Monetary Policy and Sovereign Risk in Emerging Economies (NK-Default)*

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Abstract

This paper develops a New Keynesian model with sovereign default risk (NK-Default). We focus on the interaction between monetary policy, conducted according to an interest rate rule that targets inflation, and the government's external debt, which is subject to default. High default risk increases inflation because firms raise their prices in expectation of the negative effects of a default event on productivity and prices. However, these monetary frictions also serve to discipline sovereign borrowing and reduce sovereign spreads. Our framework replicates the positive comovements of spreads with inflation, a salient feature of emerging markets data. Through counterfactual analyses, we find that about half of the increase in inflation during inflationary events in emerging markets over the last two decades can be attributed to default risk.

Keywords: sovereign default, inflation, open economy, New Keynesian theory
JEL classification: F34, F41, E52

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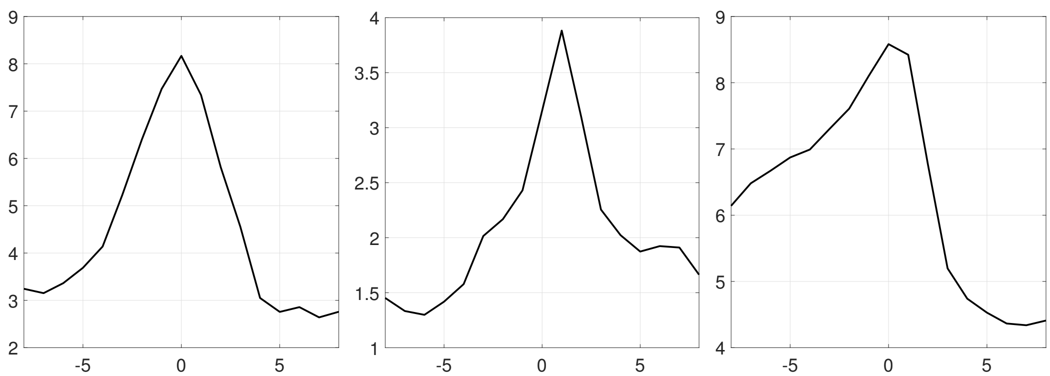
1 Introduction

Since the early 2000s, many central banks in emerging markets have achieved independence from the central government and have conquered their historically high inflation, bringing it down to single digits. As in advanced economies, the monetary policy now largely consists of setting domestic nominal interest rates to target inflation. The open economy New Keynesian monetary model is a main toolkit for emerging markets central banks, but this framework, however, abstracts from sovereign risk, a major source of economic fluctuations in these countries.[1] To bridge this gap, this paper develops a New Keynesian model that integrates sovereign risk and investigates the interplay between monetary policy and sovereign default risk. Our findings indicate that sovereign risk not only affects economic activity but can also critical amplifier of inflation.

The motivation for our work is the observation that in emerging markets inflation and sovereign risk are positively correlated. We illustrate this correlation in Figure 1, which shows the time path for inflation, sovereign spreads, and nominal rates during temporary inflation events in eight emerging-market inflation targeters over the last two decades.[2] During these events, inflation temporarily rose about 5%, sovereign spreads also rose about 2.5%, and central banks increased nominal rates to combat inflation. Notably, the elevated inflation was only temporary and all variables returned to lower levels within about a year. In this paper, we use our model to shed light on the co-movements between inflation and sovereign risk and perform counterfactual analyses to understand the role of default risk in shaping inflation dynamics.



(a) Inflation (b) Spread (c) Nominal Rate

Figure 1: Inflation Events

In our framework, default risk plays a crucial role in exacerbating inflation and amplifying monetary distortions. This is because a higher likelihood of default leads to an increase in inflation expectations, as default events are typically associated with productivity losses and elevated inflation. Similar to standard monetary models, inflation is determined by firms' pricing decisions, which satisfy a New Keynesian Philips Curve and depend on inflation expectations. In the presence of high default risk, firms tend to increase their prices in anticipation of future inflation. Tight monetary policy can dampen inflation but also affects default risk because the presence of monetary distortions curbs government borrowing, reducing sovereign spreads. Our *NK-Default* framework combines the workhorse New

[1] For example, the influential paper by Galí and Monacelli (2005) analyzes monetary policy under perfect financial markets.

[2] We define inflation events as time periods where inflation is above two standard deviations. For further information on the countries and construction of these events, refer to section 5.1.

Keynesian monetary model of Galí and Monacelli (2005) with a standard sovereign default model and delivers the positive correlations of sovereign spreads with inflation. Additionally, our counterfactual experiments show that default risk can account for a significant portion of inflation volatility in these countries.

The small open economy model we consider consists of households, firms, a monetary authority, and a government that borrows internationally. Households value consumption of domestic and foreign goods. They supply labor to intermediate goods firms that produce domestic varieties. The intermediate goods firms are subject to productivity shocks and face frictions in setting their prices, in the tradition of Rotemberg (1982). Final goods firms are competitive and use intermediate goods varieties to produce domestic output, which is consumed by domestic households and exported to the rest of the world. The monetary authority sets nominal interest rates in local currency using an interest rate rule to target domestic inflation. As in standard New Keynesian models, monetary policy and firms' pricing frictions generate monetary distortions. We measure these monetary distortions with inflation deviations and a monetary wedge that encodes inefficient productions.

The government borrows internationally by issuing long-term bonds denominated in foreign currency and transfers the proceeds from these operations to households. The focus on foreign currency debt allows us to abstract from the better-understood fiscal-monetary interaction that inflation can reduce the real value of local currency debt. The government, nevertheless, lacks the commitment to repay its debt and can choose to default. Default is associated with a decline in productivity, which reduces consumption and production and increases inflation. The price of bonds compensates risk-neutral lenders for the risk of default. In this environment, equilibrium default events lead to time-varying default risk and sovereign spreads. As is standard, the presence of default risk in long-term debt encourages overborrowing because of the incentives to dilute the value of legacy lenders.

In this environment, we identify two main channels of monetary-fiscal interactions. First, default risk in our model amplifies monetary frictions by increasing inflation and worsening the monetary wedge. When default risk is high, it triggers expectations of future high inflation and recessions. High expected inflation tends to increase current inflation through the NKPC, and low expected consumption lowers current consumption through intertemporal consumption-smoothing, which results in depressed production and a worsening of the monetary wedge. These outcomes highlight the significance of default risk in monetary outcomes, and we refer to this channel as *default amplification*. Second, these additional costs from default risk tend to constrain the government's borrowing, as it internalizes how its policies affect the private economy. The resulting monetary frictions lead to borrowing wedges in the optimal borrowing condition for the government, which reduce sovereign borrowing incentives and enforce *monetary discipline*, which reduces the government's default risk. Lowering debt is useful in our model because the government tends to overborrow and experiences too frequent costly defaults.[3]

We establish that monetary policy interacts with sovereign risk both theoretically, in simplified versions of our model, and quantitatively, in our general model parameterized to emerging market data. Our theoretical results isolate the mechanisms for default amplification and monetary discipline, in a setting with simplified preferences that are separable and quasi-linear with respect to foreign goods consumption. We show that if default events are associated with high inflation and low consumption, high default risk leads to increases in inflation and a worsening of the monetary wedge, the default amplification mechanism. We then analyze the outcomes of monetary policy in an example where the

[3] Hatchondo et al. (2016) demonstrate that in sovereign default models with long-term defaultable bonds, the government has incentives to overborrow and dilute existing bondholders and that these forces are important for explaining the sizable spreads in emerging economies.

economy experiences a one-time deviation, with monetary frictions and overborrowing incentives, from a constrained efficient baseline. Here we establish our disciplining result: tight monetary policy lowers default risk. The reason is that tight monetary policy lowers domestic consumption and increases the monetary wedge, both of which increase the costs for government borrowing, as these frictions will be exacerbated even further with default risk. The disciplining result shows that the standard prescription of strict inflation targeting for the optimal monetary policy does not hold in our environment with default risk. In fact, our third theoretical result is that a monetary policy rule that targets only default risk can achieve the constrained efficient outcome because it can solve not only the overborrowing friction but also the pricing friction.

For our quantitative results, we parameterize the NK-Default model to the data of 8 emerging market inflation targets. We show the model produces patterns for spreads, inflation, and nominal domestic rates that resemble the data. In particular, our model delivers the key positive comovement of spreads with inflation and nominal rates. We also provide empirical evidence of the disciplining mechanism using panel data by projecting sovereign spreads on monetary policy shocks, recovered from estimated monetary policy rules. We find that, in data as in the model, contractionary monetary shocks lower sovereign spreads. Finally, we use our model to study inflation events. We find that the model can replicate the paths of inflation, output, nominal rates, and spreads with a combination of low productivity shocks and expansionary monetary shocks. Using a reference model with no default, we measure the contribution of default risk for these dynamics and find that default risk accounts for about 50% of the increase in inflation during these events.

Finally, we assess the welfare implications of our model under different monetary policy rules. We compare welfare outcomes relative to a strict inflation target regime, which is a monetary policy that completely neutralizes pricing frictions and achieves the flexible price equilibrium. We consider rules that vary the weight assigned to inflation and also add an extra term that targets default risk. We find that the calibrated model is dominated by the strict inflation target outcome, but welfare can surpass it with a high enough weight on inflation. We also find that a rule with a weight on default risk can be highly beneficial, surpassing the welfare under strict inflation targeting because with this rule the volatility of spreads and inflation plummets.

Related Literature Our project builds on two distinct literature on emerging markets' business cycles: the work on sovereign default and the work on New Keynesian monetary policy. We construct a quantitative sovereign default model, in the tradition of Eaton and Gersovitz (1981), as in Aguiar and Gopinath (2006) and Arellano (2008) but with long-term debt, as in Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012). We expand this framework to incorporate production and an import-export structure. Our domestic monetary environment is close to the workhorse framework of Galí and Monacelli (2005). One methodological difference between our project and standard monetary models is that we use global methods rather than local approximations around the steady state to compute the model.

The literature on sovereign default has recently turned to questions raised by nominal rigidities. Several papers have considered environments with defaultable sovereign debt and downward rigidity of nominal wages. Na et al. (2018) first introduced this friction in a model of sovereign default and emphasize that exchange rate pegs are costly because they prevent devaluations that would adjust real wages to their efficient level. Optimal policy in their environment delivers the joint incidence of devaluations and defaults. Bianchi et al. (2018) study the role of downward rigidity of nominal wages for procyclical fiscal policies, which result from a tradeoff between fiscal policy stimulating demand but possibly increasing default risk. Our project shares the emphasis in these papers on the interaction between sovereign risk and monetary frictions but differs in important ways. First, price frictions in our model arise from optimal price setting by firms which results in a standard New Keynesian Phillips Curve, where expectations of future inflation matter for current inflation and output. These papers, in contrast, directly impose that nominal wages are downwardly rigid and abstract from the role of inflation expectations in affecting current inflation and output. Second, our modeling of monetary policy focuses on a positive theory that resembles the practice of many emerging markets central banks, which set interest rates to target inflation.[4]

A large literature, following Calvo (1988), studies the incentives of governments to default on debt that is denominated in local currency with inflation. Aguiar et al. (2013) analyze the tradeoffs generated by monetary policy credibility in a dynamic continuous-time model of self-fulfilling default crises and show that monetary policy credibility helps suppress self-fulfilling debt crises but hinders the benefits of state-contingent payments induced by inflation.[5] Hur et al. (2018) and Sunder-Plassmann (2018) also study the interaction between inflation and defaultable debt denominated in local currency. The former considers exogenous inflation, for given covariance structures with fundamentals, whereas the latter builds on a cash-and-credit model with a constant money supply. Nuno and Thomas (2019) build a continuous-time model with local currency debt and default and a discretionary choice of inflation, whereas Engel and Park (2019) analyze how default and inflation incentives shape the composition of sovereign debt between local and foreign currency. In contrast with these papers, we emphasize the joint dynamics of endogenous inflation and sovereign risk, with a monetary authority that uses interest rate rules, hence abstracting from the incentives of using monetary policy to inflate away the debt. We view this work as complementary to ours and important, especially for emerging markets that have not been able to achieve central bank independence.

The idea that monetary policy faces a tension between addressing pricing distortions and financial frictions, a thread running through our paper, is shared by a few other recent works. Itskhoki and Mukhin (2022) study the role of optimal monetary policy in fostering international risk sharing. Their result that, under certain cases, a policy that pegs the exchange rate can address both frictions resembles our result that an interest rate rule that targets default risk can address the monetary frictions as well as overborrowing incentives. Aoki et al. (2018) study a model of a small open economy with a frictional banking sector and revisit the role of monetary and macroprudential policy. They find, like us, that strict inflation targeting is dominated by policy regimes that occasionally aim at changing financial conditions.

Finally, our model's implications for exchange rates and international capital flows raise a natural comparison with the work on capital controls, exchange rates, and financial frictions in small open economies, such as Farhi and Werning (2012), Fanelli (2017), Devereux et al. (2019), and Itskhoki and Mukhin (2021).

2 Model

We consider a small open economy composed of households, final goods producers, intermediate goods firms, a monetary authority, and a fiscal government. There are three types of goods: final domestic goods, domestic intermediate varieties, and foreign imported goods. The final good is produced using all

[4]The statutory objectives, given by the legislature to central banks in inflation-targeting emerging markets, center on controlling inflation. For example, the only objective of the Monetary Policy Committee of the Central Bank of Brazil is the achievement of the inflation targets set by the National Monetary Council.

[5]Concerning the multiplicity of equilibria and the role inflation can play in selecting among them, Corsetti and Dedola (2016) focus on unconventional monetary policy whereas Bacchetta et al. (2018) analyze how interest rate rules can be used to prevent the self-fulfilling crises in the environment of Lorenzoni and Werning (2019).

varieties of differentiated intermediate goods and consumed by both domestic and foreign households. Each intermediate good variety is produced with labor.

Foreign demand for domestic goods (export demand) is given by

$$X_t = \left(\frac{P_t^d}{\varepsilon_t P_t^*}\right)^{-\rho} \xi,$$

where P_t^d is the price of domestic goods in local currency, P_t^* the price of foreign goods in foreign currency, ξ the level of overall foreign demand, ρ the trade elasticity, and ε_t the nominal exchange rate. An increase in ε_t represents a depreciation of the home currency. We assume that the law of one price holds, so we can write the price of the foreign good in local currency as $P_t^f = \varepsilon_t P_t^*$. The terms of trade e_t equal

$$e_t = \frac{P_t^f}{P_t^d} = \frac{\varepsilon_t P_t^*}{P_t^d}. \tag{1}$$

Hence, the foreign demand for domestic goods is a function of the terms of trade and the level of overall foreign demand ξ:

$$X_t = e_t^{\rho} \xi. \tag{2}$$

We normalize the foreign price P_t^* to one in all periods.

2.1 Households

Identical households consume domestic goods C_t and foreign goods C_t^f and supply labor N_t. Their preferences are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_t,\ C_t^f, N_t), \tag{3}$$

where $u(C_t,\ C_t^f, N_t)$ is the per-period utility function and β is the discount factor of households.

Taking prices as given, households choose consumption, labor supply, and holdings of domestic bonds B_t^d. These domestic bonds are denominated in local currency and can only be traded by domestic households. Households own firms and receive their profits Ψ_t. They also earn labor income $W_t N_t$ and receive government transfers T_t. Their budget constraint is given by

$$P_t^d C_t + (1+\tau_f) P_t^f C_t^f + q_t^d B_{t+1}^d \leq W_t N_t + B_t^d + \Psi_t + T_t \tag{4}$$

where q_t^d is the nominal price of domestic discount bonds and τ_f is a constant consumption tax that households pay on imports. We can characterize households' choices with the following optimality conditions:

$$-\frac{u_{N,t}}{u_{C,t}} = w_t, \tag{5}$$

$$\frac{u_{C^f,t}}{u_{C,t}} = (1+\tau_f) e_t, \tag{6}$$

$$u_{C,t} = i_t \beta \mathbb{E}_t \left[\frac{u_{C,t+1}}{\pi_{t+1}}\right]. \tag{7}$$

The real wage is $w_t = W_t / P_t^d$, the gross domestic goods inflation, hereafter *inflation*, is $\pi_t = P_t^d / P_{t-1}^d$, and the nominal domestic interest rate is the yield of the discount bond price $i_t \equiv 1/q_t^d$.

2.2 Final Goods Producers

The final good Y_t is produced using a unit measure of differentiated intermediate goods y_{jt}, $j \in [0,1]$, under perfect competition, $Y_t = \left[\int_0^1 y_{jt}^{\frac{\eta-1}{\eta}} dj\right]^{\frac{\eta}{\eta-1}}$ where η is the elasticity of substitution between intermediate goods. The optimization problem of the final goods producers yields the standard demand function

$$y_{jt} = \left(\frac{p_{jt}}{P_t^d}\right)^{-\eta} Y_t, \tag{8}$$

where p_{jt} is the price of intermediate good j at time t. The price of domestic goods P_t^d is the price index $P_t^d = \left[\int_0^1 p_{jt}^{1-\eta} dj\right]^{\frac{1}{1-\eta}}$.

2.3 Intermediate Goods Producers

Each differentiated intermediate good is produced with labor n_{jt}, using a constant returns to scale production function with productivity z_t:

$$y_{jt} = z_t n_{jt}. \tag{9}$$

Productivity depends on the aggregate shock $\tilde{z}_t$ and also on the credit standing of the government Θ_t such that $z_t = z(\tilde{z}_t, \Theta_t)$. As we see below, when credit standing is good productivity is equal to the shock, and when credit standing is bad productivity is lower.

Intermediate goods firms are monopolistically competitive and set the prices for their products, taking as given the demand (8). These firms, however, face price-setting frictions in that they have to pay a quadratic adjustment cost when they change their prices away from the target inflation rate $\overline{\pi}$, as in Rotemberg (1982). Taking as given the wage W_t and the final good price P_t^d, an intermediate firm j chooses labor and its price to maximize the present discounted value of profits,

$$\max_{\{n_{jt}, p_{jt}\}} \mathbb{E}_0 \sum_t Q_{t,0} \left\{ p_{jt} y_{jt} - (1-\tau) W_t n_{jt} - \frac{\varphi}{2} \left(\frac{p_{jt}}{p_{jt-1}} - \overline{\pi}\right)^2 P_t^d Y_t \right\},$$

subject to the production function (9). Firms discount profits using the stochastic discount factor of households, $Q_{t,0} = \beta^t \frac{u_{C,t} P_0^d}{u_{C,0} P_t^d}$, and get a labor subsidy τ.[6] The first-order condition for each firm, after imposing symmetry across all firms ($p_{jt} = P_t^d$), results in

$$(1-\tau)\frac{w_t}{z_t} = \frac{\eta-1}{\eta} + \frac{1}{\eta}\left\{ \varphi\left(\pi_t - \overline{\pi}\right)\pi_t - \mathbb{E}_t\left[\beta \frac{u_{C,t+1}}{u_{C,t}} \frac{Y_{t+1}}{Y_t} \varphi\left(\pi_{t+1} - \overline{\pi}\right)\pi_{t+1}\right]\right\}. \tag{10}$$

This equation is a standard New Keynesian Phillips Curve (NKPC) that relates inflation to a measure of contemporaneous unit cost, $(1-\tau)w_t/z_t$, and expected inflation.

2.4 Government and External Debt

The fiscal government engages in international borrowing using long-term bonds denominated in foreign currency and can default on its debt. To keep long-term debt tractable, we consider random maturity bonds, as in Hatchondo and Martinez (2009). The bond is a perpetuity that specifies a price q_t and a

[6] We follow the standard practice in the New Keynesian literature by introducing a constant subsidy designed to alleviate average inefficiencies induced by market power.

quantity ℓ_t such that the government receives $q_t\ell_t$ units of foreign currency in period t. In the following period, a fraction δ of the debt matures and the government's debt is the sum of legacy debt that has not matured $(1-\delta)B_t$ and the new issuance ℓ_t such that $B_{t+1} = (1-\delta)B_t + \ell_t$. Each unit of debt calls for a payment of $r^* + \delta$ every period.[7]

The government can default on its debt, and depending on its default history, it is in good or bad credit standing, which is encoded in Θ_t. When the government repays the debt, $D_t = 0$, credit standing is good $\Theta_t = 0$ and the government borrows and decides on the level of debt next period B_{t+1}. When the government defaults, $D_t = 1$, it avoids paying the debt but receives a direct utility cost ν_t and a temporary bad credit standing, $\Theta_t = 1$. The utility costs ν_t are i.i.d. *enforcement shocks* that control the enforceability of debt. With bad credit, the government loses access to financial markets and productivity is depressed $z(\tilde{z}_t, \Theta_t = 1) \leq \tilde{z}_t$. A government in bad credit standing regains good credit with probability ι, and reenters financial markets with zero debt obligations.

The government transfers T_t to households which equals the net receipts from its operations. Its budget constraint in local currency conditional on having a good credit standing and repaying its debt is

$$T_t + \tau W_t N_t = \varepsilon_t \left[q_t(B_{t+1} - (1-\delta)B_t) - (r^* + \delta)B_t \right] + \tau_f P_t^f C_t^f, \tag{11}$$

where the net capital inflow from debt operations is multiplied by the nominal exchange rate ε_t to convert it to domestic currency. When the government is in bad credit standing, its budget constraint is as in (11) with $B_t = B_{t+1} = 0$. The tax rates for labor and foreign goods consumption, τ and τ_f, are set as in standard New Keynesian models, to correct the markup in goods markets and allow for a static optimal tariff on imports in the steady state, such that $(1-\tau) = \frac{\eta-1}{\eta}$ and $(1+\tau_f) = \frac{\rho}{\rho-1}$.[8] Using the definition of the terms of trade (1), the government budget constraint in units of domestic goods is

$$t_t + \tau w_t N_t = e_t[q_t(B_{t+1} - (1-\delta)B_t) - (r^* + \delta)B_t] + \tau_f e_t C_t^f. \tag{12}$$

The government's objective is to maximize the present discounted value of the flow utility derived from consumption and labor by households, $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta_g^t u(C_t, C_t^f, N_t)$. The government's discount factor β_g can differ from that of the households, β. The government borrows from competitive risk-neutral international lenders that discount the future at a foreign currency rate r^*. The bond price is such that they break even in expectation, thus receiving compensation for any expected losses from default:

$$q_t = \frac{1}{1+r^*} \mathbb{E}_t \left[(1 - D_{t+1})(r^* + \delta + (1-\delta)q_{t+1}) \right]. \tag{13}$$

In states where the government does not default, $D_{t+1} = 0$, each unit of the discount bond makes a payment $r^* + \delta$, and the fraction that does not mature has market value $(1-\delta)q_{t+1}$. In states of default, the associated payoff for lenders is zero. We define the government spread as the difference in the yield-to-maturity of the bond and the international rate r^*, such that

$$\text{spread}_t = (r^* + \delta)\left(\frac{1}{q_t} - 1\right).$$

We also define default risk γ_t as the default probability the following period $\gamma_t = \mathbb{E}_t D_{t+1}$.

[7] We normalize the debt service payment of the bond to $r^* + \delta$ so that the default-free bond price for this instrument equals 1.

[8] Such tariff neutralizes the potential incentive of the government to use debt to exert market power with respect to the downward-sloping demand for the country's exports. See for example Corsetti and Pesenti (2001) for details.

2.5 The Monetary Authority

We finally describe monetary policy. The monetary authority has a commitment to its monetary regime and sets policy according to a monetary rule. In our baseline inflation target regime, the monetary authority conducts policy using a nominal interest rate rule that targets domestic goods inflation. As in standard New Keynesian models, we assume that the nominal domestic rate i_t depends on a long-run value $\bar{i}$, is subject to monetary shocks m_t and responds to the deviation of inflation from their target, π_t relative to $\overline{\pi}$

$$i_t = \bar{i}\left(\frac{\pi_t}{\overline{\pi}}\right)^{\alpha_p} m_t. \tag{14}$$

with $\alpha_p > 1$. We will also analyze our model in counterfactual monetary regimes of strict inflation targeting, where the monetary authority sets interest rates to keep $\pi_t = \overline{\pi}$, and of default risk targeting, where the monetary rule also responds to default risk.[9]

3 Equilibrium

We consider a Markov equilibrium where the government takes into account that its states and default and borrowing policies affect the allocations of the private equilibrium and the monetary authority's response. The exogenous states are the productivity and monetary shocks $s = \{\tilde{z}, m\}$, and the enforcement shock, ν. The endogenous states for the private and monetary equilibrium are the level of debt B, the credit standing Θ, and borrowing B'. Recall that the government chooses to borrow B' only when it has good credit.

When the government enters the period with good credit standing $\Theta_{-1} = 0$ and has an endogenous state of debt B, it chooses whether to default or repay the debt. The default decision determines the end-of-the-period credit standing Θ. If it enters the period with bad credit $\Theta_{-1} = 1$, the government draws a random variable Λ following a Bernoulli distribution. With probability ι, $\Lambda = 1$ and the government regains good credit $\Theta = 0$ if it chooses not to default. The evolution of credit standing is given by

$$\Theta(s, \nu, B, \Theta_{-1}) = \begin{cases} 0 & \text{if } (\Theta_{-1} = 0 \text{ and } D = 0) \text{ or} \\ & \quad (\Theta_{-1} = 1 \text{ and } \Lambda = 1 \text{ and } D = 0) \\ 1 & \text{otherwise .} \end{cases} \tag{15}$$

The private and monetary equilibrium, which we label with $\Xi(S)$, depends on the shocks, the endogenous state for debt B, the credit standing Θ, and the choice for borrowing B', because these variables affect government transfers and productivity. Let $S = \{s, B, \Theta, B'\}$ be the end-of-period state. The private and monetary equilibrium also depends on the government policy functions for future default $D' = H_D(s', \nu', B')$, borrowing $B'' = H_B(s', B')$, and the corresponding credit standing $\Theta'(s', \nu', B', \Theta)$, because of the forward-looking nature of the equilibrium.

Definition 1. *Private and Monetary Equilibrium. Given state $\{S\}$, the government policy functions for default $H_D(s', \nu', B')$, borrowing $H_B(s', B')$, the evolution of credit standing $\Theta'(s', \nu', B', \Theta)$, and the transfer function $t(S)$ consistent with the government budget constraint, the symmetric private and monetary equilibrium $\Xi(S)$ consists of*

- *Households' policies for domestic goods consumption $C(S)$, foreign goods consumption $C^f(S)$, labor $N(S)$, and domestic debt $B^d(S)$,*

[9] Recall that in the environment of Galí and Monacelli (2005) with perfect financial markets, the optimal monetary policy in an economy with only productivity shocks is strict inflation targeting.

- *Intermediate and final goods firms' policies for labor $n(S)$, inflation $\pi(S)$, and final domestic goods' output $Y(S)$ and exports $X(S)$,*
- *The wage $w(S)$, nominal domestic rate $i(S)$, and the terms of trade $e(S)$*

such that: (i) *the policies for households satisfy their budget constraint and optimality conditions (5), (6), (7);* (ii) *the policies of intermediate and final goods firms satisfy their optimization problem (8), (9), and (10);* (iii) *export demand (2) is satisfied;* (iv) *the nominal domestic rate satisfies the monetary authority's interest rate rule (14); and* (v) *labor, domestic goods, and domestic bond markets clear, and the balance of payments condition is satisfied.*

The labor market clears so that labor demanded by firms equals labor supplied by households $n = N$. Domestic bonds are in zero net supply in the economy, reflected in the market clearing condition $B^d = 0$. The resource constraint for domestic goods requires that domestic final goods' output equals domestic consumption and exports net of the adjustment costs,

$$C(S) + X(S) + \frac{\varphi}{2}(\pi - \overline{\pi})^2 Y(S) = Y(S) \tag{16}$$

where aggregate output $Y(S) = z(\tilde{z}, \Theta)\, N(S)$.

The balance of payments condition requires that net imports equal net capital inflows, which here equal the government transfer plus the labor subsidy,

$$e(S)C^f(S)(1 + \tau_f) - X(S) = t(S) + \tau w(S)N(S). \tag{17}$$

The presence of price rigidities leads to inefficient use of labor, as monopolistic firms set time-varying markups. We will make use of a *monetary wedge* to measure these distortions in production, defined as

$$1 + \text{monetary wedge} \;\equiv \frac{z(\tilde{z}, \Theta)}{w(S)} = -\frac{z(\tilde{z}, \Theta)u_C(S)}{u_N(S)}. \tag{18}$$

This wedge captures deviations from production efficiency and depends on the dynamics of current and future inflation, as seen in the NKPC equation (10). Production efficiency requires that the marginal product of labor z equals the wage w, which is the marginal rate of substitution between labor and consumption for households $-u_N/u_C$.

3.1 Government Recursive Formulation

We now describe the recursive problem of the government, which borrows long-term bonds in international financial markets and can default. The bond price, $q(s, B')$, is an endogenous function that compensates lenders for default risk. It depends on the shocks s and the borrowing level B', because these affect the probability to default. The bond price schedule that satisfies the break-even condition for lenders depends on the default and borrowing policy functions of the government,

$$q(s, B') = \frac{1}{1 + r^*}\mathbb{E}\left[(1 - H_D(s', \nu', B'))(r^* + \delta + (1 - \delta)q(s', H_B(s', B'))\right]. \tag{19}$$

We can now set up the recursive problem of the government. Let $V(s, \nu, B)$ be the value with the option to default such that

$$V(s, \nu, B) = \max_{D \in \{0,1\}} \left\{(1 - D)W(s, B) + D\left[W^d(s) - \nu\right]\right\}, \tag{20}$$

where $W(s,B)$ is the value from repaying debt, and $W^d(s) - \nu$ is the value from defaulting. Specifically, the value of repaying is

$$W(s,B) = \max_{B'} \left\{ u(C, C^f, N) + \beta_g \mathbb{E} V(s', \nu', B') \right\} \tag{21}$$

subject to the private and monetary equilibrium $\Xi(S)$ with $S = \{s, B, \Theta = 0, B'\}$ and the break-even condition for the bond price schedule (19).

If the government chooses to default, the debt B is eliminated, productivity is reduced, and the government suffers the utility cost ν. After default, with probability ι, the government regains access to the international financial markets and re-enters with zero debt. The defaulting value W^d net of the enforcement cost is given by

$$W^d(s) = \left\{ u(C, C^f, N) + \beta_g \mathbb{E} \left[\iota V(s', \nu', B' = 0) + (1-\iota) W^d(s') \right] \right\} \tag{22}$$

subject to the private and monetary equilibrium $\Xi(S)$ with $S = \{s, B = 0, \Theta = 1, B' = 0\}$.

It is convenient to write the default decision of the government as a cutoff rule based on the default cost ν. Given that default costs are i.i.d., the default decision $D(s,\nu,B)$ can be characterized by a cutoff cost $\nu^*(s,B)$ at which the repayment value is equal to the default value such that

$$\nu^*(s,B) = W^d(s) - W(s,B), \tag{23}$$

and the sovereign is indifferent between the two options. Then $D(s,\nu,B) = 1$, whenever $\nu \leq \nu^*(s,B)$ and zero otherwise. Let Φ be the cumulative distribution of ν, such that default probability given s equals $\Phi(\nu^*(s,B))$.

We now define the recursive equilibrium for the economy.

Definition 2. *Equilibrium. Given the aggregate state $\{s,\nu,B\}$, a recursive equilibrium consists of government policies for default $D(s,\nu,B)$ and borrowing $B'(s,B)$, and government value functions $V(s,\nu,B)$, $W(s,B)$, and $W^d(s)$ such that*

- *Taking as given future policy and value functions $H_D(s',\nu',B')$, $H_B(s',B')$, $V(s',\nu',B')$, $W(s',B')$, and $W^d(s')$, government policies for default and borrowing and value functions solve its optimization problem.*
- *Government policies and values are consistent with future policies and values.*

3.2 Government Borrowing

To illustrate the forces shaping debt accumulation, we manipulate the government's problem and derive its optimality condition. In this derivation, we have assumed that all functions in the government problem are differentiable.[10] Optimal foreign borrowing satisfies the following Euler equation

$$\begin{aligned} u_{C^f} \left[q + \frac{dq}{dB'} \left(B' - (1-\delta)B \right) \right] (1 - \tau_m^X) - \tau_m^C \\ = \beta_g \mathbb{E} \left\{ (1 - D') u'_{C^f} \left[r^* + \delta + (1-\delta) q' \right] \left(1 - \tau_m^{X'} \right) \right\}. \end{aligned} \tag{24}$$

which relates the marginal utility of foreign goods consumption across periods to the bond price schedule $q(s,B')$, future default and borrowing decisions D' and B'', future bond prices $q(s',B'')$, and the borrowing wedges τ_m^X and τ_m^C. Appendix A contains the explicit derivation of this equation.

[10] We do not require this assumption for the computation of the model, nor do we employ the Euler equation derived in this section for the numerical implementation.

The borrowing wedges τ_m^X and τ_m^C arise only due to monetary frictions and capture the Lagrange multipliers associated with the NKPC and domestic Euler equations. These wedges depend on B' and are present in this Euler equation because the government internalizes the consequences of its borrowing and default decisions for the monetary frictions. When the NKPC and domestic Euler are slack, these multipliers are zero and so are the borrowing wedges.

Equation (24) reflects the government's borrowing incentives, which are affected by three major forces. The first is the incentive to smooth and tilt the time path of foreign goods consumption. This force is present in standard models without default risk, like Galí and Monacelli (2005), which exhibit the following undistorted international Euler equation

$$q\, u_{Cf} = \beta_g \mathbb{E}\left[u'_{Cf}(r^* + \delta + (1-\delta)q')\right]. \tag{25}$$

Here, borrowing smooths the marginal utility of foreign goods consumption against shocks and achieves the right tilting of consumption over time, given q and β_g.

The second force affecting borrowing is the endogenous bond price schedule q and the presence of legacy long-term debt $(1-\delta)B$ as emphasized by Arellano and Ramanarayanan (2012). Bond prices decrease with borrowing due to the increased risk of default, $\frac{\partial q}{\partial B'} \leq 0$, and a higher legacy debt $(1-\delta)B$ incentivizes borrowing because lower prices dilute this debt, $-\frac{\partial q}{\partial B'}(1-\delta)B \geq 0$. Such debt dilution incentives lead to *overborrowing*, as established by Hatchondo et al. (2016). A lower discount for the government, $\beta_g < \beta$, also leads to overborrowing as discussed by Aguiar et al. (2020).[11]

The third force works through the borrowing wedges τ_m^X and τ_m^C and is unique to our model with sovereign risk and monetary frictions. Positive borrowing wedges lower incentives to borrow. The borrowing wedges tend to be large when the inflation or the monetary wedge is high. As we explore below, when a default is likely, the expectations terms in the NKPC and domestic Euler equations tend to be high, which depress activity and lead to high inflation and a high monetary wedge. As a consequence, the borrowing wedges are high, giving the government an incentive to reduce B'. Lowering B' reduces default risk, which lowers the monetary wedge and inflation. An additional benefit of reduced borrowing is that it leads to a depreciation of the terms of trade and a boost in exports which lowers the monetary wedge. In the next section we provide a sharper characterization of these interactions of monetary frictions and default risk and relate them to our two main mechanisms, default amplification and monetary discipline.

4 Interactions of Monetary Frictions with Default Risk

This section studies in more detail the interactions of monetary frictions and default risk in simplified versions of our framework. We show that default risk increases monetary distortions by increasing inflation and monetary wedges. These monetary frictions in turn discipline the government's borrowing and lower default risk.

4.1 Default Amplification

In our environment, high default risk tends to increase inflation and generate inefficiently low production. These effects amplify aggregate responses to productivity shocks resulting in more volatile inflation and

[11] Cuadra and Sapriza (2008) and Hatchondo et al. (2009) model such additional discounting as arising from high political turnover.

monetary wedge. To illustrate this tradeoff, we revisit the key equilibrium NKPC pricing condition:

$$(\pi - \overline{\pi})\,\pi = \frac{\eta - 1}{\varphi}\left(-\frac{u_N}{zu_C} - 1\right) + \frac{\beta}{Yu_C}\,\mathbb{E}\left(Y'\,u'_C\left(\pi' - \overline{\pi}\right)\pi'\right). \tag{26}$$

Firms increase inflation when future expected inflation π' and the marginal utility of consumption u'_C are high or when the monetary wedge is low ($-u_N/zu_C$ is high).[12] We want to analyze how an increase in default risk impacts inflation. Recall that during a default event, inflation is high and consumption is low due to low productivity. Hence, if the risk of a default next period is high, expectations for future inflation and the marginal utility of consumption are high, which increases the expectation term of the NKPC. Such *inflation expectation effect* from default risk gives firms incentives to raise current inflation.

In response to these inflationary pressures, the monetary policy rule calls for higher interest rates, and tight monetary policy. These high nominal domestic rates, however, depress consumption through the domestic Euler equation

$$u_C = i\beta\,\mathbb{E}\frac{u'_C}{\pi'}. \tag{27}$$

Low domestic consumption in turn tends to reduce production and increase the monetary wedge. All in all, high default risk can end up causing high inflation and monetary wedges, and tight monetary policy. We call this effect *default amplification*.[13]

To characterize more formally the default amplification that works through inflation expectations and the domestic Euler equation, we simplify the model by assuming that preferences are quasi-linear in foreign goods consumption and are given by

$$u(C, C^f, N) = \theta \log C + (1-\theta)C^f - \frac{N^{1+\frac{1}{\zeta}}}{1+1/\zeta}. \tag{28}$$

We evaluate the responses of the model to an increase in default risk $E_{s'}\Phi(\nu^*(s', B'))$ from higher government borrowing B'.[14] With these preferences consumption of foreign goods fully adjusts to accommodate net capital inflows from debt operations. Higher borrowing only affects the private and monetary equilibrium through its impact on default risk and the expectation channels: high default risk changes the expectation terms in the pricing condition (26) and the domestic Euler (27).

In the recursive equilibrium, these expectations terms are taken as given by the sovereign. Let functions $F(s, B', \Theta)$ and $M(s, B', \Theta)$ encode these expectations, such that

$$\begin{aligned} F(s, B', \Theta) =& \mathbb{E}\left[z(\tilde{z},' \Theta')N(S')u_C(S')\left(\pi(S') - \overline{\pi}\right)\pi(S')\right], \\ M(s, B', \Theta) =& \mathbb{E}\frac{u_C(S')}{\pi(S')}. \end{aligned}$$

The sovereign understands that the future state $S' = (s', B', \Theta'(s', \nu', B', \Theta), H_B(s', B'))$ depends on the future government policies and the evolution of credit standing. We assume that $F(s, B', \Theta)$ and $M(s, B', \Theta)$ are differentiable and increase with borrowing, and then analyze how changes in borrowing affect the equilibrium using a first-order Taylor expansion.

[12] We find that u'_C and Y' tend to move in opposite directions but that the net effect is dominated by the marginal utility term and that the covariance between $Y'u'_C$ and π' tend to be positive.

[13] We thank Luigi Bocola for his insightful discussion that led to much of this analysis.

[14] We could consider other shocks that increase default risk, such as news about future productivity or default costs. Our results are robust to these other sources of variation.

Assumption 1. *$\partial F(s,B',\Theta)/\partial B' \geq 0$ and $\partial M(s,B',\Theta)/\partial B' \geq 0$, and the parameters satisfy the restriction $a_0 \geq (\partial M(s,B',\Theta)/\partial B')/(\partial F(s,B',\Theta)/\partial B')$, with $a_0 = \frac{\varphi\theta}{(\eta-1)\left(1+\frac{1}{\xi}(\alpha_C+\rho(1-\alpha_C))\right)\beta\bar{i}\bar{\pi}}$ where α_C is the share of domestic consumption in output at the approximating point in the Taylor expansion.*

These assumptions imply that expected inflation and the marginal utility of consumption rise with default risk as B' rises. These properties, that hold in our quantitative model, feed through the equilibrium and affect current allocations and prices. The following proposition characterizes up to first-order the effects on default risk, inflation, the nominal domestic rate, and the monetary wedge from increased borrowing under Assumption 1 and preferences given by (28) when inflation is around the target.

Proposition 1 (Amplification)**.** *Higher borrowing increases default risk, inflation, the nominal domestic rate, and the monetary wedge.*

Proof. See Appendix B.1 □

High borrowing increases default risk, which in turn affects current outcomes for inflation and the monetary wedge. High default risk increases expectations of future inflation and marginal utility. Given monetary policy i, the NKPC calls for an increase in current inflation and the domestic Euler equation calls for a decline in current consumption, which increases the monetary wedge. The interest rate rule that targets inflation generates an increase in the nominal domestic rate which dampens the rise in inflation but amplifies the increase in the monetary wedge.

These amplification effects of government borrowing on inflation and the monetary wedge from monetary frictions in Proposition 1 crucially relies on the presence of default risk. In environments without default risk, it is easy to show that higher borrowing would have no effect on inflation, the nominal domestic rate, domestic consumption, or output. With quasi-linear preferences, fluctuations in borrowing and capital flows would be simply absorbed by foreign goods consumption.

4.2 Monetary Discipline

In our framework not only default risk amplifies monetary frictions, but monetary frictions affect government borrowing and default risk. In particular, a tight monetary policy reduces the government's borrowing incentive and lowers default risk. We call this mechanism, *monetary discipline*. Next, we further simplify the model to characterize this mechanism and to relax some of the assumed properties in Assumption 1.

We alter the environment and monetary policy, such that our main frictions, i.e. pricing friction and overborrowing, are only present in period 0. In the full model, the government has overborrowing incentive due to its discount factor β_g potentially lower than that of households β, and the existence of the long-term debt. In this simplified version, we shut down the long-term debt channel and only retain the low discount factor. Furthermore, we examine the model against a constrained efficient case with $\beta_g = \beta$.

Specifically, we consider a case when the monetary authority delivers a strict inflation target for periods $t \geq 1$ such that $\pi_t = \bar{\pi}$, while in period 0 monetary policy is given by i. Moreover, consider the case of short-term debt $\delta = 0$, and with a fiscal government less patient than households, $\beta_g < \beta$ only in period 0. In periods $t \geq 1$, $\beta_g = \beta$. We also abstract from productivity shocks, $z_t = \bar{z}$ for any t. Default, however, triggers a fall in productivity $z_d \leq \bar{z}$, as well as permanent exclusion from international financial markets and a stochastic utility cost ν with a cumulative distribution function Φ and probability distribution function ϕ. We assume the hazard function $h(\nu) = \phi(\nu)/(1-\Phi(\nu))$ strictly increases with

ν.[15] These assumptions allow us to analyze the effects of a *one-time deviation* from a constraint efficient problem. The following assumption summarizes the settings for this case

Assumption 2 (One time deviation). *For $t \geq 1$ the monetary authority implements strict inflation targeting $\pi_t = \bar{\pi}$, and the fiscal government is as patient as the household $\beta_g = \beta$. In period 0, monetary policy is i and the fiscal government is less patient than households $\beta_g < \beta$. Productivity without default is constant $z_t = \bar{z}$, debt is short term $\delta = 0$, the exclusion is permanent $\iota = 0$, the hazard of the utility cost shock $h(\nu)$ is increasing in ν, and preferences are given by (28).*

Before analyzing the one-time deviation, we characterize the problem from $t \geq 1$ on. In this reference model, pricing frictions are neutralized and the government borrows and defaults in a constrained efficient manner, as shown in Aguiar et al. (2019). In general, as shown by Aguiar et al. (2019), a model with long-term debt exhibits inefficient borrowing and default due to the lack of commitment, via debt dilution. They characterize constrained efficient borrowing and show that it can be implemented with standard, defaultable short-term bonds. In light of these results, we label this reference economy *constrained efficient*.

Constrained efficient economy In the constrained efficient economy, the monetary authority implements strict inflation targeting, neutralizing completely the pricing frictions. Therefore, the economy features an undistorted consumption-labor choice and zero monetary wedges. Furthermore, with quasi-linear C^f, optimal domestic consumption, labor, and terms of trade are independent of debt, borrowing, and default. Specifically, the optimal allocations for C^{CE}, N^{CE}, and e^{CE} satisfy the following three equations, $C + e^{\rho} = \bar{z}N$, $C = \rho/(\rho-1)e$, and efficient labor choice $N^{1/\zeta}C = \bar{z}$. We can plus these triple into the utility function, and define $u^{CE} = \log C^{CE} - \frac{(N^{CE})^{1+1/\zeta}}{1+1/\zeta} + (e^{CE})^{\rho-1}$. The problem for the constrained efficient economy can then be simplified to

$$W(B) = \max_{B'} \; u^{CE} - B + q(B')B' + \beta \left\{ \left[1 - \Phi(\nu^*(B'))\right] W(B') + \int^{\nu^*(B')} (W^d - \nu) d\Phi(\nu) \right\}. \tag{29}$$

subject to the bond price function $q(B') = \frac{1}{1+r^*}\left[1 - \Phi(\nu^*(B'))\right]$, and where the default cutoff $\nu^*(B')$ satisfies $\nu^*(B) = W^d - W(B)$.

The default value W^d is the present value of financial autarky $W^d = \max u(C_d, C_d^f, N_d)/(1-\beta)$ subject to the resource constraint $C_d + e_d^{\rho} = z_d N_d$ and balanced trade condition $e_d^{\rho} = e_d C_d^f$.

The optimal borrowing in the constrained efficient economy $B' = B^{CE}$ satisfies the following Euler equation

$$1 - h(v^*(B^{CE}))B^{CE} = \beta(1 + r^*). \tag{30}$$

Note that the optimal borrowing in this model is constant and independent of current debt B because of the linearity of utility in C^f. Let $\Phi^{CE} = \Phi(\nu^*(B^{CE}))$ be the equilibrium default risk in the constrained efficient economy.

To implement the inflation target $\bar{\pi}$, the monetary authority needs to implement a nominal rate i^{CE} to satisfy the domestic Euler equation,

$$\frac{1}{C^{CE}} = \beta i^{CE}/\bar{\pi} \left[\frac{1-\Phi^{CE}}{C^{CE}} + \frac{\Phi^{CE}}{C_d} \right].$$

This constrained efficient problem gives the economy allocations for $t \geq 1$. These allocations matter for the economy in period 0, because of the expectation terms in the NKPC, the Euler equation, the bond

[15] It is easy to find a distribution function with this property. For example, under the normally distributed ν, a sufficient condition for strictly increasing h is that the equilibrium default is less than 1/2.

price function, as well as the continuation utility. See Appendix B.2 for a detailed characterization of the constrained efficient allocations.

One-time deviation economy We now study the one-time deviation economy. Recall that in period 0, monetary policy is given by i and the government has a lower discount factor $\beta_g < \beta$. Inflation endogenously arises from NKPC condition and may not stay at the target. In period 0, conditional on not defaulting, the government solves the following problem:

$$\max_{B',C,C^f,N} u(C,C^f,N) + \beta_g \left\{ \left[1-\Phi(\nu^*(B'))\right] W^{CE}(B') + \int^{\nu^*(B')} (W^d - \nu) d\Phi(\nu) \right\} \tag{31}$$

subject to the private equilibrium conditions and the bond price function

$$\begin{aligned}
C + e^{\rho} &= \left[1 - \frac{\varphi}{2}(\pi - \bar{\pi})^2\right] zN && \text{(resource constraint)} \\
e^{\rho} &= e\left[C^f + B_0 - q(B')B'\right] && \text{(balance payments)} \\
C &= \frac{\rho}{\rho - 1} e && (\text{intra } C - C^f) \\
-\frac{u_N}{zu_C} &= 1 + \frac{1}{\eta - 1}\varphi(\pi - \bar{\pi})\pi && \text{(NPKC)} \\
\frac{1}{C} &= \frac{\beta i}{\bar{\pi}} \mathbb{E} u_{C'}(B') = \frac{\beta i}{\bar{\pi}} \left[\frac{1 - \Phi(\nu^{CE}(B'))}{C^{CE}} + \frac{\Phi(\nu^{CE}(B'))}{C_d}\right] && \text{(domestic Euler)} \\
q(B') &= \frac{1}{1+r^*}\left[1 - \Phi(\nu^{CE}(B'))\right]. && \text{(bond price)}
\end{aligned}$$

The expectation terms of this problem are those arising from the constrained efficient problem. The NKPC condition uses that in the future $\pi = \bar{\pi}$ and the expected marginal utility $Eu_{C'}(B')$ is the weighted average of marginal utility of future consumption in non-default C^{CE} and default C_d state; the bond price function and the expected marginal utility use the cutoff function ν^{CE} for arbitrary state B'; and the continuation value function is also that of the constrained efficient problem for the arbitrary state.

This problem reveals several important properties. For a given debt state B, nominal rate i, and a choice for repayment and borrowing B', the private equilibrium conditions pin down the allocations C, C^f, N and prices e, π as well as the wage w (which is substituted in the NKPC above). The private equilibrium, however, depends on the future, through the expectations of the future default and consumption function, which enter into the domestic Euler equation and bond price function.

We analyze how monetary policy i impacts the equilibrium in the one-time deviation economy. Consider a candidate nominal rate such that equation (31) delivers strict inflation targeting $\bar{\pi}$ at period 0. Let such nominal rate in period 0 be i^{ST}. With strict inflation targeting, the monetary wedge is zero and domestic consumption and labor are efficient, given by (C^{CE}, N^{CE}). Optimal borrowing, however, B^{ST}, is different from the constrained efficient borrowing since the government has an overborrowing incentive and satisfies

$$1 - h(v^*(B^{ST}))B^{ST} = \beta_g(1 + r^*). \tag{32}$$

The above Euler equation is similar as the one for the constrained efficient economy, except that in the one-time deviation, the government discounts the future more heavily β_g. Such impatient leads to overborrowing of government, $B^{ST} > B^{CE}$, and is costly to households' welfare. These results are summarized in the next lemma.

Lemma 1. *Under assumption 2, the monetary authority can deliver strict inflation targeting $\pi = \bar{\pi}$ in period 0. Under this monetary policy, default risk is higher and households' welfare is lower than in the constrained efficient outcome, $\Phi^{ST} > \Phi^{CE}$, $W^{ST} \leq W^{CE}$.*

The lemma demonstrates that it is possible for the monetary authority in period 0 to eliminate domestic pricing frictions by targeting inflation. However, the economy ends up with lower welfare than the constrained efficient case since the government borrows and defaults too much. The monetary authority, however, can set nominal rates at different levels, and given a nominal rate, the monetary wedge responds to changes in default risk. The next lemma shows that with tighter monetary policy, increases default risk increases the monetary wedge.

Lemma 2. *Under assumption 2, higher default risk $\Phi(v^*)$ increases the monetary wedge $-zu_C/u_N$, when the monetary wedge is positive.*

This result is similar to the amplification result in Proposition 1, but derived for the one-time deviation economy under assumptions on the environment summarized in Assumption 2. Higher default risk increases the future marginal utility of consumption since the expectation in (domestic Euler) places more weight on default states in which consumption is lower, $C_d \leq C^{CE}$. In response to this low expected future consumption, current consumption, C, declines. Low C also leads to a real appreciation, e decreases, which reduces exports. Labor is lower because of lower demand in both domestic and export markets. As a result, the monetary wedge increases.

We can also compare the equilibrium default risk and monetary wedges for a different stance of monetary policy in period 0. We find that tighter monetary policy disciplines the fiscal government to borrow less. The following proposition summarizes our disciplining result

Proposition 2 (Discipline)**.** *Under assumption 2, if $i > i^{ST}$, the monetary wedge is positive and the default risk is lower than with strict inflation targeting.*

High policy rate $i > i^{ST}$ depresses domestic consumption and generates a positive monetary wedge. The government internalizes its borrowing on future default risk and domestic consumption and the Euler equation for optimal borrowing reflects such effects,

$$1 - h(v^*(B'))B' - \kappa \left(\frac{\partial E u_{C'}(B')}{\partial B'} \frac{u_C}{E u_{C'}(B')} \right) = \beta_g (1 + r^*) \tag{33}$$

where κ is the multiplier on (domestic Euler) and is positively related to the monetary wedge. From Lemma 2, higher borrowing B' increases default risk and expected marginal utility, and so $\partial E u_{C'}(B')/\partial B' > 0$. Hence, the government faces an additional cost of borrowing $\kappa \left(\frac{\partial E u_{C'}(B')}{\partial B'} \frac{u_C}{E u_{C'}(B')} \right) > 0$ relative to keeping inflation at target in period 0. This term reflects the cost that higher borrowing increases expected future marginal utility and depresses current consumption and production. Hence, with a more restricted monetary policy $i > i^{ST}$, the government borrows less, and the economy ends up with a lower default rate.

Proposition 2 illustrates a tradeoff faced by the monetary authority. The monetary authority can choose strict inflation targeting to eliminate pricing frictions but at the cost of inefficient borrowing and default risk. Alternatively, it can implement a higher policy rate $i > i^{ST}$ to induce a more efficient default risk at the cost of positive monetary wedges.

A natural theoretical question within this setup is whether a sophisticated interest rate rule could be designed to eliminate both distortions at once, given the preferences of the government and households. We propose an interest rate rule that targets the default risk, and that can achieve both objectives: efficient default risk and near-zero monetary wedge. The rule, which we label as the *default risk monetary rule*, contains an intercept and dictates increasing nominal rates when default risk rises. Because of the disciplining effects of tight monetary policy in our model, this rule can curve borrowing and default risk. Moreover, the intercept of the rule can be chosen for achieving production efficiency. We show below that the default risk monetary rule can deliver both the efficient default risk and near-zero monetary wedges.

Proposition 3 (Default Risk Monetary Rule)**.** *A monetary policy rule of the form $i = \bar{i}(\Phi/\Phi^*)^{\alpha_D}$ can achieve the constrained efficient default risk Φ^{CE} and an arbitrary small monetary wedge of size ε, with a bounded and positive coefficient α^D.*

Here is the sketch of the proof. In the optimal monetary rule, we set $\bar{i} = i^{CE}$ and $\Phi^* = \Phi^{CE}$. The government's Euler is now given by

$$1 - h(\nu^*(B'))B' - \kappa \left[\frac{\partial E u_{C'}(B')}{\partial B'} \frac{u_C}{E u_{C'}(B')} + \alpha_D u_C \frac{\phi(\nu^*(B'))}{\Phi(\nu^*(B'))} \right] = \beta_g(1 + r^*).$$

It reflects the marginal cost of borrowing in terms of tightening the domestic Euler equation and the changes in monetary policy $\alpha_D \phi/\Phi$. We can implement an arbitrary monetary wedge ε and the constrained efficient borrowing by choosing α_D to satisfy the following condition

$$\kappa(\varepsilon) \left[\frac{\partial E u_{C'}}{\partial B'} \frac{u_C}{E u_{C'}\left(B^{CE}\right)} + \alpha_D u_C \frac{\phi^{CE}}{\Phi^{CE}} \right] = \left(\beta - \beta_g\right)(1 + r)$$

where κ and u_C are pinned down by monetary wedge ε. From the NKPC condition, we can find inflation

$$\frac{1}{\eta - 1} \varphi(\pi - \bar{\pi})\pi = -\frac{\varepsilon}{1 + \varepsilon}$$

which is close to its target when $\varepsilon \to 0$. Hence with targeting only default risk, the optimal monetary rule is able to fix two frictions by achieving a near-efficient domestic production and constrained efficient borrowing.

We have shown that in a simplified version of our model, high default risk can induce higher inflation and positive monetary wedges because it affects expectations of future inflation and consumption. Tight monetary policy, in turn, can discipline overborrowing incentives of the government and can curb default risk. In deriving these results, we focused on an economy where default risk affects inflation and monetary wedges only through the expectations channel, and abstracted from shocks and preferences that are concave in imported consumption. In the next section, we analyze our general model and show numerically that our key theoretical results are present and shape much of the quantitative outcomes.

5 Quantitative Analysis

We now conduct the quantitative analysis of our model and map it to data. We start by describing patterns of emerging market data. We document comovements of inflation, nominal domestic rates, and spreads, discuss properties of salient inflation events, and provide evidence of the key disciplining mechanism in the model. We then describe the parameterization of the model, discuss decision rules and impulse responses, and compare the model's implications with the data. In drawing our lessons, we also compare our baseline NK-Default model to a reference model without default risk and analyze the welfare implications of various monetary policy rules.

5.1 Data in Emerging Market Inflation Targeters

Several emerging markets successfully adopted an inflation-targeting regime as their monetary policy in the early 2000s. We analyze data from these countries and document some stylized facts on the volatility and covariation of inflation, spreads, and domestic nominal rates. We also perform an event analysis around periods of elevated inflation and provide some evidence for our disciplining mechanism, namely that tight monetary policy lowers government spreads.

	Mean		Std. Dev. Rel. Output		Correlation with Spread		
	Inflation	Spread	Inflation	Spread	Inflation	Domestic Rate	Output
Brazil	5.6	2.8	0.6	0.3	0.6	0.6	−0.6
Chile	3.3	1.4	0.9	0.2	0.3	0.2	−0.6
Colombia	4.2	2.3	0.9	0.5	0.6	0.4	−0.4
Mexico	4.1	2.2	0.4	0.3	0.2	0.1	−0.5
Peru	2.8	2.0	0.5	0.3	0.5	0.1	−0.0
Philippines	3.8	2.1	1.3	0.8	0.7	0.6	−0.6
Poland	2.0	1.1	0.9	0.4	0.4	0.2	−0.4
South Africa	5.4	2.2	1.0	0.5	0.5	0.1	−0.7
Mean	**3.9**	**2.0**	**0.8**	**0.4**	**0.5**	0.3	−0.5

Table 1: Emerging Market Inflation Targeters, Key Statistics

Stylized Facts. The adoption of inflation targeting in many emerging markets has been successful in bringing average inflation down to single digits.[16] We collect data on inflation, spreads, nominal domestic rates, and output for eight emerging markets that are inflation targeters. The sample of emerging markets consists of those in the JP-Morgan Emerging Market Bond Index (EMBI) that are inflation targeters. The data start in 2004, by which point all countries considered had adopted inflation targeting, and run through 2019.

Table 1 reports key statistics on the joint behavior of these series using quarterly series. Because of data availability, we focus on inflation based on the consumer price index (CPI) and compute it as the log difference in the index relative to four quarters prior.[17] The spreads are EMBI-based and are measured as the difference in yields between foreign currency government bonds of these emerging markets and a comparable U.S. government bond. Domestic nominal rates are the policy rates of central banks, and output is the four-quarter difference in log real gross domestic product.[18]

We highlight several stylized facts across these emerging market inflation targeters. Inflation is on average low and relatively stable for these inflation targeters. Mean inflation across these countries ranges from 2.0% to 5.6%, with an overall average of 3.5%. The standard deviation of inflation relative to that of output ranges from 0.4% to 1.3%, with an average across countries of 0.8%. This stable inflation pattern contrasts sharply with the historical experience of these countries of very high and volatile inflation. Emerging markets bond spreads are on average 2.0% and with an average standard deviation of 0.4%.

[16]See Roger (2009) and Ha et al. (2019) for more details on the implementation and performance of inflation targeting in emerging markets.

[17]We have confirmed that the main moments for domestic goods (producer price index) inflation are very similar to the CPI ones, for the countries where both are available.

[18]See Appendix ?? for detailed definitions and sources of this data.

We also report correlations of spreads with inflation, nominal domestic rates, and output. As documented in many studies, spreads are negatively correlated with output, with an average correlation of -0.5 for this sample. Correlations of spreads with nominal rates and inflation are strongly positive, on average 0.3 and 0.5, respectively. Note that domestic nominal rates are based on local currency, whereas government spreads are based on foreign currency. The different currency denomination of the underlying securities rules out that the positive correlation is due to a common inflation factor but instead can be understood as a positive correlation between inflation and default risk.

Temporary Inflation Events. Although emerging markets inflation targeters have been successful in keeping average inflation low, these countries have experienced some events during the last 20 years when inflation rose a few percentage points temporarily. Importantly, these *temporary inflation events* have been relatively subdued and inflation returned to lower levels shortly after the monetary authority tightened policy.[19] Here we illustrate the dynamics around these events and later use them in our quantitative analysis.

We classify a country as experiencing an inflation event if its inflation is unusually high, defined as two standard deviations above its mean. We center the events around the peak of inflation, where the standard deviation is calculated country by country, and analyze dynamics 2 years before and after this peak. In our panel data, we find 9 events, which at their peaks are: Brazil 2015Q4, Chile 2008Q3, Colombia 2008Q4 and 2015Q2, Mexico 2008Q4 and 2017Q4, Peru 2008Q4, Philippines 2008Q4, South Africa 2008Q3.[20]

Figure **??** plots averages across the 9 events for the dynamics of inflation, domestic nominal rates, spreads, and output, starting 8 quarters before the peak inflation and ending 9 quarters after the peak. The underlying series are standardized country by country, therefore the units in Figure **??** are standard deviations and a value of zero means the series is at its average. The top left panel illustrates the tent-like shape of inflation during the event. Inflation starts at its average level and increases close to 3 standard deviations at its peak before returning to low levels. The 3 standard deviations increase corresponds to an average of 5 percentage points. Output, in the top right panel, starts above its average and falls about 2.5 standard deviation, which corresponds to 5.8 percentage points, before recovering towards the end of the event. The lower left panel plots the domestic nominal rate. This rate increases by about 1 standard deviation and then falls at the end of the event. At its peak, the domestic nominal rate increase corresponds to 2.9 percentage points above its mean. The bottom right panel plots the spread dynamics. Spreads also feature a tent-like shape, increasing about 2.5 standard deviations at its peak, corresponding to about 2.3 percentage points, and falling back down towards the end of the event. In Appendix ??, we plot the underlying series for the 9 events, which feature similar patterns.

These event dynamics illustrate that emerging markets' inflation targeters have been successful at managing shocks that lead to temporary high inflation. The resolution of these inflation episodes is impressive as inflation decreased about 5 percentage points and returned to target in about a year. These dynamics also reinforce the stylized facts presented above. Elevated inflation tends to be accompanied by increased spreads and recessions. Monetary policy in these inflation targeters in turn responds with higher nominal rates.

We also find it notable that spreads are high during these inflation events. The standard New Keynesian model, as in Galí and Monacelli (2005), is silent about sovereign spreads, yet this empirical

[19] The recent surge of global inflation has been unusual in the time series of advanced economies inflation targeters but emerging economies inflation targeters, have experienced some temporary inflation events in the last two decades.

[20] These events tend to occur around the commodity price booms of 2008 and 2017. Our data runs through the end of 2019 and therefore do not consider the inflation post COVID-19.

regularity of emerging markets is consistent with a main mechanism in our NK-Default framework (illustrated in Section 4.2) that default risk can be an amplifying force for elevated inflation. Below we will use our quantitative model to measure and shed light on the quantitative importance of this mechanism, but before that, we also provide some empirical support for the disciplining effect.

Disciplining Effect in the Data. In this subsection we provide empirical evidence of the key disciplining mechanism in our model, derived in Section 4.2, that tight monetary policy can reduce government spreads. As we illustrated above, unconditionally nominal rates tend to rise with sovereign spreads, as seen by the overall positive correlation and during the inflation events. This unconditional positive correlation is of course silent on the direct effect of monetary policy on spreads, which is the object of interest for the disciplining effect.[21] To tease out the direct effect of monetary policy on spreads, we follow the monetary literature and first recover monetary shocks. With the estimated monetary shocks, we then project government spreads on them.

For the first step, we estimate monetary policy shocks country by country. In particular, for each country c we recover the monetary shock $m_{c,t}$ by running a standard Taylor-type regression of interest rates on inflation, output, and a lag on interest rates such that $i_{c,t} = \alpha_c^0 + \alpha_c^1 i_{t-1} + \alpha_c^2 \pi_{c,t} + \alpha_c^3 y_{c,t} + m_{c,t}$, where we allow the residuals to be autocorrelated, $m_{c,t} = \rho_c m_{c,t-1} + \varepsilon_{c,t}$ and $\varepsilon_{c,t} \sim \mathcal{N}(0, \sigma_m)$. In estimating these regressions, we follow the literature and use monthly data for policy rates $i_{c,t}$, inflation $\pi_{c,t}$, and output $y_{c,t}$. The series for nominal rates and inflation are the same as the ones used for our stylized facts, but at a monthly frequency, and as in much of the literature, we use industrial production to measure output due to the monthly availability and define it as year-to-year growth. We find that the average persistence and standard deviation of the monetary shocks across countries are 0.3 and 0.25%, respectively.

We now estimate the effects of monetary shocks on spreads with a panel regression, where we project spreads for country c at time t on a vector of monetary shocks with L lags and additional controls $Z_{c,t}$

$$\text{spr}_{c,t} = \sum_{h=0}^{L} \beta_h \varepsilon_{c,t-h}^m + \Gamma Z_{c,t} + \nu_{c,t}. \tag{34}$$

The coefficients of interest are the β_h's. We include in the controls $Z_{c,t}$ country fixed effects, contemporaneous values for output growth and inflation, as well as 6 lags for these variables. The identifying assumption is that with these control variables, the coefficients β_h tease out the effects of monetary shocks on fluctuations in spreads. We also cluster standard errors across time and country.

Figure 2 plots the estimated coefficients β_h for $h = 0, 1, ..., 6$ together with their 95 confidence band. Contractionary monetary policy shocks tend to decrease spreads. The impact effects are sizable; a positive monetary policy shock of 1% decrease spreads by about 0.4% on impact. The effects are also persistent as spreads remain below their mean for 3 periods, but the significance of the effect decreases. The median elasticity of spreads with respect to interest rates during the first quarter is -0.3, with a 95 percent confidence band between -0.6 and -0.1. In the appendix, we present robustness results of these disciplining effects with a variety of specifications for the interest rate rule, including a specification that only contains an inflation term (as in our baseline model), by varying the lag structure, and with variables that are standardized. We show that the results are robust to these different specifications.

[21] The unconditional positive correlation of nominal rates and spreads, can be understood as a third factor affecting both inflation and spreads. If, for example, productivity shocks are the main source of the business cycle (as it will be in our model) and low productivity is associated with increases in default risk and inflation, then productivity fluctuations under an inflation-targeting regime can immediately rationalize the unconditional positive correlation.

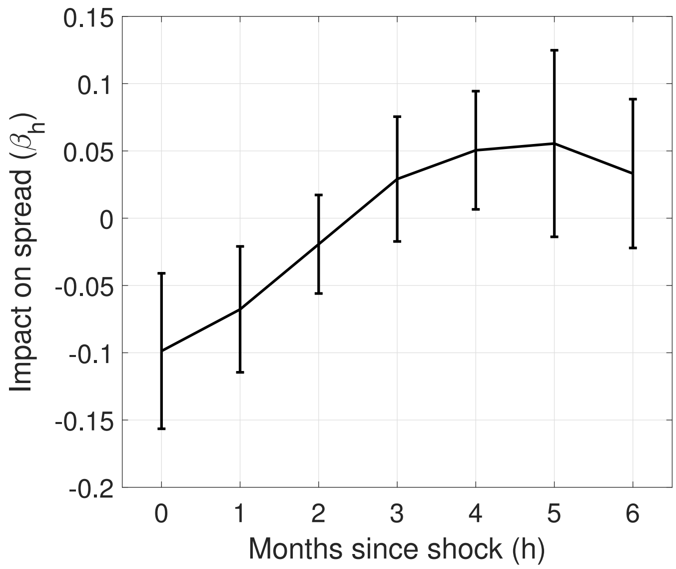


Figure 2: Monetary Discipline Estimates

5.2 Functional Forms and Parameterization

We now describe the parameterization of the quantitative model. We assume separable preferences between a composite H of domestic goods C and imported goods C^f and labor N. The per-period utility function is given by

$$u(C_t, C_t^f, N_t) = \log\left[H(C_t, C_t^f)\right] - \frac{N_t^{1+1/\zeta}}{1+1/\zeta}, \tag{35}$$

where $H(C_t, C_t^f)$ is the CES composite $H(C_t, C_t^f) = \left(\theta C_t^{\frac{\omega-1}{\omega}} + (1-\theta)(C_t^f)^{\frac{\omega-1}{\omega}}\right)^{\frac{\omega}{\omega-1}}$. Here θ controls the share of imports in consumption and ω is the elasticity of substitution between foreign and domestic goods. We can derive the consumer price index (CPI) as the price of the bundle of domestic and foreign goods consumption, $P^{\text{CPI}} = P^d\left[\theta^\omega + (1-\theta)^\omega e^{1-\omega}\right]^{\frac{1}{1-\omega}}$, and the resulting CPI inflation, $\pi^{\text{CPI}} = \frac{P^{\text{CPI}}}{P^{\text{CPI}}_{-1}}$, where the subscript $_{-1}$ denotes the previous period's value. The rate of depreciation of the nominal exchange rate is

$$\frac{\varepsilon}{\varepsilon_{-1}} = \frac{e}{e_{-1}}\frac{P^d}{P^d_{-1}} = \frac{e}{e_{-1}}\pi, \tag{36}$$

which reflects inflation and the depreciation of the terms of trade.

The model contains three shocks: productivity z_t, monetary m_t, and enforcement ν_t. We assume that productivity and monetary shocks follow AR(1) processes such that $\log\tilde{z}_t = \rho_z\log\tilde{z}_{t-1} + \sigma_z\varepsilon_{z,t}$ and $\log m_t = \rho_m\log m_{t-1} + \sigma_m\varepsilon_{m,t}$ with innovations $\varepsilon_{z,t},\ \varepsilon_{m,t} \sim \mathcal{N}(0,1)$. Following Chatterjee and Eyigungor (2012) productivity suffers a convex penalty while in default, $\max\left\{0, \lambda_0\tilde{z} + \lambda_1\tilde{z}^2\right\}$ with $\lambda_0 \le 0 \le \lambda_1$, such that

$$z(\tilde{z},\Theta) = \begin{cases} \tilde{z} & \text{, if } \Theta = 0 \\ \tilde{z} - \max\left\{0, \lambda_0\tilde{z} + \lambda_1\tilde{z}^2\right\} & \text{, otherwise.} \end{cases}$$

The model also contains enforcement shocks ν that control the relative values of repayment and default. We integrate these shocks into our computational technique following Dvorkin et al. (2018) and Gordon

(2019).[22] The shocks to the default-repayment decisions map into the model's enforcement shocks ν as a logistic distribution, where the parameter ϱ_D controls the relative importance of the shocks.

Table 2: Parameter Values

Assigned Parameters		*Parameters from Moment Matching*	
Preferences	$\omega = 0.85, \theta = 0.64, \zeta = 0.33$	Discount factor	$\beta = 0.9955$
Varieties elasticity	$\eta = 6$	Interest rate rule	$\alpha_P = 1.45$
Export demand elasticity	$\rho = 3$	Inflation target	$\overline{\pi} = 1.011$
Price adjustment cost	$\varphi = 58$	Government discount factor	$\beta_g = 0.9877$
Shock processes	$\rho_z = 0.9, \rho_m = 0.3, \sigma_m = 6e^{-4}$	Productivity volatility	$\sigma_z = 1.2\%$
International rate	$r^* = 0.5\%$	Default costs	$\lambda_0 = -0.16, \lambda_1 = 0.18$
Reentry probability	$\iota = 4.17\%$	Enforcement shock	$\varrho_D = 1e^{-4}$

Table 3: Targeted Moments

		Data	NK-Default
Means	CPI inflation	3.9	4.0
	Nominal domestic rate	5.6	5.6
	Spread	2.0	2.0
Standard Deviations	Output	2.3	2.2
	CPI inflation	1.7	1.9
	Spread	0.9	1.0
	Consumption aggregate	2.4	2.1
Correlation	(Output, Spread)	−49	−45

We consider a quarterly time frequency and set some parameters based on prior studies and others as part of a moment-matching exercise. The first set of parameters, assigned directly, includes the Frisch elasticity of labor supply ζ, the export demand elasticity ρ, the elasticity of substitution between foreign and domestic goods in consumption ω, the weight of domestic goods in consumption θ, domestic varieties' elasticity η, the international interest rate r^*, the probability of return to financial markets after default ι, the Rotemberg adjustment cost φ, the persistence and standard deviation of monetary shocks $\{\rho_m, \sigma_m\}$, and the persistence of the productivity shock ρ_z.

For the Frisch elasticity of labor supply, we choose a value of 0.33 following Galí and Monacelli (2005). This is a conservative value in line with the open economy New Keynesian literature. The export demand elasticity ρ is assigned as 3, which falls within a wide range of estimates for trade elasticity found in the literature, as reviewed by Bajzik et al. (2020). We set θ to induce an imports' share in the balanced-trade steady state of 29%, which implies $\theta = 0.64$, given the value of the elasticity of substitution, $\omega = 0.85$, following Corsetti et al. (2008). The elasticity of substitution between varieties η is set to 6 to induce a 20% markup, which is in line with the markup estimates in the literature, for example, Edmond, Midrigan,

[22] This computational technique consists of augmenting the model with taste shocks in the discrete choice tradition. The taste shocks slightly perturb the sovereign choices and help with numerical stability and robust convergence in models with long-term defaultable debt. Appendix **??** details the structure of taste shocks and their numerical properties, as well as the algorithm for the computation and simulation of the model.

and Xu (Edmond et al.) and Díez et al. (2021).

We set the Rotemberg adjustment cost using the well-known first-order equivalence between Calvo and Rotemberg pricing frictions: our varieties' elasticity of $\eta = 6$ and a Calvo frequency of price changes of roughly once per year (once every fourth quarter) imply a value for φ of 58.[23] The intercept of the interest rate rule is set to satisfy the steady-state condition $\bar{i} = \overline{\pi}/\beta$. We normalize the level of export demand ξ to 1. The international risk-free rate is 2% annually, consistent with U.S. Treasury yields, implying $r^* = 0.5\%$. We target an average length of market exclusion of roughly six years, which is an average duration of sovereign defaults based on Cruces and Trebesch (2013). We choose the persistence parameter ρ_z to be 0.9, comparable with many international real business cycle studies. The persistence and standard deviation of the monetary shock process are based on our empirical estimates. In particular, we choose the standard deviation of the monetary innovation such that the model has the same standard deviation of monetary shock as in the data.

The second set of parameters is chosen as part of the moment-matching exercise, such that our model replicates salient moments of the data in emerging markets inflation targeters. These eight parameters are the discount factors of the private sector β and of the government β_g, respectively, the inflation target $\overline{\pi}$, the interest rate rule coefficient α_P, the volatility of the productivity innovations σ_z, the parameters of the default cost function $\{\lambda_0, \lambda_1\}$, and the parameter governing the importance of the enforcement shock ϱ_D. We target average moments across the sample of emerging market inflation targeters. These moments are averages for the means and standard deviations of CPI inflation and spreads, means of nominal domestic rate, standard deviations of output and consumption, and the correlation of spread and output. Most parameters affect all moments, yet some moments are more informative for certain parameters. The average CPI inflation rate in the data is the most informative for $\overline{\pi}$. The weight on inflation in the interest rate rule α_P heavily affects the volatility of CPI inflation. The volatility of productivity shocks is the main driver of output volatility. As in standard sovereign default models, the productivity default cost parameters, the borrower's discount β_g, and the enforcement shock parameter ϱ_D are crucial for the dynamics of spreads and the volatility of consumption. The discount factor of the private sector β controls the average real domestic rate. We collect the values of all the parameters in Table 2.

Table 3 contains the results of the moment-matching exercise. Overall, we find that the model is able to replicate closely the data target moments. CPI inflation, nominal domestic rates, and spreads are reported annualized. The model matches quite closely the moments in the data. In the model and data, CPI inflation is about 3.9%, spreads are 2%, and nominal domestic rates are about 5.6%. The standard deviation of output is about 2.2%, and that for CPI inflation and consumption is a bit below 2%. The volatility of spreads is just under 1% in both the data and the model. Output is negatively correlated with spreads, with a correlation close to −50%.

5.3 Policy Rules and Impulse Response Functions

To analyze the workings of the general model, we now describe the model's policy functions and impulse response functions to monetary and productivity shocks. With these functions, we illustrate that our main mechanisms of amplification and discipline, presented in Section 4.2 for the simplified model, remain in the quantitative model.

Policy Rules. Figure 3 presents policy rules as a function of government debt B relative to mean output, for the average values of the productivity and monetary shocks, with the focus on the behavior conditional

[23] See, for example, Miao and Ngo (2021) for the mapping between the Calvo and Rotemberg parameters.

on not defaulting. To highlight the impact of default risk on policy functions, we plot two lines in each figure, a solid line for the variable of interest and a dotted line for the equilibrium one-period-ahead default risk. As shown in the dotted lines in the plots, default probabilities increase with current debt B because debt due next period $B' = H_B(s, B)$ increases with B, which makes default more likely.

Default risk affects domestic allocations in the general model through two main channels. The first channel is the expectations channel: default risk alters the expectations for the domestic Euler equation and the NKPC. This channel is the one we focused on in our theoretical analysis of Section 4.2, by considering a utility function that is linear in imported consumption C_f, which effectively makes terms of trade independent of C_f. In our quantitative model, with preferences concave on imported consumption, a second channel arises, as debt and default risk affect the terms of trade through its impact on capital flows and imported consumption. In describing the policy rules in Figure 3, we will be discussing two main regions, a region with positive default risk, for levels of B above 0.3 in the figures, where the expectations channel is mostly at play, and another one with almost no default risk, for lower debt levels, where the terms of trade channel effects dominate.

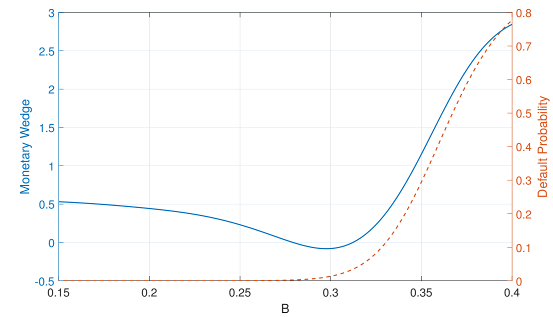
Panel 3(a) plots our key result for the monetary wedge. The monetary wedge is high when the default risk is high. Moreover, it tends to increase with B when default probabilities are positive. In this region, high default risk increases future expected marginal utility, and through the domestic Euler equation, it lowers current domestic consumption, as seen in Panel 3(e). Low demand for domestic consumption lowers labor too, as seen in 3(d) in the positive default region of B.

Inflation and nominal rates are also high when default risk is high. According to the NKPC (10), current inflation is negatively related to the monetary wedge and positively related to expected future inflation. In the low default risk region, increases in debt lead to a depreciation in the terms of trade, which tends to increase labor and inflation. In the high default region, terms of trade stabilize, and the monetary wedge rises. Overall, when comparing the high versus low default-risk region of B, high default risk is associated with high inflation, as shown in Panel 3(b). In response to the high inflation, the monetary authority implements a high nominal rate in accordance with the interest rate rule, as in Panel 3(c).

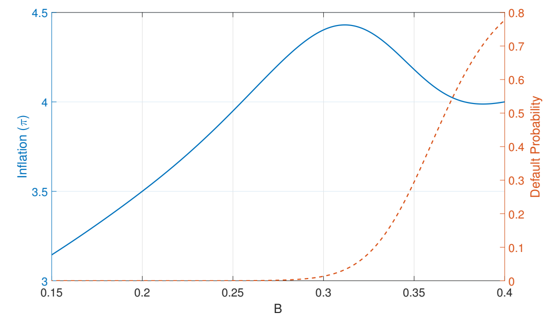
Panel 3(g) plots the policy function for capital inflows, which equals $q(s, B')(B' - (1-\delta)B) - (r^* + \delta)B$ in our model with long-term debt. With default risk encoded in the bond price schedule, government borrowing increases slower than current debt, and hence capital inflows decrease as a function of debt B, resulting in a decline in imported consumption C^f, as shown in Panel 3(f). In the low debt region with no default risk, this effect leads to a depreciation in terms of trade 3(h), which boosts export demand and raises labor.

Note the patterns in the low default region follow the results in the standard literature. As Blanchard et al. (2017) discuss, in the workhorse open-macro model, a reduction in capital inflows is expansionary because it depreciates the exchange rate, the so-called expenditure switching effect. The novel finding in our model with default risk is that a reduction in capital inflows can be recessionary, as exemplified in the high default region. This is because a reduction in capital inflows due to high default risk leads to a reduction in domestic consumption demand through the expectation channel, which in turn depresses production.

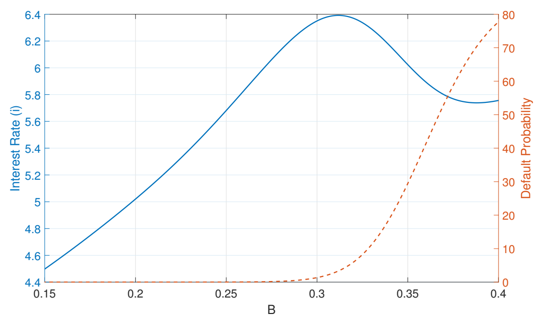
Impulse Response Functions. We now present impulse responses of the main variables of interest to monetary shocks m and productivity shocks z. To isolate the interactions between monetary frictions and default risk, we compare our findings with a reference model, labeled **NK-Reference**, which is a version of the Galí and Monacelli (2005) model with nominal rigidities and without default.



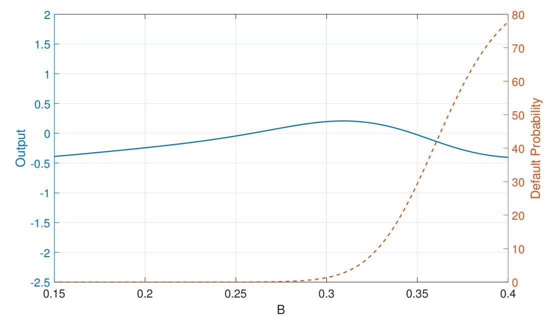
(a) Monetary Wedge



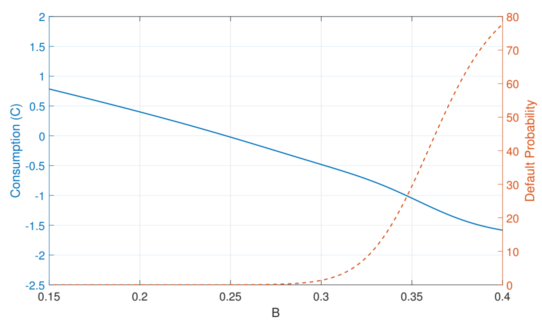
(b) Inflation



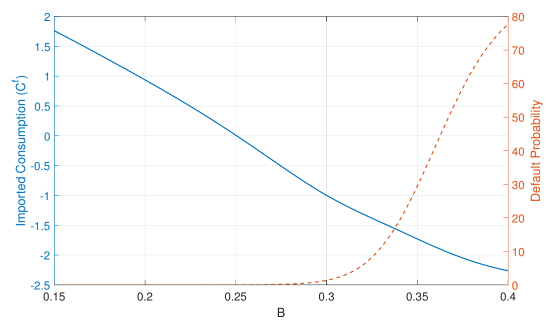
(c) Nominal Interest Rate



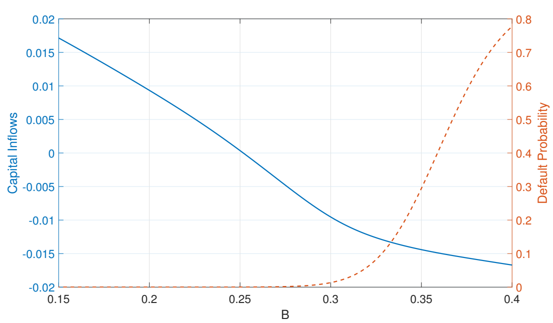
(d) Output



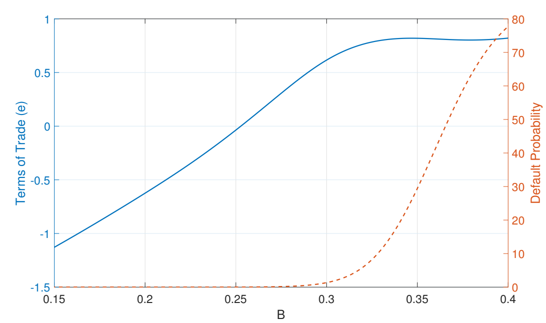
(e) Domestic Goods Consumption



(f) Imports



(g) Capital Inflows



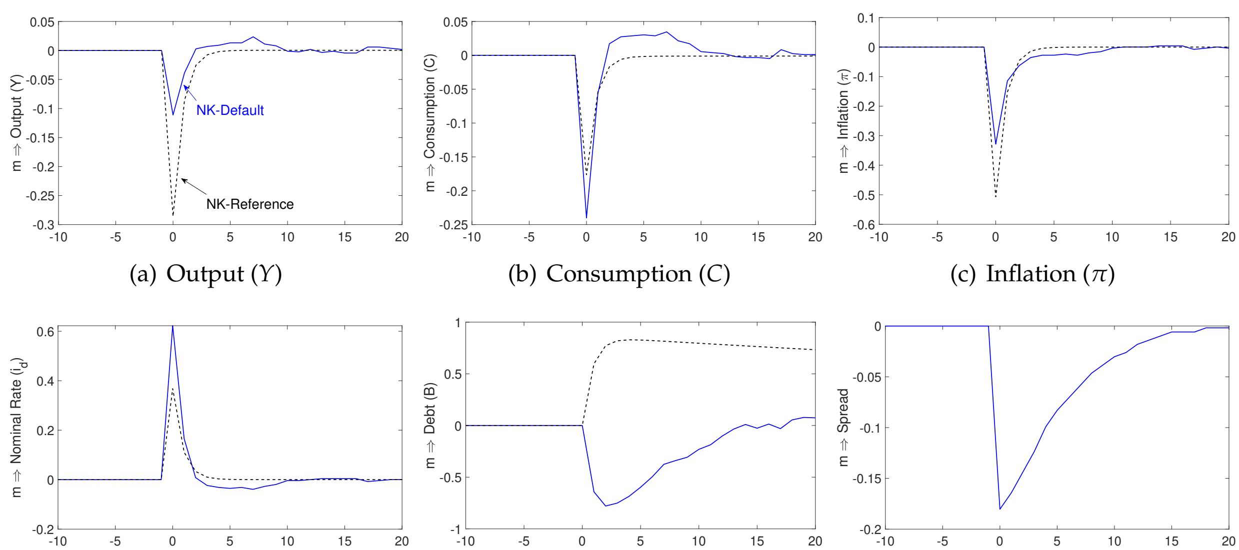
(h) Terms of Trade

Figure 3: Equilibrium Policy Functions

The equilibrium of the NK-Reference model is characterized by the same private and monetary equilibrium of our baseline model in Definition 1 and summarized in the appendix with equations (74–79), the international Euler condition (25), and an exogenous debt-elastic bond price schedule to close the model, as in Schmitt-Grohé and Uribe (2003). The debt-elastic bond price schedule is $q^*(B)^{-1} = \beta + \Gamma\left[\exp(B - \overline{B}) - 1\right]$, with Γ set to $1e^{-5}$, which gives a very loose borrowing schedule, and $\overline{B}$ set to give the same average debt level as our baseline. We solve the NK-Reference model with a first-order log-linear approximation of the equilibrium conditions, keeping all parameter values the same.

For our baseline NK-Default model, we construct the impulse response functions (IRFs) in our nonlinear model following Koop et al. (1996). We simulate a panel of 200,000 units for 1,500 periods. For the first 1,450 periods, the shocks follow their underlying Markov chain so that the cross-sectional distribution converges to the ergodic distribution of the model. In period 1,451, the impact period normalized to 0 in the plots, we shock all units by the same amount. From period 1,452 onward, shocks resume their Markov chain processes. The impulse responses plot the average across the time series. The impulse responses are computed over all units, including those with defaults. Discarding defaults from the cross-sectional average does not alter the properties of the IRFs.

We start with IRFs to the monetary shock to illustrate the discipline force in our model, namely that a high nominal interest rate lowers default risk. Figure 4 plots the impulse responses for the variables of interest. The solid blue lines are for our baseline NK-Default, and the dashed black lines are for the NK-Reference model. An increase in the monetary shock leads to the standard effects in the New Keynesian literature: temporary increases in the nominal rate and temporary decreases in inflation, output, and consumption.



(a) Output (Y) (b) Consumption (C) (c) Inflation (π)

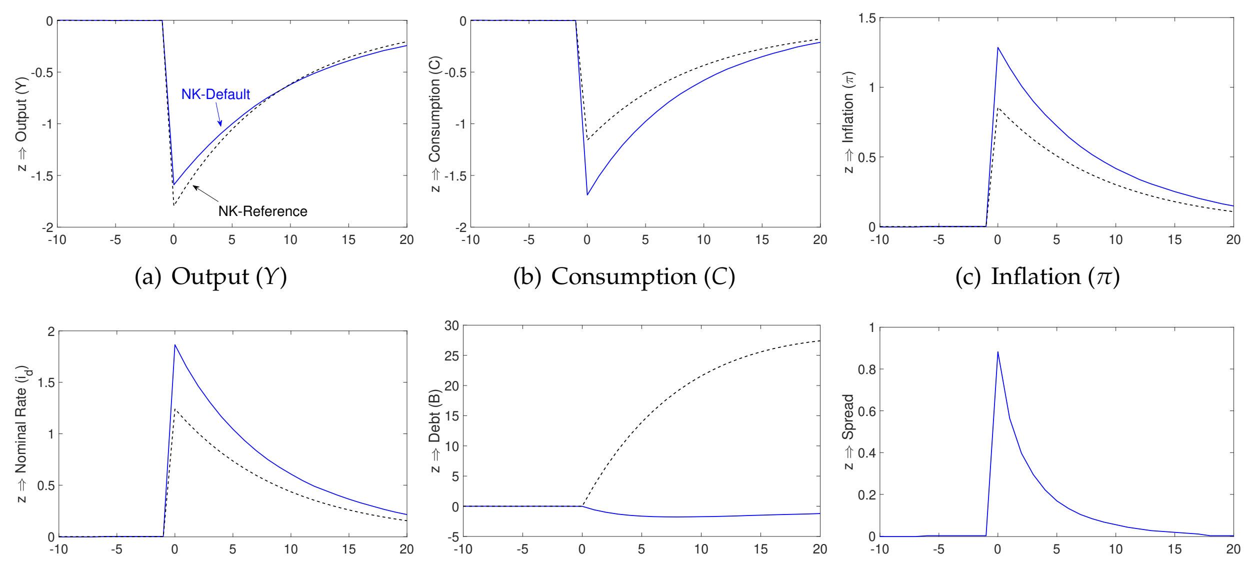
(d) Nominal Domestic Rate (i) (e) Debt (B) (f) Spread

Figure 4: Impulse Response Functions to Monetary Shocks

In an open economy, monetary shocks also impact the government's borrowing incentive. In the NK-Reference model, where there is no default risk and borrowing capacity is high, the decline in output makes the government borrow more to smooth consumption. In contrast, the NK-Default model has a monetary discipline mechanism that encourages a reduction in external borrowing. This reduction in borrowing has two effects: first, it reduces future default risk, which increases both current and future domestic consumption and labor, as shown in Section 4.2. Second, it causes capital outflows and real depreciation, resulting in a decrease in imported consumption and an increase in exports. The fall in imported consumption further lowers the demand for domestic goods because the two goods are complements in our calibration. Both forces, default risk, and capital outflows, boost domestic production. Consequently, the decline in production in the NK-Default model is only about one-third of that in the NK-Reference model. Inflation also falls by a smaller amount, and nominal rates increase more in the NK-Default model, while the rise in the labor wedge is smaller as well.

The responses in the NK-default model, illustrated in the panels 4(a) through 4(f) in Figure 4, imply that an increase in the monetary shock of about 1.1% decreases inflation by about 0.33%, output by 0.1%, and consumption by 0.23%. The tighter monetary policy reduces debt by 0.8% and spreads by about 0.18%. The impact responses imply a negative elasticity of spread with respect to the monetary shock of about 16.3% (0.18/1.1), which is within the confidence band estimate of the elasticity, in Figure 2.

We now present impulse responses to productivity shocks in Figure 5. We lower z by 1.7%, roughly 1.4 times the σ_z standard deviation. Declines in productivity lead to a decline in output of about 1.5%, comparable to the decline in the shock. Aggregate consumption declines a bit more, with consumption of domestic and foreign goods falling by 1.7% and 1.3%, respectively. As is typical in sovereign default models, low productivity tightens the bond price schedule because default is more likely in recessions, and with persistent shocks, low productivity makes future recessions more likely. The tight bond price schedule leads to higher spreads and a reduction in debt which tend to increase the volatility of consumption. Spreads rise about 0.9%, and debt contracts slowly by about 1.75% of its average value. Inflation rises about 1.3% on impact, because of the high unit cost from low productivity and the increased default risk. Nominal domestic rates rise in response to the elevated inflation, about 1.9%. These dynamics illustrate that productivity shocks lead to a strong, positive comovement of spreads with inflation and nominal rates.



(a) Output (Y) (b) Consumption (C) (c) Inflation (π)

(d) Nominal Domestic Rate (i) (e) Debt (B) (f) Spread

Figure 5: Impulse Response Functions to Productivity Shocks

The responses of the NK-Reference model are also shown in Figure 5. Output responds similarly to that of our baseline. In contrast, debt expands during the downturn. The smaller contraction of consumption reflects ample insurance possibilities via external borrowing. Inflation and nominal rates also rise in the NK-Reference model because of the higher unit cost for production, but their responses are more muted, about two-thirds of the corresponding magnitudes in the NK-Default baseline. Terms of trade appreciate more sharply in the NK-Reference model consistent with the more muted rise in inflation, and deeper decline in output due to the reduced exports.

These IRF comparisons highlight the amplification role of default risk for monetary policy. Default risk and tight borrowing conditions raise spreads and induce additional inflation and aggregate consumption volatility. These larger swings in inflation call for a more aggressive monetary policy with stronger responses of nominal domestic rates.

5.4 Business Cycle Moments, Inflation Event, and Reference Model

We now describe some additional business cycle implications of our model and compare them with the moments in the reference model without default risk. We also perform an event analysis to assess the contribution of default risk to the inflation events described in Section 5.1.

The first two columns of Table 4 report the first and second moments for the emerging market data and for simulated data from our baseline NK-Default model. The data consist of the series reported in Sections 5.1 and 5.2, for output, CPI inflation, nominal domestic rates, spreads, and aggregate consumption as well as the trade-weighted nominal exchange rate depreciation. All moments are averages across the emerging markets in our sample and reported in percentage points

Overall, the moments in the baseline model resemble emerging market data. The mean CPI inflation, nominal domestic rate, and spread, as well as the volatility of inflation, output, and spreads, are targets in our moment-matching exercise. The model delivers a volatility of the nominal rate comparable to the data, whereas it underestimates the high volatility in nominal exchange rates, reflecting the common disconnect between exchange rates and fundamentals in much of international business cycle theory. The model delivers the key stylized facts: positive correlations of inflation, nominal domestic rates, and nominal depreciation with spreads, with magnitudes in the 30–50% range. These correlations arise in our model because, across all state variables, namely, productivity z, monetary shock m, and debt B, inflation and spreads comove positively. These positive correlations are the implications of the novel amplification and disciplining mechanisms of our model. Finally, the model also delivers autcorrelations for output, inflation, spreads and nominal domestic rates, consistent with those in the data.

The third column of Table 4 reports the moments from the NK-Reference model, which by construction is silent on default risk. Average CPI inflation and the nominal domestic rate are the same as in the benchmark. The volatilities of the nominal interest rate and CPI inflation are, however, only about 72% of those in the NK-Default baseline. Default risk makes inflation more volatile because it affects expected future inflation and the monetary wedge, as discussed in Section 4.2. This comparison shows that an emerging market central bank targeting inflation must implement a more volatile interest rate policy as a result of sovereign default risk.

Inflation Event. We now perform an event analysis with our model to study the temporary inflation events in emerging markets. The main goals of this exercise are to evaluate the quantitative performance of the model in matching the dynamics observed and to assess the role of default risk in these events.

To simulate the event, we start the model in the mean stationary distribution with good credit standing and feed in a sequence of productivity and money shocks. The shocks are chosen such that the resulting paths of inflation and output in our NK-Default model best fit the data in Figure **??**. In this process, we interpolate the policy functions across the three states, the productivity and monetary shocks $\tilde{z}, m$ and the debt B, given initial conditions during 17 quarters that span the event. Note that in our non-linear model with debt as a state variable, this procedure is similar to the particle filtering problem over the entire path.

	Data	NK-Default	NK-Reference	NK-Default Alternative Rules	
				Strict Inflation	Default-Inflation
Mean					
CPI inflation	3.9	4.0	4.2	4.3	4.0
Nominal domestic rate	5.6	5.6	5.2	5.8	5.6
Spread	2.0	2.0	–	2.2	0.3
Standard Deviation					
Output	2.3	2.2	2.5	2.3	2.4
CPI inflation	1.7	1.9	1.4	0.3	0.4
Spread	0.9	1.0	–	0.7	0.04
Consumption aggregate	2.4	2.1	1.1	2.4	2.5
Nominal domestic rate	1.9	2.9	2.1	1.6	1.9
Depreciation rate	8.6	2.0	1.9	0.8	0.8
Correlations					
(Spread, Output)	−49	−45	–	−57	−48
(Spread, CPI inflation)	49	53	–	−52	−1
(Spread, Nominal domestic rate)	30	74	–	40	96
(Spread, Depreciation rate)	36	40	–	−52	−28
(CPI inflation, Output)	−19	1	22	83	69
(CPI inflation, Nominal domestic rate)	64	85	76	−26	−6
Autocorrelations (%)					
Output	84	69	70	70	71
CPI inflation	88	97	96	63	73
Spread	82	66	–	53	84
Nominal domestic rate	93	87	90	60	68

Table 4: Moments: NK-Default Alternative Monetary Policy Rules and NK-Reference

In practice, we also consider states with good credit standing, but find that the economy does not default on the resulting path.

The solid blue lines in Figure 6 represent the resulting NK-Default paths for inflation, output, nominal rates, and spreads. The dashed magenta lines correspond to the data. All series are standardized. The baseline model can replicate well the paths for output and inflation, as seen in the top two panels of the figure. Note that in our non-linear model with default risk, it is not a guarantee that there exist shocks that can deliver arbitrary paths for inflation and output, so we view this analysis as providing further validation of our model. Inflation increases temporarily, close to 2.5 standard deviations, in both model and data, and returns to low levels about a year after peaking. Output falls about 2 standard deviations and reaches its trough about 2 quarters after the peak in inflation. Moreover, through the lens of our model, the inflation event results from a combination of low productivity shocks and expansionary monetary shocks.

The lower panels of the figure compare the resulting paths of nominal rates and spreads to the data. In the model, nominal rates and spreads increase during the inflation event. At their peak, nominal rates are about 2 standard deviations above their mean, close to the data counterpart peak of 1.5 standard deviations. In the model, however, nominal rates are below their mean at the start of the event and increase fast, while in the data they are initially elevated and experience a more moderate increase. The model spread resembles closely that in the data, reaching close to 2 standard deviations above the mean at its peak. Overall, we find that the model successfully matches the dynamics of temporary inflation events in emerging markets.

We now move on to assessing the contribution of default risk to the dynamics of this event. To that end, we compare our NK-Default model to paths produced by the NK-Reference model, without default risk. For this exercise, we feed into the NK-Reference model the same paths for productivity and monetary shocks as in the baseline and start the episode in the steady state. The resulting series is the dot-dashed black lines in Figure 6. Without default risk, the increase in inflation during the event is more muted than in our baseline model. At its peak, the inflation spike is about half of the increase in the data and in our baseline model. In contrast, the decrease in output in the NK-Reference model is comparable to that observed in the data and in the baseline model with default. The lower panels show that the increase in nominal rates is smaller, reflecting the more muted inflation increase. Finally, note that the NK-Reference model does not feature any spread dynamics.

This event analysis shows that default risk can be a source of inflation volatility, as it magnifies the consequences of shocks for inflation. The contribution of default risk for inflation is substantial for these events, about 50%. Low productivity and expansionary monetary shocks tend to raise default risk, which increases expected inflation. In turn, this higher expected inflation feeds into current inflation, via the firms' current pricing decisions, and calls for more aggressive monetary policy in the form of stronger responses to nominal domestic rates.

5.5 Alternative Monetary Rules and Welfare

We have shown that default risk alters monetary transmission through its amplification and disciplining mechanisms. In this section, we study the welfare implication of various monetary policy regimes. This exercise is motivated by the standard prescription from Galí and Monacelli (2005) that strict inflation targeting is the optimal monetary policy in economies with pricing frictions and subject to productivity shocks. In this section, we find that this standard prescription does not hold in our model with default risk.

We compare our baseline NK-Default model with inflation targeting monetary rule to two different

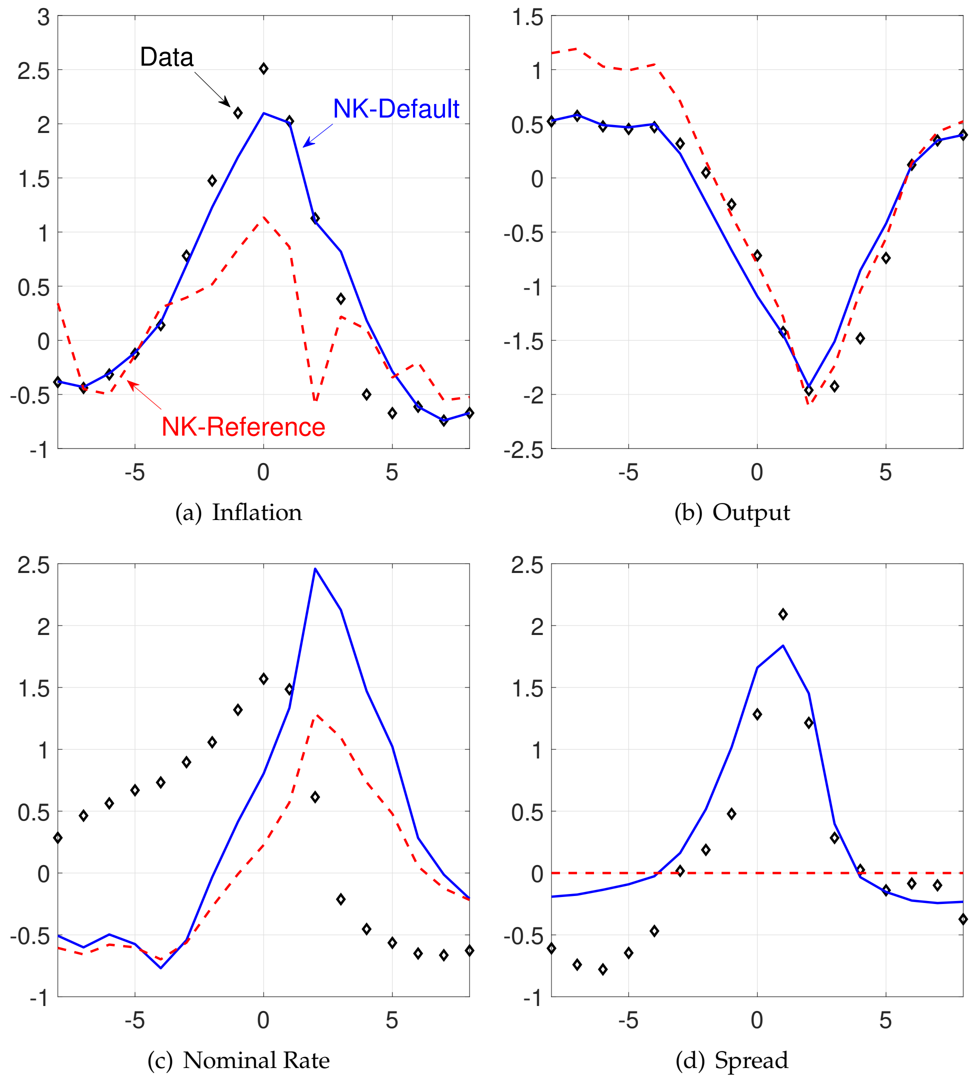


Figure 6: Inflation Event Analysis

monetary policy regimes. The first alternative we consider is that of *strict inflation targeting*. Under this regime, the monetary authority sets nominal rates such that inflation π_t is always at the target level $\overline{\pi}$. Recall that this is the optimal monetary regime in a model similar to ours but without default risk. The second alternative regime we consider is one under which the monetary policy rule also responds to default risk Φ_t. The interest rate rule under this *default-inflation targeting* regime is

$$i_t = \bar{i} \left(\frac{\pi_t}{\overline{\pi}} \right)^{\alpha_P} \left(\frac{\Phi_t}{\overline{\Phi}} \right)^{\alpha_D} m_t. \tag{37}$$

For this regime, we keep α_P at its value from the baseline model and study the consequences of varying α_D for welfare. We set a value for $\overline{\Phi}$ corresponding to a 0.25% annual default probability and value for α_D of 8. We also perform comparative statics for other values of these parameters on allocations and welfare. In this model, the monetary authority responds to default risk directly, which greatly enhances the monetary discipline mechanism. This induces less sovereign borrowing and lower spreads.

As in the theoretical analysis of Section 4.2, under strict inflation targeting, outcomes are those that would prevail under flexible prices, inflation is always at target, production is undistorted, and the monetary discipline mechanism is disabled. As a result, spreads are higher due to more aggressive sovereign borrowing.

Business Cycle Moments. Before discussing welfare, we briefly discuss the business cycle moments under alternative monetary policy regime, collected in Table 4. The second column corresponds to our NK-Default baseline model while the last two columns report the values for strict inflation targeting and the default-inflation targeting regime, respectively. Under strict inflation targeting, the mean spread is roughly 20 basis points higher than in NK-Default while CPI inflation is substantially smoother. This model exhibits two counterfactual negative correlations, between spreads and CPI inflation and between spreads and the domestic nominal rate. As PPI inflation is constant at target, all movements in CPI inflation are due to changes in the terms of trade. As a consequence, the correlation between spreads and CPI inflation reflects terms of trade movements.

Turning to default-inflation targeting, we find that spreads are significantly reduced by the presence of this additional term in the interest rate rule. One notable consequence of this additional monetary discipline is the much lower volatility of CPI inflation, about one-fifth, even though the α_P parameter is unchanged. This quantitative finding echoes Proposition 3 from our theoretical analysis of the monetary discipline, where we found that optimal monetary policy can approach the constrained efficient outcome, with an arbitrarily small monetary wedge. Such an optimal monetary policy reduces both default risk and the monetary wedge, associated with inefficient movements in inflation.

Welfare. To assess the welfare consequences of these alternative monetary policy regimes, we consider the welfare of the domestic household at a key state in the model's state space: an initial debt level of zero ($B = 0$) and all shocks are at their median levels ($z = \bar{z}, m = 0$), while the country enjoys good credit standing. We confirm that our results are robust to evaluating welfare at the median debt level of the ergodic distribution of the NK-Default model.

Figure 7 compiles our findings on welfare. In both panels, we plot welfare relative to the strict inflation-targeting regime, as the consumption equivalent percentage. In the left panel, we vary α_P, the coefficient controlling the response of domestic nominal rates to increases of inflation above target, while keeping $\alpha_D = 0$. This left panel corresponds to outcomes in our NK-Default model, with monetary policy consisting of a simple interest rate rule. The red asterisk marks the parameter values given by our baseline

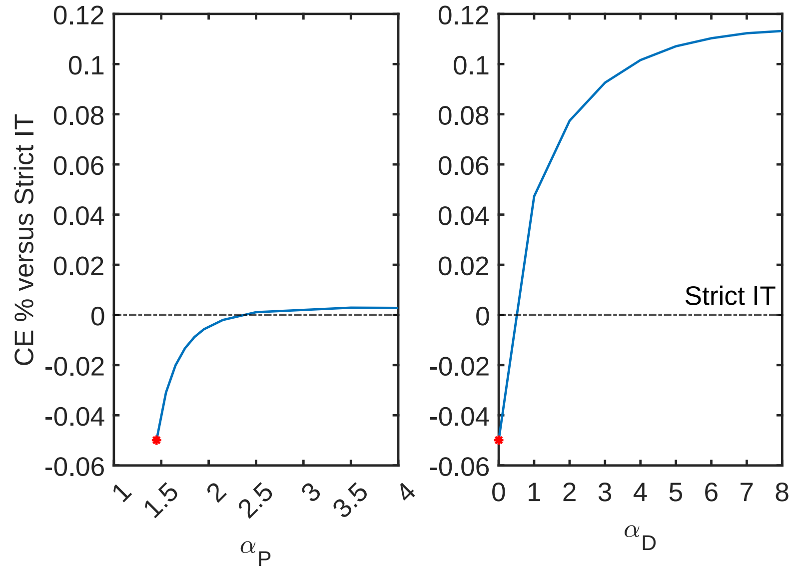


Figure 7: Welfare Comparison Across Monetary Policy Regimes

parameterization. With an α_P of 1.45, the household would prefer to reform monetary policy and adopt strict inflation targeting, even at the cost of additional default risk. Welfare increases, however, with higher α_p and surpasses that of the strict inflation targeting regime at about $\alpha_P > 2.25$. As the monetary policy rule calls for more aggressive responses to inflation, the disciplining mechanism lowers spreads enough that it is more worth it than the reduction in pricing frictions. Eventually, the gains from sharper responses to inflation flatten out as α_P approaches 4.

The right panel of Figure 7 summarizes our findings for the default-inflation targeting regime. Here, α_P is kept fixed at 1.45 and the horizontal axis varies α_D, starting at its baseline value of 0, at which the NK-Default and default-inflation targeting coincide. The potential for welfare gains from adopting this regime over strict inflation targeting is sizable. If the interest rate rule is responsive enough to default risk, for roughly $\alpha_D \geq 1$, households are better off under the default-inflation regime than under strict inflation targeting. As α_D approaches 8, welfare gains flatten out. In the limit, households would be willing to forego 10 basis points of their consumption on average, in order to adopt the default-inflation regime over strict inflation targeting, a sizable magnitude compared to typical welfare cost of business cycles calculations.

6 Conclusion

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Online Appendix to "Monetary Policy and Sovereign Risk in Emerging Economies (NK-Default)"

by Cristina Arellano, Yan Bai, and Gabriel Mihalache

A Deriving the Government Borrowing

In this appendix, we derive the government's optimal borrowing equation (24) in Section 3.2. To illustrate the government's borrowing incentives, we assume that all the policy functions are differentiable with respect to state B and that the first-order conditions are sufficient for the government's optimization problem. Conditional on not defaulting, the government chooses $\{C, C^f, N, \pi, B'\}$ to solve the following problem:

$$W(s,B) = \max\ u(C,C^f,N) + \beta_g \mathbb{E}\left\{\int_{\nu^*(s',B')} W(s',B')d\Phi(\nu') + \int^{\nu^*(s',B')} [W^d(s') - \nu']d\Phi(\nu')\right\} \tag{38}$$

subject to the constraints arising from the Private and Monetary Equilibrium

$$[\lambda]\ \ C + [C^f + (\delta + r^*)B - q(s,B')\left(B' - (1-\delta)B\right)]^{\frac{\rho}{\rho-1}} = \left[1 - \frac{\varphi}{2}\left(\pi - \overline{\pi}\right)^2\right] zN, \tag{39}$$

$$[\lambda_e]\ \ \frac{u_{C^f}}{u_C} = \frac{\rho}{\rho-1}[C^f + (\delta + r^*)B - q(s,B')\left(B' - (1-\delta)B\right)]^{\frac{1}{\rho-1}}, \tag{40}$$

$$[\kappa]\ \ \bar{i}\left(\frac{\pi}{\overline{\pi}}\right)^{\alpha_P} m\ \beta M(s,B',0) = u_C, \tag{41}$$

$$[\gamma]\ \ \frac{\eta-1}{\varphi}\left(-\frac{u_N}{zu_C} - 1\right) + \beta\frac{1}{u_C zN}F(s,B',0) = \left(\pi - \overline{\pi}\right)\pi, \tag{42}$$

where the default cutoff $\nu^*(s',B')$ equals $W^d(s') - W(s',B')$ and the functions $F(s,B',0)$ and $M(s,B',0)$ are defined in (80) and (81) with $\Theta = 0$. Here we have substituted the terms of trade e and the nominal domestic rate i using the balance of payments condition (75) and the interest rate rule (79). Let λ, λ_e, κ, and γ be the Lagrange multipliers on the resource constraint (39), the relative demand condition (40), the domestic Euler condition (41), and the NKPC condition (42), respectively. Note that a positive κ is associated with a smaller left-hand side of (41) than its right-hand side. Similarly, a positive γ is associated with a smaller left-hand side of (42) than its right-hand side.

The first-order conditions over C, C^f, N, π, and B' are

$$u_C - \lambda - \lambda_e\frac{u_{CC^f}u_C - u_{CC}u_{C^f}}{(u_C)^2} + \kappa u_{CC} + \gamma\frac{u_{CC}}{u_C}\left[-\frac{\eta-1}{\varphi}\frac{u_N}{zu_C} + \beta\frac{F(s,B',0)}{zNu_C}\right] = 0, \tag{43}$$

$$u_{C^f} - \lambda\frac{\rho}{\rho-1}e + \lambda_e\left(\frac{\rho e^{2-\rho}}{(\rho-1)^2} - \frac{u_{C^fC^f}u_C - u_{CC^f}u_{C^f}}{(u_C)^2}\right)$$
$$+ \kappa u_{CC^f} + \gamma\frac{u_{CC^f}}{u_C}\left[-\frac{\eta-1}{\varphi}\frac{u_N}{zu_C} + \beta\frac{F(s,B',0)}{zNu_C}\right] = 0, \tag{44}$$

$$u_N + \lambda\left[1 - \frac{\varphi}{2}\left(\pi - \overline{\pi}\right)^2\right]z - \gamma\left(\frac{\eta-1}{\varphi}\frac{u_{NN}}{zu_C} + \beta\frac{F(s,B',0)}{N^2zu_C}\right) = 0, \tag{45}$$

$$-\lambda\varphi(\pi - \overline{\pi})zN - \kappa\alpha_P\frac{u_C}{\pi} + \gamma(2\pi - \overline{\pi}) = 0, \tag{46}$$

$$\left[q + \frac{dq}{dB'}\left(B' - (1-\delta)B\right)\right]\left\{\lambda\frac{\rho}{\rho-1}e - \lambda_e\frac{1}{\rho-1}\frac{\rho}{\rho-1}e^{2-\rho}\right\} - \beta i\frac{\partial M}{\partial B'}\kappa - \beta\frac{1}{u_C zN}\frac{\partial F}{\partial B'}\gamma \tag{47}$$
$$= \beta_g\mathbb{E}(1-D')(r^* + \delta + (1-\delta)q(s', B''))\left\{\lambda'\frac{\rho}{\rho-1}e' - \lambda'_e\frac{1}{\rho-1}\frac{\rho}{\rho-1}(e')^{2-\rho}\right\}.$$

We can replace the multipliers λ and λ_e with (C, C^f, N, π) using the first-order conditions of (43) and (44). We can replace the term $\lambda\frac{\rho}{\rho-1}e - \lambda_e\frac{1}{\rho-1}\frac{\rho}{\rho-1}e^{2-\rho}$ on the Euler equation (47) with the borrowing wedge τ_m^X:

$$\tau_m^X = G^X\left[u_C\kappa + ((\eta-1)/\varphi + (\pi-\overline{\pi})\pi)\,\gamma\right],$$

where $G^X = \frac{\frac{u_{C^fC^f}u_C - u_{CC^f}u_{C^f}}{u_C u_{C^f}}\left(\frac{u_{CC}}{u_C^2} - \frac{u_{CC^f}}{u_C u_{C^f}}\right)}{e^{1-\rho}/(\rho-1) - \frac{u_{C^fC^f}u_C - u_{CC^f}u_{C^f}}{u_C u_{C^f}} + \frac{u_{CC^f}u_C - u_{CC}u_{C^f}}{u_C^2}} - \frac{u_{CC^f}}{u_C u_{C^f}}$. Also we define the borrowing wedge τ_m^C as

$$\tau_m^C = \beta i\frac{\partial M}{\partial B'}\kappa + \beta\frac{1}{u_C zN}\frac{\partial F}{\partial B'}\gamma.$$

Plugging the two borrowing wedges τ_m^X and τ_m^C into the equation (47), we obtain the Euler equation (24) in Section 3.2. □

B Proofs

B.1 Proof of Proposition 1

The proof consists of two parts. In the first part, we prove that higher borrowing B' increases future default risk. In the second part we show that, under Assumption 1, current inflation, the nominal domestic rate, and the monetary wedge increase with B'.

We first present the Private and Monetary Equilibrium under the quasi-linear preferences in (28). In state $S = (s, B, \Theta, B')$, the equilibrium satisfies the following conditions

$$C + (C(\rho-1)(1-\theta)/\rho\theta))^{\rho} = z(\tilde{z}, \Theta)N\left[1 - \frac{\varphi}{2}(\pi - \overline{\pi})^2\right], \tag{48}$$
$$(\pi - \overline{\pi})\pi = \frac{\eta-1}{\varphi}\left(\frac{CN^{1/\zeta}}{\theta z(\tilde{z}, \Theta)} - 1\right) + \beta\frac{C}{\theta z(\tilde{z}, \Theta)N}F(s, B', \Theta), \tag{49}$$
$$\frac{\theta}{C} = i\beta M(s, B', \Theta), \tag{50}$$
$$i = \bar{i}\left(\frac{\pi}{\overline{\pi}}\right)^{\alpha_P} m, \tag{51}$$
$$C^f = (C(\rho-1)(1-\theta)/\rho\theta))^{\rho-1} + (1-\Theta)\left[q(s, B')(B' - (1-\delta)B) - (r^* + \delta)B\right], \tag{52}$$

where we used the definition of exports, $X = e^{\rho}\xi$ with $\xi = 1$, and the relations $u_{C^f}/u_C = \rho e/(\rho-1)$, $u_{C^f} = 1-\theta$, $u_C = \theta/C$, and $-u_N = N^{1/\zeta}$. The F and M function are given by (80) and (81), respectively. In the following, we consider how B' affects default risk and the Private and Monetary Equilibrium.

Higher B' increases default risk $E_{s'}\left[\Phi(v^*(s', B'))\right]$ We first show that the government's value under repayment $W(s, B)$ is decreasing in B. Take two values B_0 and B_1 with $0 < B_0 < B_1$. We want to show $W(s, B_0) > W(s, B_1)$ for any given s. First, for a given B', the current debt B is absorbed by C^f and does not affect $\{C, N, \pi, i\}$. We can therefore write $\{C, N, \pi, i\}$ as functions of (s, B') and C^f as a function of

(s, B, B'), conditional on repayment. Hence, every B' that is feasible under B_1 is also feasible under B_0 since (C, N, π, i) remains the same under the same B', while C^f can take any real value. Moreover,

$$(1-\delta)q(s, B')B_0 + (r^* + \delta)B_0 < (1-\delta)q(s, B')B_1 + (r^* + \delta)B_1$$

since $q(s, B') \geq 0$ and $r^* + \delta > 0$. This implies $C^f(s, B_0, B') > C^f(s, B_1, B')$ for any given (s, B'). Let B_1' and B_0' be the optimal borrowing levels associated with B_1 and B_0, respectively. The following inequalities hold

$$\begin{aligned} W(s, B_1) &= u\left(C(s, B_1'), C^f(s, B_1, B_1'), N(B_1')\right) + \beta_g \mathbb{E} V(s', B_1') \\ &< u\left(C(s, B_1'), C^f(s, B_0, B_1'), N(s, B_1')\right) + \beta_g \mathbb{E} V(s', B_1') \\ &\leq u\left(C(s, B_0'), C^f(s, B_0, B_0'), N(s, B_0')\right) + \beta_g \mathbb{E} V(s', B_0') \\ &= W(s, B_0). \end{aligned}$$

Note that the first inequality holds because $C^f(s, B_0, B') > C^f(s, B_1, B')$, and the second inequality holds because under B_0, B_1' is feasible yet B_0' is the optimal choice. Hence for $B_0 < B_1$, $W(s, B_0) > W(s, B_1)$ for any given s.

The default cutoff given by $\nu^*(s, B) = W^d(s) - W(s, B)$ increases with B since the repaying value $W(s, B)$ decreases with B for any given s and the defaulting value $W^d(s)$ is independent of B. This makes the default probability $\Phi(\nu^*(s, B))$ increase with B for any given s. As a result, the default risk given by $E_{s'}\left[\Phi(\nu^*(s', B'))\right]$ increases with B'.

Higher B' increases inflation, the nominal rate, and the monetary wedge We consider the case when the the economy is in good credit standing, with $\Theta = 0$. B' impacts $\{C, N, \pi, i\}$ exclusively through its effect on the F and M functions. We approximate the system of equations (48–52) and the monetary wedge given by

$$\text{weg} = \frac{\theta z(\tilde{z}, \Theta)}{C N^{1/\zeta}} - 1$$

with a first-order Taylor expansion around the equilibrium and the monetary wedge $(\overline{C}, \overline{N}, \overline{\pi}, \overline{i}, \overline{\text{weg}})$ associated with $\overline{B'}$. We focus on a level of borrowing $\overline{B'}$ where inflation is close to target and the monetary wedge is close to zero. We solve for deviations of the equilibrium variables. In the solution, holding shocks constant $d\tilde{z} = dm = 0$, the deviation of inflation $d\pi$, nominal domestic rates di, and the monetary wedge $d$weg are

$$d\pi = a_1 \left[-\frac{1}{a_0} dM + dF \right] \tag{53}$$

$$di = \alpha_P \frac{\overline{i}}{\overline{\pi}} d\pi \tag{54}$$

$$d\text{weg} = a_2 \left[\frac{\beta \overline{i}}{\theta} dM + \alpha_P dF \right] \tag{55}$$

where the positive constants a_1 and a_2 are a convolution of parameters, $a_1 = \frac{\alpha_C}{\overline{\pi}^2 + \alpha_p \frac{\eta-1}{\varphi}(1 + \frac{1}{\zeta}(\alpha_C + \rho(1-\alpha_C)))} > 0$, $a_2 = \frac{(1+\frac{1}{\zeta}(\alpha_C + \rho(1-\alpha_C)))\alpha_C}{1 + \alpha_p \frac{\eta-1}{\varphi}(1 + \frac{1}{\zeta}(\alpha_C + \rho(1-\alpha_C)))} > 0$, and $\alpha_C = \overline{C}/\overline{N} > 0$. The deviations of inflation, nominal rates, and the monetary wedge derived in (53–55) together with Assumption 1 prove the result. □

B.2 Characterization of Constrained Efficient Allocations

We start with characterizing the reference model which has no pricing frictions and constrained-efficient borrowings. After default, the country is permanently excluded from international financial market. Hence the defaulting value is given by

$$W^d = \max_{C,C^f,N} u(C, C^f, N)/(1-\beta)$$

subject to the resource constraint and the balanced trade condition,

$$C + e^{\rho} = z_d N, \quad e^{\rho} = eC^f.$$

The optimal allocation $\{C_d, N_d, e_d\}$ satisfy the resource constraint and the two first order conditions,

$$C + e^{\rho} = g(z)N, \quad C = \rho/(\rho-1)e, \quad N^{1/\zeta}C = g(z), \tag{56}$$

with $g(z) = z_d$. Note that we substituted the derivatives $u_C = 1/C$, $u_{C^f} = 1$, and $u_N = N^{1/\zeta}$ for the utility function $u(C, C^f, N) = \log C + C^f - \frac{N^{1+1/\zeta}}{1+1/\zeta}$. Once we know e_d, we can back up the imported consumption $C_d^f = e_d^{\rho-1}$ using the balanced trade condition.

Conditional on not defaulting, the government chooses (C, C^f, N, B') to solve the following problem

$$W(B) = \max\ u(C, C^f, N) + \beta\left\{\left[1-\Phi(\nu^*(B'))\right]W(B') + \int^{\nu^*(B')}(W^d-\nu)d\Phi(\nu)\right\}, \tag{57}$$

subject to the resource constraint, balanced trade condition, and the bond price function

$$C + e^{\rho} = \bar{z}N, \quad e^{\rho} = e\left[C^f + B - q(B')B'\right], \quad q(B') = \frac{1}{1+r^*}\left[1-\Phi(\nu^*(B'))\right], \tag{58}$$

where the default cutoff $\nu^*(B')$ satisfies $\nu^*(B) = W^d - W(B)$. The optimal allocation $\{C^{CE}, N^{CE}, e^{CE}\}$ also satisfies the system of equations (56) but with $g(z) = \bar{z}$. Define a utility constant $u^{CE} = \log C^{CE} - \frac{(N^{CE})^{1+1/\zeta}}{1+1/\zeta} + (e^{CE})^{\rho-1}$, we can simplify the government's problem as

$$W(B) = \max_{B'}\ u^{CE} - B + q(B')B' + \beta\left\{\left[1-\Phi(\nu^*(B'))\right]W(B') + \int^{\nu^*(B')}(W^d-\nu)d\Phi(\nu)\right\}. \tag{59}$$

The optimal borrowing B^{CE} satisfies the following Euler equation

$$1 - h(v^*(B'))B' = \beta(1+r^*), \tag{60}$$

where h is the hazard function of enforcement shock ν with $h = \phi/(1-\Phi)$.

Note that the optimal default cutoff ν^{CE} and value W^{CE} are given by the respective functions evaluated at B^{CE} and satisfy $\nu^{CE} = W^d - W^{CE}$ and

$$W^{CE} = u^{CE} - B^{CE} + \frac{1}{1+r^*}[1-\Phi(\nu^{CE})]B^{CE} + \beta\left\{\left[1-\Phi(\nu^{CE})\right]W^{CE} + \int^{\nu^{CE}}(W^d-\nu)d\Phi(\nu)\right\}.$$

Furthermore, for arbitrary initial debt B these functions are

$$W^{CE}(B) = W^{CE} + B^{CE} - B, \quad \nu^{CE}(B) = W^d - W^{CE} - B^{CE} + B.$$

$W^{CE}(B)$ is linear and decreasing in B, and the default cutoff function $\nu^{CE}(B)$ increases with B.

B.3 Proof of Lemma 1

First, the the monetary authority can deliver $\bar{\pi}$ with $i = i^{ST}$ at period 0. In this strict inflation targeting (ST), the government solves a similar problem as (CE) but with discounting future with β_g at period 0: (C, C^f, N, B') to solve the following problem

$$\max \; u(C, C^f, N) + \beta_g \left\{ \left[1 - \Phi(\nu^*(B'))\right] W(B') + \int^{\nu^*(B')} (W^d - \nu) d\Phi(\nu) \right\}, \tag{61}$$

subject to the resource constraint, balanced trade condition, and the bond price function as in (58). The cutoff function $\nu^*(B)$ and the future value function W and W^d are the same as those in CE (59). Optimal borrowing B^{ST} satisfies the government's Euler equation

$$1 - h(v^*(B'))B' = \beta_g(1 + r^*). \tag{62}$$

To deliver $\bar{\pi}$, the monetary authority needs to pick interest rate i^{ST} to satisfy the domestic Euler equation under the optimal government's borrowing B^{ST},

$$\frac{1}{C^{CE}} = \beta i^{ST} / \bar{\pi} \left[\frac{1 - \Phi(\nu^*(B^{ST}))}{C^{CE}} + \frac{\Phi(\nu^*(B^{ST}))}{C_d} \right].$$

Note that domestic consumption is the same as CE since there are no pricing frictions in goods market and (C, N, e) solve (56) with $z = \bar{z}$ under no default.

We now prove that the default risk is higher under ST. Using the implicit function theorem, we can find the derivative of B' on β_g using the Euler (62),

$$\frac{\partial B'}{\partial \beta_g} = -\frac{1 + r^*}{(\partial h / \partial B')B' + h} < 0.$$

The derivative is negative due to the assumption $\partial h / \partial B' > 0$. The optimal borrowing for CE is under the discount factor β as shown in (60), while the optimal borrowing for ST is under β_g. Given that $\beta_g < \beta$ and $\partial B' / \partial \beta_g < 0$, we can show $B^{ST} > B^{CE}$ and $\Phi^{ST} = \Phi(\nu^*(B^{ST})) > \Phi^{CE} = \Phi(\nu^*(B^{CE}))$.

Furthermore, the welfare under strict inflation targeting is lower than that in CE. Here is the reason. Both CE and ST face the same future value, default cutoff function, and bond price schedule. B^{ST} is available under CE, however the optimal choice is B^{CE}. It must be the case that $W^{ST} \leq W^{CE}$.

B.4 Proof of Lemma 2

Here we prove that default risk increases monetary wedge for a given monetary policy i. We consider the response of the private economy to the government's borrowing B'. Higher B' pushes up the default risk, which affects consumption and production of domestic goods through the domestic Euler equation. Specifically, for any given B', domestic consumption C, labor N, monetary wedge μ, and inflation π satisfies the following four equations,

$$C + \left(\frac{\rho - 1}{\rho} C \right)^{\rho} = \left[1 - \frac{\varphi}{2} (\pi - \bar{\pi})^2 \right] \bar{z} N \tag{63}$$

$$\frac{1}{1 + \mu} = 1 + \frac{1}{\eta - 1} \varphi (\pi - \bar{\pi}) \pi \tag{64}$$

$$1 + \mu = \frac{\bar{z}}{N^{1/\xi} C} \tag{65}$$

$$\frac{1}{C} = \frac{\beta i}{\bar{\pi}} \left[\frac{1 - \Phi(\nu^*(B'))}{C^{CE}} + \frac{\Phi(\nu^*(B'))}{C_d} \right] \tag{66}$$

where we replaced exchange rate e using $C = \frac{\rho}{\rho-1}e$, and used the marginal utility $u_n = N^{1/\xi}$ and $u_C = 1/C$. From the domestic Euler equation (66), we can solve for C as a function of default risk $\Phi(\nu^*(B'))$:

$$C = \frac{\bar{\pi}}{\beta i} \frac{1}{\left[\frac{1-\Phi(\nu^*(B'))}{C^{CE}} + \frac{\Phi(\nu^*(B'))}{C_d}\right]}.$$

Given that $C^{CE} > C^d$, it is easy to see that higher default risk $\Phi(\nu^*(B'))$ lowers C. Hence, the domestic Euler equation determines domestic consumption C for any given B'.

We now prove that monetary wedge μ moves in the opposite direction with C. Hence, the monetary wedge increases with default risk. From NKPC, we can solve π as a function of monetary wedge μ[24]

$$\pi = \frac{\bar{\pi} + \sqrt{\bar{\pi}^2 - 4\frac{\eta-1}{\varphi}\frac{\mu}{1+\mu}}}{2} \equiv G(\mu). \tag{67}$$

We can then replace N in the resource constraint (63) with the definition of wedge (65),

$$C^{1+\xi} + \left(\frac{\rho-1}{\rho}\right)^{\rho} C^{\rho+\xi} = S(G(\mu))(1+\mu)^{-\xi}(\bar{z})^{1+\xi}, \tag{68}$$

where the function S is given by $S(\pi) = [1 - \varphi(\pi - \bar{\pi})^2/2]$. Given that C is pinned down by the domestic Euler equation, the condition (68) solves for the monetary wedge μ for any C.

We need to show that when C is low, the monetary wedge is high, i.e., $\partial\mu/\partial C \leq 0$. Using the implicit function theorem, we have

$$\frac{\partial\mu}{\partial C} = -(\bar{z})^{-(1+\xi)}(1+\mu)^{1+\xi}\frac{(1+\xi)C^{\xi} + (\rho+\xi)\left(\frac{\rho-1}{\rho}\right)^{\rho} C^{\rho+\xi-1}}{\xi S(\pi) + \varphi(1+\mu)(\pi-\bar{\pi})G'(\mu)}.$$

It is clear that $\partial\mu/\partial C \leq 0$ if and only if

$$\xi S(\pi) + \varphi(1+\mu)(\pi-\bar{\pi})G'(\mu) \geq 0.$$

Let the inflation cost be $\tau = \varphi/2(\pi - \bar{\pi})^2$. In equilibrium, it has to be the case that $0 \leq \tau < 1$. We can rewrite the condition for $\partial\mu/\partial C \leq 0$ as

$$1 - \tau \geq \frac{\eta-1}{\xi}(\pi-\bar{\pi})\left[\bar{\pi}^2 - 4\frac{\eta-1}{\varphi}\frac{\mu}{1+\mu}\right]^{-\frac{1}{2}}\frac{1}{1+\mu}. \tag{69}$$

It is easy to see that when $\mu \geq 0$ and so $\pi \leq \bar{\pi}$, the above condition always holds, and $\partial\mu/\partial C \leq 0$ ensues. Hence, higher default risk pushes down C and leads to a higher monetary wedge μ, when $\mu \geq 0$.

B.5 Proof of Proposition 2: (Discipline)

We first characterize the government's problem (31). Solving $e = (\rho-1)C/\rho$ from (intra $C - C^f$) and $C^f = e^{\rho-1} - B_0 + q(B')B'$ from (NPKC), and using the assumption of $u(C, C^f, N) = \log C + C^f - \frac{N^{1+1/\zeta}}{1+1/\zeta}$,

[24]Note that there are two solutions for inflation from NKPC,

$$\pi_1 = \frac{\bar{\pi} + \sqrt{\bar{\pi}^2 - 4\frac{\eta-1}{\varphi}\frac{\mu}{1+\mu}}}{2}, \quad \pi_2 = \frac{\bar{\pi} - \sqrt{\bar{\pi}^2 - 4\frac{\eta-1}{\varphi}\frac{\mu}{1+\mu}}}{2}.$$

However the inflation cost $\varphi(\pi-\bar{\pi})^2/2$ under π_2 is higher than π_1. Hence the optimal solution should be $\pi = \pi_1$.

we can simplify (31) as

$$\max_{B',C,N} \log C + \left(\frac{\rho-1}{\rho}C\right)^{\rho-1} - B_0 + q(B')B' - \frac{N^{1+1/\zeta}}{1+1/\zeta} + \beta_g \left\{ \left[1-\Phi(\nu^*(B'))\right] W(B') + \int^{\nu^*(B')} (W^d - \nu) d\Phi(\nu) \right\} \tag{70}$$

subject to the bond price function from (bond price) and the private equilibrium conditions

$$C + \left(\frac{\rho-1}{\rho}C\right)^{\rho} = \left[1 - \frac{\varphi}{2}\left(\pi - \bar{\pi}\right)^2\right] zN \qquad (\lambda, \text{resource constraint})$$

$$\frac{N^{1/\zeta}C}{z} = 1 + \frac{1}{\eta-1}\varphi\left(\pi - \bar{\pi}\right)\pi \qquad (\gamma, \text{NPKC})$$

$$\frac{\beta i}{\bar{\pi}} E u_{C'}(B') = \frac{1}{C}. \qquad (\kappa, \text{domestic Euler})$$

where $(\lambda, \gamma, \kappa)$ are multipliers for the private equilibrium conditions.

We can derive the following first-order conditions:

over C

$$\frac{1}{C}\left[1 + (\rho-1)e^{\rho-1}\right] - \lambda\left[1 + (\rho-1)e^{\rho-1}\right] - \gamma\frac{N^{1/\zeta}}{z} - \kappa\frac{1}{C^2} = 0$$

over N

$$-N^{1/\zeta} + \lambda\left[1 - \frac{\varphi}{2}\left(\pi - \bar{\pi}\right)^2\right] z - \gamma\frac{1}{\zeta}\frac{N^{1/\zeta-1}C}{z} = 0$$

Over π

$$-\lambda\varphi(\pi - \bar{\pi})zN + \gamma\frac{1}{\eta-1}\varphi(2\pi - \bar{\pi}) = 0$$

Over borrowings

$$\left[q + \frac{dq}{dB'}B'\right] - \kappa\left[\frac{\partial E u_C(B')}{\partial B'}\frac{u_C}{E u_C(B')}\right] = \beta_g(1 - \Phi(\nu^*(B'))). \tag{71}$$

We can solve γ, λ, and κ from FOC on π, N, and C:

$$\gamma = \lambda(\eta-1)\frac{\pi - \bar{\pi}}{2\pi - \bar{\pi}}zN$$

$$\lambda = \frac{N^{1/\zeta}}{z}\frac{1}{\left[1 - \frac{\varphi}{2}\left(\pi - \bar{\pi}\right)^2\right] - (\eta-1)\frac{\pi-\bar{\pi}}{2\pi-\bar{\pi}}\frac{1}{\zeta}\frac{N^{1/\zeta}C}{z}}$$

$$\kappa = \frac{C\left[1 + (\rho-1)e^{\rho-1}\right]}{1+\mu}\left\{1 + \mu - \frac{1 + \frac{1}{\left[1+(\rho-1)e^{\rho-1}\right]}(\eta-1)\frac{\pi-\bar{\pi}}{2\pi-\bar{\pi}}N^{1/\zeta+1}}{\left[1 - \frac{\varphi}{2}\left(\pi - \bar{\pi}\right)^2\right] - (\eta-1)\frac{\pi-\bar{\pi}}{2\pi-\bar{\pi}}\frac{1}{\zeta}\frac{1}{1+\mu}}\right\}. \tag{72}$$

where the monetary wedge $1 + \mu = (zu_C)/N^{1/\zeta}$.

We now prove that when $i > i^{ST}$, monetary wedge is positive $\mu > 0$. We prove by contradiction. Suppose when $i > i^{ST}$, $\mu \leq 0$. If so, $\pi \geq \bar{\pi}$ according to (γ, NPKC). Furthermore, it has to be the case that $\kappa \leq 0$ from (72) since $1 + \mu < 1$, $2\pi \geq \bar{\pi}$, the numerator of the second term larger than 1, and the denominator of the second term of (72) less than 1.

Now compare the optimal borrowing under ST with that under the one-time deviation (71). The optimal borrowing under ST satisfies

$$q(B^{ST}) + \frac{dq}{dB'}B^{ST} = \beta_g(1 - \Phi(\nu^*(B^{ST}))).$$

With $\kappa \leq 0$, it has to be the case that $B' \geq B^{ST}$ since the left-hand side of Euler captures the marginal benefit of borrowing. Given that expected marginal consumption increases with B' and high $i > i^{ST}$, this implies consumption has to be lower $C < C^{ST}$. We proved in the proof of Lemma 2, the monetary wedge decreases with C from equation (68). Hence lower consumption implies higher $\mu > 0$. Contradiction. Hence when $i > i^{ST}$, the monetary wedge is positive.

With positive monetary wedge, $\pi < \bar{\pi}$, $\kappa > 0$, which lowers the government's incentive to borrow. Hence $B' < B^{ST}$ and default risk is lower under the one-time deviation and $i > i^{ST}$.

B.6 Proof of Proposition 3

Proof. Five variables C, N, π, $\bar{i}$ and α_D solve jointly the following five equations of monetary wedge, NKPC, resource constraint, NKPC, domestic Euler, the government's Euler, i.e.,

$$\frac{1}{1+\mu} = -\frac{u_N}{zu_C} = 1 + \frac{1}{\eta - 1}\varphi\left(\pi - \bar{\pi}\right)\pi$$

$$C + \left(\frac{\rho - 1}{\rho}C\right)^{\rho} = \left[1 - \frac{\varphi}{2}\left(\pi - \bar{\pi}\right)^2\right] zN$$

$$u_C = \beta\bar{i}(\Phi(B')/\Phi^*)^{\alpha_D} Eu_C(B')$$

$$1 + \frac{dq}{dB'}\frac{B'}{q} - \kappa\left[\frac{\partial Eu_C(B')}{\partial B'}\frac{u_C}{Eu_C(B')} + \alpha_D u_C \frac{\phi(\nu^*(B'))}{\Phi(\nu^*(B'))}\right] = \beta_g(1 + r^*)$$

where κ is given by (72). For any given value of monetary wedge $\mu = \varepsilon$ and constrained efficient borrowing $B' = B^{CE}$, we have inflation given by

$$\pi^{OP} = \frac{\bar{\pi} + \sqrt{\bar{\pi}^2 - 4\frac{\eta-1}{\varphi}\frac{\varepsilon}{1+\varepsilon}}}{2}.$$

consumption C^{OP} satisfies

$$(C^{OP})^{1+\xi} + \left(\frac{\rho - 1}{\rho}\right)^{\rho}(C^{OP})^{\rho+\xi} = S(G(\varepsilon))(1+\varepsilon)^{-\xi}(\bar{z})^{1+\xi}, \tag{73}$$

labor N^{OP}

$$N^{OP} = (\bar{z})^{\xi}(1+\varepsilon)^{-\xi}(C^{OP})^{-\xi}$$

exchange rate e^{OP}

$$e^{OP} = \frac{\rho - 1}{\rho}C^{OP},$$

κ solves

$$\kappa^{OP} = \frac{C^{OP}\left[1 + (\rho - 1)(e^{OP})^{\rho-1}\right]}{1+\varepsilon}\left\{1 + \varepsilon - \frac{1 + \frac{1}{\left[1+(\rho-1)e^{\rho-1}\right]}(\eta - 1)\frac{\pi^{OP}-\bar{\pi}}{2\pi^{OP}-\bar{\pi}}(N^{OP})^{1/\zeta+1}}{\left[1 - \frac{\varphi}{2}\left(\pi^{OP} - \bar{\pi}\right)^2\right] - (\eta - 1)\frac{\pi-\bar{\pi}}{2\pi^{OP}-\bar{\pi}}\frac{1}{\zeta}\frac{1}{1+\varepsilon}}\right\}.$$

We then find $\bar{i}$ and α_D to satisfy

$$\frac{1}{C^{OP}} = \beta\bar{i}Eu_C(B^{CE})$$

$$\kappa^{OP}\left[\frac{\partial Eu_C(B^{CE})}{\partial B'}\frac{1}{C^{OP}Eu_C(B^{CE})} + \alpha_D\frac{\phi(\nu^*(B^{CE}))}{C^{OP}\Phi(\nu^*(B^{CE}))}\right] = (\beta_g - \beta)(1 + r^*).$$

Q.E.D.

B.7 Conditions for Private and Monetary Equilibrium

The private and monetary equilibrium can be summarized with the decision rules for domestic and foreign goods consumption $\{C(S), C^f(S)\}$, labor $N(S)$, inflation $\pi(S)$, the nominal domestic rate $i(S)$, and the terms of trade $e(S)$, which satisfy the following system of dynamic equations:

$$C(S) + e(S)^{\rho}\xi = \left[1 - \frac{\varphi}{2}\left(\pi(S) - \overline{\pi}\right)^2\right] z(\tilde{z}, \Theta)N(S) \tag{74}$$

$$e(S)^{\rho}\xi = e(S)[C^f(S) + (1-\Theta)((r^* + \delta)B - q(s, B')(B' - (1-\delta)B))] \tag{75}$$

$$\frac{u_{C^f}(S)}{u_C(S)} = \frac{\rho}{\rho - 1}e(S) \tag{76}$$

$$\left(\pi(S) - \overline{\pi}\right)\pi(S) = \frac{\eta - 1}{\varphi}\left(-\frac{u_N(S)}{z(\tilde{z}, \Theta)u_C(S)} - 1\right) + \frac{\beta}{u_C(S)z(\tilde{z}, \Theta)N(S)}F(s, B', \Theta) \tag{77}$$

$$u_C(S) = i(S)\beta\, M(s, B', \Theta) \tag{78}$$

$$i(S) = \bar{i}\left(\frac{\pi(S)}{\overline{\pi}}\right)^{\alpha_P} m, \tag{79}$$

where the functions F and M are the expectations in the firms' pricing condition (NKPC) and the households' Euler condition given by

$$F(s, B', \Theta) = \mathbb{E}\left[z(\tilde{z},' \Theta')N(S')u_C(S')\left(\pi(S') - \overline{\pi}\right)\pi(S')\right], \tag{80}$$

$$M(s, B', \Theta) = \mathbb{E}\frac{u_C(S')}{\pi(S')}. \tag{81}$$

The future state $S' = (s', B', \Theta'(s', \nu', B', \Theta), H_B(s', B'))$ depends on the future government policies and the evolution of credit standing given by (15). Note that when $\Theta = 1$, the expectations on the right-hand-side of (80) and (81) also include the probability of regaining a good credit ι.

The equilibrium conditions (74) to (79) are analogous to those arising in the standard New Keynesian small open economy of Galí and Monacelli (2005). The difference in our model is that the government understands that its choice of default D and borrowing B' affect the state S and the equilibrium. Moreover, the government's choices determine next period's state variables, which means that future allocations and prices also depend on the government's current choices. These future effects are encoded in the functions $q(s, B')$, $F(s, B', \Theta)$, and $M(s, B', \Theta)$.

B.8 Algorithm

REDO ME!!!

The model is subject to an AR(1) productivity shock z, which we discretize over a grid with $\#z = 21$ points spanning ± 3 standard deviations of its unconditional distribution. We also allow for a zero-probability shock to the interest rate rule m, which we discretize over $\#m = 7$ points spanning $\pm 1.5\%$. The m shock is i.i.d., with $\Pr(m = 1) = 1$ and $\Pr(m \neq 1) = 0$. We use these zero probability shocks to study the consequences of unexpected monetary tightening in the quantitative analysis. We confirm that the results are identical if instead we consider a Normally-distributed m shock with low variance. The B grid consists of $\#B = 250$ points equally spaced over $[0, 0.45]$.

The algorithm proceeds as follows:

1. We start with initial guesses for the value functions V_0, W_0^d and the bond price schedule q_0, together with guesses for the F_0 and M_0 functions and the default and borrowing policies. We assume the probability of default is 1 and $B' = B$ with probability one, everywhere in the state space.

2. We solve for for the private and monetary equilibrium (PME) everywhere in the state space, for arbitrary B'. We restrict attention to B' values that do not induce capital inflows or outflows that are "too large," for which a private and monetary equilibrium might not exist and confirm that this restriction does not bind in equilibrium:

$$| - (r^* + \delta)B + q(s, B') \left[B' - (1-\delta)B\right] | \leq 0.1 \qquad \text{(Capital Inflow Bounds)}$$

   We solve the private and monetary equilibrium via root-finding using Powell's hybrid method, on a system of two equations in two unknowns, C^f and N:

   (a) Using the current guess of $\langle C^f, N \rangle$ and the capital inflow, we compute the terms of trade e from the balance of payments condition.
   (b) We compute the implied level of exports X associated with the terms of trade e.
   (c) Given C^f and e, we can recover domestic consumption C from the relative consumption condition.
   (d) Given C and the government's borrowing choice B', we compute the domestic nominal rate i from the domestic Euler equation.
   (e) Given i, we use the interest rate rule to compute the level of PPI inflation π.
   (f) We use these quantities to compute equation residuals for the New Keynesian Phillips curve and the domestic resource constraint.

   The solution to the PME yields policy functions $C(s, B, B')$, $C^f(s, B, B')$, $N(s, B, B')$, $\pi(s, B, B')$, $i(s, B, B')$, $e(s, B, B')$.

3. We solve the PME in default similarly. In particular, in default trade is balanced and the capital inflow term is zero, and productivity is penalized. The solution constitutes policy functions in default: $C_d(s)$, $C_d^f(s)$, $N_d(s)$, $\pi_d(s)$, $i_d(s)$, $e_d(s)$.

4. Using PME results, we compute the value of the government in each state (V) and in default (W^d) and derive choice probabilities for the B' policy and default probabilities.

5. Given borrowing and default policies (probabilities), we update the bond price schedule q and the expectation functions M and F.

6. We check for the convergence of the bond price schedule, value functions, and expectation functions. We stop if values are closer than $1e^{-7}$ and prices are closer than $1e^{-5}$ in the sup norm, otherwise we fully update and iterate.

Simulation. Model statistics are computed over a simulation of 50,000 periods in length, excluding periods in default and the 20 periods (5 years) following the return to market. Without recovery, the sovereign returns to market without obligations and accumulates debt fast over the following few periods. If we include these transitional debt dynamics in the sample used to compute model moments, we find the results are largely unaltered, with the exception of the cyclical patterns of the trade balance. By including all periods outside of default, the trade balance becomes acyclical, while with our selection criterion the trade balance is countercyclical, as in the data.