Automation and Top Wealth Inequality

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Abstract

Over the last 50 years, the share of wealth held by the richest 1% of individuals in the US has increased by 30%. This paper analyzes the effects of improvements in automation technology on the rise of the top wealth share. I build an incomplete market model with entrepreneurs and a collateral constraint. Automation impacts wealth concentration through two channels. First, it decreases the severity of diseconomies of scale in the entrepreneurial sector, and, hence, it increases income concentration. Since the wealth distribution follows the income distribution, it affects wealth concentration. Second, it raises capital demand, which tightens the collateral constraint and, in turn, increases the dispersion of the return to capital. I calibrate the model to the 1968 US economy to quantitatively analyze the impact of an improvement in automation. I analyze the impact of an unexpected increase in automation technology to the 2016 level. In the model, the capital share of income equals the automation level; hence, I measure the increase in automation by the change in the capital share. In the new steady-state, the top wealth share increases by 8%. In other words, the model can explain one-fourth of the rise in the wealth share of the top 1%. In consumption equivalence terms, workers' welfare increased by 4% and entrepreneurs' welfare increased by 8%.

JEL classification: E23, J23, J3, O33.

Keywords: automation, top wealth inequality, entrepreneurship, superstars.

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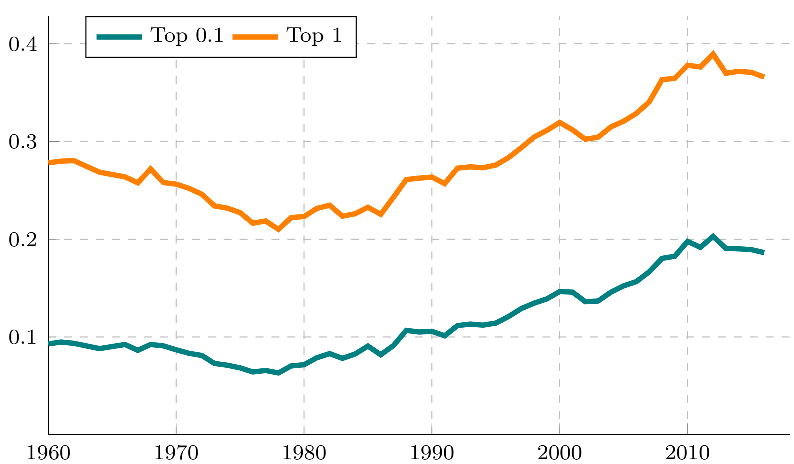
1 Introduction

Over the last 50 years, there has been a substantial increase in wealth concentration in the US (as documented by Saez & Zucman (2016) and others). Figure 1 below shows the share of wealth owned by the top 1% and the top 0.1% based on data from the World Inequality Database.[1] Since the 1960s, the wealth share of the top 1% increased from 27% to 36.5%. A more striking growth had occurred in the wealth share of the top 0.1%: it doubled from 9% to 18%. There is an ongoing debate both in the public and academic spheres about the causes of this rise in wealth concentration.

In this paper, I analyze the impact of automation on the increase in wealth concentration. Specifically, I consider two channels through which automation impacts the share of top 1%: a rise in income concentration due to higher return to entrepreneurial productivity; and an increase in the dispersion of the return to capital. I the model, the increase in automation explains one-fourth of the rise in the wealth share of the top 1% in the US.

Figure 1: Top Wealth Shares



Note: The orange line plots the share of wealth that owned by top 1%. The green line plots the share of wealth that owned by top 0.1%.
Source: World Inequality Database.

I use the term automation in a broad sense that includes robots and machines as well

[1] https://wid.world/wid-world/.

as computers and software. During the last 50 years, automation technology has significantly increased. For example, the mid-70s, when the top wealth shares started to increase, was the dawn of the information technology (IT) revolution, when the usage of computers and software in the production of a wide range of industries started to take-off. While automation substitutes some workers and suppresses their wage, it generates a higher return to individuals who own the capital and a higher return to individuals who use this automation technology. Hence, automation is an important factor that affects inequality.

In this paper, I make two contributions to the literature. First, I introduce the concept of automation technology by incorporating a task-based production function and a convex labor cost (as in Koru (2019)) into an Aiyagari model that features entrepreneurs and a financial friction, as in Quadrini (2000) and Cagetti & De Nardi (2006). I model the financial friction as a collateral constraint: entrepreneurs can only borrow up to a fraction of their own asset. I show that automation affects top wealth inequality through two channels: it increases income concentration; and it increases the dispersion of the return to capital. Second, I quantify the impact of automation on top wealth inequality by calibrating the model and analyzing the exogenous improvement in automation. I focus on entrepreneurs because business capital is an important part of the assets of the wealthiest individuals. Almost half of individuals at the top 1% of wealth distribution and income distribution own a business, and they hold one-third of their wealth in their businesses (Kuhn & Ríos-Rull, 2016; Smith et al., 2019). I model the financial friction as the collateral constraint; in other words, entrepreneurs can only borrow up to a fraction of their assets for their businesses. As documented by Cagetti & De Nardi (2006), around one-third of entrepreneurs use their assets as collateral for a business loan and almost one-fifth of entrepreneurs are denied credit. For this reason, I use collateral constraints to model the incomplete market for entrepreneurs.

I depart from the canonical model of Aiyagari by changing the production function. First, to include automation decisions, I use a task-based framework, as in Zeira (1998) and Acemoglu & Restrepo (2018). To produce the final good, entrepreneurs need to complete a measure of one of the tasks. There are two types of tasks: automated and non-automated. If a task is automated, then it can be done using capital. On the other hand, if the task is not automated, then only labor can be used to produce that task. Entrepreneurs choose what tasks to automate and this provides a notion of automation choice in the model. The automation level is defined as the share of tasks that can be automated and it is exogenously given. I analyze how an exogenous shift in the share of automated tasks impacts wealth concentration. Second, I introduce a convex cost of labor, similar to Koru (2019). This convex cost of labor leads to a production function that exhibits decreasing returns to scale. In contrast to the literature that defines the decreasing returns to scale over total production, this modeling decision links the severity of diseconomies of scale in the entrepreneurial sector to automation technology. By decreasing the dependency on labor, automation reduces the convexity of the cost function and, hence the severity of diseconomies of scale. Because the decreasing returns to scale property of the production function is related to the span-of-control of entrepreneurs (Lucas, 1978), and the span-of-control is usually measured as the number of employees (Ouchi & Dowling, 1974), it is intuitive to link the scalability to the number of employees. The convex cost of labor achieves this.

An improvement in automation technology has two impacts on wealth concentration. The first impact occurs through the change in the top of the income distribution. Because automation decreases the severity of diseconomies of scale, it increases the return to entrepreneurial skill. As Koru (2019) shows, in a static model with the same production function, when entrepreneurial productivity is distributed by a Pareto distribution, the tail of income distribution can be approximated with a Pareto tail, and the shape parameter is inversely related to the automation level. As automation technology improves, dependency on labor diminishes. This enables highly productive entrepreneurs to scale up their production more than less productive entrepreneurs. This leads to the more pronounced superstar effect of Rosen (1981) and, hence, to higher income concentration. This implies that an improvement in automation technology increases the return to the "superstar" stage and, hence, it increases wealth concentration.

The second impact of an improvement in automation on wealth concentration is through the increase in the dispersion of return to capital. An improvement in automation increases the demand for capital. However, if the collateral constraint is binding before the improvement, now the constraint will become even tighter. Hence, the return to business capital increases, therefore the incentive to save increases. Clearly, this increase in the return to business capital is higher for more productive entrepreneurs, because among them the demand for capital increases more than among low-productivity entrepreneurs. This leads to a higher dispersion in the return to capital, and that, in turn, leads to an increase in wealth concentration.

One implication of the model is that automation increases the capital intensity of firms, average firm size, and the employment share of the largest firms. Using data on European private firms, I document that in industries in which IT intensity increased at a higher rate, there is a higher increase in average firm size, average capital intensity, and employment concentration. The model can generate this positive relation between automation and firm size distribution.

To quantify the impact of automation on wealth inequality, I calibrate the model to the 1968 US economy. I analyze the impact of an unexpected improvement in automation technology and measure the change in the wealth share of the top 1%. An implication of the task-based framework is that the capital share of income is a function of the automation level (Acemoglu & Restrepo, 2018; Martinez, 2019). Similarly, in this model, the capital share of income equals the automation level. Hence, I use the capital share as a measure of the automation level. The model can match the initial steady-state well. When I increase the automation level to the 2016 level, the wealth share of the top 1% increases in the new steady-state by 8.82%, which contrasts with a 34.6% increase in the data. Thus, the model can explain one-fourth of the increase in the wealth share of the top 1%.

On the other hand, in the model, the wealth share of the top 0.1% increases by 10%. However, in reality, it doubled. In other words, the model can only explain 10% of the increase in wealth concentration at the very top. One reason why the model cannot generate high dynamics at the top is that the second channel is not relevant to very wealthy individuals. Because the collateral constraint does not bind for those individuals, an improvement in automation does not lead to a higher return to business capital. Therefore, the model does not generate this additional incentive to save among the top 0.1%.

Who gained from improvements in automation technology? To answer this question, I calculate the transition dynamics after the unexpected automation shock. I assume that automation technology increased at a constant rate for 45 years and remained constant afterward. In consumption equivalence terms, workers' welfare increased by 4%, and entrepreneurs' welfare increased by 8%. The gain occurred primarily because of the increase in overall productivity in the economy, which can be attributed to the shift of the labor force to more productive firms.

Related Literature: This paper is related to the literature on the impact of automation on the labor market. Acemoglu & Restrepo (2020) provide evidence for the impact of industrial robots on employment. Autor & Dorn (2013), and Goos et al. (2014) study the job polarization attributable to routine-biased-technological change. Hémous & Olsen (2018) Koru (2019), and Prettner & Strulik (2019) consider the income inequality. Acemoglu & Restrepo (2018) Martinez (2019), and Eden & Gaggl (2018) analyze the effect on the labor share of income of improvements in automation technology. In this paper, I focus on the impact of automation on wealth concentration.

The second strand of literature that this paper contributes is the literature on the dynamics of top wealth distribution. Piketty (2014), Saez & Zucman (2016), Kopczuk (2015), Kuhn & Ríos-Rull (2016), and Smith et al. (2020) document the increase in wealth concentration using the capitalization method, estate tax, and survey data. Hubmer et al. (2020), Kaymak & Poschke (2016), Cao & Luo (2017), and Aoki & Nirei (2017) study various channels that affect top wealth inequality.

Kaymak & Poschke (2016) consider the impact of the increase in wage inequality and the decrease in marginal tax rates. They claim that the increase in wage inequality is the main driver of top wealth inequality because the impact of the tax change is offset by the change in prices. In their analysis, they feed the observed change in wage inequality into the model, whereas in this paper the change in income concentration is a result of the change in the automation technology.

Hubmer et al. (2020), Cao & Luo (2017) and Aoki & Nirei (2017) show that a change in the income tax schedule can explain a significant part of the change in wealth concentration. Aoki & Nirei (2017) provide a micro-foundation for the heterogeneous returns to wealth and income inequality. In Aoki & Nirei (2017), a decrease in the tax leads entrepreneurs to invest in risky projects and, hence, it increases the dispersion of income. I consider a different reason for the increase in income dispersion and the heterogeneous return to wealth: a change in production technology due to an increase in automation. This paper focuses on the link between automation and wealth concentration. To understand the individual effect of automation technology I abstract from the impact of other possible explanations.

The paper most related to this paper is Moll et al. (2019), who study the impact of automation technology on income and wealth inequality. They, too, use a task-based framework, but in their model, the main mechanism is the increase in return to capital. Due to the birth and death process, only a small fraction of households live long enough to accumulate wealth exponentially. The top of the wealth distribution is populated by long-lived households. As automation advances, the return to capital increases, and, hence, households save more, which leads to an increase in the top wealth inequality. In contrast, my mechanism depends on the higher return to entrepreneurial skill. Given that more than 40% of the individuals at the top of the wealth distribution are entrepreneurs and more than two-thirds of the income source is the return to human capital (either through labor or business) (Kuhn & Ríos-Rull, 2016), this channel, too, is important. Moreover, an important fraction of top wealth owners are self-made and acquired their fortune in a short period of time. For example, half of the individuals on the 2017 Forbes 400 list are self-made billionaires (Guvenen et al., 2019).

The production function in the model builds on my companion paper Koru (2019). Using the same production function, I provide a theory that links automation to the Pareto parameter of top income distribution. I show that, in a static model, when the productivity of entrepreneurial skill is distributed by a Pareto distribution, the right tail of income distribution can be approximated with a Pareto distribution as well. The shape parameter of top income distribution is a function of the shape parameter of the productivity distribution, the automation level, and the convexity of labor cost. Moreover, he shows that the thickness of the right tail increases with the level of automation. In this paper, I focus on the impact of automation on top wealth inequality and quantify how the change in income concentration attributable to automation impacts wealth concentration.

This paper is structured as follows. Section 2 describes the model and discusses the impact of automation on wealth concentration. Section 3 provides details about calibration. Section 4 presents the results. Section 5 analyses welfare consequences and Section 6 concludes.

2 The Model

My model is based on the dynamic general equilibrium incomplete market model of Aiyagari (1994), augmented by entrepreneurial choice and financial frictions, as in Cagetti & De Nardi (2006) and Quadrini (2000). The main difference between the current model and the standard models found in the literature is the production function. There are two main differences in this production function. First, I use a task-based framework, as in Acemoglu & Restrepo (2018), that provides a notion of automation choice in the model. Second, I define the span-of-control as a function of the measure of labor, instead of total output, as in Koru (2019). This leads the severity of the diseconomies scale to be a function of automation.

2.1 Demographics and Preferences

There is a continuum of the infinitely lived individual of measure one. The utility of individuals from consumption is given by $u(c)$. Individuals discount the future at a rate of β. Individuals are subject to uninsurable labor productivity shock; however, there is no aggregate uncertainty. The labor market productivity of an individual evolves according to a Markov process. Let $p_s(s'|s)$ denote the probability density function of the next period's labor productivity s', conditional on this period's labor productivity s. Let $\mathcal{S}$ be the set of all possible levels of labor productivity.

In a given period, an individual can be either a worker or an entrepreneur. A worker supplies a unit of labor inelastically. In each period, a worker gets an entrepreneurial idea with probability p. The productivity of the idea z follows a Pareto distribution with the shape parameter μ and the scale parameter $\underline{z}$. Let $\phi(.)$ denote the pdf of the distribution of z and let $\mathcal{Z}$ denote the set of all possible values of z. If the individual implements the idea, he becomes an entrepreneur; otherwise, he remains a worker. The productivity of the idea remains constant throughout the entrepreneurship spell. At the beginning of the period, an entrepreneur decides whether to continue to operate his firm or become a worker. If he becomes a worker, he loses the idea and needs to find another one to become an entrepreneur again. With probability p_e, his business fails for some exogenous reason and he becomes a worker.

2.2 Technology

There are two production sectors: corporate and non-corporate. Firms in the non-corporate sector are owned by entrepreneurs. However, in reality, not all firms are closely held by entrepreneurs. Therefore, following Cagetti & De Nardi (2006) and Quadrini (2000), I also include a corporate sector. There is a unique homogeneous good in the economy; hence, both sectors produce the same good. Both sectors have a similar production function. The main difference is that firms in the non-corporate sector face a convex cost of labor, which leads to a production function that exhibits decreasing returns to scale.

2.2.1 Corporate Sector

I use a task-based framework similar to Zeira (1998) and Acemoglu & Restrepo (2018). To produce a final good, a measure of one of the tasks must be completed. There is no market for tasks; hence, each firm needs to complete all tasks inside the firm.

There are two types of tasks: automated and non-automated. If a task is automated, then capital and labor are perfect substitutes in production. On the other hand, if a task is not automated, then the only input in the production function is labor. I assume that the productivity of labor and capital is the same across all tasks. Let I be automation technology frontier such that any task below I is automated and any tasks above I are non-automated. Formally, the production function of task $i \in [0,1]$ is given by:

$$y_i = \begin{cases} k_i + \ell_i & \text{if } i \leq I, \\ \ell_i & \text{if } i > I, \end{cases} \tag{1}$$

Tasks are complements and they are aggregated into output by a unit elastic aggregator (i.e., Cobb-Douglas):

$$\ln Y = \int_0^1 \ln(y_i) di, \tag{2}$$

where Y is the total output. The problem of a corporate firm is:

$$\max_{\ell_i, k_i} AY - w \int_0^1 \ell_i di - (r + \delta) \int_0^1 k_i di,$$

where A is the aggregate TFP and δ is the depreciation rate.

Observe that automation is a labor replacing technology. An improvement in automation, i.e., an increase in I, means that labor can be replaced in this new automated task. However, a task complements other tasks. Therefore, even though automation replaces labor within a task, it improves the productivity of other tasks by cost reduction.

Since capital and labor are perfect substitutes, only one of them is used to produce a task. Because the productivity of capital and labor is the same across all tasks, the cheaper input is used in automated tasks. In equilibrium, because there is a positive supply of capital, it is the case that the price of capital is less than the wage; hence, only capital is used in automated tasks, i.e., $\ell(i) = 0$ for $i \leq I$. Moreover, by the symmetry of tasks, the optimal choice of capital is the same for all automated tasks and the optimal choice of labor is the same for all non-automated tasks, i.e., $k(i) = k$ for all $i \leq I$ and $\ell(i) = \ell$ for all $i > I$. Hence, the optimal solution induces to a Cobb-Douglas production function for a firm with the capital share equal to I:

$$Y = k^I \ell^{1-I}. \tag{3}$$

2.2.2 Non-corporate Sector

Entrepreneurs have access to a decreasing returns to scale production function. The production of tasks and aggregation into the final good is similar to the corporate sector. However, there is an additional convex cost that depends on the measure of labor used in the production. The profit function of an entrepreneur with productivity z is given by:

$$zAY - w\int_0^1 \ell_i di - v\left(\int_0^1 \ell_i di\right) - (r+\delta)\int_0^1 k_i di,$$

where Y is given by (2) and $v(.)$ is the convex cost with properties $v' > 0$ and $v'' > 0$.

Convex cost of labor

The main mechanism in this paper depends on the convex cost of labor. Observe that Y is constant returns to scale, and, therefore $zAY - v(.)$ is decreasing returns to scale. Here, the convex cost can be seen in a reduced form as the organization cost of labor or the hiring-firing cost of labor or search cost. For example, Koru (2019) shows that this convex cost can be micro-founded by Shapiro & Stiglitz's (1984) efficiency wage theory of the shirking model. In order to prevent labor from shirking, the entrepreneur needs to spend additional resources. Because capital does not have an incentive to shirk, it does not create any moral hazard problem; hence, this additional cost does not depend on capital. The general idea is that if entrepreneurs want to grow, they need to pay more. In this sense, a theory of a firm-size-wage premium can generate the desired result.

I assume that this convex cost of labor is only relevant in the non-corporate sector. In other words, the corporate sector can scale its production perfectly and can replicate the process that causes this cost in the non-corporate sector, whether it is vacancy posting in search friction or problem of monitoring workers or something else. This assumption leads to constant returns to scale production function in the corporate sector. Therefore, there is a representative firm in the corporate sector and I do not need to make assumptions about firm distribution, who owns these firms, and competition structure.

2.3 Financial Market

To raise capital for a business, an entrepreneur can borrow from the financial market. However, an entrepreneur needs to provide collateral in order to borrow. Hence, the amount of borrowing depends on the entrepreneur's asset. An entrepreneur can use up to λ fraction of his asset in his business; i.e.,

$$\int_0^1 k_i di \leq \lambda a, \tag{4}$$

where a is the level of asset owned by the entrepreneur and $\lambda > 1$. In another words, an entrepreneur can only rent up to $(\lambda - 1)$ fraction of his asset.

Workers cannot borrow from the financial market. Only entrepreneurs can borrow, but they can only use it in their business; they cannot consume it.

2.4 Problem of an individual

Let $V(a, s)$ denotes the lifetime value of a worker with labor productivity s and asset a and let $E(a, s, z)$ be the lifetime value of an entrepreneur with entrepreneurial productivity z.

2.4.1 Problem of a Worker

Consider a worker with labor productivity s and asset a. He earns ws as labor income and ra as capital income. With probability p, he gets an idea and decides whether to become an entrepreneur or not. With the remaining probability, he remains as a worker. The lifetime value of a worker is

$$\begin{aligned} V(a,s) = \max_{c,a'} \; u(c) + \beta \Bigg[& p \sum_{s' \in \mathcal{S}} \int_{z \in \mathcal{Z}} \left[\max\{V(a',s'), E(a',s',z')\}\right] phi(z') dz' p_s(s'|s) \\ & + (1-p) \sum_{s' \in \mathcal{S}} V(a',s') p_s(s'|s) \Bigg] \\ s.t. \quad & c + a' \leq ws + (1+r)a, \; a \geq 0, \; c \geq 0. \end{aligned} \tag{5}$$

2.4.2 Problem of an Entrepreneur

Consider an entrepreneur with entrepreneurial productivity z, labor productivity s and asset a. He chooses which tasks to automate, how much capital and labor he needs for each task, and how much to save. First, consider the profit maximization problem. For a given asset level this problem is static. Because in an automated task labor and capital are perfect substitutes, only one of the inputs is used. Therefore, if an entrepreneur automates a task, he uses only capital in that task. An entrepreneur faces two constraints. The first constraint is the automation constraint: he can only automate only the tasks that are technologically amenable to automation. In other words, the choice of automation $I^\star$ must be lower than the exogenously given automation level I. The second constraint is the financial constraint defined in equation (4).

Formally, the problem of an entrepreneur is:

$$\pi(z,a) = \max_{\substack{I^\star, \{\ell_i\}_{i\in[I^\star,1]},\\ \{k_s\}_{s\in[0,I^\star)}}} zY - w\int_{I^\star}^{1} \ell_i di - v\left(\int_{I^\star}^{1} \ell_i di\right) - (r+\delta)\int_{0}^{I^\star} k_s ds \tag{6}$$

$$\begin{aligned} s.t.\ \ & 0 \le I^\star \le I, \\ & \int_0^{I^\star} k(s)ds \le \lambda a, \\ & \ell_i \ge 0,\ \ k_s \ge 0, \end{aligned}$$

and the production function of tasks (1) and the production function of final good (2).

Then, the lifetime value of an entrepreneur is

$$E(a,s,z) = \max_{c,a'}\ u(c) + \beta\left[(1-p_e)\mathbb{E}\left[\max\{V(a',s'),E(a',s',z)\}\right] + p_e\mathbb{E}[V(a',s')]\right] \tag{7}$$

$$\begin{aligned} s.t.\quad & c + a' \le \pi(a,z) + (1+r)a, \\ & a' \ge 0,\ \ c \ge 0. \end{aligned}$$

$u(c)$ is the utility from today's consumption. With probability p_e his business will fail and he will become a worker for an exogenous reason. With probability $1-p_e$ his business will continue, however, he can still close and become a worker if his labor productivity becomes high enough. His resources today are profit, $\pi(a,z)$, and return to the asset.

2.5 Definition of Equilibrium

Now I can define a competitive equilibrium:

Definition 1. *A stationary equilibrium consists of prices w and r; lifetime value function and policy function for a worker with asset level a and labor productivity a, $V(a,s), g_w(a,s)$; a lifetime value function and policy function for entrepreneur with asset level a, labor productivity s and en-*

trepreneurial productivity z, $E(a,s,z), g_e(a,s,z)$; and an automation decision, a labor and capital demand of entrepreneur $I^\star(a,z), \ell(a,z,i), k(a,z,i)$; a labor and capital demand of corporate firms, $\ell_c(i), k_c(i)$; an optimal choice of occupation, $g_o(a,s,z)$; and a stationary distribution of individuals over asset level, labor productivity and entrepreneurial productivity $\Gamma(a,s,z)$, where $z=0$ is for workers, such that:

- *Value functions and policy functions solve* (5) *and* (7)*,*
- $I^\star(a,z), \ell(a,z,i), k(a,z,i)$ *solve entrepreneur problem* (6)*,*
- *labor and capital demand of corporate firm are given by:*
  - $\ell_c(i) = \ell_c$ *for* $i > I$*, and* $k_c(i) = k$ *for* $i \leq I$*,*
  - $A(k_c/\ell_c)^I = w$*,*
  - $A(\ell_c/k_c)^{1-I} = r + \delta$*,*
- *optimal occupational choice:* $g_o(a,s,z) = 1$ *if* $E(a,s,z) > V(a,s)$*,*
- *distribution of individuals is stationary:*

$$\begin{aligned}
\Gamma(a',s',z) = & \int \int_{\mathbb{B}_e} (1-p_e) g_o(a',s',z) p_s(s,s') \Gamma(a,s,z) db + \\
& \int \int_{\mathbb{B}_w} p g_o(a',s',z) phi(z) p_s(s,s') \Gamma(a,s,0) dads, \\
\Gamma(a',s',0) = & \int \int_{\mathbb{B}_e} [p_e p_s(s,s') \Gamma(a,s,z) + (1-p_e)(1-g_o(a',s',z)\Gamma(a,s,z)] dads + \\
& \int \int_{\mathbb{B}_w} (1-p) p_s(s,s') \Gamma(a,s,0) dads + \\
& \int_{\underline{z}}^{\infty} \int \int_{\mathbb{B}_w} p p_s(s,s') phi(z')(1-g_o(a',s',z'))\Gamma(a,s,0) dadsdz',
\end{aligned}$$

where $\mathbb{B}_w = \{(a,s)|g_w(a,s) = a'\}$*, and* $\mathbb{B}_e = \{(a,s,z)|g_e(a,s,z) = a'\}$*,*

- *labor market clears:*

$$\int \int_{I^\star(a,z)}^{1} \ell(a,z,i)d\Gamma(a,s,z)di + (1-I)\ell_c = \int s d\Gamma(a,s,0),$$

- *capital market clears:*

$$\int \int_{0}^{I^\star(a,z)} k(a,z,i)d\Gamma(a,s,z)di + Ik_c = \int a d\Gamma(a,s,z).$$

2.6 Impact of An Improvement in Automation

The main mechanism in this paper is the impact of automation on returns to entrepreneurial skills. An improvement in automation technology has two impacts on the problem of the entrepreneur. First, it relaxes the automation constraint, and, second, it tightens the collateral constraint. In this section, I discuss how these two affect wealth concentration.

To understand the impact of improvements in automation technology on wealth concentration, it is important to know why the model generates a thick wealth tail. The basic setup of Aiyagari (1994) fails to generate a thick tail because the precautionary saving motivation for rich individuals is not high enough, and this is so because they have a sufficient amount of assets to self-insure (for discussion see De Nardi & Fella (2017)). The literature discusses some additional mechanisms to generate thick wealth distribution (Benhabib & Bisin, 2018). In this model, there are two main channels: an endogenous high and persistent "superstar" income state, and a heterogeneous return to capital.

First, in the model, with a small probability, workers can draw a highly productive idea that has a high return and become a "superstar". However, in each period they face business failure risk, with probability p_e they become a worker. This creates an income risk for entrepreneurs. They earn multiples of wage income today, but tomorrow their business can fail and, consequently, suffer a drastic decrease in income. This provides a precautionary saving motive for entrepreneurs: they want to save more to smooth consumption. Castañeda et al. (2003) show that this type of large income risk for high-income earners can generate realistic income and wealth distribution.

An improvement in automation leads to a higher return in the superstar stage. Relaxing the automation constraint enables high productive entrepreneurs to scale up their production more than low productive entrepreneurs. This leads income distribution to spread out and increase the income concentration, which eventually leads to higher wealth concentration.

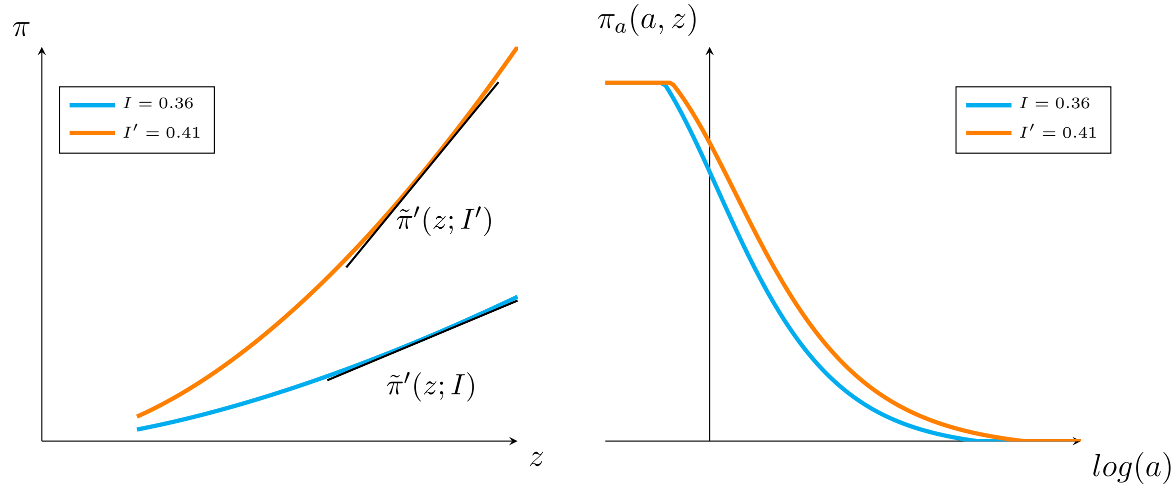
Proposition 1. *Fix prices w and r. Let $\tilde{\pi}(z; I)$ be the profit function when automation technology is given by I and without the collateral constraint, i.e., when entrepreneurs operate their businesses at the efficient level. Then, $\tilde{\pi}'(z; I') > \tilde{\pi}'(z; I)$ for $I' > I$, where $\tilde{\pi}'$ is the derivative with respect to z.*

The proposition states that an improvement in automation increases the profit of a highly productive entrepreneur more than a poorly productive entrepreneur. The intuition for this result is provided in Koru (2019). An advancement in automation relaxes the technology constraint in (6). Since this constraint is more costly for highly productive entrepreneurs, the return is higher for them, relative to low productive entrepreneurs. This is why the gap between high and low productive entrepreneurs is increasing, and it implies that the top entrepreneur's income increases substantially relative to low skilled entrepreneurs and workers. Moreover, the risk of business failure increases. Hence, the savings of highly productive entrepreneurs are higher, which leads to an increase in wealth concentration.

To see the impact clearly, assume that the convex cost of labor takes the form $v(L) = cL^\alpha$ and consider an entrepreneur with high enough assets that the collateral constraint does not bind. Then, we have a closed-form solution and the profit is then

$$\pi(a, z) = c(\alpha - 1)L^{\star\alpha} = c^{-\frac{1}{\alpha-1}}(\alpha - 1)\left[\left(\frac{z}{(r+\delta)^I}\right)^{\frac{1}{1-I}} - w\right]^{\frac{\alpha}{\alpha-1}}\left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}. \tag{8}$$

Figure 2



(a) Profit function $\tilde{\pi}(z)$

(b) Marginal return to business capital.

Observe that the profit function is convex in z and the convexity is increasing in I. In a sense, the superstar effect, wherein small change in the entrepreneurial ability increases the return substantially (Rosen, 1981), becomes more pronounced. Therefore, income distribution spreads out. In other words, this channel increases income concentration, which eventually affects wealth concentration. In Koru (2019), I show that when z is distributed by a Pareto with shape parameter μ, the right tail of the income distribution can be approximated by a Pareto distribution with shape parameter $\mu(1 - I)(\alpha - 1)/\alpha$. However, this result does not apply here because of the collateral constraint, which causes entry into entrepreneurship to depend on the asset level of individuals. Hence, the equilibrium distribution of active entrepreneurs' productivity is not the same as the distribution of z. However, the idea is similar, and the thickness of income distribution depends on the productivity distribution, automation level, and convexity of the labor cost function.

The second channel that affects the tail of the wealth distribution is the heterogeneous return to capital, which is an important channel that generates thick wealth tails (Hubmer et al., 2020; Benhabib et al., 2019). Due to the collateral constraint, return to capital is not equalized across entrepreneurs. Since entrepreneurs cannot achieve an efficient level of production, marginal productivity of capital is higher than the risk-free interest rate. Therefore, for entrepreneurs, the return to capital is higher than it is for a worker. This generates higher capital income for entrepreneurs. Furthermore, because the tightness of the collateral constraint increases with productivity, there is also a dispersion in the return to capital across entrepreneurs. A higher return to capital creates a higher incentive to save.

An improvement in automation technology increases the return to business capital for entrepreneurs whose collateral constraint binds because it increases the marginal product of capital. For an entrepreneur, a higher asset level has two benefits. First, it increases capital income through the risk-free rate. Second, it relaxes the collateral constraint, and hence, it increases the profit. Clausen & Strub (2012) prove that the envelope theorem holds in dynamic models with the occupational choice. Hence, the marginal return to higher capital today can be calculated by the envelope theorem and the first-order condition with respect to consumption, which is given by:

$$E_a(a, s, z) = [\pi_a(a, z) + (1 + r)]u'(c),$$

where subscript denotes the derivative with respect to the denoted argument. The first term on the right-hand side is the shadow cost of the collateral constraint in the entrepreneur's problem (6), and it is positive for binding entrepreneurs. Now I show that $\pi_a(a, z)$ rises with automation technology.

Proposition 2. *Fix prices w and r. Let $\pi(a, z; I)$ denote the profit function when automation technology is I. Then, the derivative of profit function with respect to a is increasing with I, i.e., $\pi_a(a, z; I') \geq \pi_a(a, z; I)$ when $I' > I$. When the automation constraint binds, this condition holds with strict inequality.*

This implies that the return to savings increases for entrepreneurs whose collateral constrain binds. This is intuitive because when the automation constraint binds, an increase in I leads to a higher marginal product of capital. However, because of the collateral constraint, the entrepreneur cannot rent more capital. Hence, the entrepreneur's incentive to save increases. Formally, the shadow cost of the collateral constraint increases with an increase of I. As the savings of entrepreneurs increases, the wealth concentration increases.

However, notice that in Figure 2b, both ends of the graph are constant, and an increase in I does not affect those regions. It is easy to see the reason for the high a. When wealth is large, the collateral constraint does not bind, and, hence, this channel disappears for wealthy entrepreneurs. In the case of a low a, the automation constraint does not bind. An entrepreneur with a low level of assets does not use all of the available automation technology because he does not have enough capital to allocate across a wide range of tasks. Instead, he uses labor. When his asset increases, he starts automating new tasks, and the overall effect on the marginal product of capital remains constant. Because the automation constraint does not bind for entrepreneurs who have a low level of assets, these entrepreneurs are not impacted by an improvement in I. Therefore, the magnitude of this channel depends on the size of these regions. If the distribution of z is concentrated on a low level of productivity, then an efficient level can be achieved very easily, and, hence, a rise in I might not have a big impact on savings.

Here, I consider partial equilibrium results by fixing prices. However, in general equilibrium prices will adjust, and, hence, the overall impact might be different. Nevertheless, in (8), the convexity of the profit function does not depend on the prices. Hence, price only affects the level; in relative terms, high productive entrepreneurs still are better off with advanced automation technology even when prices adjust.

3 Quantitative Analysis

I calibrate the model to the US economy in 1968. I choose 1968 because this is the first year for which I can calculate the entrepreneurship rate in PSID.[2] Moreover, top wealth share and labor share were stable in the 1960s, and they only started to change in the 1970s. In this regard, I believe 1968 is a good starting point. A period in the model is a year.

The aim of this paper to analyze the impact of an improvement in automation technology on top wealth inequality. After calibrating the model to 1968, I change the automation level I to

[2]Business ownership question started to ask in 1969, even though PSID starts in 1968.

the 2016 value, leaving all other parameters at the same at the calibrated values. Then, I calculate the change in the top wealth shares between the two steady states.

The main parameter in this analysis is the automation level, I. Recall that the optimal solution of the corporate sector induces to Cobb-Douglas looking production function, equation (3), with capital share I. So, I set the automation level to the capital share of income. It is important to notice that I only use the capital share in the corporate sector. First, this allows me to exogenously pin down I because the capital share in the non-corporate sector is endogenous. Hence, I cannot set it exogenously for 2016. Second, the Penn World Table splits self-employed income using the share of the non-self-employed sector's capital share. The capital share of income was 0.36 in 1968, and it was 0.41 in 2016, which is the latest year for which I have wealth inequality data.

3.1 Parametrization

This section describes the quantitative specification of the model.

Preferences: I consider the CRRA utility function, $c^{1-\sigma}/(1-\sigma)$ and the risk aversion parameter, σ, is set to 1.5. I calibrate the discount factor β to match the capital-to-GDP ratio of 3, $K/Y = 3$.

Technology: I normalized the total factor productivity A to 1. As I noted above, automation technology is set to the capital share of income. The depreciation rate, δ, is set to 5%.

I assume that the convex cost of labor is given by $v(L) = cL^{\alpha}$. Since this cost function is novel, there is no standard way to calibrate these parameters. Coefficient c determines the level of the cost function, and, therefore, the level of profit. This is clear from equation (8). Because in this model the top of the income distribution is populated by entrepreneurs, it will directly affect the share of the top 1%. Thus, I calibrate c to match the wealth share of the top 1%.

The convexity of the cost function, α, affects the size of entrepreneurs' businesses. However, due to a lack of public data on private firms that goes back to the 1960s, I consider size not in terms of employment but of capital. The underlying assumption here is that employment is positively correlated with capital size. I match the ratio of non-financial non-corporate business assets to non-financial business assets, which I calculate using the FED's Flow of Funds.[3]

Labor Productivity Process: I assume that the log of labor productivity, $\log(s)$, evolves with an AR(1) process:

$$log(s') = \rho log(s) + \epsilon, \epsilon \sim N(0, \sigma_s^2).$$

I set the autocorrelation $\rho = 0.9$ and the standard deviation of innovation to 0.2 following Guvenen et al. (2019). I use the Tauchen & Hussey (1991) method to discretize the labor productivity process.

Distribution of Ideas: There are 4 parameters for the process of ideas: the probability of getting an idea, p; the probability of exogenous exit, p_e; and the scale and the shape parameter of the Pareto distribution of z, $\underset{\sim}{z}$ and μ. I set the exogenous exit probability to 26.5%, which is the share of entrepreneurs in PSID that leave entrepreneurship status next year.

The probability of getting an idea and the scale parameter cannot be identified jointly. To see this consider the problem of a worker (5). Because $W(a, s, z)$ is increasing in z, let $z^\star(a, s)$ be the marginal productivity of the entrepreneur who is indifferent between becoming an entrepreneur and a worker. Let z' be the minimum of such $z^\star$. Assume $z' > \underset{\sim}{z}$. Since $z^\star \geq z'$, I can write the problem as

$$\begin{aligned} V(a, s) = \max_{a'} & \, u(ws + (1 + r)a - a') \\ & + \beta \left[(1 - p)V(a, y) + pP_z(z')V(a, s) + \int_{z'}^{\infty} \max\{V(a, s), W(a, s, z)\} p_z(z) dz \right]. \end{aligned}$$

[3]FRED series TABSNNB and TABSNNCB.

Now consider $p' = p(1 - P_z(z'))$. Observe that:

$$p_z(z|z > z') = g(z)/(1 - P_z(z')) = \frac{\mu \underline{z}^{\mu}}{z^{\mu+1}} \cdot \frac{z'^{\mu}}{\underline{z}^{\mu}} = \frac{\mu z'^{\mu}}{z^{\mu+1}}.$$

This implies that $z|z > z' \sim Pareto(\mu, z')$, so set $\underline{z}' = z'$. Then, the problem of a worker with new parameters is:

$$V(a,s) = \max_{a'} u(ws + (1+r)a - a') + \beta \Big[(1 - p(1 - P_z(z')))V(a,s) \\ + p(1 - P_z(z)) \int_{z'}^{\infty} \max\{V(a,s), W(a,s,z)\} p_z(z)/(1 - P_z(z)) dz \Big].$$

This is the same problem before. Hence, there is no change in the solution. Thus, for any $z < z'$, I can find $(\tilde{p}, \tilde{z})$ such that the solution is the same with $(p, \underline{z})$. Therefore, I set $\underline{z} = A = 1$. Observe that because there is no profit in the corporate sector, no one wants to become an entrepreneur when productivity is equal to A. Hence $A < z'$, so it satisfies the condition.

This leaves me with two parameters to calibrate. I calibrate the probability of getting an idea to match the entrepreneurship rate because it determines the entry into entrepreneurship. Hence, it is directly related to the share of entrepreneurs. I calculate this moment using the PSID. I define entrepreneurs as self-employed workers who own a business. The shape parameter determines the thickness of entrepreneurial productivity. It directly impacts the tail of the income distribution, which affects the tail of the wealth distribution. Therefore, I match the thickness of the wealth distribution. To this end, I calibrate the shape of the Pareto distribution to match the ratio of the top 0.1% share to the top 1% share.[4] Because I set the coefficient of convex cost of labor, c to match the share of the top 1%, instead of the relative share, I match directly the wealth share of the top 0.1%. The main idea is that c determines the level and μ determines the thickness of the top distribution.

[4]It is known, first, that top wealth distribution can be approximated by a Pareto distribution and, second, that the relative shares at the top is a function of the shape parameter. In other words, matching the relative share is similar to matching the Pareto tail.

Table 1: Exogenously Calibrated

I	capital share	0.36	Penn World Table
δ	depreciation rate	0.05	
ρ_y	labor productivity persistency	0.9	Guvenen et al. (2019)
σ_y	labor productivity variance	0.2	Guvenen et al. (2019)
σ_u	risk aversion	1.5	-
p_e	entrepreneur exit	0.265	PSID
$\underline{z}$	scale parameter for idea	1	
A	TFP	1	

Collateral Constraint: The last parameter of the model is the collateral constraint of entrepreneurs, λ. Clearly, this parameter affects how much entrepreneurs can borrow, given their asset level. Therefore, I calibrate this parameter to match the debt-to-asset ratio of the non-corporate business sector, which is obtained from the Flow of Funds.[5].

Table 1 summarizes the parametrization.

3.2 Model Fit

To sum up, I choose the probability of getting an idea, p, the discount factor β, the collateral constraint λ, the coefficient of convex labor cost c, the convexity of convex labor cost α, and the shape parameter of Pareto distribution μ to match the entrepreneurial rate, the capital-to-income ratio, the debt-to-asset ratio, the ratio of non-financial non-corporate business assets to the total non-financial business asset, the wealth share of top 1% and the wealth share of top 0.1%. As can be seen from table 2, the model matches the targeted moments well.

The model also fits the overall distribution of wealth remarkably well. Figure 3a shows the Lorenz curve for the wealth distribution above the 50th percentile both from the data and the model. The horizontal axis is the percentiles of the wealth distribution and the vertical axis is the cumulative shares of wealth. The inner plot zooms into the top 1 percentile. The model fits the

[5] FRED series TLBSNNB over TABSNNB

Table 2: Calibration Result

Parameter		Value	Target	Data	Model
Discount Factor	β	0.92	K/Y	3	3
Col. Cons	λ	1.2	Debt-to-Asset	0.19	0.19
Convexity of Cost	α	1.8	Share of NC in Capital (%)	35.6	35.28
Prob. of Idea	p	0.94	Ent Rate (%)	7.97	7.97
Coef. of cost	c	0.26	Top 1 Share (%)	27.19	27.23
Pareto shape	μ	7.87	Top 0.1 Share (%)	9.23	9.23

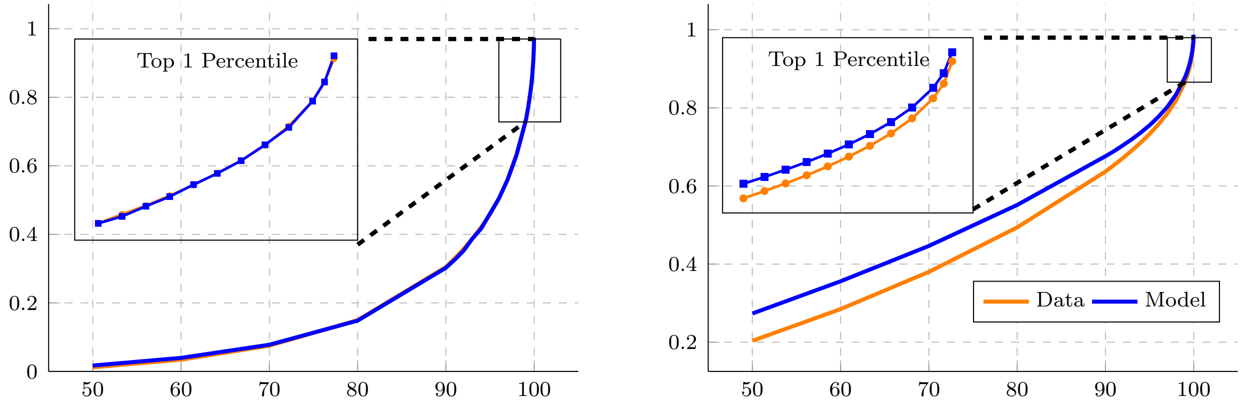
Table 3: Non-targeted Moments

	Data	Model
Income Gini	0.46	0.36
Income Bottom 50%	20.4%	27.3%
Income Top 10%	36.3%	32.4%
Income Top 1%	13.4%	12.2%
Income Top 0.1%	5.1%	4.2%
Income Top 0.01%	2%	1.3%
Wealth Gini	0.83	0.82
Wealth Bottom 50%	1.2%	1.7%
Wealth Top 10%	69.5%	69.8%
Wealth Top 0.01%	3%	2.6%

data very well: the two curves are almost on top of one other. Table 3 shows numerical values for some of the points in this graph to give a sense of the difference. Recall that I am only matching two points on the top percentile, but the model also matches the lower percentiles. As a measure for overall inequality, the Gini coefficient in the model is 0.82, whereas the same figure is 0.83 in the data.

Even though the income distribution is not targeted, the model provides a good fit for top income inequality. Figure 3b plots the Lorenz curve for income distribution. The blue line lies above the orange line, which means that for any percentile, the cumulative share below that percentile is higher in the model than in the data. In other words, the model generates lower

Figure 3: Lorenz Curve



(a) Wealth (b) Income

income inequality than the data. The Gini coefficient for income is 0.36 in the model and 0.46 in the data. However, the gap between these two lines is decreasing at the top of the distribution. Therefore, the model matches the top percentiles better than the low percentiles. This is also clear in the upper panel of table 3, which shows top shares for selected percentiles.

3.3 Testing Model Predictions

In this section, I use the data to test the model's prediction.

One of the main assertions of this paper is that automation enables entrepreneurs to scale up their production. An implication of this is that the average firm size increases with the automation level. Equation (8) shows that profit is a power function of employment. Hence, as the convexity of the profit function increases, the convexity of the optimal labor choice as a function of productivity increases. This implies that the employment share of highly productive entrepreneurs is increasing, and, thus, the average firm size in the entrepreneurial sector is increasing.

To test the hypothesis that higher automation leads to larger firms, I regress the change in firm size to a change in automation. Because this result is for the entrepreneurial sector, I only consider the employment distribution across private firms. I obtain the data from the

Table 4

	Dependent variable:		
	Δlog(Ave. FS)	Δlog(Ave. K/L)	Δlog(Top Emp Share)
	(1)	(2)	(3)
Δlog(IT Intensity)	0.581***	0.633***	0.276*
	(0.147)	(0.183)	(0.151)
Nobs	182	174	182
Note:			*p<0.1; **p<0.05; ***p<0.01

Amadeus database, which provides information about private firms in Europe. For each industry-country pair, I calculate three measures: average firm size, average capital-to-labor ratio, and the share of top 1% of firms in employment. For the automation measure, I use information technology intensity, defined by total IT capital over total capital. I construct this measure using the data from EU KLEMS. I consider the changes between 2006 and 2016 because the number of observations in the Amadeus database is significantly low for previous years.

Table 4 presents the results. All of the measures of change in firm size distribution are positively correlated with IT intensity. This implies that industries that observed a higher rate of IT growth also observed a higher rate of firm size growth. A percentage increase in the growth of IT intensity leads to around a 0.6 percent increase in the growth of the average firm size in employment and the growth of the average capital intensity of a firm. Also, the growth rate of the employment share of the top 1% of firms increases by 0.3%. Furthermore, Bessen (2017) and Brynjolfsson et al. (2008) show that in terms of sales, higher IT intensity leads to higher market concentration. Stiebale et al. (2020) estimate that the impact of robots on productivity and sales is greater in larger firms than in smaller firms. Hence, the data supports the model's prediction that automation enables entrepreneurs to grow their business.

Another prediction of the model is that labor productivity is increasing with employment size while capital productivity is decreasing with capital size. This is true because of the convex

Table 5

	log(Productivity)	
	Data	Model
log(Employment)	0.0725^{***}	3.38
	(0.005)	
log(Assets)	-0.26^{***}	-1.52
	(0.005)	

cost of labor. Given that large businesses have a higher marginal cost of labor, they also must have higher labor productivity. The opposite is true for capital because capital has no convex cost. To see this, consider the first-order condition of the entrepreneurs' problem:

$$\frac{zY}{L} = \frac{w + v'(L)}{1 - I}, \quad \frac{zY}{K} = \frac{r + \delta + \lambda\eta(z, a)}{I},$$

where $\eta(z, a)$ is the Lagrange multiplier of the collateral constraint. Given the convexity of v, labor productivity must be increasing with employment. Because the Lagrange multiplier is increasing at z and decreasing with a, it is not immediately clear how the capital productivity behaves. However, in quantitative results the relationship is negative, meaning that capital productivity is decreasing with capital size. In a world without friction and convex cost, these productivity measures would be solely a function of prices. In contrast, this model predicts that there is a positive correlation between employment size and labor productivity, whereas there is a negative correlation between capital size and capital productivity.

To test this prediction, I use firm-level data from the Amadeus database and use sales over input as a measure of factor productivity. For capital, I consider total assets. Table 5 presents the results from the regression and model outcome.

Unfortunately, the Amadeus database does not have data for 1968. Therefore I calculate the model correlation at the second steady state. However, all of the parameters are calibrated to 1968. Consequently, numerical results cannot be compared directly. The objective here is to demonstrate that the coefficients are different signs both in the data and in the model outcome.

To be clear, in the regressions in this section, I do not claim any causality. These regressions are motivated by the implication of the model, which is to show that model predictions are consistent with the data.

4 Quantitative Impact of an Increase in Automation

The aim of this paper is to understand the impact of automation on wealth concentration. To this end, I increase the automation parameter, I, to the 2016 value, leaving all other parameters constant. In 2016, the capital share of income was 41%, and so I increased I from 0.36 to 0.41.

4.1 Impact on the Wealth Distribution

Table 6 presents the equilibrium wealth distribution in the new steady state. The first and second columns show the values from the World Inequality Database for 1968 and 2016, respectively. The third column shows the percentage change between these years. The last column shows how much these changes are generated by the model, which is calculated by dividing the percentage change in the model by the percentage change in the data. In the data, the wealth share owned by the top 1% increased about one third. In the model, it increased by about 8%, which implies that the model can generate one-fourth of the observed increase in the top 1%'s wealth share. On the other hand, the wealth share of the very top is more pronounced in the data. The wealth share owned by the top 0.1% doubled between 1968 and 2016. However, the rise of the top 0.1% share in the model is not as pronounced as in the data. The model generates a 10% increase in the wealth share of the top 0.1%. As noted above, this is so because the heterogeneous return is not relevant for very rich individuals because the collateral constraint does not bind. Hence, automation does not increase the return to business capital, hence it does not generate an additional incentive to save more. All of the dynamics at the very top occur through an increase in income concentration. The impact of the heterogeneous return channel is more important for entrepreneurs for whom the

Table 6: Results - Wealth Distribution

	Data 1968	Data 2016	%Δ Data	Model Explains
Bottom 50	1.2%	0.4%	-70%	64%
Top 10	70%	71%	3%	393%
Top 1	27%	37%	35%	27%
Top 0.1	9%	19%	102%	10%
Top 0.01	3%	9%	216%	8%
Gini	0.83	0.84	1%	423%

collateral constraint is tight. Since this tightness is decreasing with wealth, this channel becomes more pronounced for lower parts of the distribution. This is clear from the change in the wealth share of the top 10%. In the data, it increases by 3%, whereas the model generates about 11% increase, which is almost 4 times higher than the data.

4.2 Impact on the Income Distribution

Table 7 shows the change in the income distribution. The model can explain one-third of the increase in the income share of the top 1% and the top 0.1%. This result is expected because, in theory, the Pareto parameter is proportional to the labor share of income when there is no collateral constraint, as proved by Koru (2019). Because the decrease in the labor share of income is a third of the decrease in the Pareto parameter of the top income distribution, it is expected that the model will also generate one-third of the observed changes in income concentration. However, this change partially affects wealth concentration. This is another reason why the model cannot generate the observed increase in the wealth share of the top 0.1%. The model can only generate a quarter of the rise in the income concentration, and a part of it translates into wealth concentration.

Table 7: Results - Income Distribution

	Data 1968	Data 2016	%Δ Data	Model Generates
Bottom 50	20.4%	12.5 %	-38.6%	18.57%
Top 10	36.3%	46.7%	29%	49%
Top 1	13.4%	19.6%	46%	39%
Top 0.1	5.1%	8.8%	73%	25%
Top 0.01	2%	3.9%	93%	28%
Gini	0.46	0.60	30%	39%

4.3 Discussion of the Results

To understand the relevance of this magnitude, I compare this result with Kaymak & Poschke (2016), who analyze the impact of change in the earnings distribution and the change in fiscal policy. They find that between 1980 and 2010, the change in the earnings distribution alone can explain 60% of the increase in the wealth share of the top 1%. In contrast, the current model can explain 25%. However, it is important to note that Kaymak & Poschke (2016) feed the change in the earnings distribution as seen in the date into their model. In contrast, in the current model, the change in income concentration is endogenous. Because income concentration leads to wealth concentration, the success of the model depends on the change in income concentration. To understand the link between top income inequality and top wealth inequality, consider the ratio of the change in the top wealth share to the change in the top income share. In the data, this ratio is 0.76 while in the model it is 0.53. In other words, given the change in income concentration, the model can explain two-thirds of the increase in top wealth inequality.

4.4 Alternative Measure of Automation

I use the change in the labor share of income to measure the increase in automation technology. However, not all of the decrease in the labor share can be accounted for by the rise in automation. Several other reasons can be at play, such as an increase in market power, market concentration, or rents in the housing sector (De Loecker et al., 2020; Autor et al., 2020; Rognlie, 2016). In this

Table 8: Results - Wealth Distribution - IT Share

	Data 1968	Data 2016	%Δ Data	Model Explains
Bottom 50	1.2%	0.4%	−70%	41%
Top 10	70%	71%	3%	203%
Top 1	27%	37%	35%	17%
Top 0.1	9%	19%	102%	5%
Top 0.01	3%	9%	216%	5%
Gini	0.83	0.84	1%	242%

section, to abstract from other explanations, I consider the change in the share of income that accrues to IT capital as a measure of the increase in automation technology. This is not a perfect measure of automation because it does not take into account robots or machines. However, it is a subset of automation technology, and, consequently, I see this as a lower bound on improvements in automation technology. Eden & Gaggl (2018), who estimate the rise in IT share, find that it increased by 3 percentage points. In contrast, in the main estimation, I consider a 5 percentage points increase. In this section, I analyzed the impact of an increase in IT share in income by increasing I to 0.39.

One thing to notice is that, because the definition of capital in the model includes automation and other types of assets, my initial calibration is not affected. Only the change in the capital share must be related to automation technology. Therefore, I do not re-calibrate the model.

Table 8 and Table 9 show the changes in wealth and income concentration. Comparing these to the benchmark results, it can be seen that the impact of automation decreased by 50% to 60%. This is expected since the change in the automation level decreased by 60%. Now, the model explains 17% of the rise in the wealth share of the top 1%.

On the other hand, the model transfers at a higher rate the change in income concentration into wealth concentration. The ratio of the change in the wealth share of the top 1% to the change in the income share of the top 1% increased from 0.53 to 0.6. This corresponds to 80% of the same ratio in the data.

Table 9: Results - Income Distribution - IT Share

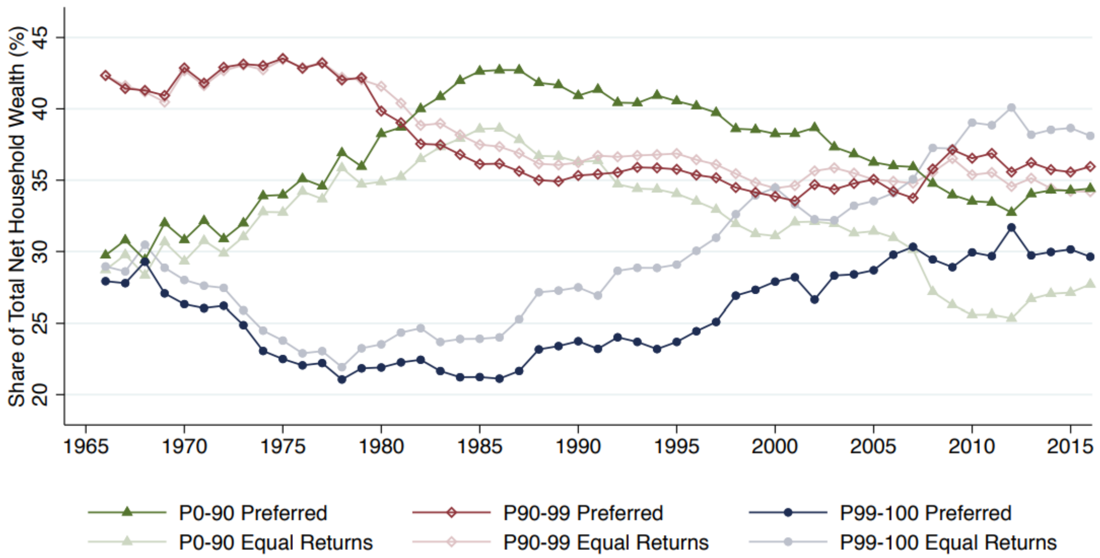
	Data 1968	Data 2016	%Δ Data	Model Generates
Bottom 50	20.4%	12.5 %	-38.6%	10.5%
Top 10	36.3%	46.7%	29%	28%
Top 1	13.4%	19.6%	46%	22%
Top 0.1	5.1%	8.8%	73%	14%
Top 0.01	2%	3.9%	93%	17%
Gini	0.46	0.60	30%	24%

4.5 Alternative Estimates of Wealth Shares

Several methods can be used to estimate the top wealth shares (Kopczuk, 2015). The data I use relies on the capitalization method (Saez & Zucman, 2016), which examines capital tax return data. By observing the realized tax payment, the level of wealth can be backed out. However, that level depends on assumptions about the return to wealth. Fagereng et al. (2020) and Bach et al. (2020) show that the return to wealth is not constant across wealth groups and that in Norway and Sweden there is significant variation in return to wealth. Since there is a positive correlation between wealth and return to wealth, a simple capitalization method overestimates the top wealth shares. Smith et al. (2020) adjust the capitalization method to incorporate return heterogeneity. Figure 4 plots alternative estimates of wealth shares estimates by Smith et al. (2020). Since my analysis focuses on the top 1%, the important plots are dark and light blue lines. The light blue line is the estimate under the assumption that return to wealth is the same for everyone, while the dark blue line is the estimate under the assumption that return to wealth increases with wealth. As can be seen, there is a significant difference between the two series. Smith et al. (2020) estimate that the increase in the top 1% share is not as pronounced: it only increases to 30%, whereas in the data it increases to 37%. (Kuhn et al., 2020) estimate the wealth shares using the Survey of Consumer Finance and their estimate is similar to the capitalization method estimation under the assumption of constant return to wealth.

Observe that both estimates are very similar until the 1980s. Thus, current calibration is a good fit for Smith et al.'s (2020) estimate. If I consider the estimate under the assumption

Figure 4: Alternative Wealth Shares Estimates



Source: Taken from Smith et al. (2020)

of heterogeneous return, the model generates almost all of the increase in wealth concentration. Therefore, I believe that the result I present in Table 6 is a lower bound for the real impact of automation on top wealth concentration.

5 Welfare Analysis

In the previous section, I considered the increase in wealth concentration. However, automation impacts prices, as does the overall productivity of the economy also. To understand the overall impact of automation, I analyze in this section the welfare gains for individuals who have different levels of asset holding and occupation. The measure that I use for welfare gain is expressed in consumption equivalent terms. Specifically, for each asset level and skill level, I calculate the required percentage increase in consumption level during each period and in each states that make individuals indifferent between a world in which automation improved and a world in which there is no improvement in automation. Formally, welfare gain in consumption equivalent terms, denoted by ν, is

$$
\begin{aligned}
&\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t u((1+\nu(a,s,0))c_t(a',s'))|a,s\right] = V_0(a,s),\\
&\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t u((1+\nu(a,s,z))c_t(a',s',z))|a,s,z\right] = E_0(a,s,z),
\end{aligned}
$$

where $V_t(a,s)$ and $E_t(a,s,z)$ is the life-time value of a worker in state (a,s) and an entrepreneur in state (a,s,z), after t periods during which automation started to improve. Observe that I do not compare two steady states. Instead, I compare life-time value in the first steady-state and life-time value in the same period when automation technology changes, taking into account the transition path of prices and value functions. Under the assumption that the utility function is CRRA, ν simplifies to:

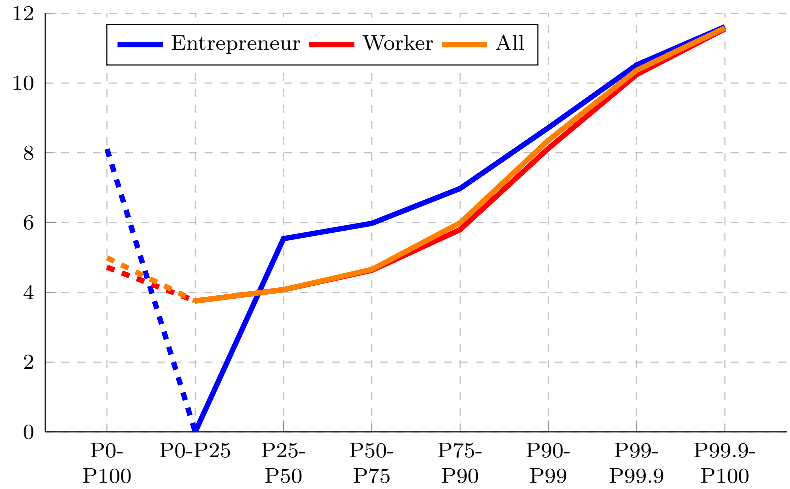
$$
\begin{aligned}
\nu(a,s,0) &= \left(\frac{V_0(a,s)}{V_{na}(a,s)}\right)^{1/(1-\sigma)} - 1,\\
\nu(a,s,z) &= \left(\frac{E_0(a,s,z)}{E_{na}(a,s,z)}\right)^{1/(1-\sigma)} - 1,
\end{aligned}
$$

where V_{na} is the value function at the initial steady state.

To compute the value functions along the transition path, I assume that automation technology improved gradually over 45 years. In other words, I let I increase from 0.36 to 0.41 at a constant rate for 45 years, and then I fix it at 0.41 thereafter. Furthermore, I assume that individuals have perfect foresight for the increase in I and its impact on prices.

For a partition of asset distribution and skill distribution, $T \in \mathcal{A} \times \mathcal{S} \times \{\mathcal{Z} \cup 0\}$, I calculate the average welfare gain $\bar{\nu}$ as the weighted average of welfare gains of individuals in that partition:

Figure 5: Welfare Gains by Occupation and Asset Level



Note: Welfare gain computed in consumption equivalent terms, $\bar{\nu}$ in equation (9). The horizontal axis is the partition on asset, where $Px - Py$ denotes the asset level between the xth and yth percentile in overall distribution in the initial steady state (not conditional on occupation).

$$\bar{\nu} = \int_T \Gamma(t)\nu(t)dt/\Gamma(T). \tag{9}$$

Figure 5 shows the welfare gain by occupation and asset level. The horizontal axis is the partition on the asset; where $Px - Py$ denotes the asset level between xth and yth percentile in the overall distribution in the initial steady-state (not conditional on occupation). The figure shows that everyone gained from automation.[6] An average worker gained around 5%, whereas an average entrepreneur gained 8%. Clearly, the gain increases by asset level, and so the wealthiest households gain the most from automation. It is intuitive that entrepreneurs are gaining more than workers, given that automation increases the return to entrepreneurial skills. Even the poorest workers gain, thanks to the shift of employment to more productive entrepreneurs. Because the top-skilled entrepreneurs can scale up their production, the share of employment in top firms increases. Hence, there is a significant increase in productivity in the economy, which leads to

[6]There is no entrepreneur who is in the lowest quartile; hence, the gain is zero there.

welfare gain for workers, too.

Some of the differences between an average worker and an average entrepreneur can be attributed to the fact that entrepreneurs on average own more assets than workers. For each partition of the asset level, entrepreneurs gain more than workers. This is expected: automation directly affects the return on the entrepreneurial skill, whereas its impact on workers is secondary. However, the gap between entrepreneurs and workers decreases with wealth. At the very top of wealth distribution, the gap is very small. This is so mainly because for individuals at the very top, business income relative to capital income is small. Therefore, whether one is an entrepreneur or a worker is not very important.

The main underlying assumption in this welfare analysis is that, for two reasons, there is no friction in the labor market for workers to change their jobs. First, there is no search friction that prevents labor reallocation across firms. Hence, workers can immediately reallocate to highly productive firms. Second, the model does not allow for different job types or occupations for workers, such as occupations. In reality, automation does not affect every occupation in the same way. If there were different types of jobs, and if it was not possible for workers to change their types of jobs, then automation would not impact everyone in the same way. For example, workers in occupations that are more prone to automation would gain less, or they could even lose. However, without any friction for labor reallocation, everyone in the economy enjoys significant gains from automation, although the benefits are concentrated among wealthy entrepreneurs.

6 Conclusion

In the last 50 years, the US has experienced a significant increase in wealth concentration. In this paper, I analyze the impact of automation technology on the change in wealth concentration. Automation has two main effects on wealth accumulation. First, it increases the income concentration because it enables entrepreneurs to scale up their production. Second, it increases the heterogeneity of return to capital.

I calibrate the model to the US economy in 1968 and then the automation technology parameter to the 2016 value, keeping everything else constant. The quantitative exercise implies that the improvements in automation technology can explain one-fourth of the rise in the wealth share of the top 1%. Taking into account the transition path, the welfare of workers in consumption equivalence terms increased by 5%, while the welfare of entrepreneurs increased by 8%. Although everyone gained thanks to the shift of the labor force to more productive firms, wealthy individuals gained more than the poor.

One drawback of the quantitative exercise is the lack of a direct measure of the convex cost of labor. Labor could have a convex cost for many reasons. But the underlying idea is that if a firm wants to get bigger, it needs to spend more resources. In a sense, the firm size wage premium can generate this convex cost. This premium is known to be decreasing – i.e., the wage gap between large firms and small firms is decreasing (Bloom et al., 2018; Cobb & Lin, 2017). This might be seen as a decrease in the convexity of labor cost, which eventually leads to a higher return to capital and entrepreneurial skill. Incorporating the change in firm size wage premium might provide a good means of analyzing the impact of change in the convex cost of labor.

In the current model, top of income distribution is populated by entrepreneurs. However, in the data, for half of the individuals in the top 1% wage is the major source of income (Smith et al., 2019). Automation might enabled wage of those people also, for example it might increase the compensation of CEOs, which might contribute to wealth inequality. For this, a more rich structure for labor productivity is needed to be incorporated to also include wage earners at the top of income and wealth distribution.

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A Proofs

Proposition 1. *Fix prices w and r. Let $\tilde{\pi}(z; I)$ be the profit function when automation technology is given by I and without the collateral constraint, i.e., when entrepreneurs operate their businesses at the efficient level. Then, $\tilde{\pi}'(z; I') > \tilde{\pi}'(z; I)$ for $I' > I$, where $\tilde{\pi}'$ is the derivative with respect to z.*

Proof. *Consider the problem of an entrepreneur in* (6) *without collateral constraint (assume a is high enough. By envelope theorem, the impact of increase in the automation technology on profit is the shadow cost of automation technology constraint. Let η be the Lagrange multiplier with that constraint, then:*

$$\tilde{\pi}_I(z; I) = \eta(z).$$

First order condition with respect to $I^\star$ is:

$$zY(ln(k(I)) - ln(\ell(I))) + w\ell(I) + v'(L)\ell(I) - (r + \delta)k(I) = \eta(z).$$

Since in optimal solution marginal rate of technical substitution is equal to relative marginal costs, this condition simplifies to:

$$zY(ln(k(I)) - ln(\ell(I))) = \eta(z).$$

Right hand side is the shadow cost of automation. Left hand side is the change in production when automation increases. Observe than left hand side is increasing with z. This is becase both zY and k/ℓ is increasing with z. In the optimal solution, $k/\ell = (w + v'(L))/(r + \delta)$. Since L is increasing with z, k/ℓ is also increasing. Hence, shadow cost of automation is increasing with z. This implies that $\tilde{\pi}_I(z; I)$ is increasing with z. By Young's theorem, second derivative is symmetric, hence $\tilde{\pi}_z(z; I)$ is increasing with I.

■

Proposition 2. *Fix prices w and r. Let $\pi(a, z; I)$ denote the profit function when automation technology is I. Then, the derivative of profit function with respect to a is increasing with I, i.e., $\pi_a(a, z; I') \geq \pi_a(a, z; I)$ when $I' > I$. When the automation constraint binds, this condition holds with strict inequality.*

Proof. *Consider the problem of an entrepreneur in* (6). *By envelope theorem, impact of increase in a is:*

$$\pi_a(a, z) = \lambda\gamma(I),$$

where $\gamma(I)$ is the Lagrange multiplier associated with the collateral constraint when automation technology is I. It is clear that when collateral constraint does not bind, $\gamma(I) = 0$. So we want to show that $\gamma(I)$ is increasing with I when collateral constraint binds. The first order condition with respect to $k(i)$ is

$$\frac{zY}{k_i} = r + \delta + \lambda\gamma(I),$$

Optimal solution yields to $Y = k^I \ell^{1-I}$, hence $Y/k = (\ell/k)^{1-I}$. Suppose than Y/k is decreasing with I, then $(1-I)log(\ell/k)$ is decreasing in I. Derivative with respect to I:

$$-log(\ell/k) + (1-I)\frac{d(\ell/k)}{dI} < 0.$$

Assume that automation constraint binds. Then $\ell < k$, which implies that $log(\ell/k) < 0$. Hence, ℓ/k is decreasing with I. We know that k is decreasing, because we are allocating the same level of capital to larger range of task. This implies that ℓ must be decreasing, hence total labor $L = (1-I)\ell$ is decreasing. Now consider the first order condition with respect to ℓ

$$\frac{zY}{\ell} = z\left(\frac{k}{\ell}\right)^I = w + v'(L).$$

Right hand side is decreasing because L is decreasing. However, left hand side is increasing *because* $(k/\ell)^I$ *is increasing. This leads to contradiction. This implies that marginal productivity of capital must be increasing with* I*, therefore the shadow cost of collateral constraint is increasing when automation technology and collateral constraint are binding.*

Now assume that automation constraint does not bind. Then $\ell = k$*, which implies that* $zY/k = z$*. Hence, marginal product of capital is constant. This implies that* $\gamma(I)$ *is constant and does not change with* I *when optimal automation level is interior.*

■