The Aggregate Effects of Bank Lending Cuts

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Abstract

A large body of cross-sectional evidence has established that cuts in the supply of bank lending affect firm outcomes and the allocation of credit. However, it is unclear what these results imply for the effect on aggregate output of a cut in aggregate bank lending. I estimate this aggregate effect using a new general equilibrium model that incorporates multibank firms, relationship banking, endogenous credit dependence, and bank market power. I use a set of cross-sectional patterns to estimate the key structural parameters of the model. The effect of an aggregate lending cut on aggregate output is large: a 1 percent decline in aggregate bank lending supply reduces aggregate output by 0.2 percent. The structure of labor and credit markets is important in reaching this answer. Under an alternative parametrization of the model that ignores input market frictions, the response of aggregate output is three times smaller. Under my preferred parametrization, the cross-sectional effects survive aggregation in general equilibrium. Instead, with frictionless input markets the cross-sectional patterns over-estimate the aggregate response by a factor of five.

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Introduction

This paper addresses the long-standing question of the effect of disruptions in bank lending on *aggregate* economic performance. Motivated by this question, a large and robust body of cross-sectional studies have produced evidence that cuts in the supply of bank lending affect firm outcomes and the allocation of credit. The gold standard empirical approach in the literature uses microdata to exploit variation in banks exposure to funding shocks, and variation in the exposure of firms and regions to different banks. Khwaja and Mian (2008), Chodorow-Reich (2014), and Huber (2018), for example, have used this approach.

Despite a large cross-sectional literature, the debate on the macroeconomic effects of bank lending cuts persists. The main reason is that cross-sectional studies are often designed to identify relative rather than aggregate effects. It is therefore unclear what their findings and mechanisms imply for aggregate output or employment. In addition, the alternative of using aggregate data to answer the question relies on strong identification assumptions, and methodologies using this approach are silent about the extensive findings of the cross-sectional literature, which are the best evidence of causal effects at our disposal.[1]

In this paper I study the effects on aggregate output of a cut in the supply of aggregate bank lending. I develop an explicit model with a tractable setup for banks, firms, and workers that allows for the main mechanisms behind the cross-sectional patterns of firm responses after a bank shock. I use those estimated patterns as the primary input to infer the aggregate response through the lens of the model. This is the first study to provide an estimate of the macroeconomic impact of cuts in the supply of bank lending informed by such evidence, characterize its determinants, and account for a wide range of facts. I

[1] There is an extensive literature on macroeconomics studying the relevance of financial frictions and financial intermediaries. These models are usually calibrated or estimated using VARs or Bayesian methods to fit first and second moments of aggregate time series, as in Gertler and Kiyotaki (2010), Gertler and Kiyotaki (2015), Christiano, Eichenbaum, and Trabandt (2015), or Del Negro, Giannoni, and Schorfheide (2015).

find that a cut in the supply of bank lending has a sizeable effect on aggregate output. This finding is the consequence of frictions in credit and input markets. Ignoring frictions in input markets would lead us to wrongly conclude that the effects on aggregate output of a bank lending cut are significantly smaller than what is implied in the cross-sectional literature.

Relative and aggregate effects would be guaranteed to be equal if the disruption of a particular bank had zero indirect effects on unexposed banks and on firms that borrowed from those healthy banks. These conditions may not hold in an environment where both banks and firms compete in different markets. Banks compete for funding and for customers, so a shock to one bank indirectly affects the balance sheet of its competitors. Firms compete in labor and goods markets, so firms will be indirectly affected even if their lenders are not directly exposed to a shock. These general equilibrium effects are transmitted through changes in aggregate prices and quantities and can make a given pattern of relative effects consistent with different aggregate responses.

To measure the aggregate effects, I use a model that speaks to these general equilibrium effects and to the cross-sectional estimates at the same time. The model has the three main mechanisms studied in the empirical literature. Two of these mechanisms are absent from textbook macroeconomic models and are a contribution of this paper. First, firms borrow from multiple banks, and the strength of bank-firm relationships is a function of how close these entities are for geographical, historical, or sectoral reasons. Second, the model allows for flexible patterns of substitution of the sources of finance for the firm. Firms may substitute funding from one bank in favor of funding from other banks or substitute away from bank credit altogether. I provide a micro-foundation for these decisions of the firm based on a discrete choice model with a tractable solution. Two parameters capture the two margins of substitution. Bank market power emerges naturally in this framework, as there are only a handful of banks in the economy.

The third main mechanism of the model allows for a labor market in which firms face an upward-sloping firm-specific (relative) supply curve. This means that in order to hire additional labor, a firm must pay a higher wage. A market in which an individual firm can hire any number of workers at the prevailing market wage rate is nested as a special limiting case. This feature used in other studies is meant to capture the difficulty in moving labor across firms, which disrupts the ability of the economy to reallocate resources across firms after an arbitrary shock.

I characterize analytically the elasticity of aggregate output and firm-level output to an exogenous increase in the cost of loans in the economy. I show that the extent of frictions that inhibit firms from finding alternative sources of finance, as well as the structure of labor and goods markets, are informative about the response of aggregate output to a disruption in the supply of bank credit. The ability of firms to borrow from different banks and to procure funding from sources other than banks counteracts the negative shock. When labor markets work without frictions the cross-sectional output effects of a bank disruption become larger.

I extend the simple model to a dynamic model and for banks to set lending rates optimally and raise funds from depositors. I solve the model using state-of-the-art numerical methods, as in Ahn et al. (2018), that allow for aggregate shocks in macroeconomic models with heterogeneous agents where the state-space is infinitely dimensional owing to the relevance of the distribution of firms to forecast prices. In particular, my model has heterogeneous firms and banks, and since banks are large, even the disruption of one bank has aggregate consequences.

I then calibrate the model, recovering the parameters that determine the extent of the two key financial frictions in the model. These parameters are the elasticity with which a firm substitutes funding from a particular bank with funding from other banks, and the elasticity with which firms avoid bank credit altogether. I recover these parameters by combining two elasticities estimated in the microdata: the cross-sectional effect of an idiosyncratic bank shock on firm credit, and the cross-sectional effect of an idiosyncratic bank shock on firm employment.

The idea behind the identification of the banking friction parameters is the following. After a bank disruption, firms that can replace funding from the affected bank with funding from other banks will experience little change credit exposure or output triggered by the shock. However, if firms cannot subsitute across banks but can avoid bank credit altogether, they will take on much less debt, but their output losses will be small. Therefore, with information about the decrease in firm credit and employment after a bank shock, it is possible to back out the two key parameters in the model. I use these moments as estimated by Huber (2018), and the methodology can be adapted to target moments in other countries and time periods.

To discipline the extent of frictions in the labor market I use two sources of evidence. First, I use direct evidence from Webber (2015), which documents an inelastic firm-specific labor supply curve. This means that to hire additional labor , a firm must pay a significantly higher wage. Second, I use the *indirect effects of bank lending cuts*. In models with flexible input markets, firms without exposure to a bank shock operating in regions where firms are highly exposed on average, outperform firms without exposure located in low-exposure regions. This is contrary to the evidence on the indirect effects reported by Huber (2018), who finds the opposite. This statistic rejects models without rigidities in the labor market and favors models with costs of reallocation of labor within the region.

I estimate an elasticity of output to lending caused by a shock to the lending supply of 0.2. This number means that a 1 percent drop in aggregate lending caused by a bank shock causes a drop of aggregate output of 0.2 percent. This elasticity depends on a number of different factors. Under an alternative parametrization of the model that ignores labor market frictions, the elasticity is three times smaller, illustrating the relevance of the structure of the economy to the result. When labor markets are flexible, the degree of banking frictions identified by the cross-sectional moments is smaller. The reason is that, for a given extent of financial frictions, more flexible labor markets imply larger cross-sectional moments after the same shock. Therefore frictions in the banking sector must be smaller when labor markets are perfect to target the same microeconomic patterns.

I compare the magnitude of the elasticity I obtain in general equilibrium with the partial equilibrium aggregations that would be obtained from a back-of-the-envelope aggregation using estimated causal effects in the cross-section. These aggregations are computed by adding up the differences in firm outcomes between each firm in the economy and comparing them with a control firm with zero direct exposure to the shock. Under my preferred parametrization of the model, the partial equilibrium aggregation is similar in size to the general equilibrium aggregation I studied. This means that general equilibrium forces in the parametrized model do not cause the effects in the cross-section to vanish. I illustrate that this is not required to be the case for this application. Under the alternative parametrization of the model with frictionless labor markets, the general equilibrium aggregation is 5 times smaller than the partial equilibrium one. However, the evidence and the model prefer combinations of the parameter space where general equilibrium effects do not cause the patterns observed in the microdata to vanish.

Literature Review

This paper addresses the long-standing question of the relevance of bank health disruptions on aggregate economic performance. Bernanke (1983) stated that cuts in the supply of bank lending make credit more expensive, potentially affecting the aggregate economy. I analyze the relevance of cuts in the supply of bank lending to firms in determining drops in aggregate production.[2]

To measure the effects of an aggregate lending cut on aggregate output, I rely on a large and robust empirical literature that inquires about the effects of bank health in a

[2] Bernanke (1983) hypothesized that large firms would be immune to cuts in bank lending. However, Benmelech, Frydman, and Papanikolaou (2019)document that large firms with maturing bonds tried to access bank loans after debt markets froze up during the Great Depression. Firms located in regions with more affected banks suffered larger employment losses during the Depression.

cross- section of firms and banks. This cross-sectional literature exploits variation in bank exposure to funding shocks and variation in the exposure of firms and regions to different banks. This body of evidence concludes that bank disruptions affect the allocation of firm credit, as in Khwaja and Mian (2008);[3] firm outcomes like employment and sales, presumably because of the existence of sticky firm-bank relationships, as in Chodorow-Reich (2014);[4] and regional outcomes, as in the seminal work by Rosengren and Peek (2000).[5]

I incorporate this literature by embedding a discrete choice problem in a macroeconomic model with heterogeneous firms. This approach is similar to the Ricardian models in the Eaton and Kortum (2002) spirit, used to characterize trade flows between countries. In particular, I use the extensions made by Dingel, Meng, and Hsiang (2019) and by Lashkaripour and Lugovskyy (2018). Instead, however, I use it to characterize the flow of credit from banks to firms and firms decisions about how much to borrow. In this setting, banks set lending rates in an imperfect competitive market. Crawford et al. (2018), Drechsler et al. (2017), Wang et al. (2018) and Xiao (2019) have incorporated bank market power in macro-finance questions.

This paper contributes to the broad literature that uses cross-sectional estimates to investigate the macroeconomic effects of aggregate shocks. The approach I follow in this study uses causal effects measured in cross-sectional settings as inputs to measure an aggregate elasticity, in this case the elasticity of aggregate output to aggregate lending. Nakamura and Steinsson (2017) survey the literature and discuss its challenges. The ap-

[3]Other examples of this literature are Gan (2007); Schnabl (2012); Iyer, Peydro, da Rocha Lopes, and Schoar (2013); Benetton and Fantino (2018); Jiminez, Mian, Peydro, and Saurina Salas (2014); Becker and Ivashina (2014) ; and Cingano, Manaresi, and Sette (2016).

[4]The strength of the relationship between firms and banks depends on the closeness of the entities, either geographically, like in Degryse and Ongena (2005), Agarwal and Hauswald (2010), Brevoort, Wolken, and Holmes (2010), and Nguyen (2019); historically, as in Huber (2018); or sectorally or culturally, as in Fisman, Paravisini, and Vig (2017). The consequences of bank-firm relationships in the cross-section has been widely studied. Examples are Darmouni (2017) and Bolton, Freixas, Gambacorta, and Mistrullu (2016). Under the null hypothesis that funds from a given bank are perfectly substitutable for the firm, an id- iosyncratic bank shock should create zero cross-sectional effects for firms that differ in their pre-existing bank relationships. This null hypothesis is rejected in the data.

[5]Other examples are Ashcraft (2005); Greenwood, Mas, and Nguyen (2014); and Huber (2018).

proach I use here is different than the one followed by an exciting growing literature on sufficient statistics that derives expressions in generic models to compute aggregate elasticities. Particularly relevant is the work of Sraer and Thesmar (2018). Here I ask what we can learn from a body of evidence that is already measured in the literature, and discuss the relevance of the mechanisms in the aggregate, instead of proposing a new set of elasticities to be measured. The cost of my approach is that it requires more structure.

1 A Model of Credit Dependence of Multi-Bank Firms

In this section I present a model that is flexible enough to incorporate the patterns observed in the data and works as a laboratory for analyzing the effect of bank health on aggregate output. The model features a continuum of firms, a discrete number of banks, and a representative household. Firms borrow from multiple banks simultaneously. Banking relationships are imperfectly substitutable in the sense that the relative demand for funding from a particular bank is downward sloping, not horizontal. Self-finance is also an imperfect substitute for bank credit.

This model is static and makes a number of simplifications that will be relaxed when the full model is presented. In particular, in this section I will take lending rates as given. The best interpretation for now is that different banks play the role of different technologies. Later in the paper, we will incorporate the problem of the banks.

1.1 Firms

There is a continuum of monopolistic competitive firms producing differentiated varieties. Each firm is indexed by j in the unit interval. The demand schedule for each variety is given by:

$$Y_{jt} = Y_t P_{jt}^{-\eta}, \tag{1}$$

where P_{jt} is the relative price of variety j, Y_{jt} is the quantity demanded of each variety and Y_t is aggregate demand. The aggregate price is set to be the numeraire.

Each firm produces by mixing a continuum of intermediates indexed by ω. Think of these intermediates as projects or tasks the firm has to complete in order to produce its differentiated product. The firm aggregates the intermediates via a CES function with elasticity of substitution σ[6]

$$Y_{jt} = \left(\int_0^1 (y_{jt}(\omega))^{\frac{\sigma-1}{\sigma}} \, d\omega \right)^{\frac{\sigma}{\sigma-1}}. \tag{2}$$

Each intermediate good ω is produced with labor in a constant-returns-to-scale production function, and a firm-wide productivity shifter z

$$y_{jt}(\omega) = z_{jt} l_{jt}(\omega). \tag{3}$$

1.2 Financing

For a given task, firms decide whether to self-finance or look for funding from a bank. Firms that choose bank financing must select an individual bank to finance each task. Different banks offer different terms, based on their comparative advantage in particular segments, and on the functioning of credit markets that cause similar projects to be priced differently across banks.

Because firms need to finance a continuum of tasks, the cost of funds for the firm, which determines its marginal costs, does not depend on the realization of the financing cost of any particular task, but on structural parameters that capture how substitutable bank credit is for self-finance, and how substitutable the credit from a particular bank is. In the two next subsections I introduce these discrete choice problems.

[6] This elasticity of substitution will end up being irrelevant for the purposes of this paper.

1.2.1 Shopping for a lower rate

The cost of financing task ω is given by $TC_j(\omega)$, which consists of the wage bill and the financing costs of financing the wage bill,

$$TC_j(\omega) = \frac{w_j}{z_j} R_j(\omega) y_j(\omega). \tag{4}$$

$R_j(\omega)$ is the interest rate firm j gets to finance ω. As part of its cost minimization problem, firm j looks for the cheapeast financing option.

In particular

$$R_j(\omega) = \min_{b,f} \left\{ \frac{R_{bf}}{\epsilon_{jbf}(\omega)} \right\}. \tag{5}$$

Here $f \in (\mathcal{B}, \mathcal{S})$ indexes a given financing sector, either banks or self-finance. And b indexes an option within a given financing option. There are $N_{\mathcal{B}}$ banks in the economy and one self-financing option. The effective cost the firm perceives if it were to choose a financing option is equal to the cost of funds of that option R, over a shifter, that captures all the idiosyncratic reasons why one option may be better for some intermediates than others. For example, some projects of the firm are very difficult to monitor, so the firm may prefer to self-finance them. Other projects benefit from the know-how of a specific expert bank, and so on.

I assume the vector $\boldsymbol{\epsilon} = \{\epsilon_{j,1,\mathcal{B}}, ..., \epsilon_{j,N_{\mathcal{B}},\mathcal{B}}, \epsilon_{j,N_{\mathcal{B}},\mathcal{B}}, ...\epsilon_{j,N_{\mathcal{S}},\mathcal{S}}\}$ is drawn from a nested Fréchet Distribution

$$F_j(\boldsymbol{\epsilon}) = \exp\left\{ - \sum_{s\in(\mathcal{B},\mathcal{S})} \bar{\varphi}_s \left(\sum_{b=1}^{N_s} T_{jb} \epsilon_{sb}^{-\theta} \right)^{\frac{\varphi}{\theta}} \right\}.$$

This distribution has been used by Dingel, Meng, and Hsiang (2019), and by Lashkaripour and Lugovskyy (2018), and it extends the Fréchet distribution common in the Ricardian model of international trade of Eaton and Kortum (2002). An analogy comes to mind from the literature in international trade. When deciding where to import from, a recepient country shops around many locations and chooses the one with the lowest effective price. These locations are cities within countries. The Fréchet shifters capture the fact that some countries have a comparative advantage in the production of some goods, and that within each country, some cities have comparative advantage. In this application, the shifters capture the firms preference to finance a given task with a given bank. It is akin to a productivity shock that depends on the financing source and the intermediate. The nested Fréchet distribution captures the variation in the advantage of financing a given task both across banks (some banks are better than others) and across financing options (some intermediates are perfect for bank financing).

The T_{jb} parameters capture the strength of the long-term banking relationship between firm j and bank b, or the absolute advantage of bank b in providing funding for firm j.

Under the assumptions stated before, we can characterize the share of expenditures financed with each bank ν_{jb} and the cost of bank credit for the firm R_{jB}:

$$R_{jB} = \left(\sum_{b \in \mathcal{B}} T_{jb} R_b^{-\theta} \right)^{-1/\theta}. \tag{6}$$

The share of borrowing needs that firm j gets from bank b is given by:

$$\nu_{jbt} = \frac{T_{jb} R_{bt}^{-\theta}}{\sum_k T_{jk} R_{kt}^{-\theta}}. \tag{7}$$

The borrowing shares depend on θ, which is the elasticity of substitution of funding from a specific bank, and on T_{ib}, which is the relative strength of the banking relationship between firm i and bank b. It is similar to the characterization made by Eaton and Kortum (2002) for international trade flows. The share of expenditures financed with the banking sector s_j, is given by

$$s_{jt} = \frac{\bar{\varphi} R_{jtB}^{-\varphi}}{\bar{\varphi} R_{jtB}^{-\varphi} + (1 - \bar{\varphi}) R_{jtS}^{-\varphi}}, \tag{8}$$

and the effective cost of funds for the firm is given by the cost of funds index R_{jt}

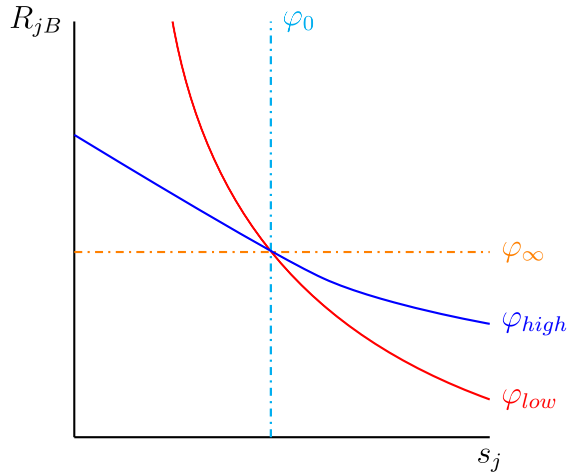

Figure 1: Share of bank financing as a Function of the cost of funds from the Banking sector

$$R_{jt} = \left(\bar{\varphi} R_{jtB}^{-\varphi} + (1-\bar{\varphi}) R_{jtF}^{-\varphi}\right)^{-1/\varphi}. \tag{9}$$

This discrete choice block is a microfoundation of the desired mix of bank borrowing that the firm chooses. When bank credit becomes more expensive ($R_{jtB} \uparrow$), the firm moves away from bank lending ($s_{jt} \downarrow$). The elasticity at which the substitution occurs is given by φ.

Figure (1) plots the share of financing from the banking sector as a function of the cost of funds for different values of φ. The figure shows that as φ increases, the relative demand schedule for bank funds becomes more elastic. In the limit, when $\varphi \to \infty$ the demand curve becomes horizontal, and firms are perfectly elastic in switching between bank funding and self-finance. On the other side, when φ becomes smaller, the share of bank financing is less sensitive to the lending rate.

When θ is higher, the demand curves for funding for a particular bank become flatter, which I show in Figure (2). In the limit, when $\theta \to \infty$ the demand curve becomes horizontal, and firms are perfectly elastic in switching between banks. On the other side, when θ tends to zero, the share of bank financing from bank b is less sensitive to bank b's lending rate.

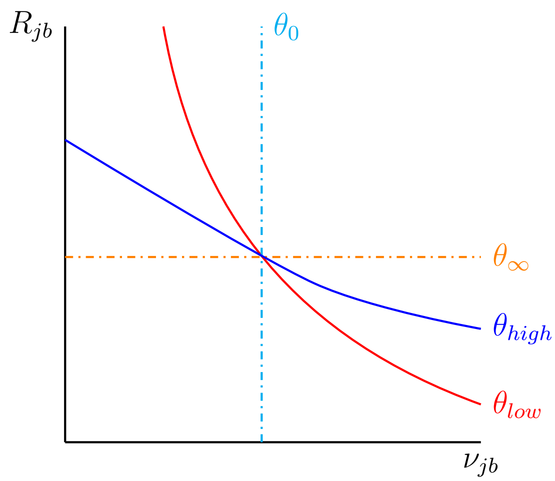

Figure 2: Figure 2: Shape of the demand curves for different values of θ. The x axis shows the market share of a particular bank, and the y axis shows the lending rate of the bank

The term T_{ib} in the equation has a different purpose than θ. Figure (3) illustrates that the relative demand curve for funds from a particular bank with a higher T is shifted to the right. That means that for two banks offering the same lending terms, a firm will borrow proportionally more from banks with higher T.

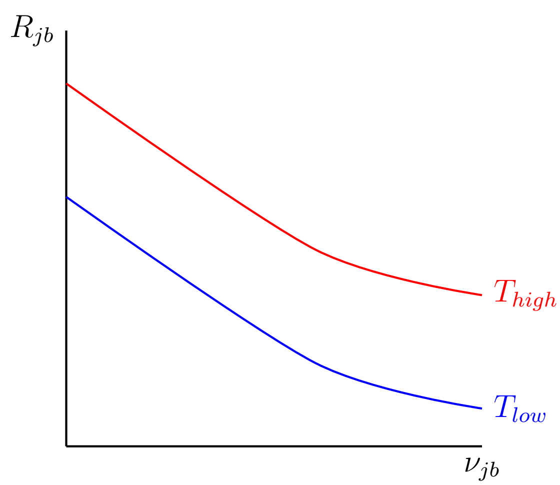

Figure 3: Shape of the demand curves for different values of T: The x-axis shows the market share of a particular bank, and the y-axis shows the difference of the interest rate between that bank and the other bank in the economy for an example where there are only two symmetric banks.

1.3 Workers

There is a representative household. It consumes and supplies labor. The household maximizes the utility function

$$U(C_t, L_t) = C_t - \frac{L_t^{\phi+1}}{1+\phi}, \tag{10}$$

Where L_t is an aggregator of the labor supply to different firms in the economy:

$$L_t = \left(\int L_{jt}^{\frac{1+\alpha}{\alpha}} dj \right)^{\frac{\alpha}{1+\alpha}}. \tag{11}$$

Workers maximize utility subject to a budget constraint $\int w_{jt}L_{jt} + \pi_{jt}dj = C_t$, where π_{jt} are the profits of firm j. Therefore households supply labor according to the following relationship:

$$L_t = w_t^{1/\phi}, \tag{12}$$

$$L_{jt} = L_t \left(\frac{w_{jt}}{w_t} \right)^{\alpha}, \tag{13}$$

where w_t is defined as L_t^{ϕ}.

This specification tells us that the disutility of working more hours for the same firm is convex. Therefore, the workers need higher pay in order to work more hours for the same firm.

When $\alpha \to \infty$, the labor market operates under a single wage rate $w_{jt} = w_t \ \forall j$. Otherwise, firms that hire more workers than average, pay wages that are higher than average.

1.4 Other Aspects of the Model

In this model, it is assumed that lending rates are exogenous. Later in the full model, I will specify the bank problem that gives rise to the lending rates in equilibrium as a function of the market structure and the ease of securing funding. I also assumed that the profits belong to the workers. The problem of the firm owners is included in the full model as well.

2 Characterization

The focus of this section is to characterize the elasticity of aggregate output to an exogenous lending rate hike of a particular bank, and to the whole banking sector.

There are two main results. First, the aggregate and cross-sectional effects of the lending rate hike of an individual bank are different, and it is a priori unclear which of them is larger. The difference in magnitude is dominated by the difference in the Frisch elasticity of the labor supply and the easiness to reallocate demand and inputs across firms. When it is easy to reallocate labor and demand across firms, then up to a second order the cross-sectional effects of output are larger. On the other side, the aggregate effects are large with an elastic labor supply. Under a perfectly inelastic labor supply curve, an increase in lending rates that raises firms marginal costs will will lead to lower wages without changing aggregate hours and production.

Second, greater frictions in the banking sector, in the form of low elasticities of substitution of funds between banks and between funding alternatives, increase the output losses caused by lending rate hikes. However, it is difficult to back out from a single cross-sectional elasticity the structural parameters that determine the response of aggregate output.

2.1 The Aggregate Effects of Loan Term Changes in One Bank

All the results in this section exploit the following assumption.

Assumption 1. *There is no sorting in bank relationships. That is, firm-level productivity z_j and the strength of bank lending relationships T_{jb} are independent. I rule out the possibility that banks suffering lending rate hikes are linked to firms with lower productivity.*

As I show in the appendix, under assumption 1, aggregate output is given by equation 14, where the expectation operator is taken across the continuum of firms:

$$Y = \left(\frac{\eta}{\eta-1}\right)^{-1/\phi} \mathbb{E}\left(z_j^{\frac{(\eta-1)(\alpha+1)}{\alpha+\eta}}\right)^{\frac{(1+\phi)(\alpha+\eta)}{\phi(\eta-1)(\alpha+1)}} \mathbb{E}\left(R_j^{\frac{-(\eta-1)\alpha}{\alpha+\eta}}\right)^{\frac{(1+\phi\eta)}{\phi(\eta-1)}} \mathbb{E}\left(R_j^{\frac{-\eta(\alpha+1)}{\alpha+\eta}}\right)^{\frac{(1-\phi\alpha)}{\phi(\alpha+1)}}. \quad (14)$$

The first results of this section hold under the following assumption:

Assumption 2. *Assume the lending terms of all banks except one are kept constant at an arbitrary level R, as is the self-financing rate. At these rates, the level of output coming from equation 14 is defined as $\bar{Y}$. For an arbitrary bank b, the lending terms are disrupted to Re^u, for a positive and sufficiently small u.*

Note that after making an assumption about the distribution of T, and setting an arbitrary level of lending terms, we can compute numerically the behavior of output according to equation 14. Assumption (2) is made in order express analytically the aggregate output effects of lending term disruptions, presented in Proposition (1).

Proposition 1. *Under Assumption (2), up to the second order, the log change of output is given by:*

$$\log Y - \log \bar{Y} \approx -\frac{1}{\phi}\bar{s}u\left(\nu_b - \theta\frac{u}{2}\Upsilon_1 - \varphi(1-\bar{s})\frac{u}{2}\Upsilon_2)\right), \quad (15)$$

where $\nu_b = \int_0^1 T_{jb}dj$ is the average market share of bank b in the symmetric equilibrium, $\Upsilon_1 = (\nu_b(1-\nu_b) - \sigma_b^2)$, and $\Upsilon_2 = (\sigma_b^2 + \nu_b^2)$ are constants.

Proof: See Appendix

Proposition (1) shows that for a sufficiently small shock u to the lending terms of one bank, the response of output depends on three terms. The first term measures the direct effect of the shock, abstracting from any substitution in funding markets. The drop in output will be proportional to the relevance of the affected bank $\bar{s}\nu_b$, weighted by the Frisch elasticity of labor supply $1/\phi$. When the labor supply is inelastic, the increase in the cost of funds in the aggregate will be compensated for by a fall in the aggregate wage, limiting the fall in output. The second term captures a counteracting force from the ability of the economy to subtsitute for the affected bank. Importantly, θ, the cross-bank elasticity of substitution, helps determine this second term. In a similar way, the third term captures the ability of the economy to avoid using bank credit altogether, which is determined by φ.

Although only accurate for small enough shocks, Proposition (2), shows that the response of output depends on observables, like the average bank-dependence of the real sector, $\bar{s}$, the average market share of the disrupted bank, μ_b, or the dispersion of the market shares, σ_b^2. It also depends on parameters that have been well studied in macroeconomics and other fields of economics, like the Frisch elasticity of labor supply (see Chetty et al. (2011)), the elasticity of substitution across goods (see Broda and Weinstein (2006)), or the firm-specific elasticity of labor supply (see Webber (2015)) . However, the output response also depends on two less-studied parameters: the elasticity of substitution of funding from a given bank θ, and the elasticity of substitution of bank-credit φ.. In later sections of the paper I discuss the strategy I use to recover these parameters from the cross-sectional evidence and use them to estimate the effects of an aggregate bank disruption.

2.2 The Aggregate Effects of Overall Loan Term Disruptions

Now I extend the results in Proposition (1) for a generalized disruption in the loan terms of all the banks. Proposition (2) presents the main result of this section, using Assumption (3).

Assumption 3. *Assume the lending terms of all banks are disrupted from R to Re^u, for a positive and sufficiently small u. Keep the self-finance rate equal to R.*

Proposition 2. *Under Assumption (3), up to a second order, the fall of output is given by:*

$$\log Y - \log \bar{Y} \approx \frac{1}{\phi}\left(-\bar{s}u + \varphi\bar{s}(1-\bar{s})\frac{u^2}{2}\right). \tag{16}$$

Proof: See Appendix

Proposition (2) shows that the elasticity of substitution between banks is irrelevant for the aggregate. If every bank offers the same loan terms, the elasticity to reallocating borrowing between banks is irrelevant up to second order. This does not mean that more generally, θ is an irrelevant parameter, since a more competitive banking sector where firms can move will change the behavior of banks in setting lending rates, but when lending rates are exogenous, then the substitution between banks is irrelevant when all banks are exposed to a shock.

However, the elasticity of substitution away from bank lending φ is still important through its second-order effect on aggregate output. Up to a first order approximation, the response of aggregate output is determined by observable and usual parameters.

2.3 The Cross-Sectional Elasticity of Bank-Funding Shocks

Under the conditions stated in Assumption (2), Proposition (3) characterizes the determinants of the cross-sectional differences in output with respect to a notional control firm that has zero exposure to the disrupted bank b ($Tcb = 0$).

Proposition 3. *Under Assumption (2), up to a second order approximation, the average cross-sectional effect on output of a lending rate hike of bank b from R to Re^u, with respect to the production of a firm with zero direct exposure to bank b $\bar{Y_{ct}}$, is given by:*

$$\mathbb{E}(\log Y_{jt} - \log Y_{ct}) \approx \frac{\eta\alpha}{\alpha+\eta}\left(-\bar{s}\nu_b u + \frac{1}{2}\theta\bar{s}u^2\left(\nu_b(1-\nu_b) - \sigma_b^2\right) + \varphi\bar{s}(1-\bar{s})\frac{u^2}{2}\left(\sigma_b^2 + \nu_b^2\right)\right), \tag{17}$$

where $\nu_b = \int_0^1 T_{jb}dj$ is the average market share of bank b in the steady state, $\sigma_b^2 = var(T_{jb})$ is the variance of market shares of bank b across firms, $\bar{s}$ is the credit dependence in the steady state, *and* $\log \bar{Y}$*, is the steady state level of output.*

Proof: See Appendix

From Proposition (3) we see that the effect is larger when the shocked bank is more important (ν_b is large), when firms are credit dependent ($\bar{s}$ is high), and when the elasticities of substitution between banks (θ) and away from bank-credit (φ) are low, and when it is easy to reallocate demand from one firm to another (η and α) are high. Note that low real rigidities, in the form of high values of η and α, increase the cross-sectional effects of bank disruptions.

2.4 The identification challenge

Even if we observe the cross-sectional effects on output, we would be missing equations to back out φ, which is the relevant variable for understanding the aggregate effects of an overall shock. In particular, many combinations of θ and φ can produce the same cross-sectional patterns.

3 Identification

In this section I use the model to illustrate how the patterns in the data identify θ and φ, the key parameters of the model. I use the insight in this section to estimate the full model I introduce in the following section. I start by introducing two cross-sectional estimates used in the literature: first, the elasticity of credit after a bank shock; second, the elasticity of a firms outcome, usually employment or value added.

I will start by introducing two cross-sectional estimates used in the literature. First, the elasticity of credit after a bank shock. Second, the elasticity of a firm outcome, usually employment or value added.

3.1 The Elasticity of Firm Production

The elasticity of firm production to a disruption in the terms of loans of bank b in the line of the experiment in the previous section can be estimated through the following regression:

$$\Delta \log \mathrm{Y}_f = \beta_0 + \beta_{output} T_{jb} + \epsilon_f, \tag{18}$$

where Δ is the difference operator between a pre-period, that I will assume to be equal to the symmetric equilibrium of the model, and pos-period, when a shock of size u that increases the interest rate of bank b from R to Re^u occurs. The independent variable is the pre-existing exposure of firm j to bank b, measured by T_{jb}.

T_{jb}, the pre-existing borrowing share of firm j with bank b is exogenous and given by historical reasons as in Huber (2018), or assumed to be endogenous but instrumented as in Chodorow-Reich (2014). The main empirical concern is that banks that are more prone to receiving funding shocks are also more likely to pick bad firms, which would induce a correlation between lending and firm outcomes even in absence of a causal link. The empirical literature has addressed that problem by using an instrumental variables (IV) approach to deal with selection. The idea is to find variation in the ability of some banks to give out loans that is not correlated with the quality of the firms they lend to.

The elasticity of production with respect to pre-existing exposure is characterized in Proposition (4)

Proposition 4. *Under assumptions (1) and (2), the regression coefficient of a regression of firm-level output growth on the pre-existing exposure, accurate up to a second-order, is given by the following expression*

$$\beta_{output} = -\frac{\eta\alpha}{\alpha+\eta}\bar{s}u\left(1 - \theta\frac{u}{2}\mathcal{M}_1 - \varphi(1-\bar{s})\frac{u}{2}.\mathcal{M}_2\right). \tag{19}$$

For constants $\mathcal{M}_1 = \left(1 - \frac{cov(T_{jb}^2, T_{jb})}{var(T_{jb})}\right) > 0$ *and* $\mathcal{M}_2 = \left(\frac{cov(T_{jb}^2, T_{jb})}{var(T_{jb})}\right) > 0$

Proof: See Appendix

Equation (19) makes clear that up to a second order, as the elasticity of substitution across banks (θ) and the elasticity of substitution away from bank credit (φ) increase, the firm-level effects on output of a bank disruption become smaller. On top of the frictions in the banking sector, the structure of the goods market (η), and the structure of labor markets α determine the cross-sectional effects of the bank disruption. When α tends to infinity, the cross-sectional effects tend to η. When both α and η tend to infinity, the cross-sectional effects diverge, since in this situation all production would take place in the firms with the lowest marginal costs. The distinction that the elasticities of substitution φ and θ have second order effects on the elasticity of output is important for this paper.

3.2 The Elasticity of Firm Borrowing

Instead of analyzing the cross-sectional effects on output, the next regression studies the effects of the bank disruptions of firm-credit. There is some variation in the specification of the regression in the literature, but we will use the following specification:

$$\Delta \log \text{Loans}_j = \beta_0 + \beta_{credit} T_{jb} + \epsilon_j, \tag{20}$$

where Δ is the difference operator between a pre-period, which I assume to be equal to the symmetric equilibrium of the model, and pos-period, when a shock of size u that increases the interest rate of bank b from R to Re^u occurs. The independent variable is the pre-existing exposure of firm j to bank b, measured by T_{jb}. Gan (2007), Khwaja and Mian (2008), Schnabl (2012), and Iyer, Peydro, da Rocha Lopes, and Schoar (2013), among others are examples of this approach.

Proposition 5. *Under assumptions (1) and (2), the regression coefficient of a regression of firm-level output growth on the pre-existing exposure, accurate up to a second order, is given by the following expression*

$$\beta_{credit} = \beta_{output} \frac{\alpha + 1}{\alpha} - \varphi(1-s)u \left(1 + \varphi \frac{u}{2} s \mathcal{M}_1 - \theta \frac{u}{2} \mathcal{M}_2\right). \tag{21}$$

For constants $\mathcal{M}_1 = \left(1 - \frac{cov(T_{jb}^2, T_{jb})}{var(T_{jb})}\right)$ *and* $\mathcal{M}_2 = \left(\frac{cov(T_{jb}^2, T_{jb})}{var(T_{jb})}\right)$

Proof: See Appendix

Proposition 5 shows that on top of the effect on output times a multiplier (first term), there is a first order effect of the elasticity of substitution of bank credit φ on firm credit. When firms are more elastic in substituting away from bank credit, credit falls by more.

3.3 Identification Argument

The elasticity of credit becomes larger (more negative) when φ is larger and when θ is smaller. The elasticity of output becomes larger when both φ and θ are smaller. Therefore it is possible to back out the values of these two coefficients once we take a stance on the other coefficients that determine the cross-sectional elasticities.

The identification argument is represented in Figure (4). The figure presents two locus of points in the space φ - θ, which produce a given estimate for the elasticity of credit and production, after taking a stance on the other parameters of the economy.

Start by placing yourself on point b_1, in the locus of β_{loan}. Now arbitrarily increase the value of φ. Since a larger φ causes the elasticity of output to be larger in absolute value, in order to keep the elasticity constant we must move θ in a direction that compensates for the change in φ. That is, we need to make θ larger, making firms more elastic with respect to a given bank such that they do not move away from bank credit by much. This argument implies that the locus of points (φ and θ) that keeps the regression coefficient β_{loan} constant is upward sloping.

Now place yourself on top of point a_1 on the locus of β_{prod}. Once again move to a larger value of φ. When firms are more elastic to substitute bank credit, the elasticity of production becomes smaller in absolute value. In order to keep its value constant, we need firms to be less able to switch from the affected lender, making θ smaller. Therefore, the locus of points is downward sloping.

This means that it is possible to find a point such as d_1, where the two loci intersect, satisfying both cross-sectional elasticities.

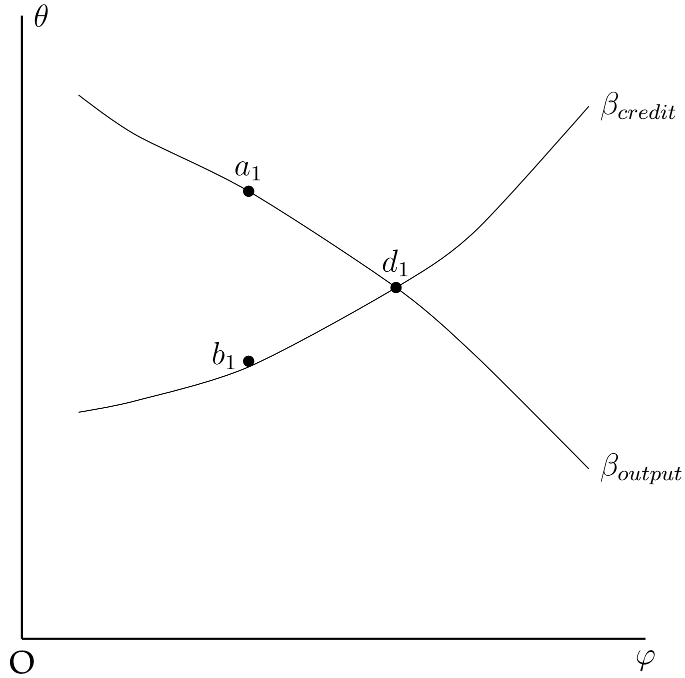

Figure 4: Identification argument for φ and θ. The figure plots the locus of points (φ and θ) that achieve a given value for β_{output} and β_{credit} after taking a stance on the other parameters that influence the values of the statistics. The intersection of the two loci gives the value of θ and φ

4 Full Model

In this section I embed the simple model in a consumption/savings model in order to make the total amount of deposits endogenous, and let banks set lending rates as a response to balance sheet disruptions. The basics of the model are the same as in the simple model, and here I only present the new blocks of the model.

Time is continuous. Space is contained in a $[0, 1]$ interval. $N_{\mathcal{B}}$ banks are uniformly spaced in this interval. Firms are distributed uniformly over space. I take as primitive of the model the closeness of firm j to bank b, and denote it by T_{jb} as in the simple model. I take the stance that Ti is a vector of size B 1, which specifies I take as primitive of the model the closeness of firm i to bank b, and denote it by T_{ib} as in the simple model. T_j, a vector of size $N_{\mathcal{B}} \times 1$ specifies the closeness of firm j with each bank, and given by $T_{j,b} = \max\{1 - \bar{d} \times d_{j,b}, 0\}$ where $d_{j,b}$ is the distance between firm j and bank b, and $\bar{d}$ is a constant that determines how the distance between a firm-bank pair affects the ease of creating banking relationships. In the extreme where $\bar{d} = 0$, firms are equally likely to borrow from banks regardless of their distance. When $\bar{d}$ increases, firms only use banks that are close to them.

Each firm is owned by an entrepreneur, with utility function $u(c_{it}) = \frac{c_{it}^{1-\gamma}}{1-\gamma}$. Each entrepreneur solves the following problem:

$$\max_{c} \mathbb{E}_0 \int_0^\infty e^{-\rho t} u(c_{it}) dt.$$

They maximize utility subject to the budget constraint:

$$\dot{a}_{it} = r_{it}^d a_{it} + \pi_{i,t}^* - c_{it}$$

That is, entrepreneurs earn interest income at rate r_{it}^d on their wealth a_{it}, earn profits $\pi_{i,t}^*$, and consume c_{it}.

The effective rate of deposits r_{it}^d is a weighted average of the deposit rates at different banks $r_{dt} = \sum_k \omega_{kt} r_{dkt}$. And weights given by $\omega_{bt} = \frac{R_{bdt}^{\chi}}{\sum_k R_{kdt}^{\chi}}$. This functional form for the deposit shares is chosen to be symmetric with the way that firms allocate their loan demand across banks.

Profits are given by $P_{jt}Y_{jt} - w_{jt}L_{jt}R_{jt}$, prices are given by $\frac{\eta}{\eta-1}MC_{jt}$, and marginal costs are given by $MC_{jt} = \frac{w_{jt}}{z_{jt}}R_{jt}$.

4.1 Banks

Banks compete by setting rates.[7] Banks understand the structure of demand of each firm, but do not internalize the aggregate consequences of their actions. That is, banks take the aggregate wage and aggregate output as given, but they understand that firms can substitute towards other banks, or substitute away from bank credit, and that firm optimal scale is decreasing in its cost of funds. I allow for banks to price-discriminate across firms.

Banks compete by setting rates. They understand the structure of demand of each firm, but do not internalize the aggregate consequences of their actions. That is, banks take the aggregate wage, and aggregate output as given, but they understand that firms substitute to other banks, that firms substitute away from bank credit, and that firm-level scale is decreasing in the cost of funds that they face. I allow for banks to price discriminate across firms.

The profits that bank b gets from its relationship with firm j are:

$$\Pi_{jb} = w_j L_j s_j \nu_{jb}(R_b - R_{bd}).$$

I am saving on the notation by eliminating the time subscript.

The first order condition

$$R_{bj} = R_{bd} \frac{\tilde{\theta}_{jb}}{\tilde{\theta}_{jb} - 1}$$

For $\tilde{\theta}_{jb} = \theta + \nu_{jb}(\varphi - \theta) + \left(\eta \frac{1+\alpha}{\alpha+\eta} - \varphi\right) \nu_{jb} s_j$,

characterizes the optimal pricing of the loans for each bank.

A bank with zero mass ($\nu_{jb} \rightarrow 0$) faces an elasticity of substitution θ, the elasticity at which firms switch banks. A monopolist bank ($\nu \rightarrow 1$)that lends to firms that are fully dependent on bank credit ($s \rightarrow 1$), faces an elasticity of substitution $\eta \frac{1+\alpha}{\alpha+\eta}$, the elasticity

[7] In this model I assume that the loan market is cleared using lending rates. In the data, banks offer multi-dimensional contracts that differ in terms, covenants, size of credit lines, on top of variation on prices. Some of this variation has been has been covered by Payne (2018) and Chodorow-Reich and Falato (2017).

at which higher costs translate into lower firm scale and correspondingly to lower loan demand. The elasticity is positive since φ,η, and θ are positive, and ν_{bj} and s_j are between zero and one. Banks charge variable markups. This is an important departure from models with constant elasticities of substitution.

The balance sheet of the bank is given by:[8]

$$\text{Loans}_{bt} = \text{Deposits}_{bt} + \text{Equity}_{bt} \tag{22}$$

Loans granted by a bank are the integral of the loans given to each firm in the economy, given by:

$$\text{Loans}_{bt} = \int_0^1 \text{Loans}_{jbt} dj = \int_0^1 s_{jt} w_{jt} L_{jt} \nu_{bjt} dj. \tag{23}$$

Similarly, deposits are equal to the integral of the deposits that the bank gets from all entrepreneurs in the economy

$$\text{Deposits}_{bt} = \int_0^1 \text{Deposits}_{jbt} dj = \int_0^1 \omega_{bt} a_{jt} dj. \tag{24}$$

I assume that Equity_{bt} is exogenous, and that banks are owned by agents outside the economy. It is simple to change that assumption on the ownership of the banking sector.

The supply of deposits at a given bank depends positively on its deposit rate, while the demand for loans depends negatively on it, through its negative relationship with the lending rate and the positive relationship between lending and deposit rates. Therefore, after a decrease in the right-hand side of the balance sheet, the bank will respond by increasing the deposit and lending rates accordingly, balancing out its balance sheet again.

The aggregate state vector is $S = (\mathbf{Equity}, X)$, where $\mathbf{Equity}$ is a $K \times 1$ vector of the equity of each of the K banks in the economy, and X is the distribution of entrepreneurs over their individual state-space $\varsigma = (z, a, \mathbf{T})$, where T is a $K \times 1$ vector that represents the

[8] In the model, I interpret Equity_{bt} as another source of funding for the bank that is different than deposits. It could well be thought of as a generic source of funding.

demand shifters for each entrepreneur's firm with respect to each bank in the economy.

1. Entrepreneur's optimization. Taking $w(s)$, $R_k(s)$, $R_k^d(s)$ as given, entrepreneurs maximize utility and their firms maximize profits.

2. Household problem. Taking $w(s)$ as given, households maximize utility

3. Banks problem. Taking $R_k^d(s)$, banks set R_k to maximize profits.

4. Market Clearing. $w(s)$, $R_k^d(s)$, are such that labor market clears $L^s = \int l(z, a, \mathbf{T})X(dz, da, d\mathbf{T})$, and banks' balance sheet holds $Deposits_k(s) + Equity_k s) = Loans_k(s)$

4.2 Solution Method

Since there are only a handful of banks, a shock to the financial conditions of a bank will create aggregate disturbances. Therefore, when agents are formulating their policy functions, they need to forecast the behavior of the input prices in the economynamely, the wage rate and the deposit rate at each bank. In order to do so, agents need to forecast the behavior of the cross-sectional distribution of entrepreneurs and banks, which is an infinite-dimensional object. I take advantage of methods developed byAhn, Kaplan, Moll, Winberry, and Wolf (2018). In particular, the solution will be globally accurate with respect to the individual state space, and will be a linear approximation with respect to the aggregate shocks.

5 Estimation

The parametrization of the model takes two steps. The majority of the parameters are calibrated. Most of these parameters are well studied and I fix them at standard values. I use microdata to calibrate a subset of parameters that are not widely used in macroeconomic models but for which we have good evidence. Then, the key parameters of the model, θ

and φ, are estimated to target the patterns observed in cross-sectional studies of the bank lending channel that were introduced in the previous sections.

I offer a preview of the results of this section. In my benchmark calibration, the values of θ and φ I estimate are low, implying low ability to adjust to bank shocks. As an illustration, Under an alternative specification of the labor market, (high α), the values of θ and φ that are consistent with the cross-sectional elasticities are large.

On top of evidence from labor economics that advocates for an economy with a low α, I use an additional cross-sectional moment from the banking literature as a sanity check. I extend the model to have two symmetric regions. In models with flexible labor markets within the region (α is high), the indirect effects of bank shocks are positive. This means that a firm without exposure to a shocked bank in a region where the average exposure to the troubled bank is high will outperform an unexposed firm in a region where the average exposure to the troubled bank is low. This prediction is at odds with the evidence, as Huber (2018) has documented. Only when there are substantial rigidities in local labor markets, the model is consistent with the sign of the indirect effect. Therefore, the model rejects the limit of high α, consistent with the micro evidence from labor economics.

5.1 Calibration of Standard Parameters

Table (1) lists the parameters that I fix throughout the estimation. The intertemporal elasticity of substitution is set to a standard value of 1/2. The Frisch elasticity of labor supply is 0.75, as suggested by Chetty et al. (2011). This value is significantly lower than what is used in most macro models. A highly elastic labor supply will increase the aggregate effects of a bank shock, by making it more difficult for wages to go down after a negative shock, increasing the elasticity of output to bank funding shocks.

I set η, the elasticity of substitution across goods equal to 4, within the range of estimates in Broda and Weinstein (2006). I set the discount rate ρ equal to 0.03 per year as in Itskhoki and Moll (2019). I set the persistence of the shock ρ_E at 0.95, consistent with

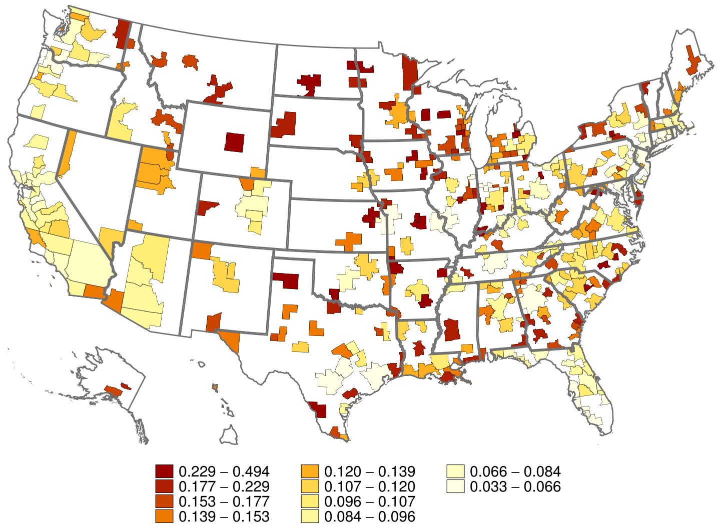

Figure 5: HHI Index for C&I Loans. Data: FFIEC. Author's calculation

the persistence used by Gertler and Kiyotaki (2015). I set the parameters of the productivity Poisson process to target the volatility of 0.056 and a persistence of 0.9 as chosen by Winberry (2018).

I set the number of banks in the economy $N_{\mathcal{B}}$ equal to 10 equal-sized banks. This number replicates the across-MSA[9] median Herfindahl-Hirschmann Index (HHI) of 0.11 coming from data from the Community Reinvestment Act (CRA) data that report business loans for 2006 in the U.S.. Figure (5) presents the dispersion in the HHI index of business and commercial loans for each MSA during 2006. Specifically, the HHI index equals $\sum_i \text{market share}_i^2$, the sum of the squares of the market shares of each bank in a given MSA. I find the number of equal-sized banks that would replicate the median HHI. This number is $\frac{1}{HHI}$. As an alternative, using call reports data at the national level, the HHI of commercial and industrial loans (C&I) for 2006 is 0.05, implying 20 equal-sized banks. However, this number underestimates the degree of concentration in C&L loans, since firms prefer banks that are closer to them (see Nguyen (2019)), and banks are concentrated in specific geographical regions. The parameter d, controls how many banking relationships each firm will have. I fix d so that firms have three banking relationships, as

[9]Metropolitan Statistical Area

Parameter	Description	Value
$1/\gamma$	Intertemporal Elasticity of Substitution	1/2
ρ	Discount Rate	0.03
η	Elasticity of Substitution - Goods Market	4
$1/\phi$	Frisch Elasticity of Labor Supply	0.75
z	Two-State Markov Process	0.9 - 1.1
λ	Intensity of Poisson productivity shock	1/3
$\mathcal{B}$	Number of Banks in the Economy	10
ρ_E	Persistence of Equity Shock	0.95
d	Distance Coefficient	3 bank relationships
χ	Elasticity of deposits to deposit rates	5

Table 1: Fixed Parameters Values: The table presents the parameters of the model that I calibrate.

reported by Huber (2018). I set χ, the parameter that governs how much deposits flow out of a bank with lower deposit rates to 5, matching the semi-elasticity reported by Drechsler et al. (2017).

5.2 Estimation of Key Parameters

Using the relative effects in the data as target moments to estimate the full model, I structurally estimate the parameters values for θ, the elasticity of substitution of firms across banks, and φ, the elasticity at which firms switch away from bank credit. The idea behind the identification is the same as exposed in the identification section, with the difference that the full model gives dynamics to simulate a simulated panel dataset, and that the model is globally accurate with respect to individual policy functions, which are more accurate than the second-order Taylor expansions we introduced before. Specifically, I simulate a panel of firms over time after a bank funding shock. With the simulated data, I run a regression analysis that replicates the cross-sectional analysis, after collapsing a set of periods before and after the shock into two bins, the pre-period and the post-period. Table (6) specifies the microeconomic targets of the calibration. For a detailed discussion of the regressions behind these moments, please refer to the identification section.

Moment	Source	Value
Bank Credit Elasticity	Huber (2018)	-0.166
Output Elasticity	Huber (2018)	-0.044

Table 2: Microeconomic Targets: Each entry specifies the target for two microeconomic moments.

5.3 Sensitiviy of Cross-Sectional Elasticities to Structural Parameters

Before showing the estimation of the model, I illustrate the effect of θ and φ in determining the cross-sectional moments and the effect of different values of α in shifting the effect of these two parameters.

Figures (6) and (7) show the effect of changing θ for two values of α, on the cross-sectional moments of credit and production, respectively, while keeping the rest of the parameters in the model fixed. As is intuitive from previous sections, a higher value of θ, by increasing the flexibility of firms on switching across banks, decreases the cross-sectional elasticities of both output and credit. In the limit, where $\theta \to \infty$, the elasticities tend to zero. Figures 6 and 7 make an additional point. Because the elasticity is larger in absolute value when labor markets do not have any frictions, the value of θ that is consistent with a given elasticity is significantly larger when $\alpha \to \infty$ than when α is low. Therefore, in order to match the same cross-sectional elasticities, θ will be lower in an economy with labor market frictions.

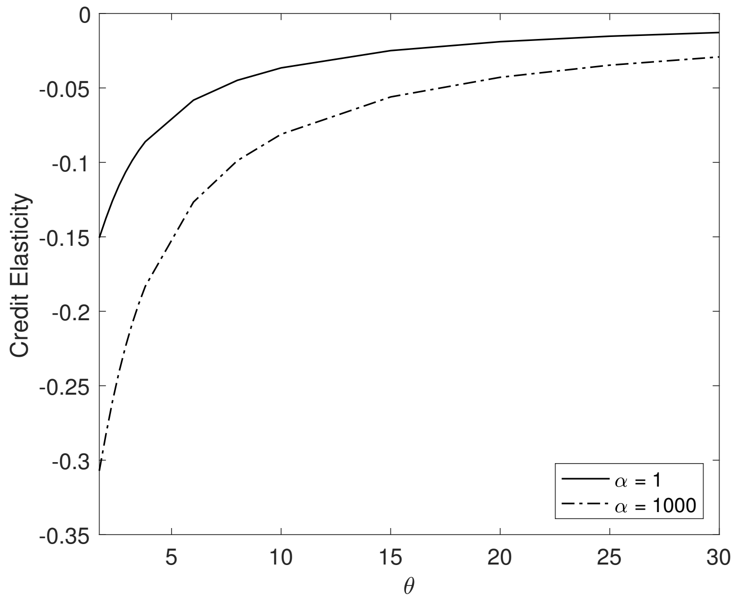

Figure 6: Effect of θ in the cross-sectional elasticity of credit for two levels of α. This figure shows the cross-sectional elasticity of credit in response to a bank shock for different values of θ, the elasticity of substitution of funding across banks. I conduct this exercise for two different values of α: first for a market with $\alpha \to \infty$, and second, for a low level of α when there are substantial difficulties in moving labor across firms.

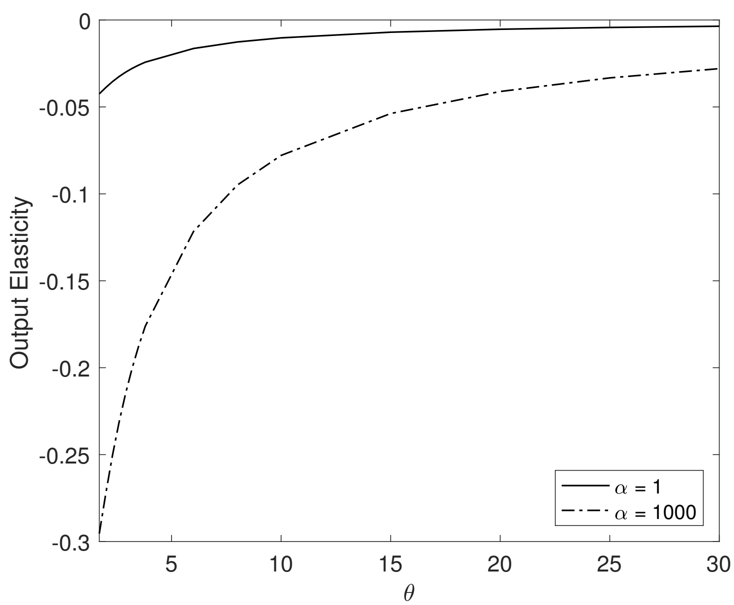

Figure 7: Effect of θ in the cross-sectional elasticity of output for two levels of α. This figure shows the cross-sectional elasticity of credit in response to a bank shock for different values of θ, the elasticity of substitution of funding across banks. I conduct this exercise for two different values of α: first for a market with $\alpha \to \infty$, and second, for a low level of α when there are substantial difficulties in moving labor across firms.

Figures (8) and (9) perform the same exercise for the elasticity at which firms move away from bank credit (φ). These figures show that the identification argument holds beyond the second order approximation we did in the simple model. When φ increasesthe output effects of the shock are smaller, but the credit effects of the same shock are larger.

With respect to α, Figure (9) shows that for frictionless labor markets, the value of φ that is consistent with a given elasticity is higher than for markets with frictions. The intuition for this result is the same as for the results that involved θ. Under a frictionless labor market, the cross-sectional effects are larger since it is easier to move labor across firms. In the case of Figure (8), when α is larger, which increases the losses of a given shock, firms move away from credit by more, explaining why the schedule of $\alpha = 1000$ is below from the schedule for $\alpha = 1$.

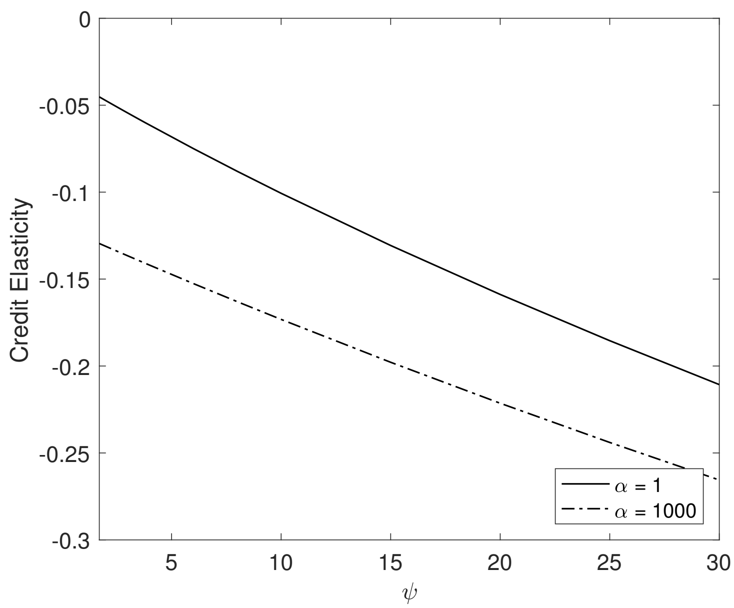

Figure 8: Effect of φ in the cross-sectional elasticity of credit for two levels of α: This figure shows the cross-sectional elasticity of credit to a bank shock for different values of φ, the elasticity of substitution from bank credit. I conduct this exercise for two different values of α. First for a frictionless labor market, where $\alpha \to \infty$. And second, for a low level of α when there are substantial frictions in the labor market.

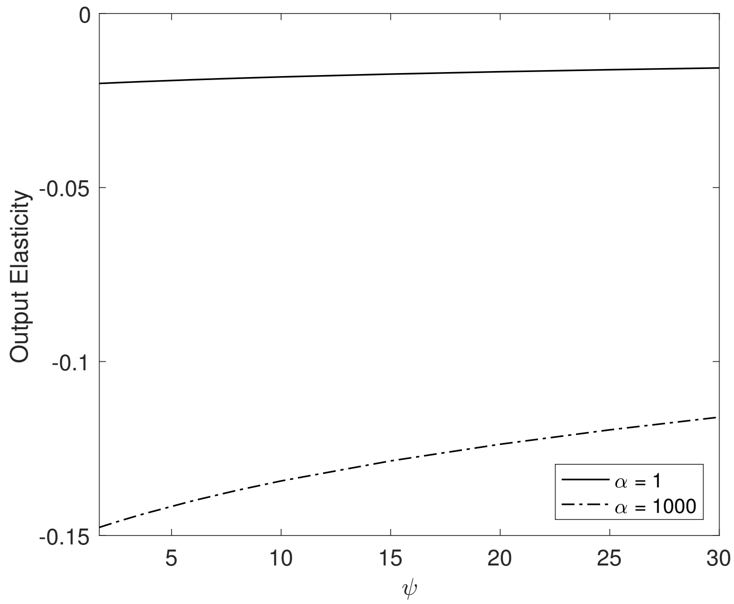

Figure 9: Effect of φ in the cross-sectional elasticity of output for two levels of α: This figure shows the cross-sectional elasticity of output to a bank shock for different values of φ, the elasticity of substitution from bank credit. I conduct this exercise for two different values of α. First for a frictionless labor market, where $\alpha \to \infty$. And second, for a low level of α when there are substantial frictions in the labor market.

6 Estimated Parameters

In this section I report the combination of θ and φ that match the values of the observed moments as reported in Table . I report the values that fit the cross-sectional moments in models where $\alpha = 1$ and $\alpha \to \infty$, with the purpose of showing that the estimated structural parameters are vastly different depending on the assumed structure of the labor market.

Parameter	Description	Value ($\alpha = 1$)	Value ($\alpha = 1000$)
θ	Substituability Across Banks	1.5	6.5
φ	Inverse credit Dependence	4.5	20

Table 3: Estimated Elasticities of Substitution

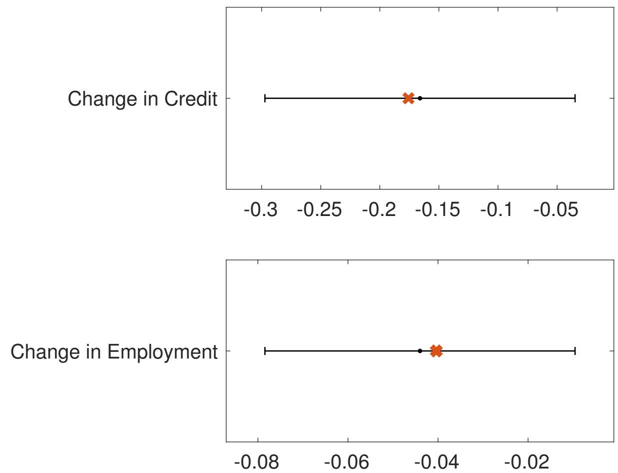

Figure 10: Targeting of cross-sectional moments - $\alpha = 1000$: This table shows the point estimate for each cross-sectional moment provided in Huber (2018), with 95 percent confidence interval bounds. The x mark shows the fit of the model.

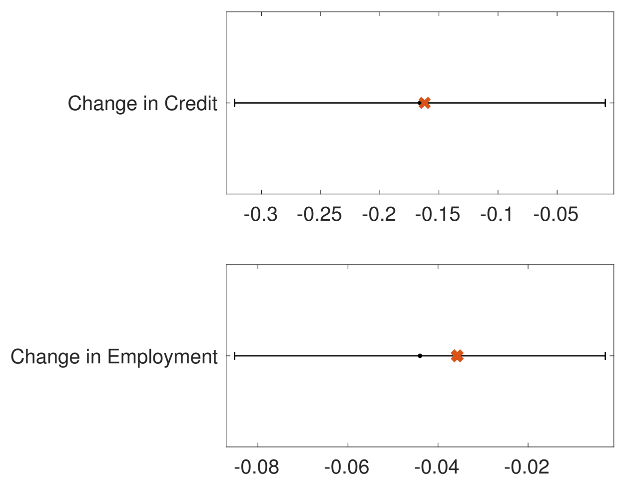

Figure 11: Targeting of cross-sectional moments - $\alpha = 1$: This table shows the point estimate for each cross-sectional moment provided in Huber (2018), with 95 percent confidence interval bounds. The x mark shows the fit of the model.

The estimated parameters in Table (3) led me to reject that firms and banks operate in markets of perfect substituability, which is the limit of $\theta \to \infty$ and $\varphi \to \infty$. The numbers in the table alone do not tell us quantitatively, how important are deviations from perfect substituability, an answer that I provide in the next section.

Table (3) makes clear the importance of the structure of the labor market. Under frictionless labor markets, the parameters are larger, implying that firms are more flexible in reacting to a bank shock. Therefore the effects of bank shocks will be lower.

We have shown how α, the parameter that governs the extent of frictions in the labor market, is important in this model. The reason is that the extent of real rigidities in the model change the extent to which demand and inputs can be reallocated across firms. When there are substantial frictions in reallocating labor across firms, the model requires substantial frictions in banking as well, in order to match the cross-sectional moments. On the other side, with frictionless labor markets, the banking sector must be relatively flexible, or the model would predict cross-sectional elasticities that are larger than the ones observed in the data. The question becomes how to distinguish across values of α. I use two sources of evidence: direct evidence on the value of , and indirect evidence showing that additional cross-sectional patterns in the banking sector reject the case of labor markets with low frictions.

In particular, I Webber (2015) document an inelastic firm-specific labor supply. This evidence has already been used in the literature by Chodorow-Reich (2014), and I show that in a more flexible model with flexible patterns of substitution of firm funding, the extent of these frictions is still important. I also use an additional cross-sectional moment, the *indirect effects* of bank lending cuts, to distinguish across models. The indirect effects measure how a firm without direct exposure to the shocked bank that operates in a region where other firms are highly exposed behaves with respect to another firm without direct exposure to the troubled bank that operates in a region where firms are not highly exposed to the troubled bank. Huber (2018) reports that the indirect effects of bank-lending cuts are negative. This means that unexposed firms in exposed regions underperform unexposed firms in unexposed regions.

I extend the model to illustrate the behavior of the indirect effects. Specifically, I extend the model to have 2 symmetric regions. The regions are segmented in the markets for goods and labor. That is, each firm produces non-tradeable goods, and people cannot move across regions. However, there is partial financial integration. Lending relationships are determined by distance, regardless of geographical barriers. Therefore, firms may borrow from banks in their home or a foreign region, but must sell their products and hire their workers in the local region. As before, the extent to which workers can move across firms *within the same region* is given by the parameter α:

$$\Delta \log Y_{jr} = \beta_0 + \beta_1 \nu_{jr,pre} + \beta_2 \nu^{-}_{jr,pre} + \epsilon_{jr}. \tag{25}$$

Equation (25) presents the regression we will run to get the reduced-form indirect effects. The dependent variable is the log change of an outcome of interest (in this case output) of firm j located in region r, and the right-hand-side variables are the pre-existing lending relationship of the same firm and the average exposure of the firms in region r. β_2 is the coefficient of interest; it captures the change in outcomes of a firm with $\nu_{jr,pre} = 0$ in a region where the average exposure is complete $\nu^{-}_{jr,pre} = 1$, with respect to a firm with zero direct exposure $\nu_{j-r,pre} = 0$ in a region $-r$ where the average exposure is also zero $\nu^{-}_{jr,pre} = 0$.

To give a clear sense of the effect of α in the model, I show the effect of different values of this parameter on the three cross-sectional patterns I have documented so far: the elasticity of credit, the elasticity of output, and the indirect effects. In order to provide a clean intuition, I fix all the other values of the parameters at arbitrary values, including θ and φ. This approach is in contrast to the previous results where I estimated φ and θ for different values of α.

Figures (12) and (13) illustrate an argument that is familiar by now. When labor markets exhibit less frictions, the direct cross-sectional effects increase in absolute value. This happens because the wedge between marginal costs between firms with and without exposure to the shock increases. As a consequence, the wedge between prices, production, and credit demand increases as well.

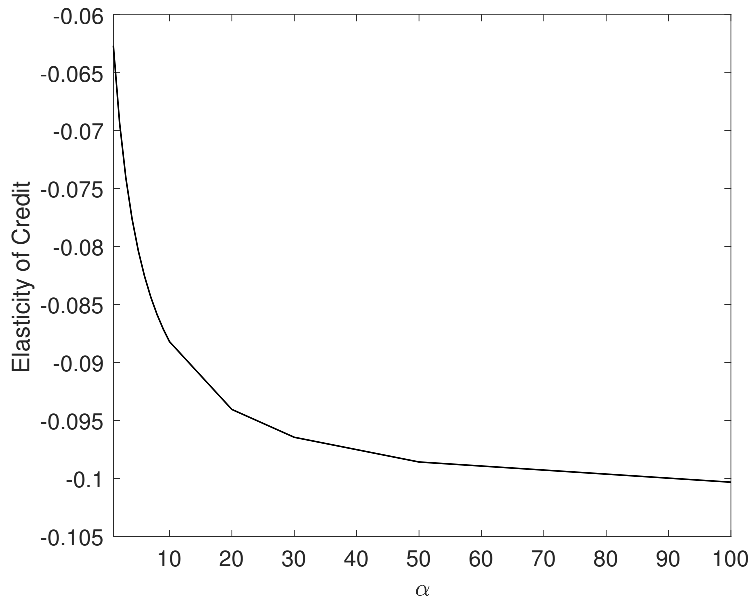

Figure 12: Sensitivity of the cross-sectional effects on credit of an idiosyncratic bank shock to α : This Figure shows the cross-sectional effect on credit to a bank shock for different values of α, the extent of frictions in the labor market. All the other parameters are fixed in their calibrated values, except θ and φ which are fixed in an arbitrary level of 5. The qualitative properties of the figure do not depend on this choice

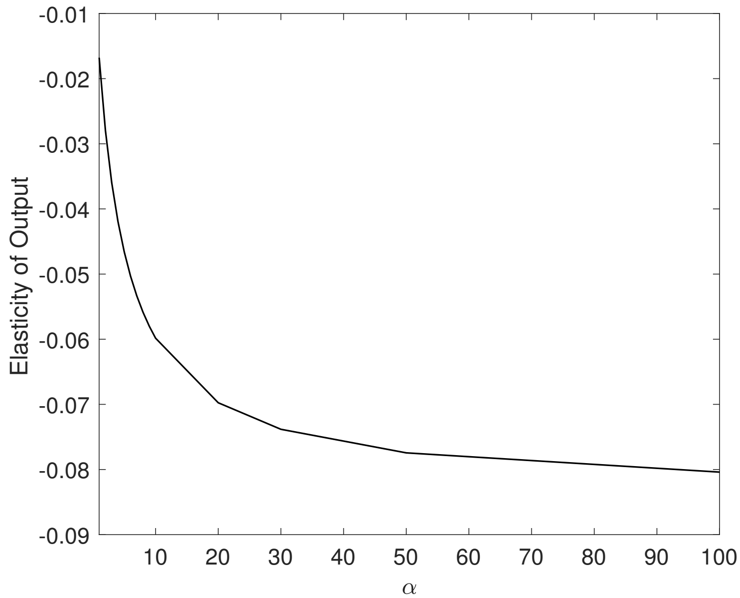

Figure 13: Sensitivity of the cross-sectional effects on output of an idiosyncratic bank shock to α. This Figure shows the cross-sectional effect on ouput to a bank shock for different values of α, the extent of frictions in the labor market. All the other parameters are fixed at their calibrated values, except θ and φ which are fixed in an arbitrary level of 5. The qualitative properties of the figure do not depend on this choice.

Figure (14) plots the indirect effects of the lending shock for different values of α. The figure makes clear that as labor markets become more efficient, the indirect effects of a lending shock become more positive. That is, an unexposed firm in an exposed region experiences a outperforms an unexposed firm in an exposed region. On the contrary, Huber (2018) reports that firms in exposed regions underperform unexposed firms in exposed regions. Although the confidence intervals on the indirect effects reported by Huber (2018) are wide, they reject positive values of the indirect effects, which means that the model rejects values of α greater than 1.

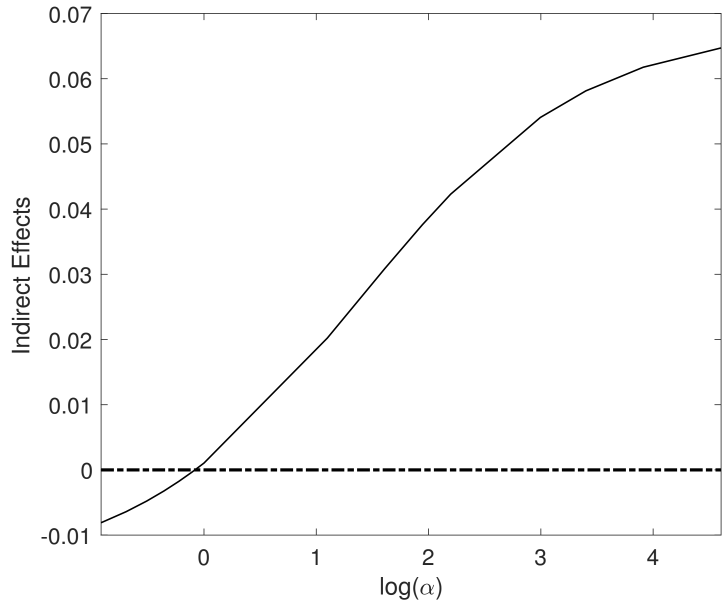

Figure 14: Sensitivity of the indirect effects on credit of an idiosyncratic bank shock to α. This figure shows the indirect effects of a bank shock for different values of α, the extent of frictions in the labor market. All the other parameters are fixed at their calibrated values, except θ and φ , which are fixed at an arbitrary level of 5. The qualitative properties of the figure do not depend on this choice.

The insight that the model rejects perfectly competitive labor markets by using the indirect effects is key in the estimation of the aggregate effects of bank shocks. As Figure (14) shows, only values of $\alpha < 1$ can rationalize negative indirect effects. Therefore, we can reject the limit of frictionless labor markets, and with it, the small elasticities of output to lending they entail.

7 Discussion

7.1 The Aggregate Effects of Bank Supply Shocks

In this section I analyze the aggregate effects of a cut in the supply of bank lending. In particular I compute the ratio between the integral of the discounted value of aggregate output drops over the integral of the discounted value of the funding shock. Formally, I compute an elasticity ε^M as follows:

$$\varepsilon^M = \frac{\int_0^T e^{-\rho t}\left(\log(Y_t) - \log(\bar{Y})\right) dt}{\int_0^T e^{-\rho t}\log(\text{Lending}_t) - \log(\bar{\text{Lending}}) dt}). \tag{26}$$

The reason to compute the elasticity of output to lending in this way is that output may exhibit different persistence than total lending, and that the shock that is feeding the economy is persistent, inducing additional responses in output and lending beyond the response on impact. Note as well that the elasticity is computed with respect to lending, not with respect to the shock. There are two reasons for this. First, the policy-relevant variable is the reduced ability of banks to make loansor to put it another way, the drop in the right-hand-side of the balance sheet of the banking sector. Second, this definition admits comparisons with back-of-the-envelope aggregations that cross-sectional studies make by abstracting from general equilibrium effects.

ε^M should be interpreted as the elasticity of output to lending caused by a shock in the supply of bank lending. It is the macroeconomic equivalent of an instrumental variables (IV) specification. In an IV, we compute regressions between two endogenous variables, and find an instrument that affects the right-hand-side variable (lending in this case), and that only affects the dependent variable (aggregate output), through its effect on lending.[10]

The result of this section is an estimation of this elasticity, and I will show the sensitivity of the elasticity for both experiments with respect to the key parameters of the model. As before, we will consider results for two extreme values of , the extent of rigidities in the labor market.

7.2 The aggregate effects of an aggregate bank shock

We start by performing an experiment in which every bank in the economy is shocked at the same time. This experiment is interesting for several reasons. One, this type of shock

[10]Computing an elasticity between two endogenous variables in macroeconomics is commonplace. The Phillips Curve slope for instance is the elasticity of inflation to unemployment caused by a demand shock. Interest rate parities relates exchange rates to interest rate differentials.

Calibration	$\alpha = 1$	$\alpha = 1000$
Benchmark (%)	19.63	6.73

Table 4: Elasticity of Aggregate Output to Aggregate Bank Lending: This table shows the elasticity of output to lending to bank lending. Each column shows the elasticity of output to bank lending for two assumptions of the labor market. One where there are meaningful frictions in the labor market ($\alpha = 1$), and for a case where labor markets are frictionless.

captures the attention of macroeconomists and policy experts. Second, it speaks to situations without meaningful heterogeneous exposure to the shock, where using the cross-section to estimate effects is implausible. However, we will inquire how the knowledge of the structural parameters we gained from the cross-sectional estimates extrapolates to an aggregate shock.

Figures (15) and (16) show the effects of the key parameters, φ and θ, in determining the output effects of an idiosyncratic shock. The x-axis of these figures is the value of one parameter, and the y-axis is the elasticity of aggregate output to aggregate lending after an aggregate bank shock. The solid line shows the preferred case when $\alpha = 1$, and the dashed line shows the case of frictionless labor markets, when $\alpha \to \infty$. The marker in each line shows the estimated value of the parameter for each case.

Figure (15) shows that higher values of φ, which decrease the extent of financial frictions, diminishes the elasticity of output to lending. Under frictionless labor markets, the estimated parameter of 20, implies that the elasticity of output to lending is one third the elasticity estimated when there are meaningful frictions in the labor market. The solid and dashed line are over the other for two indicating that other than φ, no other parameters that differ across the two parametrizations of the model α or θ change the size of the elasticity.

On the other side, Figure (16) shows that θ is not quantitatively relevant for determining the aggregate elasticity since the lines are flat around the estimated values. This is true even when θ is relevant at determining the cross-sectional responses, as shown in previous sections. This result indicates that irrespective of the value of θ, the response of output to lending is the same. It does not mean that θ is irrelevant in the aggregate. To think about this issue it is useful to remember that the elasticity of output to lending is equal to the elasticity of output to the shock, divided by the elasticity of lending to the shock. The flatness of the elasticity of output to lending indicates that the behavior of lending follows the same pattern.

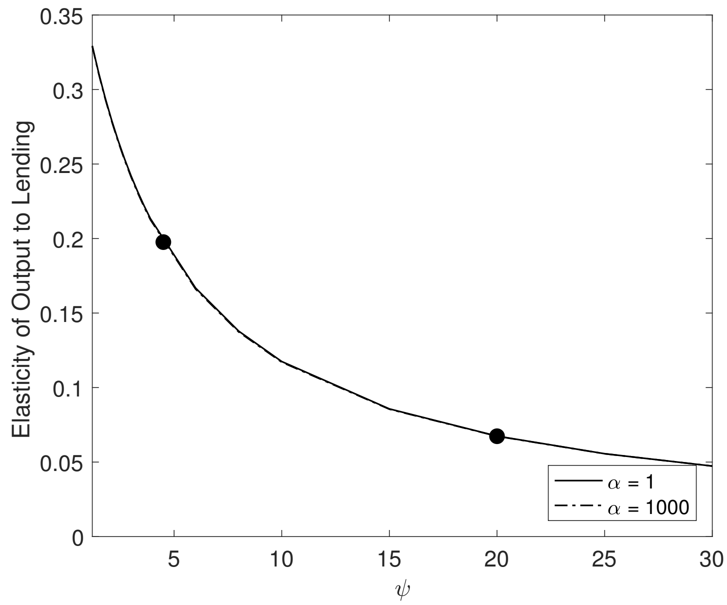

Figure 15: Sensitivity of the aggregate effects of an aggregate bank shock to φ : This Figure shows the aggregate output drop after an idiosyncratic bank shock for different values of φ, the elasticity of credit dependence. We perform this exercise for two different values of α. First for a frictionless labor market, where $\alpha \to \infty$. And second, for a low level of α when there are substantial frictions in the labor market. All the parameters are fixed in their calibrated or estimated values except for φ. The dot on each line represents the estimated value for φ and the correspondent output drop.

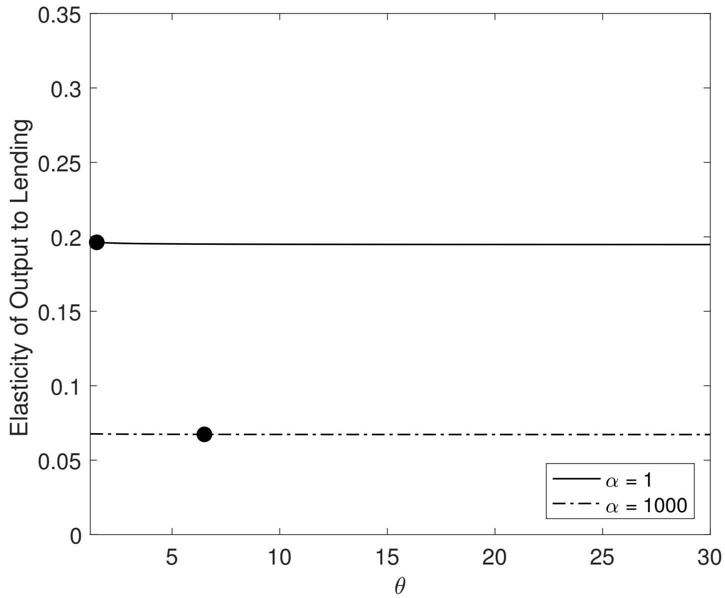

Figure 16: Sensitivity of the aggregate effects of an aggregate bank shock to θ : This Figure shows the aggregate output drop to a bank shock for different values of θ, the substituability of funds across banks. We perform this exercise for two different values of α. First for a frictionless labor market, where $\alpha \to \infty$. And second, for a low level of α when there are substantial frictions in the labor market. All the parameters are fixed in their calibrated or estimated values except for θ. The dot on each line represents the estimated value for φ and the correspondent output drop.

7.3 The aggregate effects of an idiosyncratic bank shock

So far, I presented results about the effects on aggregate output of a cut in the supply of bank lending of the whole banking sector, a truly aggregate shocks. However, idiosyncratic bank lending cuts have aggregate consequences in the model. The reason is that banks in the model are large entities. In this section I illustrate the macroeconomic effects of an idiosyncratic bank shock. I measure the elasticity of aggregate output to the cut in the supply of bank lending of one entity with the following elasticity:

$$\varepsilon^{M,b} = \frac{\int_0^T e^{-\rho t} \left(\log(Y_t) - \log(\bar{Y})\right) dt}{\int_0^T e^{-\rho t} \log(\text{Lending}_{bt}) - \log(\bar{\text{Lending}}_b) dt}). \tag{27}$$

Where $\varepsilon^{M,b}$ is the macro elasticity of output after a cut in lending of bank b. The interpretation of the elasticity is the same as before. It is the macroeconomic equivalent of an instrumental variable regression, where after taking a stance in a source of variation, we compare the effect of that shock on two exogenous variables.

The main result of this section is that opposed to the case of a truly aggregate shock, in this case, θ the elasticity of substitution of funds across different banks is important in determining aggregate outcomes. The economic intuition behind this result is clear. When one bank suffers a given shock that induces the bank to offer less attractive loan terms to its customers, the elasticity at which firms switch away from the affected bank dictates their change in marginal costs and their output as a consequence. This result is the numerical equivalent of the qualitative argument presented in the theoretical sections of the paper, that shows that when one bank is disrupted, both θ and φ are important in determining the aggregate response of output.

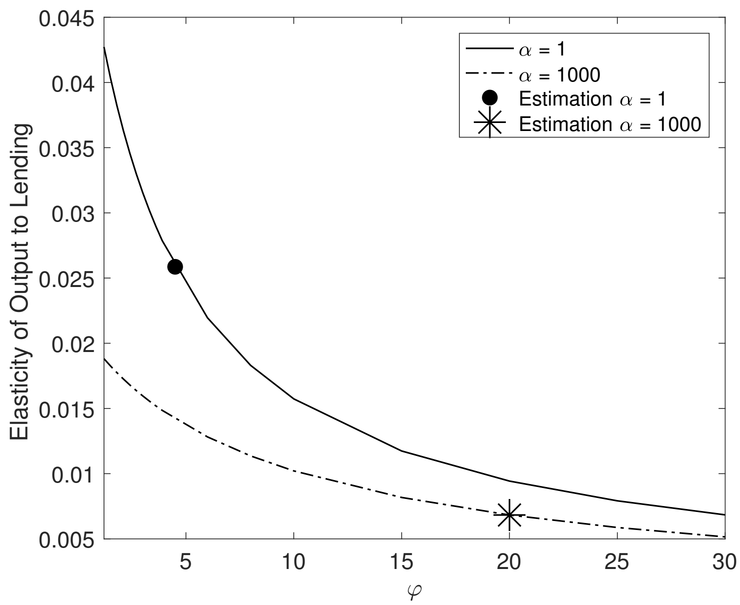

Figure 17: Sensitivity of the aggregate effects of an idiosyncratic bank shock to φ : This Figure shows the aggregate output drop after an idiosyncratic bank shock for different values of φ, the elasticity of credit dependence. We perform this exercise for two different values of α. First for a frictionless labor market, where $\alpha \to \infty$. And second, for a low level of α when there are substantial frictions in the labor market. All the parameters are fixed in their calibrated or estimated values except for φ. The dot on each line represents the estimated value for φ and the correspondent output drop.

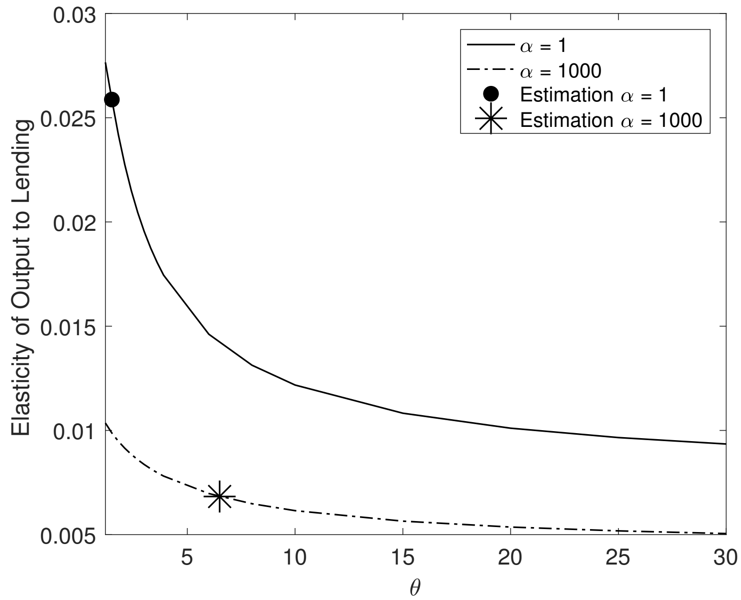

Figure 18: Sensitivity of the aggregate effects of an idiosyncratic bank shock to θ : This Figure shows the aggregate output drop to a bank shock for different values of θ, the substituability of funds across banks. We perform this exercise for two different values of α. First for a frictionless labor market, where $\alpha \to \infty$. And second, for a low level of α when there are substantial frictions in the labor market. All the parameters are fixed in their calibrated or estimated values except for θ. The dot on each line represents the estimated value for φ and the correspondent output drop.

Figure (17) shows on the x axis the elasticity of substitution away from bank credit, and on the y axis, the elasticity of aggregate output to idiosyncratic bank lending. Here, I estimate an elasticity of 0.025, which means that if the shocked bank (that had a bank share of 10 percent) cuts its lending by 1 percent, then aggregate output will fall by 0.025 percent. The figure also shows that when $\alpha \to \infty$, the case of perfect labor mobility, this elasticity would be roughly 0.007.

Figure (18) shows on the x axis the elasticity of substitution across banks, and on the y axis, the elasticity of aggregate output to idiosyncratic bank lending. This figure makes clear that θ, the elasticity of substitution across banks, is important in determining the aggregate response of aggregate output to an idiosyncratic bank shock.

The fact that the elasticity is lower is no surprise, as illustrated in the theoretical section of the paper, the effect of a disruption of one bank is weighted by its market share in the pre-period. What is worth emphasizing is that the elasticity of substitutition of funding across banks is now relevant to determine aggregate fluctuations. The estimation of the model suggests that a 10 percent drop in lending of a bank with 10 percent market share would generate a drop in aggregate activity of 0.25 percent.

7.4 Comparing General to Partial Equilibrium

An important use of the parametrized model is to compare the estimated aggregate bank-lending channel to the alternative measure when general equilibrium effects are ignored. These aggregations are important because after estimating a result in the cross-section using micro data and regression analysis, empirical researchers want to assess the potential of their findings to have aggregate implications. Empirical researchers recognize that the existence of general equilibrium effects may or may not change their findings.

To clarify concepts, Figure (19) shows three alternative escenarios illustrating how the same finding in the cross-section are consistent with different aggregate elasticities. In this illustrative example we assume there are only two groups of firms, those who are exposed directly to a shock (via their banks in our application), and those who are not. In each panel, the solid red line represents the log change in firm-output of a firm exposed to a shocked bank, while the dashed blue line represents the behavior of a firm with zero direct exposure to the affected bank.

In both cases the cross-sectional response is the same, since the difference between the red and blue lines are the same. Therefore, a back-of-the-envelope aggregation that computes the aggregate effect as the difference between exposed and unexposed firms times the average exposure to a shock in the distribution of firms, is the same for the three panels. However, the true aggregate response, which is measured by the average between the blue and the red line is not equal across the three panels. In the first panel the aggregate response is larger than the implied by the partial equilibrium aggregation. In the second panel, the aggregate response is equal to zero. In the third case, because the

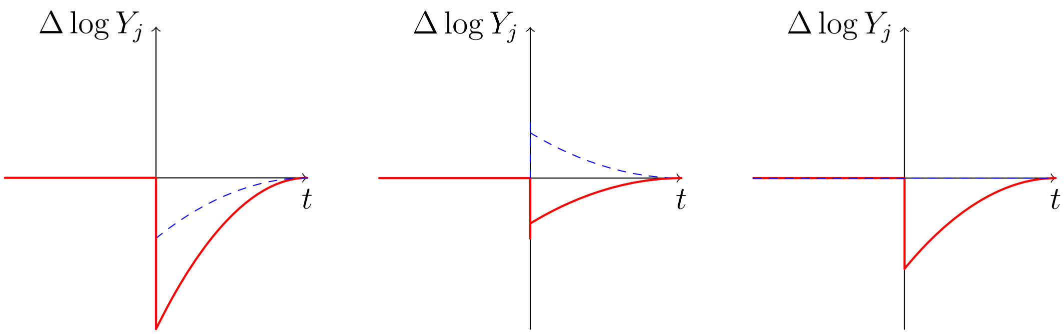

Figure 19: Polar Cases when translating cross-sectional to aggregate elasticities

aggregate response is the same as the partial equilibrium aggregation. The reason is that firms with zero direct exposure (control group) are not indirectly affected by the shock. Under those conditions, the partial equilibrium responses aggregate up.

The partial equlibrium aggregation measures the difference in any given firm outcome between each firm in the economy with respect to least exposed firm to the shock, the control firm which we denote with c. In the model we can present an intertemporal version of the partial equilibrium aggregation in present value given by the following expression

$$\varepsilon^{cs} = \frac{\int_0^T e^{-\rho t} \int_0^1 \left(\log(Y_{jt}) - \log(Y_{ct})\right) dj dt}{\int_0^T e^{-\rho t} \int_0^1 \log(\text{Borrowing}_{jt}) - \log(\text{Borrowing}_{ct}) dj dt}, \tag{28}$$

computing the equivalent of the area between the red and blue lines in Figure (19) in present value.

To compare the general and partial equilibrium aggregations, I simulate an experiment in which I shock only one bank. The parametrization of the model indicates that the partial equilibrium aggregation (ε^{cs}) is 10 percent higher than the general equilibrium response (ε^{M}). This message is important. The preferred estimation of the model, that is consistent with many patterns documented over the years in the corporate finance literature, indicates that general equilibrium forces of the model do *not* cause the micro patterns to vanish in the aggregate.

However this result does not need to hold, and it depends on the parameters we have estimated. For instance, under an alternative model with frictionless labor market frictions, the partial equilibrium aggregation is only one fifth of the general equilibrium effect. Meaning that extrapolating from cross-sectional estimates in such a world would lead researchers to overestimate the relevance of the firm credit channel by a factor of five. However, such a world with frictionless labor markets is at odds with the evidence.

Figure (20) shows how the extent of financial frictions in the model, the substitution from bank credit (φ), and the θ, change the ratio between the general equilibrium and the partial equilibrium elasticities for two parametrizations of the labor market. In particular, it shows that the General Equilibrium aggregation can be higher or lower than the partial equilibrium one as θ and φ change. It also shows that general equilibrium effects are stronger when labor markets work better, as illustrated in the theoretical sections of the paper. It shows that the ratio between general equilibrium and partial equilibrium elasticities is more or less stable, and higher for a model with input market frictions.

However, although Figure (20) presents important information with respect to the output effects of a given lending drop, it does not answer the question of whether back-of-the-envelope aggregations over or underestimate drops in output. The reason is that for the same shock, the aggregate and the cross-sectional drop in lending are different. Specifically, Figure (20) shows that for each 1 percent of a lending drop caused by the shock, output reacts by with a given elasticity. However the two aggregations differ in the percent change in lending they exploit. The general equilibrium aggregation exploits the drop in aggregate lending, while the partial equilibrium one exploits the differential change in lending across banks.

To provide a more clear view, Figure (21) shows the ratio of the output aggregations, which means the ratio of the numerators of ε^{M} and ε^{cs}. The figure makes several points. First, it shows that across the parameter space, in principle the general equilibrium effects on output can be larger, similar, or smaller than is implied by partial equilibrium estimates. However, the estimation of the model imposes restrictions on the size of the difference. By preferring a model with input market frictions rather than a model with frictionless input markets, the ratio of the output responses is around 2/3 rather than around 1/6, or put differently, the extent of labor market frictions elevates the ratio of the output effects in GE to PE aggregations by a factor of 4. Second, within worlds with frictions in input markets, the estimated parameters of financial frictions φ and θ indicate that the output drops in GE are around 70 percent those implied in PE.

So far we have considered two extreme cases. Situations where one bank is shocked which we used to gather information about the cross-sectional effects of lending cuts, and aggregate shocks, where aggregate meant that all the banks where affected by the same shock. However, in the data, bank disruptions are characterized by events that look like the combination of these two extreme cases. All the banks are to some extent affected by a funding shock, but then there is heterogeneity across banks in the exposure to the shock.

This pattern suggests an interesting question. Is the response of the economy different when the profile of shocks exhibits the "across-the-board" plus heterogeneous exposure compared to a situation with only each element separately. This question becomes interesting because the cross-sectional studies we have studied so far explot precisely the heterogeneous exposure that is on top of an aggregate shock.

To check these level effects I compare three different exercises. One in which I shock all the banks, another in which I shock only one bank, and another in which I shock all the banks at the same time, but one particular bank has a higher exposure to the shock.

In this experiment, the sum of the exogenous shock of the first and second experiment are equal to the exogenous shock of the third experiment. I check that their aggregate responses are similar in magnitude. The idiosyncratic shock experiment exhibits an aggregate fall of output of 10.27% of the one where all the banks are shocked in the same way. The aggregate shock plus heterogeneity, has an output fall that is 1.1020 times as large as the experiment where all the banks suffer in an homogeneous fashion. Since 1.1020 is roughly equal to 1 plus 0.1027, I conclude that under the lenses of the model, and the solution method I employ, the aggregate response that I get from an experiment

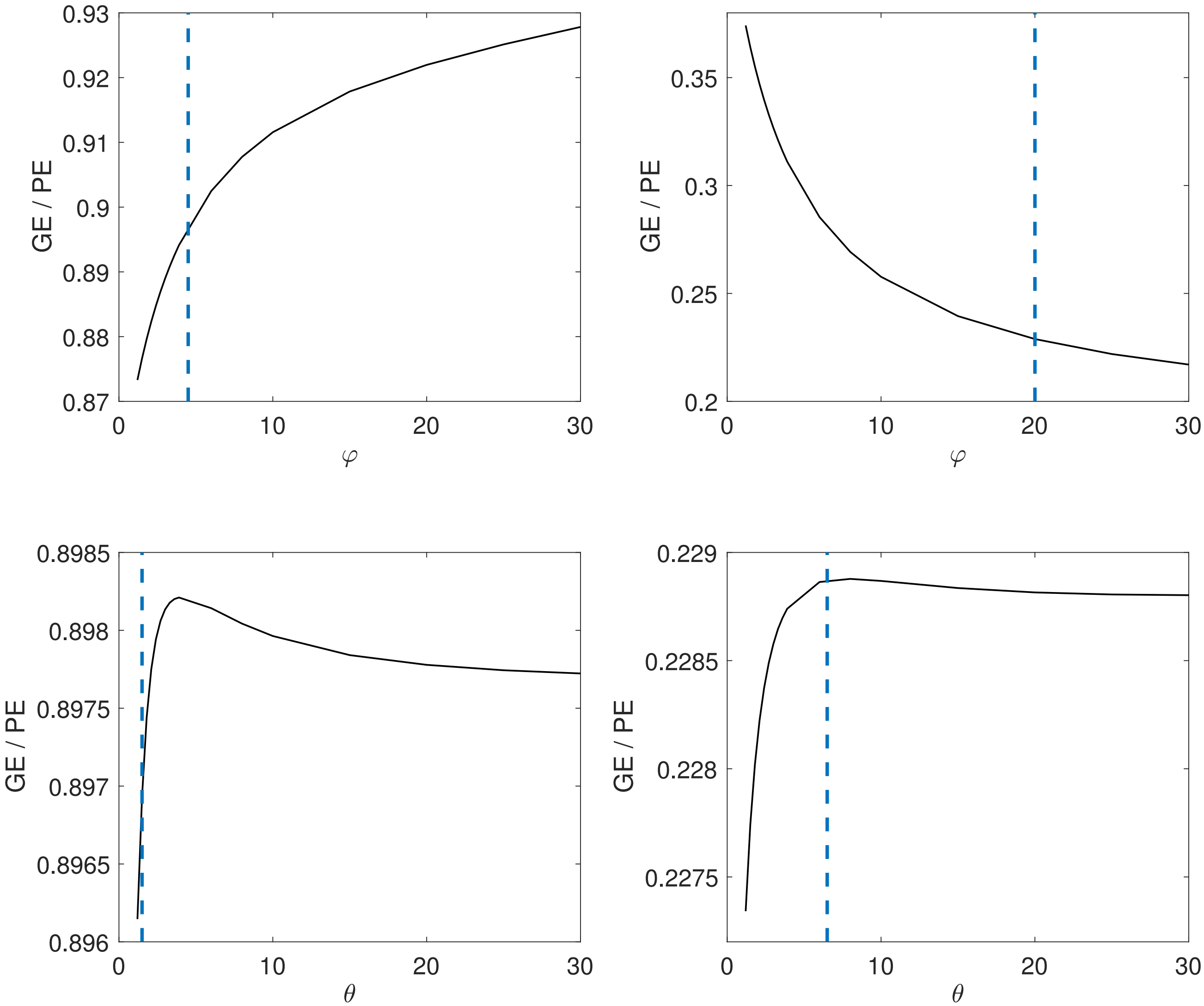

Figure 20: Ratio of the aggregate elasticity to back-of-the-envelope aggregations: This figure shows four panels. The left column shows figures when there are significant frictions in the labor market $\alpha = 1$. The right column shows the case when $\alpha \to \infty$. The top row shows results for the elasticity of substitution away from bank credit φ, while the bottom row shows results for the elasticity at which firms substitute funding from a particular bank, θ. Each panel shows the ratio between the elasticity of aggregate output to aggregate bank lending (ε^M), to the back-of-the-envelope aggregation ε^{cs}. The x axis shows the value of a parameter keeping constant all the other parameters in the parametrization.

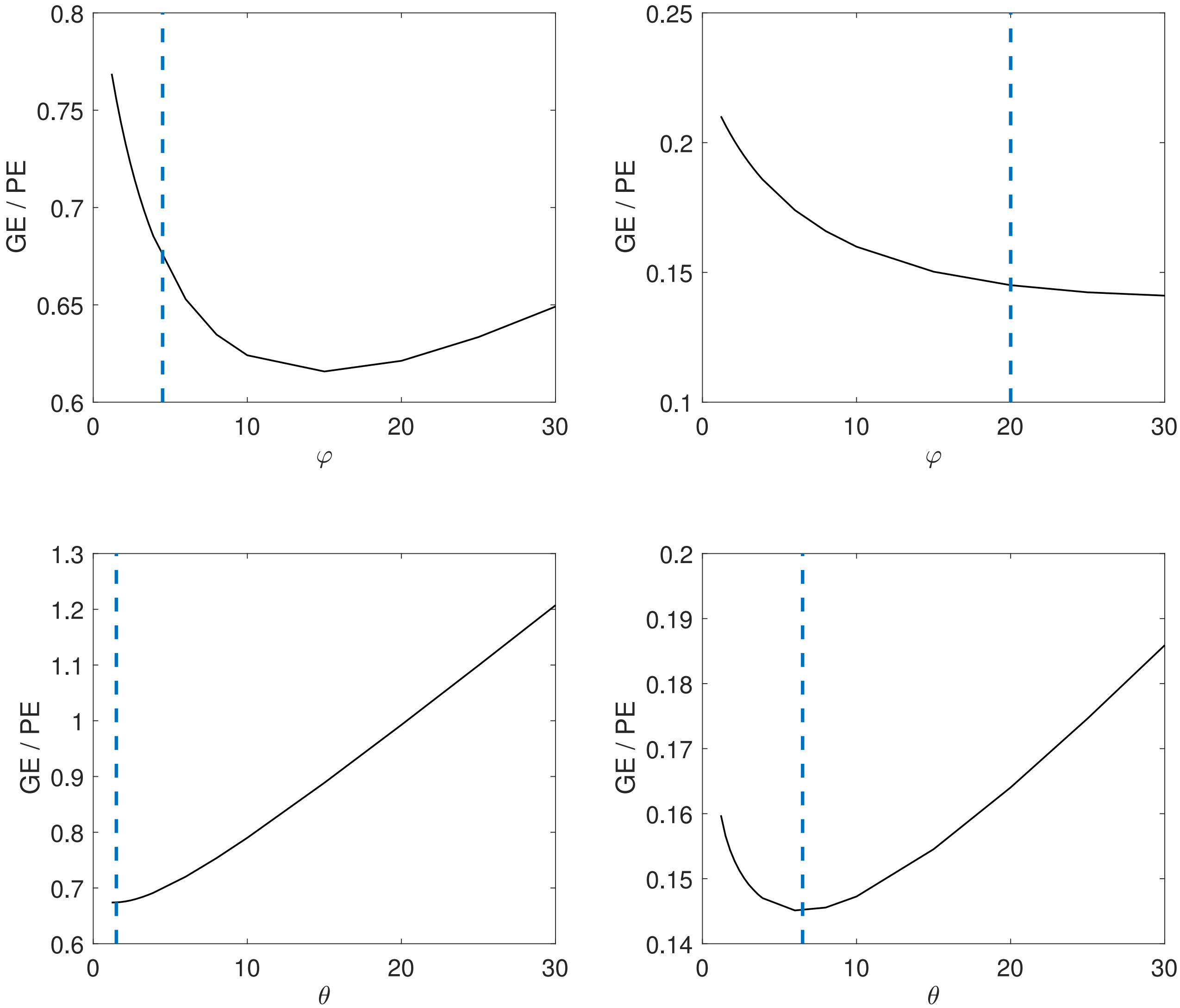

Figure 21: Ratio of the aggregate output drop with respect to back-of-the-envelope aggregations: This figure shows four panels. The left column shows figures when there are significant frictions in the labor market $\alpha = 1$. The right column shows the case when $\alpha \to \infty$. The top row shows results for the elasticity of substitution away from bank credit φ, while the bottom row shows results for the elasticity at which firms substitute funding from a particular bank, θ. Each panel shows the drop of aggregate output to the drop in output inferred from a back-of-the-envelope-aggregation. The x axis shows the value of a parameter keeping constant all the other parameters in the parametrization.

with one bank being shock is consistent with one where all the banks are shocked, and one particular bank suffers additional exposure.

8 Counterfactuals

This section performs different experiments in the model, illustrating the effects of heterogeneity, different types of shocks, and alternative policy scenarios

8.1 Bank Disruptions in Small versus Large Banks

One policy relevant question is the difference in effect of a shock that affects banks that are more or less important in the aggregate economy. The counterfactual we are analyzing is one in which we make the affected bank smaller. The source of variation I consider is to increase the importance of distance in determining firm-bank relationships. This source of variation makes the shocked bank more distant from the average firm, and therefore decreases its market share before the onset of the shock.

The y axis of Figure (22) shows the normalized drop in aggregate output relative to a situation where the affected bank has a market share of 10 percent. the x-axis shows the market share the affected bank had previous to the shock. The Figure makes a couple of points. The first one is that when the affected bank has lower market shares in the pre-period, a shock to it creates smaller aggregate effects. The second point the Figure makes is that this relationship is non-linear. In particular, the relative drop in output in the figure falls faster than the market share of the bank. The dotted line shows a reference line where the relative aggregate output drop falls as fast as the market share, highlighting that the effects of distance, or centrality of a bank in determining aggregate output drops.

Instead of using distance as a source of variation, I also explore changing the market structure of the economy. In particular I show how the aggregate effects of a shock to an individual bank change when the banking sector becomes more competitive.

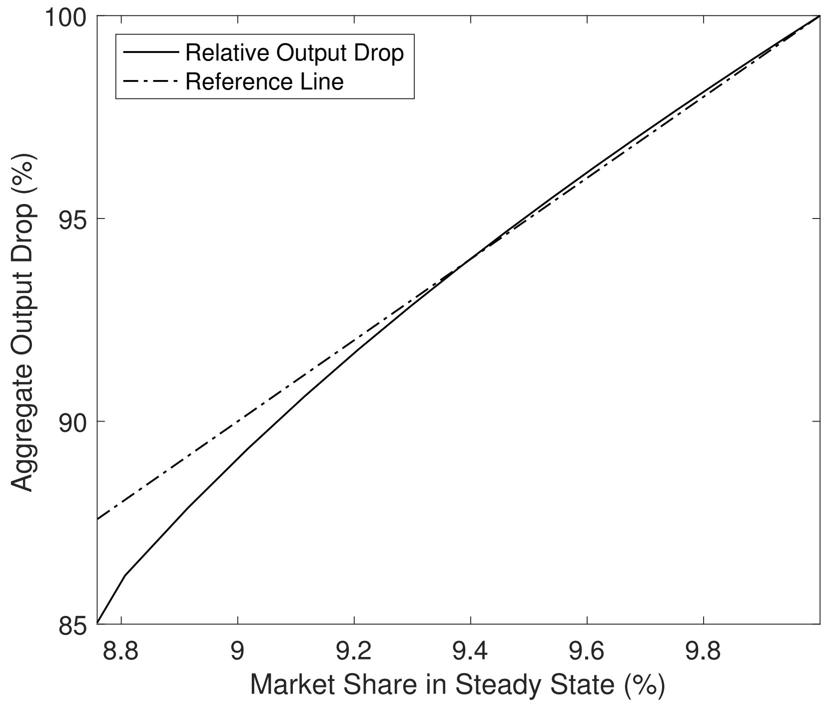

Figure 22: Effect of a shock to a bank as a function of its mean market share: The Figure plots the relative drop in aggregate output as a function of the pre-existing market share of the shocked bank. The dotted line shows a reference line, where the relative drop in output decays at the same rate that the market share in the pre-period.

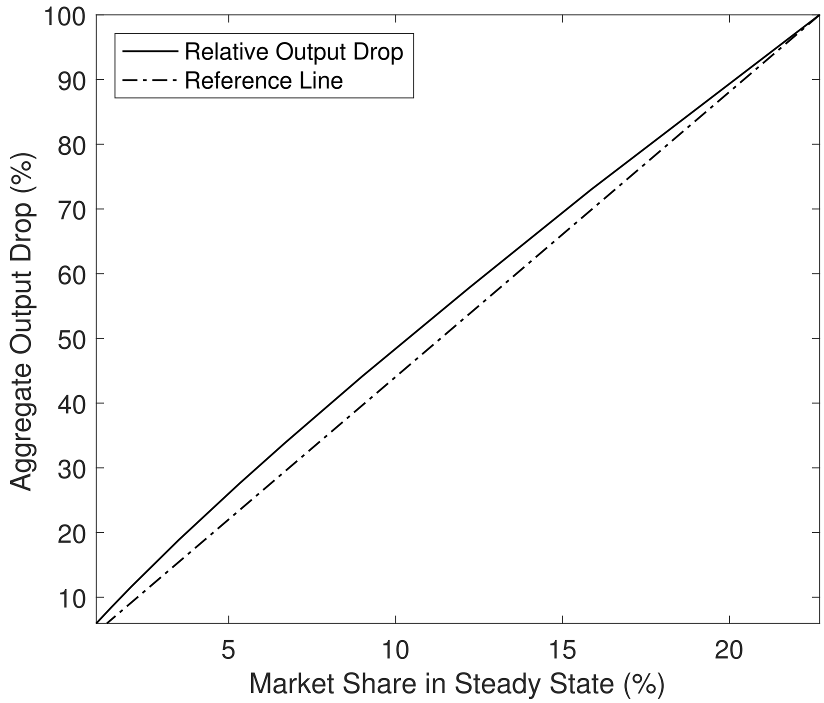

Figure 23: Effect of a shock to a bank as a function of market structure: The Figure plots the relative drop in aggregate output as a function of the pre-existing market share of the shocked bank. The underlying experiment is an increase in the number of banks in the economy. The dotted line shows a reference line, where the relative drop in output decays at the same rate that the market share in the pre-period.

Figure (23) shows the effect of having more banks in the economy. The x-axis shows the pre-existing market share of the shocked bank. In the pre-period banks are roughly the same size, so a market share of 20% translates into an economy with 5 banks for example. The economies depicted in the figure change from having 100 banks to having only five. The main message of the figure is that when the banking sector becomes more competitive, the relevance of a single bank in aggregate fluctuations diminishes more slowly than the market structure itself. The reason is that in the experiment I am considering, banks are located uniformly throughout space.[11] Therefore, many banks enter but they do not lend to the firms that had relationships with the affected bank.

[11]remember that space is not to be taken literally. It just means that some banks are closer to some firms than others, for whatever reason, not limited to geography.

9 Conclusion

The aggregate effects of cuts in the supply of bank lending are difficult to measure using aggregate time-series because bank funding disruptions coincide with other shocks that affect loan demand and output at the same time, and because banks are sensitive to drops in economic conditions creating reverse causality concerns.

Using direct and indirect evidence on the cost of reallocating inputs across firms, and on the relative effects of bank shocks on firm outcomes and credit, I conclude that the aggregate consequences of bank lending cuts are large. When lending drops by 1 percent due to a disruption in bank funding, aggregate output is reduced by 0.2 percent.

This elasticity depends on the extent of bank dependence, and this paper uses cross-sectional evidence to recover this elasticity. Although the ease with which firms can borrow from different banks is relevant in the cross-section, it is not quantitatively relevant in determining the aggregate effect of an aggregate bank shock. Taking a stance on the frictions needed to reallocate inputs and demand across firms is important, even under the experiment of an aggregate shock where all firms are shocked symmetrically. This happens because, in order to target the same cross-sectional moments, frictionless input and demand markets require banking frictions to be milder than in an economy with substantive frictions in reallocating inputs and demand.

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Appendices

A Full derivation of the model

A.1 Firms

There are a continuum of firms and a discrete number of banks. Each firm is denoted by j and banks by b. I will save in the time subscript for brevity, unless necessary. Firms face a downward-sloping demand curve

$$Y_j = Y P_j^{-\eta}$$

.

On the production side, firms produce by mixing a continuum of intermediates with a CES technology with elasticity of substitution σ. For reasons clear below, σ will be irrelevant in the model conditional on a restriction on the parameter space

$$Y_j = \left(\int_0^1 \left(y_j(\omega)\right)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}, \tag{29}$$

and each intermediate good ω is produced with labor in a constant returns to scale production function, and a firm-wide productivity shifter z

$$y_j(\omega) = z_j l_j(\omega). \tag{30}$$

Firms face a two-stage financing problem. In the first stage firms must decide whether to self-finance the task, or to look for funding in the banking sector. For tractability, I assume that firms do not observe the exact lending rates of each bank for a given task, but they can form expectations about it. This assumptions lets me to break the problem in two distinct stages and gives analytical tractability to the problem.

The Total Cost to finance intermediate ω with option $F \in \{S, B\}$ is given by:

$$TC_{jF}(\omega) = w_j l_j(\omega) R_{jF}(\omega)$$

,

and R_{jF} is described by

$$R_{jF}(\omega) = \begin{cases} \frac{R_{jS}}{\varepsilon_{jS}(\omega)} & \text{if } F = S \\ \frac{R_{jB}(\omega)}{\varepsilon_{jB}(\omega)} & \text{if } F \in B. \end{cases} \tag{31}$$

As it is clear from equation 31, each lending cost is scaled by a shifter. These shifters are meant to capture reasons why firms use bank credit for some production tasks and not for others, and they take a form that is isomorphic to a productivity shock at the firm-finance option-intermediate level. They can also be interpreted as a taste from the owner of the firm, or a source of idiosyncratic variation across intermediates. The equation also clarifies that the cost of funds from the banking sector for a particular bank depends on the task. After the firm decides to use the banking sector, it has to go to a set of the banks and ask for quotes to finance the intermediate.

As mentioned before, the firm makes the first-stage decision before getting quotes from the banks, therefore it choosess the financing option that minimize the expected value of the marginal cost of a particular task, which is made explicit by

$$MC_{jF}(\omega) = \frac{w_j}{z_j} R_{jF}(\omega)$$

.

Because the marginal cost is linear on the lending cost, the firm will use one and only one financing source for task ω. Therefore the firm picks the option

$$argmin_{S,B} \mathbb{E}(MC_{jF}(\omega)) = \frac{w_j}{z_j} \frac{R_{jF}}{\varepsilon_{jF}(\omega)},$$

where $R_{jB} = \mathbb{E}R_{jB}(\omega)$ and will be defined later. Because w_j and z_j are firm-level variables and do not depend on the financing choice for any intermediate, then the decision of the firm in this stage collapses to compute the $\min\{\frac{R_{jS}}{\varepsilon_{jS}(\omega)}, \frac{R_{jB}}{\varepsilon_{jB}(\omega)}\}$.

The terms $\varepsilon_{jF}(\omega)$ is sampled from a Fréchet distribution with CDF $F(\varepsilon) = e^{-\bar{\varphi}_F \varepsilon^{-\varphi}}$. I impose without loss that $\bar{\varphi}_S + \bar{\varphi}_B = 1$, and rename $\bar{\varphi}_B = \bar{\varphi}$.

The derivation below is standard in discrete choice models with Fréchet shifters. For a reference, see Eaton and Kortum (2002).

We start the derivation by computing the probability that $\frac{R_{jF}}{\varepsilon_{jF}(\omega)}$ is lower than an arbitrary level x.

$$P\left(\frac{R_{jF}}{\varepsilon_{jF}(\omega)} < x\right) = P\left(\frac{R_{jF}}{x} < \varepsilon_{jF}(\omega)\right) = 1 - e^{-\bar{\varphi}_F R_{jF}^{-\varphi} x^{\varphi}}$$

Now, we compute the probability that the $\min\{\frac{R_{jS}}{\varepsilon_{jS}(\omega)}, \frac{R_{jB}}{\varepsilon_{jB}(\omega)}\}$ is lower than an arbitrary level x

$$P\left(\min\{\frac{R_{jS}}{\varepsilon_{jS}(\omega)}, \frac{R_{jB}}{\varepsilon_{jB}(\omega)}\}\right) = 1 - \Pi_{S,B}\left(1 - P\left(\frac{R_{jF}}{\varepsilon_{jF}(\omega)} < x\right)\right) \tag{32}$$

$$= 1 - \Pi_{F\in S,B} e^{-\bar{\varphi}_F R_{jF}^{-\varphi} x^{\varphi}} \tag{33}$$

$$= 1 - e^{-\sum_{F\in S,B} \bar{\varphi}_F R_{jF}^{-\varphi} x^{\varphi}} \tag{34}$$

Importantly, the term $\sum_{F\in S,B} \bar{\varphi}_F R_{jF}^{-\varphi}$ is the key parameter term that determines the distribution of borrowing costs for firm j. In a similar spirit to the work of Eaton and Kortum (2002), there are three important properties. First, the share of borrowing from the banking sector is given by

$$s_j = \frac{\bar{\varphi} R_{Bj}^{-\varphi}}{\bar{\varphi} R_{Bj}^{-\varphi} + (1-\bar{\varphi}) R_{Sj}^{-\varphi}}. \tag{35}$$

Second, conditioning on the financing source does not an effect on effect on the distribution of prices. When one source is more efficient than the other, this will materialize in a higher financing share from that source, but not on a different price distribution of the terms contracted from that source. Finally, The exact cost of finance for the firm, R_j takes closed form,

$$R_j = \left(\bar{\varphi} R_{Bj}^{-\varphi} + (1-\bar{\varphi}) R_{Sj}^{-\varphi}\right)^{-\varphi}. \tag{36}$$

With this knowledge we can move to the second stage of the problem. Where for an intermediate that was decided to be financed with the banking sector, the firm decides the bank with which to borrow from.

In a similar spirit than before, the marginal cost of choosing bank b to finance task ω is given by

$$MC_{jb}(\omega) = \frac{w_j}{z_j \varepsilon_{jB}(\omega)} R_{jb}(\omega)$$

.

Where $R_{jb} = \frac{R_{jb}}{\epsilon_{jb}(\omega)}$. All the banks inherit the shifter $\varepsilon_{jB}(\omega)$. However, the draw of this shifter is irrelevant for the decision of which bank to use, since it is common to all the banks in the economy. The same happens for the firm-shifter z and the wage rate w_j. Since the marginal cost is linear in the lending rate $R_{jb}(\omega)$, then the firm chooses one and only one bank to finance intermediate ω.

The terms $\epsilon_{jb}(\omega)$ is sampled from a Fréchet distribution with CDF $F(\epsilon) = e^{-T_{jb}\epsilon^{-\theta}}$. I impose without loss that $sum_{\forall b} T_{jb} = 1$

In a similar spirit than before, we start the derivation by computing the probability that $\frac{R_b}{\epsilon_{jb}(\omega)}$ is lower than an arbitrary level x.

$$P\left(\frac{R_b}{\epsilon_{jb}(\omega)} < x\right) = P\left(\frac{R_b}{x} < \epsilon_{jb}(\omega)\right) = 1 - e^{-\bar{\theta}_F R_b^{-\theta} x^{\theta}}$$

Now, we compute the probability that the $\min\{\frac{R_{jb}}{\epsilon_{jb}(\omega)}\}$ is lower than an arbitrary level x

$$P\left(\min\{\frac{R_b}{\epsilon_{jb}(\omega)}\}\right) = 1 - \Pi_{\forall b}\left(1 - P\left(\frac{R_b}{\epsilon_{jb}(\omega)} < x\right)\right) \tag{37}$$

$$= 1 - \Pi_{\forall b} e^{-T_{jb}R_b^{-\theta}x^{\theta}} \tag{38}$$

$$= 1 - e^{-\sum_{\forall b} T_{jb}R_b^{-\theta}x^{\theta}} \tag{39}$$

Importantly, the term $\sum_{\forall b} T_{jb}R_b^{-\theta}$ is the key term that determines the distribution of bank borrowing costs for firm j. In a similar spirit to the work of Eaton and Kortum (2002), there are three important properties. First, the share of borrowing from the banking sector is given by

$$\nu_{jb} = \frac{T_{jb}R_b^{-\theta}}{\sum_{\forall b} T_{jb}R_b^{-\theta}}. \tag{40}$$

And the bank borrowing cost is given by:

$$R_{jB} = \left(\sum_{\forall b} T_{jb}R_b^{-\theta}\right)^{-1/\theta}$$

B Proofs Section 2

B.1 Derivation of Aggregate Output in the simple model

We start with the expression of firm-level labor demand. I save on the time subscript for brevity.

$$L_j = \left(\frac{\eta}{\eta-1}\right)^{-\eta} Y z_j^{\eta-1} w_j^{-\eta} R_j^{-\eta}$$

The firm takes as given the labor supply curve $w_j = w\left(\frac{L_j}{L}\right)^{\frac{1}{\alpha}}$. Plugging this relationship into the labor demand equation, we get the following:

$$L_j = \left(\frac{\eta}{\eta-1}\right)^{-\frac{\eta\alpha}{\alpha+\eta}} Y^{\frac{\alpha}{\alpha+\eta}} z_j^{(\eta-1)\frac{\alpha}{\alpha+\eta}} w^{-\eta\frac{\alpha}{\alpha+\eta}} R_j^{-\eta\frac{\alpha}{\alpha+\eta}}$$

By elevating to the power $\frac{\alpha+1}{\alpha}$, integrating over firms, and elevating to the power $\frac{\alpha}{\alpha+1}$, we get an expression for aggregate labor:

$$L = \left(\frac{\eta}{\eta-1}\right)^{-\eta} Y w^{-\eta} \mathbb{E}\left(z_j^{(\eta-1)\frac{\alpha}{\alpha+\eta}}\right)^{\frac{\alpha+\eta}{\alpha+1}} \mathbb{E}\left(R_j^{-\eta\frac{\alpha+1}{\alpha+\eta}}\right)^{\frac{\alpha+\eta}{\alpha+1}}$$

This expression is useful because it let us to plug in the aggregate labor supply equation, replacing away L

$$w = \left(\frac{\eta}{\eta-1}\right)^{-\frac{\eta\phi}{1+\eta\phi}} Y^{\frac{\phi}{1+\eta\phi}} \mathbb{E}\left(z_j^{(\eta-1)\frac{\alpha}{\alpha+\eta}}\right)^{\frac{\alpha+\eta}{\alpha+1}\frac{\phi}{1+\eta\phi}} \mathbb{E}\left(R_j^{-\eta\frac{\alpha+1}{\alpha+\eta}}\right)^{\frac{\alpha+\eta}{\alpha+1}\frac{\phi}{1+\eta\phi}}$$

Since $Y_j = z_j L_j$, then

$$Y_j = \left(\frac{\eta}{\eta-1}\right)^{-\frac{\eta\alpha}{\alpha+\eta}} Y^{\frac{\alpha}{\alpha+\eta}} z_j^{\eta\frac{\alpha}{\alpha+\eta}} w^{-\eta\frac{\alpha}{\alpha+\eta}} R_j^{-\eta\frac{\alpha}{\alpha+\eta}}$$

Taking the $\frac{\eta-1}{\eta}$ power, integrating over all the firms, and taking the power $\frac{\eta}{\eta-1}$, we get an expression for Y. By replacing the expressions we derived for L and w, we get the result

$$Y = \left(\frac{\eta}{\eta-1}\right)^{-1/\phi} \mathbb{E}\left(z_j^{\frac{(\eta-1)(\alpha+1)}{\alpha+\eta}}\right)^{\frac{(1+\phi)(\alpha+\eta)}{\phi(\eta-1)(\alpha+1)}} \mathbb{E}\left(R_j^{\frac{-(\eta-1)\alpha}{\alpha+\eta}}\right)^{\frac{(1+\phi\eta)}{\phi(\eta-1)}} \mathbb{E}\left(R_j^{\frac{-\eta(\alpha+1)}{\alpha+\eta}}\right)^{\frac{(1-\phi\alpha)}{\phi(\alpha+1)}}$$

B.2 Proof of Proposition 1

Start from equation (14). Take logs and omit the constant term and the productivity term by the assumption of no sorting.

$$\log Y \propto \frac{1+\eta\phi}{\phi(\eta-1)} \log \int_0^1 R_j^{-(\eta-1)\frac{\alpha}{\alpha+\eta}} dj + \frac{1-\alpha\phi}{\phi(\alpha+1)} \log \int_0^1 R_j^{-\eta\frac{\alpha+1}{\alpha+\eta}} dj$$

Then we will do a second-order Taylor expansion around a point where all the lending rates take value of R. According to assumption 2, all the lending rates will stay at that level, except for the lending rate of an arbitrary bank b that will suffer a disruption of its lending terms to Re^u for a positive and sufficiently small u.

$$\log Y \approx \log \bar{Y} + \frac{d \log Y}{d \log R_b} u + \frac{1}{2} \frac{d^2 \log Y}{d \log R_b^2} u^2$$

Define $\bar{X}$ as the value of variable X at the point where the lending rates are equal to R. Up to the second order:

$$\begin{aligned} R_j^{-(\eta-1)\frac{\alpha}{\alpha+\eta}} \approx \bar{R}_j^{-(\eta-1)\frac{\alpha}{\alpha+\eta}} - (\eta-1)\frac{\alpha}{\alpha+\eta}\bar{R}_j^{-(\eta-1)\frac{\alpha}{\alpha+\eta}} \bar{s}_j \bar{\nu}_{jb} u \\ - \bar{R}_j^{-(\eta-1)\frac{\alpha}{\alpha+\eta}} \left(-\theta \bar{s}_j \bar{\nu}_{jb}(1-\bar{\nu}_{jb}) - \varphi \bar{\nu}_{jb}^2 \bar{s}_j (1-\bar{s}_j) - \frac{\alpha}{\alpha+\eta} \bar{s}_j^2 \bar{\nu}_{jb}^2 \right) \frac{u^2}{2} \end{aligned} \tag{41}$$

Which includes the fact that $\frac{d\nu_{jb}}{d \log R_b} = -\theta \nu_{jb}(1-\nu_{jb})$ and that $\frac{ds_j}{d \log R_b} = -\varphi \nu_{jb} s_j (1-s_j)$. In the point around we are taking the second-order Taylor expansion, $\bar{nu}_{jb} = T_{jb}$ and $s_j = \bar{s}$, where $\bar{s}$ is the share of bank credit when all the lending rates are set at R.

$$\begin{aligned} \mathbb{E} R_j^{-(\eta-1)\frac{\alpha}{\alpha+\eta}} \approx \bar{R}_j^{-(\eta-1)\frac{\alpha}{\alpha+\eta}} \Bigg(1 - (\eta-1)\frac{\alpha}{\alpha+\eta} \bar{s} \mathbb{E}(T_{jb}) u + \theta \bar{s} \mathbb{E}(T_{jb}(1-T_{jb})) \frac{u^2}{2} \\ + \varphi \mathbb{E}(T_{jb}^2) \bar{s}(1-\bar{s}) \frac{u^2}{2} + \frac{\alpha}{\alpha+\eta} \bar{s}^2 \mathbb{E}(T_{jb}^2) \Bigg) \frac{u^2}{2} \end{aligned} \tag{42}$$

Applying logs and using the fact that u is sufficiently small such that the two

$$\begin{aligned} \log \mathbb{E} R_j^{-(\eta-1)\frac{\alpha}{\alpha+\eta}} \approx \log \bar{R}_j^{-(\eta-1)\frac{\alpha}{\alpha+\eta}} - (\eta-1)\frac{\alpha}{\alpha+\eta} \bar{s} \mathbb{E}(T_{jb}) u + \theta \bar{s} \mathbb{E}(T_{jb}(1-T_{jb})) \frac{u^2}{2} \\ + \varphi \mathbb{E}(T_{jb}^2) \bar{s}(1-\bar{s}) \frac{u^2}{2} + \frac{\alpha}{\alpha+\eta} \bar{s}^2 \mathbb{E}(T_{jb}^2) \frac{u^2}{2} \end{aligned} \tag{43}$$

By applying the same procedure to the third term in the first equation of this subsection, and renaming $\mathbb{E}(T_{jb}) \equiv \mu_b$, and $var(T_{jb}) = \sigma_{jb}^2$, yields the following expression for output:

$$\log Y = \log \bar{Y} - \frac{1}{\phi} \bar{s} \mu_b u \left(1 - \mu_b \frac{u}{2}\right) + \frac{u^2}{2} \theta \bar{s} \Omega \left(\mu_b - \sigma_b^2 - \mu_b^2\right) + \varphi \frac{u^2}{2} \bar{s}(1-\bar{s}) \Omega \left(\sigma_b^2 + \mu_b^2\right) \tag{44}$$

By substracting $\log \bar{Y}$, we get the result.

B.3 Proof of Proposition 2

Will be here soon

B.4 Proof of Proposition 3

Firm-level output can be expressed as a function of aggregate variables and firm-level shifters as in:

$$Y_j = \left(\frac{\eta}{\eta-1}\right)^{-\eta\frac{\alpha}{\alpha+\eta}} Y^{\frac{\alpha}{\alpha+\eta}} z_j^{\frac{\eta(\alpha+1)}{\alpha+\eta}} L^{\frac{\eta}{\eta+\alpha}} w^{-\eta\frac{\alpha}{\alpha+\eta}} R_j^{-\eta\frac{\alpha}{\alpha+\eta}}$$

Define as Y_j^c the level of output of firms that do not have any relationship with shocked bank b. The difference between any particular firm with a relationship with bank b and a control firm is given by:

$$\log Y_j - \log Y_j^c = -\eta\frac{\alpha}{\alpha+\eta}\left(\log R_j - \log R_j^c\right) + \eta\frac{\alpha+1}{\alpha+\eta}\left(z_j - z_j^c\right)$$

Now we take expectations across firms, and using the assumption of no sorting, we cancel out the productivity term. That is, firm-level productivity is independent of the existence of bank relationships.

$$\mathbb{E}(\log Y_j - \log Y_j^c) = -\eta\frac{\alpha}{\alpha+\eta}\mathbb{E}(\left(\log R_j - \log R_j^c\right))$$

A second-order Taylor expansion of R_j with respect to a disrpuption of the lending terms of bank b as stated in Assumption 2, around a symmetric point where all the lending rates are equal to R yields:

$$R_j \approx \bar{R}_j\left(1 + \bar{s}\bar{\nu}_{jb}u + \bar{s}^2\bar{\nu}_{jb}^2\frac{u^2}{2} - \theta\bar{s}\bar{\nu}_{jb}(1-\bar{\nu}_{jb})\frac{u^2}{2} - \varphi\bar{s}(1-\bar{s})\bar{\nu}_{jb}^2\frac{u^2}{2}\right)$$

And $\bar{R}_j^c = \bar{R}_j$. By plugging combining these expressions we get the result:

$$\mathbb{E}(\log Y_j - \log Y_j^c) = -\frac{\eta\alpha}{\alpha+\eta}\left(\bar{s}\mu_b u(1+\mu_b\frac{u}{2}) - \theta\bar{s}\mathbb{E}T_{jb}(1-T_{jb})\frac{u^2}{2} - \varphi\bar{s}(1-\bar{s})\mathbb{E}(T_{jb}^2)\frac{u^2}{2}\right)$$

C Proofs Identification Section

As shown in the previous section, firm-level output can be written as:

$$Y_j = \left(\frac{\eta}{\eta-1}\right)^{-\frac{\eta\alpha}{\alpha+\eta}} Y^{\frac{\alpha}{\alpha+\eta}} z_j^{\eta\frac{\alpha}{\alpha+\eta}} w^{-\eta\frac{\alpha}{\alpha+\eta}} R_j^{-\eta\frac{\alpha}{\alpha+\eta}} \tag{45}$$

Taking logs we get:

$$\log Y_j = -\frac{\eta\alpha}{\alpha+\eta}\log\left(\frac{\eta}{\eta-1}\right) + \frac{\alpha}{\alpha+\eta}\log Y + \eta\frac{\alpha}{\alpha+\eta}\log z_j - \eta\frac{\alpha}{\alpha+\eta}\log w - \eta\frac{\alpha}{\alpha+\eta}\log R_j \tag{46}$$

I will collapse the first, second, and fourth term into a single term called $\log \Theta_t$, which is common to all the firms, and will therefore become irrelevant in computing the result.

$$\log Y_j = \log \Theta_t + \eta\frac{\alpha}{\alpha+\eta}\log z_j - \eta\frac{\alpha}{\alpha+\eta}\log R_j \tag{47}$$

Taking temporal differences we get:

$$\Delta \log Y_j = \Delta \log \Theta_t + \eta\frac{\alpha}{\alpha+\eta}\Delta \log z_j - \eta\frac{\alpha}{\alpha+\eta}\Delta \log R_j \tag{48}$$

A second-order Taylor expansion of $\log R_j$ that coincides with assumption 2 yields:

$$\log R_j \approx \log \bar{R}_j + \bar{s}\bar{\nu}_{jb}u - \theta\bar{s}\bar{\nu}_{jb}(1-\bar{\nu}_{jb})\frac{u^2}{2} - \varphi\bar{s}(1-\bar{s})\bar{\nu}_{jb}^2\frac{u^2}{2} \tag{49}$$

Taking temporal differences with respect to a pre-period where $\log R_j = \log \bar{R}_j$, yields:

$$\Delta \log R_j \approx \bar{s}T_{jb}u - \theta\bar{s}T_{jb}(1-T_{jb})\frac{u^2}{2} - \varphi\bar{s}(1-\bar{s})T_{jb}^2\frac{u^2}{2} \tag{50}$$

Plugging this expression into equation 48 yields a second order approximation of firm-level output after one shock suffers an increase in its lending terms.

$$\Delta \log Y_j = \Delta \log \Theta_t + \eta\frac{\alpha}{\alpha+\eta}\Delta \log z_j - \eta\frac{\alpha}{\alpha+\eta}\left(\bar{s}T_{jb}u - \theta\bar{s}T_{jb}(1-T_{jb})\frac{u^2}{2} - \varphi\bar{s}(1-\bar{s})T_{jb}^2\frac{u^2}{2}\right) \tag{51}$$

The cross-sectional regression of log output changes on pre-existing exposure is the equivalent of running the following regression by OLS:

$$\Delta \log \mathrm{Y}_f = \beta_0 + \beta_{real} T_{jb} + \epsilon_f \tag{52}$$

In this setting the exposure in the preperiod to the affected bank is just T_{jb}.

The regression coefficient is given by the covariance between $\Delta \log Y_j$ and T_{jb}. Since all the firms have the same $\Delta \log \Theta_t$ regardless of their specific T_{jb}, and because we are imposing a no-sorting condition that implies $cov(\Delta \log z_j, T_{jb}) = 0$, then:

$$\beta_{real} = \frac{cov\left(\Delta \log Y_j, T_{jb}\right)}{var(T_{jb})} \tag{53}$$

$$= \frac{1}{var(T_{jb})} cov\left(-\eta \frac{\alpha}{\alpha+\eta}\left(\bar{s} T_{jb} u - \theta \bar{s} T_{jb}(1-T_{jb})\frac{u^2}{2} - \varphi \bar{s}(1-\bar{s}) T_{jb}^2 \frac{u^2}{2}\right), T_{jb}\right) \tag{54}$$

$$= -\eta \frac{\alpha}{\alpha+\eta}\bar{s}u + \theta\eta\frac{\alpha}{\alpha+\eta}\bar{s}\frac{u^2}{2}\frac{cov(T_{jb}(1-T_{jb}), T_{jb})}{var(T_{jb})} + \varphi\eta\frac{\alpha}{\alpha+\eta}\bar{s}(1-\bar{s})\frac{u^2}{2}\frac{cov(T_{jb}^2, T_{jb})}{var(T_{jb})} \tag{55}$$

$$= -\frac{\eta\alpha}{\alpha+\eta}\bar{s}u + \theta\frac{\eta\alpha}{\alpha+\eta}\bar{s}\frac{u^2}{2}\left(1 - \frac{cov(T_{jb}^2, T_{jb})}{var(T_{jb})}\right) + \varphi\frac{\eta\alpha}{\alpha+\eta}\bar{s}(1-\bar{s})\frac{u^2}{2}\left(\frac{cov(T_{jb}^2, T_{jb})}{var(T_{jb})}\right) \tag{56}$$

This is the main result. The regression coefficient in the population is larger when consumers are more elastic in reallocating demand across varieties (η higher), when labor markets work without frictions $\frac{\alpha}{\alpha+\eta}$. Both substitution across banks and substitution away from bank credit make the elasticity less negative. Note the term $\left(1 - \frac{cov(T_{jb}^2, T_{jb})}{var(T_{jb})}\right)$ in the second term that accompanies the θ. Since the shifter T are between 0 and 1, the covariance can be equal to the variance if the T terms only take either 0 or 1. When that is the case, firms are completely dependent of one bank. Therefore θ becomes irrelevant.