmission of innate ability and financial wealth transfers. Parents are altruistic towards their children, solve a unitary decision problem and can invest time and monetary resources into the human capital accumulation of children when the latter are still living with their parents. In addition, parents can transfer wealth to children directly when they leave the parental household. The government collects taxes, runs a PAYGO social security system and finances exogenous government spending and endogenous education spending with taxes and government debt, subject to an intertemporal budget constraint and a period-per-period social

[5] See, e.g., Kaplow (2020) for a discussion of the importance of disentangling the distributional effects of a reform itself from those induced by the accompanying fiscal adjustments.

security budget constraint. In general equilibrium the goods-, labor- and asset markets have to clear in every period along a policy-reform induced transition.

Relative to the standard quantitative life cycle literature our model contains three key additional features. First, households have children whose human capital accumulation during the transition from childhood to adolescence is endogenous and depends both on public schooling and private parental inputs. This element of the model is crucial for a study of education policies that differ in the extent to which primary/secondary and tertiary education is impacted and fiscal policies that impact the trade-off between market work (including participation) and time investment into children.[6] Second, generations in our model are linked through "brains and bucks", that is, human capital inputs and financial transfers from parents to children. With this model element, parents have endogenous margins of adjustment in direct response to education policy reforms. If college will be free, private inter-vivos transfers (and the accumulation of parents assets to make these transfers) will endogenously adjust. When public schools become better, private time and resource inputs can respond as well. Third, modeling both married households but also single mothers allows us to explicitly account for a group of children that disproportionally grow up in poverty and are the least likely to go to college, both because of financial constraints and due to poor pre-college academic achievement, at least on average.[7]

2.2 Individual State Variables

In order to meaningfully study the distributional consequences of the proposed policy reforms the model features rich cross-sectional heterogeneity, best described in terms of the individual state variables that characterize households. These are summarized in Table 1, including the range of values these state variables can take.

Individuals differ by age j and young households start their independent economic life as singles and with four ex-ante predetermined state variables: gender (either being a woman or a man, $g \in \{wo, ma\}$), the education of their parents s_p (which determines their cost of attending college) initial human capital (h) and initial assets (a). Upon realization of the high school dropout schock and after an individual has taken its own higher education decision s, which is subject to college dropout risk, the highest completed education level also becomes a state variable (and that of the parent ceases to be relevant). Upon labor market entry, the acquired human capital stochastically translates into a discrete-valued fixed effect γ with education-specific

[6] The presence of government transfer- and social assistance programs whose importance varies by family structure renders the explicit modeling of an extensive margin (for both partners of a married couple) important as the recent work by Guner, Kaygusuz, and Ventura (2012), Bick and Fuchs-Schündeln (2017) and Holter, Krueger, and Stepanchuk (2023) suggests.

[7] The model abstracts from single fathers, given that this group constitutes a negligible share of the population in the data.

Table 1: Individual State Variables

State Var.	Values	Interpretation
j	$j \in \{0, 1, \dots, J\}$	Model Age
g	$g \in \{wo, ma\}$	Gender
h	$h > 0$	Human Capital
a	$a \geq -\underline{a}(j, s)$	Financial Assets
s, s_p	$s \in \{hsd, hs, cod, co\}$	(Higher) Education
γ	$\gamma \in \{\gamma_l(s), \gamma_h(s)\}$	Fixed Productivity Component
η	$\eta \in \{\eta_l, \eta_h\}$	Persistent Productivity Shock
q	$q \in \{si, cpl\}$	Marital Status

support.[8] Labor productivity is also impacted by a persistent stochastic component η which is part of the state space. Finally, one period before children are born into a household, the marital status q of a household realizes and becomes a state variable, as a fraction of single households marry ($q = cpl$ for "couples") while the rest remains single ($q = si$). Finally, when children are born into a household, their human capital h becomes a state variable as well.[9] In terms of notation, for married households the education and labor productivity of both partners are state variables, and the notation s_{-g} and γ_{-g} denotes the state variable of the "other" spouse.

2.3 Demographics, Timing and Economic Decisions

Time is discrete, indexed by t and extends to infinity, where each model period corresponds to four real years. In every period t the economy is populated by J overlapping generations. Individuals survive from age j to age $j + 1$ with probability ϕ_{j+1}. Before retirement survival is certain while from the exogenous retirement age j_r onward survival risk becomes relevant, and individuals live at most until age J. Assets of households that die at age j are distributed in a lump-sum fashion among all working age households.[10] Transfers from accidental bequests are denoted by $Tr_{t,j}$.

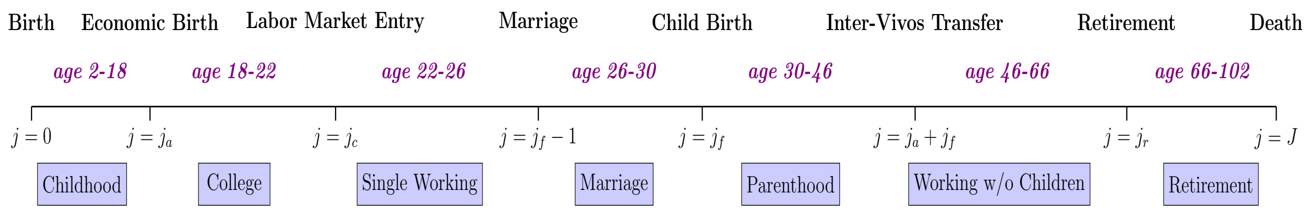
Figure 1 provides a summary of the household lifecycle. Children are born at age $j = 0$, corresponding to biological age 2 (the first two years of a child's life cycle remain un-modelled). At parental fertility age j_f a number of $\varsigma(s(wo))$ identical children is born; that number depends on the education level $s(wo)$ of the mother. The initial human capital h_0 of each child is also a function of $(q, s(wo))$. Children stay in the parental household and accumulate human capital depending on their initial human capital and the time and resource input of their parents, for

[8]The stochastic mapping from human capital h to the fixed labor productivity γ replaces the continuous state variable h with the discrete-valued variably γ, which reduces the dimensionality of the state space.

[9]At that time fixed productivity γ has replaced parental human capital, and thus there is no scope for confusion between parental and child human capital in a household.

[10]To be more precise, what is redistributed among surviving households are the accidental bequests net of the amount needed to finance private college subsidies.

Figure 1: Lifecycle: Timeline



a given level of the public schooling input, as described below. These parental investments are referred to as private human capital investments. When children leave the parental household parents give them (non-negative) inter-vivos transfers b which can be used for consumption and/or for covering college expenses.

At model age j_a (biological age 18) children form their own households. They first graduate from high school with probability $\pi^{hs}(h)$ that depends on the human capital accumulated in school. Conditional on having a high-school diploma they then make their college enrollment decision. Young adults who choose college spend one model period for education, the other group starts working directly at age j_a. Dropping out of college is both a choice and subject to dropout risk, a shock, but given the four-year periodicity of the model both happen during the same time period, at age j_a. Specifically, conditional on choosing to enroll in college there is an additional decision to become a college dropout.[11] Next, conditional on opting to continue, students successfully graduate with probability $\pi^{co}(h)$. College dropouts are assumed to have to pay only half of the tuition cost than college graduates, and they also face a correspondingly tighter borrowing limit.

After the education decision is taken all education groups draw an education group-specific fixed productivity component $\gamma(s)$ for the labor market, which, for each education group s, has two realizations, high and low. College students and dropouts can work-part time at high-school wages during the college period.[12] The probability of drawing a high realization of the fixed effect is an increasing function of acquired human capital h and education group specific, denoted by $\pi^{\gamma}(s,h)$. After education is completed all households enter the labor market. Upon labor market entry acquired human capital h seizes to be a state variable for all education groups. College graduates and college dropouts then redraw their fixed productivity component from a domain associated with their newly obtained higher education level.

[11] In our model it is possible for individuals to have higher expected lifetime utility from becoming a college dropout than being only a high school graduate (because of higher wages) and than continuing with college (because of lower tuition cost and disutility from studying).

[12] This means that both the aggregate wage level as well as the fixed productivity component are the same as for high school graduate workers.

During the working life, households make a discrete decision whether to work, and, conditional on employment, make a continuous intensive margin hours choice. One period before the fertility age, at age $j_m = j_f - 1$, households face an exogenous education specific probability of marriage, and depending on the realization of the marriage shock continue to live either as singles or as couples. Table 2 summarizes all choice variables and the ages at which these decisions are made.

Table 2: Decision Variables

Control Var.	Values	Decision Period	Interpretation
c	$c > 0$	$j \geq j_a$	Consumption
ℓ	$\ell \geq 0$	$j \geq j_a$	Hours worked (for couples $\ell(wo)$ and $\ell(ma)$)
a'	$a' \geq -\underline{a}(j,s)$	$j \geq j_a$	Asset Accumulation
i^t	$i^t \geq 0$	$j \in \{j_f, ..., j_f + j_a\}$	Time Investments (for couples $i^t(wo)$ and $i^t(ma)$)
i^m	$i^m \geq 0$	$j \in \{j_f, ..., j_f + j_a\}$	Monetary Investments
b	$b \geq 0$	$j = j_f + j_a$	Monetary Inter-vivos Transfer
s	$s \in \{hsd, hs, cod, co\}$	$j = j_a$	(Higher) Education

2.4 Human Capital

In every period during childhood human capital accumulation takes place according to the following production function:

$$h' = g\left(j, h, i^m, i^t, i^g\right), \tag{1}$$

where i^t and i^m denote parental time and monetary investment, while i^g denotes public investment. A subset of the parameters in the human capital production function will be allowed to vary by age j for calibration purposes, in order to capture differences in the relative importance of private time and resource inputs (as well as public schooling) at different stages of childhood. For married households the time investment i^t is a composite of the time inputs of both parents, which are assumed to be perfectly substitutable,

$$i^t = i^t(wo) + i^t(ma), \tag{2}$$

where $i^t(wo)$ and $i^t(ma)$ denote the time inputs of a *wo*man and of a *ma*n, respectively.

2.5 Labor Productivity

The wage of a single household at age j, of gender g with an education level s and with a fixed productivity component realization $\gamma(s)$ is given by:

$$w(s, \gamma(s), g, j) = w(s) \cdot \gamma(s) \cdot \epsilon(s, g, j) \cdot \eta \tag{3}$$

where $w(s)$ is an aggregate education specific wage component, $\gamma(s)$ is a fixed household productivity component, $\epsilon(s, g, j)$ is a deterministic gender- and education-specific productivity profile, and η denotes a potentially persistent productivity shock.

2.6 Decision Problems

All household decision problems below are cast in recursive formulation, with variables expressed in per capita terms and detrended by the rate of technological progress μ. In terms of notation, we denote the lifetime expected utility function of a household of age j and marital status q at time t after current period shocks have been realized by value functions $V_{t,j,q}(.)$ and value functions prior to the realizations of shocks in the current period by $\bar{V}_{t,j,q}(.)$. Since the set of state variables changes with age and marital status, using subscripts (j, q) makes clear that, technically speaking, value functions for different (j, q) are different functions.

2.6.1 Young Adults and the Education Decision at Age j_a

Since children born at age $j = 0$ are economically inactive until age $j = j_a - 1$ and simply accumulate human capital as a result of parental decisions, the youngest age at which economic choices are made is model age j_a (biological age 18) when children have become young adults and have formed independent households. At that age their initial state is (j_a, g, s_p, a, h), comprised of age j_a, gender g, parental education s_p, financial assets a and human capital h.

Now the tertiary education level is determined, partially by choice and partially by chance. It takes four values, as individuals can be high-school dropouts (hsd), high-school graduates (hs), college dropouts (cod) and college graduates (co). First, high school graduation is exogenous from the perspective of the newly founded household but stochastic: with probability $\pi^{hs}(h)$, with $\pi_h^{hs}(h) > 0$, the individual obtains a high-school diploma and with complementary probability it becomes a high-school dropout, with continuation expected lifetime utility $\bar{V}_{t,j_a,si}(g, hsd, a, h)$, which does not depend on parental education s_p, because the latter only affects psychological (or, utility) costs of college attendance (see below).

A high-school graduate can choose to attend college, and conditional on enrollment may either graduate or drop out. Attending college is costly, both in terms of tuition (which is potentially subsidized by the government and can be financed by student loans and parental transfers) as well as in terms of the opportunity cost of time and the psychological cost of studying, which depend on their acquired human capital h and on the education of their parents s_p. For those deciding to try to graduate from college, their ability to do so is subject to exogenous (but human-capital dependent) drop-out risk: individuals succeed in college only with probability $\pi^{co}(h)$, with $\pi_h^{co}(h) > 0$. Individuals weigh these costs against the benefits of higher wages upon college graduation. College students (both those who will graduate and those who will drop out) can work part-time at high school wages, and thus during college the productivity $\gamma(hs)$ is a relevant state variable. [13]

The college attendance choice of an age j_a single ($q = si$) individual can then be written as[14]

$$s = \begin{cases} hs & \text{if } \bar{V}_{t,j_a,si}(g, hs, a, h) \geq \bar{V}_{t,j_a,si}(g, ce, s_p, a, h) \\ ce & \text{if } \bar{V}_{t,j_a,si}(g, ce, s_p, a, h) > \bar{V}_{t,j_a,si}(g, hs, a, h), \end{cases} \quad (4)$$

where only the college enrollment value function is indexed by parental education s_p, because of the dependency of psychological attendance costs on parental education. $\bar{V}_{t,j_a,si}(g, ce, s_p, a, h)$ is the pre-dropout college *attendance* value function given by

$$\begin{aligned} \bar{V}_{t,j_a,si}(g, ce, s_p, a, h) = \max_{s \in \{cod, co\}} \big\{ & \bar{V}_{t,j_a,si}(g, cod, s_p, a, h), \\ & \pi^{co}(h) \cdot \bar{V}_{t,j_a,si}(g, co, s_p, a, h) + (1 - \pi^{co}(h)) \cdot \bar{V}_{t,j_a,si}(g, cod, s_p, a, h) \big\}, \end{aligned} \quad (5)$$

so that individuals who prefer to drop out of college (from an ex-ante perspective) always do so. Finally, the pre-college enrollment decision value function at age j_a is given by

$$\begin{aligned} \bar{V}_{t,j_a,si}(g, s_p, a, h) = & (1 - \pi^{hs}(h)) \cdot \bar{V}_{t,j_a,si}(g, hsd, a, h) \\ & + \pi^{hs}(h) \cdot \max_{s \in \{hs, ce\}} \big\{ \bar{V}_{t,j_a,si}(g, hs, a, h), \bar{V}_{t,j_a,si}(g, ce, s_p, a, h) \big\}. \end{aligned} \quad (6)$$

2.6.2 First Period of Working Life / College Period

At the beginning of the first period of independent economic life, realizations of the fixed productivity component and idiosyncratic productivity shocks are drawn. Thus, in the first period of economic life the decision problem can be split in two sub-periods. In the first sub-period, the

[13] College drop-outs also pay smaller tuition costs and face a tighter borrowing limit than college graduates.

[14] In the computational implementation we apply Extreme Value Type I (Gumbel) taste shocks to smooth out this discrete decision problem, see, e.g., Iskhakov, Jorgensen, Rust, and Schjerning (2017) for a discussion of numerical advantages of Gumbel shocks in discrete-continuous dynamic applications. The taste shifters are purely a computational device, and the scale parameter of Gumbel shocks is set to 0.05.

fixed productivity component and the persistent income shock are drawn:

$$\bar{V}_{t,j_a,si}(g,s,s_p,a,h) = \sum_{\gamma} \pi^{\gamma}(s,h) \sum_{\eta} \Pi(\eta) V_{t,j_a,si}(g,s,s_p,a,h,\gamma,\eta)$$

where γ denotes education-specific realizations of the fixed productivity component, and η is the persistent productivity shock realization. For households that neither enroll in college nor complete it, from this point in time onward acquired human capital seizes to be a state variable and is replaced by the fixed effect $\gamma(s)$. Since college students work at high school wages during the college phase, for them γ has to be redrawn at the end of the college period.

After the fixed effect is drawn, a standard consumption-savings problem with endogenous labor supply is solved. For households that choose not to enroll in college, the decision problem is identical to the one described in the next subsection 2.6.3. For households that complete college, the decision problem is slightly modified because they redraw the fixed productivity component given their newly obtained higher education level, incur psychological and financial costs of attending college, can work only up to maximum $\bar{\ell}^{ce}$ and also are allowed to borrow:

$$\begin{aligned} V_{t,j_a,si}(g,s,s_p,a,h,\gamma(hs),\eta) = \max_{c,a',\ell\leq\bar{\ell}^{ce}} \Big\{ & u(c,\ell) - F(g)_{\ell>0} - p(s,s_p,h) \\ & + \beta \sum_{\gamma'} \pi^{\gamma'}(co,h) \sum_{\eta'} \pi(\eta'|\eta) V_{t+1,j_a+1,si}(g,s,a',\gamma'(co),\eta') \Big\} \end{aligned}$$

subject to

$$\begin{aligned} a'(1+\mu) + c(1+\tau^c) + T(y(1-0.5\tau^p)) + \iota(1-\vartheta-\vartheta^{pr}) &= (a+Tr_{t,j})(1+r(1-\tau^k)) + y(1-\tau^p) \\ y &= w(s)\gamma(s)\epsilon(s,g,j)\eta\ell \\ a' &\geq -\underline{a}(s,j), c\geq 0, \ell\in[0,\bar{\ell}^{ce}]. \end{aligned}$$

where $p(s,s_p,h)$ is the psychological cost of attending college, and $\iota(1-\vartheta-\vartheta^{pr})$ is the tuition cost net of public and private subsidies. $F(g)_{\ell>0}$ denotes a fixed utility cost of working positive hours which depends on an individual's gender.

Households that enroll in college but drop out solve the same problem as above with the only difference that they are assumed to pay only half of the tuition costs.

2.6.3 Working Life Before Marriage

After completing (or not) their tertiary education single individuals enter the labor market and make labor supply as well as consumption-saving choices (c,a',ℓ), in light of their labor productivity, which is determined by the individual fixed effect γ, a deterministic education-, gender- and age-specific life cycle profile $\epsilon(s,g,j)$ and a persistent stochastic component η. With probability $\pi^{\gamma}(s,h)$ permanent productivity is $\gamma=\gamma_l(s)$ and with complementary probability it is $\gamma=\gamma_h(s)$. The wage of a single individual is then given by $w(s)\cdot\gamma(s)\cdot\epsilon(s,g,j)\cdot\eta$, where $w(s)$ is the education-specific aggregate wage per efficiency unit of labor.

During working life, households make the discrete decision whether to work, and conditional on employment endogenously choose hours worked subject to a time endowment constraint. The decision problem of singles can then be written as

$$V_{t,j,si}(g,s,a,\gamma,\eta)=\max_{c,a',\ell}\left\{u\left(c,\ell\right)-F(g)_{\ell>0}+\beta\sum_{\eta'}\pi(\eta'|\eta)V_{t+1,j+1,si}(g,s,a',\gamma,\eta')\right\} \tag{7}$$

subject to

$$\begin{aligned} a'(1+\mu)+c(1+\tau^c)+T(y(1-0.5\tau^p)) &= (a+Tr_{t,j})(1+r(1-\tau^k))+y(1-\tau^p)\\ y &= w(s)\gamma(s)\epsilon(s,g,j)\eta\ell\\ a' &\geq -\underline{\mathsf{a}}(s,j), c\geq 0, \ell\geq 0, \end{aligned}$$

where $\underline{\mathsf{a}}(s,j)$ is an age- and education-specific borrowing limit and $F(g)_{\ell>0}$ denotes a fixed, gender-specific utility cost of working positive hours. The household takes as given aggregate wages and interest rates $(w(s),r)$ as well as the proportional tax rates on consumption, asset income and labor income for social security (τ^c,τ^k,τ^p), the nonlinear labor income tax schedule $T(.)$ and well as the transfers Tr_j. Labor income taxes are levied on labor income net of employer contributions to social security $y(1-0.5\tau^p)$.

2.6.4 Marriage

Individuals remain single until at age j_m, when they face an exogenous, education-specific probability $\pi^m(s)$ of marriage, where we assume random matching so that the marriage probability only depends on own education s. Depending on the realization of the marriage shock individuals continue to live as singles or form a new married household. Since a married household is characterized by the education and wage fixed effect of both spouses as well as their combined financial asset positions (all of which are at least partially the result of endogenous choices), at age j_m-1 a single individual has to form expectations over the type of spouse it might marry (and these expectations have to be confirmed in a rational expectations equilibrium, inducing an additional equilibrium fixed point problem). Recalling that state variables of the spouse of the opposite gender are indexed by $-g$, the decision problem at model age $j_m - 1$, in anticipation of potential marriage next period, is given by:

$$
\begin{aligned}
&V_{t,j_m-1,si}(g,s,a,\gamma,\eta) = \max_{c,a',\ell} \{u(c,\ell) - F(g)_{\ell>0} \\
&+\beta(\pi^m(s)\mathbf{E}_{s(-g),a'(-g),\gamma(s(-g))} \sum_{\eta'(wo)} \Pi(\eta'(wo)) \sum_{\eta'(ma)} \Pi(\eta'(ma)) \cdot \\
&V_{t+1,j_m,cpl}(s(wo),s(ma),a'(wo)+a'(ma),\gamma(s(wo)),\gamma(s(ma)),\eta'(wo),\eta'(ma)) \\
&\qquad\qquad\qquad +(1-\pi^m(s)) \cdot \sum_{\eta'} \pi(\eta'|\eta) V_{t+1,j_m,si}(g,s,a',\gamma,\eta') \Big\}
\end{aligned}
$$

where $\mathbf{E}_{s(-g),a'(-g),\gamma(s(-g))}(\cdot) = \int (\cdot) d\Phi_{t,j_m-1,si}(-g,s,a'(-g),\gamma(s(-g)))$, i.e. the expectation over the characteristics of potential spouses is determined by the cross-sectional measure and savings policy functions of the opposite gender households in period $j_m - 1$. The maximization problem is subject to the following constraints

$$
\begin{aligned}
a'(1+\mu) + c(1+\tau^c) + T(y(1-0.5\tau^p)) &= (a + Tr_{t,j})(1 + r(1-\tau^k)) + y(1-\tau^p) \\
y &= w(s)\gamma(s)\epsilon(s,g,j)\eta\ell \\
a' &\geq -\underline{a}(s,j), c \geq 0, \ell \geq 0.
\end{aligned}
$$

2.6.5 Parenthood and Child Human Capital Accumulation

At age $j_f > j_m$ children enter single women- and married households (single men do not live with children). The number of children per household is a function of the mother's marital status and education level, and is denoted by $\varsigma(s(wo))$. All children of a household are assumed to be identical and characterized initially by a level of human capital h that depends on parental education and marriage status (s,q). As long as children are present, parents invest time and resources (i^m, i^t) into the production of new child human capital; we term these *private* human capital investments. Time investments bear a utility cost for households, which depends on the number of children $\varsigma(s(wo))$. The period utility function is therefore $u(c,\ell,i^t,s(wo))$. Finally, when children leave the household, parents can give them non-negative inter-vivos transfers b to finance tertiary education (or their consumption).

In every period during childhood, private human capital investments are combined with *public* investment into schooling to transform existing child human capital h into new human capital h' according to the following age-dependent production function $h' = g(j,h,i^m,i^t,i^g)$. For single women, the decision problem during this stage of the life cycle then is

$$V_{t,j,si}(wo,s,a,h,\gamma,\eta) = \max_{c,i^m,i^t,a',h',\ell} \Big\{ u\left(c,\ell,i^t,s(wo)\right) - F(wo)_{\ell>0} + \beta \sum_{\eta'} \pi(\eta'|\eta) V_{t+1,j+1,si}(wo,s,a',h',\gamma,\eta') \Big\} \tag{8}$$

subject to

$$\begin{aligned} a'(1+\mu) + c(1+\tau^c) + \varsigma(s)\cdot i^m + T(y(1-0.5\tau^p)) &= (a+Tr_{t,j})(1+r(1-\tau^k)) + y(1-\tau^p) \\ y &= w(s)\gamma(s)\epsilon(s,g,j)\eta\ell \\ a' &\geq -\underline{a}(j,s), c \geq 0, \ell \geq 0 \\ h' &= g(j,h,i(i^m,i^t,i^g)), i^m \geq 0, i^t \geq 0. \end{aligned}$$

Since single men are assumed not to have children present in the household, they solve the same maximization problem as in (7).

For couples, participation, hours worked and the time investment of both spouses are choice variables, and thus the dynamic programming problem of the household reads as:

$$\begin{aligned} &V_{t,j,cpl}(s(wo),s(ma),a,h,\gamma(s(wo)),\gamma(s(ma)),\eta(wo),\eta(ma)) = \\ &\max_{c,i^m,i^t(wo),i^t(ma),a',h',\ell(wo),\ell(ma)} \left\{ u\left(c,\ell(wo),\ell(wo),i^t(wo),i^t(ma),s(wo)\right)\right. \\ &-F(wo)_{\ell(wo)>0} - F(ma)_{\ell(ma)>0} + \beta \sum_{\eta'(wo)} \pi(\eta'(wo)|\eta(wo)) \sum_{\eta'(ma)} \pi(\eta'(ma)|\eta(ma)) \cdot \\ &\qquad V_{t+1,j+1,cpl}(s(wo),s(ma),a',h',\gamma(s(wo)),\gamma(s(wo)),\eta'(wo),\eta'(ma))\} \end{aligned}$$

subject to

$$\begin{aligned} &a'(1+\mu) + c(1+\tau^c) + \varsigma(s(wo))\cdot i^m + T^{cpl}(y(1-0.5\tau^p)) = (a + 2\cdot Tr_{t,j})(1+r(1-\tau^k)) + y(1-\tau^p) \\ &y = w(s(wo))\gamma(s(wo))\epsilon(s(wo),wo,j)\eta(wo)\ell(wo) + w(s(ma))\gamma(s(ma))\epsilon(s(ma),ma,j)\eta(ma)\ell(ma) \\ &a' \geq -\underline{a}(j,\max(s(wo),s(ma))), c \geq 0, \ell(g) \geq 0 \text{ for } g \in \{ma,wo\} \\ &h' = g(j,h,i(i^m,i^t,i^g)), i^t = \sum_{g\in\{ma,wo\}} i^t(g), i^t(g) \geq 0 \text{ for } g \in \{ma,wo\}, i^m \geq 0, \end{aligned}$$

where $T^{cpl}(.)$ is the labor income tax function it faces.

2.6.6 Children Leaving the Household and Inter-Vivos Transfers

When children are of age j_a (and their parents of age $j_f + j_a$) they form their own households. At the beginning this period a one time inter-vivos transfer $b \geq 0$ can be made by the altruistically motivated parents; these transfers become assets of children within the same model period.[15] In the interest of brevity, only the dynamic programs of single mothers are presented from this stage of the life cycle; the optimization problems of married couples are similar to those of singles. At age $j_a + j_f$ the dynamic program of single mothers is given by:

$$V_{t,j_a+j_f,si}(wo,s,a,h,\gamma,\eta) = \max_{c,b,a',\ell} \{u(c,\ell) - F(g)_{\ell>0}$$
$$+\beta \sum_{\eta'} \pi(\eta'|\eta) V_{t+1,j_a+j_f+1,si}(wo,s,a',\gamma,\eta') + \nu \mathbf{E}_{g^{ch}} \bar{V}_{t,j_a,si}\left(g^{ch}, s, \frac{b}{1+r(1-\tau^k)}, h\right)\right\}$$

where $\bar{V}_{t,j_a,si}\left(g^{ch}, s, \frac{b}{1+r(1-\tau^k)}, h\right)$ denotes the pre-education decision value function of a child of gender g^{ch} (which is revealed after children leave the household so that expectations are taken with respect to g^{ch}, and with probability 0.5 adult children become young women and men, respectively), parental education s, human capital h and assets $a = b/(1 + r(1 - \tau^k))$, noting that the transfer accrues to the newly independent child at the beginning of the current period and thus already earns interest concurrently. The parameter ν measures the strength of altruism of a parent. Maximization is subject to

$$c(1+\tau^c) + (1+\mu)a' + \varsigma(s) \cdot b + T(y(1-0.5\tau^p)) = (a + Tr_{t,j})(1 + r(1-\tau^k)) + y(1-\tau^p)$$
$$y = w(s)\gamma(s)\epsilon(s,g,j)\eta\ell$$
$$a' \geq -\underline{\mathsf{a}}(s,j), c \geq 0, b \geq 0, \ell \geq 0.$$

After children have left the household, parental households continue solving a consumption-savings problem with endogenous labor supply until they reach retirement:

$$V_{t,j,si}(wo,s,a,\gamma,\eta) = \max_{c,a',\ell} \left\{ u(c,\ell) - F(g)_{\ell>0} + \beta \sum_{\eta'} \pi(\eta'|\eta) V_{t+1,j+1,si}(wo,s,a',\gamma,\eta') \right\}$$

[15] Since girls and boys have different expected lifetime utilities, parents may find it optimal to condition transfers on gender. In order to avoid having to carry child gender as a state variable, we assume that parents are constrained to treat girls and boys equally, or equivalently, learn the gender of children only when the leave the household. Thus, the expected lifetime utility of their offspring entails taking expectations with respect to the gender of the child.

subject to

$$
\begin{aligned}
a'(1+\mu)+c(1+\tau^c)+T(y(1-0.5\tau^p)) &= (a+Tr_{t,j})(1+r(1-\tau^k))+y(1-\tau^p)\\
y &= w(s)\gamma(s)\epsilon(s,g,j)\eta\ell\\
a' &\geq -\underline{\mathsf{a}}(s,j), c\geq 0, \ell \geq 0.
\end{aligned}
$$

2.6.7 Retirement and Death

In retirement, that is after reaching the model age j_r, households solve a standard consumption-saving problem, receive social security benefits $pen(s,\gamma,\eta(j_r-1))$ and face mortality risk until they die for sure at maximal lifetime J. The problem is

$$
V_{t,j,si}(g,s,a,\gamma,\eta)=\max_{c,a'\geq 0}\left\{u\left(c\right)+\beta\phi(j)V_{t+1,j+1,si}(g,s,a',\gamma,\eta)\right\}
$$

subject to

$$
\begin{aligned}
a'(1+\mu)+c(1+\tau^c) &= (a+Tr_{t,j})(1+r(1-\tau^k))+pen(s,\gamma,\eta)\\
a' &\geq 0, c\geq 0,
\end{aligned}
$$

where $pen(s,\gamma,\eta(j_r-1))$ is retirement income which depends on education-specific wages $w(s)$, the persistent shock realization in the last working period before retirement η[16], the education level s and the fixed productivity component γ.

2.7 Production

A representative firm employs aggregate labor L_t and capital K_t to produce the final output good Y_t —both, K_t and Y_t are expressed in de-trended units— employing a neo-classical production function

$$
Y_t = F(K_t, \Upsilon_t L_t), \tag{9}
$$

where Υ_t is a labor augmenting technology level growing exogenously at rate μ, i.e., $\Upsilon_{t+1} = \Upsilon_t(1+\mu), \Upsilon_0 = 1$. Aggregate labor L_t at time t is the aggregate of college $L_{t,co}$ and non-college $L_{t,nc}$ labor

$$
L_t = H(L_{t,nc}, L_{t,co}), \tag{10}
$$

[16]This construction allows us to capture the progressivity embedded in the actual US social security benefit formula without carrying around another continuous state variable during working age.

where $L_{t,nc} = L_{t,hsd} + L_{t,hs} + L_{t,cod}$ are the labor units jointly supplied by high-school dropouts, high-school graduates and college dropouts. Function $H(\cdot)$ captures imperfect substitutability across education groups nc and co giving rise to education specific aggregate wages $w(s)$.

2.8 Government

The government administers a progressive labor income tax code, pays transfers to households and collects linear taxes on consumption and capital income. Aggregate labor income tax revenues net of transfers are denoted by T_t. In addition, the government spends $\alpha_j i^g$ per child on primary and secondary school education. The age profile α_j permits us to differentiate between the cost of primary and secondary school and i^g measures the scale of public education spending, and will be one key policy choice by the government. Total spending on primary and secondary public schools is denoted by E_t. The government also subsidizes tertiary education, with the share ϑ of tuition covered by the government; ϑ is the second crucial policy choice variable, and a choice of $\vartheta = 1$ represents free college. We denote by E_t^{CL} the aggregate cost of college subsidies.

In addition to the streams of education expenditures (E_t, E_t^{CL}) for primary, secondary and tertiary education the government also needs to finance an exogenous stream of non-education related expenditures G_t. To do so, the government raises revenues from taxing labor- and capital income as well as consumption, and from issuing government debt B_t. The period t flow government budget constraint then is[17]

$$E_t + E_t^{CL} + G_t + (1 + r_t)B_t = (1 + \mu)(1 + n_t)B_{t+1} + T_t + \tau_{c,t}C_t + \tau_{k,t}r_t(K_t + B_t) \quad (11)$$

The initial stock of government debt B_0 is an exogenously given initial condition (as is the initial aggregate capital stock K_0). Finally, the government also runs a pure pay-as-you-go social security system whose budget equates payroll taxes (with tax rate τ^p) to all pension benefits paid out according to the benefit formula $pen(s, \gamma, \eta)$.

[17]Recall that we define all aggregate variables as de-trended objects, so that the debt level in period $t + 1$ needs to be trend-adjusted by the growth rate of aggregate variables, $(1 + n_t)(1 + \mu)$. Also note that the population growth rate is time varying, because of education specific fertility $\varsigma(s)$ and endogenously time varying education shares, cf. Appendix B.

3 Equilibrium Definition and Computation

The key equilibrium object in our model is the cross-sectional measure $\Phi_{t,j,q}$ over household characteristics[18] $(j, q, g, s, a, h, \gamma, \eta)$. For each time period t and age j we normalize the total measure $\Phi_{t,j,q}(\cdot)$ to 1 and denote by N_j the (time-invariant) size of age cohort j.

$$\int d\Phi_{t,j,si}(g, s, a, h, \gamma, \eta) + \int d\Phi_{t,j,cpl}(s(ma), a, h, \gamma(wo), \gamma(ma), \eta(wo), \eta(ma)) = 1 \quad (12)$$

In order to clarify the distinction between the partial- and the general equilibrium versions of the model it is necessary to give a somewhat formal definition of equilibrium.

3.1 Equilibrium Definition

For given initial physical capital stock and government debt (K_0, B_0) and initial cross-sectional distributions of singles $\{\Phi_{0,j,si}(\cdot)\}_{j_a}^{J}$ and couples $\{\Phi_{0,j,cpl}(\cdot)\}_{j_m}^{J}$ a competitive equilibrium is given by sequences of household value and policy functions (for consumption, assets, labor supply, child human capital investments and bequests), aggregate capital and labor inputs, tax and transfer policies and government debt levels, aggregate prices, accidental bequests as well as household measures such that

1. In each period, household value and policy functions solve the household optimization problems, given factor prices, government policies and accidental bequests.

2. Denoting the exogenous depreciation rate of capital by δ, factor prices for capital and college- as well as non-college labor per efficiency unit satisfy

$$r_t = F_K(K_t, G(L_{t,co}, L_{t,nc})) - \delta \quad (13a)$$

$$w_{s,t} = F_{L_{s,t}}(K_t, G(L_{t,co}, L_{t,nc})), \quad \text{for } s \in \{co, nc\}. \quad (13b)$$

3. The government budget constraint (11) and the social security system budget constraint hold $\forall t$:

$$\tau_t^p \sum_{s \in \{co,nc\}} w_{s,t} L_{s,t} = \sum_{j=j_r}^{J} N_j \int pen_t(\cdot) d\Phi_{t,j,q}(\cdot) \quad (14)$$

[18] It is understood that, depending on the age j of the household as well as its marital status q, the household state space changes; for example, for couples it includes the education and fixed effect of both partners.

4. Markets clear in all periods t[19]:

$$L_{co,t} = \sum_{j_a}^{j_r-1} N_j \int \gamma\epsilon(co,g,j)\eta\ell_t(j,co,\cdot)d\Phi_{t,j,q}(co,\cdot) \tag{15a}$$

$$L_{nc,t} = \sum_{j_a}^{j_r-1} N_j \sum_{s\in\{hsd,hs,cod\}} \int \gamma(s)\epsilon(s,\cdot,j)\eta\ell_t(j,s,\cdot)d\Phi_{t,j,q}(s,\cdot) \tag{15b}$$

$$K_{t+1} + B_{t+1} = \sum N_{j=j_a}^{J} \int a'_t(j,\cdot)d\Phi_t(j,\cdot) \tag{15c}$$

$$C_t + K_{t+1}(1+n_t)(1+\mu) + CE_t + E_t + G_t = F(K_t,L_t)L_t + (1-\delta)K_t, \tag{15d}$$

where L_t was defined in (10) and CE_t are aggregate private education expenditures.

5. The marriage market clears:

$$\sum_{s(g)\in\{hsd,hs,cod,co\}} \pi^m(s(g)) \cdot \int \Phi_{t,j_m-1,si}(g,s(g),\cdot) =$$
$$\sum_{s(-g)\in\{hsd,hs,cod,co\}} \pi^m(s(-g)) \cdot \int \Phi_{t,j_m-1,si}(-g,s(-g),\cdot).$$

6. The total accidental bequests received by the working age population in period $t+1$ are equal to the total assets those passing away in period t net of private college subsidies

$$\sum N_{t+1,j}{}_{j=j_a}^{j_r-1} Tr_{t+1,j} = \sum N_{t,j}{}_{j=j_r}^{J} \int (1-\phi_j)a'_t(j,\cdot)d\Phi_{t,j,q}(\cdot) \tag{16}$$

7. The cross-sectional measures of households evolve according to the laws of motion induced by exogenous population dynamics, the exogenous Markov processes for idiosyncratic labor productivity, the exogenous distribution of transitory shocks, endogenous asset and child human capital accumulation, higher education and inter-vivos transfer decisions, both at the age of marriage and at all other ages.

8. The initial measure of newly formed households $\Phi_{t,j_a,si}(\cdot)$ at age j_a is consistent with inter-vivos transfers and human capital investment decisions of parents and the measure of economic newborns at age j_a after the higher education choice is made.

9. At age $j_m - 1$, prior to marriage, expectations of singles about characteristics of future spouses are consistent with the cross-sectional distribution of the opposite gender at age $j_m - 1$.

[19]Recall that we need to trend adjust variables in period $t+1$, cf. footnote 17.

3.2 Solution Algorithm

We propose to solve for (optimal) policy transitions in a model characterized by non-convex household maximization problems involving discrete and continuous decision variables as well as a sizable individual state space, and in which there are two nested fixed-point problems even in partial equilibrium, one emerging from the intergenerational linkages (the value function of children enters lifetime utility of their altruistic parents) and one from the marriage market equilibrium (types of pre-marriage singles are endogenous and have to match and conform to household expectations). The solution of market clearing prices in steady state and along the transition path is then relatively standard; our description here focusses on the more novel fixed point problems in steady state.[20]

Modeling of marriage requires that the marriage market clears which results in a fixed point problem in distributions. Assuming rational expectations implies that before the marriage period, expectations on assets and productivity of a future spouse should be consistent with the cross-sectional distribution of assets and productivity of the opposite gender, $\Phi_{t,j_m-1,si}(g,s,a,\gamma(s))$. Recall that due to explicitly modelled intergenerational altruism, the initial measure of economic newborns $\Phi_{t,j,si}(\cdot)$ must be consistent with inter-vivos transfers and human capital investment decisions of parents. This implies a second fixed point problem in distributions. Additionally, the value function of the child generation at age j_a should be consistent with the value function of the parental generation at age j_a which turns the finite horizon life cycle problem of each generation into an infinite horizon problem over time. Given that each iteration of the latter fixed point problem is affected by $\Phi_{t,j_m-1,si}(g,s,a,\gamma(s))$ the three fixed point problems (one in value functions and two in measures) have to be solved jointly. To deal with this multi-layer fixed point problem, we propose Algorithm 1.

Algorithm 1 Nested Fixed Point Problem

1: **Step 1:** Guess distribution of assets, fixed productivity and education for both genders at the end of period j_{m-1} (for a given skill level s), $\Phi_{t,j_m-1,si}(g,s,a,\gamma(s))$
2: **Step 2:** For given $\Phi_{t,j_m-1,si}(g,s,a,\gamma(s))$, solve for intergenerational RE equilibrium:
3: **2.1:** Solve fixed point problem in value functions (guess $V_{t,j_a,si}(\cdot)$, iterate until convergence)
4: **2.1:** Solve fixed point problem in distributions (guess $\Phi_{t,j_a,si}(\cdot)$ iterate until convergence)
5: **Step 3:** If $||\Phi_{t,j_m-1,si}(g,s,a,\gamma(s))^{\text{update}} - \Phi_{t,j_m-1,si}(g,s,a,\gamma(s))^{\text{guess}}|| < \epsilon$, EXIT, else go back to Step 1 and continue until convergence.

[20] The algorithm for the household problem is a combination of the discrete-continuous endogenous grid method described in Iskhakov, Jorgensen, Rust, and Schjerning (2017), embedded in a value function iteration algorithm that draws on Druedahl (2021).

4 Calibration

The model is calibrated to US aggregate and cross-sectional data, using a standard two-stage procedure in which a subset of the parameters is chosen outside the model based on values in the literature, and a second set of parameters is calibrated inside the model. Specifically, the key parameters governing preferences and the child human capital production function are calibrated internally so that the initial steady state general equilibrium of the model is consistent with the (child) age profile of parental time and resources investments, average hours worked, labor force participation, the cross-sectional wage- and education distributions, as well as the level of government spending and government debt. Although these parameters are determined jointly, for ease of exposition we associate below each parameter with a specific target.

4.1 Demographics

The population growth rate n is assumed to be 1% which is the average of the US annual population growth rate values in the 2000s. The number of children per mother differs by education level and is ca. 15% higher for households without a college degree, in accordance with the five most recent PSID waves. The marriage probability $\pi^m(s)$ is set to 0.51 for all education groups, which is the average marriage rate for the age range 25-35 based on PSID 2011-2019.

4.2 Preferences

For single households, the per period utility function takes the following functional form:

$$u(c, \ell) = \log(c) - \phi \frac{\ell^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} \tag{17}$$

i.e., we assume logarithmic utility.[21] Parameter ψ, that can be interpreted as the Frisch elasticity of labor supply, and is set to 0.6, following Kindermann and Krueger (2022)[22]. Finally, parameter ϕ is calibrated endogenously to match the average hours worked of $1/3$ of the time endowment.

Households composed of couples experience disutility from hours worked of both partners:

$$u(c, \ell(wo), \ell(ma)) = \log(c) - \phi \frac{\ell(wo)^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} - \phi \frac{\ell(ma)^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}}. \tag{18}$$

[21] Given the logarithmic utility assumption, the child equivalence scale parameter is irrelevant for the household problem and for brevity considerations is omitted.

[22] As Kindermann and Krueger (2022) point out this value is based on the average of estimates for men and for women.

The term capturing fixed costs of working positive hours $F_{\ell>0}$ is also calibrated endogenously to match the average share of non-participating and unemployed households of 25%.

During the model periods when children live in the parental household, time spent with children affects parental utility. We assume that the disutility from time with children enters the utility function of parents in an additively separable manner[23]:

$$u(c, \ell, i^t, s(wo)) = \log(c) - \phi \frac{\ell^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} - \kappa \frac{\varsigma(s(wo)) \cdot {i^t}^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} \tag{19}$$

where κ is calibrated to match the average household time investment into children (per week per child), and $\varsigma(s(wo))$ is the average number of children per household, which depends on the education of the mother.

For couple households, accordingly, there are additional terms capturing disutility from hours worked and time with children of the partner:

$$\begin{aligned} u(c, \ell(wo), \ell(ma), i^t(wo), i^t(ma), s(wo)) = \log(c) - \phi \left(\frac{\ell(wo)^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} + \frac{\ell(ma)^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} \right) \\ - \kappa \cdot \varsigma(s(wo)) \cdot \left(\frac{i^t(wo)^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} + \frac{i^t(ma)^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} \right) \end{aligned} \tag{20}$$

When children attend college, they experience psychological costs determined by the cost function

$$p(s, s_p; h) = \varrho(s, s_p) + \frac{1}{h}$$

where $\varrho(s, s_p)$ is a calibration parameter which depends on parental education and is 50% smaller for college dropouts than for college graduates, due to the assumed lesser time these individuals spend in college. Specifically, $\varrho(s = co, s_p < co)$ is calibrated to match the average share of college graduates of 32% while $\varrho(s = co, s_p = co)$ is chosen such that the college enrollment rate conditional on parents being college graduates equals 92% (PSID 2011-2019). Observe that the psychological cost specification above implies that the utility costs are monotonically decreasing and convex in the acquired human capital h.

Households discount utility at rate β which is chosen such that in general equilibrium the implied interest rate equals $\approx 3\%$. Utility of future generations is discounted at rate ν which governs the degree of parental altruism. Parameter ν is chosen so that average per child inter-vivos transfer is 61,200$, as implied by the 2013 Rosters and Transfers Module of the PSID (based on monetary transfers from parents to children until age 26).

[23] Bastian and Lochner (2022) based on females responses to EITC expansions point out that mothers increase their time with children not at the cost of hours worked but rather via reallocating their leisure time.

4.3 Human Capital Production Function

Innate human capital (at biological age 2) depends on parental education and marital status and for given parental background is exogenously given. The dependency of innate child ability on parental background is disciplined using child (Letter Word) test score data from the Child Development Supplement (CDS) to the PSID.

At ages $j_0, \ldots, j_a - 1$, human capital is accumulated according to a multi-layer human capital production function with imperfectly substitutable inputs:

$$h'(j) = \left(\kappa_j^h h^{1-\frac{1}{\sigma^h}} + (1-\kappa_j^h) i(j)^{1-\frac{1}{\sigma^h}}\right)^{\frac{1}{1-\frac{1}{\sigma^h}}} \tag{21a}$$

$$i(j) = \bar{A}\left(\tilde{\kappa}_j^g \left(i^g\right)^{1-\frac{1}{\sigma^g}} + (1-\tilde{\kappa}_j^g)\left(i^p(j)\right)^{1-\frac{1}{\sigma^g}}\right)^{\frac{1}{1-\frac{1}{\sigma^g}}} \tag{21b}$$

$$i^p(j) = \left(\tilde{\kappa}_j^m \left(i^m(j)\right)^{1-\frac{1}{\sigma^m}} + (1-\tilde{\kappa}_j^m)\left(i^t(j)\right)^{1-\frac{1}{\sigma^m}}\right)^{\frac{1}{1-\frac{1}{\sigma^m}}}. \tag{21c}$$

The production function features partially age dependent parameters for calibration purposes — to reflect relative differences in importance of different inputs at different stages of childhood. All inputs are divided by their respective unconditional means which allows achieving unit independence (see Cantore and Levine 2012). This normalization is accounted for by adjusting the weight parameters $\tilde{\kappa}_j^g$ and $\tilde{\kappa}_j^m$, respectively —see Appendix B.2 for details.

In the outermost nest of the production function, existing human capital h is combined with aggregate investment $i(j)$ at age j. The substitution elasticity σ^h is set exogenously to 1 for all ages (implying a Cobb-Douglas specification). The age profile for the weight parameter κ_j^h is calibrated to match the age profile of (per child) parental time investment in the data. The average κ^h, in turn, is chosen such that the average short-run college enrollment elasticity with respect to high school spending generosity matches the midpoint of the range of empirical estimates reviewed in the meta-study by Jackson and Mackevicius (2023).

In the second nest of the production function, public and private inputs $(i^g, i^p(j))$ are combined, with the substitution elasticity between the two inputs being denoted by σ^g and the age-specific weight parameters $\tilde{\kappa}^g(j)$. Our choice of i^g in the baseline calibration is described below in Section 4.6 and $i^p(j)$ is the CES aggregate of parental inputs in the innermost nest. The substitution elasticity between both inputs is set exogenously to 2.43 using the estimate provided in Kotera and Seshadri (2017). For kindergarten ages, i.e. age bin 2-6, the weight parameter is calibrated endogenously to match average parental time investment at that age. For other ages, the weight parameter is also calibrated endogenously such that the inequality in acquired human capital by family background is close to the disparity in test scores in CDS-PSID at ages 17-19.

$\bar{A}$ is a normalization parameter which is chosen such that average acquired human capital at age 18 is equal to 1.

Finally, in the innermost nest parental time and resource inputs $(i^t(j), i^m(j))$ are combined, with a substitution elasticity that is denoted by σ^m and the age-dependent weight parameter $\tilde{\kappa}^m(j)$. The substitution elasticity σ^m is fixed exogenously at the value of 1 using the estimate provided in Lee and Seshadri (2019), whereas the weight parameter $\tilde{\kappa}^m(j)$ is calibrated endogenously to match the mean and the age profile of the parental monetary input.

4.4 High School and College Dropout

The high school completion probability takes the following functional form:

$$\pi^{hs}(h) = 1 - \exp(-\lambda^{hs} h), \tag{22}$$

where λ^{hs} is a parameter calibrated endogenously to match the average share of high school dropouts in PSID data[24] of 8%. The college completion probability takes the same functional form:

$$\pi^{c}(h) = 1 - \exp(-\lambda^{c} h), \tag{23}$$

with parameter λ^c calibrated endogenously to match the average share of college dropouts in PSID data of 28%. Observe that for $\lambda^h > 0$ and $\lambda^c > 0$ this functional form specification implies that the probability of both graduating from high school and finishing college is increasing in acquired human capital.

4.5 College Tuition Costs & Borrowing Constraint of Students

Based on NCES statistics, the net tuition cost ι (tuition, fees, room and board rates charged for full-time students in degree-granting post-secondary public institutions) for one year of college in constant 2010 dollars has been on average 15,500$ during the time period 2000-2019. Following Krueger and Ludwig (2016), the maximum amount of publicly provided students loans per year is given by $11,397\$$, which is the borrowing limit for college students in the model. For college dropouts, we assume that the borrowing limit is twice as tight as for college graduates. For all ages after the college period (i.e. for all $j > j_a$) we let

$$\underline{a}(j, s > hs) = \underline{a}(j-1, s > hs)(1+r) - rp$$

[24]Education shares are based on the five recent waves of PSID: 2011, 2013, 2015, 2017 and 2019.

and compute rp such that the terminal condition $\underline{a}(j_r, s) = 0$ is met.

4.6 Education Spending

The government spends on schooling for children and pays the college subsidy for college students. According to NCES statistics, average per student spending on public schools is ca. $i^g = \$14,000$. The public college subsidy is set to $\vartheta\% = 38.8\%$ of average gross tuition costs, as in Krueger and Ludwig (2016). We also explicitly model private subsidies that are paid from accidental bequests and constitute $\vartheta^{pr}\% = 16.6\%$ of the gross tuition cost in the baseline (see Krueger and Ludwig 2016).

4.7 Labor Productivity

We use PSID data to measure labor productivity at the household level. Appendix C.1 describes our procedure in greater detail. First, we regress it on a cubic in age of the household head, time dummies, family size, a dummy for marital status of the household head, and household fixed effects. Predicting the age polynomial gives our estimates of $\epsilon(s, g, j)$. We next compute log residuals and estimate moments of the log earnings process by GMM, closely following Heathcote, Storesletten, and Violante (2010). We specify a standard process of the log residuals according to a persistent and transitory shock specification, i.e., we decompose log residual productivity $\ln(\mathrm{y}_t)$ as

$$\ln(\mathrm{y}_t) = \ln(\mathrm{z_t}) + \ln(\varepsilon_t)$$
$$\ln(\mathrm{z_t}) = \rho \ln(\mathrm{z_{t-1}}) + \ln(\nu_t)$$

where $\varepsilon_t \sim_{i.i.d} \mathcal{D}_\varepsilon(0, \sigma^2_\varepsilon)$, $\nu_t \sim_{i.i.d} \mathcal{D}_\nu(0, \sigma^2_\nu)$ for density functions $\mathcal{D}$.

To approximate this process in our model, we translate our estimates of $[\hat{\rho}, \hat{\sigma}^2_\nu, \hat{\sigma}^2_\varepsilon]$ into a four-period Markov chain with 2 states, targeting the conditional variance of $\ln(\mathrm{y_t})$, conditional on $\ln(\mathrm{z_{t-4}})$, of $\hat{\sigma}^2_\varepsilon + \hat{\sigma}^2_\nu \cdot \sum_{s=0}^{3} \hat{\rho}^{2s}$ and the autocorrelation of $\ln(y_t)$ and $\ln(y_{t-4})$ of $\rho^4 / \left(1 + \frac{\sigma^2_\varepsilon}{\sigma^2_{\ln(\mathrm{z})}}\right)$, where $\sigma^2_{\ln(\mathrm{z})} = \frac{\sigma^2_\nu}{1-\rho^2}$.

The parameter estimates and the moments of the approximation are reported in Table 3. The magnitude of our estimates, $[\hat{\rho}, \hat{\sigma}^2_\nu, \hat{\sigma}^2_\varepsilon]$, is similar to the stochastic labor productivity estimates by Heathcote, Storesletten, and Violante (2010).[25]

[25] Heathcote, Storesletten, and Violante (2010) do not include a fixed effect at the first stage and estimate a first-order autocorrelation of the persistent productivity component of 0.97, a variance of the persistent productivity shock ranging between 0.01 and 0.02, and of the transitory productivity shock between 0.04 and 0.06 (for the 1990s to 2000s).

Table 3: Stochastic Wage Process

	Estimates			Markov Chain	
Parameter	$\hat{\rho}$	$\hat{\sigma}^2_\nu$	$\hat{\sigma}^2_\varepsilon$	$\pi_{hh} = \pi_{ll}$	$[\eta_l, \eta_h]$
Estimate	0.9501	0.0109	0.0637	0.7623	$[0.6919, 1.3081]$

Notes: This table contains the estimated parameters of the residual log wage process.

The mapping of human capital into a fixed productivity component is probabilistic. The fixed effect $\gamma(s)$ can take two values, $\gamma^h(s)$ (high) and $\gamma^l(s)$ (low), respectively, for each education group. The probability of drawing a high realization $\gamma^h(s)$ is given by

$$\pi^h(h) = 1 - \exp(-h), \tag{24}$$

where h is child acquired human capital at age 18.

The education-specific permanent productivity parameters $\gamma^h(s)$ and $\gamma^l(s)$ are calibrated endogenously to ensure that for each of education group the average $\gamma(s)$ is equal to one[26], i.e.

$$\int \left(\pi^h(s,h)\gamma^h(s) + \left(1 - \pi^h(s,h)\right)\gamma^l(s)\right)\Phi(dh,s) = 1. \tag{25}$$

The education-specific spreads $\Delta^\gamma(s)$ between $\gamma^h(s)$ and $\gamma^l(s)$ are calibrated as follows. For high school dropouts and high school graduates that serve as a reference group $\Delta^\gamma(s = hsd) = \Delta^\gamma(s = hs)$ is set such that the average variance of log wages equals 0.45 as implied by PSID 2011-2019. For the other two education groups, $\Delta^\gamma(s)$ parameters are scaled relative to $\Delta^\gamma(s < sco)$ such that the ratios of human capital gradients of (lifetime) wages of college graduates and the reference group, on the one hand, and college dropouts and the reference group, on other hand, estimated with NLSY79 data (in expectation, i.e. from an ex ante perspective) are matched. Specifically, estimates of education-specific human capital gradients $\hat{\rho}(s)$ are obtained by running the following regressions:

$$\ln\left(\omega(s)\right) = \rho(s) \cdot \frac{e}{\bar{e}} + \upsilon(s),$$

where $\omega(s)$ denotes age-free education-specific wages and e measures test scores of the Armed Forces Qualification Test (AFQT) which are normalized by their mean $\bar{e}$. Finally, $\upsilon(s)$ is an education group specific error term. Table 4 shows the resulting estimates $\hat{\rho}(s)$. The estimated

[26]This ensures that the skill premia are matched.

ability (human capital) gradient is strictly increasing in education reflecting a pronounced complementarity between ability (human capital) and education.

Table 4: Ability Gradient by Education Level

Education Level	Ability Gradient
(HS- & HS)	0.4248 (0.0481)
(CL-)	0.5786 (0.0245)
(CL & CL+)	0.7298 (0.0670)

Notes: Estimated ability gradient $\hat{\rho}(s)$, using NLSY79. Standard errors in parentheses.

For $s > hs$, $\Delta^\gamma(sco)$ and $\Delta^\gamma(co)$ parameters are set such that

$$\frac{\int \left(\frac{\partial \left[\pi^h(s,h)\gamma^h(s) + \left(1-\pi^h(s,h)\right)\gamma^l(s) \right]}{\partial h} \right) \Phi(dh, s)}{\int \left(\frac{\partial \left[\pi^h(s<co,h)\gamma^h(s<co) + \left(1-\pi^h(s<co,h)\right)\gamma^l(s<co) \right]}{\partial h} \right) \Phi(dh, s<co)} = \frac{\hat{\rho}(s)}{\hat{\rho}(s<co)}.$$

Thus, for a given acquired human capital distribution, the education-specific parameters $\gamma^h(s)$ and $\gamma^l(s)$ jointly determine the dispersion of wages as well as the degree of complementarity between human capital and education in wages. From an ex ante perspective these parameters thus determine the steepness of the expected college wage premium in human capital, and from an ex post perspective they drive the realized dispersion of wages.

4.8 Aggregate Production

We assume that (9) is a Cobb-Douglas production function

$$Y_t = K_t^\alpha (\Upsilon_t L_t)^{1-\alpha},$$

where α determines the elasticity of output with respect to capital. In order to permit the possibility that a policy-induced change in the share of college graduates changes their relative wages we assume that non-college labor (including college dropouts) and college labor (i.e., college graduates) are imperfectly substitutable in production. Thus, the labor aggregator (10) is given by $L_t = (L_{t,nc}^\rho + L_{t,co}^\rho)^{\frac{1}{\rho}}$ where ρ governs the elasticity of substitution $\frac{1}{1-\rho}$ between college $L_{t,co}$ and non-college labor $L_{t,nc} = L_{t,hsd} + L_{t,hs} + L_{t,cod}$.

The capital share parameter α is set to $1/3$, a standard value in the literature, and the annual physical capital depreciation rate equals 5%. The rate of technological progress (and thus the long-run growth rate of per-capita income) μ equals 1%. Finally, the elasticity of substitution between skilled (college) and non-skilled (high school dropout, graduate and college dropout) labor is set to 3.3, following the estimate of Abbott, Gallipoli, Meghir, and Violante (2019).

4.9 Government

The government has to balance the budget of the general tax and transfer system as well as the budget of the pension system. In the scope of the general tax and transfer system budget, the government finances an exogenous stream of (non-education related) expenditures and an endogenous stream of education related expenditures (pre-tertiary and tertiary). The revenue side of the general tax and transfer system is comprised by taxes on consumption, capital and labor income. The consumption tax rate is set to 5% (see Mendoza, Razin, and Tesar 1994) while the capital income tax rate is fixed at 36%, following Trabandt and Uhlig (2011). Additionally, the government can issue debt.

Households that work positive hours in the labor market face the labor income tax schedule that is approximated using a two-parameter tax function as in Heathcote, Storesletten, and Violante (2017):

$$T(y, n > 0) = y - (1-\tau) y^{1-\xi} \tag{26}$$

where τ is the level parameter, and ξ is the progressivity parameter. The progressivity parameter is exogenously set to 0.18 for all population groups, following Heathcote, Storesletten, and Violante (2017), while the level parameter is calibrated endogenously to match the government debt to GDP ratio of 100% in the baseline.

The non-participating and unemployed households have no labor income and thus do not pay labor income taxes but receive government transfers ω that are set to 20.2% of average (full-time) earnings (CEX 2001-2007; consumption of bottom 10%). Thus, for non-working singles and couples (i.e. both spouses do not work) the tax/transfer functions are given by:

$$T(q = si, 0) = -\omega \quad \text{and} \quad T(q = cpl, 0) = -2\omega.$$

If, however, only one spouse is non-working and the other spouse supplies positive hours, then the tax function is:

$$T(q = cpl, n(g) > 0, n(g^-) = 0, y) = y - (1-\tau) y^{1-\xi} - \max\{0, 2\omega - (1-\tau) y^{1-\xi}\}. \tag{27}$$

In other words, the government guarantees the minimum income of 2ω also to the couples with only one partner supplying positive hours.

Finally, as for the the pension system, the payroll tax τ^p is set to the current legislative level of 12.4% and the actual progressivity of the pension system is taken into account.

4.10 Calibration Summary

Table 19 in Appendix C.3 summarizes the subset of parameters calibrated exogenously outside the model, and Table 20, also in the appendix, provides an overview of the second stage parameters that are calibrated endogenously within the model.

5 Quantitative Properties of Benchmark Economy

Before using the model for a counterfactual education policy analysis we view it as crucial to ensure that it has plausible short-run predictions for comparable small-scale policy interventions empirically studied and surveyed by the applied literature, e.g. Deming and Dynarski (2010), Jackson and Mackevicius (2023). To do so, it is important to decide which version and time horizon of our model to confront with these empirical estimates. The empirical literature focuses on the short-run effects, and by their small-scale nature the experiments can plausibly be assumed to have no significant impact on the economy-wide interest rate as well as the aggregate and relative wages and the government budget. Therefore we contrast the short-run, partial equilibrium model response to this empirical evidence.[27] Since the range of the empirical estimates is fairly large, our goal is not to argue that our model matches any specific study, but rather to demonstrate that the model-based statistics fall into the empirical range and, especially, does not overstate the positive impact of the education policy reforms discussed in this paper.

5.1 Increase in College Tuition Subsidies

The empirical evidence on the short-run, small scale (quasi-)experimental effect of college tuition cost on attendance and completion is quite broad. Deming and Dynarski (2010) summarize the large literature on this topic, with the upshot that an $1,000 increase in college subsidies leads to a 3-6 percentage point increase in college enrollment. Our model implies a short-run, partial equilibrium response in college enrollment of 5.1 percentage points.

[27] Partial equilibrium means that wages and interest rates as well as tax rates are held fixed when the policy changes. In contrast, we continue to assume that the marriage market clears, that is, even in partial equilibrium households adjust their beliefs about the characteristics of potential future spouses in response to the policy change.

5.2 Increase in High School Spending

In their meta-study of a large number of empirical (quasi-)experimental studies, Jackson and Mackevicius (2023) report that an increase in public high school funding by \$1,000 per pupil for four years leads to an increase in high-school completion by 0.07-3.99 percentage points and an increase in college enrollment by 0.90-5.51 percentage points. We target the midpoint of the estimates for the impact of high-school funding for college enrollment when calibrating the model, but not the response of high school graduation rates to the same intervention, leaving this prediction of the model for model validation. In partial equilibrium, the high school completion rate increases by 0.6 percentage points on impact of a $\$1,000$ increase in public high-school spending, towards the lower bound of the wide-ranged empirical estimates, indicating that in our model public schooling is not "overly" productive relative to the available evidence.

5.3 What Accounts for Differences in College Attendance in the Model?

Our structural approach also permits us to ask, from the perspective of our model, why children from rich families attend college at such a higher rate than poor children in the benchmark economy? Table 5 shows the higher education distribution of children conditional on parental background. Among children born into the "bottom" family backgrounds (those with single mothers without high-school degree) only ca. 13% attend (but may not complete) college, whereas for those born into the "top" parental backgrounds this number is 92%. In the model, the probability of attending college is determined by the joint distribution of acquired age 18 child human capital, initial assets determined by parental inter-vivos transfers and parental education (via its effect on the psychological costs of attending college).

Table 5: Education Shares by Parental Background: "Bottom" and "Top"

	$s = hsd$	$s = hs$	$s = cod$	$s = co$
$s^p = hsd$, $q = si$ ("Bottom")	0.14	0.73	0.06	0.07
$s^p = co$, $q = cpl$ ("Top")	0.07	0.02	0.41	0.51

Notes:: The "bottom" group refers to children with single mothers that dropped out of high school, and the "top" denotes children with two parents that have a college degree.

Using the structural model we can decompose this college attendance gap into the sources emerging at different points of a child's life cycle. If we give poor children the average human capital of rich children at age 18, then 50% of the college enrollment gap disappears, that is, differential human capital attainment through the school years accounts for half of the observed college enrollment differences documented in Table 5, from the perspective of the model. Of these 50%, about 1/5 is accounted for by initial differences in human capital at age 2; recall from the calibration section that even at early ages poor children have lower human capital than rich children.[28] The second main source of enrollment differences between the rich and the poor are financial transfers; if we endow poor children with the average level of inter-vivos transfers received by children at the "top", the college enrollment gap closes by a further 20%, suggesting that financial constraints are a nontrivial, but not major reason between these enrollment differences. The remaining 30% of college enrollment disparities in the baseline are attributed to the lower psychological cost experienced by children with college-educated parents. This decomposition exercise suggests that policies affecting the pre-college child human capital accumulation process can have very significant impact on the college attainment gap between the rich and the poor, both directly through child human capital accumulation and through the intergenerational propagation of education (the "better parent" effect). We demonstrate this point in the next section.

6 Results for Two Pure Policy Reforms

6.1 The Thought Experiment

For each transition thought experiment we assume that the economy is initially in steady state, that the policy reform triggering the transition is unexpected, and that the government is henceforth fully committed to the policy reform. Our benchmark reform is "Free College", a 100% subsidy of college tuition, financed by a permanent change in the labor income tax rate τ and time-varying government debt. That is, the tax parameter adjusts once and for all to ensure that the intertemporal government budget constraint is satisfied. In order to guarantee that the period-by-period budget constraint holds, government debt endogenously evolves along the transition from the initial steady state level towards the final steady state where the government debt to GDP ratio is constant. The corresponding "Better Schools" reform increases public (primary and secondary) school spending i^g permanently so that the extra expenditures have the same present discounted value as the "Free College" reform.

6.2 Transitional Dynamics: Aggregates, Distribution and Welfare

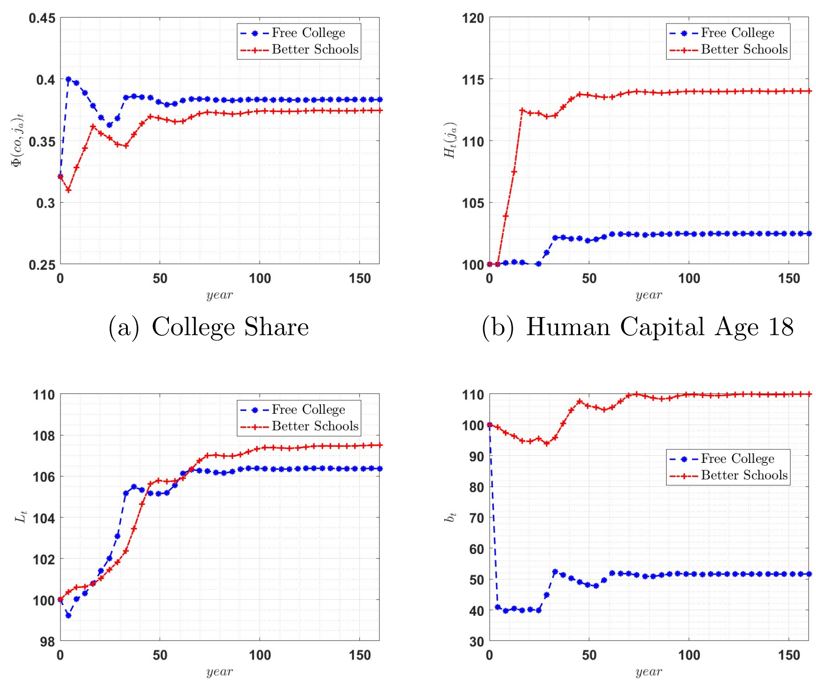
In this section we summarize the transition results from our two main policy exercises; Table 13 in Appendix A provides a summary of comparison of the initial and the final steady states to which the policy transitions converges.

[28]In our model, initial human capital differences are exogenous, and therefore we cannot speak to the sources that give rise to this initial heterogeneity.

6.2.1 Aggregate Effects

In Figure 2 we display the dynamics of the college share, aggregate labor in efficiency units (L_t), average human capital at age 18 and aggregate inter-vivos transfers over time. Panel (a) of Figure 2 demonstrates that both policies are successful in increasing the share of college graduates, although making college free does so to a larger extent in the short run. However, it also leads to many more college dropouts.[29] Furthermore, it takes time for the full impact of the reforms to take hold. Only after the third generation born after the reform has made the higher education decision and completed college (i.e., roughly 50 years after the policy change, see Figure 6, panel a, below) does the share of the population with a college degree reach its new, higher steady state level. This broad observation, which also holds true for the other aggregate variables depicted in Figure 2, reinforces the need to model transitions explicitly.

Figure 2: Aggregate Variables: General Equilibrium



(a) College Share (b) Human Capital Age 18

(c) Labor (d) IVT

Notes: Panel (a): Share of a given "birth" cohort that completes college; Panel (b): Average human capital of a given "birth" cohort at age 18 (Initial Steady State = 100); Panel (c): Aggregate labor efficiency units (Initial Steady State = 100); Panel (d): Aggregate Inter-vivos transfers (Initial Steady State = 100).

Panel (b) and (c) of Figure 2 indicate that the college expansion is achieved through very different channels in the two reforms. In the "free college" reform, as panel (b) of Figure 2

[29]There are also substantial differences in the underlying human capital and, thus, productivity distribution of the pool of new college students induced by each reforms. These differences will be discussed in Section 6.2.2 on the distributional impact of the reforms.

shows, there is only a very marginal positive human capital response within the lifespan of the first impacted generation (i.e. before the newly educated children become parents themselves). There are several conflicting forces at play that determine how parental incentives to invest in child human capital are affected by the generosity of the college subsidy. On the one hand, holding fixed the expected benefits from enrolling in college (in terms of graduation probability and earnings), the endogenous optimal human capital attendance threshold is decreasing in the college subsidy rate, which creates a disincentive for parents to invest in child pre-college human capital. On the other hand, the benefits from college (in terms of graduation probability and earnings) are increasing in the level of human capital and thus making college financially affordable creates an incentive for altruistic parents to increase their human capital investments in children so that the latter can take bigger advantage of attending college. Quantitatively, the positive and the negative investment incentive effects almost fully offset each other and acquired human capital increases only very marginally within the lifespan of one generation.

In the "better schools" reform there is a much more pronounced increase in child human capital accumulation, and some of the now better-schooled 18 year old teenagers choose to attend college when they used not to. Crucially, as panel (c) demonstrates, even those whose college attendance decisions are not affected by the reform now tend to have more human capital, and consequently are more productive in the labor market. Furthermore, since those attending college now have more human capital, under the "better schools" reform the college completion rate improves as well. Consequently, aggregate labor efficiency units rise more strongly under the "better schools" reform than under the "free college" reform.

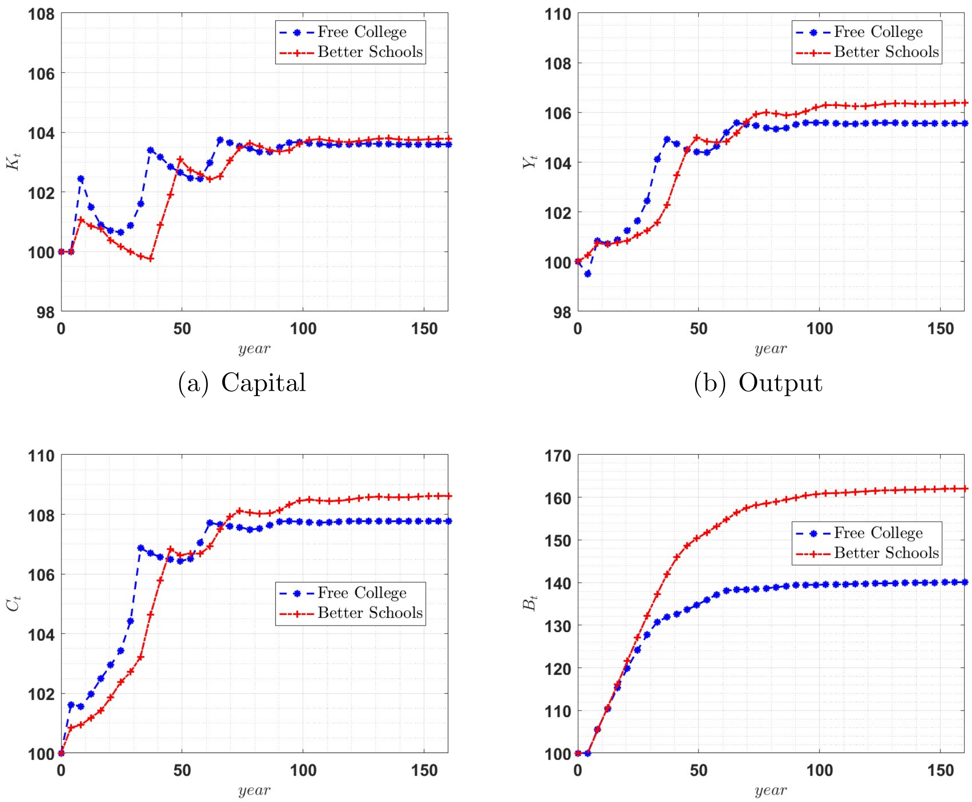
Finally, panel (d) demonstrates that private parental adjustments also significantly differ: when college is free, private inter-vivos transfers (which are mainly used for financing college tuition) collapse, which in turn reduces overall asset accumulation by parental generations.[30] The strong response in the college share, in aggregate labor as well as aggregate savings also anticipates the finding that accounting for general equilibrium factor price adjustments will have quantitatively very important aggregate, distributional and welfare consequences.[31]

Figure 3 displays the evolution of aggregate capital, output, consumption and government debt. Since capital is only mildly increasing along the transition, the time path of output roughly follows that of aggregate labor; the same is true for aggregate consumption. For the same reason, the tax base increases gradually with labor along the transition, whereas the education cost in both reforms rises immediately on impact. Therefore, the government accumulates debt along

[30] The importance of intergenerational transfers for (indivisible) human capital investment in the presence of credit market imperfections was already highlighted in Galor and Zeira (1993).

[31] The adjustments of parental resource and time investments are shown in Appendix A. The quantitative importance of parental investment adjustments is somewhat limited due to a relatively large relative weight on public schooling in the calibration of the human capital production function.

Figure 3: Aggregate Consumption, Production and Government Debt



(a) Capital (b) Output

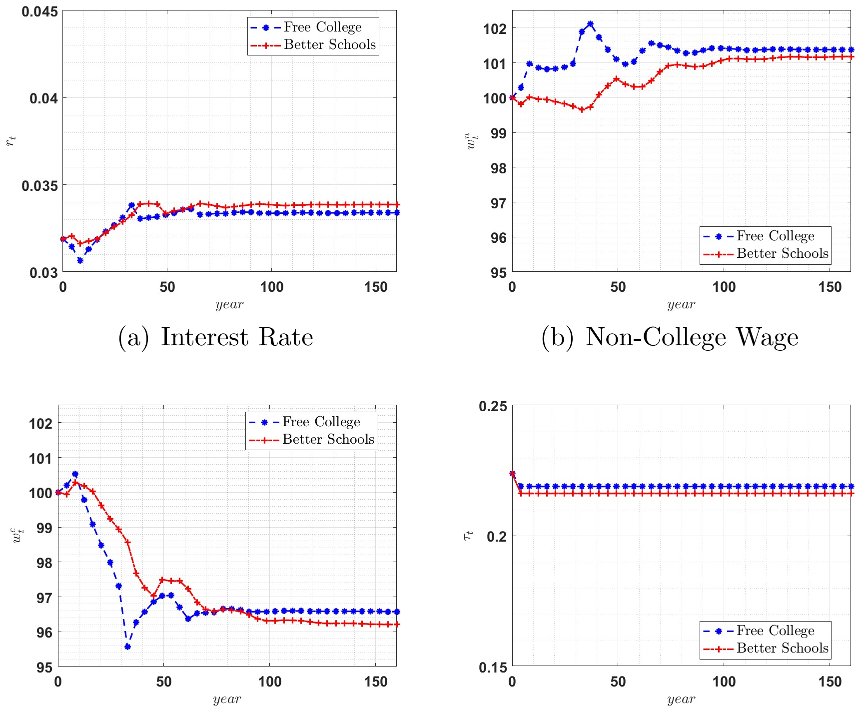
(c) Consumption (d) Debt

Notes: Panel (a): Capital; Panel (b): Output; Panel (c): Consumption; Panel (d): Government debt.

the transition, and since the "better schools" reform delivers a larger output in the long run, the capacity to service debt is more substantial in this reform. See panel (d) of Figure 3.

We cast our model in general equilibrium, and therefore interest rates, wages and taxes adjust along the transition path to ensure that the labor markets for college-educated labor, non-college labor and the assets markets clear. In Figure 4 we display the time paths of these equilibrium factor prices and taxes. We observe that on account of the increase in the aggregate labor input induced by both education reforms, the capital-labor ratio falls, the interest rate increases over time (see panel (a)), and wages per efficiency unit (not shown) fall over time. However, since college- and non-college labor are imperfect substitutes and non-college labor becomes scarcer relative to college labor, the college wage premium falls by eight percentage points under the "free college" reform, and seven percentage points under the "better schools" reform but wages of those without a college degree actually increase (see panel (b) of Figure 4). In contrast, those with a college degree see their absolute wages decline substantially (relative to the long-run balanced growth path, see panel (c)). Panel (d) shows the (once and for all) adjustment in the labor income tax rate τ required to ensure that the intertemporal government budget constraint holds. It demonstrates that both reforms generate *more* fiscal space in that the reforms are self-financing: the labor income tax rate falls, more so in the "better schools" reform.

Figure 4: Prices and Taxes in General Equilibrium



(a) Interest Rate

(b) Non-College Wage

(c) College Wage

(d) Labor Income Tax Rate τ

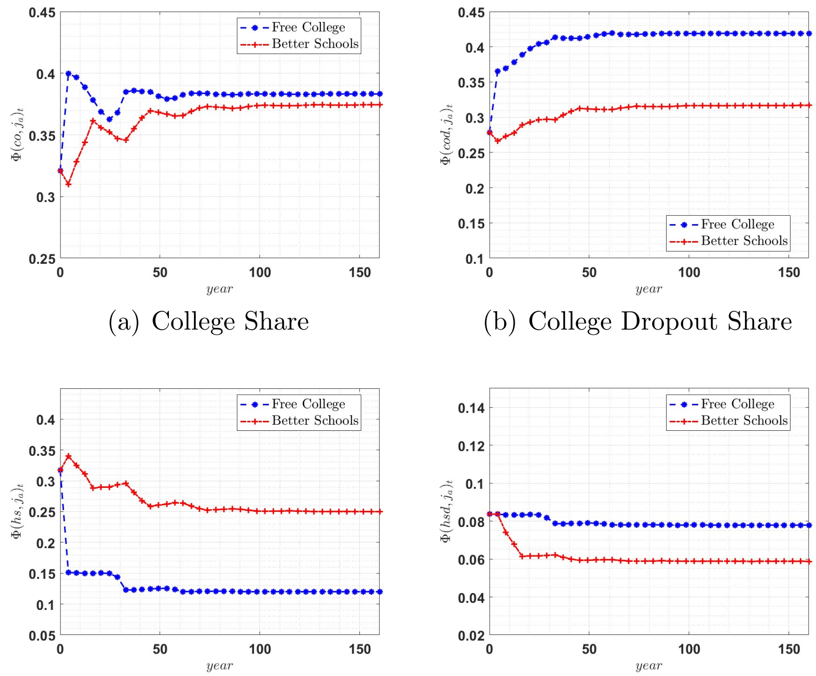
Notes: Panel (a): Interest rate; Panel (b): Non-college wage; Panel (c): College wage; Panel (d): Labor income tax level parameter τ.

6.2.2 Distributional Consequences

The two policy reforms also have substantially different distributional consequences. This is apparent from Figure 5 which complements panel (a) of Figure 2 and shows the evolution of the share of college dropouts in panel (a), those completing high-school in panel (b) and those with some, but not complete high school in panel (c). We highlight two key observations.

First, the "free college" reform does not change the share of children dropping out from high school by much (see panel (c)). In contrast, the "better schools" reform leads to a decline in the share of high-school dropouts in the population by 2.5 percentage points since the larger public investment into child human capital accumulation in school is only partially offset by lower private time and resource investments (see Appendix A.1) The improved human capital distribution at age 16 then results in a smaller share of the population dropping out of high school.

Figure 5: Higher Education Shares: General Equilibrium



(a) College Share

(b) College Dropout Share

(c) High School Share

(d) High School Dropout Share

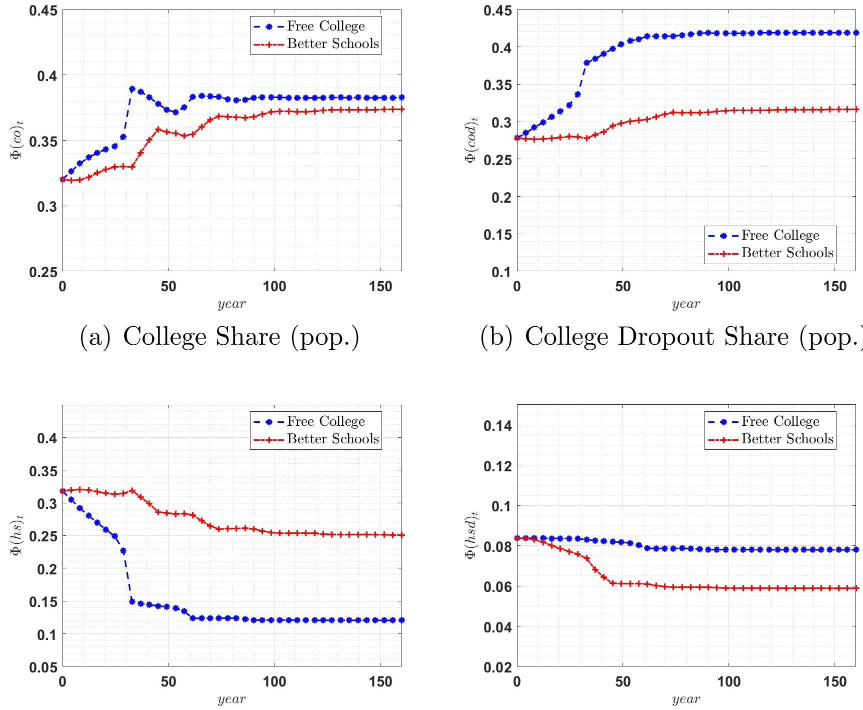
Notes: Panel (a): Share of a given "birth" cohort that completes college; Panel (b): Share of a given "birth" cohort that becomes a college dropout; Panel (c): Share of a given "birth" cohort that becomes a high school completer; Panel (d): Share of a given "birth" cohort that becomes a high school dropout.

Second, many of the additional students drawn to college under the "free college" reform do not graduate (since their human capital from high school is low and thus the chances of dropping out are high). In the long run (see Table 13 in the appendix), although the "free college" reform shifts 20 percentage points of previous high school graduates to college attendance, only about less than one third of these end up with a degree. In contrast, almost 60% of the new college attendees under the "better schools" reform (approximately 5 percentage points) graduate from college, suggesting that this reform uniformly shifts up the tertiary school attainment distribution and benefits all segments of the distribution in terms of labor-market relevant skills.[32]

Figure 6 shows the education *population* shares (as opposed to the education shares of a specific age cohort depicted in Figure 2). It displays a recurrent theme of this paper that the education reforms studied in this paper take time to materialize their full effect since the education expansion only directly impacts currently young generations that still have to go through school

[32]In partial equilibrium almost 80% of new college attendees obtain a degree under the "better schools reform".

Figure 6: Total Population Education Shares (General equilibrium)



(a) College Share (pop.) (b) College Dropout Share (pop.)

(c) High School Share (pop.) (d) High School Dropout Share (pop.)

Notes: Panel (a): Population share of college completers; Panel (b): Population share of college dropouts; Panel (c): Population share of high-school completers; Panel (d): Population share of high-school dropouts.

and/or make their higher education choices. Initially, this is a small share of the population, but over time these cohorts make an increasingly large share of the total labor force and thus the share of college-educated workers gradually increases (and that of individuals with only a high school degree declines). The extent to which this happens differs, of course, across the two reforms and is stronger for the "free college" thought experiment. In contrast, the "better schools" reform over time reduces by about a quarter the share of the population without even a high school degree, although this takes three generations, with no such effect from the "free college" reform.

6.2.3 Intergenerational Persistence

The policy reforms not only affect cross-sectional inequality, but also the intergenerational persistence of earnings and education. Table 6 displays one dimension of intergenerational mobility, showing how the earnings of children from different parental backgrounds (measured by parental education and marital status) change in response to the policy interventions. The first column shows the average lifetime earnings within a specific parental group, e.g., single parents without a high-school degree on average earn $21,297. The second column shows the average earnings of children for each parental group under the baseline policy, and the remaining two columns show the percentage change in these child earnings induced by the two policy reforms.

Table 6: Child Earnings % Change, by Parental Background

Parent Background	Baseline	FC, % Δ	BS, % Δ
$s^p = hsd$, $q = si$ (\$21,297)	\$37,358	9.72	10.01
$s^p = hs$, $q = si$ (\$37,153)	\$39,999	8.63	8.13
$s^p = cod$, $q = si$ (\$44,951)	\$44,472	7.84	5.77
$s^p = co$, $q = si$ (\$65,574)	\$59,187	-0.61	4.32
$s^p = hsd$, $q = cpl$ (\$27,043)	\$44,976	9.52	7.21
$s^p = hs$, $q = cpl$ (\$44,241)	\$47,799	7.19	5.05
$s^p = cod$, $q = cpl$ (\$59,484)	\$51,607	6.55	3.66
$s^p = co$, $q = cpl$ (\$88,968)	\$64,478	-0.17	3.62

Notes: Annual gross earnings % change relative to the baseline, averaged over the working life. In parentheses, own parental (annual, averaged over the working life) earnings are shown.

Table 6 shows that both reforms reduce the earnings gap between socio-economic groups. Interestingly, the reduction is larger in the "free college" reform than in the "better" schools reform because children from the highest socio-economic group overwhelmingly attend college even without it being free, and thus this reform does not induce earnings gains for this group. The "better schools" reform in contrast elevates the accumulation of (earnings-relevant) human capital of all children (including those at the very top and the very bottom). As a result, this reform "raises all boats" and the resulting reduction of child earnings inequality is less pronounced.

Table 7: Intergenerational Education Transition Matrix: Initial Steady State

Single Parents				
	$s = hsd$	$s = hs$	$s = cod$	$s = co$
$s^p = hsd$, $q = si$	0.14	0.75	0.05	0.06
$s^p = hs$, $q = si$	0.13	0.73	0.06	0.08
$s^p = cod$, $q = si$	0.10	0.62	0.13	0.15
$s^p = co$, $q = si$	0.07	0.02	0.43	0.48
Married Parents				
	$s = hsd$	$s = hs$	$s = cod$	$sof = co$
$s^p = hsd$, $q = cpl$	0.11	0.63	0.13	0.14
$s^p = hs$, $q = cpl$	0.09	0.51	0.19	0.20
$s^p = cod$, $q = cpl$	0.08	0.42	0.24	0.27
$s^p = co$, $q = cpl$	0.06	0.01	0.42	0.51

Notes: Intergenerational education mobility: rows show parental background, columns show conditional child education shares.

Finally, Tables 7 and 8 display the intergenerational transition matrix of educational attainment as well as the changes induced by the policy reforms, separately for single parents (on average the poorest families in the population) and for married parents with dual earners. Table 7 shows the strong intergenerational persistence of education in the benchmark economy: the share of children from parents without a high school degree going to college (and succeeding or dropping out) is only 13% for those with single parents and 28% for those with married parents. In contrast, this number rises to 92% for those with parents that have a college degree (roughly independent of marital status of the parent).

Table 8: Intergenerational Education Transition Matrix: Single Parents

	Better Schools				Free College			
	$s = hsd$	$s = hs$	$s = cod$	$s = co$	$s = hsd$	$s = hs$	$s = cod$	$s = co$
$s^p = hsd$, $q = si$	-0.04	-0.07	0.06	0.05	0.00	-0.32	0.21	0.10
$s^p = hs$, $q = si$	-0.03	-0.08	0.06	0.05	0.00	-0.34	0.23	0.10
$s^p = cod$, $q = si$	-0.03	-0.07	0.07	0.03	0.00	-0.35	0.26	0.09
$s^p = co$, $q = si$	-0.02	-0.01	-0.02	0.05	0.00	-0.01	0.01	0.00

Notes: Changes in intergenerational education mobility: rows show parental background, columns show conditional child education shares. Table entries show absolute changes in conditional child education shares relative to the benchmark economy. Positive percentage point changes relative to the baseline are marked in red, negative changes are marked in blue.

Table 8 summarizes one key dimension of the distributional consequences of the educational policy reforms, by showing how the intergenerational education transmission matrices for children with single mothers (that is, the share of children with maternal education $s^p \in \{hsd, hs, cod, co\}$ that end up with own education s) are affected by both policies (in the long run, comparing steady states).[33] The table highlights the different impact of both reforms on intergenerational persistence of education. A "free college" reform has virtually no impact on the share of children dropping out of high school (for any parental type). It is successful, however, in drawing a much larger share of those previously only completing high school into college, but close to 3/4 of these additional college goers end up dropping out of college (see the last two columns of the right panel of Table 8). Since for a majority of teenagers dropping out of college is ex-post inefficient (had they known they would not succeed, they would have opted not to attend college in the first place), the reform is not a very effective intervention of raising the share of the population with a college degree.[34]

In contrast, the "better schools" reform (left panel of Table 8) significantly reduces high-school dropout rates (see first column of the table), but is less effective in shifting previous high-school completers into attempting college. Conditional on going, however, the rise in the dropout rate is much less pronounced than in the free-college reform since teenagers under the "better schools"

[33]Note that the respective steady state is induced by the respective full policy transition. The corresponding results for married parents are contained in Appendix A.3.

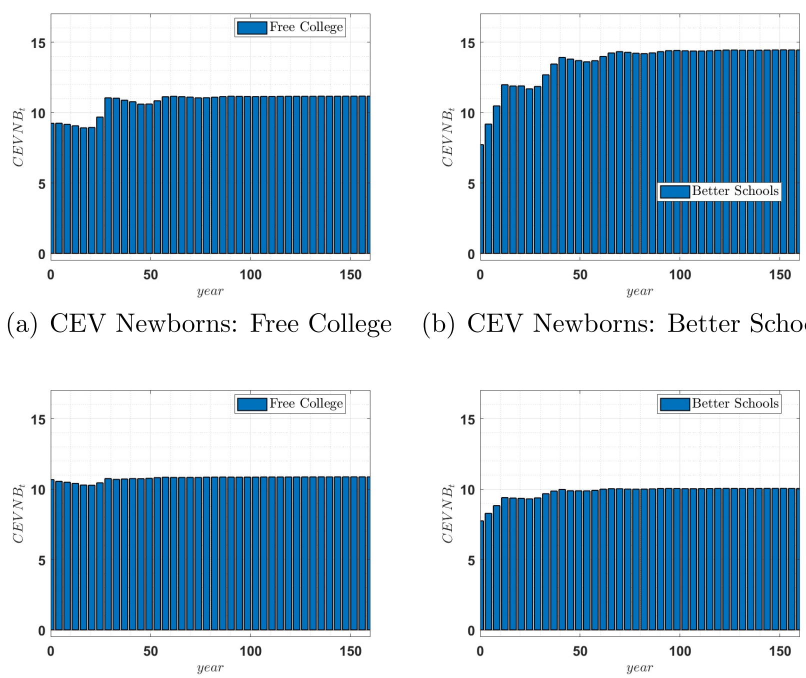
[34]In the baseline economy, 55% of all $s = cod$ households would have been better off ex post if they had not have enrolled in college. Under the "free college" reform this number falls to 52%.

reform are better prepared for college (i.e., have higher human capital which translates into lower dropout probabilities). The results from both reforms displayed here also suggest that a mixed reform that uses some of the budget to improve schools to make children more college ready and make it cheaper to attend could attain higher attendance without massively increasing dropout, and thus achieve the best of both worlds. We explore this possibility in Section 7 on the optimal (within a restricted policy set) policy reform.

6.2.4 The Welfare Consequences of the Reforms

Economically Newborn Individuals Figure 7 displays the welfare consequences of both policy reform transitions, measured as consumption equivalent variation of economically newborn individuals (i.e., based on expected lifetime utility at age 18), and plotted as a function of the period of the transition at which these individuals enter the economy (i.e., $t = 0$ means individuals becoming economically active in the first period of the transition).

Figure 7: Welfare Gains of Newborns



(a) CEV Newborns: Free College (b) CEV Newborns: Better Schools

(c) CEV NB: Free College, Φ_0 (d) CEV NB: Better Schools, Φ_0

Notes: Panel (a), (b): newborn CEV with actual period t cross-sectional distribution $\Phi_t(j_a, \cdot)$;
Panel (c), (d): newborn CEVs with initial steady state period 0 cross-sectional distribution $\Phi_0(j_a, \cdot)$.

Specifically, we ask what uniform[35] increase of consumption households born into the old steady state would require to be indifferent between the status quo steady state and to being born into the transition induced by the policy reform. The left panels are for the "free college" reform and the right panels are for the expansion of public school funding. In order to distinguish the welfare gains originating from newborns living a better life (i.e., having a larger value function for a given initial state (a, h, s_p) of a newborn and facing an improved distribution of potential marriage partners) from an improved (or worsened) distribution over these initial state variables (Φ_t vs. Φ_0), in the lower panels we display the welfare consequences that would emerge if the (endogenous) distribution over initial characteristics were to remain unchanged at Φ_0. Thus, the lower panel captures purely the welfare gains from higher lifetime utilities of the different types of economically newborn individuals, and the difference between the upper and the lower panels therefore reflects the welfare consequences of the endogenous and policy-induced shift in the distribution of the initial characteristics (human capital, financial wealth and parental education (a, h, s_p)) of the 18-year olds.[36]

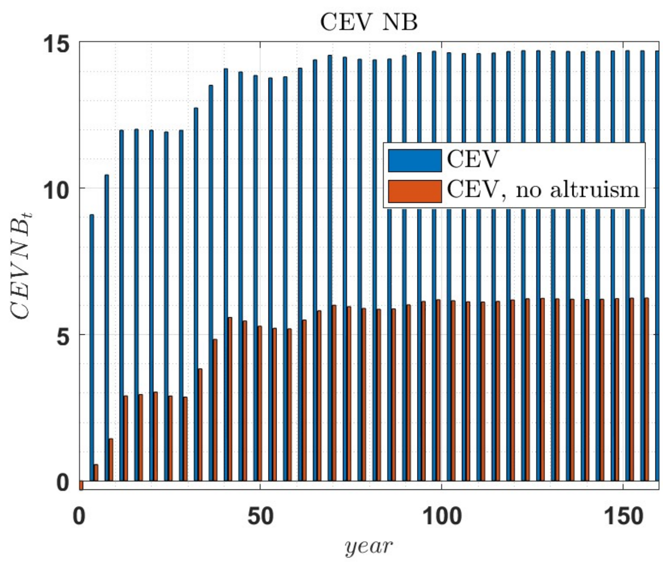
We highlight three qualitative points. First, both reforms entail substantial welfare gains for current newborns and future generations. In contrast to aggregate allocations, the welfare gains are fairly smooth across generations along the transition. The availability of government debt allows the government to smooth the short-run costs and use the long-run higher tax revenues from the education expansion to make the transition "painless" for newborns (and the majority of the currently alive). Second, as a comparison between the upper and the lower left panel reveals, the direct benefits of "free college" for 18 year-olds are partially offset by the fact that they enter adult life with fewer assets as their parents respond to the policy by adjusting inter-vivos transfers. That is, the actual welfare gains for the youth are smaller compared to a scenario where the policy is evaluated under a fixed initial distribution of wealth and human capital, at least early in the transition. Along the transition, the fact that young adults have better educated parents offsets this effect. Third, and in sharp contrast, a large part of the welfare gains with better school funding comes from an improved human capital and parental education distribution and these gains increase over time, as can be seen comparing the top and the bottom right panels. This reinforces the need for studying (debt financed) policy *transitions* when considering fundamental education reforms.

[35] Uniform across individual types —initial assets, human capital and parental education— across time and states of the world.

[36] It is understood that the lifetime utilities of newborns are also affected by changes in the marriage market distribution.

The magnitude of the welfare gains is large, in the order of 11-15% of permanent consumption.[37] It is important to note here that in our model with parental altruism a potentially large part of these welfare gains, especially for generations born early in the transition, may come from the better lives these generations expect their children (and grandchildren) to have later during the transition and in the new steady state.

Figure 8: Better Schools Reform, Welfare Gains with and without Altruism



Notes: Newborn CEV under "Better Schools" reform under two welfare metrics: accounting for intergenerational altruism (blue bars), and without accounting for lifetime utility of children (red bars).

Figure 8 shows, by birth cohort, the welfare gains from the "better schools" reform documented in the upper right panel of Figure 7 as well as those welfare gains in the absence of intergenerational altruism.[38] The key observation is that for early transition cohorts almost all of the welfare gains stem from better lives of their offspring, and even in the steady state this effects accounts for a large share (approximately 60%) of the overall welfare gains.

Intracohort Heterogeneity The welfare results thus far viewed economically newborn agents as monolithic block and abstracted from potentially important intra-cohort redistribution associated with the different policy reforms. In Table 9 we break down the welfare gains of children that were *biologically* born in the first period of the transition (and thus become economically active themselves in transition period $t = 5$), by the education level and marital status of the

[37] Daruich and Fernández (2024) document welfare consequences of fundamental reforms of the welfare system in a model with intergenerational links and altruism that are of similar magnitude.

[38] For the latter, we use the same decision rules as in the benchmark, that is, households behave as if they are altruistic, but welfare is evaluated with preferences (and thus a value function) that abstracts from altruism, i.e., sets $\nu = 0$. The results for the "free college reform are qualitatively similar.

parents. This captures precisely the heterogeneity between poor children (that is, children born into poor families) and rich children we studied in Tables 5 to 8), but here we focus on children directly after the reform (rather than those born in the new steady state).

Table 9: Newborn CEVs (age j_a) by Parental Background, Transition Period $t = 5$

	Better Schools				
	$s^p = hsd$ [t=1]	$s^p = hs$ [t=1]	$s^p = cod$ [t=1]	$s^p = co$ [t=1]	%p Δ Top/Bot
NB CEV [t=5], $q = si$	13.02	12.77	12.44	12.16	-0.86
NB CEV [t=5], $q = cpl$	12.35	11.92	11.82	11.61	-0.74
	Free College				
	$s^p = hsd$ [t=1]	$s^p = hs$ [t=1]	$s^p = cod$ [t=1]	$s^p = co$ [t=1]	%p Δ Top/Bot
NB CEV [t=5], $q = si$	11.87	11.81	11.71	8.87	-3.00
NB CEV [t=5], $q = cpl$	11.42	10.00	8.93	7.21	-4.21

Notes: The table shows newborn CEVs in transition period $t = 5$ by parental education and marital status. The last column shows the %p difference between welfare gains of children from $s^p = co$-families and $s^p = hsd$-families. Period 5 is the period in which children that were born to parents aged j_f in the first period of the reform $t = 1$ reach age 18, i.e. become economic newborns.

We observe that, consistent with the previous findings, both policy interventions engender significant welfare gains that are uniformly (across parental socioeconomic status) larger for the "Better Schools" reform than the "Free College" reform. What we want to stress here are the differences across education groups. To facilitate the discussion we not only report the welfare gains for all family backgrounds (marital status and four parental education levels), but in the last column of Table 9 the difference in welfare gains between the poorest and the richest children.

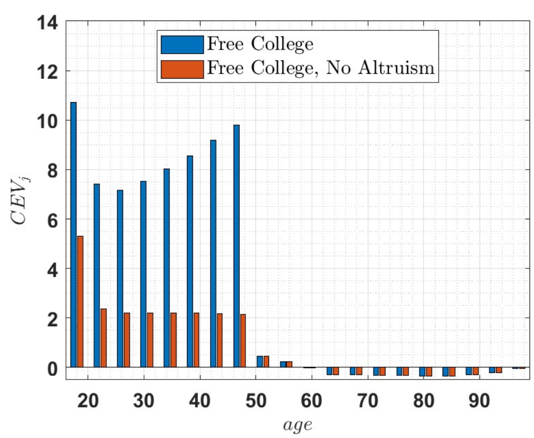
Broadly speaking, both policies are progressive in that they most strongly benefit children from poorer family backgrounds. Their pre-reforms acquired human capital in school and chances of attending and completing college are relatively worse compared to newborns from richer backgrounds prior to the reform and thus they have most to gain from the reforms. Furthermore, both reforms crowd out private time- and resource investments as well as inter-vivos transfers, and since the poorest parents do not much engage in either of these pre-reform, this crowding-out is least severe for them, explaining why the welfare gains are highest for the lowest parental education groups. Finally, general equilibrium wages of parents with a college degree are reduced relative to non-college wages under both reforms.

Comparing the progressivity across the two reforms, the welfare gains from better schools are more uniform across family background groups, whereas the free college reform shows distinctly less gains for those with college educated parents, and in this sense is more progressive. Thus, the "free college" reform reduces the lifetime utility gap between children of high school dropout parents and those with college educated parents by more than the "Better Schools" reform. This might seem surprising at first. Note, however, that in the model the economic returns to enrollment for marginal students (which tend to come from poorer families) are substantial;

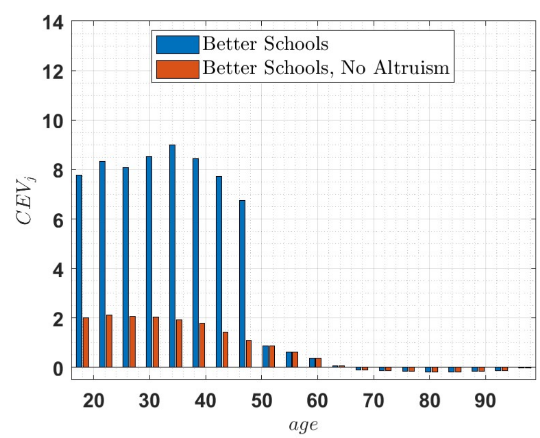
even college dropouts earn a 20% wage premium compared to high school graduates (based on the recent 5 waves of PSID). In contrast, at the top of the parental socioeconomic ladder the majority of children are infra-marginal students (they go to college no matter when), and for them the college subsidy is a pure cash transfer which parents partially offset by reducing endogenous parental inter-vivos transfers. The "Better Schools" reform in contrast raises all boats, and it is important to note that even for poor children this reform generates larger absolute welfare gains than the free college reform.

Existing Generations Of course, welfare gains for economically newborn agents might partially come at the cost of welfare losses for existing (at the time of the policy reform) generations that do not benefit directly from the reforms since their human capital accumulation and tertiary education decisions lie in the past. However, since these generations are altruistically motivated toward their children, these generations might benefit indirectly through higher expected lifetime utility of their offspring. They are also affected by GE price and government budget adjustments.

Figure 9: Welfare Gains of Currently Alive Population



(a) CEV Currently Alive Population: "Free College"



(b) CEV Currently Alive Population: "Better Schools"

Notes: CEV of currently alive population by age (blue bars show the full welfare metric accounting for intergenerational altruism, red bars show the welfare outcomes without accounting for lifetime utility of children).

In Figure 9 we summarize the welfare consequences (again measured in terms of consumption equivalent variation) for these generations (by their age, and again averaged over their relevant state variables). For both reforms we additionally show the welfare consequences when ignoring intergenerational altruism. We observe that younger generations with children still in the household also gain, mostly on account of the higher lifetime utility of their children, or the children they expect to have, in case of individuals that are younger than 26. This latter group also benefits from the fact that with some probability they will marry, and under the reform will marry someone with higher education level (than in the benchmark) and thus higher earnings which they get to share.

Older generations (those age 46 and older whose college education is completed and whose children have left the households) have smaller welfare gains, and if they are retired, might actually suffer welfare losses. This is due to general equilibrium effects: when wages fall, so do benefits from the PAYGO social security system, which offset (and for older households, dominate) the mild increase in asset returns and (if still in working age) the reduction in the labor income tax rate. Note, however, that these welfare losses are relatively mild. Thus, although neither reform constitutes a Pareto improvement (since the current old lose), it is conceivable that there is room for additional fiscal redistribution so as to avoid the welfare losses for generations older than 46 at the time of the reform that Figure 9 documents.

6.3 Discussion of Key Modeling Elements

6.3.1 Importance of General Equilibrium

To isolate the importance of changes in endogenous interest rates and (relative) wages we also conduct a sequence of partial equilibrium exercises in which we hold these endogenous prices as well as the taxes required to balance the intertemporal government budget constant. As a summary, we show that qualitatively, the aggregate and to a large degree the distributional conclusions discussed above also emerge in the absence of equilibrium price adjustments. However, endogenous interest and (relative) wage adjustments in general equilibrium make the welfare gains for newborn generations smaller (relative to partial equilibrium) and reduce the difference between the two reforms.

The most important general equilibrium effects stem from the fact that inflow of more college-educated workers into the labor market (induced by the education reforms) and their higher human capital lowers both the capital-labor ratio and the college wage premium, in turn muting the increase in the college share in general equilibrium relative to partial equilibrium. The decline in the capital-labor ratio puts downward pressure on all wages (which hurts workers) but raises the interest rate. The *relative* wage effect, which provides welcome (from the perspective of ex-ante utility) redistribution across education types, is stronger in the school expenditure expansion reform since college enrollment decisions are more sensitive to the college wage premium in that thought experiment.

In addition, an increase in the interest rate induced by the reduction of the capital-labor ratio[39] in general equilibrium mutes the crowding-out effect of the "free college" reform on inter-vivos transfers and results in a larger crowding-in under the school expenditure reform.

In Figure 10 we display the college share, human capital at age 18, aggregate efficiency units of labor as well as inter vivos transfers for the "free college" reform for five scenarios that are summarized in Table 10. Observe that the only difference between the last scenario "PE+$r+w^n+w^c$" and the benchmark "GE" are the level of labor income tax, the social security contribution rate and transfers from accidental bequests. These thought experiments seek to isolate the importance of changes in wages and changes in the real interest rate induced by the "free college" reform. Figure 11 does the same for the "Better Schools" reform.

Table 10: GE Decomposition Scenarios

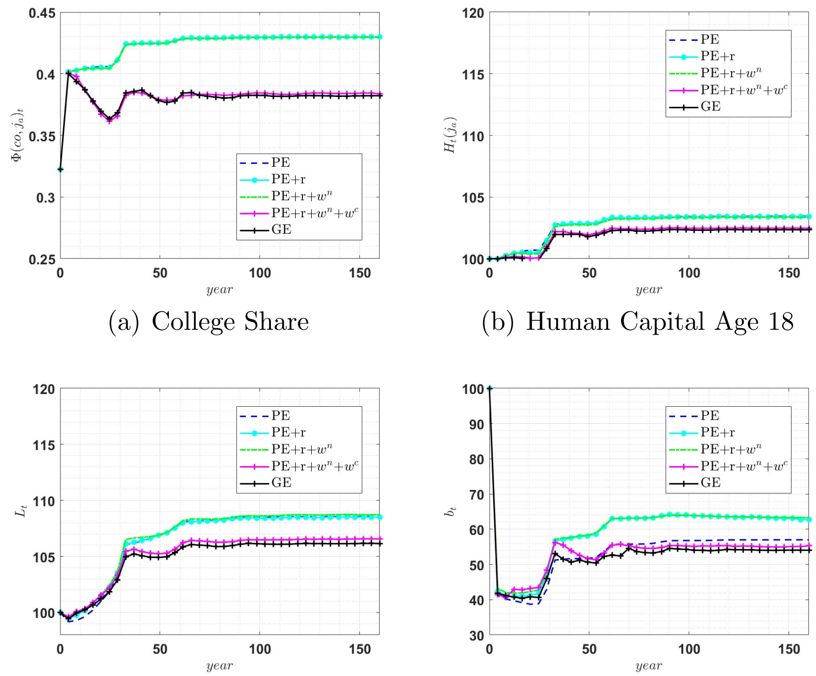
Scenario	Description
1. GE	*General equilibrium*: benchmark
2. PE	*Partial equilibrium*: prices (wages and interest rate) constant, level of labor income taxation adjusts (intertemporal government budget constraint holds)
3. PE+r	*Partial equilibrium and interest rate path from GE*: same as PE, but feed in the GE path of r_t
4. PE+$r+w^n$	*Partial equilibrium, and interest rate and non-college wage paths from GE*: same as PE+r, but feed in the GE path of $w(s<co)_t$
5. PE+$r+w^n+w^c$	*Partial equilibrium, and interest rate and wages paths from GE*: same as PE+$r+w^n$, but feed in the GE path of $w(co)_t$

Very broadly, and with some nuance for a subset of the variables, the key general equilibrium effect comes from the endogenous adjustment of college-educated labor. This is apparent from the upper left panel of the figure. The partial equilibrium response of the college share is large and increasing over time, and adjustments in the interest rates do not change that finding (as the three lines are virtually on top of each other). However, when the college wage adjusts downward (as it does in GE), after an initial massive increase this share falls and settles down at a somewhat higher level relative to the pre-reform scenario. The general equilibrium wage effects are also important for the intergenerational persistence of education: as Table 18 in Appendix A.4 shows, in partial equilibrium the "better schools" reform is actually quite successful drawing poor children with single mothers into college, but the decline in the college wage premium in general equilibrium largely offsets this effect (as we have shown above).

The (relatively small) change in the tax rate does not much affect this conclusion that it is the decline in the college wage (and thus the college wage premium) in general equilibrium that

[39]The reduction of the capital-labor ration is in turn due to the increase in effective labor as well the reduced savings incentives for privately funded education expenditures and inter-vivos transfers and the shift from capital to government debt.

Figure 10: Free College Reform, Aggregate Variables: GE Decomposition



(a) College Share (b) Human Capital Age 18

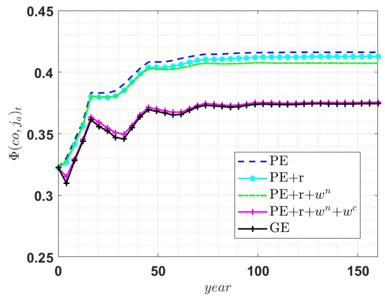
(c) Labor (d) IVT

Notes: Panel (a): Share of a given "birth" cohort that completes college; Panel (b): Average human capital of a given "birth" cohort at age 18 (Initial Steady State = 100); Panel (c): Aggregate labor efficiency units (Initial Steady State = 100); Panel (d): Aggregate Inter-vivos transfers (Initial Steady State = 100).

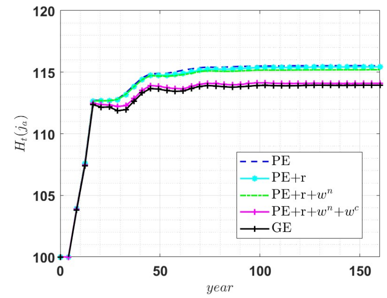
enacts the largest impact on the college share. This is also the driving force behind the smaller welfare gains of both reforms in general relative to partial equilibrium. Figures 14 and 15 as well as Table 13 in the Appendix show that, as a result of these general price movements, the welfare gains are 2.2-2.7 percentage points smaller, and the *difference* between the two reforms is also lower by 0.5 percentage points.

The endogenous interest rate increase (see again panel (a) of Figure 4) has only a minor effect on the college share, as the difference between the PE and the "PE+r" line in panel (a) is negligible. In contrast, the rise in the interest rate is more important for the intergenerational transmission of wealth, as panels (d) of Figures 10 and 11 display. Comparing the blue "PE" line with the cyan "PE+r" line shows that the increase in the interest rate in general equilibrium mitigates (but not fully offsets) the decline in inter-vivos transfers that the education reforms, especially the "free college" reform would otherwise have induced.

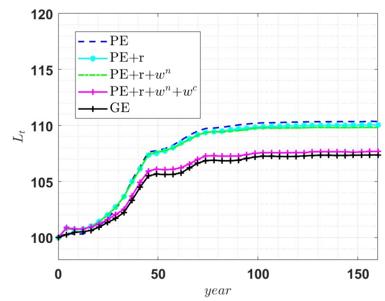
Figure 11: Better Schools Reform, Aggregate Variables: GE Decomposition



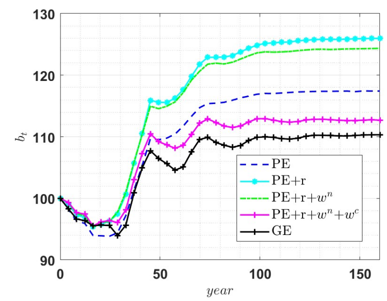
(a) College Share



(b) Human Capital Age 18



(c) Labor



(d) IVT

Notes: Panel (a): Share of a given "birth" cohort that completes college; Panel (b): Average human capital of a given "birth" cohort at age 18 (Initial Steady State = 100); Panel (c): Aggregate labor efficiency units (Initial Steady State = 100); Panel (d) Aggregate Inter-vivos transfers (Initial Steady State = 100).

6.3.2 Importance of Government Debt

We have so far documented that both reforms are (more than) budget-neutral in an intertemporal sense, but required quite significant temporary deficits and thus an increase in government debt (relative to GDP) along the transition. In Appendix A.5 we analyze the quantitative importance of access to additional government debt, by considering a world in which the reform has to be financed period by period by tax changes that ensure that the debt-to-GDP ratio in every transition period stays at its initial steady state level. This assumption on the government budget results in sizable tax increases in the short run (see upper right panel of Figure 17). Without access to additional government debt the education reforms create recessions in the short run, in that labor input, GDP and aggregate consumption fall on impact on account of the reform-induced tax increases (see Figure 17) and the welfare gains for existing and future transitional generations are substantially smaller. Interestingly, if government debt is forced to be constant, relative to GDP, in the long run the economy has less debt and lower taxes than in the benchmark economy, and consequently the newborn steady state welfare gains from the education reform are actually larger in the absence of a reform induced debt expansion (see lower left panel of Figure 17 in the Appendix).

7 Optimal Policy

Thus far, we have considered two "pure" policy reforms in isolation and have seen that both generated significant welfare gains. We have also documented that especially for children from poor families college remains largely out of reach since under a "free college" reform the low human capital acquired in primary and secondary school translates into college failure rates that are so high to make college unattractive even if free. The pure "better schools" reform, in contrast, raises human capital of the entire population, including poor children, but without some college tuition subsidies going to college remains too expensive for the poorest children.

This raises the question whether a combination of both reforms raises human capital of these children sufficiently to make college success feasible while reducing the cost to make it affordable. As transparently shown in Guerrieri (2024), the two policy instruments are relative welfare substitutes and relative welfare complements at the same time. On the one hand, both instruments decrease the human capital threshold it takes to attend college. On the other hand, increasing human capital through increased school spending increases relative wages of college youngsters and reduces their dropout probabilities, and therefore a policy reform of increased school financing may complement a policy reform of reduced college fees. Due to this dual role of the policy instruments it is a quantitative question whether our analysis of joint policies delivers a corner solution such that the "better schools" policy reform is the preferred policy choice, or an interior solution with an optimal mix of both instruments.[40]

We address this quantitative question in this section by holding constant the total cost of the reform at the level in the previous section. Specifically, let Ω be the share of the total additional government education expenditures to be allocated to "better school" financing and $1-\Omega$ be the share devoted to college tuition subsidies. The reforms in the previous section correspond to $\Omega = 0$ ("free college") and $\Omega = 1$ ("better schools). We now seek to find the $\Omega^* \in [0,1]$ that maximizes social welfare $\mathcal{W}(\Omega)$.

7.1 Measurement of Social Welfare

In our life cycle economy with current and future generations and heterogeneity within generations the choice of the social welfare function $\mathcal{W}(\Omega)$ is not obvious. We will consider two alternatives, chosen again to highlight the potential distinction and conflict between the policy impact in the short- and in the long run. Our long-run welfare measure is the expected lifetime utility of economic newborns in the steady state, where as before expectations are taken under the veil of

[40] This also reemphasizes the crucial role of our model validation in Section 5 showing that the results of our calibrated model are consistent with empirical estimates of both instruments in raising college participation.

ignorance, that is, before the initial state (a, h, s_p) is realized. This was the welfare measure the discussion in the previous section has focused on.

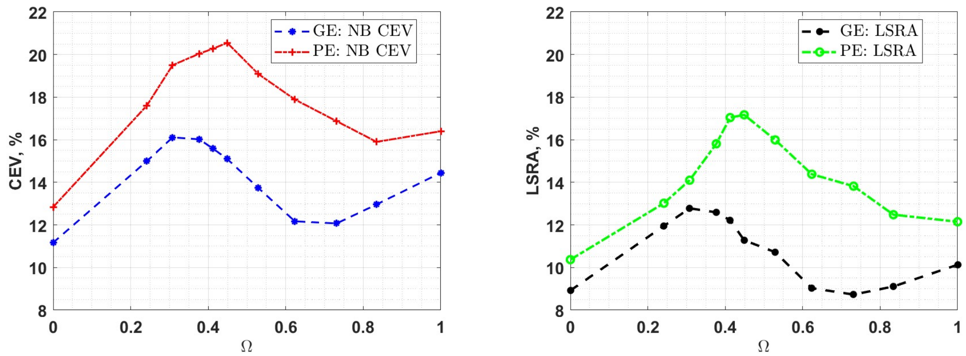
The second alternative that captures the welfare consequences both for generations living through the transition and those born into the new steady state, is based on the lump-sum redistribution authority concept (LSRA) initially introduced by Auerbach and Kotlikoff (1987). Specifically, for each individual currently alive we compute the (possibly negative) wealth transfer required to make the individual indifferent between the status quo and a specific reform. We do the same for all newborn generations along the transition path and in the new steady state. We then aggregate these transfers using the populations shares in the initial steady state (for individuals already alive) and the relative size of newborn agents (relative to the population in the initial steady state), and discount transfers along the transition using the equilibrium interest rates of the "free college" reform (so that the discounting is the same for all policy reforms considered). Finally, for comparison with other welfare measure we translate the aggregate wealth transfer into a flow consumption measure. For details how the LSRA welfare measure is constructed, see Appendix B.4.

7.2 The Optimal Policy Mix

In Figure 12 we plot the welfare gains, measured as CEV of newborns in the steady state and (flow-consumption based) LSRA as a function of the policy weight Ω, where again $\Omega = 0$ is the "free college" reform and $\Omega = 1$ corresponds to the "better schools" reforms discussed in the previous section. We do so for the general equilibrium benchmark model and, in order to interpret results, for the partial equilibrium version of the model.

Although the precise optimum Ω^* evidently depends on the adopted welfare criterion, three robust findings emerge. First, and consistent with the previous section, for all weights Ω the welfare gains of the policy reforms are quite sizable (in the order of above 10% of lifetime consumption) and larger in partial equilibrium than in general equilibrium. Second, the CEV welfare measure that focuses on steady states shows larger gains than the ones that includes transitional generations (the LSRA measure), again confirming that the full welfare benefits of the reforms take time to materialize. Finally, and most importantly for the purpose of this section, the highest welfare is attained in the interior of the policy space and spends ca. $1/3$ of its budget on better schools and the remainder on subsidizing the cost of college. Thus, while aggregate welfare is higher under the pure "better schools" reform than under the "free college" reform, it takes a larger spending share on college than on schools in the optimal mix. The reason is the

Figure 12: Joint Reforms in GE and PE: Welfare



(a) Newborn CEV (final steady-state): GE and PE

(b) LSRA: GE and PE

Notes: Ω (on the x-axis) denotes the relative weight on the i^g reform.

discrete nature of the college decision: it takes a sufficiently large college reform to move many individuals to complete college and to thereby effectively complement the schooling reform.[41]

As Table 11 shows, although all three reforms are successful in reducing the dispersion of wages and the poverty rate in the population (in the long run), relative to both extreme reforms the optimal policy mix leads to a larger reduction in the poverty rate, and has a similar effect on the variance of log wages as the better schools reform. The same is true for the intergenerational persistence of poverty, as the last column of Table 11 shows.

Table 11: Distributional Statistics: Benchmark Model and Policy Reforms

	Variance of Log Wages	Poverty Rate	Prob. of Stay in Q1
Baseline	0.4419	14.31%	21.54%
Free College	0.4326	12.82%	16.51%
Better Schools	0.4331	12.11%	15.59%
Optimal Policy	0.4381	11.54%	14.86%

Notes: The variance of log wages is computed on the sample of the entire working age population, i.e. ages 22-65. The poverty rate is computed based on the OECD poverty definition according to which the poverty threshold is set at 50% of median household disposable income, adjusted for the household size using the OECD household equivalence scale (0.5 for an additional adult, 0.3 for a child). The probability of staying in the lowest quintile of the income distribution is computed based on gross total household incomes.

[41]Smaller spending on colleges instead mainly constitutes a consumption transfer to individuals that already attend college. It is then more efficient to spend this money on schools. This mechanism also explains the non-monotonicity in Figure 12 of the gains in the share parameter Ω.

Table 12 displays the welfare gains from the age 18 perspective for children from different family backgrounds that were biologically born in the first transition period and whose initial conditions were, therefore, entirely determined under the new policy. The table is the counterpart of Table 9 for one-dimensional reforms and documents similar qualitative patterns of welfare benefits: they are progressive with respect to family background (which was determined in the initial steady state and thus not affected by the policy). Although children from all family backgrounds benefit more from the optimal policy mix than from either of the two univariate reforms alone, those born into the very bottom family backgrounds (i.e. with single mothers that dropped out of high school) benefit the most from the mixed reform relative to the one-dimensional interventions. At the heart of this result is that one policy draws more teenagers into college, and the other insures that these additional students are well-enough prepared so that dropout rates are kept in check.

Table 12: Newborn CEVs (age j_a) by Parental Background, Transition Period $t = 5$

	Optimal Mix				
	$s^p = hsd$ [t=1]	$s^p = hs$ [t=1]	$s^p = cod$ [t=1]	$s^p = co$ [t=1]	%p Δ Top/Bot
NB CEV [t=5], $q = si$	14.58	14.48	13.94	13.01	-1.57
NB CEV [t=5], $q = cpl$	13.68	12.53	12.09	11.85	-1.83

Notes: The table shows newborn CEVs in transition period $t = 5$ by parental education. The last column shows the %p difference between welfare gains of children from $s^p = co$-families and $s^p = hsd$-families. Period 5 is the period in which children that were born to parents aged j_f in the first period of the reform $t = 1$ reach age 18, i.e. become economic newborns.

8 Conclusion

In this paper we have studied the optimal combination of college tuition subsidies and school financing, for a given pre-specified budget for these reforms. To do so, we have developed a quantitative modeling framework that allows for a joint analysis of transitions induced by policies concerned with pre-labor market entry pre-distribution and post-labor market entry redistribution. Using this model we have evaluated the aggregate, distributional and welfare consequences of these reforms targeted at different stages of childhood and adolescence. We found that although individual reforms generate very significant welfare gains, a combination of both is most effective, in a welfare sense, of both curbing high-school dropout rates and encouraging college attendance without an overly large increase in college dropout rates. In absolute welfare terms, children born into the "bottom" parental backgrounds, i.e. those with single mothers without high school degree, benefit most from the optimal mix reform as compared to "free college" or "better schools" reforms alone.

Our analysis held the overall size of the education reforms constant. An analysis of the optimal *size* of the reform(s), both in the presence and the absence of a period-by-period budget balance assumption, would be informative about the importance of the "fiscal space" for the success of education reform. More broadly, we have taken the remainder of the fiscal constitution, that is, the tax-transfer system as given and invariant to the education policy reform. However, especially changes in the welfare system making transfers to the poor (which they might be used for additional education investments by the impacted families) could provide an alternative mobility enhancing policy. A quantitative analysis of a more comprehensive reform of the entire tax-transfer-education financing system, with specific focus on the performance of children of single mothers that constitute more than 20% of the current US child population[42] and are subject to particularly low upward mobility levels, would be especially relevant in this context.

Finally, our paper focused on the demand side for education, assuming the expansion of the supply of college seats and high school instruction can proceed at constant returns to scale. For the nation-wide education reforms considered this might be too optimistic an assumption, and a complete analysis would have to model the somewhat inelastic supply of teachers, professors and classrooms (as well as the incentives of educational institutions expanding their supplies) explicitly. We defer this to ongoing and future work.

[42]See e.g. the 2019 Pew Research Center study on "Religion and Living Arrangements Around the World"

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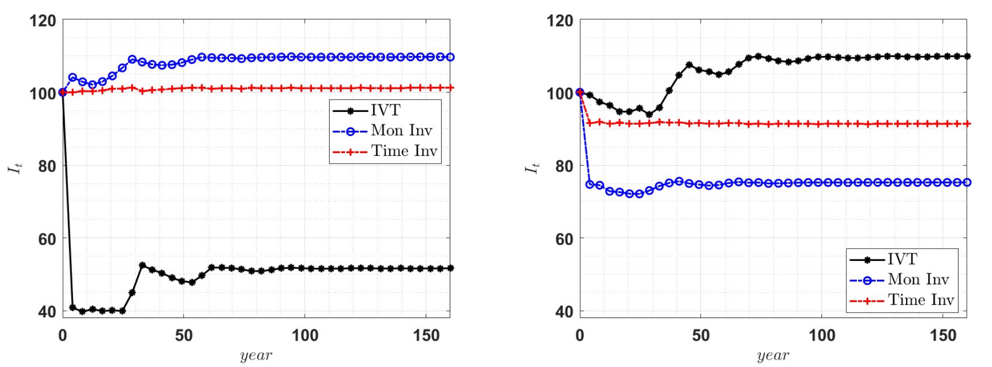
A Additional Quantitative Results

A.1 Parental Investment Responses: "Free College" and "Better Schools"

The optimality condition linking parental resource and time input choices is derived in Appendix B. The plots below show the average per child parental inputs in terms of money, time as well as inter-vivos transfers for the two main reform scenarios. These are aggregate parental input adjustments in the education reform induced transitions. In both reform scenarios, resource investment in children responds stronger in the aggregate than the time investment. From the optimality condition linking monetary and time inputs, $\frac{i_m}{i_t^{1+\psi}}$ is a decreasing function of the marginal utility of consumption. This relation at the individual level, aggregated up, moves in the same direction as the aggregate consumption. Given the optimal consumption responses, the ratio $\frac{i_m}{i_t^{1+\psi}}$ moves up in the aggregate, but the ratio $\frac{i_m}{i_t}$ moves slightly down, i.e. the positive consumption response would need to be even stronger to make the $\frac{i_m}{i_t}$ ratio move upward.

Figure 13: Parental Investments



(a) Interg. Investments: Free College

(b) Interg. Investments: Better Schools

Notes: Intergenerational transfers under "Free College" (panel (a)) and "Better Schools" (panel (b)): inter-vivos transfers, monetary and time human capital investments in children.

A.2 Comparison of Steady States: Summary Tables

Table 13: Aggregates, Prices, Taxes and Welfare: Univariate Reforms

Variable	Initial SS	Δ GE FC	Δ PE FC	Δ GE BS	Δ PE BS
$\Phi(j_a, s = co)$	32.23	5.97	10.75	5.22	9.37
$\Phi(j_a, s = cod)$	28.01	13.98	10.15	3.70	2.90
$\Phi(j_a, s = hs)$	31.41	-19.40	-20.15	-6.44	-9.56
$\Phi(j_a, s = hsd)$	8.35	-0.56	-0.75	-2.48	-2.70
HK	1.00	2.34	3.43	13.95	15.51
L	8.91	6.19	8.49	7.24	10.30
Hours	0.28	0.85	1.15	1.96	2.89
C	7.76	7.53	9.22	8.40	10.13
K	12.57	2.88	-1.32	2.80	-11.52
B	3.50	32.78	32.84	57.04	82.59
Revenues	2.56	10.55	11.38	12.86	12.86
Y	13.29	5.19	5.18	5.87	2.62
r	3.19	0.16	0.00	0.19	0.00
w	0.74	-0.93	0.00	-1.30	0.00
$\frac{w^c}{w^n}$	75.55	-8.32	0.00	-7.49	0.00
w^n	0.97	1.36	0.00	0.75	0.00
w^c	1.04	-3.44	0.00	-3.55	0.00
τ	0.22	-2.20	-4.34	-2.93	-6.33
τ^p	12.40	0.06	-0.04	-0.16	-0.41
$\frac{T(AE_0)}{AE_0}$	0.19	-0.51	-1.01	-0.68	-1.47
CEV NB	0.00	11.17	13.39	14.68	17.40
CEV alive	0.00%	4.49	5.26	4.34	5.58
LSRA	0.00%	9.24	11.01	10.13	13.74

Notes: The table summarizes long-run changes (induced by policy transitions) in the main aggregate variables, general equilibrium prices and taxes, and welfare. Changes for baseline variables initially stated in percentages are expressed in percentage points (%p), while changes for baseline variables initially in levels are shown as percentages (%).
GE refers to the full general equilibrium version of the model where wages and interest rate are endogenous as well as the government budget constraints are balanced. The PE version refers to a life cycle model with balanced government budgets, with aggregate physical capital being computed as: $K_t = A_t - B_t$, where A_t are total household assets, and B_t is debt stock (which is endogenous and results from re-balancing the intertemporal government budget constraint).

Table 14: Aggregates, Prices, Taxes and Welfare: Free College

Variable	Initial SS	ΔPE	ΔPE+r	ΔPE+$r + w^n$	ΔPE+$r + w^n + w^c$	ΔGE
$\Phi(j_a, s = co)$	32.23	10.75	10.74	10.72	6.13	5.97
$\Phi(j_a, s = cod)$	28.01	10.15	10.17	10.17	13.94	13.98
$\Phi(j_a, s = hs)$	31.41	-20.15	-20.16	-20.15	-19.48	-19.40
$\Phi(j_a, s = hsd)$	8.35	-0.75	-0.74	-0.74	-0.59	-0.56
HK	1.00	3.43	3.39	3.35	2.48	2.34
L	8.91	8.49	8.48	8.69	6.52	6.19
Hours	0.28	1.15	1.15	1.61	1.43	0.85
C	7.76	9.22	10.91	11.33	8.56	7.53
K	12.57	-1.32	7.10	7.42	4.04	2.88
B	3.50	32.84	32.84	32.84	32.84	32.78
Revenues	2.56	11.38	14.78	15.75	9.23	10.55
Y	13.29	5.18	8.05	8.30	5.80	5.19
r	3.19	0.00	0.16	0.16	0.16	0.16
w	0.74	0.00	0.00	0.00	-0.93	-0.93
$\frac{w^c}{w^n}$	75.55	0.00	0.00	0.00	-8.32	-8.32
w^n	0.97	0.00	0.00	1.36	1.36	1.36
w^c	1.04	0.00	0.00	0.00	-3.44	-3.44
τ	0.22	-4.34	-4.34	-4.34	-4.34	-2.20
τ^p	12.40	-0.04	-0.04	-0.04	-0.04	0.06
$\frac{T(AE_0)}{AE_0}$	0.19	-1.01	-1.01	-1.01	-1.01	-0.51
CEV NB	0.00	13.39	15.46	16.27	12.97	11.17
CEV alive	0.00%	5.26	5.90	6.15	5.35	4.49
LSRA	0.00%	11.01	12.88	14.08	10.61	9.24

Notes: The table summarizes long-run changes (induced by policy transitions) in the main aggregate variables, general equilibrium prices and taxes, and welfare. Changes for baseline variables initially stated in percentages are expressed in percentage points (%p), while changes for baseline variables initially in levels are shown as percentages (%).

GE refers to the full general equilibrium version of the model where wages and interest rate are endogenous as well as the government budget constraints are balanced. The PE version refers to a life cycle model with balanced government budgets, with aggregate physical capital being computed as: $K_t = A_t - B_t$, where A_t are total household assets, and B_t is debt stock (which is endogenous and results from re-balancing the intertemporal government budget constraint).

PE+r refers to the PE version where the interest rate path from the GE version is exogenously imposed - without any adjustments of other outer-loop variables. Therefore, the government budget constraints do not have to hold, and therefore the debt values (as well as the implied physical capital $K_t = A_t - B_t$ and thus also output are not shown, and marked as n/a.

PE+$r + w^n + w^c$ refers to the PE version where the interest rate and wages paths from the GE version are exogenously imposed - without any adjustments of other outer-loop variables.

Table 15: Aggregates, Prices, Taxes and Welfare: Better Schools

Variable	Initial SS	ΔPE	ΔPE+r	ΔPE+$r+w^n$	ΔPE+$r+w^n+w^c$	ΔGE
$\Phi(j_a, s = co)$	32.23	9.37	9.02	8.49	5.32	5.22
$\Phi(j_a, s = cod)$	28.01	2.90	2.52	2.47	3.72	3.70
$\Phi(j_a, s = hs)$	31.41	-9.56	-8.86	-8.31	-6.54	-6.44
$\Phi(j_a, s = hsd)$	8.35	-2.70	-2.68	-2.65	-2.50	-2.48
HK	1.02	15.51	15.42	15.19	14.11	13.95
L	8.91	10.30	10.00	9.77	7.69	7.24
Hours	0.28	2.89	2.85	3.08	2.64	1.96
C	7.76	10.13	12.76	12.95	9.87	8.40
K	12.57	-11.52	2.17	2.94	-1.97	2.80
B	3.50	82.59	82.59	82.59	82.59	57.04
Revenues	2.56	12.86	18.12	18.49	11.23	12.86
Y	13.29	2.62	7.41	7.55	4.51	5.87
r	3.19	0.00	0.19	0.19	0.19	0.19
w	0.74	0.00	0.00	0.00	-1.30	-1.30
$\frac{w^c}{w^n}$	75.55	0.00	0.00	0.00	-7.49	-7.49
w^n	0.97	0.00	0.00	0.75	0.75	0.75
w^c	1.04	0.00	0.00	0.00	-3.55	-3.55
τ	0.22	-6.33	-6.33	-6.33	-6.33	-2.93
τ^p	12.40	-0.41	-0.41	-0.41	-0.41	-0.16
$\frac{T(AE_0)}{AE_0}$	0.19	-1.47	-1.47	-1.47	-1.47	-0.68
CEV NB	0.00	17.40	20.70	21.15	16.68	14.68
CEV alive	0.00%	5.58	6.40	6.45	5.70	4.34
LSRA	0.00%	13.74	15.89	16.74	13.20	10.13

Notes: The table summarizes long-run changes (induced by policy transitions) in the main aggregate variables, general equilibrium prices and taxes, and welfare. Changes for baseline variables initially stated in percentages are expressed in percentage points (%p), while changes for baseline variables initially in levels are shown as percentages (%).
GE refers to the full general equilibrium version of the model where wages and interest rate are endogenous as well as the government budget constraints are balanced. The PE version refers to a life cycle model with balanced government budgets, with aggregate physical capital being computed as: $K_t = A_t - B_t$, where A_t are total household assets, and B_t is debt stock (which is endogenous and results from re-balancing the intertemporal government budget constraint).
PE+r refers to the PE version where the interest rate path from the GE version is exogenously imposed - without any adjustments of other outer loop variables. Therefore, the government budget constraints do not have to hold, and therefore the debt values (as well as the implied physical capital $K_t = A_t - B_t$ and thus also output are not shown, and marked as n/a.
PE+$r + w^n + w^c$ refers to the PE version where the interest rate and wages paths from the GE version are exogenously imposed - without any adjustments of other outer loop variables.

Table 16: Aggregates, Prices, Taxes and Welfare: Optimal Mix

Variable	Initial SS	ΔPE	ΔPE+r	ΔPE+$r + w^n$	ΔPE+$r + w^n + w^c$	ΔGE
$\Phi(j_a, s = co)$	32.23	13.53	13.71	13.52	7.01	6.85
$\Phi(j_a, s = cod)$	28.01	10.81	10.80	10.81	15.60	15.56
$\Phi(j_a, s = hs)$	31.41	-22.71	-22.83	-22.67	-21.07	-20.90
$\Phi(j_a, s = hsd)$	8.35	-1.63	-1.68	-1.67	-1.53	-1.51
HK	1.00	8.19	8.55	8.44	7.60	7.42
L	8.91	12.18	12.21	12.15	9.17	8.76
Hours	0.28	2.70	2.65	2.82	2.78	2.20
C	7.76	12.01	15.83	16.12	12.10	10.62
K	12.57	-9.82	4.25	4.89	-1.00	4.60
B	3.50	76.05	79.22	79.22	79.22	50.51
Revenues	2.56	14.88	18.99	19.58	10.55	12.13
Y	13.29	4.29	9.41	9.61	5.82	7.48
r	3.15	0.00	0.23	0.23	0.23	0.23
w	0.74	0.00	0.00	0.00	-1.45	-1.45
$\frac{w^c}{w^n}$	75.55	0.00	0.00	0.00	-8.54	-8.54
w^n	0.97	0.00	0.00	0.93	0.93	0.93
w^c	1.04	0.00	0.00	0.00	-3.97	-3.97
τ	0.22	-12.48	-12.48	-12.48	-12.48	-5.73
τ^p	12.40	-0.24	-0.24	-0.28	-0.03	0.04
$\frac{T(AE_0)}{AE_0}$	0.19	-2.92	-2.92	-2.92	-2.92	-1.34
CEV NB	0.00	16.77	22.78	23.40	18.33	15.81
CEV alive	0.00%	7.78	8.59	8.78	7.96	5.84
LSRA	0.00%	18.01	19.29	20.06	15.69	11.68

Notes: The table summarizes long-run changes (induced by policy transitions) in the main aggregate variables, general equilibrium prices and taxes, and welfare. Changes for baseline variables initially stated in percentages are expressed in percentage points (%p), while changes for baseline variables initially in levels are shown as percentages (%).

GE refers to the full general equilibrium version of the model where wages and interest rate are endogenous as well as the government budget constraints are balanced. The PE version refers to a life cycle model with balanced government budgets, with aggregate physical capital being computed as: $K_t = A_t - B_t$, where A_t are total household assets, and B_t is debt stock (which is endogenous and results from re-balancing the intertemporal government budget constraint).

PE+r refers to the PE version where the interest rate path from the GE version is exogenously imposed - without any adjustments of other outer-loop variables. Therefore, the government budget constraints do not have to hold, and therefore the debt values (as well as the implied physical capital $K_t = A_t - B_t$ and thus also output are not shown, and marked as n/a.

PE+$r + w^n + w^c$ refers to the PE version where the interest rate and wages paths from the GE version are exogenously imposed - without any adjustments of other outer-loop variables.

A.3 Intergenerational Persistence of Education: Married Parents

Table 17 shows the change in the intergenerational education state transition matrix for children with married parents.[43] It shows the same qualitative pattern as for children with single parents that we report in the main text. Finally, it displays the additional general equilibrium decomposition for the optimal mixed reform referenced in the main text.

Table 17: Intergenerational Education Transition Matrix: Married Parents

Increased School Funding				
	$s = hsd$	$s = hs$	$s = cod$	$s = co$
$s^p = hsd$, $q = cpl$	-0.04	-0.08	0.05	0.04
$s^p = hs$, $q = cpl$	-0.03	-0.08	0.06	0.05
$s^p = cod$, $q = cpl$	-0.03	-0.05	0.06	0.02
$s^p = co$, $q = cpl$	-0.02	-0.01	-0.02	0.05
Free College				
	$s = hsd$	$s = hs$	$s = cod$	$s = co$
$s^p = hsd$, $q = cpl$	0.00	-0.32	0.24	0.08
$s^p = hs$, $q = cpl$	0.00	-0.26	0.22	0.04
$s^p = cod$, $q = cpl$	0.00	-0.24	0.22	0.02
$s^p = co$, $q = cpl$	0.00	0.00	0.00	0.00

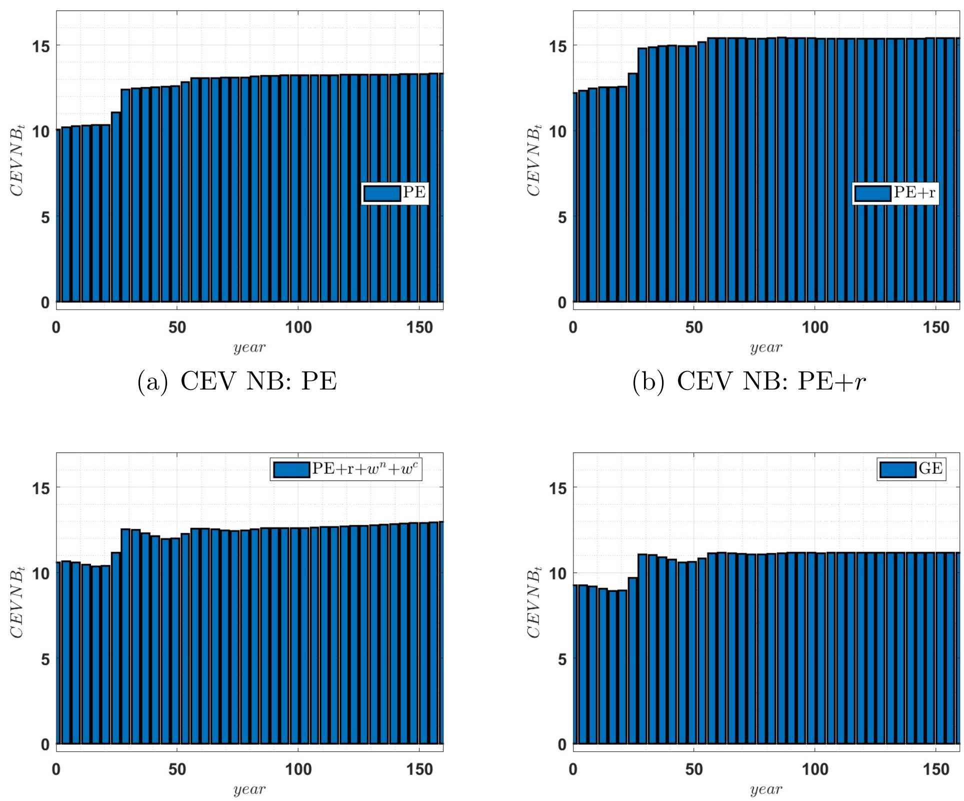
Notes: Changes in intergenerational education mobility: rows show parental background, columns show conditional child education shares. Table entries show absolute changes in conditional child education shares relative to the benchmark economy.

[43] The education level of parental households is determined by the highest educational degree obtained by either of the two parents.

A.4 Partial vs. General Equilibrium: Additional Results

In this section we show the welfare consequences of both reforms in partial equilibrium and in general equilibrium. We also display the intergenerational education transition matrix for children with single parents in partial equilibrium for the "better schools" reform, since it shows the most marked difference to the general equilibrium results in the main text.

Figure 14: Free College Reform, Welfare Gains of Newborns



(a) CEV NB: PE

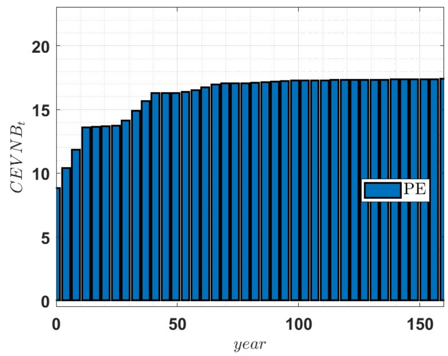
(b) CEV NB: PE+r

(c) CEV NB: PE+r+w^n+w^c

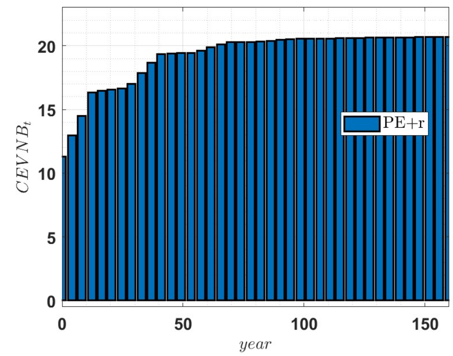
(d) CEV NB: GE

Notes: Newborn CEV. Panel (a): general equilibrium (GE); Panel (b): partial equilibrium (PE); Panel (c): PE and the interest rate path from GE (PE+r); Panel (d): PE+r and the wage paths from GE (PE+$r + w^n + w^c$).

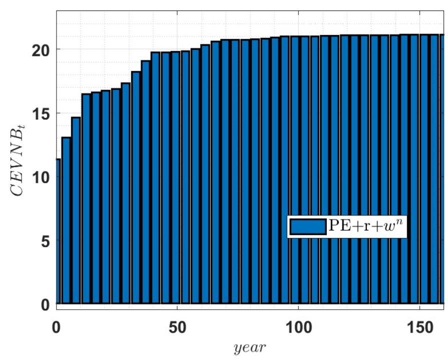
Figure 15: “Better Schools” Reform, Welfare Gains of Newborns



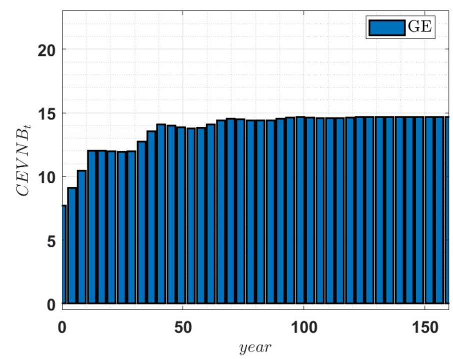
(a) CEV NB: GE



(b) CEV NB: PE



(c) CEV NB: PE+r



(d) CEV NB: PE+r+w^n+w^c

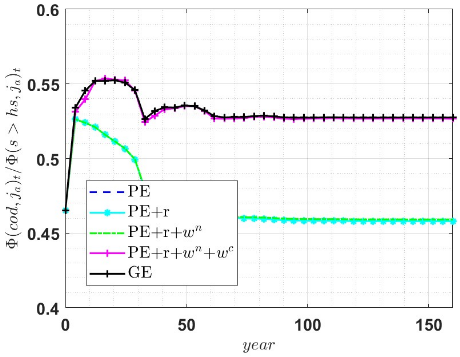
Notes: Newborn CEV. Panel (a): general equilibrium (GE); Panel (b): partial equilibrium (PE); Panel (c): PE and the interest rate path from GE (PE+r); Panel (d): PE+r and the wage paths from GE (PE+$r + w^n + w^c$).

Table 18: Intergenerational Education Transition Matrix: Single Parents in Partial Equilibrium, Change Relative to Baseline

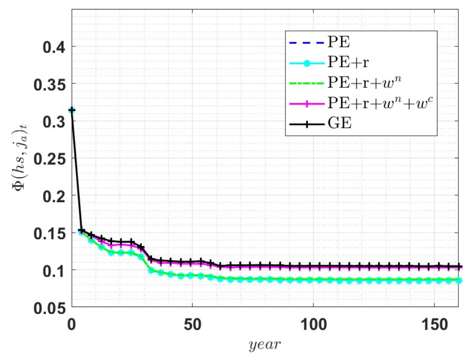
	“Better Schools”			
	$s = hsd$	$s = hs$	$s = cod$	$s = co$
$s^p = hsd$, $q = si$	-0.04	-0.08	0.05	0.06
$s^p = hs$, $q = si$	-0.03	-0.08	0.05	0.06
$s^p = cod$, $q = si$	-0.03	-0.08	0.04	0.07
$s^p = co$, $q = si$	-0.02	-0.02	-0.02	0.05

Notes: Changes in intergenerational education mobility: rows show parental background, columns show conditional child education shares. Table entries show absolute changes in conditional child education shares relative to the benchmark economy. Positive percentage point changes relative to the baseline are marked in red, negative changes are marked in blue.

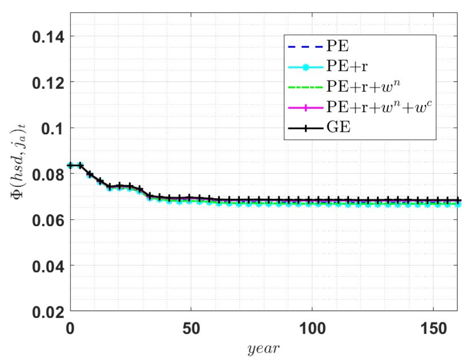
Figure 16: Mixed Reform, Education Shares



(a) College Dropout Share



(b) High School Share



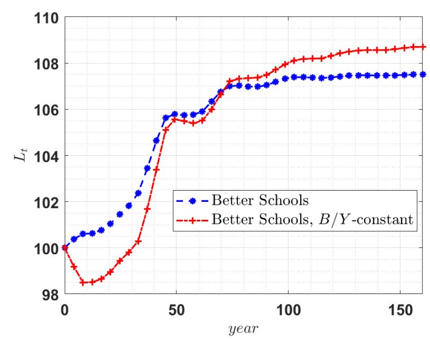
(c) High School Dropout Share

Notes: Panel (a): Share of a given “birth” cohort that becomes a college dropout; Panel (b): Share of a given “birth” cohort that becomes a high school completer; Panel (c): Share of a given “birth” cohort that becomes a high school dropout.

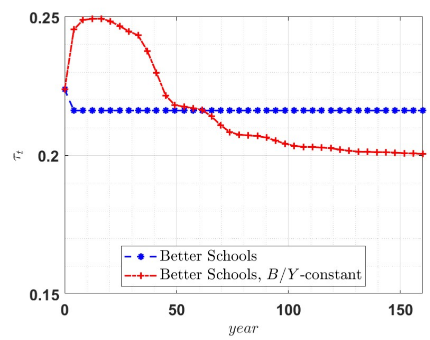
A.5 On The Importance of Government Debt

Figure 17 contrasts economic aggregates and welfare gains from the "Better Schools" reform with government debt adjustments (benchmark) and under the assumption of a constant debt to income ratio $\frac{B}{Y}$ which requires a period-by-period adjustment in the labor income tax rate.

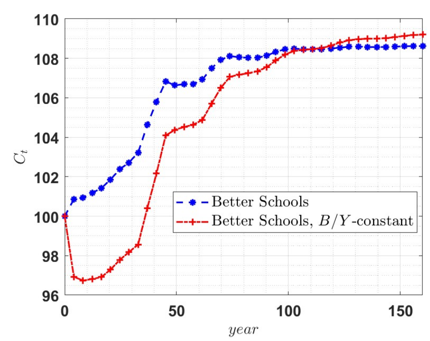
Figure 17: Comparison between Debt vs. No Debt in the "Better Schools" Reform



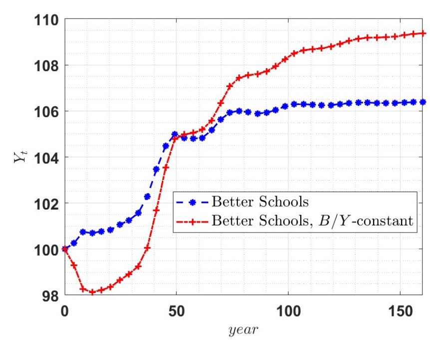
(a) Labor



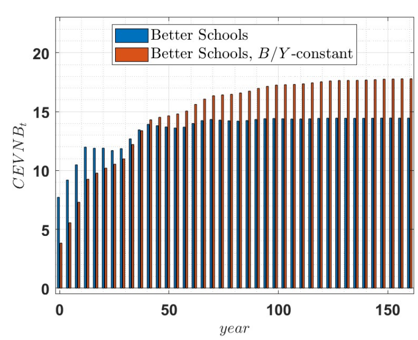
(b) Tax Level Parameter τ



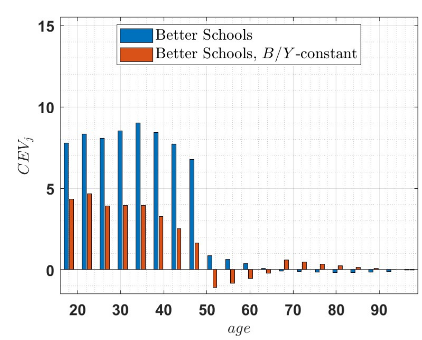
(c) Consumption



(d) GDP



(e) Newborn CEV



(f) Initially Alive CEV

Notes: The plots show transition paths of under the baseline specification (intertemporal government budget constraint), and with the specification where B/Y-ratio is being held constant at its baseline level. Panel (a): Aggregate efficiency units of labor; Panel (b): Tax function parameter τ; Panel (c): Consumption; Panel (d): Output; Panel (e): Newborn CEV; Panel (f): Currently alive population CEV.

B Quantitative Model Appendix

B.1 Population Growth

With heterogeneous fertility rates by education, $\varsigma(s(wo))$, and an endogenous evolution of education shares along the transition, the total population growth is time varying and denoted by n_t. Recall that all parents are assumed to give birth to children at the same age j_f. Starting from retirement age j_r onward households face non-zero survival risk with ϕ_j denoting the survival probability from age j to age $j+1$.

The population dynamics in every period evolve accordingly as

$$N_{t+1}(j_a) = \sum_s N_t(j_a + j_f, s, wo) \cdot \varsigma(s(wo)) \tag{28}$$

$$N_{t+1}(j+1) = N_t(j) \cdot \phi_j \tag{29}$$

$$N_{t+1} = (1+n_t)N_t \tag{30}$$

B.2 Human Capital Production Function: Normalization

The human capital production function has a multi-nested structure, cf. equation 21. The three inputs —public input i^g and parental money and time inputs $i^t(j)$ and $i^t(j)$— are normalized throughout by their unconditional means (across all child ages/grades) to account for different units. We can rewrite the second and third nests as

$$i(j) = \bar{A}\left(\Gamma^g\left(\frac{\kappa_j^g}{\Gamma^g\left(\bar{i}^g\right)^{1-\frac{1}{\sigma^g}}}\left(i^g\right)^{1-\frac{1}{\sigma^g}} + \frac{(1-\kappa_j^g)}{\Gamma^g\left(\bar{i}^p\right)^{1-\frac{1}{\sigma^g}}}\left(i^p(j)\right)^{1-\frac{1}{\sigma^g}}\right)\right)^{\frac{1}{1-\frac{1}{\sigma^g}}} \tag{31}$$

$$i^p(j) = \left(\Gamma\left(\frac{\kappa_j^m}{\Gamma\left(\bar{i}^{m,d}\right)^{1-\frac{1}{\sigma^m}}}\left(i^m(j)\right)^{1-\frac{1}{\sigma^m}} + \frac{(1-\kappa_j^m)}{\Gamma\left(\bar{i}^{t,d}\right)^{1-\frac{1}{\sigma^m}}}\left(i^t(j)\right)^{1-\frac{1}{\sigma^m}}\right)\right)^{\frac{1}{1-\frac{1}{\sigma^m}}}, \tag{32}$$

respectively, for $\Gamma^g(\kappa_j^g, \bar{i}^g, \bar{i}^p, \sigma^g) \equiv \frac{\kappa_j^g}{(\bar{i}^g)^\rho} + \frac{(1-\kappa_j^g)}{(\bar{i}^p)^\rho}$ and $\Gamma(\kappa_j^m, \bar{i}^{m,d}, \bar{i}^{t,d}, \sigma^m) \equiv \frac{\kappa_j^m}{\left(\bar{i}^{m,d}\right)^\rho} + \frac{(1-\kappa_j^m)}{\left(\bar{i}^{t,d}\right)^\rho}$ where, in general, $\Gamma^g(\cdot) \neq 1$ and $\Gamma(\cdot) \neq 1$.

We can rewrite eq. (31) and eq. (32) further as

$$i(j) = \bar{A}\tilde{\Gamma}^g\left(\tilde{\kappa}_j^g\left(i^g\right)^{1-\frac{1}{\sigma^g}} + (1-\tilde{\kappa}_j^g)\left(i^p(j)\right)^{1-\frac{1}{\sigma^g}}\right)^{\frac{1}{1-\frac{1}{\sigma^g}}} \tag{33}$$

$$i^p(j) = \tilde{\Gamma}\left(\tilde{\kappa}_j^m\left(i^m(j)\right)^{1-\frac{1}{\sigma^m}} + (1-\tilde{\kappa}_j^m)\left(i^t(j)\right)^{1-\frac{1}{\sigma^m}}\right)^{\frac{1}{1-\frac{1}{\sigma^m}}}, \tag{34}$$

where $\tilde{\Gamma}^g = (\Gamma^g)^{1-\frac{1}{\sigma^g}}$ and $\tilde{\Gamma} = \Gamma^{1-\frac{1}{\sigma^m}}$. The normalized share parameters are accordingly defined as $\tilde{\kappa^g} = \frac{\kappa^g}{\Gamma^g(\bar{i}^g)^{1-\frac{1}{\sigma^g}}}$ and $\tilde{\kappa}^m = \frac{\kappa^m}{\Gamma\left(\bar{i}^{m,d}\right)^{1-\frac{1}{\sigma^m}}}$.

B.3 Optimal Parental Human Capital Investments

Optimal Investment in Human Capital The intratemporal optimality condition for time with children is[44]

$$\begin{aligned}
\varsigma(si,s)(v^t)'\left(\varsigma(si,s)\cdot i^t\right) &= \lambda_i \frac{\partial i(i^m,i^t,i^g)}{\partial i^t} - \lambda_t \cdot \varsigma(si,s) = \lambda_h \frac{\partial g(h,i)}{\partial i}\frac{\partial i(i^m,i^t,i^g)}{\partial i^t} - \lambda_t \cdot \varsigma(si,s) \\
&= \beta \mathbf{E}_{\eta'|\eta} V_h(x',a',h')\frac{\partial g(h,i)}{\partial i}\frac{\partial i(i^m,i^t,i^g)}{\partial i^t}
\end{aligned}$$

The left hand side is the marginal cost of spending an additional time unit with children, the right hand side gives the discounted benefits, per child, of one additional unit of the final good being spent on education, where $\frac{\partial g(h,i)}{\partial i}\frac{\partial i(i^m,i^t,i^g)}{\partial i^m}$ is the marginal benefit of that spending on human capital tomorrow, and $\mathbf{E}_{\eta'|\eta}V_h(x',a',h')$ is the expected marginal benefit of a smarter child.

The intertemporal optimality condition for resource investment in children is

$$\begin{aligned}
\lambda_b \varsigma(si,s) &= \lambda_i \frac{\partial i(i^m,i^t,i^g)}{\partial i^m} = \lambda_h \frac{\partial g(h,i)}{\partial i}\frac{\partial i(i^m,i^t,i^g)}{\partial i^t} \\
\Leftrightarrow \quad \left(\beta \mathbf{E}_{\eta'|\eta} V_a(x',a',h') + \lambda_a\right)\varsigma(si,s) &= \beta \mathbf{E}_{\eta'|\eta} V_h(x',a',h')\frac{\partial g(h,i)}{\partial i}\frac{\partial i(i^m,i^t,i^g)}{\partial i^m} \\
\Leftrightarrow \quad \frac{u'\left(\frac{c}{1+\zeta_c\varsigma(si,s)}\right)}{1+\zeta_c\varsigma(si,s)}\frac{\varsigma(si,s)}{1+\tau_c} &= \beta \mathbf{E}_{\eta'|\eta} V_h(x',a',h')\frac{\partial g(h,i)}{\partial i}\frac{\partial i(i^m,i^t,i^g)}{\partial i^m}
\end{aligned}$$

The left hand side is the marginal cost of reducing spending on consumption goods by one unit, and the right hand side again gives the discounted per child benefits.

Optimal Allocation between Time and Money Taking the ratio between the first order conditions for time and money inputs yields

$$\begin{aligned}
\frac{(v^t)'\left(\varsigma(si,s)\cdot i^t\right)}{\beta \mathbf{E}_{\eta'|\eta} V_a(x',a',h') + \lambda_a} &= \frac{\frac{\partial i^p(j,i^m,i^t)}{\partial i^t}}{\frac{\partial i^p(j,i^m,i^t)}{\partial i^m}} \\
\Leftrightarrow \quad (1+\tau_c)\frac{(v^t)'\left(\varsigma(si,s)\cdot i^t\right)}{\frac{u'\left(\frac{c}{1+\zeta_c\varsigma(si,s)}\right)}{1+\zeta_c\varsigma(si,s)}} &= \frac{\frac{\partial i^p(j,i^m,i^t)}{\partial i^t}}{\frac{\partial i^p(j,i^m,i^t)}{\partial i^m}}
\end{aligned} \tag{35}$$

[44]Relative to the main text we simplify the notation and denote the value function by $V(x,a,h)$ where x subsumes all other relevant state variables.

This equation simply states that the marginal rate of substitution between time and consumption times its relative price (the consumption tax rate) equals the marginal rate of transformation in the production of inputs for human capital production.

Using the functional forms for per-period utility and human capital production function, the relation between optimal resource and time investments can be written as

$$\frac{i^m}{i^t} = \left(\frac{1}{\chi}\kappa\left(\varsigma(si,s)\cdot i^t\right)^{\frac{1}{\psi}} c(1+\tau^c)\right)^{\sigma^m} \tag{36}$$

$$\Leftrightarrow \quad \frac{i^m}{(i^t)^{1+\frac{\sigma^m}{\psi}}} = \left(\frac{1}{\chi}\kappa\left(\varsigma(si,s)\right)^{\frac{1}{\psi}} c(1+\tau^c)\right)^{\sigma^m}$$

B.4 Disentangling Efficiency and Redistribution when Measuring Welfare: The LSRA

To disentangle welfare benefits stemming from efficiency gains from those driven by redistribution, we use a wealth-based welfare criterion that follows the spirit of the lump-sum redistribution authority originally described in Auerbach and Kotlikoff (1987) and applied to a model with intragenerational heterogeneity in Nishiyama and Smetters (2005). Technically, our wealth-based measure is computed as follows. As a first step, individual-specific transfers are computed that would make the currently living households, including the period 0 newborns, indifferent between the status quo and the reform scenario. Denote those transfers by $\Psi_0(j,\cdot)$. These transfers are then aggregated up using the initial steady-state cross-sectional distribution and population shares. As a second step, for each newborn cohort in the transition and in the final steady state an ex-ante uniform transfer is computed that would make them indifferent between being born into the initial steady state or a given period of the transition (or a final steady state). We denote those transfers by Ψ_t. Finally, a present discounted value (based on the market discount rate) of these ex-ante transfers is computed and added up with the aggregate transfer to the initially alive population. Thus, the total transfer is given by:

$$W = \int \Psi_0(\cdot)d\Phi_0(\cdot) + \sum_{t=1}^{\infty}\prod_{s=1}^{t}\left(\frac{1}{1+r_s^{fc}}\right)\Psi_t \tag{37}$$

where we discount using the sequence of equilibrium interest rates of the free college reform, $\left\{r_t^{fc}\right\}_{t=1}^{\infty}$.

For expositional purposes, we follow the approach in Kindermann and Krueger (2022) and express the resulting aggregate monetary transfer as an annuity C paying a constant consumption flow in every transition period and in the final steady state:

$$C \cdot \sum_{t=0}^{\infty} \prod_{s=1}^{t} \left(\frac{1}{1 + r_s^{fc}} \right) = -W, \tag{38}$$

for $\prod_{s=1}^{0} \left(\frac{1}{1+r_s^{fc}} \right) = 1$.

Finally, we express the computed annuity value as percent of initial aggregate consumption:

$$\text{LSRA} = 100 \cdot \frac{C}{C_0} \tag{39}$$

C Model Calibration Appendix

C.1 Wage Process Estimation

Overview Our estimates of the wage process are based on PSID data for the sample years 1968-1997, using the data of Busch and Ludwig (2024). The unit of observation is the household and we use household gross labor income and household hours worked to derive a measure of labor productivity for which we then estimate a stochastic process, after controlling for household observable characteristics in a first-stage regression.

We start with annual total labor income, which includes income from wages and salaries, bonuses, and the labor part of self-employment income. We then sum up the income of the household head and spousal annual labor income, and add 50% of estimated payroll taxes[45] to the sum of head and spouse labor incomes to obtain total household pre-labor costs, which we deflate by the CPI.

We compute household hours as the sum of hours by the household head and the spouse, and finally arrive at labor productivity by dividing our measure of household labor costs by household hours.

Sample Selection We restrict the sample to household heads in the age bin 20 to 65. We drop observations with household hours less than 520 annual hours, and with household income less than 50% of a minimum annual income defined as $0.5 \cdot 520 \cdot \bar{w}$, where $\bar{w}$ is the minimum wage of a specific year. We drop observations with hourly wages less than 50% of the minimum wage in a given year, and further trim the data to cover 0.05% to 99.95% of the remaining household wage distribution. This leaves us with $106,341$ observations.

Stochastic Process and Estimation As a first stage, we regress log wages of a household on a cubic in age (of the household head), time dummies, family size, a dummy for marital status (of the household head), and household fixed effects.[46] At the second stage, we use the log residuals of the first stage, denoted by $\ln(\mathrm{y}_t)$, to estimate the stochastic component by GMM. Specifically, we assume a standard specification with persistent and transitory components for all ages $t \in \{0, ..., T\}$

$$\ln(\mathrm{y}_t) = \ln(\mathrm{z_t}) + \ln(\varepsilon_t) \tag{40a}$$

$$\ln(\mathrm{z_t}) = \rho \ln(\mathrm{z_{t-1}}) + \ln(\nu_t), \tag{40b}$$

[45] We use Taxsim to impute these payroll taxes

[46] We obtain very similar results at the second stage for a random effects specification.

where $\varepsilon_t \sim_{i.i.d} \mathcal{N}_\varepsilon(0, \sigma_\varepsilon^2)$, $\nu_t \sim_{i.i.d} \mathcal{N}_\nu(0, \sigma_\nu^2)$ and $\ln(z_{-1}) \sim_{i.i.d} \mathcal{N}_\nu(0, \sigma_{\ln(\mathrm{z})}^2)$. Thus, there are four unknown parameters $[\rho, \sigma_\varepsilon^2, \sigma_\nu^2, \sigma_{\ln(\mathrm{z})}^2]$. Following Heathcote, Perri, and Violante (2010), we estimate these four parameters by GMM using the empirical moments,

$$\hat{m}_t^0 = \mathbb{E}[\mathrm{y}_t^2], \quad \hat{m}_{t+1}^0 = \mathbb{E}[\mathrm{y}_{t+1}^2], \quad \hat{m}_t^1 = \mathbb{E}[\mathrm{y}_t \mathrm{y}_{t+1}], \quad \text{and} \quad \hat{m}_t^2 = \mathbb{E}[\mathrm{y}_t \mathrm{y}_{t+2}]$$

and the corresponding theoretical moment conditions derived from (40):

$$m_t^0 = \rho^{2((t-\bar{t})+1)} \sigma_{\ln(\mathrm{z})}^2 + \sum_{s=\bar{t}}^{t} \rho^{2(t-s)} \sigma_\nu^2 + \sigma_\varepsilon^2$$

$$m_{t+1}^0 = \rho^{2((t-\bar{t})+2)} \sigma_{\ln(\mathrm{z})}^2 + \sum_{s=\bar{t}}^{t} \rho^{2((t-s)+1)} \sigma_\nu^2 + \sigma_\varepsilon^2$$

$$m_t^1 = \rho^{2(t-\bar{t})+3} \sigma_{\ln(\mathrm{z})}^2 + \sum_{s=\bar{t}}^{t} \rho^{2(t-s)+1} \sigma_\nu^2$$

$$m_t^2 = \rho^{2((t-\bar{t})+2)} \sigma_{\ln(\mathrm{z})}^2 + \sum_{s=\bar{t}}^{t} \rho^{2(t-s)+2} \sigma_\nu^2.$$

Estimation Results The resulting estimates[47] of our main parameters of interest are the annual persistence of wages $\hat{\rho} = 0.9501$, the variance of the persistent shock $\hat{\sigma}_\nu^2 = 0.0109$ and the variance of the transitory shock $\hat{\sigma}_\varepsilon^2 = 0.0637$.

Implied 2-State Markov Chain at 4-Year Frequency We next translate these estimates into a symmetric 2-state Markov process, adjusted to the four year frequency of our model. We choose the state vector $[\eta_-, \eta_+]$ of the Markov process to match the conditional variance at the four year frequency of

$$\sigma_{\ln(\mathrm{y})}^2 \mid \ln(\mathrm{z}_{\mathrm{t}-4}) = var\left(\ln\left(\mathrm{y}_t\right) \mid \ln(\mathrm{z}_{\mathrm{t}-4})\right) = \sigma_\varepsilon^2 + \sigma_\nu^2 \cdot \sum_{s=0}^{3} \rho^{2s} = 0.1014$$

and thus $\sigma_{\ln(\mathrm{y})} \mid \ln(\mathrm{z}_{\mathrm{t}-4}) = 0.3184$. Evaluated at the stationary invariant distribution of the symmetric Markov process $\Pi = [0.5, 0.5]$, this implies that

$$\eta_- = \frac{2\exp\left(1 - \sigma_{\ln(\mathrm{y})} \mid \ln(\mathrm{z}_{\mathrm{t}-4})\right)}{\exp\left(1 - \sigma_{\ln(\mathrm{y})} \mid \ln(\mathrm{z}_{\mathrm{t}-4})\right) + \exp\left(1 + \sigma_{\ln(\mathrm{y})} \mid \ln(\mathrm{z}_{\mathrm{t}-4})\right)} = 0.6919$$

$$\eta_+ = \frac{2\exp\left(1 + \sigma_{\ln(\mathrm{y})} \mid \ln(\mathrm{z}_{\mathrm{t}-4})\right)}{\exp\left(1 - \sigma_{\ln(\mathrm{y})} \mid \ln(\mathrm{z}_{\mathrm{t}-4})\right) + \exp\left(1 + \sigma_{\ln(\mathrm{y})} \mid \ln(\mathrm{z}_{\mathrm{t}-4})\right)} = 1.3081.$$

[47] Estimating both the first stage and the second stage by education groups yields very similar estimates.

We further target the autocorrelation of our estimated process (40). Notice that the unconditional variance of $\ln(\mathrm{y})$ is

$$\sigma^2_{\ln(\mathrm{y})} = \sigma^2_{\varepsilon} + \sigma^2_{\ln(\mathrm{z})}$$

and the covariance is

$$cov\left(\ln\left(\mathrm{y}_t\right), \ln\left(\mathrm{y}_{t-4}\right)\right) = E\left[\ln\left(\mathrm{y}_t\right)\ln\left(\mathrm{y}_{t-4}\right)\right] = \rho^4 \sigma^2_{\ln(\mathrm{z})}$$

and thus the correlation is

$$corr\left(\ln\left(\mathrm{y}_t\right), \ln\left(\mathrm{y}_{t-4}\right)\right) = \rho^4 \frac{1}{1 + \frac{\sigma^2_{\varepsilon}}{\sigma^2_{\ln(\mathrm{z})}}}.$$

Since, further,

$$\sigma^2_{\ln(\mathrm{z})} = \frac{\sigma^2_{\nu}}{1-\rho^2} = 0.1140$$

we obtain

$$corr\left(\ln\left(\mathrm{y}_t\right), \ln\left(\mathrm{y}_{t-4}\right)\right) = 0.5247.$$

Finally, let the Markov transition matrix be

$$\pi\left(\eta' \mid \eta\right) = \begin{bmatrix} \kappa & 1-\kappa \\ 1-\kappa & \kappa \end{bmatrix},$$

which has autocorrelation in the stationary invariant distribution of $2\kappa - 1$. Matching the autocorrelation of the estimated process we thus obtain

$$\kappa = \frac{1 + corr\left(\ln\left(\mathrm{y}_t\right), \ln\left(\mathrm{y}_{t-4}\right)\right)}{2} = 0.7623.$$

C.2 Child Human Capital Production Function

Figure 18: Age Profiles of κ_j^h and κ_j^m

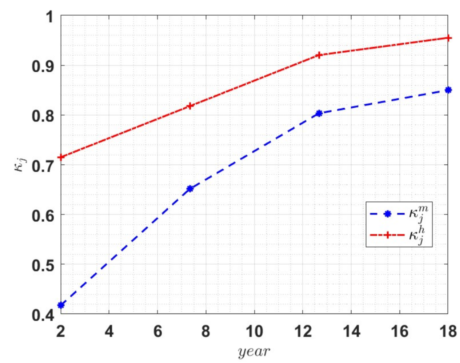
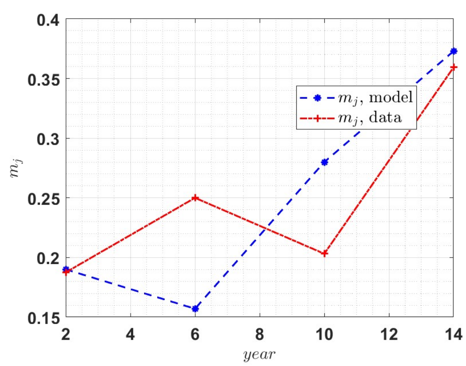
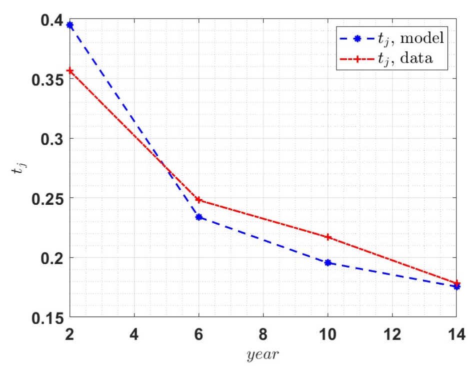


Figure 19: Age Profiles of Money and Time Parental Inputs: Model and Data



(a) Money Input



(b) Time Input

Notes: All variables are normalized by their respective means (over all age groups) which are directly targeted in the calibration.

C.3 Calibration: Summary Tables

Table 19: First Stage Calibration Parameters

Parameter	Interpretation	Value	Source (data/lit)
	Population		
$j = 0$	Age at economic birth (age 2)	0	
j_a	Age at beginning of econ life (age 18)	4	
j_c	Age at finishing college (age 22)	5	
j_{f-1}	Marriage Age (age 26)	6	
j_f	Fertility Age (age 30)	7	
j_r	Retirement Age (age 66)	16	
J	Max. Lifetime (age bin 98-101)	24	
$\{\phi_j\}$	Survival Probabilities	see main text	Life Tables SSA
n	Population Growth Rate	1%	
$\frac{\varsigma(s<co)}{\varsigma(s=co)}$	Fertility Education Gradient	1.15	PSID 2011-2019
$\pi^m(s)$	Marriage probability	[0.51 0.51 0.51 0.51]	PSID 2011 - 2019
	Preferences		
ψ	Frisch elasticity	0.6	
	Technology		
μ	Technological growth rate	1%	
α	Capital share	33.3%	
δ	Depreciation rate	5%	
ρ	Subst. Elasticity $\frac{1}{1-\rho}$	3.3	
	Labor Productivity		
$\{\epsilon(s, g, j\}$	Age Profile	see main text	PSID 1968-1997
$[\eta_l, \eta_h]$	States of Markov process	[0.6919, 1.3081]	PSID 1968-1997
π_{hl}	Transition probability of Markov process	0.7623	PSID 1968-1997
	Ability/Human Capital and Education		
ι	College tuition costs (annual, net of grans and subsidies)	15,500$	NCES (average 2000-2019)
$\underline{a}(j \in [j_a], co)$	College borrowing limit	45,590$	Krueger and Ludwig (2016)
σ^h	Elast of subst b/w human capital and CES inv. aggr.	1	Cunha, Heckman, and Schennach (2010)
σ^g	Elast of subst b/w public inv. and CES aggr. of private inv.	2.43	Kotera and Seshadri (2017)
σ^m	Elast of subst b/w monetary and time inv.	1	Lee and Seshadri (2019)
$\Phi(h(j = 0)\|s_p)$	Innate ability dist-n of children by parental education	see main text	PSID CDS I
$\underline{h}_0$	Normalization parameter of initial dist-n of initial ability	0.1248	PSID CDS I-III
	Baseline Government policy		
ϑ	Public subsidy of college education	38.8%	Krueger and Ludwig (2016)
ϑ^{pr}	Private subsidy of college education	16.6%	Krueger and Ludwig (2016)
i^g_j	Public high school education spending	$\approx 14,000$$	NCES (2000-2018)
τ_c	Consumption Tax Rate	5.0%	legislation
τ_k	Capital Income Tax Rate	36%	Trabandt and Uhlig (2011)
ξ	Labor Income Tax Progressivity	0.18	Heathcote, Storesletten, and Violante (2017)
ω	Income of non-working households	20.2% of average earnings	CEX 2001-2007 (see Holter, Krueger, and Stepanchuk 2023)
τ^p	Soc Sec Payroll Tax	12.4%	legislation
G/Y	Government consumption to GDP	13.8%	current value

Notes: First stage parameters calibrated exogenously by reference to other studies and data.

Table 20: Second Stage Calibration Parameters

Parameter	Interpretation	Value	Data Moment & Source	Moment Value
		Preferences		
β	Time discount rate	0.9949	Annual interest rate	3.00%
ν	Altruism parameter	0.5944	Average IVT transfer per child, 2013 Rosters and Transfers Module of PSID	$61,200
ϕ	Weight on hours disutility	17.9718	Average hours per household, conditional on working	1/3
F	Fixed cost of working positive hours	0.2000	Employment rate age 25-54, U.S. Bureau of Labor Statistics	83%
		Labor Productivity		
$\rho_0(s)$	Normalization parameter	$[1.01, 1.01...$ $...0.98, 0.93]$	Normalization $\mathbb{E}\gamma(s,h) = 1$	1.0
		Human Capital and Education		
κ	Weight on disutility from time inv.	0.3473	Average time inv., CDS PSID I-III	23.77
κ_j^h	Share of human capital (outermost nest)	cf. Fig. 18	Slope of time inv. and i^g-elasticity, CDS PSID I-III, and Jackson and Mackevicius (2023)	cf. Fig. 19
κ_j^m	Share of monetary input (innermost nest)	cf. Fig. 18	Average monetary inv. and slope, CDS PSID I-III	cf. Fig. 19
$\kappa_j^g,\ j > 0$	Share of government input for ages 6 and older	0.8700	Test score disparity b/w "top" and "bottom" children at age 17-19, CDS PSID I-III	26%p
κ_0^g	Share of government input for age bin 2-6	0.7647	Average time inv. age bin 2-6, CDS PSID I-III	32.40
$\bar{A}$	Investment scale parameter	1.1989	Normalization of average HK at age j_a	1.00
λ^{hs}	Slope parameter of high school graduation probability function	2.6836	Share of high school dropouts, PSID 2011–2019	8%
λ^c	Slope parameter of college completion probability function	0.6881	Share of college dropouts, PSID 2011-2019	29%
$\varrho(s^p < co)$	Psychological (utility) costs $s = co, s^p < co$	3.3272	Share of college graduates, PSID 2011-2019	32%
$\varrho(s^p = co)$	Psychological (utility) costs $s = co, s^p = co$	0.9096	Conditional fraction of group $s > hs$ with $s^p = co$, PSID 2011-2019	92%
		Government policy		
τ	Level parameter of HSV tax function	0.2246	B/Y-ratio	100%
ρ^p	Pension replacement rate	0.0883	Balance per period soc. sec. budget	

Notes: Second stage parameters calibrated endogenously by targeting selected data moments.