Welfare Assessments with Heterogeneous Individuals[*]

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Abstract

This paper develops a new approach to make welfare assessments based on the notion of Dynamic Stochastic Generalized Social Marginal Welfare Weights (DS-weights). For a large class of dynamic stochastic economies with heterogeneous individuals, we show that the aggregate welfare assessment of a DS-planner can be exactly decomposed into four components: i) aggregate efficiency, ii) risk-sharing, iii) intertemporal-sharing, and iv) redistribution. By using DS-weights, we are able to i) formalize new welfare criteria that are exclusively based on one or several of the components that we identify, and ii) revisit how welfarist (e.g., utilitarian) planners make interpersonal welfare comparisons.

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1 Introduction

Assessing whether a policy change is desirable in dynamic stochastic economies with rich individual heterogeneity and imperfect insurance is far from trivial. One significant challenge is to understand the channels — such as aggregate efficiency, intertemporal-sharing, risk-sharing, or redistribution — through which a particular normative criterion finds a policy change desirable. A different challenge is how to formally define welfare criteria that exclusively value one or several of the aforementioned channels but not others.[1]

This paper tackles both challenges by developing a new approach to making welfare assessments in dynamic stochastic economies. This approach is based on the notion of Dynamic Stochastic Generalized Social Marginal Welfare Weights (Dynamic Stochastic weights or DS-weights, for short). The introduction of DS-weights accomplishes two main objectives. First, DS-weights allow us to decompose aggregate welfare assessments of policy changes into four distinct components: aggregate efficiency, intertemporal-sharing, risk-sharing, and redistribution, each capturing a different normative consideration. Second, DS-weights allow us to systematically formalize new welfare criteria that society may find appealing. In particular, we are able to define normative criteria that are exclusively based on one or several of the four normative considerations that we identify, potentially disregarding the others.

We introduce our results in a canonical dynamic stochastic environment with heterogeneous individuals. Initially, as a benchmark, we explicitly define in our environment i) Pareto-improving policies and ii) desirable policies for a welfarist planner. While Pareto improvements seem highly desirable, they are rare to find, which forces planners/policymakers to make interpersonal welfare comparisons using a Social Welfare Function — this is the welfarist approach. However, while the welfarist approach is popular and widely applicable, it is not easy to understand how a welfarist planner exactly makes tradeoffs among individuals that are ex-ante heterogeneous along some dimension, because of the ordinal nature of individual utilities. By reviewing these well-understood approaches and treating them as benchmarks, we set the stage for the introduction of DS-weights.

In our approach, there is no social welfare objective that a planner maximizes. Instead, the primitives to make welfare assessments are DS-weights, which represent the value that society places on a marginal dollar of consumption by a particular individual i at a particular time t and along a particular history s^t. Equipped with these weights, we define a policy to be desirable when the weighted sum — using DS-weights — across all individuals, dates, and histories of the instantaneous consumption-equivalent effects of a policy is positive. By defining DS-weights marginally, we can define normative criteria that the welfarist approach cannot capture.

In order to understand how a DS-planner, that is, a planner who adopts DS-weights, carries out welfare assessments, we introduce two different decompositions. First, we introduce an individual

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[1]Recently, the Federal Reserve seems to have explicitly included cross-sectional considerations in its policy-making process — see e.g., `https://www.nytimes.com/2021/04/19/business/economy/federal-reserve-politics.html`. The approach that we develop in this paper can plausibly be used to define a mandate for a monetary authority or other policymakers that explicitly incorporates or removes cross-sectional concerns from policy assessments.

multiplicative decomposition of DS-weights. We show that, in general, the DS-weights assigned to a given individual can be decomposed into i) an individual component, which is invariant across all dates and histories; ii) a dynamic component, which can vary across dates, but not across histories at a given date; and a stochastic component, which can vary across dates and histories. Moreover, we show that there exists a unique normalized individual multiplicative decomposition of DS-weights, which is easily interpretable and has desirable properties.

Leveraging the individual multiplicative decomposition of DS-weights, we also introduce an aggregate additive decomposition of welfare assessments. Formally, we show that, in dynamic stochastic environments, the welfare assessments made by a DS-planner can be exactly decomposed into four components: i) an aggregate efficiency component, ii) a risk-sharing component, iii) an intertemporal-sharing component, and iv) a redistribution component. The aggregate efficiency component accounts for the change in aggregate consumption-equivalents across all individuals. The remaining three components of the decomposition are driven by the cross-sectional variation of each of the three elements (individual, dynamic, stochastic) of the individual multiplicative decomposition. In particular, the risk-sharing component adds up across all dates and histories the cross-sectional covariances between the stochastic component of the individual multiplicative decomposition and the change in normalized instantaneous utility at each date and history. Similarly, the intertemporal-sharing component adds up across all dates the covariances between the dynamic component of the individual multiplicative decomposition and the change in normalized net utility at each date. Finally, the redistribution component consists of a single cross-sectional covariance between the individual components of the individual multiplicative decomposition and the change in individual lifetime marginal utility from the perspective of a DS-planner. In intuitive terms, we can say that the aggregate efficiency component Ξ_{AE} captures the aggregate impact of a policy, that the risk-sharing component Ξ_{RS} captures the transitory stochastic impact across individuals of a policy, that the intertemporal-sharing component Ξ_{IS} captures the transitory deterministic impact across individuals of a policy, and that the redistribution component Ξ_{RD} captures the permanent impact across individuals of a policy.

Next, we present properties of the aggregate additive decomposition and its components for a general DS-planner. Initially, we show that a DS-planner who assigns DS-weights that do not vary across individuals at all dates and histories makes welfare assessments purely based on aggregate efficiency considerations. We also show that different components of the aggregate additive decomposition may vanish depending on which specific components of the individual multiplicative decomposition of DS-weights are invariant across individuals: if the individual multiplicative component is constant across individuals, then the redistribution component of the aggregate decomposition is zero; if the dynamic multiplicative component is constant across individuals at all dates, then the intertemporal-sharing component of the aggregate decomposition is zero; if the stochastic multiplicative component is constant across individuals at all dates and histories, then the risk-sharing component of the aggregate decomposition is zero. We highlight four implications of these results with practical relevance. First, we show that welfare assessments

in single-agent/representative-agent economies are exclusively attributed to aggregate efficiency considerations. Second, we show that welfare assessments in perfect-foresight economies (under normalized DS-weights) are never attributed to risk-sharing. Third, we show that welfare assessments in economies in which all individuals are ex-ante identical (but not necessarily ex-post) are never attributed to intertemporal-sharing or redistribution. Fourth, we show that welfare assessments in static economics (under normalized DS-weights) are exclusively attributed to aggregate efficiency or redistribution considerations. Subsequently, we show that, under normalized DS-weights, the risk-sharing, intertemporal-sharing, and redistribution components are zero whenever a given policy impacts all individuals identically. We also identify conditions on how a policy affects individuals that imply that a subset of the components of the aggregate decompositions are zero. Finally, we show that the aggregate efficiency component of the aggregate decomposition is zero in endowment economies.

Given the importance of the welfarist approach in practice, we characterize — critically, in easily interpretable consumption units — how a welfarist DS-planner makes tradeoffs across periods and histories for a given individual, and across individuals. Formally, we characterize the unique normalized individual multiplicative decomposition of DS-weights for a given welfarist planner, and discuss its implications. Armed with this decomposition, we characterize new additional properties of the aggregate additive decomposition of welfare assessments for welfarist planners. In particular, we show that all normalized welfarist planners conclude that the risk-sharing and intertemporal-sharing components are zero when markets are complete, that the intertemporal-sharing component is zero when individuals can freely trade a riskless bond, and that different normalized welfarist planners — with different Social Welfare Functions — exclusively disagree on the redistribution component. To our knowledge, the aggregate additive decomposition of welfare assessments introduced in this paper is the first welfare decomposition for which these properties — which seem highly desirable — have been established.

Since a central objective of this paper is to provide a framework to systematically formalize new welfare criteria, we describe how to use DS-weights to formalize new welfare criteria that capture particular normative objectives that society may find appealing. First, we introduce three different sets of novel DS-planners: aggregate efficiency (AE) DS-planners, aggregate efficiency/risk-sharing (AR) DS-planners, and no-redistribution (NR) DS-planners, and characterize their properties.[2] The welfare assessments made by these new planners purposefully set to zero particular components of the aggregate additive decomposition. Within each set of DS-planners, we identify a pseudo-welfarist planner as the one that represents the minimal departure relative to the normalized welfarist planner. We also introduce an α-DS-planner, a new planner that spans i) AE, ii) AR, and iii) NR pseudo-welfarist planners, as well as iv) the associated normalized welfarist planner. Finally, we explain why some new planners (AE and AR) are paternalistic, while others are not (NR). We also discuss the

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[2]For instance, the current "dual mandate" (stable prices and maximum employment) of the Federal Reserve (as defined by the 1977 Federal Reserve Act) seems to be better described by an aggregate efficiency (AE) DS-planner, rather than a normalized utilitarian planner, who would care about risk-sharing, intertemporal-sharing, and redistribution.

implications of introducing new planners for policy mandates and institutional design.

Before presenting an application of our framework, we describe additional results. First, we further decompose the components of the aggregate additive decomposition and then explain how to connect welfare assessments to measures of inequality. Next, we explain how to make welfare assessments using DS-weights in recursive environments, and show how to implement welfare assessments via an instantaneous Social Welfare Function. Finally, we briefly described additional results included in the Online Appendix. Among other results, we present a detailed comparison of how our approach relates to the widely used approach introduced by Lucas (1987) and Alvarez and Jermann (2004).

At last, we illustrate how to make welfare assessments using DS-weights in a fully specified application. In our application, we illustrate the mechanics of our approach by conducting welfare assessments of policies in a single-good economy with no financial markets. We explore two specific scenarios. Scenario 1 corresponds to an economy in which individuals with identical preferences face idiosyncratic shocks. We consider transfer policies that can potentially provide full insurance and carefully explain how, depending on the persistence of idiosyncratic risk, a normalized utilitarian planner can find such policies desirable for different reasons. In particular, when risks are transitory, the planner attributes most of the welfare gains of the policy to risk-sharing. Alternatively, when risks are very persistent, the planner attributes most of the gains to redistribution. Scenario 2 corresponds to an economy in which individuals with different preferences towards risk face aggregate shocks. We consider transfer policies that shift aggregate risk to the more risk-tolerant investors and carefully explain how a normalized utilitarian planner finds such policies desirable for different reasons depending on the state of th economy in which welfare assessments take place.

Related literature. This paper contributes to several literatures, specifically those on i) interpersonal welfare comparisons, ii) welfare decompositions, iii) welfare evaluation of policy changes in dynamic environments, and iv) institutional mandates.

Interpersonal welfare comparisons. The question of how to make interpersonal welfare comparisons to form aggregate welfare assessments has a long history in economics — see, among many others, Kaldor (1939), Hicks (1939), Bergson (1938), Samuelson (1947), Harsanyi (1955), Sen (1970) or, more recently, Kaplow and Shavell (2001), Saez and Stantcheva (2016), Hendren (2020), Tsyvinski and Werquin (2020), and Hendren and Sprung-Keyser (2020). Formally, our approach based on endogenous welfare weights is most closely related to the work of Saez and Stantcheva (2016), who introduce Generalized Social Marginal Welfare Weights. Building on their terminology, in this paper we introduce the notion of Dynamic Stochastic Generalized Social Marginal Welfare Weights (Dynamic Stochastic weights or DS-weights, for short). In static environments, our approach collapses to theirs. In dynamic stochastic environments, the use of DS-weights allows us to formalize a new, larger set of welfare criteria and to understand the normative implications for aggregate efficiency, risk-sharing, intertemporal-sharing, and redistribution of different welfare criteria, including the widely used welfarist criteria. In particular, Section 4 leverages the use of DS-weights to provide

a novel interpretation of how welfarist planners trade off welfare gains and losses across individuals in dynamic stochastic environments, a result at the heart of the question of how to make interpersonal welfare comparisons.[3]

Welfare decompositions. Our results, in particular the aggregate additive decomposition introduced in Proposition 1, contribute to the body of work that seeks to decompose welfare changes in models with heterogeneous agents. The most recent contribution to this literature is the work by Bhandari et al. (2021), who propose a decomposition of welfare changes when switching from a given policy to another that is more general than the seminal contributions of Benabou (2002) and Floden (2001).[4] A fundamental difference between these papers and ours is that, in addition to decomposing the aggregate welfare effects of a policy change, our approach allows us to define a new set of normative criteria that can be used to endow a planner/policymaker with a specific mandate. Purely from the perspective of the decomposition of welfare assessments, there are many other significant differences between the approach of Bhandari et al. (2021) and ours. For instance, their decomposition is based on a particular social welfare function (utilitarian, with individual-specific weights), while ours critically hinges on the choice of welfare weights. Also, their welfare decomposition, which is defined for non-marginal changes, relies on a Taylor expansion and is hence based on an approximation. Our welfare decomposition, which is defined for marginal policy changes, is exact, and can used to assess non-marginal changes when used as described in Section G.5 of the Online Appendix. Moreover, it should be evident that no existing decomposition satisfies Proposition 6, in which we show that all normalized welfarist planners conclude that the risk-sharing and intertemporal-sharing components are zero when markets are complete; Proposition 7, in which we show that all normalized welfarist planners conclude that intertemporal-sharing component is zero when individuals can freely trade a riskless bond; and Proposition 8, in which we show that different normalized welfarist planners exclusively disagree on the redistribution component.

Welfare assessments in dynamic stochastic models. Our results are also related to the Lucas (1987) approach to making welfare assessments in dynamic environments, in particular to its marginal formulation introduced in Alvarez and Jermann (2004). Formally, as we show in Section F of the Online Appendix, the marginal approach to making welfare assessment of Alvarez and Jermann (2004) corresponds to choosing a particular set of DS-weights. While both Lucas (1987) and Alvarez and Jermann (2004) study representative-agent environments, others have used a similar approach in environments with heterogeneity; see e.g., Atkeson and Phelan (1994), Krusell and Smith (1999), or Krusell et al. (2009), among many others. However, as highlighted by these papers, a well-known downside of the Lucas (1987) approach is that it does not aggregate meaningfully because individual welfare assessments are reported as a constant share of individual consumption. In this

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[3]Saez and Stantcheva (2016) emphasize that by using generalized weights it is possible to accommodate alternatives to welfarism, such as equality of opportunity, libertarianism, or Rawlsianism, among others. It should be evident that our approach, which nests theirs, can also accommodate these possibilities. We purposefully avoid studying these issues, since these normative approaches are rarely used in the study of dynamic stochastic economies.

[4]At an intuitive level, the decomposition proposed by Benabou (2002) and Floden (2001) is based on utilities while the decomposition of Bhandari et al. (2021) is based on allocations. Our decomposition, on the other hand, is based on marginal utilities.

paper, we show that a normalized welfarist planner — a new concept that we introduce — is able to meaningfully aggregate welfare assessments among heterogeneous individuals.

Institutional mandates. Finally, our results contribute to the literature that studies policymakers' mandates. For instance, Woodford (2003) shows in representative agent economy that endowing a monetary authority with the objective of minimizing inflation and output gaps maximizes welfare in a flow sense. Relatedly, Rogoff (1985) shows that choosing a particular planner (a conservative central banker) may be desirable in some circumstances. However, the literature on institutional mandates has eschewed cross-sectional considerations. We hope that the approach that we develop in this paper opens the door to future disciplined discussions on policy-making mandates, in particular when trading off aggregate stabilization motives against interpersonal insurance and redistribution motives.

Outline. Section 2 introduces the baseline environment and describes conventional approaches used to make welfare assessments. Section 3 introduces the notion of DS-weights, defines an individual multiplicative decomposition of DS-weights, an aggregate additive decomposition of welfare assessments, and provides general properties of such decompositions. Section 4 studies how welfarist planners make welfare assessments through the lens of DS-weights, characterizing properties of the aggregate additive decomposition in that case. Section 5 formalizes new welfare criteria that isolate different components of the aggregate additive decomposition and discusses the implications of such planners for institutional design. Section 6 further decomposes the components of the aggregate additive decomposition, explains how to connect welfare assessments to measures of inequality, describes how to make welfare assessments in recursive environments, and shows how to implement welfare assessments via an instantaneous Social Welfare Function. Section 7 illustrates how to make welfare assessments in the context of a fully specified dynamic stochastic model. All proofs and derivations are in the Appendix. The Online Appendix also includes several extensions and additional results.

2 Environment and Benchmarks

In this section, we first describe our baseline environment, which encompasses a wide variety of dynamic stochastic models with heterogeneous individuals. Subsequently, we describe the conventional approaches to making welfare assessments, setting the stage for the introduction of DS-weights in Section 3.

2.1 Baseline Environment

Our notation closely follows that of Ljungqvist and Sargent (2018). We consider an economy populated by individuals, indexed by $i \in I$. For simplicity, we assume that there is a unit measure of individuals, so $\int di = 1$, although our results apply unchanged to economies with a finite number of individuals. At each date $t \in \{0, \dots, T\}$, where $T \leq \infty$, there is a realization of a stochastic event

$s_t \in S$. We denote the history of events up to and until date t by $s^t = (s_0, s_1, \ldots, s_t)$. We denote the unconditional probability of observing a particular sequence of events s^t by $\pi_t(s^t | s_0)$. We assume that the initial value of s_0 is predetermined, so $\pi_0(s^0 | s_0) = 1$.

There is a single nonstorable consumption good — which serves as numeraire — at all dates and histories. Each individual i derives utility from consumption and (dis)utility from working, with a lifetime utility representation, starting from s_0, given by

$$V_i(s_0) = \sum_{t=0}^{T} (\beta_i)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) u_i\left(c_t^i\left(s^t\right), n_t^i\left(s^t\right)\right), \tag{1}$$

where $c_t^i(s^t)$ and $n_t^i(s^t)$ respectively denote the consumption and hours worked by individual i at date t given a history s^t; $u_i(\cdot)$ corresponds to individual i's instantaneous utility, potentially non-separable between consumption and hours; and $\beta_i \in [0, 1)$ denotes individual i's discount factor.[5] Note that Equation (1) corresponds to the time-separable expected utility preferences with exponential discounting and homogeneous beliefs commonly used in dynamic macroeconomics and finance. Note also that we purposefully allow for individual-specific preferences.

We assume that preferences are well-behaved and, for now, directly impose that $c_t^i(s^t)$ and $n_t^i(s^t)$ are smooth functions of a primitive parameter $\theta \in [0, 1]$, so

$$\frac{dc_t^i\left(s^t\right)}{d\theta} \quad \text{and} \quad \frac{dn_t^i\left(s^t\right)}{d\theta}$$

are well-defined. We interpret changes in θ as policy changes although, at this level of generality, our approach is valid for any change in primitives. This formulation allows us to consider a wide range of policies, as we illustrate in our applications.[6] In those applications — and more generally — the mapping between outcomes, $c_t^i(s^t)$ and $n_t^i(s^t)$, and policy, θ, emerges endogenously, and typically accounts for general equilibrium effects. However, for most of the paper, we can proceed without further specifying endowments, budget constraints, equilibrium concepts, etc.

We conclude the description of the baseline environment by describing several extensions, which we present in the Online Appendix. In particular, in Section F.1, we describe how to account for heterogeneous beliefs. In Section F.2, we show how our approach extends to richer preference specifications, in particular, the widely used Epstein-Zin preferences. In Section F.3, we describe how to extend our approach to environments in which preferences and probabilities directly depend on θ or in which consumption and hours worked are not differentiable everywhere. In Section F.4 we show how our results extend to economies with multiple commodities, in addition to working hours. Finally, in Section F.5 we describe how to allow for births, deaths, and related intergenerational considerations.

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[5]Following Acemoglu (2009), we refer to $V_i(\cdot)$ as *lifetime* utility and to $u_i(\cdot)$ as *instantaneous* utility. As in Ljungqvist and Sargent (2018), we use a subscript i to refer to $V_i(\cdot)$, β_i, and $u_i(\cdot)$, and a superscript i to refer to individual variables indexed by time or histories.

[6]In particular, the fact that θ is one-dimensional is not restrictive, since θ can be interpreted as the scale of an arbitrary policy variation that can differ across individuals, dates, and histories.

2.2 Benchmarks: Conventional Approaches to Welfare Assessments

Before introducing DS-weights, we first define in our environment i) Pareto-improving policies and ii) desirable policies for a welfarist planner. To that end, it is useful to characterize the change in the lifetime utility of an individual i induced by a marginal policy change, $\frac{dV_i(s_0)}{d\theta}$.

Lifetime utility effect of policy change. Starting from Equation (1), $\frac{dV_i(s_0)}{d\theta}$, which is measured in utils (utility units), can be expressed as

$$\frac{dV_i(s_0)}{d\theta} = \sum_{t=0}^{T} (\beta_i)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) \frac{\partial u_i\left(s^t\right)}{\partial c_t^i} \frac{du_{i|c}\left(s^t\right)}{d\theta}, \tag{2}$$

where we respectively denote individual i's marginal utilities of consumption and hours worked at history s^t by

$$\frac{\partial u_i\left(s^t\right)}{\partial c_t^i} \equiv \frac{\partial u_i\left(c_t^i\left(s^t\right), n_t^i\left(s^t\right)\right)}{\partial c_t^i\left(s^t\right)} \quad \text{and} \quad \frac{\partial u_i\left(s^t\right)}{\partial n_t^i} \equiv \frac{\partial u_i\left(c_t^i\left(s^t\right), n_t^i\left(s^t\right)\right)}{\partial n_t^i\left(s^t\right)},$$

and where we denote the *instantaneous consumption-equivalent effect* of the policy at date t given a history s^t, by

$$\frac{du_{i|c}\left(s^t\right)}{d\theta} \equiv \frac{\frac{du_i\left(c_t^i\left(s^t\right), n_t^i\left(s^t\right)\right)}{d\theta}}{\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}} = \frac{dc_t^i\left(s^t\right)}{d\theta} + \frac{\frac{\partial u_i\left(s^t\right)}{\partial n_t^i}}{\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}} \frac{dn_t^i\left(s^t\right)}{d\theta}. \tag{3}$$

Equation (2) shows that the impact of a policy change on the lifetime utility of individual i is given by a particular combination of instantaneous consumption-equivalent effects, which, importantly, are expressed in consumption units at a specific history. The relevance of each of these effects for $\frac{dV_i(s_0)}{d\theta}$ is determined by $(\beta_i)^t \pi_t\left(s^t \middle| s_0\right) \frac{\partial u_i\left(s^t\right)}{\partial c_t^i}$, that is, by how far in the future and how likely a given history is, and by how much individual i values (in utils) a marginal unit of consumption at that particular history. Equation (3) highlights that the instantaneous consumption-equivalent effect at a given history depends on how consumption and hours worked respond to the policy change, as well as on the rate at which an individual trades off both variables, captured by the individual marginal rate of substitution between consumption and hours worked, given by $\frac{\partial u_i\left(s^t\right)}{\partial n_t^i} \middle/ \frac{\partial u_i\left(s^t\right)}{\partial c_t^i}$.[7]

Pareto-improving policy change. Equation (2) allows us to determine whether an individual is better or worse off after a policy change. That is, when $\frac{dV_i(s_0)}{d\theta} > (<) 0$, individual i perceives to be better (worse) off after a policy change. Hence, it is possible to define a Pareto-improving policy change as follows.

Definition 1. *(Pareto-improving policy change) A policy change is strictly (weakly) Pareto-*

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[7]Note that the definition of the instantaneous consumption-equivalent effect in Equation (3) does not make use of individual optimality (i.e., the envelope theorem). However, in specific applications, exploiting individual optimality conditions can yield simple expressions for $\frac{du_{i|c}\left(s^t\right)}{d\theta}$.

improving if every individual i perceives to be strictly (weakly) better off after the policy change.
Hence, a policy change is strictly Pareto-improving when $\frac{dV_i(s_0)}{d\theta} > 0$, $\forall i$, and weakly Pareto-improving
when $\frac{dV_i(s_0)}{d\theta} \geq 0$, $\forall i$.

Note that the notion of Pareto improvement does not involve interpersonal welfare comparisons, and simply exploits the ordinal nature of utility. While Pareto improvements seem highly desirable, they are rare to find, which forces planners/policymakers to make interpersonal welfare comparisons, as we describe next.[8]

Desirable policy change for a welfarist planner. The conventional approach in economics to balance welfare gains and losses among different individuals is based on individualistic social welfare functions (SWF). As in Kaplow (2011) or Saez and Stantcheva (2016), we refer to this approach — typically traced back to Bergson (1938) and Samuelson (1947) — as the *welfarist* approach. For a welfarist planner, social welfare is a real-valued function of individuals' lifetime utilities, which we formally denote in our environment by

$$\mathcal{W}\left(\{V_i(s_0)\}_{i \in I}\right), \qquad \text{(welfarist planner)} \tag{4}$$

where $V_i(s_0)$ is defined in Equation (1) and where typically $\frac{\partial \mathcal{W}}{\partial V_i} \geq 0$, $\forall i$. As carefully explained in Kaplow (2011), the critical restriction implied by the welfarist approach is that the social welfare function $\mathcal{W}(\cdot)$ cannot depend on any model outcomes besides individual utility levels.

Different welfarist social welfare functions $\mathcal{W}(\cdot)$ have different implications for the assessment of policies. In particular, the utilitarian SWF, which adds up a weighted sum of individual utilities, is given by

$$\mathcal{W}\left(\{V_i(s_0)\}_{i \in I}\right) = \int \overline{\lambda}_i V_i(s_0)\, di, \qquad \text{(utilitarian planner)} \tag{5}$$

where $\overline{\lambda}_i$ are a set predetermined scalars, commonly referred to as Pareto weights. While the utilitarian SWF is by far the most used in practice, there exist other well-known SWF's, e.g., isoelastic (Atkinson, 1970) or maximin/Rawlsian (Rawls, 1971, 1974), among others, as we describe in Section G.3.1 of the Online Appendix.

Next, we formally define when a policy change is desirable for a welfarist planner.

Definition 2. *(Desirable policy change for a welfarist planner) A welfarist planner finds a policy*

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[8]As shown by Mas-Colell, Whinston and Green (1995) or Ljungqvist and Sargent (2018), among others, by varying the welfare weights assigned to different individuals, a planner who maximizes a utilitarian social welfare function can fully trace the Pareto frontier whenever a utility possibility set is convex, and partially when it is not. Even though characterizing Pareto frontiers is a valuable exercise, we seek to study welfare assessments generally, even away from the Pareto frontier.

change desirable if and only if $\frac{dW^{\mathcal{W}}(s_0)}{d\theta} > 0$, where

$$\begin{aligned}
\frac{dW^{\mathcal{W}}(s_0)}{d\theta} &= \int \lambda_i(s_0) \frac{dV_i(s_0)}{d\theta} di \\
&= \int \lambda_i(s_0) \sum_{t=0}^{T} (\beta_i)^t \sum_{s^t} \pi_t\left(s^t \middle| s_0\right) \frac{\partial u_i(s^t)}{\partial c_t^i} \frac{du_{i|c}(s^t)}{d\theta} di,
\end{aligned}$$

(6)

where $\lambda_i(s_0) = \frac{\partial \mathcal{W}(\{V_i(s_0)\}_{i \in I})}{\partial V_i}$, and where $\frac{dV_i(s_0)}{d\theta}$ is defined in Equation (2).

The properties of the welfarist approach have been widely studied.[9] In particular, a welfarist planner is non-paternalistic, since aggregate welfare assessments are based on individual welfare assessments, and Paretian when $\frac{\partial \mathcal{W}}{\partial V_i} \geq 0$, $\forall i$, since every Pareto-improving policy is desirable. Moreover, when individuals are ex-ante homogeneous, i.e., they have identical preferences and face an identical environment from the perspective of s_0, all welfarist planners agree on whether a policy change is desirable or not, even if individuals experience different shocks ex-post.[10]

However, because of the ordinal nature of individual utilities, it is not easy to understand how a welfarist planner exactly makes tradeoffs among individuals that are ex-ante heterogeneous along some dimension. For instance, a welfarist planner would mechanically put more weight on the gains and losses of an individual whose lifetime utility is multiplied by a positive constant factor, even though, since individual utility is ordinal, this has no impact on allocations. Relatedly, it is not clear how a welfarist planner trades off the welfare gains and losses of individuals with different preferences, endowments, or life-cycle profiles, who have access to different insurance opportunities or who face shocks driven by different stochastic processes.

By introducing Dynamic Stochastic weights, we will be able to systematically i) provide a new transparent interpretation of how a particular planner (including all welfarist planners, but also other non-welfarist planners) implicitly trade off gains and losses across individuals, dates, and histories, and ii) define new welfare criteria that capture normative objectives that society may find appealing.

3 Dynamic Stochastic Weights: Definition, Decompositions, and General Properties

In this section, we introduce a new approach to assess the desirability of policy changes, based on the notion of Dynamic Stochastic Generalized Social Marginal Welfare Weights (Dynamic Stochastic weights or DS-weights, for short).

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[9]See e.g., Mas-Colell, Whinston and Green (1995), Kaplow (2011), or Adler and Fleurbaey (2016) for recent textbook treatments. Somewhat surprisingly, dynamic and stochastic considerations are not central to the literature on policy assessments.

[10]Even in this case, it is not obvious to determine whether a welfarist planner finds a policy change desirable because of aggregate efficiency, risk-sharing, or intertemporal-sharing considerations, as we illustrate in Section 4.

3.1 Definition of DS-weights: Desirable Policy Change for a DS-Planner

We begin by formally defining when a policy change is desirable for a planner who adopts DS-weights, a "DS-planner."

Definition 3. *(Desirable policy change for a DS-planner/Definition of DS-weights) A DS-planner, that is, a planner who adopts DS-weights, finds a policy change desirable if and only if $\frac{dW^{DS}(s_0)}{d\theta} > 0$, where*

$$\frac{dW^{DS}(s_0)}{d\theta} = \int \sum_{t=0}^{T} \sum_{s^t} \omega_t^i \left(s^t \middle| s_0 \right) \frac{du_{i|c}(s^t)}{d\theta} di, \tag{7}$$

where $\frac{du_{i|c}(s^t)}{d\theta}$ denotes the instantaneous consumption-equivalent effect of the policy at date t given a history s^t, defined in Equation (3), and where $\omega_t^i(s^t| s_0) > 0$, which can be a function of all the possible paths of outcomes, denotes the DS-weight assigned to individual i at date t given a history s^t for a welfare assessment that takes place at s_0.

Equation (7) shows that, in order to carry out a welfare assessment, a DS-planner must i) know the instantaneous consumption-equivalent effect of a policy for each individual at all dates and histories, that is, $\frac{du_{i|c}(s^t)}{d\theta}$, $\forall i$, $\forall t$, $\forall s^t$, which is measured in consumption units; and ii) specify DS-weights $\omega_t^i(s^t| s_0)$ for each individual at all dates and histories, that is, $\omega_t^i(s^t| s_0)$, $\forall i$, $\forall t$, $\forall s^t$. Hence, $\frac{du_{i|c}(s^t)}{d\theta}$ and $\omega_t^i(s^t| s_0)$ are sufficient statistics for welfare analysis, which makes the computation of welfare assessments conceptually straightforward. Intuitively, a DS-planner computes the impact of a policy change in consumption units at each history for every individual and then weights those changes to form an aggregate welfare assessment. Different choices of DS-weights $\omega_t^i(s^t| s_0)$ will have different normative implications, as the remainder of this paper will show.

It is worth highlighting four features that define a DS-planner. First, note that DS-weights can be functions of model outcomes, which are typically endogenous variables. For instance, by comparing Equations (6) and (7), it follows that every welfarist planner is a DS-planner with DS-weights given by

$$\omega_t^i\left(s^t \middle| s_0\right) = \lambda_i(s_0)(\beta_i)^t \pi_t\left(s^t \middle| s_0\right) \frac{\partial u_i(s^t)}{\partial c_t^i}, \tag{8}$$

where $\lambda_i(s_0) = \frac{\partial \mathcal{W}(\{V_i(s_0)\}_{i \in I})}{\partial V_i}$. Second, by making s_0 an explicit argument of $\frac{dW^{DS}(s_0)}{d\theta}$, we emphasize that welfare assessments in dynamic stochastic economies are contingent on the state in which the assessment takes place. This observation leads to time-inconsistency of welfare assessments, a topic we revisit in Section 6.3. Third, note that we define the welfare assessment of a DS-planner in marginal form, i.e, DS-weights are marginal welfare weights. This contrasts with the welfarist approach, which takes a lifetime social welfare function as primitive — see Equation (4). In Section 6.4, we show how a DS-planner can be equivalently defined in terms of an instantaneous social welfare function with generalized (endogenous) welfare weights. Finally, note that Equation (7) allows us to define a local optimum for a DS-planner as a value of θ for which $\frac{dW^{DS}(s_0)}{d\theta} = 0$. In Section G.5, we explain how to conduct non-marginal welfare assessments.

3.2 Individual Multiplicative Decomposition of DS-weights

In Lemma 1, we introduce an individual multiplicative decomposition of DS-weights into i) individual, ii) dynamic, and iii) stochastic components.[11] This individual multiplicative decomposition of DS-weights is useful to i) provide a meaningful economic interpretation of how a planner trades off welfare gains and losses across individuals, dates, and histories, given a set of DS-weights; ii) formally define and study the aggregate additive decomposition of welfare assessments, as we show in Section 3.3; and iii) formalize welfare criteria by defining DS-weights in terms of each of its components, as we illustrate in Section 4. We also define a normalized decomposition, which is unique and has desirable properties, as we show throughout the paper.

Lemma 1. *(DS-weights: individual multiplicative decomposition; unique normalized decomposition)*
a) The DS-weights that a DS-planner assigns to an individual i can be multiplicatively decomposed into three different components, as follows:

$$\omega_t^i\left(s^t\middle|s_0\right) = \underbrace{\tilde{\omega}^i\left(s_0\right)}_{individual} \underbrace{\tilde{\omega}_t^i\left(s_0\right)}_{dynamic} \underbrace{\tilde{\omega}_t^i(s^t|s_0)}_{stochastic}, \qquad where \tag{9}$$

i) $\tilde{\omega}^i\left(s_0\right)$ corresponds to an individual component, which is invariant across all dates and histories;
ii) $\tilde{\omega}_t^i\left(s_0\right)$ corresponds to a dynamic component, which can vary across dates, but not across histories at a given date; and
iii) $\tilde{\omega}_t^i\left(s^t\middle|s_0\right)$ corresponds to a stochastic component, which can vary across dates and histories.
b) For any set of DS-weights, there exists a unique "normalized" individual multiplicative decomposition, such that
i) stochastic components add up to 1 at every date, that is, $\sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle|s_0\right)=1$, $\forall t$, $\forall i$;
ii) dynamic components add up to 1 across all dates, that is, $\sum_{t=0}^T\tilde{\omega}_t^i\left(s_0\right)=1$, $\forall i$; and
iii) individual components add up to 1 across individuals, that is, $\int\tilde{\omega}^i\left(s_0\right)di=1$.
We refer to planners who adopt this decomposition as "normalized" DS-planners.

The components of the individual multiplicative decomposition define social marginal rates of substitution for a DS-planner across individuals, dates, and histories. For instance, the stochastic component, $\tilde{\omega}_t^i\left(s^t\middle|s_0\right)$, which has the interpretation of a risk-neutral measure at date t when $\sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle|s_0\right)=1$, determines how a DS-planner values unit of consumption good across different histories s^t at date t for a given individual. The dynamic component, $\tilde{\omega}_t^i(s_0)$, which has the interpretation of a normalized discount factor when $\sum_{t=0}^T\tilde{\omega}_t^i\left(s_0\right)=1$, determines how a DS-planner values consumption across different dates for a given individual.[12] The individual component determines how a DS-planner trades off permanent gains and losses across individuals. In the case of the normalized decomposition, when $\int\tilde{\omega}^i\left(s_0\right)di=1$, it defines the units in which $\frac{dW^{DS}(s_0)}{d\theta}$ is

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[11] This individual multiplicative decomposition is inspired by Alvarez and Jermann (2005) and Hansen and Scheinkman (2009), who multiplicatively decompose pricing kernels into permanent and transitory components.

[12] Risk-neutral measures are widely used in finance (Duffie, 2001; Cochrane, 2005), while normalized discount factors are common in the study of repeated games (Fudenberg and Tirole, 1991; Mailath and Samuelson, 2006).

expressed. In particular, the individual component of individual i, $\tilde{\omega}^i(s_0)$, exactly determines the weight that a DS-planner gives to a permanent transfer of one unit of consumption good across all dates and histories to individual i, measured in units of a permanent transfer of one unit of consumption good to all individuals across all dates and histories.

It is worth highlighting that the sign of $\frac{dW^{DS}(s_0)}{d\theta}$ — and hence whether a policy change is desirable or not — is fully determined by the value of the DS-weights as a whole and not by any individual multiplicative decomposition. However, we will show that the normalized individual multiplicative decomposition is associated with desirable properties in the context of the aggregate additive decomposition that we introduce next, while unnormalized decompositions typically are not. The normalized decomposition guarantees that its components, as well as $\frac{dW^{DS}(s_0)}{d\theta}$, have a meaningful interpretation in terms of units of consumption across specific histories, dates, and individuals. In general, once the units of $\omega_t^i(s^t \mid s_0)$ and its components are defined, every individual multiplicative decomposition is unique. See Section 4, and Section G.1 of the Online Appendix for further details.

For instance, a possible individual multiplicative decomposition for an (unnormalized) welfarist planner is given by

$$\tilde{\omega}^i(s_0) = \lambda_i(s_0), \quad \tilde{\omega}_t^i(s_0) = (\beta_i)^t, \quad \text{and} \quad \tilde{\omega}_t^i\left(s^t \mid s_0\right) = \pi_t\left(s^t \mid s_0\right)\frac{\partial u_i(s^t)}{\partial c_t^i}, \tag{10}$$

where $\lambda_i(s_0) = \frac{\partial \mathcal{W}\left(\{V_i(s_0)\}_{i\in I}\right)}{\partial V_i}$. This decomposition, because it is expressed in utils, cannot be used to understand how a planner makes tradeoffs in terms of consumption units. In Section 4, we instead introduce the individual multiplicative decomposition of a normalized welfarist planner, and study its properties in detail.

3.3 Aggregate Additive decomposition of Welfare Assessments under DS-weights

Armed with the individual multiplicative decomposition of DS-weights, we now introduce an exact additive decomposition of the welfare assessments made by a DS-planner. This decomposition shows that the welfare assessment of a policy change $d\theta$ made by a DS-planner is driven by exactly four considerations: aggregate efficiency, risk-sharing, intertemporal-sharing, and redistribution.[13]

Proposition 1. *(Welfare assessments: aggregate additive decomposition) The aggregate welfare assessment of a DS-planner, $\frac{dW^{DS}(s_0)}{d\theta}$, can be decomposed into four components: i) an aggregate efficiency component, ii) a risk-sharing component, iii) an intertemporal-sharing component, and iv) a redistribution component, as follows:*

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[13]We have chosen the term risk-sharing and the (less conventional) term intertemporal-sharing to highlight that both components of the aggregate additive decomposition are driven by cross-sectional differences, via interpersonal sharing. Alternative terms, such as insurance, consumption smoothing, or intertemporal smoothing, do not have such connotation, since they are applicable to a single individual.

$$\frac{dW^{DS}(s_0)}{d\theta} = \underbrace{\sum_{t=0}^{T} \mathbb{E}_i\left[\tilde{\omega}_t^i(s_0)\right] \sum_{s^t} \mathbb{E}_i\left[\tilde{\omega}_t^i\left(s^t\middle|s_0\right)\right] \mathbb{E}_i\left[\frac{du_{i|c}(s^t)}{d\theta}\right]}_{=\Xi_{AE}\ (Aggregate\ Efficiency)}$$

$$+ \underbrace{\sum_{t=0}^{T} \mathbb{E}_i\left[\tilde{\omega}_t^i(s_0)\right] \sum_{s^t} \mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s^t\middle|s_0\right), \frac{du_{i|c}(s^t)}{d\theta}\right]}_{=\Xi_{RS}\ (Risk\text{-}sharing)}$$

$$+ \underbrace{\sum_{t=0}^{T} \mathbb{C}ov_i\left[\tilde{\omega}_t^i(s_0), \sum_{s^t} \tilde{\omega}_t^i\left(s^t\middle|s_0\right) \frac{du_{i|c}(s^t)}{d\theta}\right]}_{=\Xi_{IS}\ (Intertemporal\text{-}sharing)}$$

$$+ \underbrace{\mathbb{C}ov_i\left[\tilde{\omega}^i\left(s^0\right), \sum_{t=0}^{T} \tilde{\omega}_t^i(s_0) \sum_{s^t} \tilde{\omega}_t^i\left(s^t\middle|s_0\right) \frac{du_{i|c}(s^t)}{d\theta}\right]}_{=\Xi_{RD}\ (Redistribution)}, \tag{11}$$

where $\mathbb{E}_i[\cdot]$ and $\mathbb{C}ov_i[\cdot,\cdot]$ respectively denote cross-sectional expectations and covariances, where the history-specific term that determines the aggregate efficiency component, $\mathbb{E}_i\left[\frac{du_{i|c}(s^t)}{d\theta}\right]$, is given by

$$\mathbb{E}_i\left[\frac{du_{i|c}(s^t)}{d\theta}\right] = \int \frac{dc_t^i(s^t)}{d\theta} di + \int \frac{\frac{\partial u_i(s^t)}{\partial n_t^i}}{\frac{\partial u_i(s^t)}{\partial c_t^i}} \frac{dn_t^i(s^t)}{d\theta} di, \tag{12}$$

and where, without loss of generality, we have assumed that $\mathbb{E}_i\left[\tilde{\omega}^i(s_0)\right] = \int \tilde{\omega}^i(s_0)\, di = 1$.

The first component of the aggregate additive decomposition is the aggregate efficiency component, Ξ_{AE}. This component accounts for the aggregate instantaneous consumption-equivalent effect of the policy, expressed in consumption units. As shown in Equation (12), Ξ_{AE} adds up the changes in consumption-equivalents resulting from the marginal policy change across all dates and histories. Because Ξ_{AE} can be computed using exclusively cross-sectional averages of $\tilde{\omega}_t^i(s_0)$, $\tilde{\omega}_t^i\left(s^t\middle|s_0\right)$, and $\frac{du_{i|c}(s^t)}{d\theta}$, we refer to the this term as aggregate efficiency.[14]

The remaining three components of the aggregate additive decomposition are driven by the cross-

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[14]Note that Equation (12) can be rewritten as

$$\mathbb{E}_i\left[\frac{du_{i|c}(s^t)}{d\theta}\right] = \int \frac{dc_t^i(s^t)}{d\theta} \tau_t^i(s^t)\, di = \mathbb{E}_i\left[\frac{dc_t^i(s^t)}{d\theta}\right] \mathbb{E}_i\left[\tau_t^i(s^t)\right] + \mathbb{C}ov\left[\frac{dc_t^i(s^t)}{d\theta}, \tau_t^i(s^t)\right],$$

where $\tau_t^i(s^t) \equiv 1 + \frac{\frac{\partial u_i(s^t)}{\partial n_t^i}}{\frac{\partial u_i(s^t)}{\partial c_t^i}} \frac{\frac{dn_t^i(s^t)}{d\theta}}{\frac{dc_t^i(s^t)}{d\theta}}$, which shows that aggregate efficiency is tightly connected to labor wedges. Intuitively, policies that increases aggregate consumption contribute more to aggregate efficiency when the aggregate labor wedge is greater than 1, i.e., when $\mathbb{E}_i\left[\tau_t^i(s^t)\right] > 1$. Alternatively, policies that do not change aggregate consumption can contribute to aggregate efficiency if they increase the consumption of those individuals with higher individual labor wedges by more.

sectional variation of each of the three elements (individual, dynamic, stochastic) of the individual multiplicative decomposition of DS-weights. In particular, the risk-sharing component, Ξ_{RS}, adds up across all dates and histories the covariances between the stochastic component, $\tilde{\omega}_t^i\left(s^t \middle| s_0\right)$, and the instantaneous consumption-equivalent effect at each date and history. Similarly, the intertemporal-sharing component, Ξ_{IS}, adds up across all dates the covariances between the dynamic component, $\tilde{\omega}_t^i\left(s_0\right)$, and the (expected, under the risk-neutral measure interpretation of stochastic weights) instantaneous consumption-equivalent effect at each date. Finally, the redistribution component, Ξ_{RD}, consists of a single cross-sectional covariance between the individual component, $\tilde{\omega}^i\left(s^0\right)$, and the present discounted value — using the dynamic and stochastic components — of instantaneous consumption-equivalent effects that a DS-planner assigns to particular individual.

Before we discuss the properties of this decomposition below, it is worth making two remarks. First, the aggregate additive decomposition is exact for any marginal policy change and does not rely on any approximations. Relatedly, the decomposition can be computed using only the individual multiplicative decomposition of DS-weights — typically a function of model outcomes — and instantaneous consumption-equivalent effects.

Second, the aggregate additive decomposition is based on cross-sectional averages and covariances, and does not include covariances over future periods or histories. In Section 6.1, we further decompose the aggregate efficiency and redistribution components along those lines, developing a stochastic decomposition — see Propositions 10 and 12. There, we also provide an alternative decomposition of the risk-sharing and intertemporal-sharing components still based on cross-sectional averages and covariances.[15]

3.4 General Properties of the Aggregate Additive Decomposition

The merits of the aggregate additive decomposition introduced in Proposition 1 lie in its properties. Similarly, the names we attribute to each of the components, Ξ_{AE} through Ξ_{RD}, are only meaningful if they satisfy desirable properties. Hence, in the remainder of this section, we flesh out the properties of the aggregate additive decomposition and its components for a general DS-planner.

First, in Proposition 2, we identify conditions on DS-weights and their components under which the welfare assessments of a DS-planner i) are purely based on aggregate efficiency considerations or ii) are such that the risk-sharing, intertemporal-sharing, or redistribution components are zero.

Proposition 2. *(Properties of aggregate additive decomposition: individual-invariant DS-weights)*

a) If DS-weights $\omega_t^i\left(s^t \middle| s_0\right)$ are constant across all individuals at all dates and histories, then the welfare assessment of a DS-planner is exclusively based on aggregate efficiency considerations, i.e., $\Xi_{RS} = \Xi_{IS} = \Xi_{RD} = 0$.

b) If the stochastic component of DS-weights is constant across all individuals at all dates and histories, then $\Xi_{RS} = 0$.

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[15]The aggregate decomposition introduced in Proposition 1 is appealing because it treats systematically each of the components of the individual multiplicative decomposition. That is, Ξ_{RS} is directly determined by $\tilde{\omega}_t^i\left(s^t \middle| s_0\right)$, Ξ_{IS} by $\tilde{\omega}_t^i\left(s_0\right)$, and Ξ_{RD} by $\tilde{\omega}^i\left(s_0\right)$.

c) If the dynamic component of DS-weights is constant across all individuals at all dates, then $\Xi_{IS} = 0$.

d) If the individual component of DS-weights is constant across all individuals, then $\Xi_{RD} = 0$.

Proposition 2 shows that a DS-planner who assigns DS-weights that do not vary across individuals at all dates and histories makes welfare assessments purely based on aggregate efficiency considerations. This result bears resemblance to the classic question of defining a normative representative consumer — see e.g., Mas-Colell, Whinston and Green (1995) or Acemoglu (2009). In particular, Proposition 2a) implies that the risk-sharing, intertemporal-sharing, and redistribution components are zero in single-agent or representative-agent economies in which all individuals have the same DS-weights, i.e., DS-weights are symmetric. Parts b) through d) of Proposition 2 also show that, depending on which specific components of the individual multiplicative decomposition of DS-weights are invariant across individuals, it may be that $\Xi_{RS} = 0$, $\Xi_{IS} = 0$, or $\Xi_{RD} = 0$. These results highlight the cross-sectional nature of the risk-sharing, intertemporal-sharing, and redistribution components. Moreover, parts c) and d) of Proposition 2 respectively imply that the intertemporal-sharing and the redistribution components are always zero when individuals are ex-ante identical.

Given their practical importance, we highlight the following implications of Proposition 2 in four corollaries.[16]

Corollary 1. *(Representative-agent economies) Welfare assessments in single-agent economies or representative-agent economies in which DS-weights are symmetric are exclusively attributed to aggregate efficiency considerations, i.e.,* $\Xi_{RS} = \Xi_{IS} = \Xi_{RD} = 0$.

Corollary 2. *(Perfect-foresight economies) Welfare assessments in perfect-foresight economies in which the individual multiplicative decomposition of DS-weights is normalized are never attributed to risk-sharing, i.e.,* $\Xi_{RS} = 0$.

Corollary 3. *(Economies with ex-ante identical individuals) Welfare assessments in economies in which all individuals are ex-ante identical (but not necessarily ex-post) and DS-weights are symmetric are never attributed to intertemporal-sharing or redistribution, i.e.,* $\Xi_{IS} = \Xi_{RD} = 0$.

Corollary 4. *(Static economies) Welfare assessments in static economics in which the individual multiplicative decomposition of DS-weights is normalized are exclusively attributed to aggregate efficiency or redistribution considerations, i.e.,* $\Xi_{RS} = \Xi_{IS} = 0$.

In Proposition 3, we identify conditions on the set of policy changes under which the welfare assessments of a DS-planner i) are purely based on aggregate efficiency considerations or ii) are such that the risk-sharing, or the risk-sharing and the intertemporal-sharing components are zero.

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[16]We say that DS-weights are symmetric when two individuals with identical preferences and identical (random) paths for consumption and hours are assigned identical DS-weights. This is a natural restriction when making welfare assessments — see e.g., Mas-Colell, Whinston and Green (1995) for a discussion of symmetry. Corollaries 2 and 4 require a normalize individual multiplicative decomposition so that the choice of units of $\omega_t^i \left(s^t \middle| s_0 \right)$ and $\omega_t^i (s_0)$ does not generate meaningless cross-sectional variation when computing Ξ_{RS} and Ξ_{IS}.

Generically, a policy change will affect all four components of the aggregate additive decomposition. Hence, to guarantee that some components of the aggregate decomposition are zero, Proposition 3 identifies policies that impact all individuals identically along certain dimensions.

Proposition 3. *(Properties of aggregate additive decomposition: individual-invariant policies) Suppose that the individual multiplicative decomposition of DS-weights is normalized, so $\sum_{s^t} \tilde{\omega}_t^i \left(s^t \mid s_0 \right) = 1$, $\forall t$, $\forall i$, and $\sum_{t=0}^{T} \tilde{\omega}_t^i \left(s_0 \right) = 1$, $\forall i$.*
a) If the instantaneous consumption-equivalent effect of a policy change, $\frac{du_{i|c}\left(s^t\right)}{d\theta}$, is identical across individuals at all dates and histories, then the welfare assessment of a DS-planner is exclusively based on aggregate efficiency considerations, i.e., $\Xi_{RS} = \Xi_{IS} = \Xi_{RD} = 0$.
b) If the instantaneous consumption-equivalent effect of a policy change, $\frac{du_{i|c}\left(s^t\right)}{d\theta}$, is identical across individuals at all histories on a date, for all dates, i.e., $\Xi_{RS} = \Xi_{IS} = 0$.
c) If the instantaneous consumption-equivalent effect of a policy change, $\frac{du_{i|c}\left(s^t\right)}{d\theta}$, is identical across individuals conditional on a date and history, for all dates and histories, i.e., $\Xi_{RS} = 0$.

Proposition 3a) shows that a policy change that affects all individuals identically across all dates and histories can only affect aggregate welfare via aggregate efficiency considerations. Proposition 3b) shows that a policy change that varies over time but affects all agents identically across all histories at a given date can affect aggregate welfare via aggregate efficiency and redistribution, but not risk-sharing or intertemporal-sharing. Proposition 3c) shows that a policy change that affects all individuals identically conditional on a history taking place but that can vary across dates and individuals will have no risk-sharing component. It should be evident that, for generic DS-weights, the converse of these results also holds. That is, policy changes must affect different individuals differently if they load on the risk-sharing, intertemporal-sharing, or redistribution components of the aggregate additive decomposition.

Proposition 3 critically relies on considering a normalized individual multiplicative decomposition of (the dynamic and stochastic components of) DS-weights. As highlighted above, such normalization guarantees that the components of the individual multiplicative decomposition have meaningful units, which makes it possible to derive conditions on how policies affect individuals in terms of consumption. See Section G.1 of the Online Appendix for further details.

Finally, we show in Proposition 4 that, in an endowment economy, aggregate efficiency considerations play no role for a DS-planner when making normative assessments. We use the term endowment economy to refer to economies in which all consumption comes from predetermined endowments of the consumption good at each date and history, and individuals' instantaneous utility exclusively depends on consumption. If individual utility depended on other variables, Proposition 4 remains valid only when the sum of consumption-equivalent effects is zero.

Proposition 4. *(Properties of aggregate additive decomposition: endowment economies) In an endowment economy in which the aggregate endowment of the consumption good is invariant to policy, the aggregate efficiency component of the welfare assessment of a DS-planner is zero for any set of DS-weights, $\omega_t^i \left(s^t \mid s_0 \right)$, i.e., $\Xi_{AE} = 0$.*

Proposition 4 highlights that the aggregate efficiency component captures the impact of policies on the production side of the economy. Proposition 10 below further discusses endowment economies.

4 Normalized Welfarist Planners: Characterization and Properties

One of the challenges of the welfarist approach is to understand how a planner makes tradeoffs among heterogeneous individuals, because of the ordinal nature of individual utilities. In Section 4.1, we first show how to systematically characterize — critically, in easily interpretable consumption units — how a welfarist DS-planner makes such tradeoffs across periods and histories for a given individual, and across individuals. Next, in Section 4.2, we characterize new additional properties of the aggregate additive decomposition of welfare assessments for welfarist planners. We focus on defining and studying normalized welfarist planners because their welfare assessments satisfy highly desirable properties. In particular, we show that all normalized welfarist planners conclude that the risk-sharing and intertemporal-sharing components are zero when markets are complete, that the intertemporal-sharing component is zero when individuals can freely trade a riskless bond, and that different normalized welfarist planners — with different SWF's $\mathcal{W}(\cdot)$ — exclusively disagree on the redistribution component.

4.1 Characterization of Individual Multiplicative Decomposition for Normalized Welfarist Planners

Proposition 5 characterizes the unique normalized individual multiplicative decomposition of DS-weights for a given welfarist planner, i.e., for a given SWF, $\mathcal{W}(\cdot)$, defined in Equation (4).

Proposition 5. *(Normalized welfarist planners: individual multiplicative decomposition) The unique normalized individual multiplicative decomposition of DS-weights for a welfarist planner with SWF, $\mathcal{W}(\cdot)$, is given by*

$$\tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\middle|s_0\right) = \frac{(\beta_i)^t\,\pi_t\left(s^t\middle|s_0\right)\frac{\partial u_i(s^t)}{\partial c_t^i}}{(\beta_i)^t\sum_{s^t}\pi_t\left(s^t\middle|s_0\right)\frac{\partial u_i(s^t)}{\partial c_t^i}} = \frac{\pi_t\left(s^t\middle|s_0\right)\frac{\partial u_i(s^t)}{\partial c_t^i}}{\sum_{s^t}\pi_t\left(s^t\middle|s_0\right)\frac{\partial u_i(s^t)}{\partial c_t^i}} \tag{13}$$

$$\tilde{\omega}_t^{i,\mathcal{W}}\left(s_0\right) = \frac{(\beta_i)^t\sum_{s^t}\pi_t\left(s^t\middle|s_0\right)\frac{\partial u_i(s^t)}{\partial c_t^i}}{\sum_{t=0}^{T}(\beta_i)^t\sum_{s^t}\pi_t\left(s^t\middle|s_0\right)\frac{\partial u_i(s^t)}{\partial c_t^i}} \tag{14}$$

$$\tilde{\omega}^{i,\mathcal{W}}\left(s_0\right) = \frac{\lambda_i\left(s_0\right)\sum_{t=0}^{T}(\beta_i)^t\sum_{s^t}\pi_t\left(s^t\middle|s_0\right)\frac{\partial u_i(s^t)}{\partial c_t^i}}{\int\lambda_i\left(s_0\right)\sum_{t=0}^{T}(\beta_i)^t\sum_{s^t}\pi_t\left(s^t\middle|s_0\right)\frac{\partial u_i(s^t)}{\partial c_t^i}di}, \tag{15}$$

where $\lambda_i\left(s_0\right) = \frac{\partial\mathcal{W}\left(\{V_i(s_0)\}_{i\in I}\right)}{\partial V_i}$.

This normalization explains how a welfarist planner makes welfare assessments. First, note that the instantaneous consumption-equivalent effect of the policy at date t and history s^t, $\frac{du_{i|c}(s^t)}{d\theta}$, is

expressed in units of the consumption good (dollars) at such a history. The stochastic component, $\tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\middle|s_0\right)$, can consequently be interpreted as the marginal rate of substitution between a dollar in history s^t and a dollar across all possible histories at date t for individual i from the planner's perspective. Formally, the denominator of Equation (13) corresponds to the marginal value of transferring one dollar in every possible history at date t. For instance, if the stochastic component is 0.4 for a given individual, history, and date, a welfarist planner values equally a one-dollar transfer at that particularly history and a transfer of 0.4 dollars to the same individual across all histories at that date.

The dynamic component, $\tilde{\omega}_t^{i,\mathcal{W}}\left(s_0\right)$, can similarly be interpreted as a marginal rate of substitution between a dollar across all possible histories at date t and a dollar at date 0 for individual i from the planner's perspective. Formally, the denominator of Equation (14) corresponds to the marginal value of permanently transferring one dollar across all dates and histories. For instance, if the dynamic component is 0.3 for a given individual and date, a welfarist planner values equally — for that individual — a one-dollar permanent transfer across all histories at that particular date and a transfer of 0.3 dollars at date 0. Both the stochastic and the dynamic components are thus useful because they allow the planner to meaningfully compare the welfare impact of policy changes across dates and histories for a given individual i.

Finally, the individual component, $\tilde{\omega}_t^{i,\mathcal{W}}\left(s_0\right)$, can be interpreted as the weight that a welfarist planner assigns to welfare changes for a given individual, expressed in terms of a permanent dollar transfer across dates and histories. Formally, the denominator of Equation (15) corresponds to the marginal value of permanently transferring one dollar to each individual in the economy across all dates and histories. For instance, if the individual component is 0.2 for a given individual, a welfarist planner values equally a one-dollar permanent transfer to that individual across all dates and histories and a permanent transfer of 0.2 dollars to all individuals across all dates and histories. It follows from Equation (15) that a welfarist planner gives more weight to individuals who are more patient, whose utility function has more curvature, who have lower consumption, and for whom $\lambda^i\left(s_0\right)$ is lower.

Several implications follow from Proposition 5. First, the welfare assessment of a normalized welfarist planner has a cardinal interpretation, since it is measured in dollars at all dates and histories for all individuals. In other words, if $\frac{dW^{\mathcal{W}}}{d\theta} = 0.1$, a normalized welfarist planner concludes that a marginal policy change is equivalent to a permanent transfer to all individuals at all dates and histories of 0.1 dollars.

Second, it is possible to reformulate the dynamic and stochastic normalized components as

$$\tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\middle|s_0\right) = \frac{q_t^i\left(s^t|s_0\right)}{\sum_{s^t} q_t^i\left(s^t|s_0\right)} = \frac{\text{individual } i \text{ date-0 state-price of history } s^t}{\text{individual } i \text{ date-0 price of date-}t \text{ zero coupon bond}} \tag{16}$$

$$\tilde{\omega}_t^{i,\mathcal{W}}\left(s_0\right) = \frac{\sum_{s^t} q_t^i\left(s^t|s_0\right)}{\sum_{t=0}^T \sum_{s^t} q_t^i\left(s^t|s_0\right)} = \frac{\text{individual } i \text{ date-0 price of date-}t \text{ zero coupon bond}}{\text{individual } i \text{ date-0 price of } T\text{-consol bond}}, \tag{17}$$

where $q_t^i\left(s^t|s_0\right)$ denotes the state-price over history s^t from the perspective of individual i at date 0,

given by[17]

$$q_t^i\left(s^t|s_0\right) = (\beta_i)^t \, \pi_t\left(s^t \,\middle|\, s_0\right) \frac{\partial u_i\left(s^t\right)}{\partial c_t^i}\bigg/\frac{\partial u_i\left(s^0\right)}{\partial c_0^i}. \tag{18}$$

Equations (16) and (17) highlight that a welfarist planner makes tradeoffs across dates and histories for a given individual exclusively using the individual's own stochastic discount factor. This is a natural result, since welfarist planners are non-paternalistic.

Third, we can reformulate the individual normalized components as

$$\tilde\omega^{i,\mathcal{W}}\left(s_0\right) = \frac{\lambda_i\left(s_0\right) \frac{\partial u_i\left(s^0\right)}{\partial c_0^i} \sum_{t=0}^{T}\sum_{s^t} q_t^i\left(s^t|s_0\right)}{\int \lambda_i\left(s_0\right) \frac{\partial u_i\left(s^0\right)}{\partial c_0^i} \sum_{t=0}^{T}\sum_{s^t} q_t^i\left(s^t|s_0\right) di}, \tag{19}$$

where $q_t^i\left(s^t|s_0\right)$ is defined in Equation (19). In contrast to Equations (13) and (16), the exact form of the SWF $\mathcal{W}\left(\cdot\right)$ does impact the normalized individual components, a fact that is critical to show that welfarist planners exclusively disagree about the redistribution — see Proposition 8 below.[18]

Fourth, typically — at least when markets are incomplete (see Proposition 6 below) — we expect all four components of the aggregate additive decomposition to be non-zero for a normalized welfarist planner.

Finally, aggregate welfare assessments made by a particular welfarist planner (e.g., with a particular $\mathcal{W}\left(\cdot\right)$) are directionally invariant to whether we consider a normalized or an unnormalized individual multiplicative decomposition. That is, in both cases, both decompositions agree on whether a policy is desirable or not. However, the normalized individual multiplicative decomposition will have desirable properties, as we describe next.

4.2 Properties of Aggregate Additive Decomposition for Normalized Welfarist Planners

Since welfarist planners are particular DS-planners, it follows immediately that Corollaries 1 and 3 of Proposition 2, as well as Propositions 3 and 4, also apply — without modification — to normalized welfarist planners. However, we can further exploit the characterization of the individual multiplicative decomposition introduced in Proposition 5 to identify new desirable properties of the aggregate decomposition for normalized welfarist planners.

In particular, we show that i) all normalized welfarist planners conclude that the risk-sharing and intertemporal-sharing components are zero when markets are complete, ii) the intertemporal-sharing component is zero when individuals can freely trade a riskless bond, and iii) different normalized

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[17]Consol bonds are typically defined as fixed-income securities with no maturity date. Since we consider economies that may have a finite horizon, we define a T-consol bond as a bond that pays at every date. When $T = \infty$, the conventional definition and ours coincide.

[18]Interestingly, as we discuss in Section G.3.3 of the Online Appendix, a planner who uses a date-0 Kaldor-Hicks normalization, in which $\lambda_i\left(s_0\right) \frac{\partial u_i\left(s^0\right)}{\partial c_0^i} = 1$, implicitly assigns higher individual weights to those with higher willingness to pay for T-consol bonds, since $\tilde\omega^{i,\mathcal{W}}\left(s_0\right) = \frac{\sum_{t=0}^{T}\sum_{s^t} q_t^i\left(s^t|s_0\right)}{\int \sum_{t=0}^{T}\sum_{s^t} q_t^i\left(s^t|s_0\right)di}$, which may seem like a desirable approach.

welfarist planners — with different SWF's $\mathcal{W}(\cdot)$ — exclusively disagree on the redistribution component. To our knowledge, the aggregate additive decomposition of welfare assessments introduced in this paper is the first welfare decomposition for which these properties — which seem highly desirable — have been established.

It seems natural to conjecture that the intertemporal- and risk-sharing components of the aggregate additive decomposition depend critically on the ability of individuals to smooth consumption intertemporally and across histories. For the purposes of Proposition 6, we say that markets are complete when the marginal rates of substitution across all dates and histories are equalized across agents — this condition is endogenously satisfied in any equilibrium model in which individuals can freely trade claims that span all possible contingencies.

Proposition 6. *(Properties of normalized welfarist planners: complete markets) When markets are complete, that is, when the marginal rates of substitution across all dates and histories are equalized across agents, the intertemporal-sharing and the risk-sharing components of the aggregate welfare decomposition for a normalized welfarist planner are zero, that is, $\Xi_{RS} = \Xi_{IS} = 0$. Hence, in that case, welfare assessments made by a normalized welfarist planner are exclusively driven by aggregate efficiency and/or redistribution.*

When markets are complete, $\tilde{\omega}_t^{i,NU}\left(s^t \middle| s_0\right)$ and $\tilde{\omega}_t^{i,NU}\left(s_0\right)$ become identical across individuals, as shown by the fact that there is a unique stochastic discount factor, so $q_t^i = q_t$, $\forall i$, in Equations (16) and (17). Combined with Proposition 2b), this immediately implies that $\Xi_{RS} = \Xi_{IS} = 0$. Intuitively, a normalized welfarist planner perceives that no policy can entail welfare gains or losses coming from risk-sharing or intertemporal-sharing among individuals, since individuals can perfectly share risks and substitute intertemporally.[19]

Proposition 7. *(Properties of normalized welfarist planners: riskless borrowing/saving) When individuals are able to borrow and save freely at all times, the intertemporal-sharing component of the aggregate welfare decomposition for a normalized welfarist planner is zero, that is, $\Xi_{IS} = 0$.*

When agents are able to borrow and save freely at all times, $\tilde{\omega}_t^{i,NU}\left(s_0\right)$ becomes identical across individuals. This follows directly from Equation (16), since in that case $\sum_{s^t} q_t^i\left(s^t|s_0\right)$ is constant for all individuals. Intuitively, a normalized welfarist planner perceives that no policy can entail welfare gains or losses coming from intertemporal-sharing among individuals, since individuals can perfectly transfer resources across periods. Proposition 7 immediately implies that constraints to borrowing/saving are needed for the intertemporal sharing component to be relevant.

Proposition 8. *(Properties of normalized welfarist planners: welfarist planners only disagree about redistribution) For a given policy, the aggregate efficiency, risk-sharing, and intertemporal-sharing components of the aggregate additive decomposition are identical for all normalized welfarist planners.*

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[19]Proposition 6 suggests that the cross-sectional dispersions of the dynamic and stochastic components of DS-weights, $\mathbb{SD}_i\left[\tilde{\omega}_t^i\left(s^t \middle| s_0\right)\right]$ and $\mathbb{SD}_i\left[\tilde{\omega}_t^i\left(s^0\right)\right]$, may be natural candidates to measure the potential welfare gains from completing markets for a normalized welfarist planner — see also Proposition 13 below.

Hence, differences in welfare assessments between normalized welfarist planners are exclusively based on how they assess redistribution.

Proposition 8 follows from the fact that the individual component of the individual multiplicative decomposition, $\tilde{\omega}^{i,\mathcal{W}}(s_0)$, is the only component that depends on the exact form of $\mathcal{W}(\cdot)$. Therefore, differences in welfare assessments between welfarist planners can always be traced back to differences in the redistribution component of the aggregate additive decomposition.[20] This result crucially hinges on the fact that welfarist planners are non-paternalistic, that is, welfarist planners use individual lifetime utilities as inputs into their aggregate welfare calculations. In the next section, we encounter new "pseudo-welfarist" planners for which this property does not hold — see also Section G.3.1 of the Online Appendix.

5 New Welfare Criteria

A central objective of this paper is to provide a framework to systematically formalize new welfare criteria to assess and conduct policy. In this section, we describe how to use DS-weights to formalize new welfare criteria that capture particular normative objectives that society may find appealing. These results have the potential to allow for disciplined discussions about the mandates of independent technocratic institutions (central banks, financial regulators, other regulatory agencies, etc.).

5.1 AE/AR/NR DS-Planners

In this subsection, we introduce three different sets of novel DS-planners: aggregate efficiency (AE) DS-planners, aggregate efficiency/risk-sharing (AR) DS-planners, and no-redistribution (NR) DS-planners. The welfare assessments made by these new planners purposefully set to zero particular components of the aggregate additive decomposition. Within each set of DS-planners, we identify a pseudo-welfarist planner as the one that represents the minimal departure relative to the normalized welfarist planner.

By introducing these new planners we are able to formalize new welfare criteria that, for instance, isolate aggregate efficiency as the sole welfare objective, or that remove the desire to redistribute across individuals, among other goals. As we illustrate in our Applications, these new DS-planners are helpful not only to provide analytical characterizations, but also to characterize and compute optimal policy solutions guided by particular normative considerations.

Definition 4. *(AE/AR/NR DS-planners: definition)*
a) (Aggregate efficiency DS-planners) An aggregate efficiency (AE) DS-planner, that is, a planner who exclusively values aggregate efficiency, is a DS-planner for whom the individual, dynamic, and

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[20]Note that Proposition 8, when combined with Corollary 3 rationalizes why all normalized welfarist planners directionally agree on welfare assessments when individuals are ex-ante identical. In that case, Corollary 3 implies that the redistribution component is zero, and Proposition 8 shows that Ξ_{AE}, Ξ_{RS}, and Ξ_{IS} are identical for normalized welfarist planners.

stochastic components of DS-weights are constant across all individuals at all dates and histories. A pseudo-welfarist AE DS-planner, who values aggregate efficiency as a normalized welfarist planner, has DS-weights $\omega_t^{i,\mathcal{W},AE}\left(s^t\mid s_0\right)$ defined by

$$\tilde{\omega}^{i,\mathcal{W},AE}\left(s_0\right) = 1, \quad \tilde{\omega}_t^{i,\mathcal{W},AE}\left(s_0\right) = \mathbb{E}_i\left[\tilde{\omega}_t^{i,\mathcal{W}}\left(s_0\right)\right], \text{ and } \tilde{\omega}_t^{i,\mathcal{W},AE}\left(s^t\mid s_0\right) = \mathbb{E}_i\left[\tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\mid s_0\right)\right]. \quad (20)$$

b) (Aggregate efficiency/risk-sharing DS-planners) An aggregate efficiency/risk-sharing (AR) DS-planner, that is, a planner who exclusively values aggregate efficiency and risk-sharing, is a DS-planner for whom the individual and dynamic components of DS-weights are constant across all individuals at all dates. A pseudo-welfarist AR DS-planner, who values aggregate efficiency and risk-sharing as a normalized welfarist planner, has DS-weights $\omega_t^{i,\mathcal{W},AR}\left(s^t\mid s_0\right)$ defined by

$$\tilde{\omega}^{i,\mathcal{W},AR}\left(s_0\right) = 1, \quad \tilde{\omega}_t^{i,\mathcal{W},AR}\left(s_0\right) = \mathbb{E}_i\left[\tilde{\omega}_t^{i,\mathcal{W}}\left(s_0\right)\right], \text{ and } \tilde{\omega}_t^{i,\mathcal{W},AR}\left(s^t\mid s_0\right) = \tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\mid s_0\right). \quad (21)$$

c) (No-redistribution DS-planners) A no-redistribution (NR) DS-planner, that is, a planner who exclusively values aggregate efficiency, risk-sharing, and intertemporal-sharing, but disregards redistribution, is a DS-planner for whom the individual component of DS-weights is constant across all individuals. A pseudo-welfarist AR DS-planner, who values aggregate efficiency, risk-sharing, and intertemporal-sharing as a normalized welfarist planner, has DS-weights $\omega_t^{i,\mathcal{W},NR}\left(s^t\mid s_0\right)$ defined by

$$\tilde{\omega}^{i,\mathcal{W},NR}\left(s_0\right) = 1, \quad \tilde{\omega}_t^{i,\mathcal{W},NR}\left(s_0\right) = \tilde{\omega}_t^{i,\mathcal{W}}\left(s_0\right), \text{ and } \tilde{\omega}_t^{i,\mathcal{W},NR}\left(s^t\mid s_0\right) = \tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\mid s_0\right). \quad (22)$$

Formally, an AE DS-planner adopts components of the individual multiplicative decomposition of DS-weights that are individual invariant. The pseudo-welfarist AE DS-planner sets these components exactly equal to the cross-sectional average of those used by a normalized welfarist planner.[21] An AR DS-planner only makes the individual and dynamic components individual invariant, while the pseudo-welfarist AR DS-planner further preserves the stochastic component used by the normalized welfarist planner. A NR DS-planner only makes the individual component individual invariant, while the pseudo-welfarist NR DS-planner further preserves the dynamic and stochastic components used by the normalized welfarist planner.

We formalize the properties of these new planners for the components of the aggregate additive decomposition in Proposition 9. Table 1 summarizes its results.

Proposition 9. *(AE/AR/NR DS-planners: properties)*
a) For an AE DS-planner, the risk-sharing, intertemporal-sharing, and redistribution components of the aggregate additive decomposition are zero, that is, $\Xi_{RS} = \Xi_{IS} = \Xi_{RD} = 0$. The aggregate

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[21]It is straightforward to consider other AE DS-planners that are not pseudo-welfarist. For instance, one could choose the following weights:

$$\tilde{\omega}^{i,AE}\left(s_0\right) = 1, \quad \tilde{\omega}_t^{i,AE}\left(s_0\right) = \overline{\beta}^t, \text{ and } \tilde{\omega}_t^{i,AE}\left(s^t\mid s_0\right) = \pi_t\left(s^t\mid s_0\right),$$

for some $\overline{\beta}$, plausibly $\overline{\beta} = \int \beta_i di$. This is helpful because, in some applications, DS-planners that are not pseudo-welfarist may be easier to operationalize.

Table 1: New Welfare Criteria: Summary

DS-Planners	Ξ_{AE} Aggregate Efficiency	Ξ_{RS} Risk-sharing	Ξ_{IS} Intertemporal-sharing	Ξ_{RD} Redistribution
Aggregate Efficiency (AE)	✓	$=0$	$=0$	$=0$
Aggregate Efficiency/Risk-Sharing (AR)	✓	✓	$=0$	$=0$
No-Redistribution (NR)	✓	✓	✓	$=0$
Welfarist ($\mathcal{W}$)	✓	✓	✓	✓

Note: Table 1 summarizes the properties of the aggregate additive decomposition for the DS-planners introduced in Definition 4. These properties follow from Proposition 9.

efficiency component, Ξ_{AE}, is identical for a pseudo-welfarist AE DS-planner and its associated normalized welfarist planner.

b) For an AR DS-planner, the intertemporal-sharing and redistribution components of the aggregate additive decomposition are zero, that is, $\Xi_{IS} = \Xi_{RD} = 0$. The aggregate efficiency and risk-sharing components, Ξ_{AE} and Ξ_{RS}, are identical for a pseudo-welfarist AR DS-planner and its associated normalized welfarist planner.

c) For a NR DS-planner, the redistribution component of the aggregate additive decomposition is zero, that is, $\Xi_{RD} = 0$. The aggregate efficient, risk-sharing, and intertemporal-sharing components, Ξ_{AE}, Ξ_{RS}, and Ξ_{IS}, are identical for a pseudo-welfarist NR DS-planner and its associated normalized welfarist planner.

Proposition 9 shows that the new DS-planners, by making the individual components of DS-weights invariant across individuals, dates, and/or histories, are defined to directly exploit the properties of the aggregate additive decomposition characterized in Proposition 2. Moreover, the pseudo-welfarist planners are defined so as to exactly preserve the value of its components relative to the associated welfarist planner along the dimensions in which they are not zero. This is useful in practice because it allows us to interpret specific sums of the components the aggregate decomposition of a welfarist planner as the welfare assessment made by a pseudo-welfarist planner.

Given its practical importance, we formally state this result as Corollary 5.

Corollary 5. *(Pseudo-welfarist planners as components of welfarist aggregate additive decomposition) Particular sums of the components of the aggregate additive decomposition of welfare assessments for a given welfarist planner have the interpretation of welfare assessments for particular pseudo-welfarist DS-planners.*

Interestingly, it is not possible to define a new pseudo-welfarist planner for whom exclusively the risk-sharing and intertemporal-sharing components are zero, as we show in Section G.2 of the Online Appendix. To guarantee that $\Xi_{RS} = \Xi_{IS} = 0$, a planner would need $\omega_t^i(s_0)$ and $\tilde{\omega}_t^i(s^t | s_0)$ to be individual-invariant, which would interfere with ensuring that the value of Ξ_{RD} is the same as for a welfarist planner. A similar logic applies to other combinations of the different components.

Nonetheless, it is certainly possible to define new planners that are not pseudo-welfarist but that exclusively value aggregate efficiency and redistribution.

5.2 α-DS-planners

The new planners that we introduce in Definition 4 by no means exhaust the set of new planners that one can define using DS-weights. For instance, it is possible to define a new planner that spans i) AE, ii) AR, and iii) NR pseudo-welfarist planners, as well as iv) the associated normalized welfarist planner. We refer to this planner as an α-DS-planner.

Definition 5. *(α-DS-planner: definition) An α-DS-planner is a DS-planner for whom the individual, dynamic, and stochastic components of DS-weights are linear combinations of the components of a normalized welfarist planner and the component of an AE pseudo-welfarist planner. An α-DS-planner has DS-weights $\omega_t^{i,\mathcal{W},\alpha}\left(s^t \middle| s_0\right)$ defined by*

$$\tilde{\omega}_t^{i,\mathcal{W},\alpha}\left(s^t \middle| s_0\right) = (1 - \alpha_2)\,\tilde{\omega}_t^{i,\mathcal{W},AE}\left(s^t \middle| s_0\right) + \alpha_2\tilde{\omega}_t^{i,\mathcal{W}}\left(s^t \middle| s_0\right)$$

$$\tilde{\omega}_t^{i,\mathcal{W},\alpha}\left(s_0\right) = (1 - \alpha_3)\,\tilde{\omega}_t^{i,\mathcal{W},AE}\left(s_0\right) + \alpha_3\tilde{\omega}_t^{i,\mathcal{W}}\left(s_0\right)$$

$$\tilde{\omega}^{i,\mathcal{W},\alpha}\left(s_0\right) = (1 - \alpha_4)\,\tilde{\omega}^{i,\mathcal{W},AE}\left(s_0\right) + \alpha_4\tilde{\omega}^{i,\mathcal{W}}\left(s_0\right),$$

where $\boldsymbol{\alpha} = (\alpha_2, \alpha_3, \alpha_4)$, and where $\alpha_2 \in [0,1]$, $\alpha_3 \in [0,1]$, $\alpha_4 \in [0,1]$.

Depending on the value of $\boldsymbol{\alpha}$, an α-DS-planner behaves as a particular pseudo-welfarist planner or as a combination of pseudo-welfarist planners. In particular, as we show in Section G.2 of the Online Appendix, when $\boldsymbol{\alpha} = (0,0,0)$, we have an AE DS-planner; when $\boldsymbol{\alpha} = (1,0,0)$, we have an AR DS-planner; when $\boldsymbol{\alpha} = (1,1,0)$, we have a NR DS-planner; and when $\boldsymbol{\alpha} = (1,1,1)$, we have a welfarist planner.

By varying $\boldsymbol{\alpha}$, it is possible to model planners who care about the different components to different degrees. Moreover, estimating $\boldsymbol{\alpha}$ from actual policies in the context of a particular policy problem has the potential to uncover the weights that a particular policymaker attaches in practice to the different components of the aggregate additive decomposition. Dávila and Schaab (2022) leverage this observation to develop an "inverse optimum" approach in the context of monetary policy.

5.3 Paternalism and Institutional Design

In Figure 1, we summarize the relations between the different planners studied in this section. We conclude this section with two remarks.

Remark 1. *(Paternalistic vs. Non-paternalistic DS-planners; AE and AR planners are paternalistic)* It is important to highlight that AE and and AR DS-planners are paternalistic, in the sense that their welfare assessments do not take as an input changes in the lifetime utilities/valuations
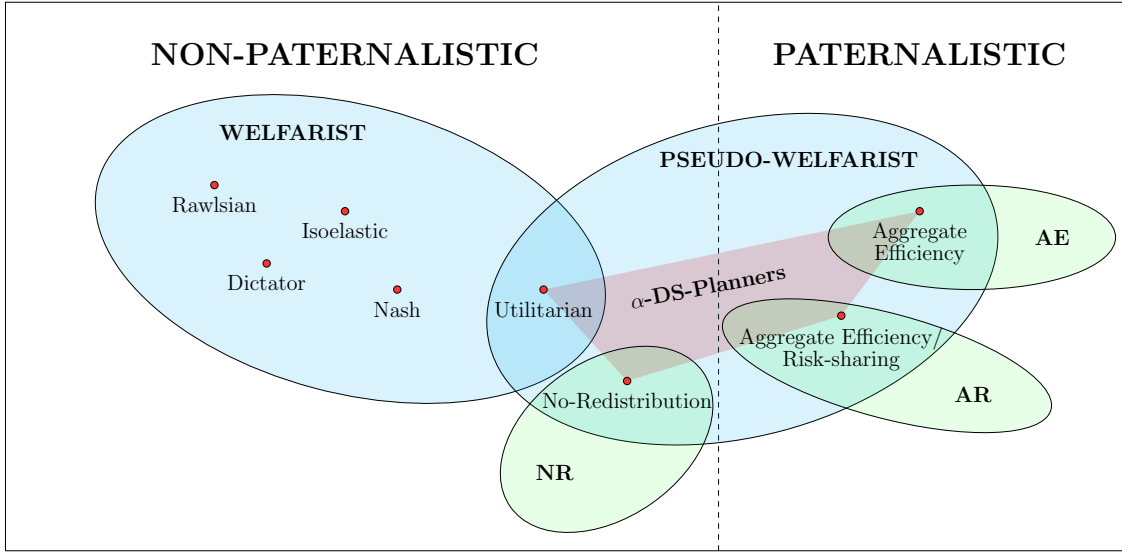
DS-PLANNERS

Figure 1: DS-Planners: Summary

Note: Figure 1 summarizes the relations between the different planners studied in Section 5 paper. The vertical dashed line separates non-paternalistic planners from paternalistic planners. All welfarist planners, as well as no-redistribution (NR) planners, are non-paternalistic. Aggregate efficiency (AE) and aggregate efficiency/risk-sharing (AR) planners are paternalistic. Some pseudo-welfarist planners are non-paternalistic (welfarist, NR), while others are paternalistic (AE, AR). In this figure, the $\boldsymbol{\alpha}$-DS-planners are pseudo-welfarist with respect to the utilitarian planner.

of individuals.[22] In these cases, a planner and an individual may have different assessments of whether a policy change is welfare improving or not for that individual. However, NR DS-planners are not paternalistic. Intuitively, the welfare assessments of any planner who respects individual preferences must value intertemporal-sharing and risk-sharing considerations as long as individuals do. Redistributional concerns are independent of whether a planner respects individuals' desires for interpersonal sharing. Therefore, if a planner wants to make welfare assessments that do not value intertemporal-sharing or risk sharing, such planner must necessarily be paternalistic.

Remark 2. *(Implications for policy mandates and institutional design)* The framework developed in this paper has the potential to guide the design of independent technocratic institutions. In practice, such institutions must be given a "mandate", much like defining a set of DS-weights. Therefore, a society may want to consider designing independent technocratic institutions that

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[22]As explained in Section G.3.1 of the Online Appendix, a non-paternalistic planner makes welfare assessments according to

$$\frac{dW^{NP}(s_0)}{d\theta} = \int \phi_i(s_0) \frac{dV_i(s_0)}{d\theta} di,$$

where $\phi_i(s_0)$ are functions of all possible paths of outcomes and where $\frac{dV_i(s_0)}{d\theta}$ is defined in Equation (2). The key distinction between a welfarist and a non-paternalistic planner is that, for welfarist planners $\phi_i(s_0)$ must take the particular form $\frac{\partial \mathcal{W}(\{V_i(s_0)\}_{i \in I})}{\partial V_i}$, where $\mathcal{W}(\cdot)$ is a SWF of the form described in Equation (4). Non-paternalistic planners can set $\phi_i(s_0)$ freely.

have some normative considerations in their mandate but not others, along the lines of the logic we have developed in this section. For instance, the current "dual mandate" (stable prices and maximum employment) of the Federal Reserve (as defined by the 1977 Federal Reserve Act) seems to be better described by an aggregate efficiency DS-planner, rather than a welfarist planner, which would care about cross-sectional considerations. Alternatively, an institution like the Federal Emergency Management Agency (FEMA) has as part of its mandate to "support the Nation in a risk-based, comprehensive emergency management system", which unavoidably involves risk-sharing considerations.

6 Additional Results

In this section, we include additional results. We first further decompose the components of the aggregate additive decomposition and then explain how to connect welfare assessments to measures of inequality. Next, we explain how to make welfare assessments using DS-weights in recursive environments, and show how to implement welfare assessments via an instantaneous Social Welfare Function. Finally, we briefly described additional results included in the Online Appendix.

6.1 Decomposing the components of the aggregate additive decomposition

Here, we further decompose and provide additional insights into the four components of the aggregate additive decomposition. For the aggregate efficiency and the redistribution components, we provide new stochastic decompositions. For the risk-sharing and intertemporal-sharing components, we provide alternative cross-sectional decompositions.

Aggregate efficiency (Ξ_{AE}). It is important to highlight that the aggregate efficiency component Ξ_{AE} includes aggregate valuation considerations. We formalize this insight by further decomposing the aggregate efficiency component of the aggregate additive decomposition into an expected aggregate efficiency component and an aggregate insurance component.

Proposition 10. *(Aggregate efficiency component: stochastic decomposition) The aggregate efficiency component of the aggregate additive decomposition, Ξ_{AE}, can be decomposed into i) an expected aggregate efficiency component, Ξ_{AE}, and ii) an aggregate insurance component, Ξ_{AE}, as follows:*

$$\Xi_{AE} = \underbrace{\sum_{t=0}^{T} \overline{\omega}_t\left(s_0\right) \mathbb{E}_0\left[\overline{\omega}_t^\pi\left(s^t \middle| s_0\right)\right] \mathbb{E}_0\left[\frac{d\overline{u}_{i|c}\left(s^t\right)}{d\theta}\right]}_{=\Xi_{AE} \text{ (Expected Aggregate Efficiency)}} \tag{23}$$
$$+ \underbrace{\sum_{t=0}^{T} \overline{\omega}_t\left(s_0\right) \mathbb{C}ov_0\left[\overline{\omega}_t^\pi\left(s^t \middle| s_0\right), \frac{d\overline{u}_{i|c}\left(s^t\right)}{d\theta}\right]}_{=\Xi_{AE} \text{ (Aggregate Insurance)}},$$

where we define $\overline{\omega}_t(s_0) = \mathbb{E}_i\left[\tilde{\omega}_t^i(s_0)\right]$, $\overline{\omega}_t^\pi(s^t|s_0) = \frac{\mathbb{E}_i\left[\tilde{\omega}_t^i(s^t|s_0)\right]}{\pi_t(s^t|s_0)}$, *and* $\frac{d\overline{u}_{i|c}(s^t)}{d\theta} = \mathbb{E}_i\left[\frac{du_{i|c}(s^t)}{d\theta}\right]$, *and where* $\mathbb{E}_0[\cdot]$ *and* $\mathbb{C}ov_0[\cdot,\cdot]$ *denote expectations and covariances conditional on* s_0.

The expected aggregate efficiency component, Ξ_{AE}, captures the discounted expectation over time and histories of the aggregate instantaneous consumption-equivalent effect of the policy change. The aggregate insurance component, Ξ_{AE}, captures whether aggregate efficiency gains take place in histories that a DS-planner values more in aggregate terms. It should be evident that aggregate insurance, Ξ_{AE}, based on aggregate covariances over histories, is logically different from the risk-sharing and intertemporal-sharing components, Ξ_{RS} and Ξ_{IS}, based on cross-sectional covariances.

In practical terms, the welfare gains associated with eliminating aggregate business cycles in a representative-agent economy, as in the policy experiment of Lucas (1987), fully arise from aggregate insurance considerations, that is, Ξ_{AE}. Note that both the expected aggregate efficiency and the aggregate insurance components incorporate discounting via $\overline{\omega}_t(s_0)$, so policy changes that front-load gains from expected aggregate efficiency or aggregate insurance are more desirable.

Risk-sharing and intertemporal-sharing components (Ξ_{RS} and Ξ_{IS}). While Propositions 2 through 4 establish desirable properties of the aggregate additive decomposition, it is possible to provide alternative formulations of the risk-sharing and intertemporal-sharing components. In Proposition 11 we further decompose the intertemporal-sharing component into a pure intertemporal-sharing component, a weight concentration component, and a policy-weights coskewness component. We also show a new identity that the sum of the risk-sharing and intertemporal-sharing components, $\Xi_{RS} + \Xi_{IS}$, must satisfy.

Proposition 11. *(Risk-sharing/intertemporal-sharing components: alternative cross-sectional decompositions)*
a) The intertemporal-sharing component of the aggregate additive decomposition, Ξ_{IS}, can be decomposed into i) a pure intertemporal-sharing component, Ξ_{IS}, ii) a weight concentration component, Ξ_{IS} and iii) a policy-weights coskewness component, Ξ_{IS} as follows:

$$\Xi_{IS} = \underbrace{\sum_{t=0}^T \sum_{s^t} \mathbb{E}_i\left[\tilde{\omega}_t^i\left(s^t\middle| s_0\right)\right] \mathbb{C}ov_i\left[\tilde{\omega}_t^i(s_0), \frac{du_{i|c}(s^t)}{d\theta}\right]}_{=\Xi_{IS}\ (Pure\ Intertemporal\text{-}sharing)}$$

$$+ \underbrace{\sum_{t=0}^T \sum_{s^t} \mathbb{C}ov_i\left[\tilde{\omega}_t^i(s_0), \tilde{\omega}_t^i\left(s^t\middle| s_0\right)\right] \mathbb{E}_i\left[\frac{du_{i|c}(s^t)}{d\theta}\right]}_{=\Xi_{IS}\ (Weight\ Concentration)}$$

$$+ \underbrace{\sum_{t=0}^T \sum_{s^t} \mathbb{E}_i\left[\left(\frac{du_{i|c}(s^t)}{d\theta} - \mathbb{E}_i\left[\frac{du_{i|c}(s^t)}{d\theta}\right]\right)\left(\tilde{\omega}_t^i(s_0) - \mathbb{E}_i\left[\tilde{\omega}_t^i(s_0)\right]\right)\left(\tilde{\omega}_t^i\left(s^t\middle| s_0\right) - \mathbb{E}_i\left[\tilde{\omega}_t^i\left(s^t\middle| s_0\right)\right]\right)\right]}_{=\Xi_{IS}\ (Policy\text{-}weights\ Coskewness)}.$$

$$(24)$$

b) The sum of the risk-sharing and the intertemporal-sharing components, $\Xi_{RS} + \Xi_{IS}$, can be decomposed into i) a weight concentration component, Ξ_{23}^C and ii) an interpersonal-sharing component, Ξ_{23}^I as follows:

$$\Xi_{RS} + \Xi_{IS} = \underbrace{\sum_{t=0}^{T} \sum_{s^t} \mathbb{C}ov_i \left[\tilde{\omega}_t^i (s_0), \tilde{\omega}_t^i \left(s^t \middle| s_0 \right) \right] \mathbb{E}_i \left[\frac{du_{i|c} (s^t)}{d\theta} \right]}_{= \Xi_{23}^{WC} \; (Weight \; Concentration)}$$

$$+ \underbrace{\sum_{t=0}^{T} \sum_{s^t} \mathbb{C}ov_i \left[\tilde{\omega}_t^i (s_0) \, \tilde{\omega}_t^i \left(s^t \middle| s_0 \right), \frac{du_{i|c} (s^t)}{d\theta} \right]}_{= \Xi_{23}^I \; (Interpersonal\text{-}sharing)}, \tag{25}$$

where $\Xi_{23}^{WC} = \Xi_{IS}$, defined above, and where $\Xi_{23}^I = \Xi_{RS} + \Xi_{IS} + \Xi_{IS}$.

The first component of Ξ_{IS} introduced in Proposition 11a), Ξ_{IS}, can be interpreted as capturing pure intertemporal-sharing considerations. The major difference between Ξ_{IS} and Ξ_{IS} is that the former is based on cross-sectional covariances of the dynamic component of DS-weights with the expected — interpreting the stochastic weights as probabilities — instantaneous consumption-equivalent effect of the policy at a given date. The latter, on the other hand, is based on the expectation of cross-sectional covariances of the dynamic component of DS-weights with the actual instantaneous consumption-equivalent effect of the policy. Formally, the difference between Ξ_{IS} and Ξ_{IS} is captured by the remaining two components, which we describe next.

The second component of Ξ_{IS} introduced in Proposition 11a), Ξ_{IS}, can be interpreted as capturing the welfare gain (loss) associated with policies that increase aggregate instantaneous consumption-equivalent when the dynamic and stochastic components of DS-weights are positively (negatively) correlated across individuals. While one may consider including Ξ_{IS} in the aggregate efficiency component, there are two good reasons not to do so. First, it would require knowledge of the cross-section of the dynamic and stochastic components of DS-weights, which goes against expressing the aggregate efficiency component exclusively as a function of aggregate statistics. Second, as implied by Proposition 6, for the case of welfarist planners, $\Xi_{IS} = 0$ when markets are complete. This fact highlights that Ξ_{IS} necessarily relies on imperfect insurance across individuals, which makes this term unsuitable to capture aggregate efficiency considerations.

The third component of Ξ_{IS} introduced in Proposition 11a), Ξ_{IS}, is exactly based on the coskewness between i) the dynamic component of DS-weights, ii) the stochastic component of DS-weights, and iii) the instantaneous consumption-equivalent effect of a policy. Coskewness is a measure of how much three random variables jointly change. For instance, note that Ξ_{IS} could be non-zero even when $\mathbb{C}ov_i \left[\tilde{\omega}_t^i (s_0), \tilde{\omega}_t^i \left(s^t \middle| s_0 \right) \right] = 0$ and, consequently, $\Xi_{23}^{WC} = 0$. Also, coskewness is zero when the random variables are multivariate normal (Bohrnstedt and Goldberger, 1969), so it relies on higher-order moments.[23] Note also that if one of $\tilde{\omega}_t^i (s_0)$, $\tilde{\omega}_t^i \left(s^t \middle| s_0 \right)$, or $\frac{du_{i|c}(s^t)}{d\theta}$ is constant across all

---

[23]We expect these terms to be in important in models that emphasize higher moments of the distribution of individual
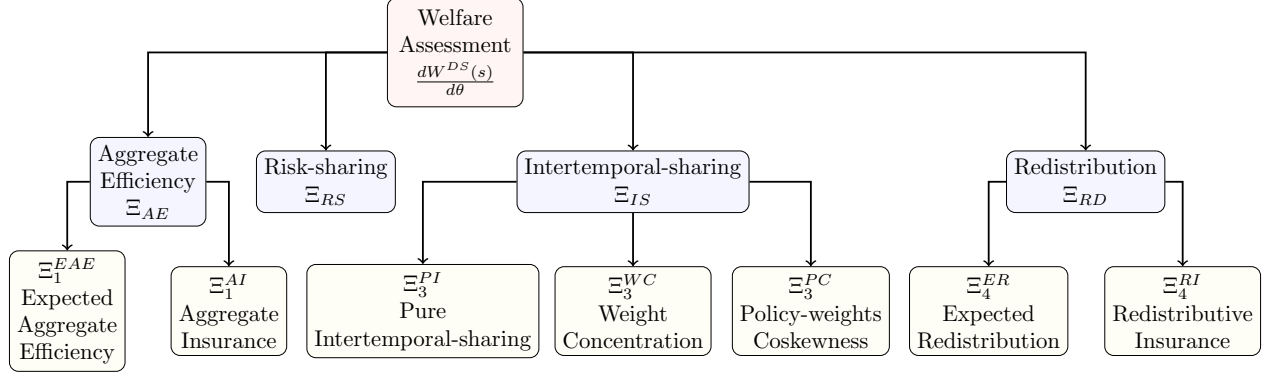
Figure 2: Aggregate additive decomposition

Note: Figure 2 illustrates the aggregate additive decomposition of welfare assessments for a general DS-planner, and how its four components can be further decomposed. See Propositions 1, 10, 11, and 12 for formal definitions of each of the terms.

individuals, then $\Xi_{23}^{WC} = 0$.

Proposition 11b) simply provides an alternative decomposition of the sum of risk-sharing and intertemporal-sharing. Its first component is exactly the weight concentration component just described, $\Xi_{23}^{WC} = \Xi_{IS}$, while the second component corresponds to the sum of risk-sharing, Ξ_{RS}, pure intertemporal-sharing, Ξ_{IS}, and policy-weights coskewness, Ξ_{IS}. At times, this alternative decomposition may provide additional insights relative to the one in Proposition 1.

Redistribution component (Ξ_{RD}). Similarly to the aggregate efficiency component, the redistribution component Ξ_{RD} is shaped by valuation considerations, in this case at the individual level. Here, we decompose the redistribution component of the aggregate additive decomposition into an expected redistribution component and a redistributive insurance component.

Proposition 12. *(Redistribution component: stochastic decomposition) The redistribution component of the aggregate additive decomposition, Ξ_{RD}, can be decomposed into i) an expected redistribution component, Ξ_{RD}, and ii) a redistributive insurance component, Ξ_{RD}, as follows:*

$$
\Xi_{RD} = \underbrace{\mathbb{C}ov_i\left[\tilde{\omega}^i\left(s^0\right), \sum_{t=0}^{T}\tilde{\omega}_t^i(s_0)\,\mathbb{E}_0\left[\tilde{\omega}_t^{i,\pi}\left(s^t\middle|s_0\right)\right]\mathbb{E}_0\left[\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]\right]}_{=\Xi_{RD}\ (Expected\ Redistribution)}
$$
$$
+ \underbrace{\mathbb{C}ov_i\left[\tilde{\omega}^i\left(s^0\right), \sum_{t=0}^{T}\tilde{\omega}_t^i(s_0)\,\mathbb{C}ov_0\left[\tilde{\omega}_t^{i,\pi}\left(s^t\middle|s_0\right), \frac{du_{i|c}\left(s^t\right)}{d\theta}\right]\right]}_{=\Xi_{RD}\ (Redistributive\ Insurance)},
$$

where we define $\overline{\omega}_t^{i,\pi}\left(s^t\middle|s_0\right) = \frac{\tilde{\omega}_t^i\left(s^t\middle|s_0\right)}{\pi_t\left(s^t\middle|s_0\right)}$, and where $\mathbb{E}_0\left[\cdot\right]$ and $\mathbb{C}ov_0\left[\cdot,\cdot\right]$ denote expectations and covariances conditional on s_0.

risks (e.g., Guvenen, Ozkan and Song (2014)).

The expected redistribution component, Ξ_{RD}, captures the perceived gains for a DS-planner from changes in the expected instantaneous consumption-equivalent effect of the policy change. When individuals with a high individual component of DS-weights, $\tilde{\omega}^i\left(s^0\right)$, have higher expected instantaneous consumption-equivalent effect, a planner attributes this to the redistribution component. The redistributive insurance component, Ξ_{RD}, captures whether individual gains from the policy change take place in histories that are more desirable for individuals with higher individual component of DS-weights, $\tilde{\omega}^i\left(s^0\right)$. In practical terms, the redistributive insurance component will be non-zero when a policy improves individual insurance for individuals with a higher individual component of DS-weights.[24]

6.2 Inequality, bounds, and welfare assessments

Concerns related to inequality often take a prominent role when assessing policies. Our aggregate additive decomposition provides direct insights into which particular forms of inequality matter for the determination of aggregate welfare assessments and each of their components. Formally, in Proposition 13, we provide bounds for the risk-sharing component, the intertemporal-sharing component, and the redistribution component defined in Proposition 1 based on the cross-sectional dispersion of DS-weights and policy effects.[25] These bounds are helpful in practice because they can be computed using univariate statistics, i.e., cross-sectional standard deviations, and do not require the joint distribution of DS-weights and normalized consumption-equivalent effects, which are necessary to compute cross-sectional covariances (a multivariate statistic).

Proposition 13. *(Cross-sectional dispersion bounds) The value of the risk-sharing, the intertemporal-sharing, and the redistribution components defined in Proposition 1 satisfy the following bounds:*

$$\left|\Xi_{RS}\right| \leq \sum_{t=0}^{T} \mathbb{E}_i\left[\tilde{\omega}_t^i\left(s^0\right)\right] \sum_{s^t} \mathbb{SD}_i\left[\tilde{\omega}_t^i\left(s^t\middle| s_0\right)\right] \times \mathbb{SD}_i\left[\frac{du_{i|c}\left(s^t\right)}{d\theta}\right] \tag{26}$$

$$\left|\Xi_{IS}\right| \leq \sum_{t=0}^{T} \mathbb{SD}_i\left[\tilde{\omega}_t^i\left(s^0\right)\right] \times \mathbb{SD}_i\left[\sum_{s^t} \tilde{\omega}_t^i\left(s^t\middle| s_0\right) \frac{du_{i|c}\left(s^t\right)}{d\theta}\right] \tag{27}$$

$$\left|\Xi_{RD}\right| \leq \mathbb{SD}_i\left[\tilde{\omega}^i\left(s^0\right)\right] \times \mathbb{SD}_i\left[\sum_{t=0}^{T} \tilde{\omega}_t^i\left(s^0\right) \sum_{s^t} \tilde{\omega}_t^i\left(s^t\middle| s_0\right) \frac{du_{i|c}\left(s^t\right)}{d\theta}\right], \tag{28}$$

where $\mathbb{SD}_i\left[\cdot\right]$ denotes a cross-sectional standard deviation.

Proposition 13 shows that the magnitude of each of the three components considered here is determined (bounded above) by i) the cross-sectional dispersion of the different components

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[24]Note that the redistribution component, Ξ_{RD}, can be positive or negative for Pareto-improving policies. This can occur if different individuals are differentially affected by the policy and if a DS-planner has different individual multiplicative components for different individuals.

[25]It should be clear that cross-sectional variances and standard deviations can only bound the welfare effect of policies. Equation (11) shows that cross-sectional covariances exactly determine each of the components of the aggregate additive decomposition.

of DS-weights, $\mathbb{SD}_i\left[\tilde{\omega}_t^i\left(s^t\middle|s^0\right)\right]$, $\mathbb{SD}_i\left[\tilde{\omega}_t^i\left(s^0\right)\right]$, and $\mathbb{SD}_i\left[\tilde{\omega}^i\left(s^0\right)\right]$, as well as ii) the cross-sectional dispersion of the instantaneous consumption-equivalent effect of the policy, effectively $\mathbb{SD}_i\left[\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]$. Consequently, inequality considerations do matter for the aggregate assessments of policies via the cross-sectional dispersion of DS-weights or the impact of a policy by itself.

Proposition 13 is helpful for three reasons. First, it shows that normative criteria with highly dispersed DS-weights have the potential to generate a large welfare effect of policies via risk-sharing, intertemporal-sharing and redistribution. Second, by computing the cross-sectional dispersion of the different components of DS-weights for a given criterion, it shows that it is possible to understand the potential scope that inequality may play when determining the risk/intertemporal-sharing and redistribution components of aggregate welfare assessments. Finally, Proposition 13 shows that the risk-sharing, intertemporal-sharing and redistribution components depend on the extent to which which policies impact different individuals differently. That is, the more $\frac{du_{i|c}\left(s^t\right)}{d\theta}$ varies across individuals, dates, and/or histories, the more likely dispersion in DS-weights matters for welfare assessments.

6.3 Recursive formulation

Up to now, we have defined DS-weights for a sequence formulation of a dynamic stochastic economy. Here, we describe how to operationalize DS-weights in recursive environments, which are widely used in practice. As in Ljungqvist and Sargent (2018), we denote possible recursive states by s and s'.[26]

Proposition 14. *(Recursive formulation) Suppose that individual consumption and hours are exclusively a function of the current realization of s_t and do not depend on the full history leading to those outcomes, so that $c_t^i\left(s^t\right) = c^i\left(s_t\right) = c^i\left(s\right)$ and $n_t^i\left(s^t\right) = n^i\left(s_t\right) = n^i\left(s\right)$. Then, it is possible to express $\frac{dW^{DS}(s_0)}{d\theta}$, as defined in Equation (7), as follows:*

$$\frac{dW^{DS}\left(s_0\right)}{d\theta} = \int \omega_0^i\left(s^0|s_0\right)\frac{d\hat{V}_{i,0}^{DS}\left(s_0\right)}{d\theta}di, \tag{29}$$

where $\frac{d\hat{V}_{i,t}^{DS}(s)}{d\theta}$ has the following recursive representation:

$$\frac{d\hat{V}_{i,t}^{DS}\left(s\right)}{d\theta} = \frac{du_{i|c}\left(s\right)}{d\theta} + \hat{\beta}_{i,t}\sum_{s'}\hat{\pi}_{i,t}\left(s'|s\right)\frac{d\hat{V}_{i,t+1}^{DS}\left(s'\right)}{d\theta}, \tag{30}$$

---

[26]Note that in recursive economies individuals with idiosyncratic (and potentially aggregate) states (i.e., Aiyagari or Krusell-Smith style economies) individuals can be ex-ante heterogeneous at the time of making a welfare assessment for two different reasons. First, individuals can be different because of predetermined reasons (e.g., individuals have different time-invariant preferences or face shocks that come from different distributions). Second, individuals can be different because they are at a different idiosyncratic state (e.g., individuals have different endowments or asset holdings at the time of the welfare assessment, even though they face identical problems starting from a given idiosyncratic state). This is an important observation to interpret correctly some of the results in this paper. For instance, Corollary 3 of Proposition 2 only applies when all individuals are identical because of predetermined reasons and when they all have the same initial state. Obviously, ex-post, individuals will also be heterogeneous if they experience different shocks. In the notation used in this section, ex-ante heterogeneity of either form is captured by the index i.

where $\hat{\beta}_{i,t}$ and $\hat{\pi}_{i,t}(s'|s)$ correspond to a twisted discount factor and a twisted set of transition probabilities of the form:

$$\hat{\beta}_{i,t} = \frac{\tilde{\omega}_{t+1}^i(s_0)}{\tilde{\omega}_t^i(s_0)} \qquad and \qquad \hat{\pi}_{i,t}(s'|s) = \frac{\tilde{\omega}_{t+1}^i(s'|s_0)}{\tilde{\omega}_t^i(s|s_0)}. \tag{31}$$

For Equation (30) to be a valid recursive representation, it must be that $\hat{\beta}_{i,t}$ is exclusively a function of time and s_0 and that $\hat{\pi}_{i,t}(s'|s)$ is exclusively a function of time, s, and s_0, but not of the full histories.

Proposition 14 shows that, in order to make a welfare assessment at a state s_0, a DS-planner must compute the date-0 DS-weights for all individuals, $\omega_0^i(s^0|s_0)$, as well as the value of $\frac{d\hat{V}_{i,0}^{DS}(s_0)}{d\theta} = \frac{\frac{dV_i^{DS}(s_0)}{d\theta}}{\tilde{\omega}_0^i(s_0)\tilde{\omega}_0^i(s^0|s_0)}$, which can be computed recursively following Equation (30). Intuitively, it is possible to find a recursive representation for $\frac{d\hat{V}_{i,t}^{DS}(s)}{d\theta}$, because it is expressed in units of consumption good at state s. In fact, $\frac{d\hat{V}_{i,t}^{DS}(s)}{d\theta}$ has the interpretation of an asset pricing equation for an asset that pays $\frac{du_{i|c}(s)}{d\theta}$ units of consumption good to individual i in state s.

It is worth highlighting that the set of DS-weights that admits a recursive representation is smaller than the set of DS-weights that can be expressed non-recursively. In particular, $\hat{\beta}_{i,t}$ and $\hat{\pi}_{i,t}(s'|s)$, which are ratios components of the individual decomposition of DS-weights cannot depend on histories, although they may be time-dependent. Interestingly, even in a fully recursive economy, the recursive representation of $\frac{d\hat{V}_{i,t}^{DS}(s)}{d\theta}$ is typically time-dependent, because the state in which the welfare assessment takes place will anchor the future values of the dynamics and stochastic components of the individual multiplicative decomposition for a DS-planner. Only in particular cases it is possible to find a time-independent recursive representation, as we discuss next.

As we show in the Online Appendix, when $\pi(s'|s)$ is Markov, we can express $\hat{\beta}_{i,t}$ and $\hat{\pi}_{i,t}(s'|s)$ for a normalized welfarist planners as follows:

$$\hat{\beta}_{i,t}^{\mathcal{W}} = \beta_i \underbrace{\frac{\sum_{s'}\pi_{t+1}(s'|s_0)\frac{\partial u_i(s')}{\partial c^i}}{\sum_s \pi_t(s|s_0)\frac{\partial u_i(s)}{\partial c^i}}}_{=\text{dynamic correction}} \equiv \frac{1}{R_{i,t}^f} \quad and \quad \hat{\pi}_{i,t}^{\mathcal{W}}(s'|s) = \pi(s'|s)\underbrace{\frac{\frac{\frac{\partial u_i(s')}{\partial c^i}}{\sum_{s'}\pi_{t+1}(s'|s_0)\frac{\partial u_i(s')}{\partial c^i}}}{\frac{\frac{\partial u_i(s)}{\partial c^i}}{\sum_s \pi_t(s|s_0)\frac{\partial u_i(s)}{\partial c^i}}}}_{=\text{stochastic correction}} \equiv \pi_{i,t}^{\star}(s'|s).$$

$$\tag{32}$$

In this case, Equation (30) can be literally interpreted as a cum-dividend asset pricing equation, since $\hat{\beta}_{i,t}^{\mathcal{W}} \equiv 1/R_{i,t}^f$ has the interpretation of individual i's one-period forward rate between dates t and $t+1$, and $\hat{\pi}_{i,t}^{\mathcal{W}}(s'|s) \equiv \pi_{i,t}^{\star}(s'|s)$ has the interpretation of individual i's risk-neutral probability between dates t and $t+1$. As we show in the Online Appendix, Equation (30) is time-independent for normalized welfarist planners and NR pseudo-welfarist planners.[27] However, Equation (30) is time-dependent for AR and AE pseudo-welfarist planners. In our application, which we formulate

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[27]Note that the product $\hat{\beta}_i^{\mathcal{W}}(s) \cdot \hat{\pi}_i^{\mathcal{W}}(s'|s)$ corresponds to the state-price assigned at state s by individual i to state

recursively, we further illustrate how to use DS-weights in recursive environments.

6.4 Instantaneous SWF formulation

As explained in Section 2.2, the conventional approach to making welfare assessments relies on defining a Social Welfare Function that takes individual lifetime utilities as arguments. In this paper, we have shown that an approach based on generalized marginal DS-weights defined over instantaneous consumption-equivalents allows us to consider a larger class of normative objectives. In this section, we show that it is possible to interpret $\frac{dW^{DS}(s_0)}{d\theta}$, defined in Equation (7), as the first-order condition of a planner with a particular Social Welfare Function that i) takes as arguments individuals' instantaneous utilities, not lifetime utilities, and ii) features generalized (endogenous) welfare weights.

Formally, a linear instantaneous Social Welfare Function, which we denote by $\mathcal{I}(\cdot)$, is a linear function of individuals' instantaneous utilities, given by

$$\mathcal{I}\left(\left\{u_i\left(c_t^i\left(s^t\right), n_t^i\left(s^t\right)\right)\right\}_{i,t,s^t}\right) = \int \sum_{t=0}^{T} \sum_{s^t} \lambda_t^i\left(s^t\right) u_i\left(c_t^i\left(s^t\right), n_t^i\left(s^t\right)\right) di, \tag{33}$$

where the instantaneous Pareto weights $\lambda_t^i\left(s^t\right)$ define scalars that are individual-, date-, and history-specific.[28] Proposition 15 shows that welfare assessments made under DS-weights correspond to the first-order condition of a planner whose objective function is given by a particular linear instantaneous SWF. It also shows that any local optimum can be found as the first-order condition of a planner who maximizes a linear ISWF, where DS-weights are evaluated at the optimum.

Proposition 15. *(Linear instantaneous SWF formulation) For any set of DS-weights, there exist instantaneous Pareto weights $\left\{\lambda_t^i\left(s^t\right)\right\}_{i,t,s^t}$ such that $\frac{dW^{DS}(s_0)}{d\theta}$, defined in Equation (7), corresponds to the first-order condition of a planner who maximizes a linear instantaneous SWF $\mathcal{I}(\cdot)$ with instantaneous Pareto weights $\lambda_t^i\left(s^t\right) = \omega_t^i\left(s^t;\theta\right)/\frac{\partial u_i\left(s^t;\theta\right)}{\partial c_t^i}$. Moreover, at a local optimum, in which $\frac{dW^{DS}(s_0)}{d\theta} = 0$, there exist instantaneous Pareto weights $\left\{\lambda_t^i\left(s^t\right)\right\}_{i,t,s^t}$ such that the optimal policy satisfies the first-order condition formula of a linear instantaneous SWF $\mathcal{I}(\cdot)$, defined in Equation (33). The instantaneous Pareto weights in that case are evaluated at the optimum, so $\lambda_t^i\left(s^t\right) = \omega_t^i\left(s^t;\theta^\star\right)/\frac{\partial u_i\left(s^t;\theta^\star\right)}{\partial c_t^i}$, where $\theta^\star$ denotes the value of θ at the local optimum.*

Proposition 15 is helpful because it shows how to reverse-engineer Pareto weights of a linear

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s':

$$\hat{\beta}_{i,t}^{\mathcal{W}} \cdot \hat{\pi}_{i,t}^{\mathcal{W}}\left(s'\,\middle|\,s\right) = \beta_i \pi\left(s'\,\middle|\,s\right) \frac{\partial u_i\left(s'\right)}{\partial c^i}/\frac{\partial u_i\left(s\right)}{\partial c^i}.$$

[28] At times, it may be more convenient to define a linear instantaneous SWF $\mathcal{I}(\cdot)$ as follows:

$$\mathcal{I}\left(\left\{u_i\left(c_t^i\left(s^t\right), n_t^i\left(s^t\right)\right)\right\}_{i,t,s^t}\right) = \int \sum_{t=0}^{T} \sum_{s^t} (\beta_i)^t \pi_t\left(s^t\,\middle|\,s_0\right) \lambda_t^i\left(s^t\right) u_i\left(c_t^i\left(s^t\right), n_t^i\left(s^t\right)\right) di.$$

Both formulations are fully exchangeable in the baseline environment considered in this paper.

instantaneous SWF from DS-weights, while guaranteeing that any local optimum can be interpreted as the solution to the maximization of a particular linear instantaneous SWF. Because the instantaneous Pareto weights $\lambda_t^i(s^t)$ are evaluated at the optimum $\theta^\star$, they are taken as fixed in the maximization of an linear instantaneous SWF. In practice, it is impossible to define the instantaneous Pareto weights $\lambda_t^i(s^t)$ without first having solved for the optimum using our approach that starts with DS-weights as primitives. Relatedly, it is typically impossible to translate DS-weights into instantaneous Pareto weights that are invariant to θ and the rest of the environment.[29]

6.5   Summary of additional results

In Section G of the Online Appendix, we discuss additional results. First, we provide a systematic dimensional analysis of DS-weights and its components, illustrating why the choice of units if critical to make meaningful welfare assessments. Second, we expand on how the approach that we develop in this paper relates to other approaches used to make welfare assessments. In particular, we revisit different welfarist SWF's, we describe how our results relate to Saez and Stantcheva (2016) and the Kaldor (1939)/Hicks (1939) compensation principle, we show how the consumption-equivalent approach of Lucas (1987)/Alvarez and Jermann (2004) can be seen as using a particular set of DS-weights that are related to the DS-weights used by welfarist planners but do not allow for aggregation, and we explain how allowing for transfers can be interpreted as restricting or partially selecting a set of DS-weights. Finally, we explain how to make use of DS-weights in optimal policy problems using both primal and dual methods, and discuss how to make global welfare assessments.

7   Application: Transfer Policies under Incomplete Markets

In this section, we illustrate how to make welfare assessments using DS-weights in a fully specified application. The purpose of this application is to illustrate the mechanics of our approach in a tractable dynamic stochastic environment.

After defining a common economic environment, we consider two different scenarios. Scenario 1 corresponds to an economy in which individuals with identical preferences face idiosyncratic risk. In this case, we consider transfer policies that can move the economy towards full insurance. Scenario 2 corresponds to an economy in which individuals with different preferences face aggregate risk. In this case, we consider transfer policies that shift aggregate risk to the more risk-tolerant investors. In both scenarios, we carefully explain the channels through which normalized welfarist planners find such policies desirable or not.

Common Environment.   We consider an economy with two types of individuals (individuals, for short), with each corresponding to half of the population. Both individuals have time-separable

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[29]For the purpose of showing that it is possible to define a DS-planner via a well-defined SWF with generalized (endogenous) weights, it is sufficient to consider *linear* instantaneous SWF's. There is scope to explore further the welfare implications of using more general instantaneous SWF, or even SWF's directly defined over consumption, hours, or other commodities.

Table 2: Summary of scenarios

	Uncertainty	Preferences	Endowment $y^i(s)$		Policy $T^i(s)$		Consumption $c^i(s)$	
			$y^1(s)$	$y^2(s)$	$T^1(s)$	$T^2(s)$	$c^1(s)$	$c^2(s)$
#1	Idiosyncratic	Common	$\bar{y}+\varepsilon(s)$	$\bar{y}-\varepsilon(s)$	$-\varepsilon(s)$	$\varepsilon(s)$	$\bar{y}+\varepsilon(s)(1-\theta)$	$\bar{y}-\varepsilon(s)(1-\theta)$
#2	Aggregate	Heterogeneous	$\bar{y}+\varepsilon(s)$	$\bar{y}+\varepsilon(s)$	$-\varepsilon(s)$	$\varepsilon(s)$	$\bar{y}+\varepsilon(s)(1-\theta)$	$\bar{y}+\varepsilon(s)(1+\theta)$

Note: Instantaneous utility for both investors is given by $u_i(c) = \frac{c^{1-\gamma_i}}{1-\gamma_i}$. Our benchmark parameterization is given by $\beta = 0.975$, $\bar{y} = 1$, $\varepsilon(H) = 0.25$, $\varepsilon(L) = -0.25$, and $\rho = 0.95$. If preferences are common, $\gamma_1 = \gamma_2 = 2$. If preferences are heterogeneous, we assume that individual 1 is more risk averse, so $\gamma_1 > \gamma_2$, where $\gamma_1 = 5$ and $\gamma_2 = 2$.

constant relative risk aversion (CRRA) preferences with exponential discounting. We formulate individual lifetime utility recursively as follows:

$$V_i(s) = u_i\left(c^i(s)\right) + \beta \sum_{s'} \pi(s'|s) V_i(s'), \quad \text{where} \quad u_i(c) = \frac{c^{1-\gamma_i}}{1-\gamma_i},$$

where $V_i(s)$ and $c^i(s)$ respectively denote the lifetime utility and the consumption of individual i in a given state s; s and s' denote possible states, and $\pi(s'|s)$ is a Markov transition matrix, described below; β is a discount factor, equal for both individuals; and $u_i(c)$ denotes the instantaneous utility function of an individual i. A higher CRRA coefficient γ_i is mechanically associated with a lower willingness to substitute consumption intertemporally.

There is a single nonstorable consumption good (dollar), which serves as numeraire. We consider a extreme form of incomplete markets: no financial markets. Hence, in the absence of policy transfers, individuals consume their endowments. The consumption of individual i at state s is given by their endowment $y^i(s)$, and a transfer, $\theta T^i(s) \gtreqless 0$, where $\theta \in [0,1]$ scales the size of the transfers at all dates/states. Hence, the budget constraint of individual i in state s is given by

$$c^i(s) = y^i(s) + \theta T^i(s), \tag{34}$$

where the form of $y^i(s)$ and $T^i(s)$ varies in each scenario considered. Given the lack of financial markets, the equilibrium definition is trivial, so Equation (34) also defines equilibrium consumption for individual i. We further assume that the transfers net out in the aggregate, so $T^1(s) + T^2(s) = 0$. This assumption will immediately imply that aggregate efficiency is 0 for any policy.

Uncertainty in this economy is captured by a two-state Markov chain, with states denoted by $s = \{L, H\}$, standing for a low (L) and a high (H) realization of $y^1(s)$ (for individual 1) and a transition matrix given by

$$\Pi = \begin{pmatrix} \rho & 1-\rho \\ 1-\rho & \rho \end{pmatrix},$$

where $\rho \in [0,1]$. Table 2 summarizes the assumptions on $y^i(s)$ and $T^i(s)$ made in each scenario. In

this model, since $\frac{du_{i|c}(s^t)}{d\theta} = T^i(s)$, welfare assessments are simply given by

$$\frac{dW^{DS}(s_0)}{d\theta} = \int \sum_{t=0}^{\infty} \sum_{s^t} \omega_t^i \left(s^t \middle| s_0 \right) T^i(s) \, di. \tag{35}$$

7.1   Scenario 1: Idiosyncratic risk, homogeneous preferences

Environment.   In our first scenario, we assume i) that both individuals have identical preferences, so $\gamma_1 = \gamma_2 = \gamma$, and ii) that they exclusively face idiosyncratic risk. Formally, we assume that

$$y^1(s) = \overline{y} + \varepsilon(s) \quad \text{and} \quad y^2(s) = \overline{y} - \varepsilon(s),$$

where $\overline{y} > 0$, and where $\varepsilon(L) = -\varepsilon(H)$. We consider the welfare assessment of a full-insurance transfer policy. Formally, we set $T^1(s) = -\varepsilon(s)$ and $T^2(s) = \varepsilon(s)$, so individual consumption takes the form

$$c^1(s) = \overline{y} + \varepsilon(s)(1-\theta) \quad \text{and} \quad c^2(s) = \overline{y} - \varepsilon(s)(1-\theta).$$

Under this policy, when $\theta = 1$, both individuals are fully insured. Note that aggregate consumption does not depend on s or θ since $\int c^i(s) \, di = \overline{y}$.

Results.   We adopt the following parameters: $\beta = 0.95$, $\overline{y} = 1$, $\varepsilon(H) = 0.25$, $\varepsilon(L) = -0.25$, and $\gamma_1 = \gamma_2 = 2$. Importantly, we make the endowment processes persistent, by setting $\rho = 0.975$ as our benchmark. In Figure 4, we compare how welfare assessments change when the endowment process is extremely persistent ($\rho = 0.999$) and fully transitory ($\rho = 0.5$).[30] As a benchmark, we consider a normalized utilitarian planner with equal weights. In Figure 5 we compare how welfare assessments change when we consider a normalized isoelastic planner.

Individual multiplicative decomposition of DS-weights.   In Figure 3, we start by showing the components of the individual multiplicative decomposition of DS-weights for a normalized utilitarian planner for each of the individuals when $\theta = 0.25$. Several insights emerge.

First, Figure 3 clearly illustrates that the DS-weights have time-dependence, despite the fact that we consider a model that is recursive and stationary. This occurs because the shocks are persistent.

Second, the plots of the dynamic components show that a normalized utilitarian planner overweights earlier periods for those individuals who initially have a low endowment/high marginal utility. As reference we include the value of $(1-\beta)\beta^t = \beta^t/\sum_{t=0}^{\infty}\beta^t$, which corresponds to the dynamic weight for a hypothetical individual with linear marginal utility, i.e., when $u_i'(c^i(s)) = 1$. Importantly, since dynamic weights must add up to 1 over time, overweighting initial periods for individuals with low endowment/higher marginal utility necessarily implies underweighting periods later in the future.

Third, the plots of the stochastic components show that a normalized utilitarian planner initially overweights those states which are more likely given the initial state, although eventually the impact

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[30]We use $\rho = 0.999$ so it makes for an easier illustration of the results. We could have used $\rho = 1$ instead.
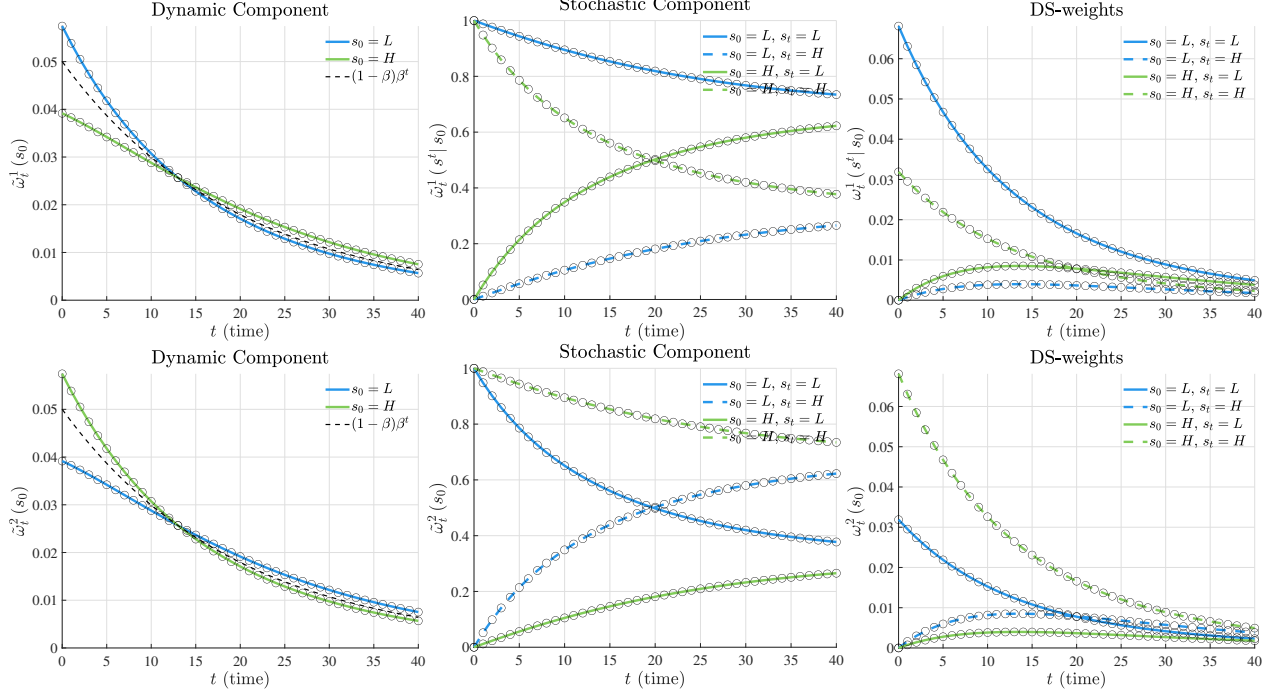
Figure 3: Individual multiplicative decomposition of DS-weights (Scenario 1)

Note: Figure 3 shows the components of the individual multiplicative decomposition of DS-weights for a normalized utilitarian planner, defined in Proposition 5. We assume that $\theta = 0.25$, although all figures are qualitatively similar when $\theta \in [0, 1)$. The top row shows each of the components for individual 1, while the bottom row shows them for individual 2. The left plots show the dynamic component, $\tilde{\omega}_t^i (s_0)$, for different values of t for different initial states, $s_0 = \{H, L\}$. For reference, we also show the dynamic weight for a hypothetical individual with linear marginal utility, given by $(1 - \beta) \beta^t = \beta^t / \sum_t \beta^t$. Note that the sum under each of the curves adds up to 1. The middle plots show the stochastic component, $\tilde{\omega}_t^i \left(s^t \middle| s_0 \right)$, for different values of t, for different initial states, $s_0 = \{H, L\}$, and for different final states, $s_t = \{H, L\}$. The right plots show the actual DS-weights, $\omega_t^i \left(s^t \middle| s_0 \right)$, also for different values of t, and different initial and final states: $s_0 = \{H, L\}$ and $s_t = \{H, L\}$. The parameters are $\theta = 0.25$, $\beta = 0.95$, $\bar{y} = 1$, $\varepsilon (H) = 0.25$, $\varepsilon (L) = -0.25$, $\rho = 0.975$, and $\gamma_1 = \gamma_2 = 2$. The individual component of DS-weights are $\tilde{\omega}^1 (s_0 = L) = 1.186$ and $\tilde{\omega}^2 (s_0 = L) = 0.814$ when an assessment takes place at $s_0 = L$; and $\tilde{\omega}^1 (s_0 = H) = 0.814$ and $\tilde{\omega}^2 (s_0 = H) = 1.186$ when the assessment takes place at $s_0 = H$.

of the initial state dissipates. More importantly, in the long run (although also in the short run), regardless of the initial state, the stochastic components are higher for those states in which an individual has a lower endowment/higher marginal utility.

Fourth, the individual components of the DS-weights further capture the differences in the marginal valuation of transfers among individuals for different initial states. A normalized utilitarian planner values a hypothetical permanent transfer at all dates/states towards the individual with low endowment at s_0 at 1.186, and towards the individual with a high endowment at 0.814. The plot of DS-weights simply combines multiplicatively the dynamic, stochastic, and individual components just discussed.

Aggregate additive decomposition of welfare assessments. In Figure 3, we show the components of the aggregate additive decomposition of welfare assessments for a normalized utilitarian planner for three different parametrizations: $\rho = \{0.5, 0.975, 0.999\}$. We exclusively consider the initial state

$s_0 = L$ since the aggregate welfare assessments are identical in both states. A different set of insights emerge from the aggregate additive decomposition.

First, as formally shown in Proposition 3, the aggregate efficiency component is zero, that is, $\Xi_{AE} = 0$. This occurs because we study an endowment economy for which aggregate consumption is invariant to the policy.

Second, a normalized utilitarian planner always finds it optimal to increase transfers until $\theta = 1$, which corresponds to full-insurance. Moreover, we show that all three remaining motives, risk-sharing, intertemporal-sharing, and redistribution contribute qualitatively to that conclusion. Hence, in this scenario, all pseudo-welfarist planners would agree on an optimal policy of $\theta^\star = 1$. When $\theta = 1$, markets are effectively complete, which implies that both risk and intertemporal-sharing components are zero, that is, $\Xi_{RS} = \Xi_{IS} = 0$. This is consistent with Propositions 6 and 7. When $\theta = 1$, both individuals have identical consumption paths, so $\Xi_{RD} = 0$. This is consistent with Corollary 3.

Third, the nature of endowment shocks, in particular whether such shocks are transitory or permanent, has a significant impact on the aggregate additive decomposition of welfare assessments. When shocks are transitory ($\rho = 0.5$), the planner attributes most of the welfare gains to risk-sharing, with intertemporal sharing playing a much smaller role and redistribution being virtually zero. When shocks are persistent ($\rho = 0.975$), part of the welfare gains are now attributed to redistribution, which is now larger than intertemporal-sharing, although risk-sharing is still the most important component. When shocks are almost permanent ($\rho = 0.999$), the planner attributes most of the welfare gains to redistribution, with risk-sharing and intertemporal-sharing playing a much smaller role.

In Figure 5, we show the marginal assessment of normalized welfarist planners for different values of the redistribution coefficient ϕ of an isoelastic social welfare function, given by

$$
\mathcal{W}\left(\{V_i\left(s_0\right)\}_{i \in I}\right) = \left(\int a_i \left(-V_i\left(s_0\right)\right)^\phi di\right)^{1/\phi}.
$$

We consider three cases: $\phi \in \{1, 5, 10\}$, where the utilitarian benchmark corresponds to $\phi = 1$.[31] Consistently with Proposition 8, differences in welfare assessments among normalized welfarist planners are exclusively based on how they assess the redistribution component. Intuitively, higher values of the curvature parameter ϕ are associated with more disperse individual components of DS-weights, which in turn increase the redistribution component of the aggregate decomposition. Moreover, we show the value of the sum of the risk-sharing and intertemporal-sharing components, $\Xi_{RS} + \Xi_{IS}$, which is invariant to the value of ϕ — in fact, it corresponds to the assessment of a pseudo-welfarist NR DS-planner. This figure illustrates an important conclusion of this paper, which is that the choice of SWF does not to impact the aggregate efficiency, risk-sharing, and intertemporal-

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[31]Our definition of isoelastic SWF is somewhat since lifetime utilities are negative for CRRA individuals. Our formulation, in which $\phi \geq 1$, guarantees that the SWF is concave, implying that a planner prefers individual utilities to be less disperse. See Section G.3.1 of the Online Appendix for further details.
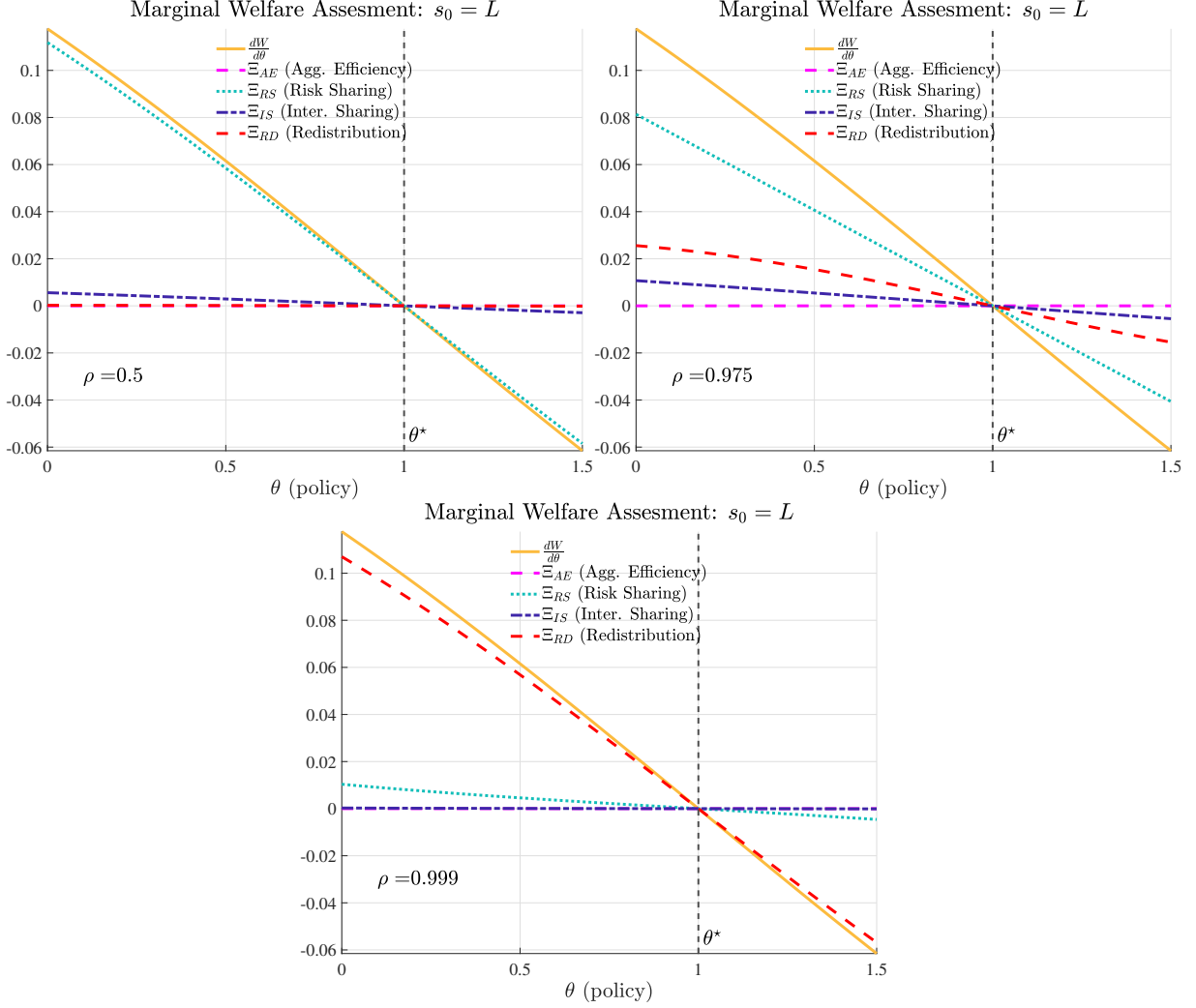
Figure 4: Aggregate additive decomposition of welfare assessments (Scenario 1)

Note: Figure 4 shows the marginal welfare assessment of a normalized utilitarian planner, $\frac{dW}{d\theta}$, and the components of its aggregate additive decomposition, as defined in Proposition 5. The left plot corresponds to the assessment when $s_0 = L$, while the right plot corresponds to the assessments when $s_0 = H$. Due to the symmetry of the model, both aggregate assessments are identical regardless of the state in which they are made. The solid line is computed as described in Equation (35). The dashed and dotted lines are computed as described in Equation (11), where the DS-weights are given by in Equations (13), (14), and (15). Note that $\frac{dW}{d\theta} = \Xi_{AE} + \Xi_{RS} + \Xi_{IS} + \Xi_{RD}$. The parameters are $\beta = 0.95$, $\overline{y} = 1$, $\varepsilon(H) = 0.25$, $\varepsilon(L) = -0.25$, $\rho = 0.975$, and $\gamma_1 = \gamma_2 = 2$.
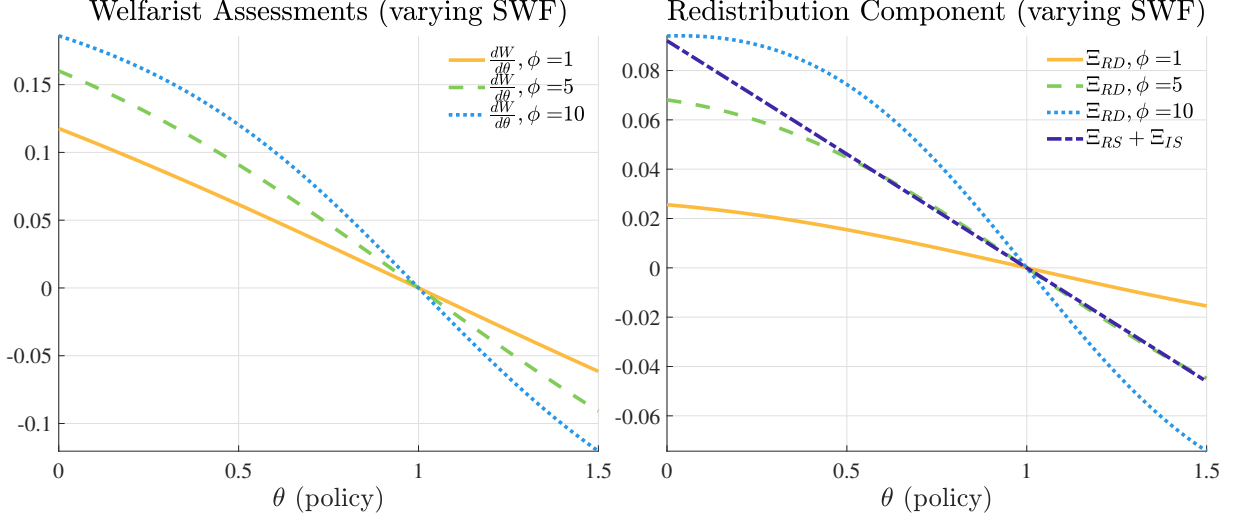
Figure 5: Aggregate additive decomposition; comparison of welfarist planners (Scenario 1)

Note: The left panel of Figure 5 shows the marginal welfare assessment of normalized welfarist planners with social welfare function

$$\mathcal{W}\left(\{V_i(s_0)\}_{i \in I}\right) = \left(\int a_i\left(-V_i(s_0)\right)^\phi di\right)^{1/\phi},$$

for $\phi \in \{1, 5, 10\}$. The utilitarian benchmark corresponds to $\phi = 1$. The right panel of Figure 5 shows the redistribution component, Ξ_{RD}, for such planners, as well as the sum of the risk-sharing and intertemporal-sharing components for either of them, since $\Xi_{RS} + \Xi_{IS}$ is identical in all three cases. In this economy, $\Xi_{AE} = 0$ at all times. Consistently with Proposition 8, differences in welfare assessments among normalized welfarist planners are exclusively based on how they assess the redistribution component. The parameters are $\beta = 0.95$, $\overline{y} = 1$, $\varepsilon(H) = 0.25$, $\varepsilon(L) = -0.25$, $\rho = 0.975$, and $\gamma_1 = \gamma_2 = 2$.

sharing components of a normalized welfarist DS-planner.

7.2   Scenario 2: Aggregate risk, heterogeneous preferences

Environment. In our second scenario, we assume i) that some individuals are more risk-averse/unwilling to substitute intertemporally than others, and ii) that all endowment risk is aggregate. In particular, we assume that individual 1 is more risk averse than individual 2, so $\gamma_1 > \gamma_2$. Formally, we assume that

$$y^1(s) = \overline{y} + \varepsilon(s) \quad \text{and} \quad y^2(s) = \overline{y} + \varepsilon(s),$$

where $\overline{y} \geq 0$, and where $\varepsilon(L) = -\varepsilon(H)$. We consider the welfare assessment of a transfer policy that shifts the amount of risk borne by individual 1 to individual 2. Formally, we set $T^1(s) = -\varepsilon(s)$ and $T^2(s) = \varepsilon(s)$, so individual consumption takes the form

$$c^1(s) = \overline{y} + \varepsilon(s)(1 - \theta) \quad \text{and} \quad c^2(s) = \overline{y} + \varepsilon(s)(1 + \theta).$$

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Figure 6: Individual multiplicative decomposition of DS-weights (Scenario 2)

Note: Figure 6 shows the components of the individual multiplicative decomposition of DS-weights for a normalized utilitarian planner, defined in Proposition 5. We assume that $\theta = 0.25$, although all figures are qualitatively similar when $\theta \in [0, 1)$. The top row shows each of the components for individual 1, while the bottom row shows them for individual 2. The left plots show the dynamic component, $\tilde{\omega}_t^i(s_0)$, for different values of t for different initial states, $s_0 = \{H, L\}$. For reference, we also show the dynamic weight for a hypothetical individual with linear marginal utility, given by $(1 - \beta)\beta^t = \beta^t / \sum_t \beta^t$. Note that the area under each of the curves adds up to 1. The middle plots show the stochastic component, $\tilde{\omega}_t^i(s^t | s_0)$, for different values of t, for different initial states, $s_0 = \{H, L\}$, and for different final states, $s_t = \{H, L\}$. The right plots show the actual DS-weights, $\omega_t^i(s^t | s_0)$, also for different values of t, and different initial and final states: $s_0 = \{H, L\}$ and $s_t = \{H, L\}$. The parameters are $\theta = 0.25$, $\beta = 0.95$, $\overline{y} = 1$, $\varepsilon(H) = 0.25$, $\varepsilon(L) = -0.25$, $\rho = 0.975$, $\gamma_1 = 5$, and $\gamma_2 = 2$. The individual component of DS-weights are $\tilde{\omega}^1(s_0 = L) = 1.125$ and $\tilde{\omega}^2(s_0 = L) = 0.875$ when an assessment takes place at $s_0 = L$; and $\tilde{\omega}^1(s_0 = H) = 1.027$ and $\tilde{\omega}^2(s_0 = H) = 0.973$ when the assessment takes place at $s_0 = H$.

Under this policy, when $\theta = 1$, individual 1 is fully insured, at the expense of increasing the consumption fluctuations of individual 2 in response to aggregate shocks. In this scenario, aggregate consumption varies with the aggregate state, but not with θ, since $\int c^i(s)\,di = \overline{y} + \varepsilon(s)$.

Results. With the exception of risk aversion, set to $\gamma_1 = 5$ and $\gamma_2 = 2$, we use the same parameters as in Scenario 1: $\beta = 0.95$, $\overline{y} = 1$, $\varepsilon(H) = 0.25$, $\varepsilon(L) = -0.25$. As in the benchmark parameterization of Scenario 1, we set $\rho = 0.975$, so endowment shocks are persistent. Once again, we consider a normalized utilitarian planner with equal weights.

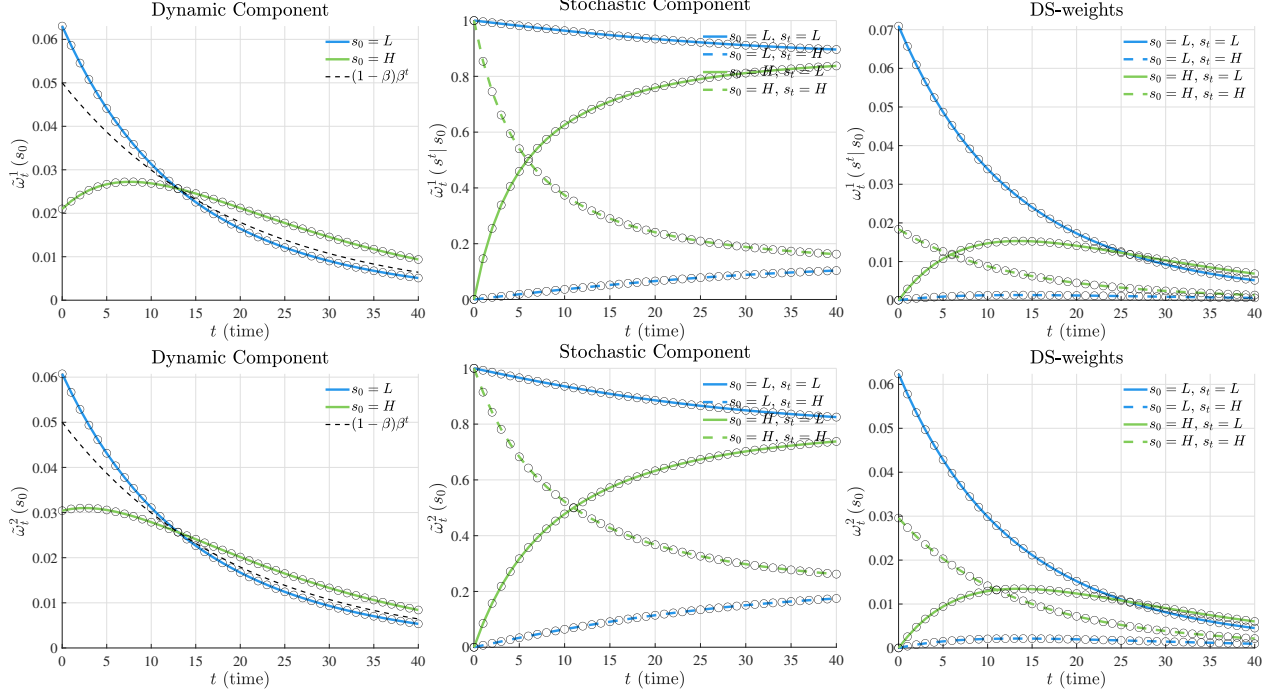*Individual multiplicative decomposition of DS-weights.* In Figure 6, we show the components of the individual multiplicative decomposition of DS-weights for a normalized utilitarian planner for each of the individuals when $\theta = 0.25$. This new scenario is associated with new insights.

First, the plots of the dynamic components show that a normalized utilitarian planner overweights

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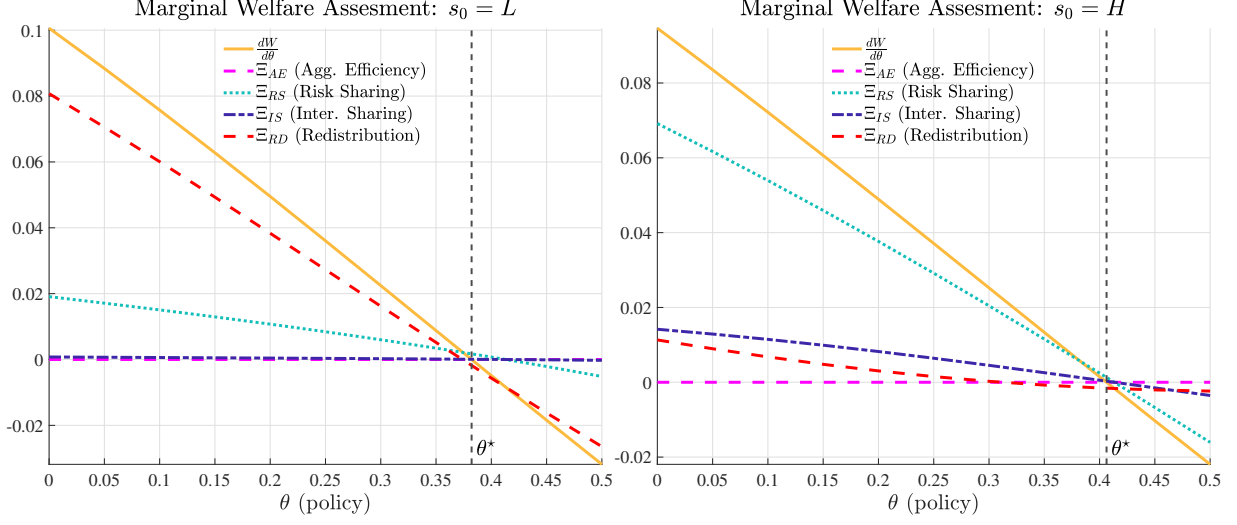
Figure 7: Aggregate additive decomposition of welfare assessments (Scenario 2)

Note: Figure 7 shows the marginal welfare assessment of a normalized utilitarian planner, $\frac{dW}{d\theta}$, and the components of its aggregate additive decomposition, as defined in Proposition 5. The left plot corresponds to the assessment when $s_0 = L$, while the right plot corresponds to the assessments when $s_0 = H$. Note that $\frac{dW}{d\theta} = \Xi_{AE} + \Xi_{RS} + \Xi_{IS} + \Xi_{RD}$. The parameters are $\beta = 0.95$, $\overline{y} = 1$, $\varepsilon(H) = 0.25$, $\varepsilon(L) = -0.25$, $\rho = 0.975$, and $\gamma_1 = 5 > \gamma_2 = 2$.

earlier periods for *all* individuals when the aggregate endowment is low (graphically, the solid blue line is above the black dashed line for both individuals when $s_0 = L$; this is not the case in Scenario 1). As one would expect, it does so more for individual 1, with the higher curvature coefficient $\gamma_1 = 5$. Note, for instance, that $\tilde{\omega}_0^1(s_0 = L) > \tilde{\omega}_0^2(s_0 = L)$ and that $\tilde{\omega}_0^1(s_0 = H) < \tilde{\omega}_0^2(s_0 = H)$.

Second, as in Scenario 1, the plots of the stochastic components show that a normalized utilitarian planner overweights those states which are more likely, given the initial state. More importantly, in the long run (although also in the short run), regardless of the initial state, the stochastic components give relatively more weight to those states in which an individual has a lower endowment/higher marginal utility, but differentially more for the individual 1, with the highest curvature coefficient $\gamma_1 = 5$. Note, for instance, that $\tilde{\omega}_\infty^1(s_t = L) > \tilde{\omega}_\infty^2(s_t = L)$ and that $\tilde{\omega}_\infty^1(s_t = H) < \tilde{\omega}_\infty^2(s_t = H)$.

Third, the individual components of the DS-weights still capture differences in the marginal valuation of permanent transfers among individuals for different initial states. However, in this scenario this differences are mostly driven by the differences in preferences between individuals. Unlike in scenario 1, a normalized utilitarian planner gives more value to a hypothetical permanent transfer towards individual 1 at all states, since $\tilde{\omega}^1(s_0 = L) > \tilde{\omega}^2(s_0 = L)$ and $\tilde{\omega}^1(s_0 = H) > \tilde{\omega}^2(s_0 = H)$. This result illustrates how by computing the individual component it is possible to determine the implicit desire for redistribution of a utilitarian planner.

*Aggregate additive decomposition of welfare assessment*s. In Figure 7, we show the components of the aggregate additive decomposition of welfare assessments for a normalized utilitarian planner. As in Scenario 1, because we study an endowment economy for which aggregate consumption is invariant to the policy, the aggregate efficiency component is zero, that is, $\Xi_{AE} = 0$. There is a new

set of insights.

First, we show that a normalized utilitarian planner finds it optimal to increase transfers until some value of $\theta^\star$, regardless of whether the optimal policy is determined from $s_0 = L$ or $s_0 = H$. This should not be surprising, since transferring aggregate risk to the individual most willing to bear such a risk seems desirable. Interestingly, the reason for why a planner finds desirable to increase θ until $\theta^\star$ differs with the initial state of the economy. When $s_0 = L$, we show that a normalized utilitarian planner mostly attributes welfare gains to redistribution (Ξ_{RD}), followed by risk-sharing (Ξ_{RS}), with intertemporal-sharing (Ξ_{IS}) barely playing a role. Instead, when $s_0 = H$, we show that a normalized utilitarian planner mostly attributes welfare gains to risk-sharing (Ξ_{RS}), followed by redistribution (Ξ_{RD}) and intertemporal-sharing (Ξ_{IS}).

These findings are intuitive. When $s_0 = L$, consumption is persistently lower, which amplifies differences in curvature between individuals on a persistent basis. This is reflected in the large redistribution component. Building on the insights of Proposition 13, one can trace these results to the cross-sectional dispersion of the different components of DS-weights. In particular, Figure 7 illustrates how the cross-sectional dispersion of the individual component is significantly higher when $s_0 = L$, which explains why the redistribution component is more important when $s_0 = L$. Alternatively, Figure 7 reflects that the cross-sectional dispersion of the dynamic and the stochastic components is higher when $s_0 = H$.

Finally, note that at the optimal $\theta^\star$ for both $s_0 = L$ and $s_0 = H$, the normalized utilitarian planner perceives Ξ_{RS} to be positive and Ξ_{RD} to be negative and greater in magnitude than Ξ_{RS}, which is also positive. This implies that both pseudo-utilitarian NS and NR DS-planners would choose a level of $\theta^\star$ higher than the normalized utilitarian planner, regardless of the state in which the assessments is made. This results illustrates that, in general, different pseudo-utilitarian DS-planners would disagree in the choice of optimal policies.

8 Conclusion

In this paper, we have introduced the notion of Dynamic Stochastic Generalized Social Marginal Welfare Weights (Dynamic Stochastic weights or DS-weights, for short) and explored their properties. First, we have shown that DS-weights are useful for decomposing aggregate welfare assessments of policy changes into four distinct components: aggregate efficiency, intertemporal-sharing, risk-sharing, and redistribution. Second, we have shown that by using DS-weights it is possible to formalize a new, larger set of welfare criteria that society may find appealing. In particular, we have been able to define normative criteria that are exclusively based on one or several of the components that we identify, potentially disregarding the others.

Retrospectively, our definition of a normalized utilitarian planner — based on DS-weights — opens the door to revisiting the exact rationales that have justified particular welfare assessments in existing work. Looking forward, we hope that our approach informs ongoing and future discussions on policy-making mandates, in particular when trading off aggregate stabilization objectives against

interpersonal insurance and redistribution objectives.

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APPENDIX

A Proofs and Derivations: Section 3

Proof of Lemma 1. (DS-weights: individual multiplicative decomposition; unique normalized decomposition)

Proof. By offering a constructive proof of part b), we automatically show that it is always possible to construct an individual multiplicative decomposition, in particular a normalized one. Let us start with a set of DS-weights $\breve{\omega}_t^i \left(s^t \middle| s_0 \right) > 0$, defined for each individual, date, and history. After multiplying and dividing by $\sum_{s^t} \breve{\omega}_t^i \left(s^t \middle| s_0 \right)$, $\sum_{t=0}^T \sum_{s^t} \breve{\omega}_t^i \left(s^t \middle| s_0 \right)$, and $\int \sum_{t=0}^T \sum_{s^t} \breve{\omega}_t^i \left(s^t \middle| s_0 \right) di$, we reach the following identity:

$$\underbrace{\frac{\breve{\omega}_t^i \left(s^t \middle| s_0 \right)}{\int \sum_{t=0}^T \sum_{s^t} \breve{\omega}_t^i \left(s^t \middle| s_0 \right) di}}_{=\omega_t^i(s^t|s_0)} = \underbrace{\frac{\sum_{t=0}^T \sum_{s^t} \breve{\omega}_t^i \left(s^t \middle| s_0 \right)}{\int \sum_{t=0}^T \sum_{s^t} \breve{\omega}_t^i \left(s^t \middle| s_0 \right) di}}_{=\tilde{\omega}^i(s_0)} \underbrace{\frac{\sum_{s^t} \breve{\omega}_t^i \left(s^t \middle| s_0 \right)}{\sum_{t=0}^T \sum_{s^t} \breve{\omega}_t^i \left(s^t \middle| s_0 \right)}}_{=\tilde{\omega}_t^i(s_0)} \underbrace{\frac{\breve{\omega}_t^i \left(s^t \middle| s_0 \right)}{\sum_{s^t} \breve{\omega}_t^i \left(s^t \middle| s_0 \right)}}_{\equiv\tilde{\omega}_t^i(s^t|s_0)},$$

which defines an individual multiplicative decomposition since $\omega_t^i \left(s^t \middle| s_0 \right)$ and $\breve{\omega}_t^i \left(s^t \middle| s_0 \right)$ are identical from the perspective of Definition 4, but for a normalization regarding the choice of units. It is straightforward to show that $\sum_{t=0}^T \tilde{\omega}_t^i \left(s_0 \right) = 1$, $\sum_{s^t} \tilde{\omega}_t^i \left(s^t \middle| s_0 \right) = 1$, and $\int \tilde{\omega}^i \left(s_0 \right) di = 1$, which concludes the proof. □

Proof of Proposition 1. (Welfare assessments: aggregate additive decomposition)

Proof. Combining Equations (7) and (9), the definition of a desirable policy change for a DS-planner can be expressed as follows:

$$\frac{dW^{DS}\left(s_0\right)}{d\theta} = \int \tilde{\omega}^i \left(s_0 \right) \frac{dV_i^{DS}\left(s_0\right)}{d\theta} di = \mathbb{E}_i \left[\tilde{\omega}^i \left(s^0 \right) \frac{dV_i^{DS}\left(s_0\right)}{d\theta} \right], \tag{36}$$

where

$$\frac{dV_i^{DS}\left(s_0\right)}{d\theta} = \sum_{t=0}^T \tilde{\omega}_t^i \left(s_0 \right) \sum_{s^t} \tilde{\omega}_t^i \left(s^t \middle| s_0 \right) \frac{du_{i|c}\left(s^t\right)}{d\theta}, \tag{37}$$

Hence, we can first decompose $\frac{dW^{DS}(s_0)}{d\theta}$ as follows:

$$\frac{dW^{DS}\left(s_0\right)}{d\theta} = \underbrace{\mathbb{E}_i \left[\tilde{\omega}^i \left(s^0 \right) \right]}_{=1} \mathbb{E}_i \left[\frac{dV_i^{DS}\left(s_0\right)}{d\theta} \right] + \underbrace{\mathbb{C}ov_i \left[\tilde{\omega}^i \left(s^0 \right), \frac{dV_i^{DS}\left(s_0\right)}{d\theta} \right]}_{=\Xi_{RD}} \tag{38}$$

where we use the fact that — without loss of generality, but for the choice of units — we can set

$\mathbb{E}_i\left[\tilde{\omega}^i\left(s^0\right)\right] = \int \tilde{\omega}^i\left(s^0\right) di = 1$, and where Ξ_{RD} satisfies

$$\Xi_{RD} = \mathbb{C}ov_i\left[\tilde{\omega}^i\left(s^0\right), \sum_{t=0}^{T}\tilde{\omega}_t^i\left(s_0\right)\sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle|s_0\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}\right].$$

Next, we can decompose $\mathbb{E}_i\left[\frac{dV_i^{DS}(s_0)}{d\theta}\right]$ as follows:

$$\mathbb{E}_i\left[\frac{dV_i^{DS}\left(s_0\right)}{d\theta}\right] = \mathbb{E}_i\left[\sum_{t=0}^{T}\tilde{\omega}_t^i\left(s_0\right)\sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle|s_0\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]$$

$$= \sum_{t=0}^{T}\mathbb{E}_i\left[\tilde{\omega}_t^i\left(s_0\right)\right]\mathbb{E}_i\left[\sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle|s_0\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}\right] + \underbrace{\sum_{t=0}^{T}\mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s_0\right), \sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle|s_0\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]}_{=\Xi_{IS}}$$

$$= \sum_{t=0}^{T}\mathbb{E}_i\left[\tilde{\omega}_t^i\left(s_0\right)\right]\sum_{s^t}\left(\mathbb{E}_i\left[\tilde{\omega}_t^i\left(s^t\middle|s_0\right)\right]\mathbb{E}_i\left[\frac{du_{i|c}\left(s^t\right)}{d\theta}\right] + \mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s^t\middle|s_0\right), \frac{du_{i|c}\left(s^t\right)}{d\theta}\right]\right) + \Xi_{IS}$$

$$= \underbrace{\sum_{t=0}^{T}\mathbb{E}_i\left[\tilde{\omega}_t^i\left(s_0\right)\right]\sum_{s^t}\mathbb{E}_i\left[\tilde{\omega}_t^i\left(s^t\middle|s_0\right)\right]\mathbb{E}_i\left[\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]}_{=\Xi_{AE}}$$

$$+ \underbrace{\sum_{t=0}^{T}\mathbb{E}_i\left[\tilde{\omega}_t^i\left(s_0\right)\right]\sum_{s^t}\mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s^t\middle|s_0\right), \frac{du_{i|c}\left(s^t\right)}{d\theta}\right]}_{=\Xi_{RS}} + \Xi_{IS}$$

$$= \Xi_{AE} + \Xi_{RS} + \Xi_{IS}. \tag{39}$$

Proposition 1 follows immediately after combining Equations (38) and (39). $\qquad\square$

Proof of Proposition 2. (Properties of aggregate additive decomposition: individual-invariant DS-weights)

Proof. a) If DS-weights $\omega_t^i\left(s^t\middle|s_0\right)$ do not vary across individuals, parts b), c), and d) below are valid.
b) If the stochastic components, $\tilde{\omega}_t^i\left(s^t\middle|s_0\right)$, do not vary across individuals at all dates and histories, then

$$\mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s^t\middle|s_0\right), \frac{du_{i|c}\left(s^t\right)}{d\theta}\right] = 0, \forall t, \forall s^t \implies \Xi_{RS} = 0.$$

c) If the dynamic components, $\tilde{\omega}_t^i\left(s_0\right)$, do not vary across individuals at all dates, then

$$\mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s_0\right), \sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle|s_0\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}\right] = 0, \forall t \implies \Xi_{IS} = 0.$$

d) If the individual components, $\tilde{\omega}^i\left(s_0\right)$, do not vary across individuals, then

$$\mathbb{C}ov_i\left[\tilde{\omega}^i\left(s^0\right), \sum_{t=0}^{T}\tilde{\omega}_t^i\left(s_0\right)\sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle|s_0\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}\right] = 0 \implies \Xi_{RD} = 0.$$

Corollary 1 follows from part a). Corollary 2 follows from part b), since $\tilde{\omega}_t^i(s^t|s_0) = 1$, $\forall t$, $\forall i$, in perfect foresight economies. Corollary 3 follows from part d). $\qquad \square$

Proof of Proposition 3. (Properties of aggregate additive decomposition: individual-invariant policies)

Proof. Note that $\sum_{t=0}^T \tilde{\omega}_t^i(s_0)$ and $\sum_{s^t} \tilde{\omega}_t^i(s^t|s_0) = 1$ imply that $\sum_{t=0}^T \tilde{\omega}_t^i(s_0) \sum_{s^t} \tilde{\omega}_t^i(s^t|s_0) = 1$.
a) If $\frac{du_{i|c}(s^t)}{d\theta} = g(\cdot)$, where $g(\cdot)$ does not depend on i, t, or s^t, then

$$\mathbb{C}ov_i\left[\tilde{\omega}^i(s^0), \sum_{t=0}^T \tilde{\omega}_t^i(s_0) \sum_{s^t} \tilde{\omega}_t^i(s^t|s_0)\right] \frac{du_{i|c}(s^t)}{d\theta} = 0 \implies \Xi_{RD} = 0.$$

And the results from parts b) and c) also apply.
b) If $\frac{du_{i|c}(s^t)}{d\theta} = g(t)$, where $g(t)$ may depend on t, but not on i or s^t, then

$$\mathbb{C}ov_i\left[\tilde{\omega}_t^i(s_0), \sum_{s^t} \tilde{\omega}_t^i(s^t|s_0)\right] \frac{du_{i|c}(s^t)}{d\theta} = 0 \implies \Xi_{IS} = 0.$$

And the result from part c) also applies.
c) If $\frac{du_{i|c}(s^t)}{d\theta} = g(t, s^t)$, where $g(t, s^t)$ may depend on t and s^t, but not on i, then

$$\mathbb{C}ov_i\left[\tilde{\omega}_t^i(s^t|s_0), \frac{du_{i|c}(s^t)}{d\theta}\right] = 0 \implies \Xi_{RS} = 0.$$

$\qquad \square$

Proof of Proposition 4. (Properties of aggregate additive decomposition: endowment economies)

Proof. In an endowment economy, Equation (11) simply corresponds to

$$\mathbb{E}_i\left[\frac{du_{i|c}(s^t)}{d\theta}\right] = \int \frac{dc_t^i(s^t)}{d\theta} di = 0,$$

where the last equality follows from the fact that aggregate consumption is equal to the aggregate endowment, and hence fixed and invariant to θ, that is, $\frac{d\int c_t^i(s^t)di}{d\theta} = 0$. $\qquad \square$

B Proofs and Derivations: Section 4

Proof of Proposition 5. (Normalized welfarist planners: individual multiplicative decomposition)

Proof. Starting from Equation (6), note that we can express $\frac{dV_i(s_0)}{d\theta}$ as follows:

$$\frac{dV_i(s_0)}{d\theta} = \sum_{t=0}^{T}(\beta_i)^t \sum_{s^t}\pi_t\left(s^t\mid s_0\right)\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}\frac{du_{i|c}\left(s^t\right)}{d\theta}$$

$$= \sum_{t=0}^{T}(\beta_i)^t \sum_{s^t}\pi_t\left(s^t\mid s_0\right)\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}\sum_{t=0}^{T}\frac{(\beta_i)^t \sum_{s^t}\pi_t\left(s^t\mid s_0\right)\frac{\partial u_i(s^t)}{\partial c_t^i}\frac{du_{i|c}(s^t)}{d\theta}}{\sum_{t=0}^{T}(\beta_i)^t \sum_{s^t}\pi_t\left(s^t\mid s_0\right)\frac{\partial u_i(s^t)}{\partial c_t^i}}$$

$$= \sum_{t=0}^{T}(\beta_i)^t \sum_{s^t}\pi_t\left(s^t\mid s_0\right)\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}\sum_{t=0}^{T}\underbrace{\frac{(\beta_i)^t \sum_{s^t}\pi_t\left(s^t\mid s_0\right)\frac{\partial u_i(s^t)}{\partial c_t^i}}{\sum_{t=0}^{T}(\beta_i)^t \sum_{s^t}\pi_t\left(s^t\mid s_0\right)\frac{\partial u_i(s^t)}{\partial c_t^i}}}_{=\tilde{\omega}_t^i(s_0)}\sum_{s^t}\underbrace{\frac{(\beta_i)^t \pi_t\left(s^t\mid s_0\right)\frac{\partial u_i(s^t)}{\partial c_t^i}}{(\beta_i)^t \sum_{s^t}\pi_t\left(s^t\mid s_0\right)\frac{\partial u_i(s^t)}{\partial c_t^i}}}_{=\tilde{\omega}_t^i(s^t\mid s_0)}\frac{du_{i|c}\left(s^t\right)}{d\theta}$$

$$= \sum_{t=0}^{T}(\beta_i)^t \sum_{s^t}\pi_t\left(s^t\mid s_0\right)\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}\sum_{t=0}^{T}\tilde{\omega}_t^i(s_0)\sum_{s^t}\tilde{\omega}_t^i\left(s^t\mid s_0\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}, \tag{40}$$

where we define dynamic and stochastic components of DS-weights as in Equations (13) and (14). Hence, we can express $\frac{dW^{\mathcal{W}}(s_0)}{d\theta}$ — with appropriately normalized units — as follows:

$$\frac{\frac{dW^{\mathcal{W}}(s_0)}{d\theta}}{\int \lambda_i \sum_{t=0}^{T}(\beta_i)^t \sum_{s^t}\pi_t\left(s^t\mid s_0\right)\frac{\partial u_i(s^t)}{\partial c_t^i}di} = \int \tilde{\omega}^i(s_0)\sum_{t=0}^{T}\tilde{\omega}_t^i(s_0)\sum_{s^t}\tilde{\omega}_t^i\left(s^t\mid s_0\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}di,$$

where we define the individual component as in Equation (15):

$$\tilde{\omega}^i(s_0) = \frac{\sum_{t=0}^{T}(\beta_i)^t \sum_{s^t}\pi_t\left(s^t\mid s_0\right)\frac{\partial u_i(s^t)}{\partial c_t^i}}{\int \lambda_i \sum_{t=0}^{T}(\beta_i)^t \sum_{s^t}\pi_t\left(s^t\mid s_0\right)\frac{\partial u_i(s^t)}{\partial c_t^i}di}.$$

It is straightforward to verify that $\sum_{s^t}\tilde{\omega}_t^i\left(s^t\mid s_0\right) = 1$, $\forall t$, $\forall i$; that $\sum_{t=0}^{T}\tilde{\omega}_t^i(s_0) = 1$, $\forall i$, and that $\int \tilde{\omega}^i(s_0)\,di = 1$, which concludes the proof. Note that by multiplying and dividing the dynamic and stochastic components of a given individual by his marginal utility of consumption at 0, we recover Equations (16) and (17):

$$\tilde{\omega}_t^i(s_0) = \frac{(\beta_i)^t \sum_{s^t}\pi_t\left(s^t\mid s_0\right)\frac{\partial u_i(s^t)}{\partial c_t^i}/\frac{\partial u_i(s^0)}{\partial c_0^i}}{\sum_{t=0}^{T}(\beta_i)^t \sum_{s^t}\pi_t\left(s^t\mid s_0\right)\frac{\partial u_i(s^t)}{\partial c_t^i}/\frac{\partial u_i(s^0)}{\partial c_0^i}} = \frac{q_t^i\left(s^t\mid s_0\right)}{\sum_{s^t}q_t^i\left(s^t\mid s_0\right)}$$

$$\tilde{\omega}_t^i\left(s^t\mid s_0\right) = \frac{\pi_t\left(s^t\mid s_0\right)\frac{\partial u_i(s^t)}{\partial c_t^i}/\frac{\partial u_i(s^0)}{\partial c_0^i}}{\sum_{s^t}\pi_t\left(s^t\mid s_0\right)\frac{\partial u_i(s^t)}{\partial c_t^i}/\frac{\partial u_i(s^0)}{\partial c_0^i}} = \frac{\sum_{s^t}q_t^i\left(s^t\mid s_0\right)}{\sum_{t=0}^{T}\sum_{s^t}q_t^i\left(s^t\mid s_0\right)}.$$

$\square$

Proof of Proposition 6. (Properties of normalized welfarist planners: complete markets)

Proof. When markets are complete, there is a unique stochastic discount factor, which implies that $q_t^i\left(s^t\mid s_0\right) = q_t\left(s^t\mid s_0\right)$, $\forall i$. From Equations (16) and (17), it follows immediately that $\tilde{\omega}_t^i(s_0)$

and $\tilde{\omega}_t^i\left(s^t\middle|s_0\right)$ are invariant across all individuals at all dates and states. Hence, Parts b) and c) Proposition 2 guarantee that $\Xi_{RS} = \Xi_{IS} = 0$. $\qquad\square$

Proof of Proposition 7. (Properties of normalized welfarist planners: riskless borrowing/saving)

Proof. When individual can freely borrow and save, it must be the case that the price/valuation of a riskless bond is identical for all individuals, which implies that $\sum_{s^t} q_t^i\left(s^t|s_0\right)$ is identical across individuals. Hence, from Equation (17), it follows immediately that $\tilde{\omega}_t^i\left(s_0\right)$ is invariant across all individuals at all dates. Hence, Part c) Proposition 2 guarantees that $\Xi_{IS} = 0$. $\qquad\square$

Proof of Proposition 8. (Properties of normalized welfarist planners: welfarist planners only disagree about redistribution)

Proof. Note that Equations (13) and (14) do not depend on $\mathcal{W}\left(\cdot\right)$, while Equation (15) does. This fact, along with Proposition 1, immediately imply that Ξ_{AE}, Ξ_{RS}, and Ξ_{IS} identical for all welfarist planner, but Ξ_{RD} is not. $\qquad\square$

C   Proofs and Derivations: Section 5

Proof of Proposition 9 (AE/AR/NR DS-planners: properties)

Proof. a) This result follows from part a) of Proposition 3, since $\tilde{\omega}_t^i\left(s^t\middle|s_0\right)$, $\tilde{\omega}_t^i\left(s_0\right)$, and $\tilde{\omega}^i\left(s_0\right)$ do not vary across individuals. Note that Ξ_{AE} is identical for the pseudo-welfarist AE DS-planner and its associated normalized welfarist planner, since

$$\mathbb{E}_i\left[\tilde{\omega}_t^{i,\mathcal{W},AE}\left(s_0\right)\right] = \mathbb{E}_i\left[\tilde{\omega}_t^{i,\mathcal{W}}\left(s_0\right)\right] \text{ and } \mathbb{E}_i\left[\tilde{\omega}_t^{i,\mathcal{W},AE}\left(s^t\middle|s_0\right)\right] = \mathbb{E}_i\left[\tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\middle|s_0\right)\right].$$

b) This result follows from parts c) and d) of Proposition 3, since $\tilde{\omega}_t^i\left(s^t\middle|s_0\right)$ and $\tilde{\omega}_t^i\left(s_0\right)$ do not vary across individuals. Note that Ξ_{AE} and Ξ_{RS} are identical for the pseudo-welfarist AR DS-planner and its associated normalized welfarist planner, since

$$\mathbb{E}_i\left[\tilde{\omega}_t^{i,\mathcal{W},AR}\left(s_0\right)\right] = \mathbb{E}_i\left[\tilde{\omega}_t^{i,\mathcal{W}}\left(s_0\right)\right] \text{ and } \tilde{\omega}_t^{i,\mathcal{W},AR}\left(s^t\middle|s_0\right) = \tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\middle|s_0\right).$$

c) This result follows from part d) of Proposition 3, since the individual components $\tilde{\omega}^i\left(s_0\right)$ do not vary across individuals. Note that Ξ_{AE}, Ξ_{RS}, and Ξ_{IS} are identical for the pseudo-welfarist NR DS-planner and its associated normalized welfarist planner, since

$$\tilde{\omega}_t^{i,\mathcal{W},NR}\left(s_0\right) = \tilde{\omega}_t^{i,\mathcal{W}}\left(s_0\right) \text{ and } \tilde{\omega}_t^{i,\mathcal{W},NR}\left(s^t\middle|s_0\right) = \tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\middle|s_0\right).$$

$\qquad\square$

ONLINE APPENDIX

Section D of this Online Appendix includes proofs and derivations for Section 6. Section E includes additional results for Application 1. Section F includes several extensions and Section G contains additional results.

D   Proofs and Derivations: Section 6

Proof of Proposition 10. (Aggregate efficiency component: stochastic decomposition)

Proof. Starting from the definition of the aggregate efficiency component in Equation (11), we can express Ξ_{AE} as follows:

$$\Xi_{AE} = \sum_{t=0}^{T} \mathbb{E}_i \left[\tilde{\omega}_t^i (s_0) \right] \sum_{s^t} \mathbb{E}_i \left[\tilde{\omega}_t^i \left(s^t \middle| s_0 \right) \right] \mathbb{E}_i \left[\frac{du_{i|c}(s^t)}{d\theta} \right]$$

$$= \sum_{t=0}^{T} \overline{\omega}_t \sum_{s^t} \overline{\omega}_t \left(s^t \middle| s_0 \right) \frac{d\overline{u}_{i|c}(s^t)}{d\theta},$$

where we define $\overline{\omega}_t (s_0) = \mathbb{E}_i \left[\tilde{\omega}_t^i (s_0) \right]$, $\overline{\omega}_t \left(s^t \middle| s_0 \right) = \mathbb{E}_i \left[\tilde{\omega}_t^i \left(s^t \middle| s_0 \right) \right]$, and $\frac{d\overline{u}_{i|c}(s^t)}{d\theta} = \mathbb{E}_i \left[\frac{du_{i|c}(s^t)}{d\theta} \right]$. Multiplying and dividing by $\pi_t \left(s^t \middle| s_0 \right)$ at every history, we can express and decompose Ξ_{AE} as follows:

$$\Xi_{AE} = \sum_{t=0}^{T} \overline{\omega}_t (s_0) \sum_{s^t} \pi_t \left(s^t \middle| s_0 \right) \frac{\overline{\omega}_t \left(s^t \middle| s_0 \right)}{\pi_t \left(s^t \middle| s_0 \right)} \frac{d\overline{u}_{i|c}(s^t)}{d\theta} = \sum_{t=0}^{T} \overline{\omega}_t (s_0) \mathbb{E}_0 \left[\overline{\omega}_t^{\pi} \left(s^t \middle| s_0 \right) \frac{d\overline{u}_{i|c}(s^t)}{d\theta} \right]$$

$$= \underbrace{\sum_{t=0}^{T} \overline{\omega}_t (s_0) \mathbb{E}_0 \left[\overline{\omega}_t^{\pi} \left(s^t \middle| s_0 \right) \right] \mathbb{E}_0 \left[\frac{d\overline{u}_{i|c}(s^t)}{d\theta} \right]}_{=\Xi_{AE}} + \underbrace{\sum_{t=0}^{T} \overline{\omega}_t (s_0) \mathbb{C}ov_0 \left[\overline{\omega}_t^{\pi} \left(s^t \middle| s_0 \right), \frac{d\overline{u}_{i|c}(s^t)}{d\theta} \right]}_{=\Xi_{AE}},$$

which corresponds to Equation (23) in the text. $\qquad\square$

Proof of Proposition 11. (Risk-sharing/intertemporal-sharing components: alternative cross-sectional decompositions)

Proof. Here we make use of the following property of covariances (Bohrnstedt and Goldberger, 1969):

$$\mathbb{C}ov \left[X, YZ \right] = \mathbb{E} \left[Y \right] \mathbb{C}ov \left[X, Z \right] + \mathbb{E} \left[Z \right] \mathbb{C}ov \left[X, Y \right] + \mathbb{E} \left[(X - \mathbb{E} \left[X \right]) (Y - \mathbb{E} \left[Y \right]) (Z - \mathbb{E} \left[Z \right]) \right],$$

where X, Y, and Z denote random variables. Applying this property to $\mathbb{C}ov_i \left[\tilde{\omega}_t^i (s_0), \tilde{\omega}_t^i \left(s^t \middle| s_0 \right) \frac{du_{i|c}(s^t)}{d\theta} \right]$,

we find that

$$\mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s_0\right),\tilde{\omega}_t^i\left(\left.s^t\right|s_0\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]=\underbrace{\mathbb{E}_i\left[\tilde{\omega}_t^i\left(\left.s^t\right|s_0\right)\right]\mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s_0\right),\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]}_{\sim PI}$$

$$+\underbrace{\mathbb{E}_i\left[\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]\mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s_0\right),\tilde{\omega}_t^i\left(\left.s^t\right|s_0\right)\right]}_{\sim WC}$$

$$+\underbrace{\mathbb{E}_i\left[\left(\frac{du_{i|c}\left(s^t\right)}{d\theta}-\mathbb{E}_i\left[\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]\right)\left(\tilde{\omega}_t^i\left(s_0\right)-\mathbb{E}_i\left[\tilde{\omega}_t^i\left(s_0\right)\right]\right)\left(\tilde{\omega}_t^i\left(\left.s^t\right|s_0\right)-\mathbb{E}_i\left[\tilde{\omega}_t^i\left(\left.s^t\right|s_0\right)\right]\right)\right]}_{\sim PC},$$

which immediately yields Equation (24) in the text after adding up over dates and states. Equation (25) follows immediately after using once again the same property of covariances on $\mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s_0\right)\tilde{\omega}_t^i\left(\left.s^t\right|s_0\right),\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]$. $\qquad\square$

Proof of Proposition 12. (Redistribution component: stochastic decomposition)

Proof. We can express $\frac{dV_i^{DS}(s_0)}{d\theta}$, defined in Equation (37), as follows:

$$\frac{dV_i^{DS}\left(s_0\right)}{d\theta}=\sum_{t=0}^T\tilde{\omega}_t^i\left(s_0\right)\mathbb{E}_0\left[\frac{\tilde{\omega}_t^i\left(\left.s^t\right|s_0\right)}{\pi_t\left(\left.s^t\right|s_0\right)}\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]$$

$$=\underbrace{\sum_{t=0}^T\tilde{\omega}_t^i\left(s_0\right)\mathbb{E}_0\left[\tilde{\omega}_t^{i,\pi}\left(\left.s^t\right|s_0\right)\right]\mathbb{E}_0\left[\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]}_{=\frac{dV_i^{DS,ER}(s_0)}{d\theta}}+\underbrace{\sum_{t=0}^T\tilde{\omega}_t^i\left(s_0\right)\mathbb{C}ov_0\left[\tilde{\omega}_t^{i,\pi}\left(\left.s^t\right|s_0\right),\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]}_{=\frac{dV_i^{DS,RI}(s_0)}{d\theta}}.$$

Hence, we can express Ξ_{RD} as follows:

$$\Xi_{RD}=\mathbb{C}ov_i\left[\tilde{\omega}^i\left(s^0\right),\frac{dV_i^{DS}\left(s_0\right)}{d\theta}\right]=\underbrace{\mathbb{C}ov_i\left[\tilde{\omega}^i\left(s^0\right),\frac{dV_i^{DS,ER}\left(s_0\right)}{d\theta}\right]}_{\Xi_{RD}}+\underbrace{\mathbb{C}ov_i\left[\tilde{\omega}^i\left(s^0\right),\frac{dV_i^{DS,RI}\left(s_0\right)}{d\theta}\right]}_{\Xi_{RD}},$$

which corresponds to Equation (15) in the text. $\qquad\square$

Proof of Proposition 13. (Cross-sectional dispersion bounds)

Proof. Equations (26) through (28) follow from applying the Cauchy–Schwarz inequality, which states that $\left|\mathbb{C}ov\left[X,Y\right]\right|\leq\sqrt{\mathbb{V}ar\left[X\right]}\sqrt{\mathbb{V}ar\left[Y\right]}$ for any pair of square integrable random variables X

and Y. When applied to the relevant elements of Ξ_{RS}, Ξ_{IS}, and Ξ_{RD}, we find that

$$\mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s^t\middle|s_0\right),\frac{du_{i|c}\left(s^t\right)}{d\theta}\right] \leq \sqrt{\mathbb{V}ar_i\left[\tilde{\omega}_t^i\left(s^t\middle|s_0\right)\right]}\sqrt{\mathbb{V}ar_i\left[\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]}$$

$$\mathbb{C}ov_i\left[\tilde{\omega}_t^i\left(s_0\right),\sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle|s_0\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}\right] \leq \sqrt{\mathbb{V}ar_i\left[\tilde{\omega}_t^i\right]}\sqrt{\mathbb{V}ar_i\left[\sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle|s_0\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]}$$

$$\mathbb{C}ov_i\left[\tilde{\omega}^i\left(s^0\right),\sum_{t=0}^{T}\tilde{\omega}_t^i\left(s_0\right)\sum_{s^t}\tilde{\omega}_t^i\left(s^t\middle|s_0\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}\right] \leq \sqrt{\mathbb{V}ar_i\left[\tilde{\omega}^i\left(s^0\right)\right]}\sqrt{\mathbb{V}ar_i\left[\sum_{t=0}^{T}\sum_{s^t}\tilde{\omega}_t^i\tilde{\omega}_t^i\left(s^t\middle|s_0\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]}.$$

These three inequalities, when combined with the definitions of Ξ_{RS}, Ξ_{IS}, and Ξ_{RD} in Equation (11), immediately imply Equations (26) through (28) in the text. $\qquad\square$

Proof of Proposition 14. (Recursive formulation)

Proof. Starting from Equation (36), note that we can express $\frac{dW^{DS}(s_0)}{d\theta}$ as follows:

$$\frac{dW^{DS}\left(s_0\right)}{d\theta} = \int \tilde{\omega}^i\left(s_0\right)\frac{dV_i^{DS}\left(s_0\right)}{d\theta}di$$

$$= \int \underbrace{\tilde{\omega}^i\left(s_0\right)\tilde{\omega}_0^i\left(s_0\right)\tilde{\omega}_0^i\left(s^0\middle|s_0\right)}_{=\omega_0^i\left(s^0\middle|s_0\right)}\underbrace{\frac{\frac{dV_i^{DS}\left(s_0\right)}{d\theta}}{\tilde{\omega}_0^i\left(s_0\right)\tilde{\omega}_0^i\left(s^0\middle|s_0\right)}}_{=\frac{d\hat{V}_{i,0}^{DS}\left(s_0\right)}{d\theta}}di$$

$$= \int \omega_0^i\left(s^0\middle|s_0\right)\frac{d\hat{V}_{i,0}^{DS}\left(s_0\right)}{d\theta}di.$$

Note that we can also express $\frac{d\hat{V}_{i,0}^{DS}(s_0)}{d\theta}$ as follows:

$$\frac{d\hat{V}_{i,0}^{DS}\left(s_0\right)}{d\theta} = \sum_{t=0}^{T}\frac{\tilde{\omega}_t^i\left(s_0\right)}{\tilde{\omega}_0^i\left(s_0\right)}\sum_{s^t}\frac{\tilde{\omega}_t^i\left(s^t\middle|s_0\right)}{\tilde{\omega}_0^i\left(s^0\middle|s_0\right)}\frac{du_{i|c}\left(s_t\right)}{d\theta}$$

$$= \underbrace{\frac{\tilde{\omega}_0^i\left(s_0\right)}{\tilde{\omega}_0^i\left(s_0\right)}}_{=1}\underbrace{\frac{\tilde{\omega}_0^i\left(s^0\middle|s_0\right)}{\tilde{\omega}_0^i\left(s^0\middle|s_0\right)}}_{=1}\frac{du_{i|c}\left(s_0\right)}{d\theta} + \sum_{t=1}^{T}\frac{\tilde{\omega}_t^i\left(s_0\right)}{\tilde{\omega}_0^i\left(s_0\right)}\sum_{s^t}\frac{\tilde{\omega}_t^i\left(s^t\middle|s_0\right)}{\tilde{\omega}_0^i\left(s^0\middle|s_0\right)}\frac{du_{i|c}\left(s_t\right)}{d\theta}$$

$$= \frac{du_{i|c}\left(s_0\right)}{d\theta} + \frac{\tilde{\omega}_1^i\left(s_0\right)}{\tilde{\omega}_0^i\left(s_0\right)}\left(\underbrace{\frac{\tilde{\omega}_1^i\left(s_0\right)}{\tilde{\omega}_1^i\left(s_0\right)}}_{=1}\sum_{s^1}\frac{\tilde{\omega}_1^i\left(s^1\middle|s_0\right)}{\tilde{\omega}_0^i\left(s^0\middle|s_0\right)}\frac{du_{i|c}\left(s_1\right)}{d\theta} + \sum_{t=2}^{T}\frac{\tilde{\omega}_t^i\left(s_0\right)}{\tilde{\omega}_1^i\left(s_0\right)}\sum_{s^t}\frac{\tilde{\omega}_t^i\left(s^t\middle|s_0\right)}{\tilde{\omega}_0^i\left(s^0\middle|s_0\right)}\frac{du_{i|c}\left(s_t\right)}{d\theta}\right)$$

$$= \frac{du_{i|c}\left(s_0\right)}{d\theta} + \frac{\tilde{\omega}_1^i\left(s_0\right)}{\tilde{\omega}_0^i\left(s_0\right)}\left(\sum_{s^1}\frac{\tilde{\omega}_1^i\left(s^1\middle|s_0\right)}{\tilde{\omega}_0^i\left(s^0\middle|s_0\right)}\underbrace{\left(\frac{du_{i|c}\left(s_1\right)}{d\theta} + \sum_{t=2}^{T}\frac{\tilde{\omega}_t^i\left(s_0\right)}{\tilde{\omega}_1^i\left(s_0\right)}\sum_{s^t\middle|s^1}\frac{\tilde{\omega}_t^i\left(s^t\middle|s_0\right)}{\tilde{\omega}_1^i\left(s^1\middle|s_0\right)}\frac{du_{i|c}\left(s_t\right)}{d\theta}\right)}_{=\frac{d\hat{V}_{i,1}^{DS}\left(s_1\right)}{d\theta}}\right),$$

which immediately implies Equation (30) in the text, since this derivation is valid starting from any state s_0.

The definitions of $\hat{\beta}_{i,t}^{\mathcal{W}}$ and $\hat{\pi}_{i,t}^{\mathcal{W}}$ follow immediately after combining Equations (13) and (14) with Equation (31). Note that the product $\hat{\beta}_i^{\mathcal{W}}(s) \cdot \hat{\pi}_i^{\mathcal{W}}(s'|s)$ corresponds to the state-price assigned at state s by individual i to state s':

$$\hat{\beta}_{i,t}^{\mathcal{W}} \cdot \hat{\pi}_{i,t}^{\mathcal{W}}(s'|s) = \beta_i \pi(s'|s) \frac{\partial u_i(s')}{\partial c^i} / \frac{\partial u_i(s)}{\partial c^i},$$

and that this state-price is time-independent. This observation, combined with the definition of the pseudo-utilitarian NR planner, implies the claim that Equation (30) is time invariant for welfarist and pseudo-welfarist NR planners. $\qquad \square$

Proof of Proposition 15 (Linear instantaneous SWF formulation)

Proof. Note that, for a planner with a linear instantaneous SWF, it must be that

$$\frac{d\mathcal{I}(\cdot)}{d\theta} = \int \sum_{t=0}^{T} \sum_{s^t} \lambda_t^i\left(s^t\right) \frac{\partial u_i\left(s^t\right)}{\partial c_t^i} \frac{du_{i|c}\left(s^t\right)}{d\theta} di, \qquad (\text{OA1})$$

where $\frac{du_{i|c}(s^t)}{d\theta}$ is defined in Equation (3). The results for both the marginal welfare assessment and the optimum follow immediately by comparing Equation (7) to Equation (OA1), where the following relation must be satisfied:

$$\lambda_t^i\left(s^t\right) = \frac{\omega_t^i\left(s^t\right)}{\frac{\partial u_i(s^t)}{\partial c_t^i}}.$$

$\qquad \square$

E   Application: Additional Figures

Figures OA-1 and OA-2 are the counterparts of Figure 3 in the text when $\rho = 0.999$ and $\rho = 0.5$. When $\rho = 0.999$, the components of the individual multiplicative decompositions evolve extremely slowly. Given the extreme persistence of the shocks, all of the welfare gains from increasing θ arise from redistribution (Ξ_{RD}). When $\rho = 0.5$, endowments shocks are fully transitory, and the components of the individual multiplicative decomposition barely have any time-dependence. In this case, the welfare gains from increasing θ arise mostly from risk-sharing. The gains from redistribution are nonzero, but very small, since they are only driven by marginal utility differences at $t = 0$.
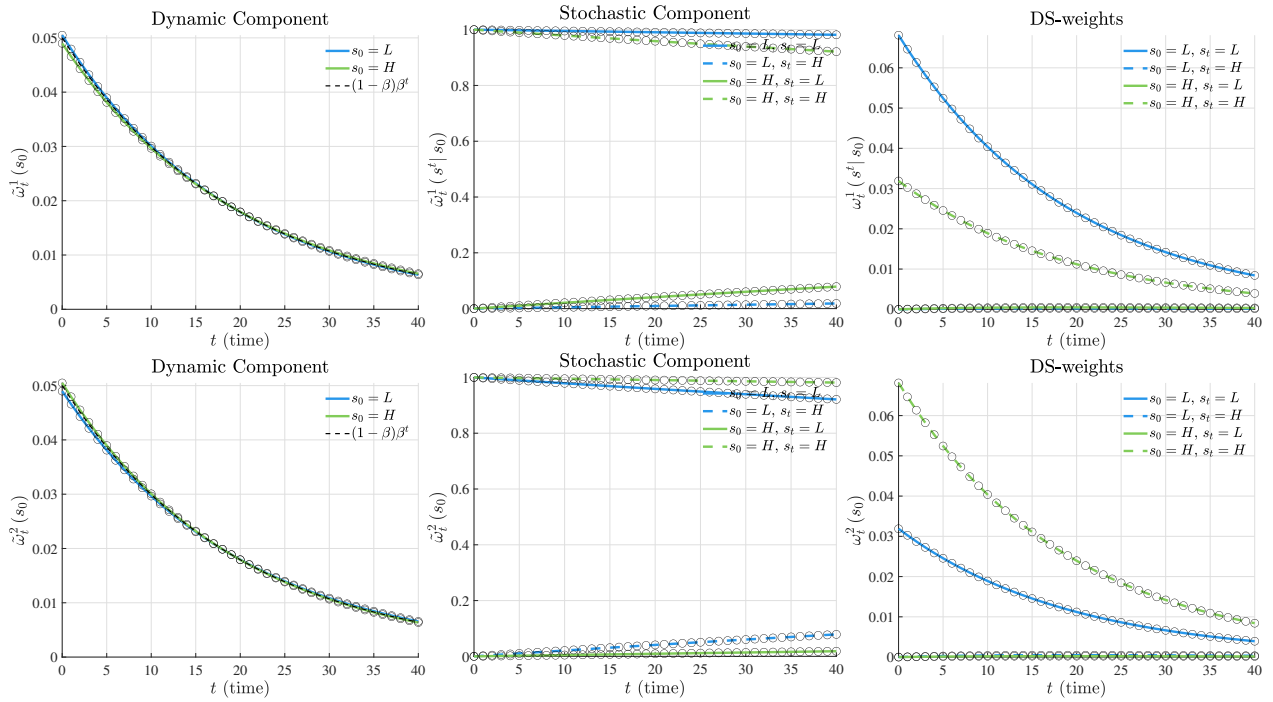
Figure OA-1: Individual multiplicative decomposition of DS-weights (Scenario 1; $\rho = 0.999$)

Note: Figure OA-1 is the counterpart of Figure 3 in the text when endowment shocks are extremely persistent ($\rho = 0.999$). The individual component of DS-weights in this case are $\tilde{\omega}^1 (s_0 = L) = 1.362$ and $\tilde{\omega}^2 (s_0 = L) = 0.638$ when an assessment takes place at $s_0 = L$; and $\tilde{\omega}^1 (s_0 = H) = 0.638$ and $\tilde{\omega}^2 (s_0 = H) = 1.362$ when the assessment takes place at $s_0 = H$.
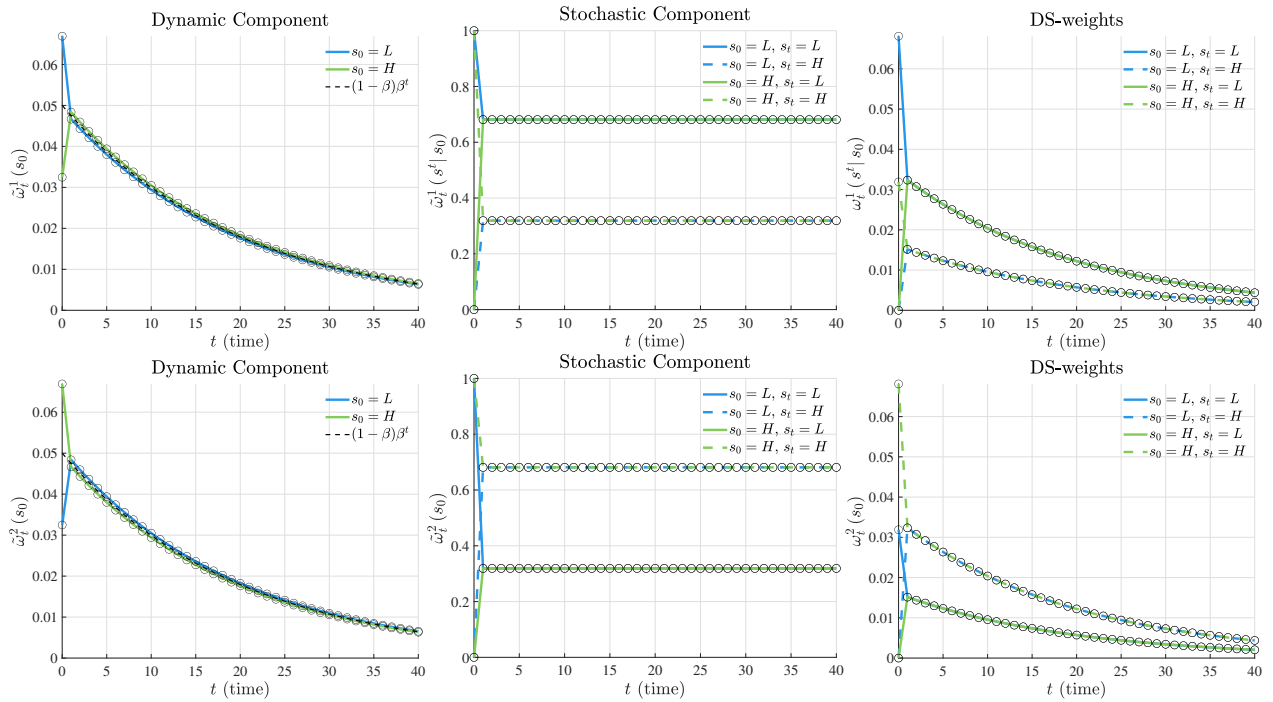
Figure OA-2: Individual multiplicative decomposition of DS-weights (Scenario 1; $\rho = 0.5$)

Note: Figure OA-2 is the counterpart of Figure 3 in the text when endowment shocks are fully temporary ($\rho = 0.5$). The individual component of DS-weights in this case are $\tilde{\omega}^1 (s_0 = L) = 1.362$ and $\tilde{\omega}^2 (s_0 = L) = 0.638$ when an assessment takes place at $s_0 = L$; and $\tilde{\omega}^1 (s_0 = H) = 0.638$ and $\tilde{\omega}^2 (s_0 = H) = 1.362$ when the assessment takes place at $s_0 = H$.

F   Extensions

F.1   Heterogeneous beliefs

In this section, we show how to use DS-weights to make paternalistic and non-paternalistic welfare assessments in environments with heterogeneous beliefs.[32] Note that the notion of paternalism used here is fully consistent with the formal definition given in Footnote 22. To model heterogeneous beliefs, instead of Equation (1), we assume instead that individual preferences take the form

$$V_i\left(s_0\right) = \sum_{t=0}^{T}\left(\beta_i\right)^t \sum_{s^t} \pi_t^i\left(s^t\middle| s_0\right) u_i\left(c_t^i\left(s^t\right), n_t^i\left(s^t\right)\right), \tag{OA2}$$

where $\pi_t^i\left(s^t\middle| s_0\right)$, denotes the beliefs held by individual i over histories, which are now individual-specific.

In this case, a non-paternalistic planner would substitute $\pi_t^i\left(s^t\middle| s_0\right)$ for $\pi_t\left(s^t\middle| s_0\right)$ whenever it appears in Equations (8) through (22). Alternatively, a paternalistic planner who imposes a single-belief would substitute some planner's belief, $\pi_t^P\left(s^t\middle| s_0\right)$, which is invariant across individuals, for $\pi_t\left(s^t\middle| s_0\right)$ whenever it appears in Equations (8) through (22).[33]

F.2   Recursive preferences

In this section, we show how to use DS-weights in the context of economies with recursive preferences. In particular, we consider the widely used Epstein-Zin preferences, which we define recursively as follows:

$$V_i\left(s\right) = \left(\left(1 - \beta_i\right)\left(u_i\left(c^i\left(s\right), n^i\left(s\right)\right)\right)^{1-\frac{1}{\psi_i}} + \beta_i\left(\sum_{s'} \pi\left(s'|s\right)\left(V^i\left(s'\right)\right)^{1-\gamma_i}\right)^{\frac{1-\frac{1}{\psi_i}}{1-\gamma_i}}\right)^{\frac{1}{1-\frac{1}{\psi_i}}},$$

where γ_i modulates risk aversion and ψ_i modulates intertemporal substitution. We use s and s' to denote any two recursive states (Ljungqvist and Sargent, 2018).

In this case, we can recursively express the welfare effect of a policy change, measured in lifetime

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[32]A recent literature has explored how to make normative assessments in environments with heterogeneous beliefs. See, among others, Brunnermeier, Simsek and Xiong (2014), Gilboa, Samuelson and Schmeidler (2014), Dávila (2020), Blume et al. (2018), Caballero and Simsek (2019), and Dávila and Walther (2021).

[33]At times, it makes sense to reinterpret heterogeneous beliefs as state-dependent preferences. In that case, $V_i\left(s_0\right)$ can be expressed as

$$V_i\left(s_0\right) = \sum_{t=0}^{T}\left(\beta_i\right)^t \sum_{s^t} \pi_t\left(s^t\middle| s_0\right) u_i\left(c_t^i\left(s^t\right), n_t^i\left(s^t\right); s^t\right).$$

All our results remain valid in the case of state-dependent preferences.

utils (utility units), as follows:

$$\frac{dV_i\,(s)}{d\theta} = \frac{\partial V_i\,(s)}{\partial c^i\,(s)} \frac{du_{i|c}\,(s)}{d\theta} + \sum_{s'} \frac{\partial V_i\,(s)}{\partial V^i\,(s')} \frac{dV_i\,(s')}{d\theta}, \tag{OA3}$$

where

$$\frac{\partial V_i\,(s)}{\partial c^i\,(s)} = (1 - \beta_i)\,(V_i\,(s))^{\frac{1}{\psi}}\,(u_i\,(s))^{-\frac{1}{\psi_i}}\,\frac{\partial u_i\,(s)}{\partial c^i}$$

$$\frac{\partial V_i\,(s)}{\partial V^i\,(s')} = \beta_i\,(V_i\,(s))^{\frac{1}{\psi}}\left(\sum_{s'} \pi\,(s'|s)\left(V^i\,(s')\right)^{1-\gamma_i}\right)^{\frac{\gamma_i - \frac{1}{\psi_i}}{1-\gamma_i}} \pi\,(s'|s)\left(V^i\,(s')\right)^{-\gamma_i},$$

and where $\frac{du_{i|c}(s)}{d\theta}$ is defined as in Equation (3). The structure of Equation (OA3) immediately implies that $\frac{dV_i(s)}{d\theta}$ can be expressed as a linear transformation of instantaneous consumption-equivalent effects, $\frac{du_{i|c}(s)}{d\theta}$, which in turn guarantees that the definition of a DS-planner in Equation (6) can also be used in the context of economies with recursive preferences.

Note that it is straightforward to define normalized DS-weights when considering normalized welfarist planners, as in Section 4. In particular, Equations (16), (17), and (19) remain valid, and the one-period version of Equation (18), from which it is straightforward to compute state-prices for any date/state, becomes

$$q^i\,(s'|s) = \frac{\frac{\partial V_i(s)}{\partial c^i(s')}}{\frac{\partial V_i(s)}{\partial c^i(s)}} = \frac{\frac{\partial V_i(s)}{\partial V^i(s')}\frac{\partial V^i(s')}{\partial c^i(s')}}{\frac{\partial V_i(s)}{\partial c^i(s)}} = \beta_i \pi\,(s'|s)\left(\frac{V_i\,(s')}{H\,(s)}\right)^{\frac{1}{\psi}-\gamma_i}\left(\frac{c^i\,(s')}{c^i\,(s)}\right)^{-\frac{1}{\psi_i}}\frac{\frac{\partial u_i(s')}{\partial c^i}}{\frac{\partial u_i(s)}{\partial c^i}},$$

where $H\,(s) \equiv \left(\sum_{s'} \pi\,(s'|s)\left(V^i\,(s')\right)^{1-\gamma_i}\right)^{\frac{1}{1-\gamma_i}}$. Finally, note that it is straightforward to define DS-weights for even more general preferences, including preferences are not time-separable or recursive.

F.3 Policy changes that affect probabilities

In this section, we describe how to use DS-weights in environments in which policy changes affect probabilities. Starting from Equation (2), note that we can express $\frac{dV_i(s_0)}{d\theta}$ as follows

$$\frac{dV_i\,(s_0)}{d\theta} = \sum_{t=0}^{T} (\beta_i)^t \sum_{s^t} \pi_t\left(s^t\big|s_0\right) \frac{\partial u_i\,(s^t)}{\partial c_t^i}\left(\frac{du_{i|c}\,(s^t)}{d\theta} + \frac{d\ln\pi_t\left(s^t\big|s_0\right)}{d\theta}\frac{u_i\,(c_t^i\,(s^t),n_t^i\,(s^t))}{\frac{\partial u_i(s^t)}{\partial c_t^i}}\right).$$

Hence, we can use the following definition of a DS-planner in this case.

Definition 6. *(Desirable policy change for a DS-planner) A DS-planner, that is, a planner who adopts DS-weights, finds a policy change desirable in an environment in which policies can also*

affect probabilities if and only if $\frac{dW(s_0)}{d\theta} > 0$, where

$$\frac{dW^{DS}(s_0)}{d\theta} = \int \sum_{t=0}^{T} \sum_{s^t} \omega_t^i \left(s^t \middle| s_0 \right) \left(\frac{du_{i|c}(s^t)}{d\theta} + \frac{d\ln \pi_t(s^t \middle| s_0)}{d\theta} \frac{u_i(c_t^i(s^t), n_t^i(s^t))}{\frac{\partial u_i(s^t)}{\partial c_t^i}} \right) di,$$

where $\frac{d\ln \pi_t(s^t \middle| s_0)}{d\theta} = \frac{\frac{d\pi_t(s^t \middle| s_0)}{d\theta}}{\pi_t(s^t \middle| s_0)}$.

Identical results apply in the case in which policy changes directly affect preferences. See Dávila and Goldstein (2021) for an application of the results of this paper to an environment in which policy changes have a discontinuous impact on payoffs.

F.4 Multiple commodities

In this section, we expand on how to use DS-weights in environments with multiple commodities. To do so, we define a generalized version of Equation (1), which includes multiply commodities, indexed by $\ell \in L$, as follows:

$$V_i(s_0) = \sum_{t=0}^{T} (\beta_i)^t \sum_{s^t} \pi_t \left(s^t \middle| s_0 \right) u_i \left(\left\{ c_t^{i,\ell}\left(s^t \right) \right\}_{\ell \in L} \right).$$

At this level of generality, the different commodities can represent hours worked, as in the baseline environment, different consumption goods, flow utility from housing, or any variable that directly impacts instantaneous utility. Without loss, we treat commodity 1 as the numeraire for the purpose of making welfare assessments, so we can express $\frac{dV_i(s_0)}{d\theta}$ as follows:

$$\frac{dV_i(s_0)}{d\theta} = \sum_{t=0}^{T} (\beta_i)^t \sum_{s^t} \pi_t \left(s^t \middle| s_0 \right) \frac{\partial u_i(s^t)}{\partial c_t^{i,1}} \sum_{\ell \in L} \frac{du_{i|c^1}(s^t)}{d\theta},$$

where $\frac{\partial u_i(s^t)}{\partial c_t^{i,\ell}} \equiv \frac{\partial u_i\left(\left\{ c_t^{i,\ell}(s^t) \right\}_{\ell \in L} \right)}{\partial c_t^{i,\ell}(s^t)}$ and where the instantaneous commodity-1-equivalent effect of the policy at date t given a history s^t, is given by $\frac{du_{i|c^1}(s^t)}{d\theta}$, where

$$\frac{du_{i|c^1}(s^t)}{d\theta} \equiv \frac{\frac{du_i\left(\left\{ c_t^{i,\ell}(s^t) \right\}_{\ell \in L} \right)}{d\theta}}{\frac{\partial u_i(s^t)}{\partial c_t^{i,1}}} = \frac{dc_t^{i,1}(s^t)}{d\theta} + \sum_{\ell \in L} \frac{\frac{\partial u_i(s^t)}{\partial c_t^{i,\ell}}}{\frac{\partial u_i(s^t)}{\partial c_t^{i,1}}} \frac{dc_t^{i,\ell}(s^t)}{d\theta}.$$

In general, when there are multiple commodities, it is necessary to account for the marginal rates of substitutions between those commodities and the commodity chosen as numeraire. It is worth making two remarks in this context. First, note that it is possible to choose a particular bundle of commodities as numeraire (e.g., this is natural when preferences are isoelastic and there is well-defined price-index). However, for the purpose of making welfare assessments, it must be that his

normalization is consistent across all individuals. Second, note that the choice of numeraire will not change the directional welfare assessment or a welfarist planner, but it can have an impact on the units of such assessments, as well as on the value of the components of the aggregate additive decomposition.

F.5 Intergenerational considerations

In this section, we describe how to use DS-weights in environments with births, deaths, bequest motives, and related considerations, which non-trivially affect welfare assessments — see Calvo and Obstfeld (1988), Farhi and Werning (2010), Heathcote, Storesletten and Violante (2017), or Phelan and Rustichini (2018). The most direct way of extending our baseline environment, is to interpret the set of individuals I considered in the baseline model as the set all individuals i) alive or ii) yet to be born from the perspective of s_0. Under that interpretation, $\frac{du_{i|c}(s^t)}{d\theta}$ is non-zero only for those alive at a given history, so definition 3 applies unchanged.

Bequest motives, altruism, warm-glow preferences, social discounting or similar considerations only impact welfare assessments via the choice of DS-weights. For instance, a welfarist planner who values future generations directly placing a positive weight on their welfare and that in turn perceives an effective social discount rate lower than the private one, can be modeled by choosing a particular set of DS-weights. While do not explore that possibility in this paper, there is scope to use the law of total covariance to internationally decompose the cross-sectional components of the aggregate additive decomposition.

G   Additional Results

G.1   Dimensional analysis

This paper puts great emphasis on the units in which different variables are defining. In this section, we carefully describe the units of the different components of individual multiplicative decomposition for a normalized welfarist planner and for a general DS-planner

Welfarist planners.   As we discuss in the text, the units of our formulation of DS-weights for the case of the normalized utilitarian planner have a clear interpretation in terms of dollars at different dates and histories. Here, we provide a systematic dimensional analysis (de Jong, 1967) of the welfare assessments made by a normalized utilitarian planner. We denote the units of a specific variable by $\dim(\cdot)$, where, for instance, $\dim\left(c_t^i\left(s^t\right)\right) =$ dollars at history s^t, where we interchangeably use dollars and units of the consumption good.

First, note that the units of $\tilde{\omega}_t^i\left(s^t\middle|s_0\right)$, $\tilde{\omega}_t^i\left(s_0\right)$, and $\tilde{\omega}^i\left(s_0\right)$, as defined in Equations (13), (14), and (15), are respectively given by

$$\dim\left(\tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\middle|s_0\right)\right) = \frac{\frac{\text{instantaneous utils at } s_0 \text{ for individual } i}{\text{dollars at history } s^t}}{\frac{\text{instantaneous utils at } s_0 \text{for individual } i}{\text{dollars at date } t}} = \frac{\text{dollars at date } t}{\text{dollars at history } s^t}$$

$$\dim\left(\tilde{\omega}_t^{i,\mathcal{W}}\left(s_0\right)\right) = \frac{\frac{\text{instantaneous utils at } s_0 \text{ for individual } i}{\text{dollars at date } t}}{\frac{\text{instantaneous utils at } s_0 \text{ for individual } i}{\text{dollars at all dates and histories}}} = \frac{\text{dollars at all dates and histories}}{\text{dollars at date } t}$$

$$\dim\left(\tilde{\omega}^{i,\mathcal{W}}\left(s_0\right)\right) = \frac{\frac{\text{instantaneous utils at } s_0}{\text{dollars at all dates and histories for individual } i}}{\frac{\text{instantaneous utils at } s_0}{\lambda_i(s_0) \text{ dollars at all dates and histories for all individuals}}}$$

$$= \frac{\lambda_i\left(s_0\right) \text{ dollars at all dates and histories for all individuals}}{\text{dollars at all dates and histories for individual } i}.$$

Note that $\lambda_i\left(s_0\right)$ in the numerator of $\tilde{\omega}^{i,\mathcal{W}}\left(s_0\right)$ does not change its units.[34] Note also that the term $\left(\beta_i\right)^t \pi_t\left(s^t\middle|s_0\right)\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}$, which defines the numerator of $\tilde{\omega}_t^i\left(s^t\middle|s_0\right)$, is measured in instantaneous utils at s_0 per dollars at history s^t for individual i, since

$$\dim\left(\left(\beta_i\right)^t\right) = \frac{\text{instantaneous utils at } s_0 \text{ for individual } i}{\text{instantaneous utils at history} s^t \text{ for individual } i}$$

$$\dim\left(\frac{\partial u_i\left(s^t\right)}{\partial c_t^i}\right) = \frac{\text{instantaneous utils at history } s^t \text{ for individual } i}{\text{dollars at history } s^t},$$

and probabilities, like $\pi_t\left(s^t\middle|s_0\right)$, are unitless, where the same logic applies to the remaining elements of $\tilde{\omega}_t^i\left(s^t\middle|s_0\right)$, $\tilde{\omega}_t^i\left(s_0\right)$, and $\tilde{\omega}^i\left(s_0\right)$.

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[34]From the perspective of aggregation of lifetime utilities, which takes places through the individual component $\tilde{\omega}^i\left(s_0\right)$, any welfarist planner has $|I| + 1$ degrees of freedom: the planner can give different weights to each of the $|I|$ individual assessments, and it can further normalize the units of aggregate welfare.

Consequently, it follows that

$$\dim\left(\tilde\omega_t^{i,\mathcal{W}}\left(s^t\middle|s_0\right)\right) = \dim\left(\tilde\omega^i\left(s_0\right)\tilde\omega_t^i\left(s_0\right)\tilde\omega_t^i\left(s^t\middle|s_0\right)\right) \tag{OA4}$$
$$= \frac{\lambda_i\left(s_0\right)\text{ dollars at all dates and histories for all individuals}}{\text{dollars at history }s^t}.$$

In words, the DS-weights $\tilde\omega_t^{i,\mathcal{W}}\left(s^t\middle|s_0\right)$ translates dollars at history s^t into $\lambda_i\left(s_0\right)$ dollars at all dates and histories for all individuals.

Second, note that the units of $\frac{du_{i|c}\left(s^t\right)}{d\theta}$ are given by

$$\dim\left(\frac{du_{i|c}\left(s^t\right)}{d\theta}\right) = \frac{\frac{\text{instantaneous utils at history }s^t\text{ for individual }i}{\text{unit of policy change}}}{\frac{\text{instantaneous utils at history }s^t\text{ for individual }i}{\text{dollars at history }s^t}} = \frac{\text{dollars at history }s^t}{\text{unit of policy change}}, \tag{OA5}$$

which follows directly from Equation (14).

Finally, combining Equations (OA4) and (OA5), it follows that

$$\dim\left(\frac{dW^{\mathcal{W}}\left(s_0\right)}{d\theta}\right) = \dim\left(\omega_t^i\left(s^t\middle|s_0\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}\right) = \frac{\lambda_i\left(s_0\right)\text{ dollars at all dates and histories for all individuals}}{\text{unit of policy change}}.$$
$$\tag{OA6}$$

Hence, the units of $W^{\mathcal{W}}$ for a normalized utilitarian planner are dollars paid to all individuals at all dates and histories. That is, if $\frac{dW^{NU}}{d\theta} = 7$ for a given policy change, the welfare gain associated with a marginal unit policy change is equivalent to paying 7 dollars to all individuals in the economy at all dates and states.

General DS-planners. The dimensional analysis in the case of general planners is similar. In this case, the welfare units of $\tilde\omega_t^{i,DS}\left(s^t\middle|s_0\right)$ can be directly computed as

$$\dim\left(\tilde\omega_t^{i,DS}\left(s^t\middle|s_0\right)\right) = \frac{\text{units of }W^{DS}}{\text{dollars at history }s^t}.$$

In this case, it is also possible to compute the units of each of the components of the individual multiplicative decomposition as we just did for welfarist planners. By doing so, it becomes clear that the units of each of the components of the individual multiplicative decomposition any normalized DS-planner (including those who are not welfarist) are identical.

Undesirable properties of unnormalized decompositions. As briefly explained in the text, using unnormalized individual multiplicative decompositions of DS-weights can be problematic in the context of the aggregate additive decomposition, since unnormalized decompositions are expressed in utils. This is not the case for normalized decompositions since these always make tradeoffs in dollar units.

For instance, if one were to set $\lambda_i\left(s_0\right) = 1$, $\forall i$, in the decomposition presented in Equation (10), the redistribution component of the aggregate additive decomposition would be zero, $\Xi_{RD} = 0$.

This result captures the fact that an unnormalized equal-weighted utilitarian planner is indifferent between redistribution across individuals in utility terms. Hence, by directly adding up utils, we would fail to capture the idea that a utilitarian planner does desire to redistribute resources (in consumption units) towards individuals with low marginal utility — see e.g., Salanie (2011). Similarly, if individual discount factors are identical, that is, $\beta_i = \beta$, $\forall i$, a welfarist planner under the decomposition presented in Equation (10) will conclude that intertemporal-sharing is zero, that is, $\Xi_{IS} = 0$, regardless of the form of the policy under consideration. Equally important, the dynamic and stochastic weights for a welfarist planner defined as in Equation (10) need not add up to 1. Hence, according to Proposition 3, even when the instantaneous consumption-equivalent effect of a policy change is identical across individuals at all dates and histories, an unnormalized utilitarian planner would typically find non-zero intertemporal-sharing components and redistribution components of the aggregate additive decomposition. This is is another undesirable property of the unnormalized utilitarian welfare criterion.

An alternative date-0 normalization. One of the contributions of this paper is to introduce the notion of a normalized planner — see Lemma 1, as one for which the stochastic, dynamic, and individual components of the multiplicative decomposition add up to 1 across specific dimensions. However, one can think of alternative normalizations. For instance, one may consider normalizing the individual welfare effect of a policy change by date-0 marginal utility. In that case, it is possible to decompose the weights of a welfarist planner as follows:

$$\tilde{\omega}_t^{i,\mathcal{W}}\left(s^t \,\middle|\, s_0\right) = \frac{\pi_t\left(s^t \,\middle|\, s_0\right) \frac{\partial u_i\left(s^t\right)}{\partial c_t^i}}{\sum_{s^t} \pi_t\left(s^t \,\middle|\, s_0\right) \frac{\partial u_i(s^t)}{\partial c_t^i}} = \frac{q_t^i\left(s^t | s_0\right)}{\sum_{s^t} q_t^i\left(s^t | s_0\right)} \tag{OA7}$$

$$\tilde{\omega}_t^{i,\mathcal{W}}\left(s_0\right) = \frac{(\beta_i)^t \sum_{s^t} \pi_t\left(s^t \,\middle|\, s_0\right) \frac{\partial u_i\left(s^t\right)}{\partial c_t^i}}{\frac{\partial u_i(s^0)}{\partial c_t^i}} = \sum_{t=0}^{T} \sum_{s^t} q_t^i\left(s^t | s_0\right) \tag{OA8}$$

$$\tilde{\omega}^{i,\mathcal{W}}\left(s_0\right) = \frac{\lambda_i\left(s_0\right) \frac{\partial u_i(s^0)}{\partial c_t^i}}{\int \lambda_i\left(s_0\right) \frac{\partial u_i(s^0)}{\partial c_t^i} di}, \tag{OA9}$$

This decomposition satisfies $\sum_{s^t} \tilde{\omega}_t^i\left(s^t | s_0\right) = 1$, $\forall t$, $\forall i$, and $\int \tilde{\omega}^i\left(s_0\right) di = 1$, but it is clear that $\sum_{t=0}^{T} \tilde{\omega}_t^i\left(s_0\right) \neq 1$. In terms of units, this decomposition adds up individual welfare effects according to $\tilde{\omega}^{i,\mathcal{W}}\left(s_0\right)$, once they are expressed in date-0 dollars, which may seem reasonable. However, in this case Proposition 3a) will note be valid if using this normalization. In particular, the redistribution component of the aggregate decomposition will note be zero for policies that are invariant across all individuals at all dates and histories.

G.2 α-DS-Planners

After substituting the definition of the components of the DS-weights, we can explicitly express welfare assessments for a α-DS-planner as follows:

$$\frac{dW^{\mathcal{W},\alpha}(s_0)}{d\theta} = \underbrace{\sum_{t=0}^{T} \mathbb{E}_i\left[\left(1-\alpha_3^i\right)\tilde{\omega}_t^{i,\mathcal{W},AE}(s_0) + \alpha_3^i\tilde{\omega}_t^{i,\mathcal{W}}(s_0)\right] \sum_{s^t} \mathbb{E}_i\left[\left(1-\alpha_2\right)\tilde{\omega}_t^{i,\mathcal{W},AE}\left(s^t\big| s_0\right) + \alpha_2\tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\big| s_0\right)\right] \mathbb{E}_i\left[\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]}_{=\Xi_{AE} \text{ (Aggregate Efficiency)}}$$

$$+ \underbrace{\sum_{t=0}^{T} \mathbb{E}_i\left[\left(1-\alpha_3^i\right)\tilde{\omega}_t^{i,\mathcal{W},AE}(s_0) + \alpha_3^i\tilde{\omega}_t^{i,\mathcal{W}}(s_0)\right] \sum_{s^t} \mathbb{C}ov_i\left[\left(1-\alpha_2\right)\tilde{\omega}_t^{i,\mathcal{W},AE}\left(s^t\big| s_0\right) + \alpha_2\tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\big| s_0\right), \frac{du_{i|c}\left(s^t\right)}{d\theta}\right]}_{=\Xi_{RS} \text{ (Risk-sharing)}}$$

$$+ \underbrace{\sum_{t=0}^{T} \mathbb{C}ov_i\left[\left(1-\alpha_3^i\right)\tilde{\omega}_t^{i,\mathcal{W},AE}(s_0) + \alpha_3^i\tilde{\omega}_t^{i,\mathcal{W}}(s_0), \sum_{s^t}\left(\left(1-\alpha_2\right)\tilde{\omega}_t^{i,\mathcal{W},AE}\left(s^t\big| s_0\right) + \alpha_2\tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\big| s_0\right)\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}\right]}_{=\Xi_{IS} \text{ (Intertemporal-sharing)}}$$

$$+ \underbrace{\mathbb{C}ov_i\left[\left(1-\alpha_4\right)\tilde{\omega}^{i,\mathcal{W},AE}(s_0) + \alpha_4\tilde{\omega}^{i,\mathcal{W}}(s_0), X\right]}_{=\Xi_{RD} \text{ (Redistribution)}}, \tag{OA10}$$

where

$$X = \sum_{t=0}^{T}\left(\left(1-\alpha_3\right)\tilde{\omega}_t^{i,\mathcal{W},AE}(s_0) + \alpha_3\tilde{\omega}_t^{i,\mathcal{W}}(s_0)\right)\sum_{s^t}\left(\left(1-\alpha_2\right)\tilde{\omega}_t^{i,\mathcal{W},AE}\left(s^t\big| s_0\right) + \alpha_2\tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\big| s_0\right)\right)\frac{du_{i|c}\left(s^t\right)}{d\theta}.$$

Note that the notion of α-DS-planner introduced in Definition 4 is designed so that the following properties are satisfied:

$$\mathbb{E}_i\left[\tilde{\omega}_t^{i,\mathcal{W},\alpha}\left(s^t\big| s_0\right)\right] = \tilde{\omega}_t^{i,\mathcal{W},AE}\left(s^t\big| s_0\right) = \mathbb{E}_i\left[\tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\big| s_0\right)\right]$$
$$\mathbb{E}_i\left[\tilde{\omega}_t^{i,\mathcal{W},\alpha}(s_0)\right] = \tilde{\omega}_t^{i,\mathcal{W},AE}(s_0) = \mathbb{E}_i\left[\tilde{\omega}_t^{i,\mathcal{W}}(s_0)\right]$$
$$\mathbb{E}_i\left[\tilde{\omega}^{i,\mathcal{W},\alpha}(s_0)\right] = \tilde{\omega}^{i,\mathcal{W},AE}(s_0) = \mathbb{E}_i\left[\tilde{\omega}^{i,\mathcal{W}}(s_0)\right].$$

Hence, Equation (OA10) implies that when $\alpha = (0,0,0)$, we have an AE pseudo-welfarist DS-planner; when $\alpha = (1,0,0)$, we have an AR pseudo-welfarist DS-planner; when $\alpha = (1,1,0)$, we have a NR pseudo-welfarist DS-planner; and when $\alpha = (1,1,1)$, we have a welfarist planner. We summarize this results in Table OA-1.

Table OA-1: α-DS-planner: Special cases

$(\alpha_2,\alpha_3,\alpha_4)$	$\tilde{\omega}_t^i\left(s^t\big| s_0\right)$	$\tilde{\omega}_t^i(s_0)$	$\tilde{\omega}^i(s^0)$	Planner
$(1,1,1)$	$\tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\big| s_0\right)$	$\tilde{\omega}_t^{i,\mathcal{W}}(s_0)$	$\tilde{\omega}^{i,\mathcal{W}}(s_0)$	$\mathcal{W}$
$(1,1,0)$	$\tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\big| s_0\right)$	$\tilde{\omega}_t^{i,\mathcal{W}}(s_0)$	$\tilde{\omega}^{i,\mathcal{W},AE}(s_0)$	NR
$(1,0,0)$	$\tilde{\omega}_t^{i,\mathcal{W}}\left(s^t\big| s_0\right)$	$\tilde{\omega}_t^{i,\mathcal{W},AE}(s_0)$	$\tilde{\omega}^{i,\mathcal{W},AE}(s_0)$	NS
$(0,0,0)$	$\tilde{\omega}_t^{i,\mathcal{W},AE}\left(s^t\big| s_0\right)$	$\tilde{\omega}_t^{i,\mathcal{W},AE}(s_0)$	$\tilde{\omega}^{i,\mathcal{W},AE}(s_0)$	AE

Note: Note that all the α-DS-planners considered in this table are pseudo-welfarist.

However, note that there are other possible extreme combinations of α that one may want to

consider, these are the following:

$$\{(1,0,1),(0,1,0),(0,1,1),(0,0,1)\}. \tag{OA11}$$

The problem with the $\boldsymbol{\alpha}$'s in Equation (OA11) is that, as long as one of the first two elements of $\boldsymbol{\alpha}$ are 0, the redistribution component will be different from the redistribution component of the relevant welfarist planner. Hence, these choices of $\boldsymbol{\alpha}$ are not pseudo-welfarist. Hence, those $\boldsymbol{\alpha}$-DS-planners will not pseudo-welfarist, but there are perfectly valid DS-planners.

G.3 Relation to existing welfare approaches

G.3.1 Welfarist Social Welfare Functions

In addition to the utilitarian SWF, defined in Equation (5), there are other welfarist SWF's that are at times used in specific applications — see e.g., Mas-Colell, Whinston and Green (1995), Kaplow (2011), or Adler and Fleurbaey (2016) for details. Here we briefly described those.

The *isoelastic* SWF, commonly traced back to Atkinson (1970), is given by

$$\mathcal{W}\left(\{V_i(s_0)\}_{i\in I}\right) = \left(\int a_i\,(V_i(s_0))^\phi\,di\right)^{1/\phi},$$

where the (inequality) coefficient ϕ is typically restricted to lie in $[-\infty,1]$, so that $\mathcal{W}(\cdot)$ is concave when $V_i(s_0) > 0$, and where it is typically assumed that $\int a_i di = 1$, and that $a_i \geq 0, \forall i.$[35] Limiting cases of the isoelastic SWF correspond to the other four widely used SWF's. First, when $\phi \to 1$, the isoelastic SWF becomes the conventional *utilitarian* SWF. In that case:

$$\mathcal{W}\left(\{V_i(s_0)\}_{i\in I}\right) = \int a_i V_i(s_0)\,di.$$

Second, when $\phi \to 0$, the isoelastic SWF becomes the *Nash* (Cobb-Douglas) SWF. In that case:

$$\mathcal{W}\left(\{V_i(s_0)\}_{i\in I}\right) = \int (V_i(s_0))^{a_i}\,di.$$

Third, when $\phi \to -\infty$, the isoelastic SWF becomes the *Rawlsian/maximin* (Leontief) SWF. In that case:

$$\mathcal{W}\left(\{V_i(s_0)\}_{i\in I}\right) = \min\left\{\dots,\frac{V_i(s_0)}{a_i},\dots\right\}.$$

Finally, when the isoelastic SWF gives positive weight to a single individual, it can be interpreted

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[35]Note that, for an isoelastic SWF, $\frac{\partial \mathcal{W}}{\partial V_i} = a_i\left(\frac{V_i}{\mathcal{W}}\right)^{\phi-1}$. More importantly $\frac{\frac{\partial \mathcal{W}}{\partial V_i}}{\frac{\partial \mathcal{W}}{\partial V_j}} = \frac{a_i}{a_j}\left(\frac{V_i}{V_j}\right)^{\phi-1}$. When lifetime utilities are negative, it is possible to define an isoelastic SWF of the form

$$\mathcal{W}\left(\{V_i(s_0)\}_{i\in I}\right) = \left(\int a_i\,(-V_i(s_0))^\phi\,di\right)^{1/\phi},$$

by considering $\phi \in [1,\infty]$.

as a *dictatorial* SWF. In that case:

$$\mathcal{W}\left(\{V_i\left(s_0\right)\}_{i \in I}\right) = V_1\left(s_0\right).$$

Note that all of these SWF are Paretian, although the Rawlsian/maximin and the dictatorial SWF's are not strictly Paretian.[36]

G.3.2 Relation to Saez and Stantcheva (2016) and Kaldor/Hicks compensation principle

It is straightforward to define welfare assessments in our framework that are based on the approach introduced by Saez and Stantcheva (2016).

Definition 7. *(Desirable policy change for a planner who uses generalized social marginal welfare weights (Saez and Stantcheva, 2016)) A planner who uses generalized social marginal welfare weights finds a policy change desirable if and only if $\frac{dW^{SS}(s_0)}{d\theta} > 0$, where*

$$\frac{dW^{SS}\left(s_0\right)}{d\theta} = \int h_i\left(\cdot\right) \frac{dV_i\left(s_0\right)}{d\theta} di, \tag{OA12}$$

where $h_i\left(\cdot\right) > 0$, $\forall i \in I$, are a collection of individual-specific positive functions, and where $\frac{dV_i(s_0)}{d\theta}$ is defined in Equation (2).

By comparing Equation (OA12) with Equation (6), it is evident that the approach based on generalized social marginal welfare weights introduced in Saez and Stantcheva (2016) is more general than the welfarist approach. The key difference between the two approaches is that for welfarist planners the functions $h_i\left(\cdot\right)$ are restricted to take the form

$$h_i\left(\cdot\right) = \frac{\partial \mathcal{W}\left(\{V_i\left(s_0\right)\}_{i \in I}\right)}{\partial V_i},$$

while $h_i\left(\cdot\right)$ can take many other values under the Saez and Stantcheva (2016) approach. Saez and Stantcheva (2016) show that their approach can capture alternatives to welfarism, such as libertarianism or equality of opportunity. It is also evident from definition 7 that a planner who uses generalized social marginal welfare weights is not paternalistic, since welfare assessments are based on individual lifetime welfare effects, $\frac{dV_i(s_0)}{d\theta}$.

Finally, it is worth highlighting that the classic Kaldor/Hicks (Kaldor, 1939; Hicks, 1939) compensation principle can be formalized in marginal form by setting $h_i = 1$, $\forall i$ in Equation (OA12) — see e.g., (Hendren, 2020). This observation implies that the Kaldor/Hicks welfare criterion can also be formalized as a particular DS-planner. In fact, when the Kaldor/Hicks compensation is defining in units of a permanent dollars (dollars across all dates and states), the Kaldor/Hicks welfare criterion exactly correspond to the pseudo-welfarist NR planner introduced in Section 5

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[36]A planner with an isoelastic SWF is strictly Paretian when $\phi > -\infty$ if $a_i > 0$, $\forall i$.

G.3.3 Relation to Lucas (1987)/Alvarez and Jermann (2004)

It is common in papers that study the welfare consequences of policies in dynamic and stochastic environments to compute welfare gains/losses of policies as in Lucas (1987), who measures the welfare gains associated with a policy change — specifically, the welfare gains associated with eliminating business cycles. Since our approach is built on marginal arguments, we connect instead our results to those in Alvarez and Jermann (2004), who provide a marginal formulation of the approach in Lucas (1987).

    While the Lucas (1987)/Alvarez and Jermann (2004) approach is easily interpretable in representative agent economies, it has the pitfall that it cannot be meaningfully aggregated when there are heterogeneous individuals. See, for instance, how Atkeson and Phelan (1994), Krusell and Smith (1999), or Krusell et al. (2009) carefully avoid aggregating welfare gains/losses for different individuals.

    To illustrate these arguments, here we consider a policy change for a given individual i, who could be a representative agent or not. Formally, we consider a special case of the environment laid out in Section 3, in which an individual i has preferences given by

$$V_i\left(s_0\right) = \sum_{t=0}^{T} \left(\beta_i\right)^t \sum_{s^t} \pi_t\left(s^t \mid s_0\right) u_i\left(c_t^i\left(s^t\right)\right).$$

We suppose that the consumption of individual i at date t and history s^t can be written as

$$c_t^i\left(s^t\right) = \left(1-\theta\right) \underline{c_t^i}\left(s^t\right) + \theta \overline{c_t^i}\left(s^t\right),$$

where both $\underline{c_t^i}\left(s^t\right)$ and $\overline{c_t^i}\left(s^t\right)$ are sequences measurable with respect to history s^t. The sequence $\underline{c_t^i}\left(s^t\right)$ can be interpreted as a given initial consumption path (when $\theta = 0$) and the sequence $\overline{c_t^i}\left(s^t\right)$ can be interpreted as a final consumption path (when $\theta = 1$). In the case of Lucas (1987), $\theta = 1$ corresponds to fully eliminating business cycles.

    First, we compute the marginal gains from marginally reducing business cycles, as in Alvarez and Jermann (2004). Next, we compute the marginal gains from marginally reducing business cycles using an additive compensation.

Multiplicative compensation. Lucas (1987) proposes using a time-invariant compensating variation, expressed multiplicatively as a constant fraction of consumption at each date/history as follows

$$\sum_{t=0}^{T} \left(\beta_i\right)^t \sum_{s^t} \pi_t\left(s^t \mid s_0\right) u_i\left(c_t^i\left(s^t\right)\left(1+\lambda\left(\theta\right)\right)\right) = \sum_{t=0}^{T} \left(\beta_i\right)^t \sum_{s^t} \pi_t\left(s^t \mid s_0\right) u_i\left(\left(1-\theta\right) \underline{c_t^i}\left(s^t\right) + \theta \overline{c_t^i}\left(s^t\right)\right),$$

$$\tag{OA13}$$

where $\lambda\left(\theta\right)$ implicitly defines the welfare gains associated with a policy indexed by θ; the exact

definition in Lucas (1987) exactly corresponds to solving for $\lambda \left(\theta = 1 \right)$.[37]

Following Alvarez and Jermann (2004), we can compute the derivative of the RHS of Equation (OA13) as follows:

$$\frac{d \left(\left(\beta_i \right)^t \sum_{s^t} \pi_t \left(s^t | s_0 \right) u_i \left(\left(1 - \theta \right) \underline{c}_t^i \left(s^t \right) + \theta \overline{c}_t^i \left(s^t \right) \right) \right)}{d\theta} = \sum_{t=0}^{T} \left(\beta_i \right)^t \sum_{s^t} \pi_t \left(s^t | s_0 \right) u_i' \left(\left(1 - \theta \right) \underline{c}_t^i \left(s^t \right) + \theta \overline{c}_t^i \left(s^t \right) \right) \frac{d c_t^i \left(s^t \right)}{d\theta}$$

(OA14)

where here $\frac{d c_t^i \left(s^t \right)}{d\theta} = \overline{c}_t^i \left(s^t \right) - \underline{c}_t^i \left(s^t \right)$.

Analogously, we can also compute the derivative of the LHS of Equation (OA13) as follows:

$$\frac{d \left(\sum_{t=0}^{T} \left(\beta_i \right)^t \sum_{s^t} \pi_t \left(s^t | s_0 \right) u_i \left(c_t^i \left(s^t \right) \left(1 + \lambda \left(\theta \right) \right) \right) \right)}{d\theta} = \sum_{t=0}^{T} \left(\beta_i \right)^t \sum_{s^t} \pi_t \left(s^t | s_0 \right) u_i' \left(c_t^i \left(s^t \right) \left(1 + \lambda \left(\theta \right) \right) \right) c_t^i \left(s^t \right) \lambda' \left(\theta \right).$$

(OA15)

Hence, combining Equations (OA14) and (OA15) and solving for $\frac{d\lambda}{d\theta} = \lambda' \left(\theta \right)$, yields the marginal cost of business cycles, as defined in Alvarez and Jermann (2004). Formally, we can express $\frac{d\lambda}{d\theta}$ as follows

$$\begin{aligned} \frac{d\lambda}{d\theta} = \lambda' \left(\theta \right) &= \frac{\sum_{t=0}^{T} \left(\beta_i \right)^t \sum_{s^t} \pi_t \left(s^t | s_0 \right) u_i' \left(\left(1 - \theta \right) \underline{c}_t^i \left(s^t \right) + \theta \overline{c}_t^i \left(s^t \right) \right) \frac{d c_t^i \left(s^t \right)}{d\theta}}{\sum_{t=0}^{T} \left(\beta_i \right)^t \sum_{s^t} \pi_t \left(s^t | s_0 \right) u_i' \left(c_t^i \left(s^t \right) \left(1 + \lambda \left(\theta \right) \right) \right) c_t^i \left(s^t \right)} \\ &= \sum_{t=0}^{T} \sum_{s^t} \omega_t^i \left(s^t | s_0 \right) \frac{d c_t^i \left(s^t \right)}{d\theta}, \end{aligned}$$

(OA16)

where the second line shows how to reformulate $\frac{d\lambda}{d\theta}$ in terms of DS-weights given by

$$\omega_t^i \left(s^t | s_0 \right) = \frac{\left(\beta_i \right)^t \pi_t \left(s^t | s_0 \right) u_i' \left(\overline{c}_t^i \left(s^t \right) + \theta \overline{\Delta c_t^i} \left(s^t \right) \right)}{\sum_{t=0}^{T} \left(\beta_i \right)^t \sum_{s^t} \pi_t \left(s^t | s_0 \right) u_i' \left(c_t^i \left(s^t \right) \left(1 + \lambda \left(\theta \right) \right) \right) c_t^i \left(s^t \right)}.$$

(OA17)

Additive compensation. Here, we would like to contrast the approach in Lucas (1987) to one that relies on a time-invariant compensating variation, expressed additively in terms of consumption at each date/history as follows:

$$\sum_{t=0}^{T} \left(\beta_i \right)^t \sum_{s^t} \pi_t \left(s^t | s_0 \right) u_i \left(c_t^i \left(s^t \right) + \lambda \left(\theta \right) \right) = \sum_{t=0}^{T} \left(\beta_i \right)^t \sum_{s^t} \pi_t \left(s^t | s_0 \right) u_i \left(\left(1 - \theta \right) \underline{c}_t^i \left(s^t \right) + \theta \overline{c}_t^i \left(s^t \right) \right).$$

(OA18)

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[37]Note that one could also define an alternative compensation as

$$\sum_{t=0}^{T} \left(\beta_i \right)^t \sum_{s^t} \pi_t \left(s^t | s_0 \right) u_i \left(c_t^i \left(s^t \right) \right) = \sum_{t=0}^{T} \left(\beta_i \right)^t \sum_{s^t} \pi_t \left(s^t | s_0 \right) u_i \left(\left(\overline{c}_t^i \left(s^t \right) + \theta \overline{\Delta c_t^i} \left(s^t \right) \right) \left(1 + \lambda \left(\theta \right) \right) \right).$$

In this case, we can follow the same steps as above to find the counterpart of Equation (OA16), which is given by

$$\frac{d\lambda}{d\theta} = \lambda'(\theta) = \frac{\sum_{t=0}^{T}(\beta_i)^t \sum_{s^t} \pi_t(s^t| s_0) u_i'\left((1-\theta)\,\underline{c}_t^i(s^t) + \theta\overline{c_t^i}(s^t)\right)\frac{dc_t^i(s^t)}{d\theta}}{\sum_{t=0}^{T}(\beta_i)^t \sum_{s^t} \pi_t(s^t| s_0) u_i'\left(c_t^i(s^t) + \lambda(\theta)\right)c_t^i(s^t)}$$

$$= \sum_{t=0}^{T}\sum_{s^t}\omega_t^i\left(s^t\Big| s_0\right)\frac{dc_t^i(s^t)}{d\theta}, \tag{OA19}$$

where the second line shows how to reformulate $\frac{d\lambda}{d\theta}$ in terms of DS-weights given by

$$\omega_t^i\left(s^t\Big| s_0\right) = \frac{(\beta_i)^t \pi_t(s^t| s_0) u_i'\left((1-\theta)\,\underline{c}_t^i(s^t) + \theta\overline{c_t^i}(s^t)\right)}{\sum_{t=0}^{T}(\beta_i)^t \sum_{s^t} \pi_t(s^t| s_0) u_i'\left(c_t^i(s^t) + \lambda(\theta)\right)}. \tag{OA20}$$

Comparison and implications. We focus on comparing Equations (OA16) and (OA19) in the case of $\theta = 0$ — similar insights emerge when $\theta \neq 0$. When $\theta = 0$, Equations (OA17) and (OA20) become

$$\omega_t^i\left(s^t\Big| s_0\right) = \frac{(\beta_i)^t \pi_t(s^t| s_0) u_i'\left(\underline{c}_t^i(s^t)\right)}{\sum_{t=0}^{T}(\beta_i)^t \sum_{s^t} \pi_t(s^t| s_0) u_i'\left(\underline{c}_t^i(s^t)\right)\underline{c}_t^i(s^t)} \quad (\text{multiplicative} \Rightarrow \text{Lucas/Alvarez-Jermann}) \tag{OA21}$$

$$\omega_t^i\left(s^t\Big| s_0\right) = \frac{(\beta_i)^t \pi_t(s^t| s_0) u_i'\left(\underline{c}_t^i(s^t)\right)}{\sum_{t=0}^{T}(\beta_i)^t \sum_{s^t} \pi_t(s^t| s_0) u_i'\left(\underline{c}_t^i(s^t)\right)}. \quad (\text{additive} \Rightarrow \text{normalized welfarist DS-planner}) \tag{OA22}$$

Two major insights emerge from Equations (OA21) and (OA22). First, the DS-weights defined for the additive case in Equation (OA22) exactly correspond to the dynamic and stochastic components of DS-weights for a normalized utilitarian planner, as defined in Equations (13) and (14). Second, note that the denominator of the DS-weights in the multiplicative case includes $\underline{c}_t^i(s^t)$ at all dates and histories. This captures the fact that the welfare assessment is computed as a fraction of consumption at each date/history, not in units of the consumption good/dollars. The presence of $\underline{c}_t^i(s^t)$ in the denominator is what complicates the aggregation of welfare assessments that follow Lucas (1987), because the denominator of the multiplicative decomposition is expressed on units of permanent transfer of $\underline{c}_t^i(s^t)$ units of consumption for individual i across dates and histories, not simply a transfer of dollars.

Relation to EV, CV, and CS. Finally, note that the analysis in this section illustrates how the marginal approach relates to the conventional approaches in classic demand theory: equivalent variation (EV), compensating variation (CV), and consumer surplus (CS).

The approach of Lucas (1987)/Alvarez and Jermann (2004), and the alternative version described

in Footnote 37 are the dynamic counterpart of compensating and equivalent variations, expressed in proportional terms, in a dynamic stochastic environment. Hence, the analysis of this section shows that a DS-planner can be used to operationalize the counterpart of all three notions — either proportionally or additively — in dynamic stochastic environments. As expected, these considerations only matter away from the $\theta = 0$ case. However, the most straightforward approach to make global assessments, described in Section G.5, corresponds to a consumer surplus approach.

G.3.4 Relation to welfare assessments that involve transfers

Finally, it was worth discussing how having the ability to costlessly transfer resources across individuals impact the welfare assessments of a DS-planner. To do so, we consider an environment in which a DS-planner has access to a set of transfers $T_i^i\left(s^t\right)$, so that investors budget constraints have the form

$$c_t^i\left(s^t\right) = T_i^i\left(s^t\right) + \ldots.$$

In that case, it follows immediately that

$$\frac{dW^{DS}\left(s_0\right)}{dT_i^i\left(s^t\right)} = \omega_t^i\left(s^t \middle| s_0\right).$$

Hence, having transfers available will endogenously impose restrictions across the DS-weights of different individuals. For instance, a planner who can transfer resources freely across all individuals, at all dates and states have identical DS-weights across all individuals. Given Proposition 2, this implies that this planner will only value aggregate efficiency. Similar conclusions can be reached when a DS-planner only has a access to a subset of transfers.

G.4 Optimal policy problems using DS-weights

Throughout most of the paper we have focused on how to make welfare assessments. Here, we show how it is straightforward to use DS-weights in the context of optimal policy problems, both in primal and in dual forms. To do so, we consider an environments in which a planner chooses a set of policy instruments $\boldsymbol{\tau}$ to maximize social welfare, which depends on allocations $\boldsymbol{X}\left(\boldsymbol{\tau}\right)$. We consider two possibilities.

First, we consider a primal problem, in which a planner maximizes social welfare $W\left(\boldsymbol{X}\left(\boldsymbol{\tau}\right)\right)$, subject to a set of implementability conditions, $\boldsymbol{H}\left(\boldsymbol{X}, \boldsymbol{\tau}\right)$.[38] Consistent with Section 6.4, we assume that $W\left(\boldsymbol{X}\left(\boldsymbol{\tau}\right)\right)$ corresponds to an instantaneous SWF. In this case, the planner solves

$$\min_{\boldsymbol{\lambda}} \max_{\boldsymbol{X}, \boldsymbol{\tau}} W\left(\boldsymbol{X}\right) + \boldsymbol{\lambda} \boldsymbol{H}\left(X, \boldsymbol{\tau}\right),$$

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[38]While social welfare is a scalar, bold variables can be vectors/matrices.

with optimality conditions for $\boldsymbol{\tau}$ given by

$$\frac{\partial W}{\partial \boldsymbol{X}} + \boldsymbol{\lambda} \frac{\partial \boldsymbol{H}}{\partial \boldsymbol{X}} = 0. \tag{OA23}$$

Second, we consider a dual problem, in which a planner maximizes social welfare $W\left(\boldsymbol{X}^{\star}\left(\boldsymbol{\tau}\right)\right)$, where $\boldsymbol{X}^{\star}\left(\boldsymbol{\tau}\right)$ denotes the equilibrium mapping implicitly defined as $\boldsymbol{H}\left(\boldsymbol{X}^{\star}\left(\boldsymbol{\tau}\right), \boldsymbol{\tau}\right) = 0$. In this case, the planner solves

$$\max_{\boldsymbol{\tau}} W\left(\boldsymbol{X}^{\star}\left(\boldsymbol{\tau}\right)\right),$$

with optimality conditions for $\boldsymbol{\tau}$ given

$$\frac{\partial W}{\partial \boldsymbol{X}} \frac{d\boldsymbol{X}^{\star}}{d\boldsymbol{\tau}} = 0. \tag{OA24}$$

In both cases, it is necessary to characterize $\frac{\partial W}{\partial \boldsymbol{X}}$ to find optimal policies. Hence, by defining $\frac{\partial W}{\partial \boldsymbol{X}}$ as in Definition 3, it is straightforward to find optimal policies for different DS-planners. As a final remark, note that, consistently with Section 6.4, it is important to understand that one cannot define a conventional SWF from the onset, DS-weights must be introduced at the marginal level in Equations (OA23) and (OA24).

G.5 Global welfare assessments

In the body of the paper, we have focused on marginal welfare assessments. However, one may be interested in exploring the impact of non-marginal welfare assessments. To do so, we assume that any policy change can be scaled by $\theta \in [0, 1]$, where θ corresponds to no policy change, while $\theta = 1$ corresponds to a global non-marginal change.

Formally, we can define a non-marginal welfare change as follows:

$$W^{DS}\left(s_0; \theta = 1\right) - W^{DS}\left(s_0; \theta = 0\right) = \int_0^1 \frac{dW^{DS}\left(s_0; \theta\right)}{d\theta} d\theta,$$

where we explicitly make θ an argument of $\frac{dW^{DS}(s_0;\theta)}{d\theta}$, as follows:

$$\frac{dW^{DS}\left(s_0; \theta\right)}{d\theta} = \int \sum_{t=0}^{T} \sum_{s^t} \omega_t^i \left(s^t \middle| s_0; \theta\right) \frac{du_{i|c}\left(s^t; \theta\right)}{d\theta} di. \tag{OA25}$$

Equation (OA25) points towards two different issues that need to be dealt with when using DS-weights to make global assessments. First, it is important to understand how $\frac{du_{i|c}(s^t;\theta)}{d\theta}$ changes with θ. Computing this term is straightforward, involves no normative assessment, and can be easily operationalized by solving for the equilibrium of an economy for different values of θ.

Second, it is important to understand how the DS-weights and its components (individual, dynamic, stochastic), vary with θ. For instance, a normalized utilitarian planner could select DS-weights normalized at $\theta = 0$, or normalized along the path of θ. This issue is closely related to the

distinction between consumer surplus, equivalent variation, and compensating variation in classic demand theory. In our applications, we assume that the DS-weights are computed for each value of θ, which is akin to adopting a consumer surplus approach, as we discuss in Section F of the Online Appendix. This is most straightforward approach to making global assessments, although one could use the same methodology as Alvarez and Jermann (2004) to consider equivalent/compensating variation-like global assessments.