Disagreement in Market Index Options*

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August 2023

Abstract

We generate new evidence on disagreement among traders in the S&P 500 options market from high-frequency intraday price and volume data. Inference on disagreement is based on a model where investors observe public information but agree to disagree on its interpretation; disagreement among investors is captured by the volume-volatility elasticity. For options, there are two natural variables related to disagreement: moneyness and tenor, which we relate to disagreement about the distribution of the market index at different quantiles and times. The estimated volume-volatility elasticity equals unity for options near the money and close to expiration, which is consistent with the case of no disagreement among investors. In contrast the elasticity estimates decrease with increases in the absolute value of moneyness, indicating investors have a higher disagreement about rare events. Likewise the elasticity decreases with increases in tenor, implying in higher investors' disagreement about more distant events.

Keywords: SPX options, market index, high-frequency data, disagreement, volume-volatility elasticity, public information. **JEL Code**: G4.

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Acknowledgements: We are grateful to Tim Bollerslev and Andrew Patton, as well as the financial econometrics group at Duke for their invaluable comments and advice; we benefited from conversations with Leonardo Salim, Taehoon Kim, Peter Horvath, and especially Viktor Todorov.

1 Introduction

This paper studies disagreement among investors in the S&P 500 options market. We consider a model where different investors interpret information about an asset's payoff. These investors agree to disagree about the interpretation of the information, which leads to investors trading the asset. In this framework, investors' disagreement about new information is revealed by analyzing the relation between trade volume and price volatility. The model implies that the elasticity is equal to unity in the case of no disagreement among investors, and that the elasticity should decrease as disagreement increases. Using high-frequency data on market index options we estimate the volume-volatility elasticity and study how the elasticity and measures of disagreement covary relative to the model predictions.

According to Cochrane (2016): "Volume is the great unsolved problem of financial economics." The problem stems from the no-trade theorem[1] of Milgrom and Stokey (1982), which implies that stock markets should see no trade. However, the reality of financial markets is that of a huge volume of trades taking place every year. Indeed, the United States' stock market turnover has been above 100% per year since 1998[2], implying that a value equivalent to the entire stock market changes hands at least once a year.

The relationship between trade volume and prices (or volatility) has been previously analyzed before. Karpoff (1987) surveys the earlier empirical evidence and argues that understanding the relationship between trade volume and prices is important due to four reasons. First, it allows us to distinguish between different hypotheses about the structure of financial markets. Second, trade volume can be key to quantifying the relevancy of a news event. Third, volume is related to stochastic variance and can be used to understand the distribution of speculative prices. Fourth, volume in the equities market can have an impact on the futures market. The surveyed empirical works find that there is a positive contemporaneous correlation between trade volume and returns (see tables 1 and 2 of Karpoff (1987)), and Karpoff concludes that:

> It is likely that observations of simultaneous large volumes and large price changes [...] can be traced to their common ties to information flows [...].

An early explanation that ties volumes and return magnitude to information flows is that of Tauchen and Pitts (1983). Tauchen and Pitts (1983) provide a statistical explanation for the positive correlation between volume and return magnitue. Tauchen and Pitts argue that changes in price and in volume are both driven by a process that models the rate of information arrival. In their model, squared returns and volume are both positively related to the information arrival process, which leads to the positive correlation between volatility and volume found by many empirical works. Their model also implies that both the variance of price changes and trading volume depends on the extent to which investors disagree about new information (see equations

4, 5 and 6 in Tauchen and Pitts (1983)). The explanation based on the common information arrival process is known as the mixture of distribution hypothesis (MDH). The MDH provides a statistical explanation for the empirical findings, but it lacks formal model generating economic channel between new information and the relation of volume and volatility.

Other explanations that provide an economic channel are discussed in Cochrane (2016) and in Hong and Stein (2007). Cochrane (2016) argues that many of the explanations provided by the literature cannot account for the large trade volume observed in financial markets:

> The sheer volume of trading is the puzzle. All these non-information mechanisms life-cycle, preference shocks, rebalancing among heterogeneous agents [...], preference shifts, generate trading volume. But they do not generate the astronomical magnitude and concentration of volume that we see.

The arguments by Hong and Stein (2007) also revolve around the same idea: the volume in rational models is due to liquidity needs and portfolio rebalancing, which cannot account for the total volume observed in financial markets.

Hong and Stein (2007) survey another strand of the literature on volume and prices that could account for the huge volume observed in the markets. This strand of the literature considers heterogeneous-agent models where agents disagree about new information (disagreement models). Different disagreement models have incorporated mechanisms that allow for gradual information flow (news arrive at different investors at different times), limited attention (investors process only a subset of all public information) and heterogeneous priors (different investors interpret same news differently). But, at the core of this class of models is the idea that different investors have different beliefs. When investors receive new information, their differences in beliefs leads to disagreement about the interpretation of the news. This disagreement between the investors generates an additional motive for trading, other than changes in the equilibrium price. And the trading due to the disagreement results in changes to prices and trading volume. Hong and Stein (2007) argue that disagreement models are compelling since they model the joint behavior of prices and volume, while providing the underlying mechanism (difference in beliefs) that connects new information to volatility and volume.

Kandel and Pearson (1995) develop a formal tractable disagreement model. In the Kandel-Pearson model, investors trade a risky asset and observe public information. Each investor uses a different economic model to interpret the news, which leads to disagreement about the value of the risky asset and generates trading. The trading generated by disagreement is not directly dependent on changes in the asset price. Therefore, in the Kandel-Pearson model, it is possible to have high trade volume even when prices do not change.

Bollerslev et al. (2018) extend the implications of Kandel and Pearson (1995) and find an expression that directly connects the disagreement between investors to the elasticity between trade volume and price volatility. The expression that connects disagreement to the volume-volatility elasticity has two main implications. The first is that the volume-volatility elasticity is equal to unity only when investors do not disagree. The second is that the volume-volatility elasticity decreases when the disagreement between investors increase. Bollerslev et al. (2018) verify these two implications for the market index. The authors take into account proxies of investor's disagreement and estimate a volume-volatility elasticity equal to unity and that decreases when the disagreement proxies increase.

We study the connection between disagreement, trading volume, and price volatility in the context of the S&P 500 options market. There are several benefits to analyzing disagreement in the options market. First, options have two measures that are plausibly directly related to disagreement: moneyness and tenor (maturity). We argue that moneyness and tenor speak to the distribution of the market index at different quantiles and at different time horizons. Thus, by analyzing options at different values of moneyness, it is possible to uncover investors' disagreement about different parts of the distribution of the market returns. Similarly, by analyzing options at different values of tenor, it is possible to uncover investors' disagreement about the distribution of the market returns at different time horizons.

Second, the risk profile of investors in the options market is different from that of investors in the stock market. We argue that market index options are often used as instruments for hedging. In this case, investors in the options market with hedged portfolios are less exposed to moves in the market index. While moves in the market index are the main source of risk for investors in the stock market, the same is not true for investors with hedged portfolios. However, investors with hedged portfolios are still exposed to other sources of risk, like moves in the volatility. Therefore, options data recover investors disagreement about these sources of risk other than moves in the market index.

We present new evidence about the behavior of the volume-volatility elasticity as a function of the moneyness and tenor of options. We find that the volume-volatility elasticity decreases with increases in the absolute value of moneyness, which is interpreted as new information leading to more disagreement about tail events of the market index. Additionally, we find that the elasticity decreases with increases in tenor, implying that new information leads to increasingly more disagreement about the future distribution of the market index. Lastly, we estimate the volume-volatility elasticity in a nonparametric setting and obtain results that confirm our initial findings. We use simple parametric models to fit the data and use statistical inference protected by a second-step nonparametric analysis.

Issues of return predictability are not of direct focus in this paper. As mentioned in Atmaz and Basak (2018), the relation between future stock returns and disagreement is ambiguous. Some studies have found a positive relation and others have found a negative relation. In exploratory work (not reported) we experimented with predictability regressions where the dependent variable was the future index 30-day return and the explanatory variable was the implied disagreement, as measured by the estimated volume-volatility elasticity evaluated at different moneyness levels and tenor.[3] The results were not very trustworthy. Full sample, the disagreement coefficients were slightly negative with HAC-adjusted t-statistics hovering around -2.0. While the sign is more or less consistent with the recent evidence of Golez and Goyenko (2022), the results were not stable and varied in sign and (slight) statistical significance in various sub-periods. The lack of robustness makes us unwilling to report results in tabular form and leave return predictability in the context of volume-volatility to future research.

The remainder of this paper organized as follows. Section 2 briefly discusses the related literature. Section 3 discusses the Kandel-Pearson model, the volume-volatility formula developed by Bollerslev et al. (2018), and the interpretation of the elasticity as disagreement about new information. Section 4.1 motivates the use of moneyness and tenor as measures of disagreement, and discusses their interpretation with respect to the volume-volatility elasticity. Section 4.2 discusses measuring options volatility and argues for the use of a measure of residual volatility when computing the volume-volatility elasticity. Section 4.3 describes the high-frequency options data used in the empirical section and discusses how to avoid measurement errors due to microstructure noise. Section 4.4 discusses the parametric specification used to estimate the volume-volatility elasticity. Section 5 presents the main empirical findings using data on options where the underlying is the S&P 500 index. This section also discusses findings from a nonparametric estimation procedure that confirms the main findings from the parametric setting. Section 6 concludes the paper.

2 Literature Review

The relationship between volume and volatility in the options market has been studied in different contexts. Several works in the literature analyze the trading volume of options and its information content. A strand of the literature studies the relationship between options volume and returns in the stock market and tries to establish whether there is a lead-lag relationship between both markets. Easley et al. (1998) develop a model where informed investors trade in the stock market and in the options market in equilibrium. The model predicts that stock market returns lead[4] trading volume in the options market due to hedging, which is supported by the authors' empirical findings. However, the authors also find evidence that the volume of some options lead the returns of the stock market.

The finding that trades in the options market influences the stock market is supported by several other works. Pan and Poteshman (2006) use a data set that identifies the types of investors in the options market and whether their investments open new positions or close existing ones. Pan and Poteshman find evidence that the trade volume of options predicts stock returns. The predictability is stronger when using the volume of out-of-the-money (OTM) options compared to in-the-money

(ITM) options. The predictability is also stronger when there is a higher concentration of informed investors. However, the predictability is limited to firm-specific options and is not present for market index options. The authors argue that the return predictability is due to firm-specific nonpublic information.

Cao et al. (2005) study the information content in options volume prior to takeover announcements. Cao, Chen and Griffin find that volume imbalances in the options market prior to takeover announcements predicts next-day stock returns. They also find that most trades take place with slightly OTM call options, since they provide the highest leverage. However, the authors find that options volume provide no predictability during times with no takeovers.

Hu (2014) analyze whether order imbalances in the options market leads to imbalances in the stock market. The author argues that due to the lower liquidity of the options market, when investors take new positions in options, the market maker often needs to hold the options position until expiration. Market makers then hedge their positions by trading in the stock market. Thus, an order imbalance in the options market generates and imbalance in the stock market. Hu finds that imbalances in the options market can predict next-day stock returns, and that the predictability is stronger for options that are close-to-the-money or in-the-money and for stocks of smaller companies.

Ge et al. (2016) study whether trade volume that open new positions in options can predict stock returns. The authors find that volume that opens positions in calls predict positive stock returns, and volume that opens positions in puts predict negative stock returns. The authors argue that the embedded leverage of options is what attracts informed investors to the options market and is what drives the predictability of the stock returns.

Cai and Du (2018) investigate the connection between options trading volume and stock factors. The authors argue that stocks with higher beta (higher correlation to the market index) also have higher volatility, which, due to short-sales constraints, leads to the stock being overvalued. On the other hand, high-volatility stocks have a higher option demand due to hedging. The authors argue that these two aspects link options trading volume to overvalued stocks that have lower expected returns. Cai and Du find a negative relationship between the trading volume of options and the stock returns. The authors also find that a strategy that sells stocks with high options trading volume and buys stocks with low options trading volume generates positive abnormal returns.

Ryu and Yang (2018) analyze the information content of options trading volume in the context of international markets. Ryu and Yang use a data set that specifies the type of investor (foreign or domestic) and whether their investments open or close positions. The authors find that trades by foreign investment firms that open a new position predict next-day stock returns, but the trades of domestic firms do not have predictive power. The predictability is greater when OTM and short-term options are traded by the foreign investment firms. The predictability also increases with the volume of orders and when the underlying asset has short-sale restrictions.

Although most papers argue in favor of price discovery in the options market, Muravyev et al. (2013) present evidence contrary to that. The authors analyze the price movements of stocks and options when there are deviations from the put-call parity. They find that stock prices do not adjust when deviations from the parity occur. Instead, the price of the associated option moves to eliminate the deviation from the parity. The authors argue that since prices implied by options do not affect the equity markets, options prices do not contain information that is not already incorporated in the stock prices.

Another strand analyzes the relationship between options volume and stock volatility. Ni et al. (2008) investigate whether options volume has information about future stock realized volatility. Ni, Pan and Poteshman argue that traders with information about stock volatility would trade in the options market, implying that excess demand for exposure to volatility should predict volatility in the stock market. To analyze this implication, the authors construct a measure of volatility demand based on the number of open positions on calls and puts. They find that volatility demand predicts stock volatility for up to five days. The authors also find that volatility demand has a price impact on options at moments of greater information asymmetry, which is consistent with the idea that market makers update prices to protect against investors with superior volatility information.

Wang (2013) finds evidence similar to that of Ni et al. (2008) but for the market index (S&P 500). Wang uses data on VIX and SPX options to analyze the trade volume impact. The author argues that trading in VIX options is purely motivated by trading information on market volatility. Wang finds that trade volume of VIX calls have information about the future realized volatility of the market index. The author also finds a stronger predictability when there is higher uncertainty and information asymmetry in the market. The author also argues that the predictability is weaker when using the volume of SPX options, since trading in SPX options is impacted by factors other than volatility.

Other works study how the trading volume of options relates to the release of new information. Cao and Ou-Yang (2009) develop a model of trading based on differences of opinion about public information. The work is purely theoretical, but their model implies that new information leads to an increase in trading volume both in the stock and options markets. In the stock market, the volume increases at the time the new information is made available and decreases slowly, but in the options market the increase in trade volume occurs even before the arrival of the information and decreases quickly.

Choy and Wei (2012) argue that trading volume in options it not due to pure information trading, but rather due to difference of opinions. The authors use EADs to control for informed trading, and the stock returns after the announcements serve as the "ultimate revelation" of private information. Choy and Wei argue that if options trading is purely informational, then there should be no change in trading volume before and after EADs with no new information (announcements where returns were close to zero). However, if there is a change in trading volume even after controlling for information, then it should be due to other factors, possibly disagreement between investors. The authors find that information alone is not enough to justify all the trading, an argue that trading around the EADs are speculative and dominated by retail investors. They also find that any predictability of post-announcement stock returns by pre-announcement options volume is explained by pre-announcement stock returns, which also indicates a lack of informed trading in options. The authors then use proxies for differences of opinion and find that they can explain the trading volume of options both in the cross-section and in time-series.

Lemmon and Ni (2011) study the trading volume of stock and market index options in relationship to stock returns and sentiment. The authors argue that stock options are traded by unsophisticated investors who use options to speculate on future stock returns, while index options are traded for hedging demands of sophisticated investors. The authors find that sentiment and past market returns are both positively related to the trading volume of stock options, but not to the volume of market index option. The authors argue that the differences in the relationship between volume is due to the differences in the composition of traders in stock and market index options. They argue this is an indication that assets traded due to speculation are influenced by sentiment, while assets traded for other motives are less impacted by changes in sentiment.

Andreou et al. (2018) build a measure of disagreement based on options trading volume and use it to explain stock returns. The authors argue that investors that speculate on directional moves trade at a moneyness level that corresponds to their views. Optimist investors buy calls with high strikes, while pessimist investors buy puts with low strikes. In this context, if investors differ in opinion there will be a range of pessimist and optimist investors, leading to trade volume across a wide range of strike prices. However, if there is no disagreement, investors views are going to be concentrated and trade will occur over a smaller range of strike prices. Thus, the authors use the dispersion of trading volume across the different strikes as a measure for differences of opinion among investors. Now, under short-sales constraint, a higher disagreement in the options market implies the underlying asset is overvalued and its future expected returns should be small. The authors compute the disagreement measure for different stocks and find that the measure is persistent from month to month. They also find that stocks with high disagreement underperform stocks with low disagreement, and the relation is stronger for options with higher short-sale costs.

In summary, several works find evidence of price discovery in the options market. The evidence comes from the options of firms rather than options on the market index. The literature argues that informed investors choose to trade on the options market due to the leverage embedded on options. There is also evidence that options contain information about future stock volatility and also about future market volatility, and the same information-based explanation applies. The evidence on understanding the trading volume of options indicates that options trading is concentrated around the release of new information and is mostly due to disagreement among investors.

The current paper relates to the existing literature that argues in favor of disagreement among investors as an explanation for the trading volume of options. We use the trading volume of options as a way to measure disagreement among investors, and, in this sense, the current work is related to Andreou et al. (2018). However, to measure the disagreement among investors, we use the disagreement model of Kandel and Pearson (1995) and the extension by Bollerslev et al. (2018). [5] This approach requires the use of volatility in addition to the options volume to measure disagreement, but allows us to investigate how the disagreement changes with the time-frame and the distribution of market returns.

3 Interpreting the Relation between Trade Volume and Price Volatility

The starting point is the model of Kandel and Pearson (1995), in which investors disagree about the interpretation of new information. Bollerslev et al. (2018) extend the findings from Kandel and Pearson (1995), and provide an interpretation to the volume-volatility elasiticty. This section discusses the Kandel-Pearson model and the interpretation of the volume-volatility elasticity proposed by Bollerslev et al. (2018). If the reader is familiar with both works, this section can be skipped.

3.1 Trade Motivated by Disagreement About New Information

In Kandel and Pearson (1995) the authors explore the relationship between the trading volume of an asset and its returns. More specifically, the authors seek to understand why the trading volume in financial markets is huge even in periods where prices do not change. The authors argue that this happens due to differences in the interpretation of new information by traders.

In the setting of Kandel and Pearson (1995), there is a competitive market with a risk-free and a risky asset. The risk-free asset has a zero return rate, but the risky asset has a random future payoff of X. There are two types of traders trading in this market: type 1 and type 2. The proportion of type 1 traders is given by $\alpha \in [0, 1]$, and that of type 2 traders is given by $1 - \alpha$.

Each trader has prior beliefs about the random payoff X of the risky asset. Traders of type 1 believe $X \overset{d}{\sim} \mathcal{N}\left(X_1, Z_1^{-1}\right)$, while traders of type 2 believe $X \overset{d}{\sim} \mathcal{N}\left(X_2, Z_2^{-1}\right)$. The assumption that $X_1 > X_2$ is used without loss of generality. The different types of traders expect different payoffs for the risky asset ($X_1 \neq X_2$). They disagree about the average payoff, and this disagreement captures the optimism or pessimism of each trader. The variables Z_1 and Z_2 represent the precision of the traders with regards to their beliefs about the expected asset payoff. The difference in precision captures how heterogeneous is the confidence of the different investors with respect to their beliefs.

The model consists of three time periods. In period one the traders maximize their expected utility given their beliefs about the payoff of the risky asset, which is realized at the third period. The investors have a negative exponential utility function $U(w) = -\exp(-\lambda w)$, where λ is the common risk aversion of the traders. A competitive market equilibrium is achieved, and the risky asset price P_1 is such that it is available in zero net supply. The demand for the risky asset for each type of investor at period 1 is given by:

$$\text{Type 1} : m_{1,1} = (X_1 - X_2) \cdot \alpha \frac{Z_1 Z_2}{\lambda \hat{Z}} \tag{1}$$

$$\text{Type 2} : m_{2,1} = (X_2 - X_1) \cdot (1 - \alpha) \frac{Z_1 Z_2}{\lambda \hat{Z}} \tag{2}$$

where $\hat{Z} \equiv \alpha Z_1 + (1 - \alpha) Z_2$.

Assume, without loss of generality, that investors of type 1 have a higher expectation for the payoff of the risky asset than investors of type 2. Then, in equilibrium, investors of type 1 hold the asset ($m_{1,1} > 0$), while those of type 2 sell it ($m_{2,1} < 0$). The amount demanded (or supplied) by each investor depends on their confidence about their beliefs. For example, as the confidence of type 2 investors decrease ($Z_2 \approx 0$), the amount supplied by these investors converges to zero.

In the second period, the traders observe a public signal $L = X + \varepsilon$ about the risky asset payoff. The signal is a noisy version of the actual payoff. However, each type of investor interprets the signal differently. The difference in interpretation stems from each investor's assumption about the noise term ε. Traders of type 1 believe that $\varepsilon \overset{d}{\sim} \mathcal{N}\left(\mu_1, b_1^{-1}\right)$, while traders of type 2 believe that $\varepsilon \overset{d}{\sim} \mathcal{N}\left(\mu_2, b_2^{-1}\right)$. The parameters μ_1 and μ_2 represent the traders expectations about the impact of the noise on the public signal. The confidence of the investors about the impact of the noise on the public signal is captured by b_1 and b_2.

Different investors know that they have different interpretations of the signal, but they agree to disagree about the signal's interpretation. The investors take the new information into account and update their beliefs about the risky asset's future payoff. The updated expectations of the risky asset's payoff are given by:

$$\text{Type 1} : \mathbb{E}_1\left(X|L\right) = \rho_1 X_1 + (L - \mu_1)(1 - \rho_1), \rho_1 \equiv \frac{Z_1}{Z_1 + b_1} \tag{3}$$

$$\text{Type 2} : \mathbb{E}_2\left(X|L\right) = \rho_2 X_2 + (L - \mu_2)(1 - \rho_2), \rho_2 \equiv \frac{Z_2}{Z_2 + b_2} \tag{4}$$

After observing the new information, each investor expects that the asset payoff will be a combination of their prior beliefs and the information obtained from the public signal. Notice that each investor subtracts from the public signal what they believe is the expected value of the noise term. The combination of the priors with the new information depends on the investors' confidence regarding their priors and regarding the noise term. For example, if the precision of investors of type 1 regarding their prior beliefs is high (high Z_1), then when these investors observe the new information, they will revise their expectations about the payoff, but will attribute a higher weight to their prior beliefs instead of the new information. However, if these investors have a very high precision regarding the noise term (high b_1), then their revised expectations will be mostly derived from the new information.

The investors also revise their precision regarding the expected payoff, which is now given by:

$$\text{Type 1} : \text{Var}_1\left(X|L\right) = \frac{1}{Z_1 + b_1} \tag{5}$$

$$\text{Type 2} : \text{Var}_2\left(X|L\right) = \frac{1}{Z_2 + b_2} \tag{6}$$

Investors' with a high precision on their priors and on the impact of noise in the new information will have more confidence on their updated expectations about the risky asset's payoff.

Given the investors updated beliefs, the traders again maximize their expected utility and a new competitive market equilibrium is achieved. The new price for the risky asset is denoted by P_2 and is such that the risky asset is available in zero net supply. The investors now have different demands for the risky asset, which can be used to compute the trading volume due to the arrival of the public signal. The trading volume is simply the change in the investors' demand for the risky asset from the first to the second period. Denote the 2nd-period demand for the investor of type 1 by $m_{1,2}$, and $m_{2,2}$ for the type-2 investor. Then, the trading volume is:

$$\text{Volume} = |m_{1,2} - m_{1,1}| = |m_{2,2} - m_{2,1}| \tag{7}$$

Given the demands for the second period, it is possible to compute the trading volume as a function of the change in the risky asset's price:

$$\text{Volume} = |\beta_0 + \beta_1(P_2 - P_1)| \tag{8}$$

$$\beta_0 = \frac{\alpha(1-\alpha)}{\lambda(\alpha b_1 + (1-\alpha)b_2)} b_1 b_2 (\mu_2 - \mu_1)$$

$$\beta_1 = \frac{\alpha(1-\alpha)}{\lambda(\alpha b_1 + (1-\alpha)b_2)} (Z_2 b_1 - Z_1 b_2)$$

Equation 8 above relates the risky asset's trade volume to two terms: a term (β_0) that captures disagreement among the investors and a term ($\beta_1 \Delta P$) that depends on the return of the risky asset.

The equation above dictates the relation between trading volume, returns and new information. The return (ΔP) of the risky asset depends on the public signal made available on the second period and on how the different investors interpret the signal. The effect of the signal implies that there are two cases to be analyzed when it comes to understanding Equation 8. The first case is when the public signal leads to a change in the price of the risky asset: $\Delta P \neq 0$. In this case, the trading volume depends on how big the asset return is. Given $\beta_1 \neq 0$ and fixing some β_0, the model implies that high returns should be associated with high trading volumes.

The second case is when the public signal does not lead to a change in the price of the risky asset: $\Delta P = 0$. In this case, the trade volume is not necessarily zero and depends only on the term β_0. The value of β_0 is a function of the investors disagreement about the interpretation of the public signal. As long as the traders disagree about the mean of ε, that is $\mu_1 \neq \mu_2$, the value of β_0 will be non-zero. In addition, as the disagreement between investors increases (higher $|\mu_1 - \mu_2|$), so does the trading volume.

The Kandel-Pearson model generates an additional motive for trading other than returns. The model implies that even when returns are zero, the disagreement among investors regarding the interpretation of public information generates trading volume. Furthermore, as the disagreement between investors increases, the relationship between trading volume and returns weakens. And, for a sufficiently high disagreement, most of the trading volume is driven by disagreement rather than returns. Thus, at times when investors' disagree about the interpretation of new public information, we expect a weak relationship between volume and returns. But, at times when investors' agree about the interpretation of new public information, we expect a strong relationship between volume and returns.

3.2 Measuring Disagreement by Volume-Volatility Elasticity

Bollerslev et al. (2018) propose a weaker version of the volume equation from Kandel and Pearson (1995) (Equation 8). Bollerslev, Li and Xue argue that while the volume equation summarizes the relationship between trade volume and price changes in response to the release of new information, it also establishes an exact relationship between random quantities, which is a condition too strong to impose. A more realistic condition, according to the authors, is that Equation 8 should hold on average, resulting in a moment condition.

The setting in Bollerslev et al. (2018) simplifies the model from Kandel and Pearson (1995) by assuming both types of investors have the same precision when interpreting the new information: $h \equiv b_1 = b_2$. This simplification implies that β_0 and β_1 can be written as:

$$\beta_0 = r\alpha(1-\alpha)h(\mu_2 - \mu_1) \tag{9}$$

$$\beta_1 = r\alpha(1-\alpha)(Z_2 - Z_1) \tag{10}$$

where $r \equiv 1/\lambda$ is the degree of risk tolerance of traders. Now the disagreement regarding the public signal is captured by β_0 via $\mu_2 - \mu_1$.

Given Equation 8 and the additional assumption that $\Delta P \overset{d}{\sim} \mathcal{N}(0, \sigma)$, the authors derive an expression for the expected trading volume in function of the volatility σ:

$$\mathbb{E}[\text{Volume}] = \frac{2}{\pi} |\beta_1| \, \sigma \exp\left(\frac{-\beta_0^2}{2\beta_1^2\sigma^2}\right) + |\beta_0| \left(2\Phi\left(\frac{|\beta_0|}{|\beta_1| \, \sigma}\right) - 1\right) \tag{11}$$

where Φ is the cumulative density function of a standard normal distribution. The equation above relates the expected trading volume to the volatility of returns, but also the terms β_0 and β_1.

While it is possible to interpret the equation for the expected volume in terms of the disagreement between investors (β_0), a much clearer expression can be obtained by computing the elasitcity between the trading volume and the price volatility:

$$\text{Volume-Volatility Elasticity} = \frac{1}{1 + \psi\left(\frac{|\beta_0|}{|\beta_1|\sigma}\right)} = \frac{1}{1 + \psi\left(\frac{h}{\sigma} \cdot \frac{|\mu_2 - \mu_1|}{|Z_2 - Z_1|}\right)} \tag{12}$$

where ψ is an increasing function with $\psi(0) = 0$ and $\lim_{x \to \infty} \psi(x) = \infty$. The denominator in the elasticity equation is a function of the investors' disagreement regarding new information: $|\mu_2 - \mu_1|$.

The expression for the volume-volatility elasticity has two important features. First, the elasticity is no greater than one and is equal to one only when $\mu_2 = \mu_1$. That is, the volume-volatility elasticity is maximized only when there is no disagreement among traders regarding the public signal. Second, the elasticity is a decreasing function of the level of disagreement. Thus, a higher level of disagreement (larger $|\mu_2 - \mu_1|$) implies in a smaller volume-volatility elasticity.

Bollerslev et al. (2018) analyze the implications of the volume-volatility elasticity regarding disagreement between investors. The authors study the behavior of the volume-volatility elasticity for the market index around macroeconomic news announcements. Initially, the authors estimate a volume-volatility elasticity below unity, which is in line with the predictions of Equation 12. The authors then control for disagreement by using proxies based on the survey of professional forecasters and economic uncertainty. When disagreement is accounted for, the volume-volatility elasticity estimates are close to unity, and the disagreement proxies have a negative impact on the elasticity.

This paper also focuses on understanding disagreement between investors via the volume-volatility elasticity, but it centers the analysis on the options market. Next, we discuss the advantages of analyzing disagreement in context of options.

4 Empirical Methodology

4.1 Disagreement in Options Markets

Our study of disagreement in options markets is motivated by two key variables that are plausibly directly related to disagreement: moneyness and tenor. We start by discussing insights available from core asset pricing theory and proceed to a more detailed description of the empirics.

Buraschi et al. (2014a,b) provide explicit general equilibrium frameworks for considering disagreement in options markets, and, in particular, why we should expect disagreement to be related to options' moneyness and tenor. In their general equilibrium frameworks, optimistic and pessimistic traders have different views about the distributions of cash flows into the indefinite future. The equilibrium state price density features an endogenous stochastic volatility that is unambiguously increasing with respect to disagreement variables. Therefore, contingent claims prices embed a nonzero premium for disagreement risk. Option markets provide means for the optimistic traders to write nonlinear hedging contracts, i.e. put options, for the pessimistic traders. Because of the risk premium, the magnitude of the volatility smile and the volatility risk premium will thereby relate to disagreement. This connection is evident across the panels in Figure 4 on page 116 of Buraschi et al. (2014b), where it is seen that the volatility risk premium is quite sensitive to disagreement away from the money and relatively insensitive at the money. As for tenor, disagreement in these general equilibrium models is a key determinant of implied volatility at any horizon. Thus, we naturally expect the term structure of implied volatilities to connect to the term structure of disagreement about expected future cash flows.

4.1.1 Moneyness and Tenor: Measurements

Consider an option with strike price K and time to expiration τ. Let S denote the current price of the underlying (in this case the S&P 500 index) and σ the underlying's volatility. Then, we can define the moneyness of this option by:

$$\text{Moneyness} = \frac{\ln \frac{K}{S}}{\sigma\sqrt{\tau}} \tag{13}$$

This measure is commonly referred to as the normalized moneyness of an option, but henceforth we refer to it simply as the moneyness of an option.

The numerator in the expression above is the logarithm of the strike-to-underlying ratio, which we refer to as simply the strike-to-underlying ratio. The strike-to-underlying ratio is negative for out-of-the-money (OTM) puts, where $S > K$, but the ratio is positive for in-the-money (ITM) puts, where $S < K$. Notice that for call options the sign of the strike-to-underlying ratio inverts: OTM calls ($S < K$) have positive ratios, while ITM calls ($S > K$) have negative ratios. In this work we use only data on put options (see discussion in section 4.3). Thus, in what follows, references to options with negative moneyness should be understood as OTM put options while positive moneyness refer to ITM put options.

The strike-to-underlying ratio represents by how much the price of the underlying asset would have to move so that the option is exactly at-the-money (ATM). Specifically, the ratio is the return required so that the price of the underlying asset is equal to the strike price. The denominator of the moneyness standardizes the strike-to-underlying ratio.

Now, consider σ to be a measure of the yearly volatility of the underlying asset's price, and let τ measure the time to expiration of the option, also in years. Then, the term $\sigma\sqrt{\tau}$ represents the volatility of the underlying asset scaled to the total time duration of the option. When we divide the strike-to-underlying ratio by $\sigma\sqrt{\tau}$, we still obtain by how much the underlying asset has to move for the option to be ATM, but in units of standard deviations. Therefore, we interpret an option's moneyness as the number of standard deviations the underlying price needs to move for the option to be exactly ATM. For example, a put option with moneyness equal to -3 would need the underlying price to decrease by three times its standard deviation for the option to be ATM.

The tenor of an option is simply the amount of time until its expiration date. In a similar fashion to moneyness, the tenor can be interpreted as the amount of time left for an option to expire ATM. Therefore, the probability that an option with some tenor will expire ATM depends on the distribution of returns in the same time period. Options with shorter tenors depend on the distribution of returns over short time horizons, while options with longer tenors depend on the distribution of returns over longer time horizons.

4.1.2 Interpretations and Empirical Predictions

In this work, we use options where the underlying asset is the S&P 500 index, known as SPX options. The price of an SPX option can be computed as the risk-neutral expectation of the option's payoff. Let $\mathbb{Q}$ denote the risk-neutral distribution as implied by, say, the model as in Buraschi et al. (2014a,b).[6] Then the price of an SPX put option is given by:

$$P_t(K,T) = \exp(-(r-q)T)\mathbb{E}_t^{\mathbb{Q}}[\max\{K - S_T, 0\}] \tag{14}$$

where we denote the price of the market index at time t by S_t, K as the strike price of the put option, T its tenor (time to expiration), and r and q denote the corresponding interest rate and dividend yield. The equation above shows that the price of a put option is given by the risk-neutral expectation of its payoff. The payoff of the options depends on the difference between the strike price and the market index at a later time T. And the expected value of the payoff depends on the distribution of the market index at expiration time.

Consider a put option where the strike price K is very low in comparison to the current market index value S_t. For this option to expire in-the-money, the value of the market index would have to decrease substantially before time T. Therefore, the price of this put option greatly depends on the left-tail of the probability distribution of the market index at time T. As the strike price increases, so does the option's moneyness. Indicating a higher likelihood of the option expiring at-the-money. The dependence of the option's price on the left-tail of the distribution of S_T decreases. At the same time, the price of the option depends increasingly more on the center of the distribution of S_T. On the other extreme, if the strike price K is very high in comparison to the current market index value S_t, then the option's moneyness will be positive and high. If the strike price is very high, then the price of the option is more dependent on the right tail of the distribution of S_T.

The distance between the strike price and the market index is captured by the option's moneyness. Therefore, moneyness measures how much the price of an option depends on the different parts of the distribution of the market index. From this perspective, depending on the moneyness of the options, new information regarding the market index would lead to disagreement regarding different quantiles of the distribution of S_T. For this reason, we interpret moneyness as a measure of disagreement about different parts of the distribution of the market index. In addition, due to the difficulty of analyzing tail events, we expect new information to generate more disagreement regarding the tails of the distribution of the market index. Therefore, we expect a negative relationship between the absolute value of moneyness and the volume-volatility elasticity.

A similar explanation applies to the tenor of an option. The option pricing equation shows that the expectation depends on the distribution of S_T, which depends on the time to expiration T. Therefore, the pricing of the option at different tenors depends on how the distribution of S_T changes for different values of T. From this perspective, the tenor captures disagreement regarding the distribution of the market index at different times. Since events far in the future are subject to more uncertainty than events about to happen[7], the value of options with longer times to expiration are subject to more uncertainty than options with short tenors. This is especially true for the case of the market index, where the uncertainty regarding the future price increases with time.

A reviewer noted that the large option gamma at the money implies delta-sensitivity and possible heightened disagreement; however, there is a compelling argument for reduced disagreement at the money. Todorov (2019) shows that, under plausible regularity conditions, the implied volatility of at the money options reveals directly the spot volatility of the underlying asymptotically as tenor goes to zero. In effect, this means that the short-dated at-the-money options reveals the local spot volatility (up to manageable econometric issues) to traders, and all options traders would perforce agree on the local volatility. By way of essentially eliminating volatility uncertainly, the scope for disagreement about option value at the money. Of course the argument does not carry over to long dated options whose values are influenced by the path of volatility over the life of the option, model uncertainty, and a host of other factors. Thus, we expect a negative relationship between tenor and the volume-volatility elasticity.

The moneyness and tenor are readily available for any option. These two measures are directly related to disagreement and are used in the analysis of the volume-volatility elasticity. While options have these direct measures of disagreement, the volatility of an option is not as straightforward to measure. This is discussed in the next section.

4.2 Measure of Volatility for Options

An option is an asset with a payoff that depends on the value of another asset, the underlying. Even though there is this dependence, we can think of an option as an asset on its own. In this case, the volatility of an option is just the volatility of this option's prices.

Assume prices of options can be observed at the high-frequency. Specifically, assume that prices are observed at equi-spaced intervals within each day and for several days. Then, for each day, it is possible to compute high-frequency returns for each option using the high-frequency price observations. Let $r_{i,t}^{\text{option}}$ denote the geometric return of the price of an option at a time interval i of day t over a short time interval, e.g., 3-minute or 5-minute. Then, the volatility of this option for a given day can be estimated using the realized variance estimator (see Barndorff-Nielsen and Shephard (2002)):

$$\mathrm{RV}_t^{\text{option}} = \sum_i \left(r_{i,t}^{\text{option}} \right)^2 \tag{15}$$

where the summation is across all time intervals within a day. $\mathrm{RV}_t^{\text{option}}$ can be computed each day for each option and relies on the use of high-frequency returns. The realized variance is an estimate of the daily variance for a given option at day t.

Another viewpoint is that an option is fundamentally linked to its underlying. Indeed, the value of an option is partially derived from the current price and volatility of the underlying asset. In the Black-Scholes world, this connection is made explicit by the Black-Scholes Delta:

$$\Delta_t^{Black-Scholes} \equiv \frac{\partial \mathrm{P}_t}{\partial S_t} = \Phi \left(\frac{\ln \frac{S_t}{K} + (r + \sigma^2/2)(T-t)}{\sigma \sqrt{T-t}} \right) \tag{16}$$

where P_t is the price of a put option at time t with some strike price K and expiration time T, and S_t is the underlying price at time t. In this case, a price change in the underlying translates into a price change in the option. This connection implies that the volatility of an option can be divided into two terms. The first term is the price volatility of an option due to changes in the underlying price. In our case, the underlying is the market index, so we denote this first term as the impact of the market volatility. The second term is the price volatility of an option due to other changes, such as changes in the underlying volatility. This second term is denoted the residual volatility. Thus, an option's volatility can be split as:

$$RV_t^{option} = RV_t^{\text{impact of market volatility}} + RV_t^{\text{residual volatility}} \tag{17}$$

Splitting the volatility in these two terms is relevant because different economic agents are interested in different parts of the total volatility. According to Hull (2003), traders in financial institutions maintain delta-neutral portfolios, but cannot maintain portfolios neutral to other risks (other Greeks) due to the difficulty of finding financial instruments to hedge with at competitive prices. This suggests that portfolios of financial institutions are hedged against changes in the market index (via delta-hedging), but are still exposed to the residual volatility. This point of view is supported by Ni et al. (2008), who argue that market makers can delta-hedge directional moves but cannot avoid volatility risk. Since the position of market makers is delta-neutral, it is possible that the option volatility relevant to these agents is not the total volatility (RV_t^{option}), but actually the residual volatility ($RV_t^{\text{residual volatility}}$). On the other hand, the total option volatility would be relevant to speculators that trade options for directional moves, since the position of these agents would not be delta-neutral.

This work assumes that the sophisticated agents trading in the SPX options market are either maintaining a delta-neutral position or are speculators that trade on volatility information (as in Ni et al. (2008)). This is partially supported by the fact that financial institutions (sophisticated agents) maintain delta-neutral portfolios (Hull (2003)), that there is a high demand for SPX puts due to the hedging demand of institutional investors (Bollen and Whaley (2004)), and that unsophisticated traders account only for 3% of the total volume of trades in the SPX options market (Lemmon and Ni (2011)).

The discussion above motivates using only the residual part of the volatility when computing the volume-volatility elasticity. To do so, we map the moves in an option's price into moves due to changes in the option's underlying and moves due to other (residual) changes:

$$r_{i,t}^{\text{option}} = \beta r_{i,t}^{\text{market}} + \varepsilon_i \tag{18}$$

To recover the impact of the market volatility on the option volatility, we estimate the equation above separately for each option and for each day in the sample. That is, fixed a day (fixed t) and fixed an option (fix the strike price and the tenor), we estimate the regression above using all of the high-frequency returns available in that day. we then use the residuals to recover the impact of the market volatility and compute the residual volatility:

$$RV_t^{\text{impact of market volatility}} \equiv \sum_{i=1}^{n} \left(\hat{\beta} r_{i,t}^{\text{market}} \right)^2 \tag{19}$$

$$RV_t^{\text{residual volatility}} \equiv \sum_{i=1}^{n} \hat{\varepsilon}_i^2 \tag{20}$$

In the next sections, we use the residual volatility as the measure of volatility for an option. It represents how much variation in an option's price is due to factors not directly hedged by sophisticated agents.

4.3 Data

This work uses data on S&P 500 options (SPX options) made available by the Chicago Board Options Exchange (CBOE). SPX options are European style and are cash-settled with a multiplier of $100 on the market index. The multiplier implies that if the S&P 500 is at the 2,867.24 level, then the notional value of one option is $286,724.00. The data set contains bid and ask quotes and trade volume sampled every 1-minute for all options. The data spans 2007 to 2016, totaling 2458 days in the sample after cleaning procedures[8]. To provide some sense of the scale of the analysis and data cleaning, there were 3,479,352,243 records in the raw data, and a number of techniques had to be employed to clean and condense the set.

The focus of the analysis is on put options (call options are excluded), since these options are heavily traded and provide a natural hedge against market downturns (Bollen and Whaley (2004)).

The strike price of options is readily available. However, to obtain the tenor of SPX options, special care needs to be taken due to differences in the types of expiration. There are two types of SPX options: standard and weekly. The standard SPX options have expiration dates set for the 3rd Friday of every month, and the actual expiration time is at the market open (9:30 a.m. Eastern Time). The weekly SPX options, however, have expiration dates at multiple days every week, but their actual expiration time is at the close of the trading day (4:00 p.m. ET)[9]. To obtain precise tenors for the SPX options, we compute the tenor at the minute level taking into account the differences in expiration times, and then convert the units to days.

In the Kandel-Pearson model investors disagree about a public signal. In Bollerslev et al. (2018), the public signals considered are macroeconomic announcements, and the authors compute the elasticity from data around the announcement times. In this work, however, we consider public signals as being the information disseminated throughout a trading day. Therefore, we analyze the volume and volatility at the daily frequency, which uncovers investors' disagreement about the information they observed throughout the day.

The bid and ask quotes of each option are used to compute mid-prices. we estimate an option's residual variance from its high-frequency intra-day mid-prices. However, it is known that high-frequency prices can be affected by microstructure noise, which leads to high biases in variance estimates. To mitigate the effect of microstructure noise, we sparsely sample the mid-prices to the 5-minute frequency. Sparse sampling is a standard practice in the literature and, in this case, is supported by a volatility signature plot for SPX options.

The volatility signature plot is displayed in Figure 1. The figure shows the average realized volatility for different options of different moneyness and computed using data sampled at different frequencies. The sampling frequency of the data varies from 1-minute (left) up to 20-minutes (right). The volatility signature plot can indicate whether microstructure noise affects the return observations (see Zhou (1996), Andersen et al. (2000) and Hansen and Lunde (2006)). If there is no microstructure noise, then the volatility estimate should converge to a value as the sampling frequency increases. In other words, as we move from right to left in the figure, the plot should be flat if there is no microstructure noise. However, if microstructure noise is present, then the volatility estimate would diverge as the sampling frequency increases.

The figure indicates that the realized volatility estimator is stable for sampling frequencies as high as 5-minutes. For sampling frequencies above 5-minutes, notice the sharp increase in the volatility estimates for out-of-the-money options, which indicates the presence of microstructure noise at higher sampling frequencies. To avoid the effect of microstructure noise, we sparsely sample the options prices to the 5-minute frequency[10].

To compute the residual volatility of each option, we also use data at the 5-minutes frequency. The residual volatility is estimated separately for each option and for each day in the sample, as discussed in Section 4.2. The moneyness of each option is computed daily at the end of the day.

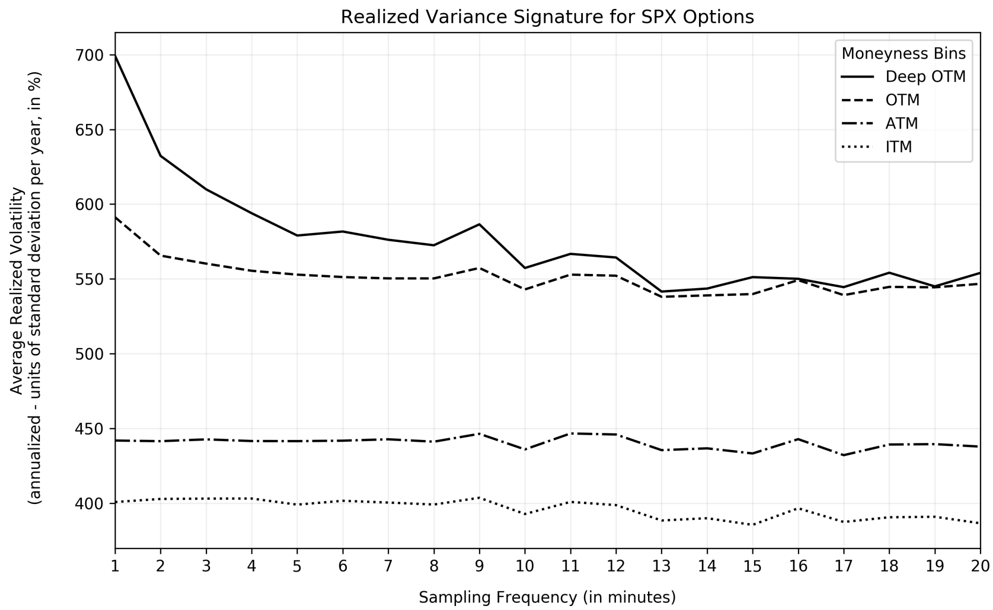


Figure 1: The figure shows the average realized volatility (annualized in percentage) for options of different moneyness. The realized volatilities are computed using different sampling frequencies. The sampling frequency varies from 1-minute up to 20-minutes. Higher sampling frequencies are at the left side of the figure, while lower sampling frequencies are at the right side of the figure. The realized volatility is estimated as the sum of squared returns over each day for each option. The figure shows an increase in the volatility for out-of-the-money options when the sampling frequency is below 5-minutes, indicating the presence of microstructure noise.

The trade volume is computed daily for each option and is given by the option's total volume of trades at that day. As discussed in Tauchen and Pitts (1983), the presence of a trend in the trading volume data could mislead the results of the volume-volatility analysis. We verify that indeed a trend exists in the data. To mitigate the trend's effects, we detrend the volume with a moving average based on the volume on the previous 252 days.

The data set on SPY was obtained from TickData[11], and it contains the last closing price of every 5-minute interval for all days from 2007 to 2016.

This study also considers two measures of sentiment. We want to explore the impact of sentiment on the disagreement between investors. The idea is that at times of optimism investors disagree less about new information, while at times of pessimism new information leads to greater disagreement.

The first sentiment measure is from Baker and Wurgler (2006) and Baker and Wurgler (2007)[12].

Baker and Wurgler construct a sentiment measure based on the principal component of various market proxies for sentiment. Specifically, they consider the dividend premium of stocks, the closed-end fund discount, the first-day return and trade volume on IPOs and the equity issues over total new issues[13]. The authors find evidence that their sentiment measure explains stock returns on the cross-section (increase in expected return on speculative and difficult-to-arbitrage stocks when sentiment increases, but the opposite for safe and easy-to-arbitrage stocks), is correlated with an equal-weighted market index and predicts future average returns (lower average return for speculative stocks when past sentiment was high). The sentiment index is available at the monthly frequency from 2007 to September of 2015.

The second sentiment measure is the FEARS index from Da et al. (2014). The FEARS (Financial and Economic Attitudes Revealed by Search) index is based on the volume of internet search queries for various economics-related terms, such as "gold prices", "recession" and "crisis". The authors aggregate the search volume for different terms into an index where the terms are chosen according to how well they explain negative returns. The authors show that a high level of FEARS is related to lower contemporaneous returns but predict higher future returns (over the next few days), and that the index explains volatility and the flow of mutual funds from equity to bonds. The authors argue that, contrary to sentiment measures based on market proxies, FEARS is not the equilibrium outcome of the economy. The authors also argue that FEARS is based on agents' attitudes and is possibly more truthful than indices based on surveys. FEARS is available at the daily frequency from 2007 to 2016.

The next section discusses the empirical methodology to analyze disagreement in the SPX options market.

4.4 Regression Specification

Disagreement about new information is revealed by the volume-volatility elasticity. According to Equation 12, if there is no disagreement between investors, then the volume-volatility elasticity should be unity. However, if there is disagreement, then the elasticity should be less than unity. Additionally, the elasticity should decrease to the extent that disagreement increases.

To analyze these implications, we consider the following regression specification:

$$\ln \text{Volume} = (\alpha_0 + \alpha_1' X_0) + (\beta_0 + \beta_1' X_1) \cdot \ln \text{Residual Volatility} + \varepsilon \tag{21}$$

where α_0, β_0 are scalars, α_1, β_1 and X_0, X_1 are vectors, and the terms in X_0 and X_1 include different control and explanatory variables. By estimating the log-log regression above, we directly recover the volume-volatility elasticity. The elasticity is given by $\beta_0 + \beta_1' X_1$. Of particular importance to the analysis are the variables included in X_1. These variables are plausibly related to disagreement, which allow us to evaluate the predictions regarding disagreement from the estimated β_1

coefficients. If the variables in X_1 capture investors' disagreement in the options market, then we expect the estimated value of β_0 to be close to unity, while those of β_1 to be negative.

The variables included in X_1 are the absolute value of moneyness, tenor and an interaction term between both. The absolute value of moneyness indicates how far an option is from being at-the-money, and is related to disagreement about the different quantiles of the distribution of the market index. For example, options far from the money will have values of moneyness far from zero, which indicates that the price of such options depend on the tails of the distribution of the market index. Since the probabilities of tail events are hard to predict, the impact of new information on the tails of this distribution should generate more disagreement. An underlying assumption is that the effect of decreases or increases in the moneyness have the same impact on the elasticity. This is plausible because the distribution of high-frequency returns is symmetric, and ITM puts are equivalent to OTM calls.[14]

We also expect the effect of tenor on elasticity to be negative, since higher tenor is related to uncertainty about the future distribution of the underlying asset. The interaction between the absolute value of moneyness and tenor allows us to capture the effect of moving away from the money on options that are close to expiration and on options that are far from expiration. Since options that are far from the money and have higher tenor are also subject to more uncertainty, we expect the interaction term between the absolute value of moneyness and tenor to be negative.

The next section discusses the empirical results.

5 Empirical Results

The main interest of the empirical analysis is to investigate the elasticity between trading volume and volatility under the predictions of Kandel and Pearson (1995) and the developments of Bollerslev et al. (2018). The volume-volatility elasticity can be interpreted as the investors' disagreement with respect to new public information. Equation 12 implies that the volume-volatility elasticity is equal to unity in the case of no disagreement among investors. However, in the case where investors do disagree, the volume-volatility elasticity should be smaller than unity. The equation also implies that the volume-volatility elasticity should decrease as investors' disagreement increases.

The estimates for the volume-volatility elasticity are presented in Table 1 below. The reported standard errors are cluster-robust.

In column I, there are no additional explanatory variables, so the elasticity estimate is directly given by the estimated value of β_0. The elasticity estimate in the first column (I) is significant, but also smaller than unity. This finding is in line with the predictions of the disagreement model, and indicates disagreement among investors in the options market.

The regressions reported in columns II, III, IV and V, include additional explanatory variables that are plausibly related to disagreement. The variables included are moneyness and tenor (trans-

Table 1: *The table reports the volume-volatility elasticity estimates including different exaplanatory variables for the elasticity. The estimates are based on the specification in Equation 21. The estimated values of α_0 and α_1 are not reported for brevity. Estimates are based on the full sample of cleaned data, spanning 2007 to 2016. Moneyness is measured in units of standard deviations and tenor is measured in days. The standard errors are cluster-robust as in Petersen (2009).*

	I	II	III	IV	V
Baseline Estimates					
Elasticity (β_0)	0.230	0.503	0.635	0.911	1.025
	(0.024)	(0.027)	(0.076)	(0.083)	(0.143)
Explanatory Variables in Elasticity (β_1)					
$\lvert$Moneyness$\rvert$		-0.203		-0.203	-0.289
		(0.015)		(0.014)	(0.060)
$\ln$ Tenor			-0.117	-0.118	-0.150
			(0.020)	(0.021)	(0.038)
$\lvert$Moneyness$\rvert \cdot \ln$ Tenor					0.024
					(0.016)
Adjusted R^2 (in %)	22.52	23.55	22.95	23.99	24.01
Number of Observations			446053		

formed and interacted) and are allowed to impact the elasticity. In these columns, the elasticity is given by a combination of β_0 and the parameters β_1.

In column II, the absolute value of moneyness is included in the regression. The elasticity estimate is now given by $0.5 - 0.2\,|\text{Moneyness}|$. By including a disagreement measure in the regression, the baseline elasticity (when $|\text{Moneyness}| \approx 0$) estimate increases from 0.23 to 0.5. This increase suggests that moneyness captures a portion of the disagreement between investors. The baseline estimate also indicates that investors disagree on the interpretation of news and their impact on the distribution of the market index.

The coefficient for the absolute value of moneyness in column II is significant and negative. As moneyness moves away from zero, the elasticity estimate decreases, which is in line with our interpretation of the relationship between moneyness and disagreement. The negative coefficient for the absolute value of moneyness indicates that disagreement among investors increases for options far-from-the-money (either OTM or ITM puts). That is, for values of moneyness away from zero, the elasticity estimate is evidence of a higher disagreement with respect to the tails of the market index distribution. This is consistent with the idea that there is more uncertainty about the tails of the distribution of the market returns.

In column III, the tenor is included as an additional explanatory variable in the elasticity. The elasticity estimate is now given by $0.635 - 0.117 \ln \text{Tenor}$. The inclusion of tenor as an explanatory variable for elasiticy pushes the baseline elasticity estimate closer to unity. The estimated coefficient for the tenor is significant and negative. The negative coefficient is also in line with our expectations and indicates that investors' disagreement increases for options with longer times to expiration. The coefficient for the tenor is consistent with the idea that there is more uncertainty about the distribution of market returns over longer time-horizons than over shorter time-horizons.

In column IV, both moneyness and tenor are included as explanatory variables in the elasticity. The elasticity estimate is now given by $0.911 - 0.203 |\text{Moneyness}| - 0.117 \ln \text{Tenor}$. Notice that the estimates of the impact of moneyness and tenor on the elasticity are similar to the estimates in columns II and III. However, the baseline estimate (0.911) is now statistically indistinguishable from unity. The estimated value of β_0 indicates that there is no disagreement for options that are at-the-money (zero moneyness) and about to expire (tenor of 1 day). In terms of the distribution of the market index, the baseline elasticity indicates a lack of disagreement about new information when it comes to the distribution of the market index in the near future.

In column V, an interaction term between the absolute value of moneyness and the tenor is added to the regression. The baseline elasticity estimate is 1.025, which is also statistically indistinguishable from unity. Again, the baseline elasticity estimate indicates that there is no disagreement for options about to expire and that are at-the-money. The coefficient of the interaction term between moneyness and tenor is positive. A positive coefficient implies that the effect of moneyness is attenuated for options that have a long time to expiration. In other words, while there is more disagreement on options that expire far in the future, the disagreement is less dependent on how far-from-the-money an option is. The attenuation of the impact of moneyness is consistent with the idea that over longer horizons the number of big moves in the market index gets averaged out, so that there is less disagreement for options far-from-the-money. The coefficient for the interaction term, however, is statistically insignificant.

Table 2 displays the average values of moneyness and tenor for the options in the sample.

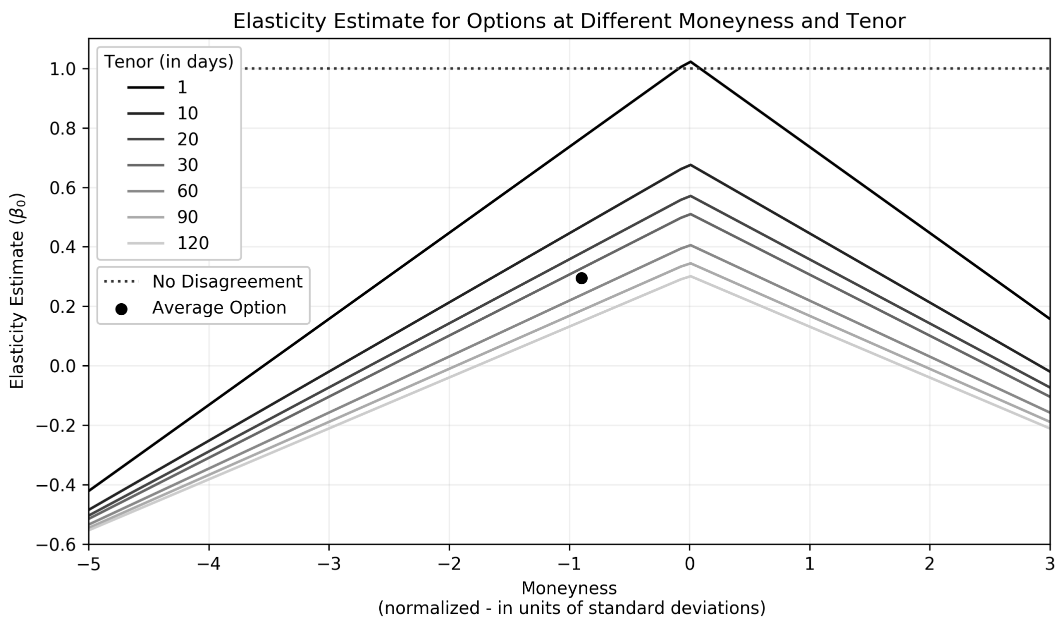
Table 2: *The table displays the average value of moneyness and tenor for a representative option.*

Average Value	
$\lvert$Moneyness$\rvert$	1.33
ln Tenor	3.34

Options with moneyness and tenor similar to the ones displayed in Table 2 are referred to as typical options. The typical option is slightly far-from-the-money, requiring the market index to move by slightly more than one standard deviation for the option to expire exactly ATM. Additionally, the time to expiration of the typical option is about 28 days. Under the estimated parameters in column IV of Table 1, the volume-volatility elasticity estimate for the typical option is 0.24, contrasted to the baseline of 0.91. The disagreement indicated by the elasticity estimate is derived in part from the typical option being either OTM or ITM and in part from its time to expiration. The moneyness indicates disagreement about the higher quantiles of the distribution of the market index, while the tenor indicates the uncertainty regarding the future value of the market index.

Using the parameter estimates from Column IV in Table 1, we compute the volume-volatility elasticity for different values of moneyness and tenor. The values are displayed in Figure 2.

Figure 2: *The figure shows the volume-volatility elasticity estimates for different values of moneyness and tenor. Fixing tenor, the elasticity decreases as options move away from the money. Fixing moneyness, the elasticity decreases as the time to expiration increases.*



The case of no-disagreement occurs for options that are ATM and that are about to expire (in a day). For any fixed tenor, there is a symmetric decrease in elasticity as options move away from-the-money. Since the tenor is fixed, this decrease in elasticity is interpreted as an increase in disagreement regarding the tails of the distribution of market returns. Fixing any moneyness level, there is a large drop in elasticity as the tenor increases, indicating a heightened disagreement regarding the future distribution of market returns. Lastly, notice that the decrease in elasticity when the tenor increases is smaller for options far-from-the-money than it is for options closer-to-the-money.

5.1 Impact of Sentiment on Elasticity

The volume-volatility elasticity was estimated taking into account the effect of options moneyness and tenor. While moneyness and tenor capture more specific aspects of the disagreement between investors, we also want to consider the impact of measures that capture a more general level of disagreement. To do so, we include two measures of sentiment as control variables when estimating the volume-volatility elasticity.

The first measure is the sentiment index from Baker and Wurgler (2006) and Baker and Wurgler (2007), henceforth referred to as the BW sentiment index. The BW sentiment index captures investors' sentiment by extracting a principal component from various market proxies for sentiment. The second measure is the FEARS index from Da et al. (2014). The FEARS index measures the Internet search volume for terms with negative economic connotations. A high value of FEARS means a higher than average search volume for these negative terms. While the BW sentiment index offers a sentiment measure based on market quantities, the FEARS index recovers sentiment from an alternative source only indirectly related to the financial markets.

We expect that when investors are optimistic, there should be less disagreement about new information, but when investors are pessimistic that new information leads to more disagreement. Therefore, we presume that the impact of the BW sentiment on the elasticity will be positive. However, for the FEARS index, we expect a negative impact on the elasticity, since higher FEARS means lower sentiment.

The elasticity estimate including the effect of the two sentiment measures are reported in Table 3 below.

The elasticity estimates are in line with the previous findings. The elasticity is the highest for at-the-money options that are about to expire (in a day). And, the volume-volatility elasticity decreases with increases in the absolute value of moneyness and with increases in tenor. This indicates investors disagree about the interpretation of information as it relates to the tails of the distribution of the market returns, and as it relates to the future distribution of returns.

The impact of the sentiment measures in the elasticity estimate is as expected. At times of optimism, there is an increase in the elasticity, indicating that new information leads to less disagreement when investors are optimistic. Notice that optimism is captured by positive values in the BW index or negative values in the FEARS index. At times of pessimism, however, there is a decrease in the elasticity, which indicates that new information leads to more disagreement when investors are pessimistic. Observe that the effect of both sentiment measures are significant, which is a sign that sentiment measured from textual sources (such as the FEARS index) can

Table 3: *The table reports the volume-volatility elasticity estimates including different exaplanatory variables for the elasticity. The estimates are based on the specification in Equation 21. The explanatory variables for the elasticity include the sentiment measure from Baker and Wurgler (2006) and Baker and Wurgler (2007), and the FEARS index from Da et al. (2014). The estimation is based on a partial sample of cleaned data, spanning 2007 to 2015. The estimated values of* α_0 *and* α_1 *are not reported for brevity. Moneyness is measured in standard deviations, tenor is measured in days, and the sentiment variables are normalized. The standard errors are cluster-robust as in Petersen (2009).*

	I	II	III	IV
Baseline Estimates				
Elasticity (β_0)	0.965	0.955	0.969	0.959
	(0.104)	(0.104)	(0.104)	(0.104)
Explanatory Variables for Elasticity (β_1)				
|Moneyness|	-0.237	-0.240	-0.240	-0.242
	(0.020)	(0.020)	(0.020)	(0.020)
ln Tenor	-0.086	-0.079	-0.085	-0.078
	(0.026)	(0.026)	(0.026)	(0.026)
BW		0.073		0.072
		(0.014)		(0.014)
FEARS			-0.056	-0.057
			(0.011)	(0.011)
Adjusted R^2 (in %)	26.44	26.76	26.47	26.80
Observations		256453		

complement the sentiment obtained from market proxies (as in the BW index).

5.2 Time Variation in Disagreement

Next, we analyze whether the relationship between volume and volatility changes throughout the years. we estimate the volume-volatility relationship as before, but slice the data into two-year periods. The estimates are presented in Table 4 below.

The baseline estimates vary from period to period, but are all statistically indistinguishable from unity (except for the 2011-2012 period). When controlling for moneyness and tenor, the baseline estimates are consistently close to unity, which is in line with the case of no disagreement among investors. The baseline estimates for the volume-volatility elasticity can be interpreted as the disagreement for options close-to-the-money and about to expire. In other words, those elasticities represent investors' disagreement about the next-day distribution of market returns.

Table 4: *The table reports the volume-volatility elasticity estimates including different exaplanatory variables for the elasticity. The estimates are based on the specification in Equation 21. The estimation is done on a bi-yearly basis, from 2007 to 2016. The estimated values of α_0 and α_1 are not reported for brevity. Moneyness is measured in standard deviations and tenor is measured in days. The standard errors are cluster-robust as in Petersen (2009).*

	2007-2008	2009-2010	2011-2012	2013-2014	2015-2016
Baseline Estimates					
Elasticity (β_0)	1.308	1.177	1.325	1.069	0.989
	(0.132)	(0.205)	(0.115)	(0.236)	(0.146)
Explanatory Variables in Elasticity (β_1)					
$\lvert$Moneyness$\rvert$	-0.287	-0.451	-0.376	-0.343	-0.244
	(0.076)	(0.098)	(0.053)	(0.111)	(0.057)
ln Tenor	-0.114	-0.120	-0.133	-0.087	-0.177
	(0.035)	(0.058)	(0.032)	(0.066)	(0.039)
$\lvert$Moneyness$\rvert \cdot \ln$ Tenor	-0.032	0.039	0.011	0.015	0.026
	(0.022)	(0.028)	(0.018)	(0.032)	(0.015)
Adjusted R^2 (in %)	26.54	25.42	23.58	35.78	24.61
Observations	40850	30391	35109	72692	267011

The coefficient estimate for the absolute value of moneyness is significant and negative for all time periods. The coefficient varies from period to period, being the highest in the 2009-2010 period, and the lowest in the period 2015-2016. The estimates indicate that the disagreement about the tails of the market index distribution could be time varying. However, the standard errors do not allow us to conclude that the estimated coefficients for the absolute value of moneyness are statistically different from one another.

The estimated coefficients for the tenor are negative for all periods, but not significant on 2009-2010 and 2013-2014. The findings indicate that disagreement increases with tenor, suggesting greater uncertainty about the market index farther into the future. However, the relationship is weakened at some of the periods, indicating a lack of disagreement as tenor increases.

The findings in Table 12 are in accordance with the previous findings. The elasticity estimate is indifferent from unity for options close-to-the-money and about to expire. The additional explanatory variables, moneyness and tenor, are directly related to the volume-volatility elasticity and drive the baseline estimates to unity.

5.3 Total Variance

The previous discussions are based on the volume-volatility elasticity. The elasticity was estimated using the residual volatility of options. In this section, we show that it is possible to obtain similar conclusions when computing the volume-volatility elasticity using the total option variance (residual and non-residual terms combined). The elasticity estimates are presented in Table 5.

Table 5: *The table reports the volume-volatility elasticity estimates including different explanatory variables for the elasticity. The estimates are based on the specification in Equation 21, but uses the total variance to estimate the elasticity. The estimated values of α_0 and α_1 are not reported for brevity. Estimates are based on the full sample of cleaned data, spanning 2007 to 2016. Moneyness is measured in units of standard deviations and tenor is measured in days. The standard errors are cluster-robust as in Petersen (2009).*

	I	II	III	IV	V
Baseline					
Elasticity (β_0)	0.052	0.358	0.803	1.117	1.337
	(0.027)	(0.051)	(0.097)	(0.104)	(0.195)
Explanatory Variables for Elasticity (β_1)					
|Moneyness|		-0.217		-0.219	-0.383
		(0.025)		(0.020)	(0.082)
ln Tenor			-0.206	-0.207	-0.269
			(0.026)	(0.027)	(0.053)
|Moneyness| $\cdot$ ln Tenor					0.045
					(0.022)
Adjusted R^2 (in %)	21.71	22.27	22.67	23.25	23.30
Number of Observations			446053		

The baseline estimates follow the same pattern as the estimates in Table 1. When moneyness and tenor are not included in the regression (column I), the volume-volatility elasticity estimate is indistinguishable from zero. This low coefficient indicates that investors' disagree about the interpretation of new information.

In column II, the absolute value of moneyness is included as an explanatory variable for the elasticity. The inclusion of moneyness pushes the baseline estimate closer to one. The coefficient for the absolute value of moneyness is significant and negative. The negative coefficient indicates more disagreement on the interpretation of news as it relates to the tails of market returns.

In column III, tenor is added as an explanatory variable in the elasticity. Again, the inclusion of tenor pushes the baseline estimate closer to one. In this case, the baseline elasticity is actually statistically indistinguishable from unity. The baseline of unity is the case of no disagreement among investors, and suggests that tenor captures disagreement among investors. The coefficient for the tenor is significant and negative, which indicates there is more disagreement about options with longer times to expiration.

In column IV, both moneyness and tenor are included as explanatory variables for the elasticity. In this case, the baseline estimate is also indistinguishable from unity. The effect of moneyness is not negligible, which indicates that both moneyness and tenor capture disagreement between investors. The coefficients for moneyness and tenor are similar to the coefficients estimated when the variables are included one at a time.

In column V, an interaction term is added to the regression. Like in 1, the coefficient for the interaction term is positive, but statistically indistinguishable from zero.

Overall, the estimates using the total variance of options presents the same evidence as the estimates using the residual variance. There is disagreement among investors, and there is greater disagreement about the tails of the market index distribution. The disagreement also increases with increases in the time frame.

The fact that the estimates above were similar to the previous results is also inline with the argument that the relevant part of the volatility for sophisticated traders is the residual volatility. It is possible to think about the residual volatility as being a non-redundant part of the volatility. Indeed, it always possible to use the market index to delta-hedge a portfolio, eliminating the risk of market moves. The similarity of elasticity results also supports the view that the residual volatility is the non-redundant part of the volatility.

5.4 Non-parametric Results

The results presented before are based on the parametric estimation of Equation 21. Now, we use a nonparametric approach to recover the volume-volatility elasticities. The idea is to allow for a more flexible impact of moneyness and tenor on the elasticity estimates. To do so, we group options by moneyness and tenor. The intervals for moneyness range from -5 to 3, which is the range of moneyness levels available in the data. The intervals for tenor range from 1 day to 120 days, which is also the range of tenors available in the data.

After grouping options by moneyness and tenor, we estimate the volume-volatility elasticity for each group separately. Within each group, the elasticity is estimated following the regression specification of Equation 21. The procedure is nonparametric in the sense that all estimated parameters are allowed to vary from group to group.

The volume-volatility elasticity estimates are presented in Figure 3 below.

The first plot on the left displays the volume-volatility elasticity estimates for options that have

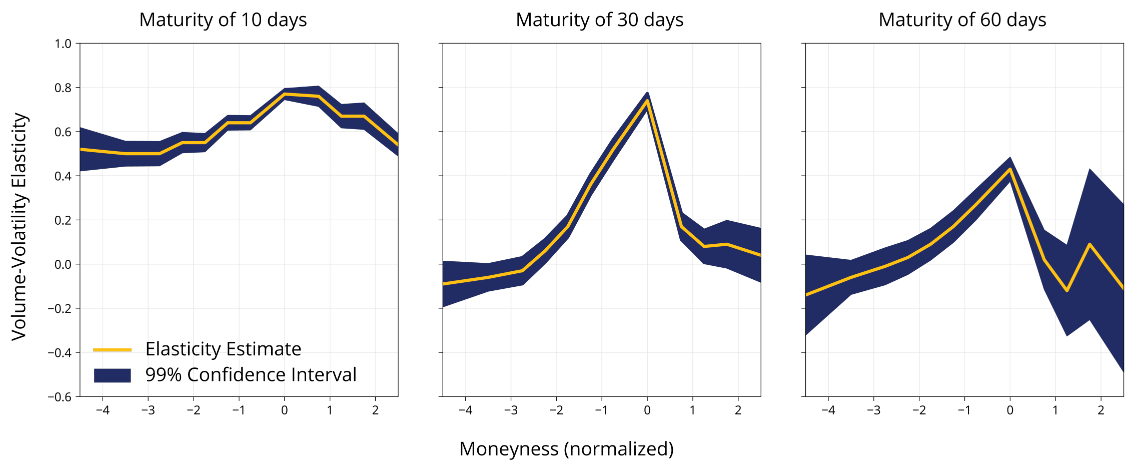


Figure 3: *The figure reports volume-volatility elasticity estimates based on a nonparametric procedure (regressions group by group). Options are groupbed by moneyness and tenor in order to obtain the estimates. The estimates are based on the full sample of cleaned data, spanning 2007 to 2016. Moneyness is measured in units of standard deviation and tenor is measured in days. Lines (yellow) display the elasticity estimates, while the shaded regions (blue) are 99% confidence intervals. Confidence intervals are computed using cluster-robust standard errors as in Petersen (2009). Each plot displays the volume-volatility elasticity as a function of moneyness, while maintaining a fixed tenor.*

a maturity of around 10 days. In that plot, moneyness varies from around -5 to around 3. The volume-volatility elasticity is computed using data for all options at a given value of moneyness (for example, around -4) and with tenor around 10 days. Observe that the elasticity is highest for options close to the money. Although the elasticity is not unity, it achieves the highest level for options with short tenor and that are close-to-the-money. This is inline with the previous findings: investors' disagreement is the lowest when it relates to the distribution of market returns in the short-run. In the parametric estimates, the elasticity is close to unity when moneyness is zero and tenor is one. However, the parametric estimates reflect the projection of the data on the model and take into account the downward trend when tenor increases. Therefore, the elasticity close to unity is possibly just an extrapolation of the negative effect of the tenor.

The first plot on the left also reveals a decrease in the elasticity as moneyness moves away from zero. This decrease in elasticity occurs irrespective of the tenor category. Indeed, observe in the center and right plots in Figure 3, the elasticity decreases as moneyness moves away from zero. Notice, however, that for options with maturity of around 10 days, the decrease in tenor is weaker compared to options with higher maturities. This is consistent with the idea that the probability of big moves in a short span of time is small, so that new information should not lead to higher disagreement about the tails in the short-run.

The plot in the center displays the elasticity estimates for options with maturity of around 30 days. The elasticity is highest for options with moneyness close to zero, and sharply decreases as moneyness moves away from zero. This confirms the previous findings. Notice that the decrease is sharp, and the elasticity estimates are zero for options with moneyness lower than -2.5 and higher than 1.5. Contrary to the parametric specification, the elasticity estimates do not decrease much beyond zero. In fact, the confidence intervals indicate the elasticities are all indistinguishable from zero for moneyness values far from zero. Interestingly, this is one of the implications from the model: the volume-volatility elasticity should be bounded between one and zero. And, the bounds for the elasticity were not imposed during the estimation procedures.

The last plot in Figure 3 displays the elasticities estimated from options that expire in around 60 days[15] The pattern is similar to that of options that expire in around 30 days. The elasticity is highest for options with moneyness around zero, but drops sharply as moneyness moves away from zero. Notice, however, that the highest elasticity in this case is almost half of the highest elasticity in the center plot. This indicates an increase in disagreement about the center of the distribution of market returns as the tenor increases. This is consistent with the previous estimates, which indicated that increases in tenor lead to decreases in elasticity.

The plots in Figure 3 also indicate a sharper drop in the elasticity when moneyness moves from zero to positive values (from ATM to ITM) than when it moves to negative values (from ATM to OTM). This implies that the release of new information generates more disagreement about the right-tail of the market returns than about the left-tail.

Lastly, there is an overall decrease in elasticity as the tenor increases. But, for options with moneyness far from zero, the decrease in elasticity is attenuated when the tenor increases. Indeed, notice that there is a sharp decrease in the elasticity when tenor goes from around 10 days to around 30 days. But, there is little decrease in the elasticity when the tenor further increases to around 60 days. The lack of a decrease in elasticity indicates that new information does not generate more disagreement about the valuation of far-from-the-money options with higher maturities.

To better understand the impact of tenor on the elasticity, Figure 4 plots the elasticity as a function of tenor and fixing moneyness at different levels.

All plots in Figure 4 display a similar behavior. The elasticity sharply decreases when the tenor increases. This is consistent with the parametric estimates and indicates higher disagreement about future market returns. Options that are ITM (moneyness around 1) or deep OTM (moneyness around -2) display a sharp in elasticity when tenor moves from 10 days to 30 days. Further increases in tenor bear little impact on the elasticity, since it is already statistically indistinguishable from zero. This indicates a sharp increase in disagreement about the tails of the market returns in a horizon of 30 days.

Now, consider the plots for options that are slightly OTM (moneyness around -1) or ATM (moneyness around 0). The volume-volatility elasticity for these options decreases almost linearly

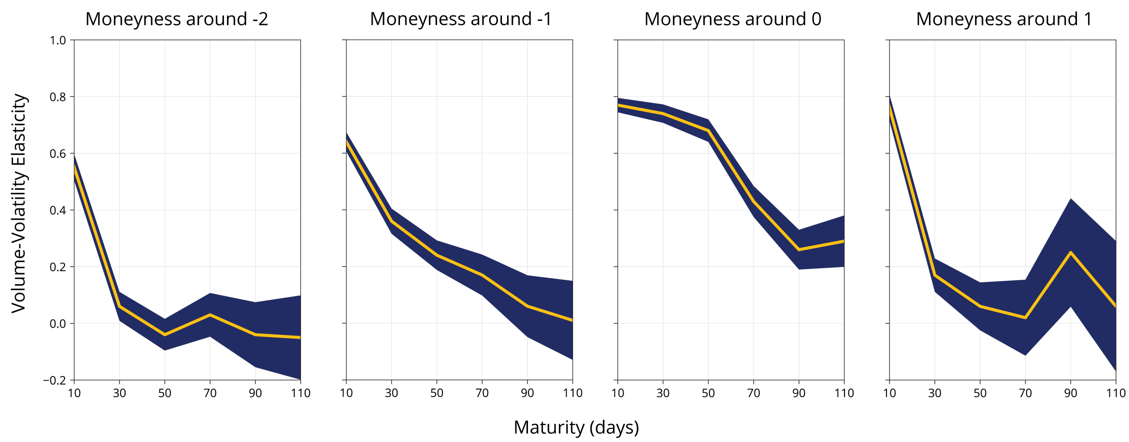


Figure 4: *The figure reports volume-volatility elasticity estimates based on a nonparametric procedure (regressions group by group). Options are groupbed by moneyness and tenor in order to obtain the estimates. The estimates are based on the full sample of cleaned data, spanning 2007 to 2016. Moneyness is measured in units of standard deviation and tenor is measured in days. Lines (yellow) display the elasticity estimates, while the shaded regions (blue) are 99% confidence intervals. Confidence intervals are computed using cluster-robust standard errors as in Petersen (2009). Each plot displays the volume-volatility elasticity as a function of tenor, while maintaining a fixed moneyness.*

when tenor increases. For options that are ATM, the decrease in elasticity from 10 to 60 days in small, consistent with the previous findings. Overall, these findings indicate there is an almost linear (or piece-wise linear) relationship between disagreement and tenor.

Lastly, observe that the elasticity for ATM options achieves its minimum for tenors higher than 90 days. This minimum, however, is higher than zero. This indicates that disagreement about averages is not as high as disagreement about tails, even for high maturities.

Overall, the results presented above support the findings from the parametric estimates. Investors disagree about the interpretation of new information, and this disagreement increases with the time horizon and moneyness.

6 Conclusion

In the Kandel-Pearson model, there are two motives for investors to trade a risky asset: changes in the asset's prices and disagreement between investors regarding new information. Bollerslev et al. (2018) explicitly connects the disagreement between investors to the elasticity between trade volume and price volatility, resulting in two main implications. First, the volume-volatility elasticity is equal to unity only when there is no disagreement between traders. Second, the volume-volatility elasticity decreases if the disagreement between investors increases. This paper analyzed these two implications in the context of the S&P 500 options market.

We presented new empirical evidence from the options market that is consistent with the implications from the Kandel-Pearson model. We argued that moneyness and tenor are directly related to the volume-volatility elasticity, and can be interpreted in terms of the distribution of the market index. We estimated a volume-volatility elasticity close to unit for options that are at-the-money and about to expire, which is consistent with the case of no disagreement among investors. We also found that the volume-volatility elasticity decreases with increases in the absolute value of moneyness, implying that new information leads to more disagreement regarding the tails of market returns. Lastly, we found that the elasticity decreases with increases in tenor, implying that new information leads to increasingly more disagreement about future values of the market index. All findings were supported by nonparametric estimates.

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Notes

1. See Mancini (2001) (pages 30 to 37) for a discussion of many no-trade theorems.

2. See `https://fred.stlouisfed.org/series/DDEM01USA156NWDB` for the stock market turnover ratio. The turnover ratio measures the total value of stocks traded in a year divided by the total market capitalization. The turnover indicates how much of the stock market exchanges hands in a year.

3. Specifically, the points of evaluation were (1) at-the-money and 1-day maturity, along with (2) 3-sigma out of the money and 60-day expiration.

4. Most of the literature on the lead-lag relationship between options and stocks finds that trading in options leads trading in stocks. In Easley et al. (1998), the authors find that stock returns lead options' trade volume. Their finding is reasonable, since options are often used for hedging. Now, the work by Stephan and Whaley (1990) also finds that stocks lead options, but in the case of options' transaction prices. Specifically, Stephan and Whaley find that stock returns lead price changes in the options market by 15 minutes on average. However, the findings by Stephan and Whaley (1990) have been contested by Chan et al. (1993). Chan, Chung and Johnson argue that the finding is due to the use of transaction prices (instead of bid-ask quotes) combined with infrequent trades, which generate the spurious relationship found in Stephan and Whaley (1990). Thus, the majority of the literature explores the impact of the options market on the stock market.

5. Our approach does not use the elegant dynamic C-CAPM models of Buraschi and Jiltsov (2006) and Atmaz and Basak (2018), which do not directly connect uncertainty to the volume-volatility elasticity as in Bollerslev et al. (2018). Exploration of dynamic volatility-volume relationships within the context of C-CAPM is too far afield for the present paper but well worth exploring in future research.

6. In model free terms, the existence of a risk-neutral measure is assured under no-arbitrage conditions. See, for example, Section 6K of Duffie (2001).

7. Forecasting the expectation of a stochastic process may or may not be more difficult as the forecast horizon increases. Indeed, for a moving average process the mean-squared error of the forecast converges to the unconditional variance

of the series as the forecasting horizon increases. However, for a unit-root process the mean-squared error of the forecast diverges as the forecasting horizon increases (see page 440 of Hamilton (1995)). Since the market index is a unit-root process, its forecasting error increases with the forecasting horizon.

8. See Section A.1 in the Appendix for details.

9. For details see CBOE (2014).

10. Section A.2 in the Appendix displays a more detailed version of the volatility signature plot with finer categories for moneyness.

11. See `https://www.tickdata.com`

12. Data sourced from Wurgler's website.

13. Originally, the authors also included NYSE turnover as a measure of liquidity in the index. However, they recently dropped it from the sentiment index because turnover is no longer a meaningful measure of liquidity due to high-frequency trading.

14. Further discussion takes place in Section 5.4. Also, an alternative specification where the negative and positive moneyness are allowed to have different impacts on the elasticity is explored in Section A.3 of the Appendix.

15. Section A.4 in the Appendix displays a figure with estimates for all groups omitted above. The groups were omitted to simplify the exposure, and because they display the same behaviour as the plots from Figure 3.

A Appendix

A.1 Cleaning Procedures

The computation of the residual volatility of options uses high-frequency data on the S&P 500 index options (SPX), and on the SPDR S&P 500 exchange-traded fund (SPY). The data set on the SPX options was obtained from the Chicago Board Options Exchange (CBOE). The set contains quotes at the 1-minute interval level from 2007 to 2016. The market hours are from 9:30 am to 4:15 pm EST, resulting in 405 quotes per day. Partial trading days are removed from the sample (23 in total), and we base the analysis on the remaining 2458 days. On 28 trading days, there are missing values for the first minute or two at the market open. These missing values are back-filled with the first non-zero prices of the same day. As noted in Section 4.3, the options data are sparsely sampled to 5-minutes to mitigate the impact of microstructure noise on volatility estimates.

The data set on SPY was obtained from TickData[6]. The set contains the last closing price of every 5-minute interval, from 2007 to 2016.

The time to expiration of an option is one of the variables used to compute its moneyness. We follow the methodology of CBOE (2014) to compute the time to maturity in minutes, while dealing with the differences between AM and PM settled options and options that expire on holidays. We compute the moneyness of all options daily at the end of the day. Then, we select options with moneyness ranging from -5 to 3, which captures a wide variety of options while eliminating options that have pricing issues due to extremely low liquidity.

Lastly, the composition of expiration dates of SPX options varies throughout the years and could lead to issues in prices and estimation of the residual volatility. The history of SPX options is relevant in understanding how so. SPX options were launched in 1987, and originally expired every 3rd Friday of the month. Their increasing success warranted the expansion of SPX options to include new expiration dates. In 2005, CBOE introduced SPX Weeklys, which expire on all other Fridays. However, at launch, these options had much lower liquidity than the original SPX options, but their liquidity increased substantially over the years[16]. Now, because the residual volatility is estimated from the prices of options, it is imperative to have prices that directly reflect the real value of options. For this reason, options with low liquidity, or higher spreads, are disregarded in favor of options with high liquidity. In the case of SPX options, the liquidity is related to the expiration dates, since the original SPX options had higher liquidity than the Weeklys at launch, and for some years after that. To take into account this difference, we only include options that do not have stale prices. An option is said to have stale prices if its intra-day prices do not change on more than 50% of the time intervals within a day. These options are eliminated from the sample.

A.2 Volatility Signature for SPX Options

Figure 5 displays the volatility signature plot for the market options using a finer division for moneyness. The options are categorized by moneyness, and their realized variances are computed for different sampling frequencies.

The figure shows an increase in the volatility for out-of-the-money options when the sampling frequency is below 5-minutes, indicating the presence of microstructure noise.

Observe that there is no increase in the realized variance when the sampling frequency increases for options that are ITM, ATM and slightly OTM. This indicates a lack of bias in the variance estimates due to microstructure noise. However, for options with moneyness smaller than -2.5, there is a clear indication of microstructure noise, since sampling frequencies smaller than 5-minutes lead to an ever increasing estimates of variance. Overall, a sampling frequency of 5-minutes provides a good trade-off between avoiding microstructure noise and estimating the realized variance with enough precision.

A.3 Asymmetric Effect of Moneyness

The parametric specification analyzed in Section 5 assumed the effect of moneyness on elasticity was symmetrical. That is, increases in moneyness away from zero had the same impact as decreases in moneyness. However, the nonparametric results from 5.4 indicate that the decrease in elasticity is higher when moneyness increases away from zero than from when it decreases. We re-estimate

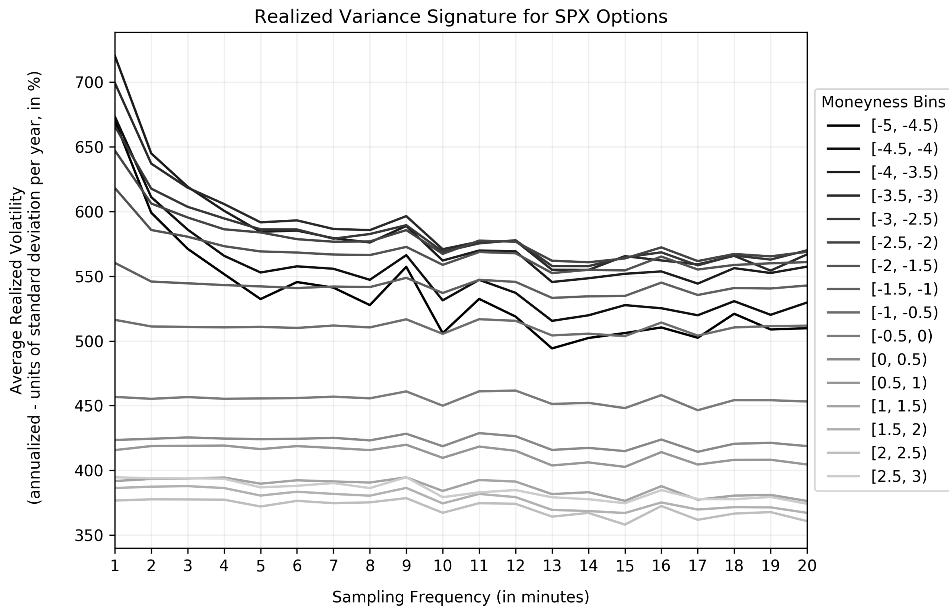

Figure 5: *The figure shows the average realized volatility (annualized in percentage) for options of different moneyness. The realized volatilities are computed using different sampling frequencies. The sampling frequency varies from 1-minute up to 20-minutes. Higher sampling frequencies are at the left side of the figure, while lower sampling frequencies are at the right side of the figure. The realized volatility is estimated as the sum of squared returns over each day for each option.*

the parametric specification allowing the impact of moneyness to be asymmetrical. Table 6 below displays the estimates.

The table shows results similar to the nonparametric findings. The absolute value of moneyness has a negative impact on the elasticity. However, the impact for positive moneyness is stronger than that of negative moneyness. This indicates a higher disagreement about the right tails of market returns.

Figure 6 below is a visual representation of the elasticity estimates from Table 6.

The characteristics are similar to the nonparametric results, and also display a sharp drop in elasticity (higher increase in disagreement) as moneyness moves from zero to positive values.

A.4 Additional Elasticity Figures

Table 6: *The table reports the volume-volatility elasticity estimates including different explanatory variables for the elasticity. The estimates are based on a modified version of the specification in Equation 21. The specification below allows for asymmetry in the impact of moneyness on the elasticity. The estimated values of α_0 and α_1 are not reported for brevity. Estimates are based on the full sample of cleaned data, spanning 2007 to 2016. Moneyness is measured in units of standard deviations and tenor is measured in days. The standard errors are heteroscedasticity robust as in White (1980).*

	Parameter Estimate	Standard Error
Baseline Estimates		
Elasticity (β_0)	1.018	0.013
Explanatory Variables for Elasticity (β_1)		
$1\{Moneyness > 0\} \cdot \lvert\text{Moneyness}\rvert$	-0.225	0.007
$1\{Moneyness \leq 0\} \cdot \lvert\text{Moneyness}\rvert$	-0.487	0.009
ln Tenor	-0.147	0.004
$\lvert\text{Moneyness}\rvert \cdot \ln \text{Tenor}$	0.013	0.002
Adjusted R^2 (in %)	24.30	
Number of Observations	446053	

Figure 6: *The figure shows the volume-volatility elasticity estimates for different values of moneyness and tenor. Fixing tenor, the elasticity decreases as options move away from the money. Notice, however, an asymmetric effect on positive moneyness. Fixing moneyness, the elasticity decreases as the time to expiration increases.*

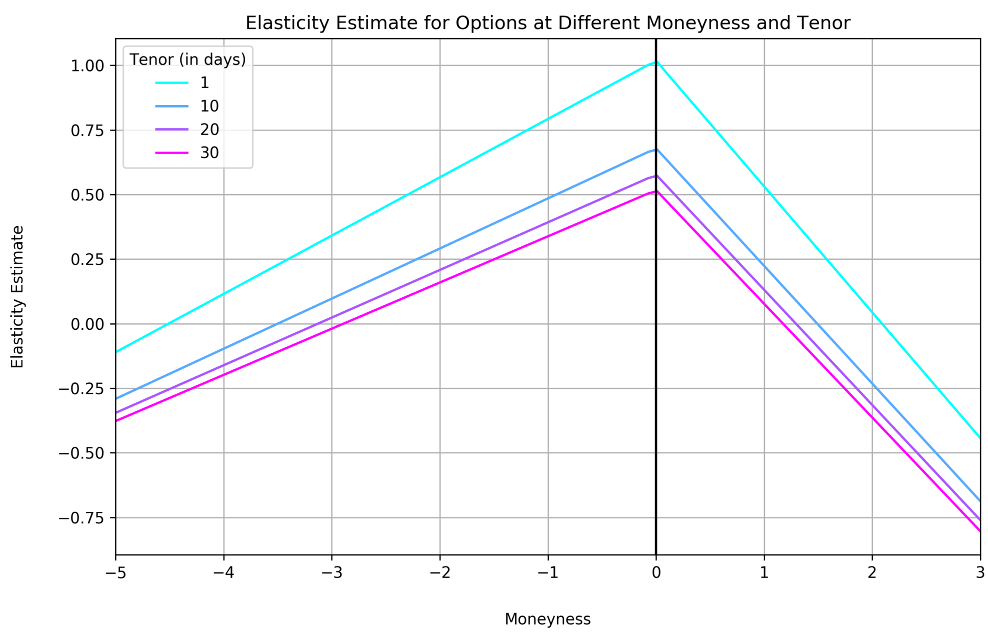

Figure 7: *The figure reports volume-volatility elasticity estimates based on a nonparametric procedure (regressions group by group). Options are grouped by moneyness and tenor in order to obtain the estimates. The estimates are based on the full sample of cleaned data, spanning 2007 to 2016. Moneyness is measured in units of standard deviation and tenor is measured in days. Lines (yellow) display the elasticity estimates, while the shaded regions (blue) are 99% confidence intervals. Confidence intervals are computed using cluster-robust standard errors as in Petersen (2009). Each plot displays the volume-volatility elasticity as a function of moneyness, while maintaining a fixed tenor.*

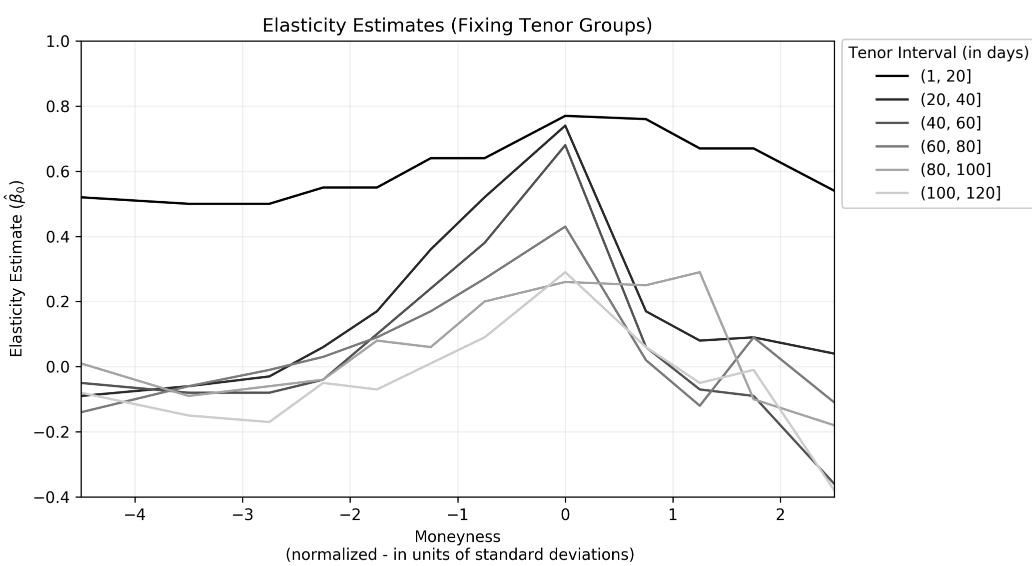

Figure 8: *The figure reports volume-volatility elasticity estimates based on a nonparametric procedure (regressions group by group). Options are groupbed by moneyness and tenor in order to obtain the estimates. The estimates are based on the full sample of cleaned data, spanning 2007 to 2016. Moneyness is measured in units of standard deviation and tenor is measured in days. Lines (yellow) display the elasticity estimates, while the shaded regions (blue) are 99% confidence intervals. Confidence intervals are computed using cluster-robust standard errors as in Petersen (2009). Each plot displays the volume-volatility elasticity as a function of moneyness, while maintaining a fixed tenor.*

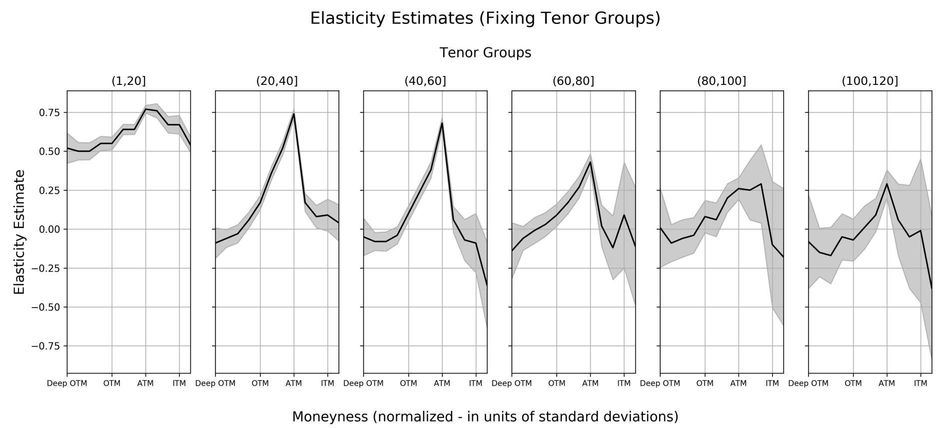

Figure 9: *The figure reports volume-volatility elasticity estimates based on a nonparametric procedure (regressions group by group). Options are groupbed by moneyness and tenor in order to obtain the estimates. The estimates are based on the full sample of cleaned data, spanning 2007 to 2016. Moneyness is measured in units of standard deviation and tenor is measured in days. Lines (yellow) display the elasticity estimates, while the shaded regions (blue) are 99% confidence intervals. Confidence intervals are computed using cluster-robust standard errors as in Petersen (2009). Each plot displays the volume-volatility elasticity as a function of tenor, while maintaining a fixed moneyness.*