**Syllabus for Econ 897**

Summer 2024

**Instructors:**

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**Schedule:**

July 8th - August 16th, 2024, Monday through Friday

Time: 10:00 AM - 12:00 PM and 1:30 PM - 3:30 PM (ET) each day

Location: Meyerson Hall – MEYH 36MK 107

**Final (Waiver) Exam:**

August 19th (Mon), 2024, 1:00 PM - 4:00 PM (3 Hours) in MEYH 36MK 107

**Course Webpage:**

`https://economics.sas.upenn.edu/graduate/course-information/summer-math-camp`

**Textbooks:**

- For real analysis, there is no required textbook (though there is a file with notes that will be shared during the course). Suggested reading is *Real Mathematical Analysis* by Charles C. Pugh, Springer 2010 or *Real Analysis for the Undergraduate with an invitation to Functional Analysis* by Matthew Pons, Springer 2014.
- For algebra, there will be notes for all required materials. Suggested reading is *Linear Algebra Done Right* by Sheldon Axler, Springer 2015.
- For optimization, *Linear and Nonlinear Programming* by David G. Luenberger and Yinyu Ye, Springer 2016 or *A First Course in Optimization Theory* by Rangarajan K. Sundaram, Cambridge 1996.
- For probability, *Statistical Inference* by Casella and Berger (CB), Thomson Learning 2002. This one will be used again in ECON 7310.

In addition, you might find some of the following books useful to supplement different parts of the lectures.

The classic *Principles of Mathematical Analysis* by Walter Rudin is a good reference. You can also have a look at the first chapters of *The Nature and Origins of Modern Mathematics: an Elementary Introduction* by Andy McLennan (link). A short summary in optimization is also contained in the appendix of Mas-Colell, Whinston, Green (MWG): *Microeconomic Theory*. This is also a recommended book for Econ 7100.

If you are not familiar with the elementary set theory and structure of proofs, we strongly suggest that you read the section 1.1 in Pugh.

**Exams and Homework:**

There will be graded tests every Friday to cover the materials covered in the corresponding week. Additionally, each instructor might give quizzes and will post problem sets. These might influence your grade as well. We strongly recommend you to try to solve the questions before you see solutions. The waiver exam is the final for all three parts of Econ 897. Your grade will be determined as the average of your grades for all three parts.

## Outline of the Course:

**Part I.** Weeks 1 and 2

(a) Mathematical logic
   i. Formal language: notation
   ii. Mathematical proofs
   iii. Sets and binary relations
   iv. Functions: injections, surjections, bijections
   v. Cardinality: countable and uncountable sets
   vi. Cantor's theorem and cardinality of $\mathbb{R}$

(b) Vector spaces and structures on them
   i. Normed spaces

ii. Metric spaces

iii. Sequences: limits and convergence in metric spaces

iv. Completeness

v. Banach fixed point theorem

(c) Topology

i. Topology of metric spaces

ii. Bases: axiomatic definition of topology

iii. Sequences and topology

iv. Continuous functions

v. Compactness

vi. Heine-Borel and related theorems

vii. Connected and path-connected sets

(d) Correspondences

i. Continuity of correspondences

ii. Closed Graph Theorem

iii. Fixed point theorems (without proof)

**Part II.** Weeks 3 and 4

(a) Differentiation: (Pugh, §3.1)

i. Definition and basic properties

ii. Mean value theorem and inverse function theorem

iii. Higher order derivatives and Taylor's theorem

(b) Linear Algebra:

i. Determinants (Sundaram, §1.3; Pugh, Chapter 5 Appendix E)

ii. Vector spaces and linear mappings (Sundaram, Appendix C.1; Pugh §5.1)

iii. Inner product and orthogonality (Sundaram, Appendix C.2)

iv. Eigenvalues and eigenvectors (Axler, $\S 4 - 5$)

(c) Analysis in $\mathbb{R}^n$:

i. Partial and total derivatives (Pugh, §5.2)

ii. Higher derivatives (Pugh, §5.3)

iii. Implicit and inverse functions (Pugh, §5.5)

iv. Convex Sets and Separating Hyperplane Theorems (MWG, Appendix M.G)

v. Concavity and Quasi-Concavity (Sundaram, §7.1-7.2 and §8.1-8.3)

**Part III.** Weeks 5 and 6

(a) Optimization Theory in $\mathbb{R}^n$:

i. Intro to Optimization (LY, §1)

ii. Unconstrained Optimization (LY, §7; MWG, Appendix M.J)

iii. Equality Constraints and the Theorem of Lagrange (LY, §11; MWG, Appendix M.K)

iv. Inequality Constraints and the Theorem of Kuhn-Tucker (LY, §11.8; MWG, Appendix M.K)

v. Convexity in Optimization (Sundaram, §7)

vi. Quasi-concavity in Optimization (Sundaram, §8)

vii. Intro to Dynamic Programming

viii. The Maximum Theorem (Sundaram, §9.2)

ix. The Envelope Theorems (MWG, Appendix M.L)

(b) Brief Review of Basic Probability Theory

i. Intro to Measure and Probability Theory (CB, §1; Pugh §6)

ii. Transformations and Moments (CB, §2)

iii. Common Families of Distributions (CB, §3.2-§3.3)

iv. Multiple Random Variables (CB, §4)

v. Sampling and Statistics (CB, §5.1-§5.4)

vi. Convergence (CB, §5.5-§5.8)