Microeconomic Theory I
Preliminary Examination
University of Pennsylvania

August 5, 2019

Instructions

You have 2.5 hours to answer all questions.

This exam has 4 questions and a total of 100 points.

Answer each question in a **SEPARATE** exam book.

If you need to make additional assumptions, state them clearly.

Be concise.

Write clearly if you want partial credit.

Good luck!

1. (25 pts) A competitive firm has a concave production function $f(z_1, z_2)$ satisfying $f(z) > f(z')$ for all $z \gg z'$. It gives rise to the cost function

$$c(w, q) = q \min\{w_2, \tfrac{1}{3}(2w_1 + w_2)\}.$$

   (a) (8 pts) Find the firm's profit function, $\pi(p, w)$.

   (b) (8 pts) Find the firm's the conditional factor demand, $z(w, q)$, for $w_1 \neq w_2$.

   (c) (9 pts) Find the production function $f(z)$.

2. (25 pts) An investor has a C^2 strictly concave and strictly increasing Bernoulli utility function $u : \mathbb{R} \to \mathbb{R}$. She has initial wealth $w > 0$ which she must invest in assets 1 and 2. The return on asset i is $\tilde{r}_i$, a random variable continuously distributed on $[0, 2]$. Letting $x_i \geq 0$ be the amount she invests in asset i, her expected utility from $x = (x_1, x_2)$ is

$$U(x) := \mathbb{E}u(\tilde{r}_1 x_1 + \tilde{r}_2 x_2).$$

   (a) (10 pts) Show that U is strictly concave.

   (b) (15 points) Using the fact that U is strictly concave, and assuming $\tilde{r}_1$ and $\tilde{r}_2$ are identically and independently distributed, characterize the investor's optimal portfolios, i.e., the x^* pairs that maximize $U(x)$ subject to the constraints $x \in \mathbb{R}^2_+$ and $x_1 + x_2 = w$.

3. (25 pts) Three hunters will hunt for deer tomorrow in a game park with exactly one deer. The deer is certain to be caught. There are thus three states of the world tomorrow: in state $s = 1, 2, 3$, the deer is caught by hunter s. Letting ω^i denote hunter i's initial endowment of contingent deer meat, we have

$$\omega^1 = (1, 0, 0),\ \omega^2 = (0, 1, 0),\ \omega^3 = (0, 0, 1).$$

Today (date $t = 0$) they arrange for how the meat from the deer caught tomorrow (date $t = 1$) will be shared. The utility function of hunter i is

$$U^i(x^i) = \Sigma_{s=1}^{3} \pi_s^i u^i(x_s^i),$$

where x_s^i is his consumption of deer meat in state s, and π_s^i is his belief probability that state s will occur. Assume u^i is concave, continuously differentiable, and strictly increasing. Assume also that the hunters agree that the state probability vector is $(1/2, 1/4, 1/4)$ (hunter 1 is twice as likely to catch a deer as is either of the other two).

   (a) (5 pts) Assuming each u^i is strictly concave, show that at any interior Pareto efficient allocation, hunter 1 consumes the same amount of deer meat regardless of who catches the deer.

   (b) (5 pts) Are there Pareto efficient allocations in which hunter 1 consumes nothing?

   (c) (5 pts) Again assuming each u^i is strictly concave, will hunter 2 and hunter 3 consume the same amount at an interior Walrasian equilibrium?

   (d) (5 pts) Now assume all hunters are risk neutral, and find a Walrasian equilibrium.

   (e) (5 pts) Now assume hunter 1 is strictly risk averse but hunters 2 and 3 are risk neutral. What can you say about a Walrasian equilibrium in this case?

4. (25 pts) Walrasian equilibrium with production.

   (a) (5 pts) State precisely the definition of a Walrasian equilibrium for an economy with production.

   (b) (5 pts) Under standard assumptions on preferences and interior endowments, what conditions on the production technology are sufficient for a Walrasian equilibrium with production to exist? (Little if any credit will be given for trivial conditions such as "the production set is empty".)

   (c) (7 pts) Give an example in which one of the conditions on the technology you gave in (b) is not satisfied, and a Walrasian equilibrium does not exist.

   (d) (8 pts) Give an example in which the condition on the technology that failed in your answer to (c) fails but nevertheless an equilibrium exists.