Information Design in Common Value Auction with Moral Hazard: Application to OCS Leasing Auctions*

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Abstract

This paper explores the extent to which information design can increase the auctioneer's revenue in the US offshore oil/gas lease auctions. In our setting, firms first submit cash bids in a first-price sealed-bid auction; the winning firm then decides whether to explore a tract, and the government receives a royalty payment on the production value of the tract. We first document that when all the bids are revealed to the winning bidder after the auction, the exploration rate is positively correlated with the losing bids, suggesting that the winning bidder utilizes the rivals' bids to infer their private information on the tract's production value. We then characterize the equilibrium bidding strategy in an environment in which the auctioneer can design and commit to how to reveal information on the losing bids to the winning bidder. Our counterfactual exercises show that information design can significantly improve the auctioneer's revenue. For example, fully withholding information on the losing bids results in a $80 million increase in the auctioneer's annual revenue.

Keywords: Information Design; Common-Value Auction
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1 Introduction

Many economic questions involve understanding how to alter an economic agent's behavior. Besides using monetary transfers, another useful way of changing behavior is strategically controlling the agent's information, thus affecting the action he takes. Since the work of Kamenica and Gentzkow (2011), there is a rapidly growing theoretical literature on how to optimally design information revelation policy. However, there is a dearth of empirical research quantifying the potential benefit of information design.

This paper provides an empirical analysis of how information on the losing bids should be strategically revealed in a common value auction.[1] Our setting is the US Outer Continental Shelf (OCS) leasing auction from 2000 to 2019. In an OCS leasing auction, the auctioneer (the government) auctions off the right to explore and produce oil and gas on federal offshore tracts via first-price sealed-bid auctions. In addition to receiving the winning cash bid from the auction, the government also charges a royalty on the production value of oil and gas extracted from the tract, and unexplored tracts do not produce oil and gas. Therefore, the government's revenue also depends on whether the winning bidder conducts exploratory drilling.

Two empirical observations render it potentially beneficial for the auctioneer to restrict the winning bidder's access to the losing bids' information. First, in our data period, the government fully discloses all the bids' values and the identities of the bidders, and the exploratory drilling rate is low—only 24.5% of leased tracts were explored. This low exploration rate is an ongoing concern for the federal government, and multiple policy adjustments aimed at improving the exploratory drilling rate have been proposed.[2] Second, our reduced-form analysis highlights a strong and positive correlation between the losing bids and the winning bidder's exploration likelihood. According to our regression results, a 1% increase in the second-highest bid leads to a 3.9 percentage point increase in the probability of exploratory drilling. This suggests that the winning bidder utilizes the rival firms' bids to infer the rivals' private information on the tract's production value. As a comparison, a 1% increase in the winning bid leads to only a 4.5 percentage point increase in the probability of exploratory

[1] The idea of controlling the information about the losing bids in auctions is not new. For example, in 2009, the FDIC stopped releasing the identity of losing bidders and the second-highest bid in the failed bank acquisition auctions, which sparked outrage from industry insiders (Fajt, 2009). Similarly, bidders on Google Ads auction were concerned about the lack of transparency on the information of losing bids (Joseph, 2020). In some cases, losing bidders prefer to withhold information on their bids. For instance, several US states block the release of information on their offers to host Amazon's second headquarter.

[2] For example, the 1995 Deepwater Royalty Relief Act provided large royalty suspension volumes to lessees of deep water tracts to promote exploration, development, and production.

drilling. As a firm's bid is increasing in its expectation of the tract's production value, this implies that the winning firm's exploration decision is similarly affected by the (inferred) information of its rival firms as by its own private information.

In this paper, we empirically study the effects of various bid disclosure policies on the government's revenue in the OCS leasing auctions. To do so, we first estimate a model of first-price sealed-bid pure common-value auction (à la Milgrom and Weber (1982)) for an offshore tract lease in which the winner also makes an exploratory drilling decision after the auction. The auctioneer is the government. Before the auction, each firm (bidder) receives a private signal about the production value of the tract. After the auction, all the submitted bids are revealed, and the winner observes the realization of the cost to engage in exploratory drilling in the tract. Based on this additional information, the winning firm then decides whether to engage in exploratory drilling. If a tract is explored, the tract's actual production value is realized, and the firm pays a royalty on the production value to the government based on a rate announced before the auction.

Any empirical analysis of information design requires knowing the agents' posterior beliefs. In our case, this is equivalent to identifying the firm's posterior belief of the production value of the tract conditional on the bidders' private signals. Our other objects of interest include the joint distribution of the firms' private signals and the distribution of the (post-auction) exploration cost. We identify these objects using two datasets. The first dataset is a publicly available dataset on the US offshore auction outcomes and the tracts' drilling and production activities post-auction. The second dataset is a proprietary dataset on all the contracts between the oil/gas firms and the offshore rig company employed by the oil/gas firms to engage in drilling. Subsequently, we use our estimates to conduct counterfactual analysis on the government's expected revenue when, before the auction, it can commit to a disclosure policy of the losing bids.

In our model, we can nonparametrically identify the firms' posterior beliefs of the production value of the tract conditional on the bidders' private signals. The main empirical challenge in identifying these posterior beliefs is that we need to separate the variation in the realized exploration cost, which is unobserved, from the variation in the observed exploratory drilling rate. We do this by constructing a cost shifter using the rig costs in past rig contracts associated with the nearby tracts. We show that this cost shifter is informative of the exploration decision and, by construction, independent of the latent tract's production value. Under the assumption that every bidder's prior belief (before observing its private signal) about the production value of the offshore tract is the observed distribution of the tracts' production values, we can identify the joint distribution of the bidders' private signals, the posterior belief of the winning bidder, and the distribution of the exploration cost.

We estimate the model in three steps. In the first step, for each auction, we estimate the bidders' prior beliefs using the realized production quantities and the exploration outcomes of the other tracts that were explored before the auction. In the second step, we use the observed bid distribution to construct the bidders' private signals. In the third step, we estimate the model using GMM, matching the realized bids and exploration decisions with the model's predictions.

Our contribution in this paper is two-fold. First, we provide a characterization of the equilibrium bidding strategy in such a common-value first-price auction model with ex post moral hazard for a large class of disclosure policies for the losing bids. Under a policy of full disclosure of the losing bids, the equilibrium bidding strategy resembles that of a standard common value auction, exhibiting the *"winner's curse"* phenomenon: winning the auction indicates that the winning firm is overly optimistic of the production value of the tract; anticipating this effect, every firm depresses its bid. When the losing bids are not expected to be fully disclosed after the auction, there are two effects on the equilibrium bidding strategy. The first effect is a negative effect on the bids, which arises because the winning firm will be making its exploration decision under less information, thus decreasing every firm's expected value of winning the auction. The second effect, which is positive, is more subtle. Regardless of the disclosure policy, winning the auction with a bid of b always reveals that all the losing bids are lower than b. Therefore, winning an auction with a higher bid increases the winner's optimism about the (undisclosed) values of the losing bids and, hence, its expectation of the tract's production value; in turn, this incentivizes the firm to bid higher. However, the second effect is always dominated by the first effect; thus, the bids are always lower than when there is full disclosure of the losing bids.

Our second contribution is quantifying the gains from various bid disclosure policies in the OCS leasing auctions. Withholding information on the losing bids creates a trade-off between the government's bid revenue and the royalty revenue. As the winning firm responds to "bad news" (i.e., low losing bids) by not engaging in exploratory drilling, by strategically pooling information between "goods news" (i.e., high losing bids) and bad news, the auctioneer can increase the expected exploratory drilling rate. In turn, the high royalty revenue could offset the loss in the bid revenue from withholding information about the losing bids. Preliminary results from the counterfactual analysis reveal that, under a complete non-disclosure policy of the losing bids, the bidders decrease their bids by 1.77%. However, the post-auction exploration rate increases by a 0.3 percentage point, which leads to a 1.50% increase in the government's royalty revenue; to achieve this increase in exploration rate with full disclosure of the losing bids, the government has to decrease its royalty rate by 37%. Overall, by committing not to disclose any information on the losing bids, the government earns an expected net increase in annual profit of 1.01%, which amounts to approximately $80 million each year.

2 Literature Review

Our paper contributes to the empirical literature on auction design with ex post actions. Earlier works, such as Athey and Levin (2001), Lewis and Bajari (2011), and Bajari et al. (2014), study the equilibrium bidding strategy in scoring auctions in which each bid is an incentive contract. For example, Athey and Levin (2001) study timber auctions in which firms bid a per-unit price for each timber species, and bidders have private information about volumes of species. In these papers, however, there is no ex post uncertainty. More recently, Bhattacharya et al. (2018) studies auction design for onshore oil auctions in which they model the winning bidders observing the production value of the lease ex post and strategically delaying production in response to oil price uncertainty. In our OCS setting, the incentive to strategically delay production is not a first-order issue because the production duration of an offshore lease can be longer than 30 years. Instead, we focus on the effect of auction outcomes on the winning bidder's belief about the tract's latent production value and subsequent drilling decisions.

Our paper also complements the literature on the role of information in auctions. Theoretically, Milgrom and Weber (1982) and Eső and Szentes (2007) study the auctioneer's information disclosure policy when the auctioneer has access to exogenous signals that are affiliated with the bidders' valuations. Empirically, Takahashi (2018); Allen et al. (2019), and Krasnokutskaya et al. (2020) study environments in which the bidders are uncertain about the scoring rule of an auction with multidimensional bids. In these papers, the uncertainty in the scoring rule stems from the auctioneer having private information on the project, and whether this information is revealed to the bidder has implications on the bidding strategies. In contrast, in our paper, we study the strategic decision of an auctioneer to disclose the rival bidders' information that is privately revealed to the auctioneer through their sealed bids.

In addition, our paper contributes to the literature on the identification of auction models.

Hendricks et al. (2003) and Athey and Haile (2007) establish the identification result for a standard pure common value when the ex post values are observed, utilizing the bid inversion method by Guerre et al. (2000). However, in our setting, the ex post values are not perfectly observed because only the explored tracts can be productive, and the exploration decision is endogenous to the latent production value. Therefore, the bid information and the realized production value are insufficient to achieve identification in our setting. Instead, much of our identification results leverage the variation in the observed exploration decisions.

Finally, our paper is related to the literature on the role of information on strategic exploration decisions in the oil and gas market. Porter (1995)and Hendricks and Porter (1996) document the determinants of exploratory drilling in the US offshore oil and gas auctions from 1954 to 1979, focusing on the information spillover among neighboring leases and the information effect from auction outcomes. Lin (2009, 2013) and Hodgson (2018) study the strategic decision of oil and gas companies to delay exploration to free ride on the information from the exploration outcomes of neighboring leases. In our paper, we abstract from the timing of the exploration decision and focus on the impact of auction outcomes on whether exploration occurs and how it affects the optimal auction design.

3 Institutional Setting and Data

We use the publicly available data from the Bureau of Ocean Energy Management (BOEM) on the OCS auctions from 2000 to 2019.[3] Each year, the BOEM holds sales in which the right to drill and extract the oil and gas from each tract is auctioned. If there is no drilling activity within a fixed lease term, the ownership of the tract reverts to the government.

During this period, all of the available unleased areas are available to be auctioned. The auctions' format is first-price sealed-bid with an announced minimum bid and a fixed royalty rate. The royalty rate determines the amount of post-production revenue that the winning bidder must pay to the government should production occurs. The government might choose to reject the highest bid if it deems that there is not enough competition for the lease. During our data period, the government rejects the highest bid only when there is one bidder.

Prior to the auction, potential bidders first acquire seismic information from a geophysical company, which are then processed by geophysicists at the oil and gas firms and will be further updated as new data arrive. The methods used by the bidders to process the data are proprietary. After the auction's outcome is revealed, the winning bidder decides whether

[3]Hendricks and Porter (2014) provides an excellent overview of the OCS auctions and how they evolve.

to conduct exploratory drilling to determine whether the tract contains profitable natural resources. If the outcome of the exploratory well is successful, the firm may decide to drill development wells to start production, and the result of this development stage is more certain than that of the exploration stage.

Oil and gas firms are allowed to bid jointly unless they are placed on a restricted joint bidders list. This list consists of producers whose production is above a threshold for either crude oil or natural gas for the prior production period. The bidders in the restricted joint bidders list are also not allowed to bid separately if they have an agreement with another restricted bidder that will result in joint ownership of an OCS lease during the 6-month bidding period. They are also prohibited from making any pre-bidding agreement to convey any potential lease interest to any person on the list of restricted joint bidders. For our analysis, we do not differentiate between joint bids and single bids.

Table 3.1: Summary statistics of 2000-2019 auctions

Statistic	N	Mean	St. Dev.	Max	Min
First Bid	2,510	5,089,434.000	11,732,752.000	157,111,000	10,100
Second Bid	2,510	2,066,882.000	6,180,480.000	84,391,221	4,235
N Bids	2,510	2.706	1.298	13	2
Oil Production	288	1,819,866.000	5,571,431.000	42,879,958.000	0.000
Gas Production	288	11,498,482.000	27,151,315.000	253,421,389.000	0.000
Explored	2,510	0.245	0.430	1	0

Table 3.1 presents the summary statistics of the auction outcomes and subsequent exploration and production outcomes from 2000 to 2019 with at least 2 bidders. There is substantial bid heterogeneity both across and within the auctions. For example, the average highest bid is more than twice as high as the average second bid. The BOEM data also include information on all well activities, such as when a well was drilled and when it was abandoned, and production activities of each well. The exploration rate during this period is low: only 24.5% of leases were explored, among which 46.8% were subsequently developed.[4]

We supplement the BOEM data with the rig contract data from Rystad, which document all the rig contracts that were active at any point in the period from 2000 to 2019. Rigs are a vital capital required for well drilling and are not owned by the oil and gas companies who

[4]For the purpose of our analysis, we will rely on all available production data, which cover 1954 to 2019, to estimate the tracts' oil and gas priors. Table D.2 in the Appendix presents the summary statistics of this data period.

are the tracts' leaseholders. They are often contracted on a daily rate for a period between 3 months and a year. The data specify the tender date, the commencement date, the parties involved in the contracts, the daily rig rates, and the contract scope for each rig contract. Table D.3 in the Appendix summarizes the average duration and costs of rig contracts.

Table 3.2: Correlation between the exploratory drilling rate and the auction outcomes

	Explored		
	(1)	(2)	(3)
Log First Bid	0.057***	0.047***	0.045***
	(0.010)	(0.010)	(0.010)
Log Second Bid	0.048***	0.038***	0.039***
	(0.012)	(0.012)	(0.012)
N Bids	0.007	0.006	0.005
	(0.008)	(0.007)	(0.007)
Year FE	Yes	Yes	Yes
Area FE	Yes	Yes	Yes
Neighbor Characteristics	No	Yes	Yes
Winning Bidder Characteristics	No	No	Yes
Observations	2,510	2,316	2,316
Adjusted R^2	0.242	0.314	0.318

Table 3.2 shows the correlation between the exploration decisions and the first and second bids. In this regression, we control for the neighboring tracts' characteristics such as whether a neighboring tract was explored and whether that happens before or after the auction.[5] We also control for the winning bidder's characteristics, including whether the winning bidder owns neighboring leases, and whether these leases have been explored and productive. The results suggest that, conditional on the winning bid, the firms are more likely to explore when the second bid is higher, which is consistent with a model in which the firms use auction outcomes to update their beliefs about the tract's profitability before deciding whether to explore. Furthermore, the magnitude of the effect of the second-highest bid on the exploration rate is similar to that of the winning firm's own bid, suggesting that

[5]For each tract, we define 'neighboring tracts' using a similar metric as in Hendricks et al. (2003), which are tracts within 0.11 degree of latitude and 0.12 degree of longitude.

the winning firm's exploration decision is similarly affected by the inferred information of its rivals as by its own private information.

In our benchmark model, we assume that each winning bidder considers its exploration decision independently between tracts. However, because the exploration outcomes on the adjacent tracts are usually positively correlated, the leaseholders of the adjacent tracts might choose to explore cooperatively to mitigate potential losses from over drilling inefficiencies (Hendricks and Porter, 2014). To measure the extent to which firms cooperate on their exploration decisions among neighboring leases, we look at the unexplored (explored) leases whose owners got into ownership agreements with nearby explored (unexplored) tracts' leaseholders after the auction. The results show that 13.2% of leases in our sample might exhibit some cooperative behavior among at least some owners. In the Appendix, we do a robustness check of our analysis by excluding these subsample of leases.

4 Model

We first present a model of a common-value auction with ex post moral hazard under full revelation of the bids ex post. In this section, we abstract from some empirically relevant details, which will be introduced in Section 5.

The federal government (auctioneer) wants to auction off the lease of an offshore tract via a first-price sealed-bid auction. There are $N \geq 2$ firms/bidders. The expected production value during the lease term is a random variable Q, and the common prior belief about Q is represented by the distribution F^Q, which has a full support on $[0, \bar{q}]$, where $\bar{q} < \infty$. Prior to the auction, each firm $i \in \{1, \ldots, N\}$ receives a *private* signal S_i about Q, which represents each firm's private information about the potential production value in the tract. Let $\boldsymbol{S} = (S_1, \ldots, S_N)$ and $f^{\boldsymbol{S}}(\boldsymbol{S}|q)$ be the joint density of the firms' signals conditional on $Q = q$; therefore, $f^{\boldsymbol{S}}(\boldsymbol{S}) = \int_0^{\bar{q}} f^{\boldsymbol{S}}(\boldsymbol{S}|q)\, dF^Q(q)$. Let the expected production value conditional on the information of every firm be denoted by

$$V(s_1, \ldots, s_N) := E[Q|S_1 = s_1, \ldots, S_N = s_N].$$

We normalize the signals such that $V(0, \ldots, 0)$ is 0.

Besides the winning bid, the government also charges a royalty rate of $r < 1$ on the tract's production value, where r is announced before the auction. To extract the natural resources from the tract, the firm must incur an exploration cost. This cost is realized only *after* the auction — for example, such exploration costs depend on the physical and geological characteristics of the tract and can be fully assessed by a firm only after it has won the auction and gained access to the tract. We let C denote the random variable of the exploration cost and F^C be its distribution with a full support on $[0, \bar{c}]$, where $\bar{c} < \infty$. F^C is common knowledge.

To summarize, suppose that firm i wins the auction with a bid of b. Under the realizations $Q = q$ and $C = c$, if firm i explores the tract, then its overall profit is $(1 - r)\, q - c - b$, and the government's profit is $b + rq$. On the other hand, if firm i does not explore, its profit is $-b$, whereas the government profit is b. The game proceeds as follows:

1. Each firm i *privately* observes the realization of S_i.
2. All the firms simultaneously submit a sealed bid to the auctioneer.
3. The government announces the winner, who is the highest bidder; the winner then pays its bid. In the event of a tie, the winner is randomly chosen from among the highest bidders.
4. The auctioneer reveals all the submitted bids.
5. The winner observes the realization of the exploration cost C.
6. The winner decides whether to explore the tract. If the winner decides to explore, it incurs C, and the value of Q is realized. Subsequently, the firm collects Q and pays rQ to the government. On the other hand, if the firm decides not to explore, the game ends.

Assumption 1. *The following conditions hold:*

1. C and Q are independent.

2. For all q in the support of F^Q, $f^{\boldsymbol{S}}(S_1, \ldots, S_N | q)$ has a full support over $[0, \bar{s}]^N$ and is symmetric in the signals.[6] *This implies that $V(\cdot)$ is also symmetric.*

3. S_1, S_2, …, and S_N are affiliated — i.e., for any $\boldsymbol{s} = (s_1, \ldots, s_N)$ and $\boldsymbol{s}' = (s'_1, \ldots, s'_N)$,

$$f^{\boldsymbol{S}}(\boldsymbol{s} \vee \boldsymbol{s}')\, f^{\boldsymbol{S}}(\boldsymbol{s} \wedge \boldsymbol{s}') \geq f^{\boldsymbol{S}}(\boldsymbol{s})\, f^{\boldsymbol{S}}(\boldsymbol{s}'),$$

where $\boldsymbol{s} \vee \boldsymbol{s}'$ and $\boldsymbol{s} \wedge \boldsymbol{s}'$ are the component-wise maximum (i.e., join) and minimum (i.e., meet) between $\boldsymbol{s}$ and $\boldsymbol{s}'$, respectively.

[6]Since $f^{\boldsymbol{S}}(\cdot|q)$ has a full support, by Bayes rule, $f^Q(q|\boldsymbol{S} = \boldsymbol{s}) = \frac{f^{\boldsymbol{S}}(\boldsymbol{S}|q)}{f^{\boldsymbol{S}}(\boldsymbol{S})}$.

4. $V(S_1, \ldots, S_N)$ *is increasing in* S_1 *to* S_N.

Assumption 1.1 assumes that the realization of the exploration cost in Stage 5 does not change the winning bidder's posterior belief about the tract's production value. In our setting, the exploration cost is mainly determined by the available rigs' locations and the tract's water depth. Therefore, the realization of the exploration cost is unlikely to carry any additional information about the tract's production value beyond the information observed by the bidders prior to the auction. The symmetry assumption on $V(\cdot)$ in Assumption 1.2 implies that the bidders are symmetric. Furthermore, the bidders' identities do not matter to the winning bidder's posterior belief about the tract's production value. Assumption 1.3 is a standard assumption in the literature to ensure the monotonicity of the symmetric equilibrium.[7] Finally, Assumption 1.4 is a normalization in which higher private signals are associated with a higher production value.

4.1 Equilibrium Analysis

Each firm's strategy consists of a bidding function $\beta_i : S_i \to \mathbb{R}_+$ and an exploration decision (conditional on winning the auction) that depends on the realization of C and all the information about the expected production value after the auction. The winner of the auction will explore if and only if its expected value of $(1-r)\,Q$ conditional on all the available information is higher than the realization of C. Given the simplicity of the drilling decision, we only need to focus on the equilibrium bidding functions in our equilibrium analysis. Henceforth, we abuse terminology and refer to an equilibrium as only the profile of the bidding functions of the equilibrium strategies. We will restrict our attention to symmetric Bayes Nash equilibria in which each firm i bids according to the same bidding function $\beta : S_i \to \mathbb{R}_+$, where β is strictly increasing.

To derive the equilibrium, first, let Y_i denote the first order statistics of the $N-1$ random variables S_j for $j \neq i$ (i.e., $Y_i = \max_{j \neq i} S_j$). Let $G^Y(Y_i|s)$ denote the distribution of Y_i conditional on $S_i = s$, and let $g^Y(Y_i|s)$ be the corresponding density; let $\boldsymbol{Z}_i$ denote the $(N-2)$-dimensional random vector $\left(Z_i^{(2)}, Z_i^{(3)}, \ldots, Z_i^{(N-1)}\right)$, where $Z_i^{(k)}$ denote the k-order statistics, and let $G^{\boldsymbol{Z}}(\boldsymbol{Z}_i|s, y)$ denote the distribution of $\boldsymbol{Z}_i$ conditional on $S_i = s$ and $Y_i = y$, and let $g^{\boldsymbol{Z}}(\boldsymbol{Z}_i|s, y)$ be the corresponding density; finally, let $G^{Y,\boldsymbol{Z}}(Y_i, Z_i|s)$ denote the joint distribution of (Y_i, Z_i) conditional on $S_i = s$, and let $g^{Y,\boldsymbol{Z}}(Y_i, Z_i|s)$ be the corresponding density.

[7] Affiliation is a strong notion of positive correlation. See Milgrom and Weber (1982) for more details.

Next, let $v(s,y)$ be firm i's expected profit from winning the auction conditional on $S_i = s$ and $Y_i = y$, where the expectation is taken at the start of Stage 2. To derive $v(s,y)$, first note that from firm i's perspective at the start of Stage 2, the bids of the other firms are random variables. Let $B_i^{(1)}$, $B_i^{(2)}$, ..., $B_i^{(N-1)}$ denote these random variables, where $B_i^{(k)}$ is the k-th highest bid of the other firms, and let $\chi_\beta\left(B_i^{(2)},\ldots,B_i^{(N-1)}|s,y\right)$ denote their joint distribution conditional on $S_i = s$ and $Y_i = y$ when firm i conjectures that all the other firms bid according to a strictly increasing bidding function β. This implies that

$$\begin{aligned}
\chi_\beta\left(b_i^{(2)},\ldots,b_i^{(N-1)}|s,y\right) &= \Pr\left[B_i^{(2)} \le b_i^{(2)},\ldots,B_i^{(N-1)} \le b_i^{(N-1)}|S_i = s, Y_i = y\right] \\
&= \Pr\left[Z_i^{(2)} \le \beta^{-1}\left(b_i^{(2)}\right),\ldots,Z_i^{(N-1)} \le \beta^{-1}\left(b_i^{(N-1)}\right)|S_i = s, Y_i = y\right] \\
&= G^{\boldsymbol{Z}}\left(\beta^{-1}\left(b_i^{(2)}\right),\ldots,\beta^{-1}\left(b_i^{(N-1)}\right)|s,y\right)
\end{aligned}$$

Conditional on $S_i = s$ and $Y_i = y$ and observing that $B_i^{(2)} = b_i^{(2)}$, ..., $B_i^{(N-1)} = b_i^{(N-1)}$ at Stage 4, firm i's expectation of the production value is $V\left(s,y,\beta^{-1}\left(b_i^{(2)}\right),\ldots,\beta^{-1}\left(b_i^{(N-1)}\right)\right)$, and firm i explores if and only if the realization of C is less than

$$(1-r)\,V\left(s,y,\beta^{-1}\left(b_i^{(2)}\right),\ldots,\beta^{-1}\left(b_i^{(N-1)}\right)\right)$$

. Let

$$\psi(V) := \int_0^{\bar{c}} \max\{\,0\,,\,(1-r)\,V - c\}\,dF^C(c) \;=\; \int_0^{(1-r)V} \left[(1-r)\,V - c\right] dF^C(c) \quad (1)$$

$$\Longrightarrow\; \psi'(V) = (1-r)\,F^C\left((1-r)\,V\right) \quad (2)$$

Therefore,

$$\begin{aligned}
v(s,y) &= \int_{b_i^{(2)},\ldots,b_i^{(N-1)}} \psi\left(V\left(s,y,\beta^{-1}\left(b_i^{(2)}\right),\ldots,\beta^{-1}\left(b_i^{(N-1)}\right)\right)\right) d\chi_\beta\left(b_i^{(2)},\ldots,b_i^{(N-1)}|s,y\right) \\
&= \int_{\boldsymbol{z}} \psi\left(V\left(s,y,\boldsymbol{z}\right)\right) dG^{\boldsymbol{Z}}\left(\boldsymbol{z}|s,y\right) \quad (3)
\end{aligned}$$

where the second line follows from a change of variable of $\beta\left(Z_i^{(k)}\right) = B_i^{(k)}$ for $k = 2,\ldots,N-1$.[8]

Consequently, at the start of Stage 2, if $S_i = s$ and firm i conjectures that all the other

[8] Throughout, we adopt the natural notation that, for any function T, $\int_{\boldsymbol{z}} T(\boldsymbol{z})\,d\boldsymbol{z} = \int_{z^{(2)}}\int_{z^{(3)}}\ldots\int_{z^{(N-1)}} T\left(z^{(2)},z^{(3)},\ldots,z^{(N-1)}\right) dz^{(N-1)}\ldots dz^{(3)}dz^{(2)}$.

firms bid according to β, firm i's expected overall profit from bidding b is

$$\int_0^{\beta^{-1}(b)} [v(s,y) - b]\, g^Y(y|s)\, dy \tag{4}$$

Let $L(s'|s) := \exp\left(-\int_{s'}^{s} \frac{g^Y(t|t)}{G^Y(t|t)} dt\right)$.

Proposition 1. *When all the bids are fully disclosed after the auction, the increasing and symmetric Bayes Nash equilibrium is uniquely*

$$\beta^{FD}(s) = \int_0^s v(s', s')\, dL(s'|s). \tag{5}$$

The proof of Proposition 1 is found in Appendix A. In the proof, we show that, for any s, $L(s'|s)$ is a proper distribution that is first order stochastically dominated by the distribution $G^Y(s'|s' \leq s)$. Therefore, $\beta^{FD}(s) < E[v(s', s')|s' \leq s]$ for all $s > 0$. This bidding strategy thus reflects the firm's understanding of the "winner's curse" — winning the auction indicates that the firm is overly optimistic of the tract's production value; anticipating this effect, the firm depresses its bid.[9]

5 Identification and Estimation

This section introduces two empirically relevant features that were not included in the theoretical model in Section 4. We then discuss the identification and estimation strategies.

5.1 Additional Empirical Features

First, in our data, a lease can produce both oil and gas. Therefore, we decompose the oil and gas quantity in each tract into oil quantity, Q^o (measured in barrels), and gas quantity, Q_g (measured in MCF). The total value of each tract is given by

$$Q = Q^o P^o + Q^g P^g,$$

where P^o and P^g are the prices of oil and gas, respectively. We choose P^o and P^g to be the annual average offshore U.S. Gulf Coast crude oil first purchase price and the Henry Hub spot price in the year of the auction, respectively.

[9]See Milgrom and Weber (1982) and Krishna (2009) for an illustration.

Second, our theoretical model also does not take into account that the firms' profits, in reality, also depend on their discount factors and the volatility in the prices of oil and gas. In addition, paying the exploration cost C only reveals the production value Q to the firm, but to fully extract Q, the firm must also pay additional development costs. Thus, we introduce two additional cost components, $\delta < 1$ and $\kappa \geq 0$, which are known to all the bidders prior to the auction, and they are objects to be identified. δ represents the impact of the discount factor and the price volatility on the present value of the tract's production value, and κ is a multiplicative cost that represents any potential development cost incurred after exploratory drilling. Therefore, if the firm engages in exploratory drilling after winning the auction with a bid of b and observing that the exploration cost is $C = c$ and the production value is $Q = q$, its profit is $(\delta(1-r) - \kappa)q - c - b$, where $\delta(1-r) - \kappa$ is assumed to be positive.

5.2 Identification

For our analysis, for each tract t, the objects of interest are the functions $\{F_t^Q, F_t^{\boldsymbol{S}}, F_t^C, V_t\}$ and the constants $\{\kappa_t, \delta_t\}$. We observe the equilibrium bids $\boldsymbol{B}_t$, the exploration decisions, and Q_t if drilling occurred. Throughout this subsection, to simplify notations, we will exclude each auction's observed characteristics, and we describe how to incorporate them in Subsection 5.3.

To identify the prior F_t^Q, we assume that the prior belief is, on average, correct; hence, F_t^Q is identified directly from the production data of the explored tracts.

Next, to identify the joint distribution of the signals $F_t^{\boldsymbol{S}}$, we first group all the auctions with a similar prior together and estimate the bid distribution associated with this prior. As it is WLOG to normalize the marginal distribution of S_{it} to be $U[0,1]$ for all i,[10] a signal associated with a bid b is thus the value of the bids' CDF evaluated at b. Therefore, the joint distribution of the signals $F_t^{\boldsymbol{S}}$ is also identified. In turn, the conditional distributions of the order statistics, $G_t^{\boldsymbol{Z}}(\boldsymbol{Z}|S, Y)$ and $G_t^{\boldsymbol{Y}}(Y|S)$, are also identified.

Next, to identify F_t^C, V_t, κ_t, and δ_t. we make the following assumptions:

Assumption 2. *There exists a cost shifter instrument I_t such that*

- *The exploration cost of tract t is*

$$C_t = C_t^0 \zeta(I_t)$$

[10] By Sklar's theorem, by normalizing the signals to be uniformly distributed, the joint distribution that we identify is the copula of the true joint distribution.

where $\zeta : \mathbb{R} \to \mathbb{R}$ is a differentiable and non-constant function, C_t^0 a random variable with a strictly increasing distribution F^{C^0}, and C_t^0, I_t, and Q_t are pairwise independent.

- *The development cost of tract t is*

$$\kappa_t = \kappa(I_t),$$

where $\kappa : \mathbb{R} \to \mathbb{R}$ is a differentiable function.

I_t is a tract-level observed variable that affects the exploration and development costs but does not carry any information about the tract's production value Q_t. C_t^0 represents the aspect of the exploration cost whose distribution F^{C^0} is common across all the tracts. Henceforth, we refer to C_t^0 as the *base* exploration cost.

Assumption 3.

1. *$\delta_t = \delta \ \forall t$.*

2. *$V_t(\cdot)$ is differentiable almost everywhere.*

We assume that the discount factor δ_t is common across all the tracts. $V_t(\cdot)$ is the expected production value conditional on all the signals; therefore, the smoothness condition on $V_t(\cdot)$ is an assumption on the distribution of Q_t conditional on $\boldsymbol{S}_t$.

Proposition 2. *Under Assumptions 2-3, F^{C^0}, $\zeta(\cdot)$, $V_t(\cdot)$, $\kappa(\cdot)$, and δ are identified.*

The proof of Proposition 2 relies on the observed exploration rate in the data conditional on $\boldsymbol{B}_t$ and I_t. The exploration rate depends on (i) the firm's posterior of the tract's production value, which is determined solely by the firms' private signals (which we identified above), and (ii) the exploration and development costs. Since the instrument I_t affects the exploration cost distribution F_t^C and the development cost κ_t but not the posterior of the tract's production value, we can identify F^{C^0}, $\zeta(\cdot)$, $V_t(\cdot)$, $\kappa(\cdot)$, and δ up to a scale. These objects are then pinned down using the bid inversion method of Guerre et al. (2000) and the fact that, by Bayes' theorem, the posterior distribution of Q_t must average back to the prior F_t^Q.

Our identification argument differs from the standard identification argument in the auction literature. In a common-value framework, it is widely known that the joint distribution of the signals and the value of the object being auctioned (the production value Q_t in our case) can be identified directly from the bid information and the realized ex post value of the object. This argument does not apply to our case because Q_t is unobserved when the winning bidder chooses not to explore the tract, and the exploration decision is endogenous to the tract's latent production value (Athey and Haile, 2007). In addition, the identification argument in the literature usually relies on normalizing the bidder's signal to be his value from marginally winning the auction—i.e., when the true signal is $S_{it} = s$, the normalized signal is $\tilde{S}_{it} := v(s, s)$. However, due to the unknown distribution of the exploration cost, the bid information alone is insufficient to identify $V_t(\cdot)$, which is required for our counterfactual analysis. Therefore, much of our identification argument relies on the variation in the exploration decisions between the auctions with a similar prior and how these decisions vary with the bids to identify $V_t(\cdot)$.

5.3 Estimation

5.3.1 Constructing priors F_t^Q

We use the realized production and exploration outcomes to construct the firms' prior belief for each tract. The prior beliefs' construction focuses on accounting for the correlation in adjacent tracts' exploration and production outcomes, which is an important feature of this market.

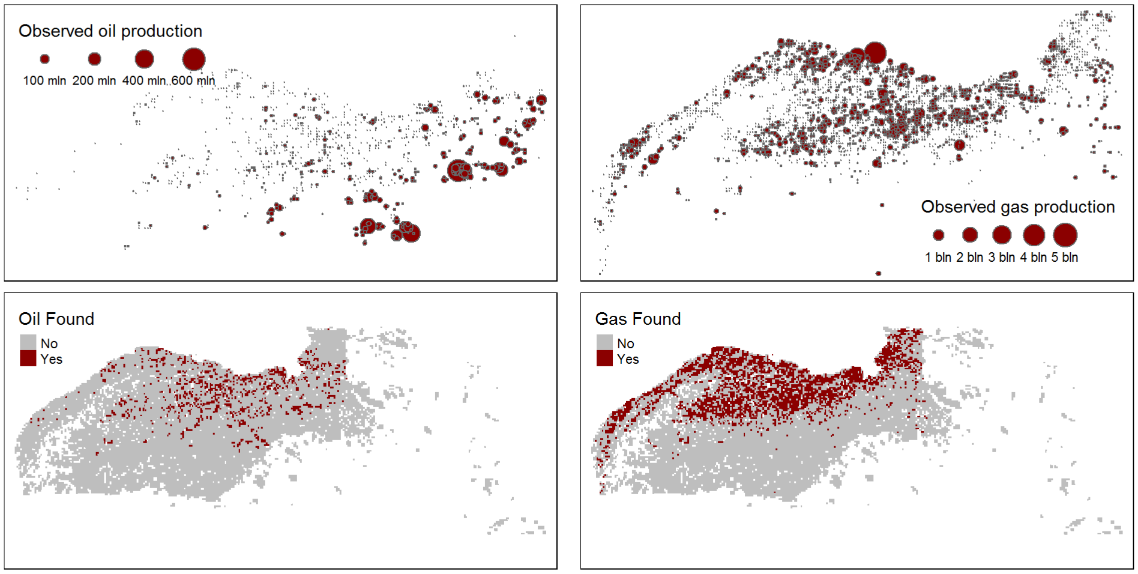
The locations of explored leases with and without production are displayed in Figure 5.1. We assume that once production begins at a tract, all the potential bidders can observe that tract's production value. All the tracts that were explored but not developed are assumed to have $Q_t^o = Q_t^g = 0$. For the leases that have expired—which we term 'completed' leases—the oil and gas quantities in the associated tract are the total amount of oil and gas the tract has produced. For tracts that are still in production at the end of our data period, we construct a measure of the total amount of oil and gas that will be produced using the production data of the previous 2 years. Further details of our construction of Q_t^o and Q_t^g are found in the Appendix.

To construct F_t^Q, we first construct the prior for the oil and gas quantities, denoted by $F_t^{Q^g}$ and $F_t^{Q^o}$, respectively. For each prior $F_t^{Q^i}$, $i \in \{o, g\}$, the distribution of the quantity of resource i is assumed to follow a mixture distribution as follows:

$$\begin{cases} Q_t^i = 0 \text{ with probability } \frac{\exp(\beta_0^i + \nu_{0t}^i)}{1+\exp(\beta_0^i + \nu_{0t}^i)} \\ Q_t^i \sim Gamma(\cdot, \cdot) \text{ with probability } \frac{1}{1+\exp(\beta_0^i + \nu_{0t}^i)} \end{cases} \tag{6}$$

β_0^i is a parameter to be estimated. ν_{0t}^i is assumed to be a Gaussian random field with a

Figure 5.1: Observed oil and gas production



Matérn correlation function, where the covariance between ν_{0t}^i and $\nu_{0t'}^i$ is

$$\sigma_0^{i\,2}\left(\gamma_0^i||s_t - s_{t'}||\right) K_1\left(\gamma_0^i||s_t - s_{t'}||\right), \tag{7}$$

where σ_0^i and γ_0^i are parameters to be estimated, $||s_t - s_{t'}||$ denotes the distance between the centroid of tract t and tract t', and $K_1(.)$ is the modified Bessel function of the second kind with a smoothness parameter of one (Krainski et al., 2018). Next, the Gamma distribution in Eq. (6) is assumed to have mean μ_t^i and standard deviation $\frac{(\mu_t^i)^2}{\phi^i}$, where

$$\log \mu_t^i = \beta_1^i + \nu_{1t}^i,$$

in which ϕ^i and β_1^i are parameters to be estimated, and ν_{1t}^i is also a Gaussian random field similarly defined as in Eq. (7), with parameters σ_1^i and γ_1^i. To estimate γ_0^i, we follow the literature and, instead, estimate the range value $\rho_0^i = \frac{\sqrt{8}}{\gamma_0^i}$. We do the same thing for γ_1^i—i.e., estimate $\rho_1^i = \frac{\sqrt{8}}{\gamma_1^i}$.

Thus, in summary, the parameters that need to be estimated are $\{\beta_0^i, \sigma_0^i, \rho_0^i, \phi^i, \beta_1^i, \sigma_1^i, \rho_1^i\}$. We estimate these parameters using R-INLA (Rue et al., 2009).[11] The estimated parameters of the priors are summarized in Table D.1 in the Appendix.

Since the bidders observe only the production outcomes that have been realized prior

[11] The package can be downloaded at `www.r-inla.org`. I thank David Childers for introducing me to R-INLA.

to the auction, for each tract t, we use only the data of the tracts that have either begun production or expired before tract t's auction date. This naturally implies that the earlier tracts have less informative priors than the later tracts. Table 5.1 summarizes the prior probability of having oil/gas and the prior mean quantity of oil/gas if oil/gas is found, taking into account each tract's exploration date. The estimated priors show substantial heterogeneity in both the probability of a tract having natural resources and the production value of the tract (if any).

Table 5.1: Estimated priors F^{Q^o}, F^{Q^g}

Statistic	N	Mean	St. Dev.	Max	Min
Bernoulli Prior Mean (Oil)	2,510	0.102	0.108	0.553	0.001
Bernoulli Prior Mean (Gas)	2,510	0.236	0.274	0.884	0.001
Gamma Prior Mean (Oil)	2,510	24,198,967.000	24,648,355.000	113,176,544.000	49,128.330
Gamma Prior Mean (Gas)	2,510	57,962,001.000	27,224,458.000	321,959,317.000	9,152,942.000

5.3.2 Estimation

Our estimation procedure is conducted in two steps. In the first step, we use kernel estimation to estimate the CDF of the bids, which then gives us the estimated private signals of the bidders. In the second step, we estimate the parameters of the model using GMM. Our first set of moments is the derivative of the sum of the squared errors between the predicted bids and the actual bids. The second set of moments is the derivative of the likelihood function of the binary drilling outcomes. Since our first step relies on the estimated signals, the standard errors are computed by bootstrap. More details are found in the Appendix.

Number of potential bidders. In our model, we did not consider reserve prices in the auction nor the possibility that some bidders did not bid because their optimal bids are below the reserve prices; therefore, we set N_t to be the number of bids for tract t observed in the data. This assumption is innocuous for the following reasons. First, in our setting, the reserve prices are determined solely by each tract's water depth ($\leq$ 400m and $> 400m$) and the auction year (before 2013 and after 2013), and the water depth and the auction year are already taken into account in our estimation as auction-level observed heterogeneities, which we will describe in more details later. Moreover, when the reserve price is not 0, the only difference that affects our estimation procedure is that the number of potential bidders is not the same as the number of bids observed. To account for this difference, we only have to change the interpretation of the signals accordingly, with the lowest signal ($S_{it} = 0$) being interpreted as the signal whose bids is the reserve price.

We divide the sample into three subsamples: (1) auctions with $N_t = 2$, (2) auctions with $N_t = 3$, and (3) auctions with $N_t > 3$. We estimate the parameters for each subsample separately. Thus, we are not imposing any parametric assumptions on how the parameters of the model vary with the number of potential bidders N_t.

Cost Shifter Instrument. We use the rig rates of the rig contracts signed within one year of and prior to tract t's auction date to construct the cost shifter instrument I_t. Since the rig contract is signed between a rig company and an operator, which is an oil/gas company appointed by the leaseholder(s) to determine exploration and production decisions, a rig can be moved between different tracts with the same operator. Therefore, we identify a set of matching leases with the same operator and having drilling activities during the contract's length for each rig contract. I_t is then defined as the average rig rate across all the operators weighted by the distance between the matching leases and tract t. Thus, our cost shifter instrument reflects the rig market condition at the tract's location before the auction occurs, and a higher I_t is associated with a lower exploration rate. Moreover, I_t is unlikely to carry additional information about Q_t because the construction of I_t uses the information about nearby tracts, which has already been taken into account in F_t^Q. Table 5.2 shows that the constructed cost shifter instrument is indeed negatively correlated with the exploration rate, providing evidence that our constructed cost shifter instrument is relevant.

Table 5.2: Relevance of Cost Instrument

	Explored	
	(1)	(2)
Log Average Rig Rate	-0.811^{***}	-0.595^{***}
	(0.062)	(0.065)
Number of Bids	0.057^{***}	0.059^{***}
	(0.006)	(0.006)
Area FE	No	Yes
Observations	2,510	2,510
Adjusted R^2	0.082	0.171

Auction Heterogeneity. To accommodate auction-level observed heterogeneities, we follow the method of Haile et al. (2003) to 'homogenize' the bids. In our benchmark estimates, the auction-level observed heterogeneities are year fixed effects, the royalty rate, and the tract's water depth. To implement Haile et al. (2003)'s approach, we first regress the log of the observed bids on year fixed effects and water depth fixed effects ($\leq$ 400m or $>$ 400m) for each subsample with the same royalty rate. The residuals are then treated as the log of the bids of auctions with homogeneous observed characteristics, conditional on I_t and F_t^Q. The homogenized bids preserve the optimality of the original bids when the observed heterogeneity commonly affects both the exploration cost and the tract's expected production value $V_t(\cdot)$ in a multiplicatively separable manner.

Signal Estimation. We use kernel estimation to estimate the CDF of the bids, denoted by $F^B(\boldsymbol{B}|I_t, F_t^Q)$, using the homogenized bids. To reduce the conditional variables' dimension, we make a simplification assumption that the prior mean μ_{Qt} is a sufficient statistic for the priors' heterogeneity.

$$\mu_{Qt} = \mu_t^g P^g(1 - \mu^{0g}) + \mu_t^o P^o(1 - \mu^{0o})$$

where μ^{0g} and μ^{0o} are the probability that $Q_t^g = 0$ and $Q_t^o = 0$, respectively, as defined in Eq. (6). We estimate $F^B(\boldsymbol{B}|I_t, \mu_{Qt})$ using the Gaussian kernel and least-squares cross-validation method to select bandwidths (Li et al., 2013). The CDF of the bids give us our estimates of $\boldsymbol{S}_t$.

Joint distribution of $\boldsymbol{S}$ and Q. For each tract, we assume a Gaussian copula for the joint distribution of $(S_1, S_2, ..., S_N, Q)$ as follow. We drop the tract identifier t for notational convenience.

$$F^{\boldsymbol{S},Q}(S_1, S_2, ..., S_N, Q) = \Phi_R\left(\Phi^{-1}\left(F^S(S_1)\right), ..., \Phi^{-1}\left(F^S(S_N)\right), \Phi^{-1}\left(F^Q(Q)\right)\right)$$

where Φ_R is the joint CDF of a multivariate normal distribution with mean 0 and covariance matrix R, and Φ^{-1} is the inverse of the CDF of a standard normal distribution. Since the bidders are ex ante symmetric, R must satisfy the following constraints:

$$\begin{cases} R_{ij} = R_{ik} & \text{for } i, j, k \leq N \\ R_{ii} = R_{jj} & \text{for } i, j \leq N \\ R_{i,N+1} = R_{j,N+1} & \text{for } i \leq N \end{cases}$$

The conditional distribution of $\left(\Phi^{-1}\left(F^{Q^o}(Q^o)\right), \Phi^{-1}\left(F^{Q^g}(Q^g)\right)\right)|\boldsymbol{S}$ is thus

$$\mathcal{N}\left(\left[\begin{array}{c} M^o \sum_i S_i \\ M^g \sum_i S_i \end{array}\right], \Sigma_Q\right) \tag{8}$$

where

$$\begin{aligned}
M^o &= \frac{R_{1,N+1}}{R_{11}+R_{12}} \\
M^g &= \frac{R_{1,N+2}}{R_{11}+R_{12}} \\
\Sigma_Q &= \left[\begin{array}{cc} R_{N+1,N+1} & 0 \\ 0 & R_{N+2,N+2} \end{array}\right] - N\frac{1}{R_{11}+R_{12}}\left[\begin{array}{cc} R^2_{1,N+1} & R_{1,N+1}R_{1,N+2} \\ R_{1,N+1}R_{1,N+2} & R^2_{1,N+2} \end{array}\right]
\end{aligned}$$

Cost Distribution. We parameterize ζ and κ as B-spline functions of I_t. For the distribution of the base exploration drilling cost, we discretize the support of $\log C^0$ to be $\{c_1^0, c_2^0, ..., c_J^0\}$, which is chosen by the econometrician. We then parameterize the distribution to be

$$F^{C^0}(c) = \sum_{j=1}^{J} p_j \Phi\left(\frac{\log(c) - c_j^0}{\sigma^{C^0}}\right)$$

where $p_j \in [0,1]\ \forall j$ and $\sum_j p_j = 1$. p_j and σ^{C^0} are parameters to be estimated. This parameterization's key property is that it does not restrict the CDF of the base drilling cost to be either concave or convex.

5.4 Estimation Results

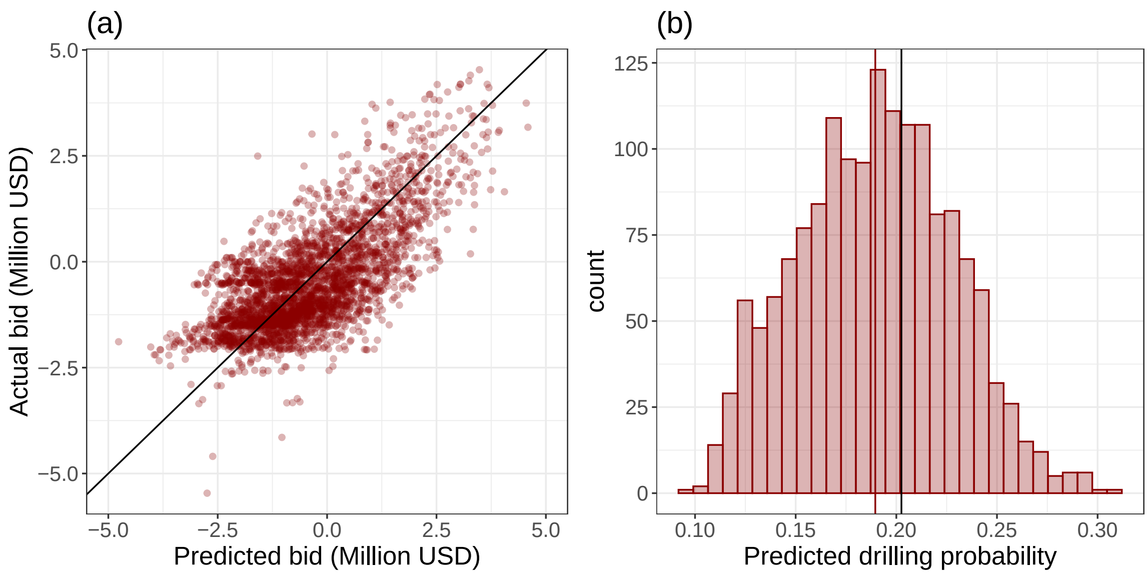
In this subsection, we discuss the estimation results for $N = 2$.[12]

Our model fit is summarized in Figure 5.2. In panel (a), we plot the predicted bids, computed using our equilibrium bidding strategy under full disclosure (Proposition 1), against the actual bids. Our predicted median bid is \$687K, compared to \$765K in the data. Our predicted first quantile and third quantile of the bid distribution are \$346K and \$2.02M, respectively. These also fit well with the data (\$361K and \$1.88M, respectively). In panel (b), we plot the distribution of the predicted drilling probability in which the red vertical line is the mean of the distribution, and the black line is the mean drilling rate in the data.

[12] I am working on estimating the parameters for the $N = 3$ and $N \geq 3$ subsamples, and the results will be included in a future version.

Our data predicted a 0.18 drilling rate, comparable to the 0.20 drilling rate in the data for the case of $N = 2$.

Figure 5.2: Model fit



6 Alternative Bid Revelation Policies

In this section, we conduct multiple counterfactual exercises to estimate the effect of alternative disclosure policies of the losing bids.

One the one hand, when the losing bids are not fully disclosed after the auction, the winning firm will determine its exploration decision based on less information, thus increasing its chances of making a 'mistake' and decreasing its value of winning the auction. In turn, this leads to every firm decreasing its bid. On the other hand, withholding information on the losing bids can increase the expected exploration drilling rate, which increases the government expected royalty revenue.

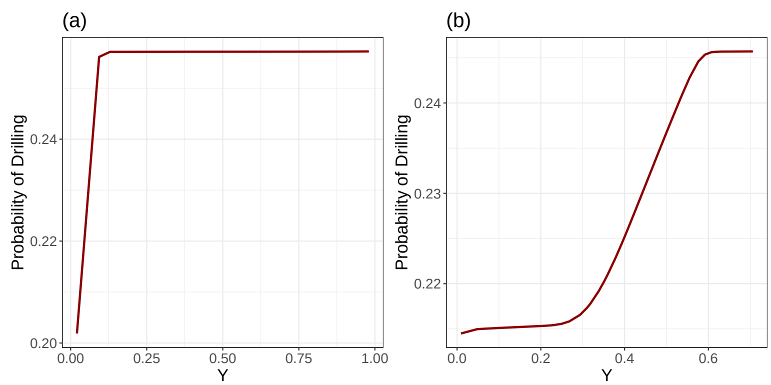
To provide a heuristic explanation of the latter effect, suppose that there are two firms. Let $\pi(s, y)$ denote the probability of the winning firm engaging in exploratory drilling when the higher signal is s and the lower signal is y.[13] When the losing bid is fully disclosed after the auction, the winning firm perfectly infers the value y; therefore, conditional on s, the expected exploration probability is $E[\pi(s, Y)|s] = \int \pi(s, y) dG^Y(y|s)$. In contrast,

[13]i.e., in the model in Section 4, $\pi(s, y) = \Pr[(1 - r)V(s, y) - c \geq 0] = F^c((1 - r)V(s, y))$.

if the losing bid is not revealed, the winning firm makes its exploration decision based on its *expectation* of the losing signal; therefore, the expected exploration probability becomes $\pi(s, E[Y|s]) = \pi(s, \int y dG^Y(y|s))$. If $\pi(s, y)$ is strictly concave in y, then by Jensen's inequality, $\pi(s, E[Y|s]) > E[\pi(s, Y)|s]$.

Figure 6.1 plots the function $\pi(s, y)$ for two different auctions in our data (under some fixed values of s). In panel (a), π is concave in y; therefore, by the argument above, completely withholding information on the losing bid in this auction always increases the exploration rate. In panel (b), π is not concave everywhere; nevertheless, by an analogous argument, the expected exploration rate can still be increased by only pooling information within the concave region of π—i.e., only the losing bids associated with a signal of $y \geq 0.3$ are pooled together.

Figure 6.1: Example of changes in drilling probabilities under FD

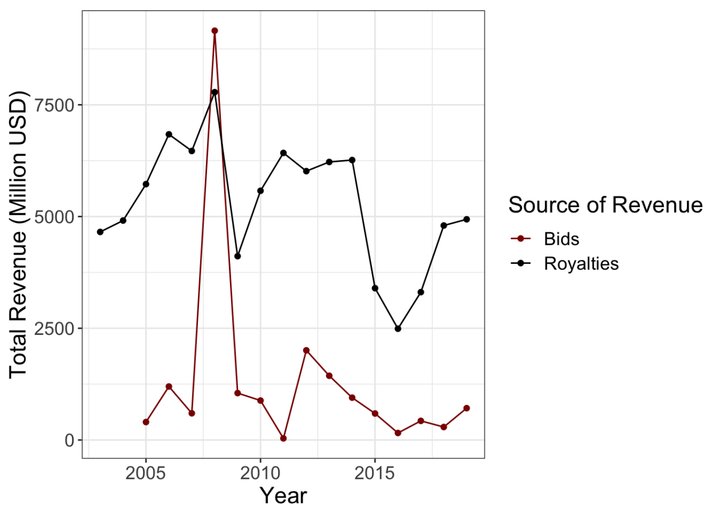


In Subsection 6.1, we present the counterfactual analysis and empirical results for the case in which information on the losing bids are completely withheld after the auction. Subsequently, in Subsection 6.2, we present the framework for the counterfactual analysis of more general form of disclosure policies of the losing bids; the empirical results for these disclosure policies will be presented in a future draft.

We are particularly interested in the effect of the disclosure policy on the equilibrium bids, the equilibrium exploration rate, and the government's revenue. Because our model abstracts from how the government discounts the royalty payments and our estimates yield only predictions on the undiscounted royalty payments, we rely on the current composition of the source of government revenue (Figure 6.2) to determine the net effect of each counterfactual policy on the government's yearly revenue. Between 2003 and 2020, the payment from the cash bids (royalties) accounts for 15% (85%) of the total revenue. Henceforth, we use these weights to compute the net effect of our policies on the government's total revenue.

For example, if a disclosure policy causes the cash bid revenue to decrease by 20% and the royalty revenue to increase by 10%, then the overall net change in the government's revenue is $-0.15 \times 20\% + 0.85 \times 10\% = 5.5\%$

Figure 6.2: Current composition of source of revenue



6.1 Non-disclosure Policy

In this exercise, we consider what happens when the auctioneer announces at the start of the auction that it will no longer reveal the losing bids in Stage 4 of the game in Section 4. We call this the "*non-disclosure auction*," which contrasts with the "*full-disclosure auction*" considered in in Section 4, in which the auctioneer commits to reveal all the submitted bids in Stage 4. We first characterize the equilibrium bidding strategy and then summarize the empirical findings.

6.1.1 Equilibrium Analysis

As before, we focus on symmetric Bayes Nash equilibria of the non-disclosure auction in which each firm i bids according to a strictly increasing function $\beta : S_i \to \mathbb{R}_+$. Let

$$\begin{aligned}\hat{V}_0\left(s,y\right) :=& E\left[V\left(S_i, Y_i, \boldsymbol{Z}_i\right)|S_i = s, Y_i \leq y\right] \\ =& \frac{1}{G^Y\left(y|s\right)} \int_{\boldsymbol{z}} \int_0^y V\left(s, y', \boldsymbol{z}\right) g^{Y,\boldsymbol{Z}}\left(y', \boldsymbol{z}|s\right) dy' d\boldsymbol{z}\end{aligned} \tag{9}$$

$$\begin{aligned}\bar{V}_0(s,y) :=& E\left[V(S_i,Y_i,\boldsymbol{Z}_i)\,|S_i=s,Y_i=y\right] \\ =& \int_{\boldsymbol{z}} V(s,y,\boldsymbol{z})\, g^{\boldsymbol{Z}}(\boldsymbol{z}|s,y)\, d\boldsymbol{z}\end{aligned} \tag{10}$$

Suppose that firm i wins the auction with a bid of b. In Stage 4, unlike in the full-disclosure auction, firm i does not know the realization of $B_i^{(k)}$ in the non-disclosure auction, except the inference that $B_i^{(2)} \leq b$. If firm i conjectures that all the other firms bid according to β, its posterior density of $(Y_i, \boldsymbol{Z}_i)$ is $\frac{g^{Y,\boldsymbol{Z}}(Y_i,\boldsymbol{Z}_i|s)}{\Pr(\beta(Y_i)\leq b|s)} = \frac{g^{Y,\boldsymbol{Z}}(Y_i,\boldsymbol{Z}_i|s)}{G^Y(\beta^{-1}(b)|s)}$. Therefore, firm i's expectation of Q conditional on $S_i = s$ and winning the auction with a bid of b is

$$\int_{\boldsymbol{z}}\int_0^{\beta^{-1}(b)} V(s,y,\boldsymbol{z})\,\frac{g^{Y,\boldsymbol{Z}}(y,\boldsymbol{z}|s)}{G^Y(\beta^{-1}(b)\,|s)} dyd\boldsymbol{z} = \hat{V}_0\left(s,\beta^{-1}(b)\right)$$

In turn, at the start of Stage 2, conditional on $S_i = s$, firm i's expected profit from winning the auction with a bid of b is $\psi\left(\hat{V}_0\left(s,\beta^{-1}(b)\right)\right)$, where ψ is defined in Eq. (1). Consequently, at the start of Stage 2, firm i's expected profit from bidding b is

$$G^Y\left(\beta^{-1}(b)\,|s\right)\left[\psi\left(\hat{V}_0\left(s,\beta^{-1}(b)\right)\right) - b\right] \tag{11}$$

Proposition 3. *In the non-disclosure auction, the increasing and symmetric Bayes Nash equilibrium is uniquely*

$$\beta^{ND}(s) = \int_0^s \left(\psi\left(\hat{V}_0(s',s')\right) + \left[\psi'\left(\hat{V}_0(s',s')\right)\left(\bar{V}_0(s',s') - \hat{V}_0(s',s')\right)\right]\right) dL(s'|s)\,. \tag{12}$$

Moreover,

- $\beta^{ND}(s) < \beta^{FD}(s)$ *for all* s *— i.e., the equilibrium bids in the non-disclosure auction are lower than that in the full-disclosure auction.*
- *for every* s*, there exist* y *and* $\boldsymbol{z}$ *such that* $\hat{V}_0(s,s) > V(s,y,\boldsymbol{z})$ *— i.e., there exist realizations of signals and* C *in which the winning firm drills in equilibrium in the non-disclosure auction but does not drill in equilibrium in the full-disclosure auction .*

To explain the difference between $\beta^{ND}(\cdot)$ with $\beta^{FD}(\cdot)$ in Eq. (5), first recall that $v(s',s')$ is firm i's expected profit from winning the full-disclosure auction when conditioned on $S_i = Y_i = s'$. In the non-disclosure auction, firm i's expected profit from winning when conditioned on $S_i = Y_i = s'$ — i.e., the analog of $v(s',s')$ — is $\psi\left(\hat{V}_0(s',s')\right)$. Therefore, the first part of $\beta^{ND}(s)$ is the part of the equilibrium bidding strategy that reflects the winner's curse, as explained earlier. The second term (in the square bracket of Eq. (12)) represents an incentive to bid higher, which is not present in the equilibrium bidding strategy of the full-disclosure auction. This additional incentive arises because if firm i wins the auction with a higher bid, it becomes more optimistic about the value of Y_i and, hence, the expected oil/gas value in the tract. By contrast, in the full-disclosure auction, this incentive is absent because there is no uncertainty (in equilibrium) about the value of Y_i after the auction.

Finally, Proposition 3 also illustrates the auctioneer's tradeoff from concealing the losing bids. First, one can verify that $v(s', s') > \psi\left(\hat{V}_0(s', s')\right)$ — the value of winning the full-disclosure auction is higher than the value of winning the non-disclosure auction because the winning firm acts with more information about the oil/gas value after the full-disclosure auction. Moreover, the additional incentive to bid higher in the non-disclosure auction (as described above) is not sufficient to offset the difference between $v(s', s')$ and $\psi\left(\hat{V}_0(s', s')\right)$. Therefore, on the one hand, the equilibrium bids are always higher in the full-disclosure auction than in the non-disclosure auction; on the other hand, it is possible that the post-auction drilling rate is higher in the non-disclosure auction. Thus, it is not clear which of the two auctions gives the auctioneer a higher expected total profit.

6.1.2 Counterfactual Outcomes

We explore the impacts of a non-disclosure policy in which the winning bidder does not receive any information on the losing bids in Figure 6.3 and Table 6.1. As our current estimation is using only the auctions with 2 bidders, our counterfactual results here are also for the same subsample. In this exercise, we fix the realization of the bidders' private signals and compute the distribution of the non-disclosure policy's impact across all the auctions in our data. The equilibrium bids are computed using our result in Proposition 3. The winning bidder's probability of drilling is straightforwardly computed from the model, and the undiscounted royalty payment is calculated using all the information available to the auctioneer, i.e., all the bidders' private signals. We compare the drilling probability of the non-disclosure policy to the expected outcome of a full disclosure policy over all the realizations of the second-highest signals.

The changes in the probability of drilling is highlighted in panel (a). For most tracts, a non-disclosure policy increases the likelihood of drilling. The average drilling probability increases by 0.3 percentage point, and, for some tracts, the drilling probability increases by more than 1.5 percentage points (recall that the average drilling rate under full disclosure is about 20%). As a result, the royalty revenue increases by 1.5%, on average, per tract,

Figure 6.3: Changes in bids and drilling probabilities from full disclosure to non-disclosure

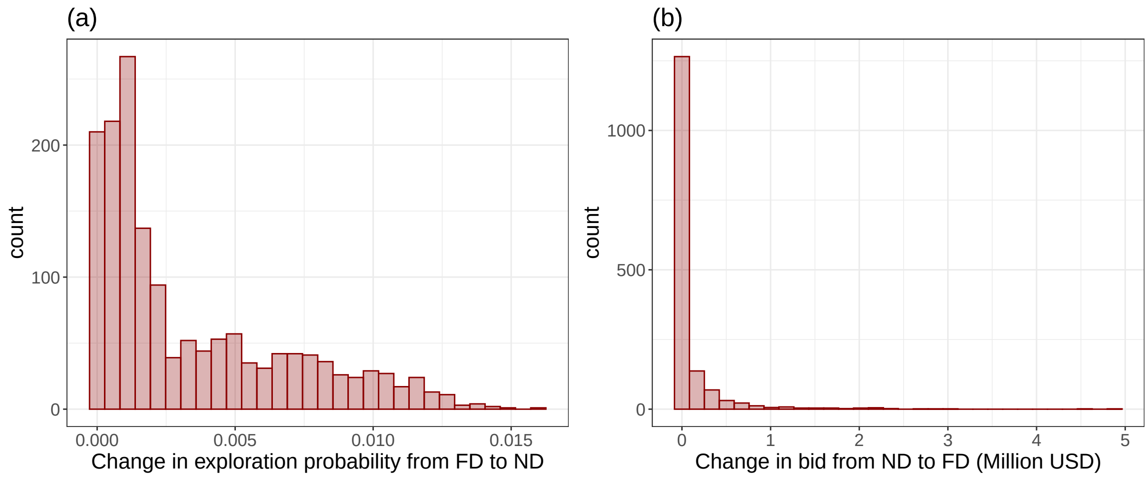


Table 6.1: Estimated effects on revenue from full disclosure to non-disclosure

	Average	SD	25pct	75pct
Change in Bids	-0.112	0.009	-0.050	-0.001
Pct Change in Bid	-1.771	0.050	-3.010	-0.273
Change in Drilling Probability	0.003	0.0001	0.001	0.005
Pct Change in Royalty Revenue	1.499	0.040	0.271	2.643
Change in Royalty Revenue	1.555	0.134	0.032	0.699
Change in Net Revenue	1.443	0.128	0.029	0.628

which is equivalent to \$1.55M per tract. Panel (b) illustrates our result in Proposition 3: the equilibrium bids under non-disclosure is always lower than those under full disclosure. On average, the non-disclosure bids are 1.77% lower than the full disclosure bids, which is equivalent to \$112K. Using the weights mentioned above, this is equivalent to an expected 1.01% increase in the federal government's annual net revenue, which is equivalent to an \$80M annual increase in revenue.

To determine the magnitude of a change in the information disclosure policy relative to other policy tools, we also conduct a counterfactual exercise in which the royalty rate is reduced to 10%. Currently, the royalty rate is either 12.5%, 16.67%, or 18.75%, and the average royalty rate across tracts in our sample is approximately 16%. We focus on the royalty rate in this exercise as a useful benchmark because the royalty rate has been the most utilized policy tool by the government to increase the exploration rate. Our estimates show that the reduction of the royalty rate to 10% leads to an 8.09% increase in bids and a 0.3 percentage point increase in the probability of drilling than our full disclosure benchmark. This increase in the exploration rate from the royalty reduction is comparable to the increase in the exploration rate from changing to a non-disclosure policy. However, due to the reduction in the royalty rate, the royalty revenue is reduced by 40%.

6.2 Partial Information Revelation of Bids Ex Post

Next, instead of considering the winner of the auction either observing all the submitted bids or not observing anything, we allow the auctioneer to commit to more general types of bid disclosure policies. Below, we first extend our baseline model to incorporate such disclosure policies and characterize the equilibrium bid. The counterfactual results will be included in a future version.

Formally, we define a *disclosure policy* D as a double $(\mathcal{M}, \mu)$, where $\mathcal{M}$ is a compact metric space of messages, and $\mu : \mathbb{R}_+^{N-1} \to \Delta(\mathcal{M})$ is a measurable map from the set of bids to $\Delta(\mathcal{M})$, which denotes the set of Borel probability measures over M, endowed with the weak* topology. Fixing a set of N bids received by the auctioneer and ordering them in decreasing order, with $b^{(1)} \geq b^{(2)} \geq \cdots \geq b^{(N)}$, $\mu\left(\cdot|b^{(2)}, \ldots, b^{(N)}\right) \in \Delta(\mathcal{M})$ is the probability measure of the messages that are sent to the winner after the auction — i.e., the bidder who submitted $b^{(1)}$.

The modifications to the game are as follows: at the start of Stage 1, the auctioneer announces the disclosure policy $D = (M, \mu)$. Subsequently, in Stage 4, instead of observing all the losing bids, the winner now receives only a message from the auctioneer according to μ. Upon receiving the message, the winning firm then updates its belief about the values of the bids submitted by the other firms using Bayes rule before making its drilling decision in the subsequent stage.

Although the disclosure policy is an endogenous choice of the auctioneer (and a point of interest in our subsequent counterfactual exercises), our equilibrium analysis here takes the disclosure policy as given and only considers the equilibrium of the subgame after the disclosure policy is announced. Therefore, we do not specify the set of disclosure policies available to the auctioneer.[14]

Our formulation of a disclosure rule is flexible, but there are still two substantial restric-

[14] Allowing the auctioneer the access to the "universal" set of disclosure policies would lead to the paradox of specifying a "set of all sets" that the auctioneer can optimize over. More pertinently, the feasibility of implementing any particular disclosure policy depends on the commitment power that the auctioneer possesses, which, in turn, depends on the context. Therefore, we do not take up this issue in our general framework.

tions. First, the message received is independent of the highest bid. This eliminates any strategic concerns of firms choosing their bids to influence the message that it receives upon winning. In terms of information provision, this restriction implies that the message does not carry any information on the winning bid, but this is without loss of generality because the message is meant for only the winning firm, who clearly knows its own bid. Second, the message carries only information about the values of the losing bids but not information about the identity of the firm that places each bid — this is natural because the firms are ex ante identical and we focus on only symmetric equilibria.

We provide a few examples of simple disclosure policies to fix idea. Let $\boldsymbol{b}$ denote an arbitrary set of bids $\left(b^{(2)}, \ldots, b^{(N)}\right) \in \mathbb{R}_+^{N-1}$, ordered in a decreasing order as above.

"Truth-or-noise" disclosure: $\mathcal{M} = \{\phi\} \cup \mathbb{R}_+^{N-1}$; for any $\boldsymbol{b}$, $\mu\left(\boldsymbol{b}|\boldsymbol{b}\right) = \lambda$, and $\mu\left(\phi|\boldsymbol{b}\right) = 1 - \lambda$. Here, the auctioneer fully discloses all the losing bids with a probability of λ, and does not disclose any information with the complement probability.

Disclosure of the k highest bids: $\mathcal{M} = \mathbb{R}_+^k$, where $k \leq N-1$; for any $\boldsymbol{b}$,

$$\mu\left(\left(b^{(2)}, \ldots, b^{(k+1)}\right) \mid \left(b^{(2)}, \ldots, b^{(N)}\right)\right) = 1.$$

Here, the auctioneer discloses only the k highest losing bids

Threshold Disclosure: $\mathcal{M} = \{\phi\} \cup \mathbb{R}_+ \cup \mathbb{R}_+^2 \cup \cdots \cup \mathbb{R}_+^{N-1}$; for any $\boldsymbol{b}$, if $b^{(2)} < \alpha$, then $\mu\left(\phi|\boldsymbol{b}\right) = 1$; if $b^{(N)} \geq \alpha$, then $\mu\left(\boldsymbol{b}|\boldsymbol{b}\right) = 1$; if $b^{(2)} \geq \alpha$ and $b^{(N)} < \alpha$, then $\mu\left(\left(b^{(2)}, \ldots, b^{(j)}\right)|\boldsymbol{b}\right) = 1$, where $b^{(j)}$ is the lowest bid that is above α. Here, the auctioneer discloses only the bids that are higher than α.

Disclosure of a summary statistics: $\mathcal{M} = \mathbb{R}$; for any $\boldsymbol{b}$, $\mu\left(\xi\left(\boldsymbol{b}\right)|\boldsymbol{b}\right) = 1$. where $\xi\left(\boldsymbol{b}\right)$ is a summary statistics function, such as the mean or the median of $\boldsymbol{b}$. Here, the auctioneer discloses only a summary statistics of the losing bids.

6.2.1 Equilibrium Bids

Fix a disclosure rule $D = (\mathcal{M}, \mu)$. Note that, besides information from the message, the knowledge that a firm has won the auction with a bid of b also provides information that all the other bids are lower than b, and this is potentially a piece of information that is not revealed by the message (e.g., in the non-disclosure case). Therefore, the winning firm essentially updates her belief twice — first, upon knowing that its bid is the highest, and, second, upon receiving the message. Because the firms are Bayes-rational, the order of belief-updating is inconsequential, and it is more instructive to consider the winning firm updating its belief using the message first, before using the knowledge that it is the winning firm.

Accordingly, let $B_i^{(1)}$, $B_i^{(2)}$, ..., $B_i^{(N-1)}$ denote the random variables representing the bids of all firms $j \neq i$, where $B_i^{(k)}$ is the k-th highest bid among them. Let $P_{D,\beta}^{Y,\boldsymbol{Z}}(Y_i, \boldsymbol{Z}_i | S_i; m)$ be firm i's *posterior* joint distribution of (Y_i, Z_i), conditioned on $S_i = s$, firm i observing a message m that is generated according to the probability measure $\mu\left(\cdot | B_i^{(1)}, B_i^{(2)}, \ldots, B_i^{(N-1)}\right)$, and firm i conjecturing that every other firm bids according to an increasing function $\beta(\cdot)$.[15] In addition, let $P_{D,\beta}^{Y}(Y_i | S_i, m)$ be the associated posterior distribution of Y_i.[16] When firm i further updates its belief based on the knowledge that its bid b_i is the winning bid, its posterior belief is

$$P_{D,\beta}^{Y,\boldsymbol{Z}}(Y_i, \boldsymbol{Z}_i | S_i; m, b_i) = \begin{cases} \frac{P_{D,\beta}^{Y,\boldsymbol{Z}}(Y_i, \boldsymbol{Z}_i | S_i, m)}{P_{D,\beta}^{Y}(\beta^{-1}(b_i) | S_i, m)} & \text{if } Y \leq \beta^{-1}(b_i) \\ 1 & \text{if } Y > \beta^{-1}(b_i) \end{cases} \tag{13}$$

Note that if $B_i^{(2)}$ is always perfectly revealed, then $P_{D,\beta}^{Y,\boldsymbol{Z}}(Y_i, \boldsymbol{Z}_i | S_i; m, b_i) = P_{D,\beta}^{Y,\boldsymbol{Z}}(Y_i, \boldsymbol{Z}_i | S_i; m)$ for all b_i — i.e., firm i's posterior is independent of its own bid.

Therefore, conditional on $S_i = s$, firm i winning the auction with a bid of b and receiving message m, firm i's expected value of the oil/gas reserve at the end of Stage 4 is

$$\tilde{V}_{D,\beta}(s; m, b) := \frac{1}{P_{D,\beta}^{Y}(\beta^{-1}(b) | s, m)} \int_{\boldsymbol{z}} \int_0^{\beta^{-1}(b)} V(s, y, \boldsymbol{z}) \bar{P}_{D,\beta}^{Y,\boldsymbol{Z}}(dy, d\boldsymbol{z} | s; m),$$

where $\bar{P}_{D,\beta}^{Y,\boldsymbol{Z}}(\cdot | s; m)$ denote the probability *measure* associated with the distribution $P_{D,\beta}^{Y,\boldsymbol{Z}}(\cdot | s; m,)$.[17] In turn, conditional on $S_i = s$, $Y_i = y$ and firm i winning the auction with a bid of b, firm

[15]i.e., given D, β and $S_i = s$, for any $\left(Y_i, \hat{\boldsymbol{Z}}_i\right) = (\hat{y}, \hat{\boldsymbol{z}})$ and Borel set $M \subset \mathcal{M}$,

$$\int_{y_i \in [0,\bar{s}]} \int_{\boldsymbol{z}_i \in [0,\bar{s}]^{N-2}} \left[\int_{m \in M} P_{D,\beta}^{Y,\boldsymbol{Z}}(\hat{y}, \hat{\boldsymbol{z}} | s, m)\, d\mu\left(m | \beta(y_i), \beta\left(z_i^{(2)}\right), \ldots, \beta\left(z_i^{(N)}\right)\right)\right] h(y_i, \boldsymbol{z}_i | s)\, d\boldsymbol{z}_i dy_i$$
$$= \int_{y_i \leq \hat{y}} \int_{\boldsymbol{z}_i \leq \hat{\boldsymbol{z}}} \mu\left(M | \beta(y_i), \beta\left(z_i^{(2)}\right), \ldots, \beta\left(z_i^{(N)}\right)\right) h(y_i, \boldsymbol{z}_i | s)\, d\boldsymbol{z}_i dy_i.$$

[16]i.e., $P_{D,\beta}^{Y}(Y_i | S_i, m) = P_{D,\beta}^{Y,\boldsymbol{Z}}(Y_i, \boldsymbol{Z}_i | S_i; m)\Big|_{\left(Z_i^{(2)}, Z_i^{(3)}, \ldots, Z_i^{(N-1)}\right) = (\bar{s}, \bar{s}, \ldots, \bar{s})}$.

[17]i.e., $\bar{P}_{D,\beta}^{Y,\boldsymbol{Z}}\left([y, y'], \left[z^{(2)}, z^{(2)\prime}\right], \ldots, \left[z^{(N-1)}, z^{(N-1)\prime}\right] | s; m\right) = P_{D,\beta}^{Y,\boldsymbol{Z}}\left(y', z^{(2)\prime}, \ldots, z^{(N-1)} | s; m\right) - P_{D,\beta}^{Y,\boldsymbol{Z}}\left(y, z^{(2)}, \ldots, z^{(N-1)\prime} | s; m\right)$

i's expected profit from the perspective at the start of Stage 2 is

$$\tilde{v}_{D,\beta}(s,y;b) := \int_m \psi\left(\tilde{V}_{D,\beta}(s;m,b)\right) R_{D,\beta}(dm|s,y),$$

where, $R_{D,\beta}(\cdot|s,y) \in \Delta(\mathcal{M})$ denotes the ex ante probability measure over the set of messages that firm i receives after winning the auction when conditioned on $S_i = s$, $Y_i = y$, and every firm $j \neq i$ using the bidding strategy β.[18] Therefore, at the start of Stage 2, if the disclosure policy is D, $S_i = s$, and firm i conjectures that every other firm $j \neq i$ bids according to β, firm i's expected profit from bidding b is

$$\int_0^{\beta^{-1}(b)} \left[\tilde{v}_{D,\beta}(s,y;b) - b\right] g^Y(y|s)\, dy. \tag{14}$$

Let

$$\begin{aligned}
\hat{V}^*_{D,\beta}(s,y;m) :=& E_{P^{Y,Z}_{D,\beta}(\cdot|s,m)}\left[V(s,Y_i,Z_i)\,|Y_i \leq y\right] \\
\bar{V}^*_{D,\beta}(s,y;m) :=& E_{P^{Y,Z}_{D,\beta}(\cdot|s,m)}\left[V(s,Y_i,Z_i)\,|Y_i = y\right],
\end{aligned}$$

where $E_{P^{Y,Z}_{D,\beta}(\cdot|s,m)}$ denote the expectation operator with respect to the distribution $P^{Y,Z}_{D,\beta}(Y_i,Z_i|s,m)$.[19] Let

$$\Delta^*_{D,\beta}(s,y;m) := \bar{V}^*_{D,\beta}(s,y;m) - \hat{V}^*_{D,\beta}(s,y;m),$$

and let $H^Y_{D,\beta}(Y_i|S_i,m)$ denote the reverse hazard rate associated with the distribution $P^Y_{D,\beta}(Y_i|S_i,m)$.[20]

Proposition 4. *Fix a disclosure policy D. If $\beta(\cdot)$ is an increasing and symmetric Bayes Nash equilibrium of the auction, then $\beta(\cdot)$ satisfies*

$$\beta(s) = \int_0^s \left[v^*_{D,\beta}(s',s') + w^*_{D,\beta}(s',s')\right] dL(s'|s) \quad \forall s \in [0,\bar{s}], \tag{15}$$

where

$$v^*_{D,\beta}(s,y) := \int_m \psi\left(\hat{V}^*_{D,\beta}(s,y;m)\right) R_{D,\beta}(dm|s,y),$$

[18] i.e., for any Borel set $M \subset \mathcal{M}$, $R_{D,\beta}(M|s,y) = \int_{\boldsymbol{z}} \mu\left(M|\beta(y), \beta\left(z^{(2)}\right), \ldots \beta\left(z^{(N)}\right)\right) g^{\boldsymbol{z}}\left(z^{(2)}, \ldots, z^{(N)}|s,y\right) d\boldsymbol{z}$

[19] It is useful to note that $\tilde{V}_{D,\beta}(s;m,b) = \hat{V}^*_{D,\beta}\left(s,\beta^{-1}(b);m\right)$.

[20] The reversed hazard rate is the ratio of the probability density/mass function and the cumulative distribution function. We take the convention that if $P^Y_{D,\beta}(y|s,m) = 0$, then $H^Y_{D,\beta}(y|s,m) = 0$.

$$w^*_{D,\beta}(s,y) := \int_0^y \left(\int_m \left[\psi' \left(\hat{V}^*_{D,\beta}(s,y;m) \right) H^Y_{D,\beta}(y|s,m) \, \Delta^*_{D,\beta}(s,y;m) \right] R_{D,\beta}(dm|s,y') \right) \frac{g^Y(y'|s)}{g^Y(y|s)} dy'$$

Moreover, if a function $\beta(\cdot)$ satisfies Eq. (15) and $v^*_{D,\beta}(s,y)+w^*_{D,\beta}(s,y)$ is increasing in both s and y, then $\beta(\cdot)$ is an increasing and symmetric Bayes Nash equilibrium of the auction.

As observed in the non-disclosure auction previously, when there is ex post uncertainty about the value of Y_i, there is an additional incentive for firm i to bid higher. This effect arises in Eq. (15) through the term $w^*_{D,\beta}$. To see this, note that if $P^Y_{D,\beta}(Y_i|s,m)$ is a degenerate distribution,[21] then $\hat{V}^*_{D,\beta}(s,y;m) = \bar{V}^*_{D,\beta}(s,y;m)$ for all y, which implies that $\Delta^*_{D,\beta}(s,y;m) = 0$. If this holds for all m — i.e., there is never uncertainty about Y_i after every message — then the term $w^*_{D,\beta}(s,y)$ is always zero.

7 Conclusion

In this paper, we empirically study how information on the losing bids should be strategically revealed to the winning bidder in the context of the US OCS auctions and quantify the potential revenue gain from such an information mechanism. In an OCS auction, the auctioneer (the government) auctions off the right to extract oil and gas on federal offshore tracts via a first-price sealed-bid auction. In addition to receiving the winning cash bid from the auction, the government also charges a royalty on the tract's production value. The winning bidder decides whether to explore the tract, which is a costly decision. Unexplored tracts do not produce oil and gas; therefore, the winning bidder's post-auction action also affects the auctioneer's payoff.

Two features in this setting suggest the importance of considering a bid disclosure policy as part of the auction's design. First, the low exploratory rate of these leases has been an ongoing issue that adversely affects the federal's government revenue from these leases. Second, the current policy allows the winning bidder to observe all the losing bids. Our reduced-form analysis offers encouraging evidence highlighting the significance of rival firms' private information, inferred by the winning bidder through the rivals' bids, on the winning bidder's exploration decision. According to our analysis, a 1% increase in the second-highest bid leads to a 3.9 percentage point increase in the probability of exploratory drilling. As a comparison, a 1% increase in the winning bid leads to only a 4.5 percentage point increase in the probability of exploratory drilling, thus implying that the winning firm's exploration

[21] i.e., there exists $\hat{y}$ such that $P^Y_{D,\beta}(\hat{y}|s,m) = 1$ and $P^Y_{D,\beta}(y|s,m) = 0$ for all $y < \hat{y}$.

decision is similarly affected by the information of its rival firms as by its own private information.

We first construct and estimate a model of first-price sealed-bid pure common value auction in which the winner also chooses whether to explore the tract after the auction at a cost after observing all the losing bids. We show that, by combining the bid data, the variation in the exploration rate across auctions, and an exploration cost shifter instrument, the firms' posterior beliefs of the production value of the tract conditional on all the bidders' private signals are nonparametrically identified.

We then extend the model to the case in which the auctioneer can use an alternative bid disclosure policy, for example, not disclosing any losing bids (non-disclosure) or disclosing only summary statistics of the losing bids. We provide a characterization of the equilibrium bidding strategy for a large class of disclosure policies of the losing bids. Our counterfactual analysis reveals that an alternative bid disclosure policy can significantly impact the government's revenue. For example, taking into account the policy's effect on the equilibrium bids, a complete non-disclosure policy increases the government's annual revenue by approximately $80 million.

A Proofs for Section 4 and Section 6

As the full-disclosure and non-disclosure auctions are special cases of disclosure policies, we can use Proposition 4 to derive their equilibria. Therefore, we first provide the proof of Proposition 4 before proving Propositions 1 and 3.

A.1 Proof of Proposition 4

Proof. Fix any strictly increasing $\beta(\cdot)$ and suppose that all firms $j \neq i$ play according to β. Firm i's expected profit from bidding b when $S_i = s$ is in Eq. (14). Differentiating Eq. (14) with respect to b, we get the first-order necessary condition (FOC) of

$$\begin{aligned} 0 = & -G^Y\left(\beta^{-1}(b)\,|s\right) + \frac{1}{\beta'\left(\beta^{-1}(b)\right)}\left[\tilde{v}_{D,\beta}\left(s, \beta^{-1}(b); b\right) - b\right] g^Y\left(\beta^{-1}(b)\,|s\right) \\ & + \int_0^{\beta^{-1}(b)} \frac{\partial \tilde{v}_{D,\beta}(s, y; b)}{\partial b} g^Y(y|s)\, dy. \end{aligned} \tag{16}$$

Note that

$$\begin{aligned} \tilde{v}_{D,\beta}\left(s, \beta^{-1}(b); b\right) &= \int_m \psi\left(\hat{V}^*_{D,\beta}\left(s, \beta^{-1}(b); m\right)\right) R_{D,\beta}\left(dm|s, \beta^{-1}(b)\right) = v^*_{D,\beta}\left(s, \beta^{-1}(b)\right) \\ &= v^*_{D,\beta}\left(s, \beta^{-1}(b)\right) \end{aligned}$$

and

$$\frac{\partial \tilde{v}_{D,\beta}(s, y; b)}{\partial b} = \int_m \psi'\left(\hat{V}^*_{D,\beta}\left(s, \beta^{-1}(b); m\right)\right)\left[\frac{\partial \tilde{V}_{D,\beta}(s; m, b)}{\partial b}\right] dR_{D,\beta}(m|s, y)$$

where

$$\begin{aligned} &\frac{\partial \tilde{V}_{D,\beta}(s; m, b)}{\partial b} \\ &= \left(\frac{1}{\beta'\left(\beta^{-1}(b)\right)}\right)\left[\frac{\int_{\boldsymbol{z}} V\left(s, \beta^{-1}(b), \boldsymbol{z}\right) \bar{P}^{Y,\boldsymbol{Z}}_{D,\beta}\left(\beta^{-1}(b), d\boldsymbol{z}|s; m\right)}{P^Y_{D,\beta}\left(\beta^{-1}(b)\,|s, m\right)} - H^Y_{D,\beta}\left(\beta^{-1}(b_i)\,|s, m\right)\tilde{V}_{D,\beta}(s; m, b)\right] \\ &= \left(\frac{1}{\beta'\left(\beta^{-1}(b)\right)}\right) H^Y_{D,\beta}\left(\beta^{-1}(b_i)\,|s, m\right)\left[\bar{V}^*_{D,\beta}\left(s, \beta^{-1}(b); m\right) - \hat{V}^*_{D,\beta}\left(s, \beta^{-1}(b); m\right)\right] \end{aligned}$$

Therefore, $\int_0^{\beta^{-1}(b)} \frac{\partial \tilde{v}_{D,\beta}(s,y;b)}{\partial b} g^Y(y|s)\,dy = \frac{g^Y\left(\beta^{-1}(b)|s\right)}{\beta'(\beta^{-1}(b))} w^*_{D,\beta}\left(s, \beta^{-1}(b)\right)$. In turn, FOC (16) becomes

$$0 = -G^Y\left(\beta^{-1}(b)|s\right) + \frac{g^Y\left(\beta^{-1}(b)|s\right)}{\beta'(\beta^{-1}(b))}\left[v^*_{D,\beta}\left(s, \beta^{-1}(b)\right) + w^*_{D,\beta}(s,y) - b\right] \tag{17}$$

At a symmetric equilibrium in which every firm plays β, FOC (17) must be satisfied at $b = \beta(s)$, which implies that

$$\begin{aligned} & 0 = -G^Y(s|s) + \frac{g^Y(s|s)}{\beta'(s)}\left[v^*_{D,\beta}(s,s) + w^*_{D,\beta}(s,s) - \beta(s)\right] \\ \iff \quad & \beta'(s) + \beta(s)\frac{g^Y(s|s)}{G^Y(s|s)} = \left[v^*_{D,\beta}(s,s) + w^*_{D,\beta}(s,s)\right]\frac{g^Y(s|s)}{G^Y(s|s)} \end{aligned} \tag{18}$$

Eq. (18) is a linear ordinary differential equation (ODE). It is well-know that the solution to a linear ODE of the form $\beta'(s) + \beta(s)\xi(s) = \gamma(s)\xi(s)$ is

$$\theta(s)\beta(s) = \theta(0)\beta(0) + \int_0^s \gamma(s')\xi(s')\theta(s')\,ds',$$

where $\theta(s) = \exp\left(\int_0^s \xi(s')\,ds'\right)$. Using the boundary condition of $\beta(0) = 0$, we thus have

$$\begin{aligned} \beta(s) &= \int_0^s \left[v^*_{D,\beta}(s',s') + w^*_{D,\beta}(s',s')\right]\left(\frac{g^Y(s'|s')}{G^Y(s'|s')}\frac{\exp\left(\int_0^{s'}\frac{g^Y(t|t)}{G^Y(t|t)}dt\right)}{\exp\left(\int_0^{s}\frac{g^Y(t|t)}{G^Y(t|t)}dt\right)}\right)ds' \\ &= \int_0^s \left[v^*_{D,\beta}(s',s') + w^*_{D,\beta}(s',s')\right]\left(\frac{g^Y(s'|s')}{G^Y(s'|s')}\exp\left(-\int_{s'}^{s}\frac{g^Y(t|t)}{G^Y(t|t)}dt\right)\right)ds' \\ &= \int_0^s \left[v^*_{D,\beta}(s',s') + w^*_{D,\beta}(s',s')\right]dL(s'|s). \end{aligned} \tag{19}$$

This proves the first (necessity) part of the proposition.

Next, to prove the sufficiency part, suppose that $v^*_{D,\beta}(s',s') + w^*_{D,\beta}(s',s')$ is also increasing. Note that for any s, $L(0|s) = 0$, $L(s|s) = 1$ and $L(\cdot|s)$ is increasing; therefore, $L(\cdot|s)$ is a CDF with a support on $[0,s]$. Moreover, if $\hat{s} > s$, then $L(y|\hat{s}) < L(y|s)$ for all y — i.e., first-order stochastic dominance. In turn, because the integrand term in Eq. (19), $\beta(s)$ is (indeed) an increasing function. to show that bidding $b = \beta(s)$ is indeed firm i's best response against all other firms playing β, first note that bidding above $\beta(\bar{s})$ is worse than bidding $\beta(s)$; therefore, firm i's best response (if it exists) is a bid in $[\beta(0), \beta(\bar{s})]$. Suppose that firm i bids $\beta(\tilde{s})$, where $\tilde{s} \neq s$. Its expected profit (using Eq. (14)) is

$$U(\tilde{s}|s) := \int_0^{\tilde{s}} \left[\tilde{v}_{D,\beta}(s,y;\beta(\tilde{s})) - \beta(\tilde{s})\right] g^Y(y|s)\,dy.$$

Let

$$\begin{aligned}\Delta(\tilde{s}|s) &= \frac{\partial U(\tilde{s}|s)}{\partial \tilde{s}} \\ &= -G^Y(\tilde{s}|s)\beta'(\tilde{s}) + \left[\tilde{v}_{D,\beta}(s,\tilde{s};\beta(\tilde{s})) - \beta(\tilde{s})\right] g^Y(\tilde{s}|s) + \beta'(\tilde{s})\int_0^{\tilde{s}} \frac{\partial \tilde{v}_{D,\beta}(s,y;\beta(\tilde{s}))}{\partial b} g^Y(y|s)\,dy \\ &= G^Y(\tilde{s}|s)\left(\left[v^*_{D,\beta}(s,\tilde{s}) + w_{D,\beta}(s,\tilde{s}) - \beta(\tilde{s})\right]\frac{g^Y(\tilde{s}|s)}{G^Y(\tilde{s}|s)} - \beta'(\tilde{s})\right).\end{aligned}$$

From FOC (18), $\Delta(\tilde{s}|\tilde{s}) = 0$. If $\tilde{s} > s$, then $v^*_{D,\beta}(s,\tilde{s}) + w_{D,\beta}(s,\tilde{s}) < v^*_{D,\beta}(\tilde{s},\tilde{s}) + w_{D,\beta}(\tilde{s},\tilde{s})$; moreover, because the signals are affiliated, $\frac{g^Y(\tilde{s}|s)}{G^Y(\tilde{s}|s)} < \frac{g^Y(\tilde{s}|\tilde{s})}{G^Y(\tilde{s}|\tilde{s})}$. Together with $\Delta(\tilde{s}|\tilde{s}) = 0$, we have $\Delta(\tilde{s}|s) < 0$. On the other hand, if $\tilde{s} < s$, then $v^*_{D,\beta}(s,\tilde{s}) + w_{D,\beta}(s,\tilde{s}) > v^*_{D,\beta}(\tilde{s},\tilde{s}) + w_{D,\beta}(\tilde{s},\tilde{s})$ and $\frac{g^Y(\tilde{s}|s)}{G^Y(\tilde{s}|s)} > \frac{g^Y(\tilde{s}|\tilde{s})}{G^Y(\tilde{s}|\tilde{s})}$, which implies that $\Delta(\tilde{s}|s) > 0$. Therefore, bidding $\beta(s)$ is firm i's best response. □

A.2 Proof of Proposition 1

Proof. The full-disclosure auction is equivalent to an auction with a disclosure policy of $\mathcal{M} = \mathbb{R}_+^{N-1}$ and $\mu(\boldsymbol{b}|\boldsymbol{b}) = 1$ for all $\boldsymbol{b}$. Under this disclosure policy, for any $m = \left(b^{(2)},\ldots,b^{(N)}\right)$, for any β, the posterior belief $P^{Y,\boldsymbol{Z}}_{D,\beta}(Y_i,\boldsymbol{Z}_i|S_i,m)$ is always a Dirac measure on $\left(Y_i, Z_i^{(2)},\ldots,Z_i^{(N-1)}\right) = \left(\beta^{-1}\left(b^{(2)}\right),\beta^{-1}\left(b^{(3)}\right),\ldots,\beta^{-1}\left(b^{(N-1)}\right)\right)$. This implies that

$$\hat{V}^*_{D,\beta}\left(s,y;\left(b^{(2)},\ldots,b^{(N)}\right)\right) = V\left(s,y,\beta^{-1}\left(b^{(3)}\right),\ldots,\beta^{-1}\left(b^{(N-1)}\right)\right)$$

and $\bar{V}^*_{D,\beta}(s,y;m) = \hat{V}^*_{D,\beta}(s,y;m)$. Therefore, $w^*_{D,\beta}$ is always the zero function. Next, abusing notation slightly, let $R_{D,\beta}(\cdot|s,y)$ denote the probability distribution (instead of measure) over the set of messages. For any m in which $b^{(2)} = \beta(y)$, $R_{D,\beta}\left(\left(b^{(2)},b^{(3)},\ldots,b^{(N)}\right)|s,y\right) = G^{\boldsymbol{Z}}\left(Z_i^{(2)} = \beta^{-1}\left(b^{(3)}\right),\ldots,Z_i^{(N-1)} = \beta^{-1}\left(b^{(N)}\right)|s,y\right)$. Therefore,

$$\begin{aligned}&v^*_{D,\beta}(s,y) \\ =&\int_{\beta^{-1}\left(b^{(3)}\right),\ldots,\beta^{-1}\left(b^{(N-1)}\right)} \psi\left(V\left(s,y,\beta^{-1}\left(b^{(3)}\right),\ldots,\beta^{-1}\left(b^{(N-1)}\right)\right)\right) dG^{\boldsymbol{Z}}\left(\beta^{-1}\left(b^{(3)}\right),\ldots,\beta^{-1}\left(b^{(N)}\right)|s,y\right)\end{aligned}$$

$$=v\left(s,y\right),$$

as defined in Eq. (3). Therefore, by Proposition 4, if there exists an increasing and symmetric equilibrium, then it must be $\beta^{FD}\left(\cdot\right)$. Moreover, if $v\left(s,y\right)$ is strictly increasing in both its arguments, then $\beta^{FD}\left(\cdot\right)$ is indeed an equilibrium, and this holds because V is strictly increasing and the signals are affiliated. □

A.3 Proof of Proposition 3

Proof. The non-disclosure auction is equivalent to an auction with a disclosure policy of $\mathcal{M}=\{\phi\}$ and $\mu\left(\phi|\boldsymbol{b}\right)=1$ for all $\boldsymbol{b}$. This implies that, for any β, $R_{D,\beta}\left(\cdot|s,y\right)$ is a Dirac measure on ϕ, and $P_{D,\beta}^{Y,\boldsymbol{Z}}\left(Y_i,\boldsymbol{Z}_i|S_i,m=\phi\right)=G^{Y,\boldsymbol{Z}}\left(Y_i,\boldsymbol{Z}_i|S_i\right)$. Therefore, for all β, $\hat{V}_{D,\beta}^{*}\left(s,y;\phi\right)=\hat{V}_0\left(s,y\right)$ (as defined in Eq. (9)), and $\bar{V}_{D,\beta}^{*}\left(s,y;\phi\right)=\bar{V}_0\left(s,y\right)$ (as defined in Eq. (10)). In turn,

$$v_{D,\beta}^{*}\left(s,y\right):=\psi\left(\hat{V}_0\left(s,y\right)\right),$$

and, because $H_{D,\beta}^{Y}\left(y|s,\phi\right)=\frac{g^Y(y|s)}{G^Y(y|s)}$,

$$\begin{aligned}
w_{D,\beta}^{*}\left(s,y\right)=&\int_0^y\psi'\left(\hat{V}_0\left(s,y\right)\right)\frac{g^Y\left(y|s\right)}{G^Y\left(y|s\right)}\left[\bar{V}_0\left(s,y\right)-\hat{V}_0\left(s,y\right)\right]\frac{g^Y\left(y'|s\right)}{g^Y\left(y|s\right)}dy'.\\
=&\psi'\left(\hat{V}_0\left(s,y\right)\right)\frac{1}{G^Y\left(y|s\right)}\left[\bar{V}_0\left(s,y\right)-\hat{V}_0\left(s,y\right)\right]\int_0^y g^Y\left(y'|s\right)dy'\\
=&\psi'\left(\hat{V}_0\left(s,y\right)\right)\left[\bar{V}_0\left(s,y\right)-\hat{V}_0\left(s,y\right)\right]
\end{aligned}$$

Therefore, by Proposition 4, if there exists an increasing and symmetric equilibrium, then it must be $\beta^{ND}\left(\cdot\right)$. Moreover, if $\psi\left(\hat{V}_0\left(s,y\right)\right)+\psi'\left(\hat{V}_0\left(s,y\right)\right)\left[\bar{V}_0\left(s,y\right)-\hat{V}_0\left(s,y\right)\right]$ is strictly increasing in both its arguments, then $\beta^{ND}\left(\cdot\right)$ is indeed an equilibrium; as before, this holds because V is strictly increasing and the signals are affiliated.

Next, to show that $\beta^{ND}\left(s\right)<\beta^{FD}\left(s\right)$, note that

$$\begin{aligned}
&\psi'\left(\hat{V}_0\left(s',s'\right)\right)\left[\bar{V}_0\left(s',s'\right)-\hat{V}_0\left(s',s'\right)\right]\\
=&\int_0^{(1-r)\hat{V}_0(s',s')}\left(1-r\right)\left[\bar{V}_0\left(s',s'\right)-\hat{V}_0\left(s',s'\right)\right]dF^C\left(c\right)\\
=&\int_0^{(1-r)\hat{V}_0(s',s')}\left(\left(1-r\right)\bar{V}_0\left(s',s'\right)-c\right)dF^C\left(c\right)-\int_0^{(1-r)\hat{V}_0(s',s')}\left(\left(1-r\right)\hat{V}_0\left(s',s'\right)-c\right)dF^C\left(c\right)\\
<&\psi\left(\bar{V}_0\left(s',s'\right)\right)-\psi\left(\hat{V}_0\left(s',s'\right)\right)
\end{aligned}$$

Therefore,

$$\begin{aligned}
\psi\left(\hat{V}_0\left(s',s'\right)\right)+\psi'\left(\hat{V}_0\left(s',s'\right)\right)\left[\bar{V}_0\left(s',s'\right)-\hat{V}_0\left(s',s'\right)\right] <&\psi\left(\bar{V}_0\left(s',s'\right)\right)\\
=&\psi\left(E\left[V\left(S_i,Y_i,\boldsymbol{Z}_i\right)|S_i=s',Y_i=s'\right]\right)\\
<&E\left[\psi\left(V\left(S_i,Y_i,\boldsymbol{Z}_i\right)\right)|S_i=s',Y_i=s'\right]\\
=&v\left(s',s'\right),
\end{aligned}$$

where the inequality in the third line follows from ψ being convex. Therefore, $\beta^{FD}(s) < \beta^{ND}(s)$. The last point that there exist y and $\boldsymbol{z}$ such that $\hat{V}_0(s,s) > V(s,y,\boldsymbol{z})$ holds trivially. □

B Proof for Section 5

B.1 Proof of Proposition 2

Proof. All of the objects to be identified in this proof are conditional on F^Q and N. For notational convenience, we drop $|F^Q$, N, and the tract's identifier from the conditional distribution.

Conditional on I, firm i's profit maximization problem if the firm bids b is

$$\begin{aligned}
&\arg\max_b \int_0^{\beta^{-1}(b)} \left[v(s,y|I)-b\right] dG^Y(y|s)\\
\implies\ & v(s,s|I) = b + \frac{\tilde{G}(b|b,I)}{\tilde{g}(b|b,I)} \qquad (20)
\end{aligned}$$

where $\tilde{G}(m|b,I)$ is the conditional distribution of the second-highest bid M when $M=m$ conditional on the highest bid being $B=b$. The density of this distribution is $\tilde{g}(m|b,I)$.

Let F^B denote the marginal distribution of firm i's bid B_i. F^B is the same for all the firms because the firms are symmetric. Hence, firm i's private signal is $S_i = F^B(B_i)$. Eq. (20) becomes

$$v(s,s|I) = F^{B^{-1}}(s) + \frac{\tilde{G}(F^{B^{-1}}(s)|F^{B^{-1}}(s),I)}{\tilde{g}(F^{B^{-1}}(s)|F^{B^{-1}}(s),I)}$$

Let $\Omega(s,I)$ denote the right-hand side of the above equation. $\Omega(\cdot)$ is identified from the data.

Let $\Delta(\boldsymbol{s}, I)$ be the probability that exploration occurs conditional on I and the firms' private signals $\boldsymbol{S} = \boldsymbol{s}$. $\Delta(\cdot)$ is also identified from the data.

Let $\phi(I) = \frac{\zeta(I)}{\delta(1-r)-\kappa(I)}$, the winner's exploration rate conditional on all the firms' signals $\boldsymbol{S} = \boldsymbol{s}$ and the cost shifter I is

$$\Delta(\boldsymbol{s}, I) = F^{C^0}\left(\frac{V(\boldsymbol{s})}{\phi(I)}\right) \tag{21}$$

Conditional on $S_i = s$, $Y_i = s$, using the definition of $v(s,s)$,

$$\Omega(s, I) = \begin{cases} \frac{\zeta(I)}{\phi(I)} \int^s \int^z \Delta(s, s, \tilde{\boldsymbol{z}}, I) \frac{\partial}{\partial \tilde{\boldsymbol{z}}} V(s, s, \tilde{\boldsymbol{z}})\ d\tilde{\boldsymbol{z}}\ dG^{\boldsymbol{Z}}(\boldsymbol{z}|s,s) & \text{If } N > 2 \\ \frac{\zeta(I)}{\phi(I)} \int^s \Delta(s', s', I) \frac{\partial}{\partial s'} V(s', s')\ ds' & \text{If } N = 2 \end{cases} \tag{22}$$

We prove by contradiction. Suppose that there exists $\left\{\tilde{V}(\cdot), \tilde{\zeta}(\cdot), \tilde{F}^{C^0}, \tilde{\phi}(I), \tilde{\delta}\right\}$ and $\left\{V(\cdot), \zeta(\cdot), F^{C^0}, \phi(I), \delta\right\}$ that both satisfy Eq. (21) and Eq. (22). Since F^{C^0} and $\tilde{F}^{C^0}$ are both strictly increasing and differentiable, for any $i \in \{1, .., N\}$

$$\frac{\frac{\partial}{\partial s_i} \log \tilde{V}(\boldsymbol{s})}{\frac{\partial}{\partial s_i} \log V(\boldsymbol{s})} = \frac{\frac{\partial}{\partial I} \log \tilde{\zeta}(I)}{\frac{\partial}{\partial I} \log g(I)}\ \forall \boldsymbol{s}, I, i = 1, 2, ..., N$$

Therefore, there exists two constants $k_0, k_1 > 0$ such that

$$\begin{aligned} \tilde{\phi}(I) &= k_1 \phi(I)^{k_0} \\ \tilde{V}(\boldsymbol{s}) &= k_1 V(\boldsymbol{s})^{k_0} \end{aligned}$$

We first prove $k_0 = 1$ by contradiction. WLOG, assume that $k_0 > 1$.

We consider two cases:

Case 1: $N > 2$**.** From Eq (22), conditional on $S_i = s$

$$\begin{aligned} 0 &= \int^s \int^z \Delta(s, s, \tilde{\boldsymbol{z}}, I) \left[\frac{\zeta(I)}{\phi(I)} \frac{\partial}{\partial \tilde{\boldsymbol{z}}} V(s, s, \tilde{\boldsymbol{z}}) - \frac{\tilde{\zeta}(I)}{\tilde{\phi}(I)} k_1 \frac{\partial}{\partial \tilde{\boldsymbol{z}}} V^{k_0}(s, s, \tilde{\boldsymbol{z}})\right]\ d\tilde{\boldsymbol{z}}\ dG^{\boldsymbol{Z}}(\boldsymbol{z}|s,s) \\ &= \int^s \int^z \Delta(s, s, \tilde{\boldsymbol{z}}, I) \left[\frac{\partial}{\partial \tilde{\boldsymbol{z}}} V(s, s, \tilde{\boldsymbol{z}}) \left(\frac{\zeta(I)}{\phi(I)} - \frac{\tilde{\zeta}(I)}{\tilde{\phi}(I)} k_1 k_0 V^{k_0 - 1}(s, s, \tilde{\boldsymbol{z}})\right)\right]\ d\tilde{\boldsymbol{z}}\ dG^{\boldsymbol{Z}}(\boldsymbol{z}|s,s) \end{aligned}$$

$$\geq \int^{s}\int^{z} \Delta\left(s,s,\tilde{\boldsymbol{z}},I\right)\left[\frac{\partial}{\partial\tilde{\boldsymbol{z}}}V(s,s,\tilde{\boldsymbol{z}})\left(\frac{\phi(I)}{\zeta(I)}-k_1k_0V^{k_0-1}(s,s,\tilde{\boldsymbol{z}})\right)\right]\, d\tilde{\boldsymbol{z}}\; dG^{\boldsymbol{z}}(\boldsymbol{z}|s,s) \tag{23}$$

where the last inequality is due to $\frac{\tilde{\phi}(I)}{\tilde{\zeta}(I)}\leq 1$. Due to the affiliation property of $\boldsymbol{S}$, $\frac{\partial}{\partial\tilde{\boldsymbol{z}}}V(s,s,\tilde{\boldsymbol{z}}) > 0$. Since $V(\boldsymbol{0})=0$, for $S_i=s$ sufficiently small, the right hand side of Eq. (23) is positive, which is a contradiction.

Case 2: $N=2$**.** From Eq (22), conditional on $S_i=s$

$$0=\int^{s}\Delta\left(s',s',I\right)\left(\frac{\zeta(I)}{\phi(I)}\frac{\partial}{\partial s'}V(s',s')-\frac{\tilde{\zeta}(I)}{\tilde{\phi}(I)}k_1\frac{\partial}{\partial s'}V^{k_0}(s',s')\right)\, ds' \tag{24}$$

Similar to the $N>2$ case, for $S_i=s$ sufficiently small, the right hand side of Eq. (24) is positive, which is a contradiction.

Therefore, $k_0=1$. Since $\mathbb{E}(Q)=\mathbb{E}V(\boldsymbol{S})$, $k_1=1$. Combining with Eq. (22), this implies $\zeta(I)=\tilde{\zeta}(I)$. Therefore, $\left\{V(\cdot),\zeta(\cdot),\phi(\cdot),F^{C^0}\right\}$ are identified. Since we have variation in r, δ and $\kappa(\cdot)$ are also identified. $\square$

C More Details on Estimation

C.1 Estimate oil capacity of productive wells that are still in production

We assume that once leases begin production, the lease' oil and gas quantities are perfectly observed by all potential bidders. For leases that have expired —which we term 'completed' leases—the oil and gas quantities are the total amount of oil/gas the lease has produced. For leases that have not expired by the end date of our data set, we use the following regression on completed lease to predict the total amount of oil and lease in the tract:

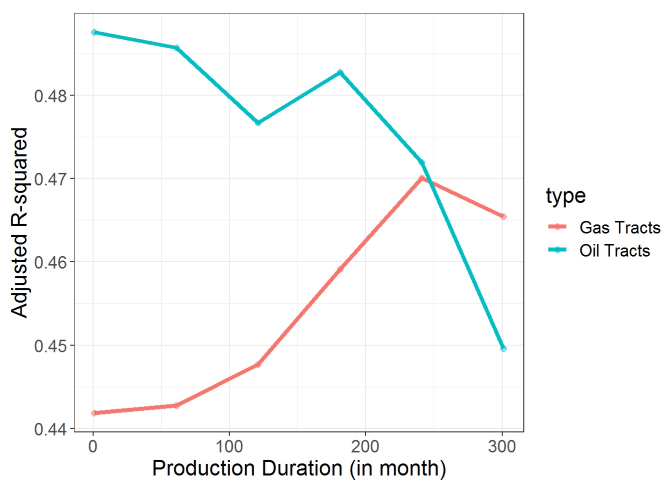
$$\log\left(\sum_{\tau=T}^{\infty}Q_t^{i,T}\right)=\beta_0+\sum_{j=1}^{\underline{T}}\beta_jQ_t^{i,T-j}+\alpha X_t+\gamma_T+\epsilon_t^i \tag{25}$$

where $i\in\{o,g\}$, and $Q_t^{i,\tau}$ is the total amount of reserve type i produced by tract t after τ months of production, and X_t are tract characteristics. We allow the regression coefficients $\{\beta_0,\beta_{j,j=1,2,\ldots,\underline{T}},\alpha,\gamma\}$ to be dependent on the range of T. In our benchmark estimates, $\underline{T}=2$ years, and we estimate regressions separately for $T<5$ years, $T<10$ years, ..., up to 30 years. Table C.1 summarizes the number of completed and uncompleted tracts in our data, and figure C.1 shows the model fit of (25).

Table C.1: Summary of tract types

Tract type	N obs	Average Production Duration (in month)
Uncompleted oil tracts with sufficient data	770	335.4
Completed oil tracts	691	157.6
Uncompleted oil tracts without sufficient data	56	9
Uncompleted gas tracts with sufficient data	724	335.4
Completed gas tracts	2786	147.9
Uncompleted gas tracts without sufficient data	40	10.7

Figure C.1: Oil and gas production model fit



D Tables and Figures

Table D.1: Estimated parameters of the priors

	β_0^o	β_1^o	ϕ_1^o	ρ_1^o	$(\sigma_1^o)^2$	ρ_0^o	$(\sigma_0^o)^2$
Estimate	-6.560	14.799	0.385	2.412	1.923	2.080	3.108
SE	1.124	0.873	0.016	0.874	0.370	0.432	0.505

	β_0^g	β_1^g	ϕ_1^g	ρ_1^g	$(\sigma_1^g)^2$	ρ_0^g	$(\sigma_0^g)^2$
Estimate	-5.437	17.555	0.659	0.440	0.891	2.284	3.337
SE	1.074	0.113	0.016	0.084	0.060	0.391	0.468

Table D.2: Summary statistics of all available auctions and production data

Statistic	N	Mean	St. Dev.	Max	Min
First Bid	4,006	3,773,542.000	9,583,399.000	157,111,000.000	10,100.000
Second Bid	4,006	1,555,405.000	5,015,138.000	84,391,221.000	4,235.000
N Bids	4,006	2.730	1.298	13.000	2.000
Oil Production	4,119	4,702,255.000	19,445,629.000	581,873,368.000	0.000
Gas Production	4,117	38,546,829.000	80,892,812.000	1,100,616,644.000	0.000
Explored	30,757	0.282	0.450	1	0

Table D.3: Summary statistics of rig contracts

Statistic	Mean	N	St. Dev.
Contract Duration (Days)	143.53	4,649	275.47
Daily Rate	90,821.19	4,649	113,670.80
Total Rig Cost	27,551,327.00	4,649	121,521,975.00

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