Skill Formation and the Trouble with Child Non-Cognitive Skill Measures

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Abstract

Research on child skill formation and related policies typically rely on parentreported measures of child non-cognitive skills. In this paper, we show that parental assessments of child non-cognitive skills are directly affected by the skills of the parents. We develop a dynamic model of child and parental skills formation that accounts for this contamination and show how standard estimates of the production of skills are affected. We use our model to illustrate how parental contamination of child non-cognitive skill measures affects the estimated impact of child development policies that also have a significant impact on parental skills.

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1 Introduction

Many studies have shown that human capital skills measured in early childhood are strong predictors of important adult outcomes such as completed education, earnings, and health (Todd & Wolpin, 2007; Flavio Cunha & Masterov, 2006; Moffitt *et al.*, 2011; Gabriella Conti & Pinto, 2016). As a consequence, the success of publicly funded policies such as universal child care, Head Start (for the US) or Sure Start (for the UK) are often measured according to their effects on children's skills. Recently, there has been a growing interest in the impact such policies have on a particular aspect of human capital, child non-cognitive skills.¹ The reason for this increased attention is that non-cognitive skills aid in the development of cognitive skills throughout early childhood and also significantly impact labor market outcomes directly (Cunha *et al.*, 2010; Agostinelli & Wiswall, 2016; Attanasio *et al.*, 2019; Heckman *et al.*, 2006). Yet, how do we measure child non-cognitive skills, such as emotional stability, motivation, and self-regulation, when children are too young to understand complex questions and provide accurate reports of their attitudes and behaviors?

To date, much of the analysis regarding child skill formation relies on household surveys where parents assess the emotional and social behavior of their child.² These assessments are naturally subjective and can be influenced not only by the non-cognitive skill of the child but also by the skills and traits of the parents. The trouble with relying on parentreported measures of child skills is most acute in a dynamic setting, where parental skills, the source of contamination, can be seen as a direct input in the production of future child skills. In these models, an important parameter is the effect of parental skills on child skills in the next period. The interpretation of this parameter becomes more challenging if child non-cognitive skills are affected by parental skills simply due to the way they are measured.

¹See Deming (2009) or Baker & Milligan (2015) for example.

²This is true for the US, where the most commonly used data for this type of analysis is the National Longitudinal Mother-Child Supplement (Deming, 2009; Cunha *et al.*, 2010), as well as the UK, where researchers have mainly relied on the Millennium Cohort Study (Del Bono *et al.*, 2016; Hernández-Alava & Popli, 2017).

The possibility that parental skills influence measures of child non-cognitive skills has important implications beyond skill production estimates. Consider a policy such as the introduction of universal child care. Assume that this policy has no effect on child skills but does impact parental skills, such as maternal mental health (Haeck *et al.*, 2019; Yamaguchi *et al.*, 2018; Baker *et al.*, 2008); assume also that parental skills have no impact on future child skills. If parental skills directly influence the *measures* of child non-cognitive skills available to the researcher, even a thorough and well-executed evaluation will indicate an impact of the policy on child non-cognitive development when there is none. While in general we expect child care policies to impact child skill directly, the same logic applies. Our understanding of a policy's direct impact on the child can be seriously contaminated by how the policy also affects parents.

In this paper, we develop a model that produces consistent estimates of the true distribution and evolution of child skills even when parental measures of child non-cognitive skills are directly contaminated. By contamination we mean that parental measures of child non-cognitive skills are influenced by the parents' own skills and traits. Our estimation strategy relies on the availability of multiple evaluators of child non-cognitive skills. For this reason we turn to the Millennium Cohort Study (MCS). The MCS is a large prospective study of infants born between 2000 and 2002 in the United Kingdom. Successive interviews took place when the children were 3, 5, 7, 11, 14 and 17 years old, and include cognitive and non-cognitive assessments of several areas of child development.³ Specifically, the non-cognitive skills of the child are evaluated by parents in every wave, but also by interviewers or teachers at different points in time such that we have measures from different evaluators in every time period. An additional attractive feature of the MCS is the availability in every wave of information on parental non-cognitive skills. This allows us to jointly model the dynamics of child and parental skills and to consider the implications of contamination in the type of setting where it is most relevant. The data also offers the opportunity to examine the role of fathers in child skill formation and to analyze the importance of feedback effects running from child non-cognitive skills to

³We focus on surveys between the ages of 3 and 11.

parental non-cognitive skills.

We begin with some simple descriptive analysis that illustrates the subjective nature of child non-cognitive skill measures. We first look at the contemporaneous correlation between mother-reported child non-cognitive skills and maternal non-cognitive skills and compare it with the correlation between teacher (or interviewer) reported child non-cognitive skills and maternal non-cognitive skills. We see that maternal non-cognitive skill measures are highly predictive of child non-cognitive skill measures only when using child non-cognitive skill measures *reported by the mother*. By contrast, maternal non-cognitive skills have significantly less predictive power when using teacher (or interviewer) reported measures of child non-cognitive skill. Similarly, maternal non-cognitive skills are strong predictors of the child non-cognitive skill measures only when the mother, not the father, reports on the child's non-cognitive skill. When the father is the main survey respondent, it is the father's non-cognitive skill measures that best predict the child non-cognitive skills. These patterns suggest that the identity and attributes of the child's assessor may significantly impact the measures of child non-cognitive skills.

While the descriptive analysis reveals that contamination is a concern, it is not wellsuited to solving the problem. To address contamination, we estimate a model of skills formation similar in spirit to Cunha *et al.* (2010) and Agostinelli & Wiswall (2016). The key innovation in our setting is to relax some of the typical assumptions related to the measurement model of child non-cognitive skills. In particular, we allow parent, teacher, and interviewer reported measures of child non-cognitive skills to be contaminated. This contamination can be viewed as a component of the skill measurement error that can be correlated across non-cognitive skill measures and, in the case of parents, with the measures of other skills. These features differentiate our measurement model from others in the literature, where it is assumed that there is no contemporaneous correlation in the measures other than through child non-cognitive skills (in Cunha *et al.* (2010), measurement error can be correlated across different time periods). We show that our model is identified under the assumption that the contamination in the child non-cognitive skill measures reported by different assessors are independent of each other. We estimate the model using a two-step approach similar to Agostinelli & Wiswall (2016) and Attanasio *et al.* (2019). Estimates of the measurement equations show that a significant share of the variation in parent-reported measures of child non-cognitive skills are driven by parental skills. Similarly, the measurement error in teacher reported measures of child non-cognitive skill are highly correlated within period. The estimated skill technology parameters indicate high levels of persistence in skill formation, a significant role of child non-cognitive skills on cognitive skills, and a large impact of paternal cognitive skills on child cognitive skills. There is also evidence that child non-cognitive skills influence the evolution of parental mental health.

To understand how contamination impacts estimates of skill production, we compare the results of our preferred model with two alternatives. The first alternative uses only parental measures of child non-cognitive skills, where it is not possible to allow for contaminated measures since only one evaluator is available. The second alternative takes into account both parent-reported and teacher (or interviewer) reported measures but assumes there is no contamination in any measure. We find important differences between our preferred specification and the two alternatives. For example, we find that the persistence of child non-cognitive skills is 15% higher when only parent measures of child non-cognitive skills are used. This occurs because maternal and paternal skills are themselves highly persistent and these skills contaminate estimates of child skills through the measures in all periods.

In addition to affecting estimates of child skill dynamics, relying on contaminated measures of child non-cognitive skills can have broader implications in terms of policy evaluation. If a child or household policy influences parental skills and parental skills contaminate the available *measures* of child non-cognitive skills, estimates of the policy effect on child skills could be partly spurious. In the final section of the paper we illustrate this point through two simulation exercises. We first implement a hypothetical policy where only initial maternal non-cognitive skill is increased by one standard deviation. Strictly as a result of contamination, child non-cognitive skill measures increase by as much as 0.15 standard deviations in the initial period. Over time, increased maternal skills directly impact child non-cognitive skills, but around half of the effect after eight years is the result of contamination. In a second simulation exercise, we consider a hypothetical policy where both initial maternal and child non-cognitive skill are increased by 0.2 standard deviations. This simulated policy is motivated by the findings in Baker *et al.* (2008), where an increase in the availability of free child care in Quebec had an impact on both maternal and child non-cognitive skills of approximately the same size. Even in this case, contamination can be responsible for up to a quarter of the increase in measured child non-cognitive skills.

Our paper is the first to propose a methodology to tackle systematic error in measures of child non-cognitive skills in a dynamic model of skills formation. The existing literature in economics has largely ignored this issue (Cunha et al., 2010; Agostinelli & Wiswall, 2016; Attanasio et al., 2019). One possible explanation for this are data constraints, since dealing with the contamination of parental-reported measures of child non-cognitive skills requires the availability of measures from multiple informants. While this type of data is frequently collected and analyzed in the psychological and psychometric literature on child development and is increasingly preferred to single-informant data (Kraemer et al., 2003; De Los Reyes et al., 2015; Martel et al., 2017), it is usually less readily available in large and representative surveys. One notable exception is the work conducted by Johnston et al. (2014), who use the 2004 Survey of Mental Health of Children and Young People in Britain to examine the effects of child mental health on education outcomes. The survey provides measures of child mental health reported by parents, teachers, and the children themselves, who are all assumed to be biased informants. Here the setting is static and identification relies on the availability of diagnostic assessments from a panel of psychiatric experts, who are assumed not to be affected by systematic bias.

Our study also adds to a growing literature that explores the impacts of maternal mental health on children. A substantial part of this literature investigates maternal stress in pregnancy or during the post-natal period on child development using either quasi-natural experiments - which create exogenous variation in maternal psychological well-being (Black *et al.*, 2016; Persson & Rossin-Slater, 2018) - or randomised control trials (Baranov *et al.* , 2017). A contribution of our work is to also consider the potential impact of children on parental mental health, a link that has been mainly emphasized in medical studies (Kuhn & Carter, 2006; Davis & Carter, 2008; Choe et al., 2014; Hastings, 2002).

The rest of the paper proceeds as follows. Sections 2 and 3 present the data and preliminary evidence that child non-cognitive skill measures are contaminated. In Section 4 we present a model of skill dynamics meant to handle this contamination and discuss identification assumptions. Section 5 describes estimation and presents the results. Section 6 evaluates how contamination impacts policy evaluation. Section 7 concludes.

2 Data

The data for our analysis are from the Millennium Cohort Study (MCS). The MCS is a large prospective study of infants born between 2000 and 2002 in the United Kingdom. It is by construction representative of the overall UK population of newborns.⁴ The first wave of data collection took place when the infants were around 9 months old and includes data on 18,552 children. The sampling design allowed for over-representation of areas with high levels of childhood deprivation and high proportions of ethnic minorities (Plewis 2007).

At the first interview, the main respondent was asked about pregnancy, birth, infant health, infant development, their own mental health, health behavior and the family social and economic circumstances. Successive interviews took place when the children were 3, 5, 7, 11, 14 and 17 years old. Data for the present analysis is restricted to interviews up to age 11, i.e. the last year of primary school. During each sweep of the study, the children were administered a series of cognitive assessments by a specially trained interviewer. At age 5, 7 and 11 the study collected additional information about the child's academic ability and socio-emotional development as reported by the main primary school teacher.

Sample selection — We operate a number of sample restrictions. First, we consider only singleton children and families where the initial interview was not obtained by proxy, starting with a sample of 19,048 children who are interviewed for the first time during the first or second wave of the study. We then focus on two-parent families, as our analysis takes

 $^{^{4}}$ Infants born on eligible dates in eligible areas were selected from the Child Benefit Register, a universal benefit in the UK at the time.

into consideration maternal as well as paternal measures of cognitive and non-cognitive skills. This restricts the sample to 14,648 children/families. We then delete a few cases where there is missing information on: gender, ethnicity, maternal age and other basic family demographics (14,598 children). As cognitive outcomes start being recorded at age 3, we consider only families that appear in the study at least once from the second wave onwards (i.e. delete families that only reply to the first interview). Due mainly to this restriction, our sample size drops to 12,530 children.

Table 1 below shows basic demographic and family background variables for the initial sample of singleton births, the sub-sample of two-parent families, and our estimation sample. For ease of comparison we show descriptive statistics using information collected at the time of the earliest available interview (columns 1-3). This is conducted when the child was 9 months old, but for less than 5% of cases it takes place at age 3 of the child as some new families were recruited in the study at wave 2. As we can see, there is very little difference in the gender and ethnic composition of these samples, as well as their geographical distribution. The age of the child (in years) at the first interview is also virtually identical. There is however a significant difference with respect to socio-demographic characteristics. Mothers are usually older and more educated among the sample of two-parent families.⁵ As we would expect, this positive selection is a bit more marked for our final estimation sample (column 4), as low socio-economic status families are less likely to respond to the second interview and stay in the panel. Paternal characteristics appear however to differ much less across these samples.

Child cognitive outcomes — The cognitive skills of the child are measured using tests administered by the interviewer as well as teacher assessments. In each wave of the survey, at least two measures of child cognitive skills are available (see Table 2). The tests ad-

⁵Education is measured as years of schooling. This measure is obtained assuming that: (i) all individuals with no qualifications have left school before the end of compulsory education (with either 8, 9 or 10 years of schooling depending on their date of birth), (ii) those with a certificate of secondary education have left school at the end of the compulsory schooling period (with either 9, 10 or 11 years of schooling depending on their date of birth), (iii) those with 0-levels or equivalent qualifications have 11 years of schooling, (iv) those with A-levels or equivalent qualifications have 13 years of schooling, (v) those with a diploma in higher education have 14 years of schooling, and those with a degree or higher level of education have 15 years of schooling.

ministered by the interviewer during the early years come mainly from the British Ability Scales or BAS (Elliott *et al.*, 1996). In addition, the survey children were assessed at age 3 according to some of the components of the Bracken School Readiness Assessment (Bracken, 2002). This is usually considered a good indicator for success in formal education. We also use information obtained at age 7 from a variant of the National Foundation for Educational Research (NFER) Progress in Mathematics test. At age 11, child cognitive skills are assessed through the Cambridge Gambling Task (CGT) and the Spatial Working Memory (SWM) task (Robbins *et al.*, 1998).⁶

Our measures of child cognitive skills also include teacher assessments. The first of these is the Early Years Foundation Stage Profile (FSP). This assessment is performed by the teacher during the reception year, when UK children are aged 5, and describes each child's development and learning achievements in the following areas: (i) personal, social and emotional development; (ii) communication, language and literacy; (iii) problem solving, reasoning and numeracy; (iv) knowledge and understanding of the world; (v) physical development; (vi) creative development. We sum up all scales in all areas, excluding (i) and (v), to construct a measure of cognitive skills. At age 7 and 11, we use information from a teacher questionnaire, which reports the teacher's evaluation of the MCS child across the main subjects (English, Math, and Science).

Child non-cognitive outcomes — At each interview, the main respondent - who is the mother of the child in the vast majority of cases, but could also be the father or another member of the family - is asked to complete the Child Strengths and Difficulties Questionnaire (SDQ). The SDQ is a behavioral screening questionnaire designed to measure psychological adjustment in children aged 3 to 16 (Goodman, 1997). The questionnaire identifies five different components: (i) hyperactivity/inattention, (ii) conduct problems, (iii) emotional symptoms, (iv) peer problems, and (v) pro-social behavior. We take the first four sub-scores as our main measures of respondent-reported child non-cognitive skills. Another measure

⁶All measures collected by the interviewer were obtained using Computer Assisted Personal Interviewing (CAPI) by interviewers who were specifically trained, but did not have a psychology background. Where appropriate, our analysis uses age-adjusted ability scores, which reflect the raw score and the difficulty of the items administered.

reported by the respondent is captured by selected items from the Child Social Behavior Questionnaire (CSB), measuring the child's ability to perform tasks independently, to concentrate, and to control his/her emotional responses (Melhuish *et al.*, 2004). A set of questions to measure child cooperative behavior was introduced at age 7 (see Table 3).

At age 3, the interviewer reported his/her own assessment of child behavior as part of a module aimed at providing information on the general conditions of the assessment. The interviewer was asked whether the child had been fidgety, focused, disruptive, etc., during the assessment and the interview. There are 10 questions in total. Two questions were aimed at capturing extreme behavior (i.e. child is dangerous, child is disruptive), others were combined using principal component analysis to obtain two additional summary scores, one centered on focus and attention, the other on the child's cooperative behavior.

Lastly, we consider teacher assessments of the child's social and emotional development. There is one area of the FPS assessment measured at age 5 (see above) which is relevant in this case: (i) personal, social and emotional development. This is further subdivided into three components: (a) dispositions and attitudes, (b) social development, and (c) emotional development. Each of them is scored between 0-9 and is considered separately. At age 7 and 11, teachers are administered the teacher-version of the SDQ, from which we take four components: (i) hyperactivity/inattention, (ii) conduct problems, and (iii) emotional symptoms and (iv) peer problems. In each wave of the survey, at least three teacher and/or interviewer measures of child non-cognitive skills are available.

Maternal and paternal non-cognitive measures — The MCS provides measures of the mental health of the respondent and his/her partner at each interview. Specifically, the main respondent and the partner (if present) were asked to provide answers to questions from a shortened version of the Kessler questionnaire, a screening device frequently used to diagnose mental illness (Kessler *et al.*, 2003). A higher score on these items indicates the presence of psychological distress or depression. We also use the answer to a question on life satisfaction, scaled from 1-10 (1 being "completely dissatisfied" and 10 being "completely satisfied"). Maternal and paternal cognitive measures — In the MCS there are relatively few high quality measures of respondent and partner cognitive skill. Respondents report their own and partner's academic qualification, which we transform into years of schooling (see footnote 5 for more details). In the first wave of the survey, respondents are also asked about any difficulties they or their partner have reading books, filling out forms, or performing everyday math. However, these binary measures are relatively uninformative since approximately 95% of respondents and partners indicate no problems with these activities. Finally, in the age 14 MCS survey, respondents and partners are asked to complete a word task. The results of this word task are highly correlated with years of schooling, but are not used here as the word task is completed by only half of our estimation sample. As a result, we rely primarily on years of schooling to measure parental cognitive skill.

3 Preliminary Evidence of Distortions and Impacts

In this section we provide descriptive evidence of the presence of contamination in child non-cognitive skill measures. We first extract the principal factor for child and parental skills in each survey wave using the measures discussed in the previous section.⁷ The skill proxies we generate are noisy versions of the true underlying skills and are functions of different skill measures in different years. Additionally, if even one of the underlying skill measures is missing, the skill proxy will also be missing. The dynamic model we introduce in the next section will handle these concerns. Here we simply provide suggestive evidence of contamination in child non-cognitive skill measures that drive our modeling choices in subsequent sections.

There are four types of individuals who report on a child's non-cognitive skills: mothers, fathers, interviewers, and teachers. An interviewer assesses the child at the end of the age 3 survey, while a teacher assesses the child in all subsequent surveys.⁸ In every wave either the mother or father evaluates the child's non-cognitive skills. To illustrate that the identity

⁷As mentioned previously, parental cognitive skill is simply equal to parental years of schooling. The key patterns we discuss in this section are unchanged if we replace years of schooling with separate dummy variables for each level of academic qualification.

⁸In the UK, all children begin school by the age of 5.

of the survey respondent can influence the assessment of the child, we create separate child non-cognitive skill proxies for each type of respondent. We then explore how these assessorspecific, non-cognitive skill proxies relate to child cognitive skills and parental cognitive and non-cognitive skills. The basic idea is that if each type of respondent provides dedicated measures of the child's true non-cognitive skills, the resulting skill proxies should project similarly on the other skill proxies.

Table 4 provides the first piece of evidence that the identity of the assessor can impact child non-cognitive skill measures. In the first two columns, we report the coefficients that result when we use the child non-cognitive skill proxy generated using the mother's responses as the dependent variable. In the third and fourth columns we use the child noncognitive skill proxy generated from the teacher/interviewer responses as the dependent variable.⁹ For both sets of equations the key regressors are the child's cognitive skill proxy and the mother's skill proxies. As discussed in the previous section, child cognitive skills are measured objectively, and the maternal skill proxies are from cognitive and non-cognitive responses the mother provides.¹⁰ All of the skill proxies in the regressions are measured contemporaneously and we include observations from four different periods, from t = 1 to t = 4 (corresponding to ages 3, 5, 7, and 11).

Child non-cognitive skills should be strongly correlated with the child cognitive skills and maternal skills since they are all interrelated. However, the key result in Table 4 is the difference in the conditional correlations across the skill measures according to the type of assessor. When we utilize the mother's measures of child non-cognitive skill, there is a strong relationship between child non-cognitive skills and mother non-cognitive skills. When we utilize the teacher/interviewer's measures of child non-cognitive skills, the coefficient on the mother non-cognitive skills declines by approximately 80%.¹¹ Additionally, the coefficient on the child cognitive skill increases significantly when we use the teacher/interviewer-generated child non-cognitive skill. While we cannot determine which

⁹At age 3 we use interviewer responses, and at ages 5, 7, and 11 we use teacher responses.

¹⁰When constructing the child cognitive skill proxy we do not include teachers' evaluations of the child cognitive abilities since we are concerned they may also be contaminated. In the full statistical model we employ these measures and account for potential contamination directly.

¹¹All skill measures are standardized to have a mean of zero and a standard deviation equal to one.

respondent, mother or teacher, is closer to the truth, we can certainly conclude that the assessments differ significantly.

One reason the assessments might differ, apart from contamination, is that parents and teachers are reporting about a different underlying latent skill. In fact, not all parent and teacher measures of child non-cognitive skills overlap. However, when we construct the child's non-cognitive skill factor using only those measures that are common between parents and teachers, we find the same patterns. Moreover, similar results are obtained when we replace the principal factor with the underlying SDQ measures.¹² Both results are in Appendix Table 1, which shows quite clearly that contamination is relevant for many possible dimensions of a child's non-cognitive skills.

In Table 5 we push the contamination idea one step further by looking at cases where the father is the main survey respondent and reports on the child non-cognitive skills. Although there are few cases and the sample is not representative of the total population, this exercise is useful for illustrative purposes. The first column of Table 5 uses the mother-reported measure of child skills as the dependent variable, but here we add paternal cognitive and non-cognitive skills to the regression. Similar to the results from Table 4 we find that mother non-cognitive skills are highly correlated with child non-cognitive skills. Paternal skills are also correlated, but the strength of this relationship is significantly weaker. The second column reports results from a similar regression but uses the teacher-reported measures of child non-cognitive skills as the dependent variable instead. As before, we see that the correlation between maternal skills and child non-cognitive skills is reduced, indeed now maternal and paternal skills are similarly related to child non-cognitive skills. The third column uses the father-reported measures of child non-cognitive skills. The striking result here is that father non-cognitive skills now appear to be strongly correlated to child non-cognitive skills.

¹²Specifically, we combine the SDQ sub-scores on 'emotional symptoms' and 'peer problems' into a measure of *internalizing* behaviour, and the SDQ sub-scores on 'hyperactivity/inattention' and 'conduct problems' into a measure of *externalizing* behaviour (Goodman *et al.*, 2010).

¹³One potential concern would be that when the father is the main respondent, this is a signal that the father plays a more prominent role in the household. This could result in a stronger link between the non-cognitive skill of the child and that of the father. However, we examined the link between child non-cognitive skill and parental non-cognitive skill for the *same* children when the *mother* is the main

coefficients across columns one and three are statistically significant, despite the fact that we have a small number of observations where the father is the primary respondent.

The finding that the non-cognitive skills of the reporting parent is always strongly related to the non-cognitive skills of the child suggests that parental measures of child noncognitive skills are influenced by parental skills or parental characteristics and therefore contaminated. This contamination means that parent-reported measures are correlated within a period and across time. Of course, teacher-reported measures of child non-cognitive skills might also be influenced by teacher perceptions or characteristics, introducing a correlation across all teacher-reported measures within the same period. If the goal is to learn about skill evolution and dynamic skill complementarity, how should one proceed when potentially all of the available measures of child non-cognitive skills are flawed? In the next section we develop a framework that will allow us to address the contamination issues under certain assumptions. However, prior to presenting the model, we first illustrate that using any of the contaminated measures to estimate a simple skill technology will likely result in misleading findings.

Table 6 presents estimates of child non-cognitive skills transition functions using combinations of mother and teacher-reported measures of child non-cognitive skills. The first column shows that if a researcher were to use only measures of child non-cognitive skills reported by the mother, child non-cognitive skills would be highly persistent (the persistence coefficient is 0.617). Additionally, there is little evidence that child cognitive skills beget non-cognitive skills, nor do maternal cognitive skills appear to be important. In column three we estimate the same model, using teacher/interviewer-reported child non-cognitive skills as the dependent variable. Here we find that the persistence in child non-cognitive skill is significantly smaller, exhibiting a coefficient of 0.221. We also find that child and maternal cognitive skills are significantly more important for the evolution of child noncognitive skills when we rely on the teacher-reported measures. Finally, columns (2) and (4) include both the mother and teacher-reported measures as inputs in the technology. The takeaway from these regressions is that both measures matter, regardless of whose

respondent (in a different wave of the study). We found that the pattern in column one re-emerges, i.e. if is the mother non-cognitive skill that is most strongly related to the child non-cognitive skill.

measure of child non-cognitive skill we use as the outcome. This suggests that both assessors are providing useful information.¹⁴ However, depending on which measure of child non-cognitive skills a researcher mainly relies upon, the technology estimates can be quite different. ¹⁵

The analysis in this section illustrates the inherent subjectivity of child non-cognitive skill measures and the potential impact that this type of systematic measurement error may have on estimates of the skills production function. However, no definitive conclusions can be reached at this point. As mentioned above, the skill proxies we use are noisy versions of the true underlying skills and the level of noise might differ across periods. Moreover, it remains unclear how much contamination there is in the parent-reported measures as opposed to the teacher-reported measures. The model we present in the next section proposes a new framework which is able to address these issues and allows us to quantify not only the level of contamination but also what the impact of ignoring it is.

4 The Model

4.1 Setup

To analyze in detail the dynamics of child skill development and parental emotional wellbeing, we follow a procedure similar to Cunha et al. (2010). In our model, a child and her parents are followed for T periods.¹⁶ Each child is characterized by cognitive and noncognitive skills, which are unobserved (to the econometrician). Parents are characterized

 $^{^{14}}$ The correlation between parent and teacher proxies of child non-cognitive skills is 0.33. Among all the child and parental skill proxies, this is the highest pairwise correlation for both measures.

¹⁵Though we do not report them here, we also examined how estimates of the transition function for child cognitive and mother non-cognitive skills vary according to the nature of the assessor. For child cognitive skills, we find that mother non-cognitive skills become important only when we utilize the teacher-reported measures of child non-cognitive skills. This makes sense since the mother-reported measures of child non-cognitive skills. In the mother non-cognitive skill transitions, we find evidence of feedback from child to mother when using mother-reported measures of child non-cognitive skills. This feedback effect essentially disappears when we use instead the teacher-reported measures.

¹⁶We formulate the model in the context of a two parent-single child household. For estimation we restrict to two parent households and include controls for being the first born child.

by observed cognitive skills and unobserved non-cognitive skills.¹⁷ The skills of the child in household *i* are denoted by (C_{it}, N_{it}) , where *C* indicates cognitive skill and *N* indicates non-cognitive skill. The cognitive and non-cognitive skills of the parents in household *i* are given by (C_i^P, N_{it}^P) , where $P \in \{M, F\}$ for maternal and paternal skills, respectively. All (child and parental) non-cognitive skills are assumed to evolve over time. Child cognitive skills also change over time, while parental cognitive skills are assumed to be constant.¹⁸

At t = 1, the first period in our model (corresponding to the interview conducted when the child was 3 years old), six unobservable skills are drawn from a joint density. We represent the initial skill draw for household *i* according to

$$S_{i1} \sim F_1,\tag{1}$$

where $S_{i1} = (C_{i1}, N_{i1}, C_i^M, N_{i1}^M, C_i^F, N_{i1}^F)$ is a six-dimensional vector. Child and parental skills evolve over time according to

$$S_{it+1} = f_t (S_{it}) + v_{it+1} \text{ and } v_{it+1} \sim F_{t+1}, \tag{2}$$

with the restriction that $C_{it+1}^P = C_{it}^P = C_{i1}^P$. Since parental cognitive skills are timeinvariant, f_t is essentially a four-dimensional function that represents the law of motion or production function of future child skills and parent non-cognitive skills.

The function f_t can be flexibly specified to allow for the self-productivity of skills, and dynamic complementarities or substitutability across skills types. In other words, maternal and paternal skills can have an impact on child skills that can be an increasing or decreasing function of past child skills. As is standard in this literature, child cognitive skills can foster non-cognitive skills and vice-versa. Maternal and paternal non-cognitive skills are allowed to evolve over time as a function of all other skills, including their child's cognitive and

¹⁷The model can be generalized to the case of unobserved parental cognitive skills. However, since the MCS contains only one informative measure of parental cognitive skills, we cannot treat the underlying skills as unobserved and we need to impose the strong assumption that parental cognitive skills are observed.

¹⁸This assumption, mainly driven by a lack of time varying measures of parental cognitive skills, is justified by the fact that parents are in a stage of life where cognitive skills are fully formed and likely stable over time.

non-cognitive skills. The four elements of the shock, v_{it+1} , can be correlated with each other. While we define f_t and F_t to be general functions. In the estimation section we discuss the parametric assumptions we employ to estimate the model.

The above framework does not explicitly include observed household characteristics in the production of skill. However, in practice we purge our skill measures of demographic and household variables (see footnote 19 for additional detail) such that the unobserved skill components discussed above are orthogonal to these characteristics. Thus, we are implicitly allowing demographics to affect the evolution of skill but in a linearly separable fashion. While it might be important to analyze how demographics interact with unobservable skills, for computational convenience this is left for future research.

4.2 Skill Measures

Child cognitive and non-cognitive skills and parental non-cognitive skills are not directly observed, but multiple noisy measures of these skills are available. We assume that the measurements are generated as follows:

$$M_{ijt}^C = \mu_{jt}^C + \alpha_{jt}^C C_{it} + \epsilon_{ijt}^C \tag{3}$$

$$M_{ijt}^{N^P} = \mu_{jt}^{N^P} + \alpha_{jt}^{N^P} N_{it}^P + \epsilon_{ijt}^{N^P} \quad \text{for } P \in \{M, F\}$$

$$\tag{4}$$

where the above equations refer to measures of child cognitive skills and maternal and paternal non-cognitive skills.¹⁹ The errors, ϵ , are assumed to be independent across individuals, measures, and time periods, and the total number of measures j for each type of skill can vary. Note that there is no measurement equation for parental cognitive skills since we assume these skills are observed.

¹⁹In practice, we first residualize all MCS skill measures (including the child non-cognitive skill measures discussed below and parental cognitive skill measures) by regressing them on child age in months, gender, ethnicity, an indicator for first born child, parent's age at time of birth, and indicators for UK region. We do this because child and parental characteristics may impact the measures directly. However, these observable characteristics can also capture observable components of skill. In this case, our production technology and measurement system is consistent with a model where observables affect skill accumulation but in a linearly separable fashion from the unobserved skill components. Note that if we estimate a version of the model where we do not residualize the MCS skill measures, the impulse response and policy counterfactuals are mostly unaffected.

In the previous section, we present suggestive evidence that parent-reported measures of child non-cognitive skills are likely to be correlated with other key variables, such as parental skills. It is also possible that parent-reported measures of child non-cognitive skills are correlated over time. Thus, the measurement equations for parent-reported measures of child non-cognitive skills can be written as:

$$M_{Pijt}^{N} = \alpha_{P,1jt}^{N} N_{it} + \alpha_{P,2jt}^{N} C_{i}^{P} + \alpha_{P,3jt}^{N} N_{it}^{P} + \alpha_{P,4jt}^{N} \theta_{i} + \epsilon_{P,ijt}^{N}, \qquad (5)$$

where C_i^P and N_{it}^P are parental cognitive and non-cognitive skills, and θ_i is a familyspecific factor that is time invariant. The latter produces a correlation across all parentreported child skills measures which is fixed over time. This might arise if, conditional on parental skills, some parents are more or less likely to classify certain child behaviors as problematic.²⁰

In addition to parents, other survey participants report on the child's non-cognitive skills. These are typically the child's school teachers, although at t = 1 (age 3) this is the interviewer since children are not yet in school. The presence of multiple assessors is key to identify contamination in parent-reported measures of child non-cognitive skills, as we will show. Notice, however, that identification does not rely on the assumption that these additional measures are free of contamination. Indeed, we can allow for correlated measurement error in the teacher-reported measures:

$$M_{T,ijt}^N = \alpha_{T,1jt}^N N_{it} + \alpha_{T,2jt}^N T_{it} + \epsilon_{T,ijt}^N.$$
(6)

Here the random component T_{it} is assumed to be independent of all other variables, and is independent across individuals and time periods but constant across contemporaneous measures for the same individual.²¹ The presence of T_{it} accounts for the idea that some

²⁰The error term $\epsilon_{P,ijt}$ is assumed to be independent across individuals, measures, and time periods.

²¹An important assumption here is that the random teacher effect is independent of all other skills. One concern might be that a teacher's evaluation of child non-cognitive skills is contaminated by child cognitive skills, since cognitive skills development is a primary focus of schooling. This would mean that T_{it} is correlated with C_{it} . However, in Appendix Table 2 we show that the relationship between child cognitive and non-cognitive skills are very similar when we use teacher-reported or interviewer-reported measures of

teachers are more or less likely to classify certain child behaviors as problematic conditional on the child's true non-cognitive skills. The independence of T_{it} across time periods is justified by the fact that a child's teacher typically changes every year.

While the measurement system we employ to identify the dynamic latent factor model is broadly similar to the ones used in the previous literature, what distinguishes our approach is the presence of a within-period correlation in measurement error across measures and the general lack of a dedicated measure of child non-cognitive skills. Although the lack of a dedicated measure is not necessarily a problem for identification (Carneiro *et al.*, 2003), the contemporaneous correlation in measurement error is prohibitive. Previous researchers have illustrated that it is possible to allow for correlated measurement error across periods (Cunha *et al.*, 2010) but not across contemporaneous measures. In the next section we discuss how the availability of multiple assessors of child non-cognitive skills allows us to identify the joint distribution of skills despite the contamination described above.

4.3 Identification

In this section, we briefly describe our approach to identification. A more formal treatment is provided in Appendix A. The components of the model that need to be identified include: the production function (f_t) , the distribution of skill shocks (F_t) , and all the parameters of the measurement equations. The key challenge is to pin down the joint distribution of unobserved skills, S_i . Because parental cognitive skills are observed, they pose no threat to the identification of our model and for the sake of brevity are excluded from the discussion.²² Once the joint density of S_i is identified, we can identify the law of motion of skills, or production function, as the expectation of one skill conditional on past skills. Suppose

child non-cognitive skills. Interviewers are likely less focused on cognitive skills development, suggesting that our assumption is reasonable. A second source of dependence could arise through the sorting of teachers and students. If students are sorted to classrooms based on teachers' characteristics that are relevant for non-cognitive skill reports, our independence assumption would be violated. In Appendix Table 3 we show that the link between child cognitive skills and teacher-reported non-cognitive skills is unaffected when we add observed or unobserved teacher characteristics (these are only available for period 3, i.e. when the child is 7 years old). This indicates that sorting is unlikely to be a concern.

²²As indicated above, a generalization that included unobserved parental cognitive skills would be straightforward in the presence of multiple measures of those skills.

for a moment that we know the joint density of all skills S_i , then for skill Y_{it+1} where $Y \in \{C, N, N^M, N^F\}$ define:

$$f_{t+1}^{Y}(Y_{it}) \equiv E(Y_{it+1}|S_{it})$$
(7)

where the mean of v_{it+1} is normalized to zero. We can then recover $v_{it+1}^Y = Y_{it+1} - E(Y_{it+1}|S_{it})$ and identify F_t using the distribution of v_{it+1}^Y .

Although it is clear that f_t and F_t can be identified when the joint distribution of S_i is known, S_i is unobservable. To identify the joint distribution of S_i , we use the measurement model described in the previous section. Following the approach of Cunha *et al.* (2010), it is straightforward to show that the first and second moments of all unobserved skills *other than* the child non-cognitive skills can be identified by taking the appropriate covariances between measures. For example, after normalizing the loading factor to one on the first measure of each skill the following four covariances,

$$Cov(M_{i1t}^C, M_{i1\tau}^C) = \sigma_{t,\tau}^C \qquad \text{for } t \neq \tau$$
(8)

$$Cov(M_{ijt}^C, M_{i1\tau}^C) = \alpha_{jt}^C \sigma_{t,\tau}^C \qquad \text{for } t \neq \tau$$
(9)

$$Cov(M_{ijt}^C, M_{i1t}^C) = \alpha_{jt}^C \sigma_t^C \tag{10}$$

$$Cov(M_{i1t}^C, M_{i1\tau}^{N^P}) = \sigma_{t,\tau}^{CN^P} \quad \text{for } P \in \{M, F\}$$

$$\tag{11}$$

can be used to identify the covariance of child cognitive skills over time, the loading factor on child cognitive skills across all measures, the variance of child cognitive skills each period, and how child cognitive skills varies with parent non-cognitive skills. A similar set of covariances can be used to identify the second moments related to parental non-cognitive skills.²³

Identifying the second moments related to child non-cognitive skills is more challenging. All the measures related to child non-cognitive skills have additional unobservables that are common across multiple measures. If we took a strategy similar to the one above we would not be able to isolate terms related only to N_{it} . For example, the covariance

²³The variances and covariances of parental cognitive skills are trivially identified since we assume they are observed. If instead parental cognitive skills were unobserved, but multiple measures were available, the same strategy would apply for identifying the relevant loading factors and variances.

between two teacher-reported measures in the same period will also contain the variance of T_{it} . Two assumptions are needed to identify the second moments related to the child's noncognitive skills: (1) the contamination in the parent-reported measures are independent of the contamination in the teacher-reported measures, and (2) the contamination in the teacher-reported measures are independent over time.

Once all the first and second moments of S_i have been identified, we show that the joint distribution of S_i is non-parametrically identified. This is crucial if we want to allow skills evolution to be non-linear. The proof broadly follows Theorem 1 of Cunha *et al.* (2010), though we need to modify it slightly to account for the correlated errors in the child non-cognitive measures.

5 Estimation and Results

Following Attanasio *et al.* (2019) and Agostinelli & Wiswall (2016), estimation of the model proceeds in two steps. First, we flexibly estimate the joint distribution of child and parental skills, along with all the parameters of the measurement system. Second, we draw from the estimated skill distribution and estimate the skill technology.

5.1 Joint Distribution of Skill and Measurement Parameters

The joint distribution of child and parental skills across all time periods is governed by $f_t(\cdot)$ and $F_t(\cdot)$ as defined in the previous section. However, in the first step of the estimation process we do not take a stand on these two functions. Instead, we assume that all child and parental skills are jointly distributed according to a mixture of normal distributions. The flexibility of the mixture distribution puts few restrictions on the precise form that $f_t(\cdot)$ and $F_t(\cdot)$ ultimately take. We parameterize these functions in the second step of our estimation approach.

The joint distribution of child and parental skill takes the following form,

$$g(S_i) = \pi g_1(\gamma_1, \Sigma_1) + (1 - \pi) g_2(\gamma_2, \Sigma_2),$$
(12)

where g_k is a normal density with mean γ_k and variance Σ_k for k = (1, 2).²⁴ π is the mixture weight and determines the probability that a household is drawn from either $g_1(\cdot)$ or $g_2(\cdot)$. Σ_k is constrained to be a symmetric matrix with a positive main diagonal. There are 16 unobserved skills: child cognitive and non-cognitive in each of four periods, and maternal and paternal non-cognitive skill in each of four periods. There are also two observed skills, maternal and paternal cognitive skill. As a result, γ_k is a vector of 18 unknown means, while Σ_k is a matrix with 18 unknown variances, and 144 unknown covariances.²⁵ All of these parameters will be estimated in the first step.

In addition to estimating the parameters that govern the joint distribution of skill, the first step also yields estimates of the measurement equation parameters. The functional form for the measurement equations is provided in equations (3)-(6).²⁶ However, we also need to specify the distributions of the measurement errors (ϵ_{ijt}^C , $\epsilon_{ijt}^{N^P}$, $\epsilon_{P,ijt}^N$, and $\epsilon_{T,ijt}^N$ for $P \in \{M, F\}$) and the teacher (T_{it}) and parent (θ_i) random effects. The measurement errors are normally distributed with means equal to zero and variances to be estimated. The teacher and parent random effects are standard normals to set their scale without loss of generality. All of these components are assumed to be independent of each other and all skills.

Finally, we make one additional restriction to ease the computational burden. Rather than estimate separate loading factors and measurement variances for maternal and paternal measures of child non-cognitive skill, we constrain all of the paternal parameters to be scalars of the maternal parameters. For example, the father's loading on the child's noncognitive skill in period t, $\alpha_{F,1jt}$, is equal to $\alpha_{F,1t} \times \alpha_{M,1jt}$ for all j. Similarly, the loading on the father's cognitive skill in the child's non-cognitive skill measures, $\alpha_{F,2jt}$, is given by $\alpha_{F,2t} \times \alpha_{M,2jt}$ for all j. Notice that we allow these scaling factors to vary by period.

²⁴As discussed in the identification section, we need to normalize the mean of skill to zero. To do this we set $\gamma_2 = \frac{\pi}{1-\pi} \gamma_1$.

 $^{^{25}}$ Although we assume parental cognitive skill is observed, we still estimate the parameters of the two mixing distributions that give rise to the observed distribution.

²⁶At t = 2, t = 3 and t = 4 (i.e. age 5, 7, and 11), the teacher also provides a measure of the child's cognitive skill. We assume that this measure includes not only the child's cognitive skill, but also the random teacher effect. This generates a correlation across these measures that is driven in part by the teacher.

We estimate the skill and measurement parameters outlined above using maximum likelihood. The likelihood contribution of household i is based strictly on the observed measures and skills associated with household i. Define M_i as the vector of all measurements and observed skills associated with household i across all time periods.²⁷ This vector can be written as

$$M_i = \mathcal{A}S_i^+ + \epsilon_i \tag{13}$$

where A is a matrix with 23 columns and as many rows as the total number of skill measurements for household *i*. The number of columns, 23, is the sum of the 18 skills and 5 random effects associated with measurement contamination. Therefore, with probability π the vector of measurements will be normally distributed with a mean of $\mu_1^M = A\gamma_1$ and a covariance matrix $\Sigma_1^M = A\Sigma_1 A' + \Sigma_{\epsilon}$. With probability $1 - \pi$, the vector of measurements will be normally distributed with a mean $\mu_2^M = A\gamma_2$ and a covariance matrix $\Sigma_2^M =$ $A\Sigma_2 A' + \Sigma_{\epsilon}$. Σ_{ϵ} is the (diagonal) covariance matrix of the measurement error. The loglikelihood contribution of household *i* is then given by

$$L_i(\gamma_1, \Sigma_1, \mu_2, \Sigma_2, \pi, \mathbf{A}, \Sigma_\epsilon) = \pi h(M_i; \mu_1^M, \Sigma_1^M) + (1 - \pi) h(M_i; \mu_2^M, \Sigma_2^M)$$
(14)

where $h(\cdot)$ is the multivariate normal density function with the given mean and variance. We maximize the likelihood using a quasi-Newton optimization algorithm.²⁸

The maximum likelihood procedure yields estimates of the joint skill distribution $(\hat{\gamma}_k, \hat{\Sigma}_k, \hat{\pi})$ and of the measurement related parameters (A and Σ_{ϵ}). The former are used primarily as an input into the second estimation step so we do not discuss these results directly. However, we can use the estimated measurement parameters to provide insight into the size of the parental and teacher distortions present in the measures of child non-

²⁷In practice there are missing measures across households and periods.

²⁸We do not observe teacher identifiers for each period in our sample, and therefore assume each child has a different teacher. For the one period we can observe teacher identifiers (t = 3) we find that the average teacher is observed with fewer than two children in the sample. Thus, our assumption regarding different teachers is not particularly restrictive. At t = 1, interviewers assess child non-cognitive skill. On average, each interviewer is observed with more than thirty children. We have experimented with versions of the likelihood that allows for correlations in child skills within interviewer and found little change in the estimated parameters. For that reason we pursue the simpler model.

cognitive skill.

In Table 7, we present the estimated fraction of the variance for each measure that is the result of the true underlying skill and distortion. These fractions do not add to one since part of the variation in each measure is also the result of measurement error. The first panel of the table presents the signal strength of the child cognitive skill measures based upon standardized exams. By construction, there is no contamination in these measures. In the first three periods most of the measures are close to 50% signal, while in the final period the measures are quite a bit noisier. Between t = 2 and t = 4 teachers are also asked to evaluate the cognitive skill of the child. This is a subjective measure which we allow to be affected by the teacher random effect. In the second panel of Table 7 we show that there is a non-trivial amount of noise in these measures that will be correlated with the teacher reported measures of child non-cognitive skill.

The third and fourth panels of the table show the degree of contamination in the parent and teacher reported measures of child non-cognitive skill. The share of the variance in these measures resulting from parent skill, parent random effects, or teacher random effects are quite large. For example, when parents respond regarding the hyperactivity of their child, the share of the variation stemming from distorting components ranges from 12.4% to 18.6% across period. The share of contamination in the teacher responses are even larger. In general, parent reported measures of child non-cognitive skill provide a better signal than teacher reported measures. This is consistent with parents observing their children over longer periods of time and in different social contexts.

5.2 Skill Evolution

In a second step, we focus on estimating the skill transition functions, $f_t(S_{it})$. Although S_{it} is unobserved, we can simulate S_i for a large number of potential households using the previously obtained estimates of the mixture distribution $(\hat{\gamma}_k, \hat{\Sigma}_k, \text{ and } \hat{\pi})$. Estimating a skill transition function using the simulated data simply requires a functional form for $f_t(S_{it})$.

For our preferred specification we assume that the skill transition function takes the

following form,

$$Y_{it+1} = \beta_{Y,1}^t S_{it} + \beta_{Y,2}^t (Y_{it} \times S_{it}) + v_{it+1}^Y$$
(15)

for $Y \in \{C, N, N^M, N^F\}$.²⁹ S_{it} represents the full vector of child and parental skill at time t, while Y_{it+1} and Y_{it} are scalars that reflect the level of specific skill Y in periods t+1 and t respectively. The vector $\beta_{Y,1}^t$ captures the linear impact of past skills on next period skill, while the vector $\beta_{Y,2}^t$ captures how the impact of previous skills varies with the level of skill Y in the previous period. The latter term allows for non-linearity in skill evolution over time, capturing potential patterns of substitutability and complementarity across skills.³⁰ Notice that the parameters of the skill transition function are allowed to vary by period.

Estimates of the skill transition functions are presented in Appendix Table 4.³¹ Skills are not standardized (the observed measures are), so it is difficult to compare effect sizes either across columns or rows. In the next section we provide a more intuitive interpretation of the estimates by exploring how a shock to skills in one period impacts skills in a future period in terms of standardized measures. However, we present the coefficient estimates themselves to highlight some basic overarching patterns. First, self-productivity is generally high. This means, for example, that the strongest predictor for child cognitive skills is child cognitive skills in the previous period. There is also evidence of cross-productivity in skill formation. Especially in the early years, child non-cognitive skills appear to foster her cognitive skills, but this is not so much the case later on. Additionally, the non-cognitive skill of the child seems to have a statistically significant impact on the evolution of the non-cognitive skill of the parents, especially the mother.

Finally, there is evidence of non-linearity in the skill transition functions, though it appears rather small in magnitude. For example, at older ages the marginal impact of child

²⁹Recall that parental cognitive skills are assumed to be fixed and thus we do not model their evolution. ³⁰We have also estimated log-CES type specifications similar to the ones employed in Cunha et al. (2010). While we cannot compare the coefficients of these models, the policy simulations we present in the next section do not change significantly if we use the log-CES specification. We prefer to use the non-linear specification in equation (16) as the parameter have a more immediate interpretation.

³¹Standard errors are calculated through a bootstrap procedure. We randomly sample with replacement from our original sample, and repeat the first estimation step. We then simulate data using the estimated skill distribution based on the bootstrap sample, and re-estimate the production function. We do this 30 times and calculate the standard deviation of the skill technology estimates across bootstrap samples.

cognitive skill at time t on child non-cognitive skill at time t + 1 is decreasing in the level of child non-cognitive skill at time t. There are also statistically significant interactions between parental non-cognitive skill and child non-cognitive skill in producing parental non-cognitive skill in the next period. Again, the non-linear effects are small.

6 Assessing the Impact of Distortions

While estimates of the skill transition and measurement equations are informative, it is difficult to fully understand how distortions in child non-cognitive measures impact our understanding of skill production. In this section, we pursue two types of counterfactuals to illustrate the importance of accounting for distortions in parental and teacher reports of child non-cognitive skill.

6.1 Impulse Response

The first set of counterfactual exercises we pursue is to increment child and parental skills in various periods and observe how final skills are affected. These impulse response functions provide an intuitive way to understand the relationships between skills across various periods of childhood. We compare the results from our baseline model with two alternative models that neglect contamination issues.

The first alternative model is one where we ignore the availability of teacher-reported, child non-cognitive skill measures. Thus, we re-estimate the entire model relying only on parental measures of child non-cognitive skills. In this model we do not include any measurement distortions in child non-cognitive skill measures since it is not possible to identify the related parameters when measures come from one type of assessor only. Most research on child non-cognitive skill development is estimated in such a fashion since most datasets lack teacher interviews.

Even when teacher interviews are available, a naive approach would be to simply combine their assessments of the child with parental assessments and estimate a standard model. We investigate whether this approach yields significantly different conclusions regarding the production of skill.

Tables 8 and 9 show how boosting child and parental skills at t = 1 (age 3) impacts average child and parental skills at t = 4 (age 11) for our main, parent-only, and parentteacher models.³² All skill boosts are standardized to reflect a one standard deviation increase, and the resulting impact is standardized according to the relevant skill distribution in the final period. As an example, the first number in Table 8 indicates that a one standard deviation increase in child cognitive skill at t = 1 leads to a 0.578 standard deviation increase in child cognitive skill at t = 4. This effect includes not just a self-productivity effect, but all the cross-skill effects accumulating over time.

There are a number of interesting findings in Table 8. Focusing first on our main specification, we find that child cognitive skill and paternal cognitive skill have the largest impact on average child cognitive skill at t = 4. For child non-cognitive skill at t = 4, it is child non-cognitive skill at t = 1 and maternal non-cognitive that have the largest impact.

When we compare our main specification with the models that ignore contamination in child non-cognitive skill measures, we see important differences. When only parental measures are employed, the self-productivity of child skills is about 25% larger. Also, we find a much smaller role for maternal non-cognitive skills in fostering child non-cognitive skills. These findings are not surprising in light of the fact that when only parental measures are available, the mother's skills - which are highly persistent - are partially absorbed by the child non-cognitive skills. In the model that takes into account both teacher and parent measures of child non-cognitive skills but ignores contamination, the persistence of child non-cognitive skill is similar to our preferred specification. This occurs because the parental and teacher distortions tend to balance out. A significant portion of the variation in the teacher reported measures of child non-cognitive skills is related to the teacher and does not persist over time, which tends to push down the persistence measure. The estimated cross-productivity of skill is also affected by this, and as a result is much smaller. Finally, fathers are found to be more important for child non-cognitive skill development in both

³²We can also investigate the skill impacts at t = 2 and t = 3 after increasing skills at t = 1. Alternatively we can boost skills at ages t = 2 and examine the subsequent effects. We focus on the endpoints for illustration purposes.

models that ignore contamination. This happens because the father's skills can act as a proxy for the true underlying skill of the child since they are highly correlated.

Table 9 focuses instead on the impact skill changes at t = 1 have on parental noncognitive skills at t = 4. Again we see that the self-productivity of skill is high. Boosting maternal or paternal non-cognitive skill when the child is 3 leads to approximately a third of a standard deviation increase in the corresponding skill when the child is age 11. Most interesting is the fact that the non-cognitive skill of the child can influence the non-cognitive skill of the mother. A one standard deviation increase in child non-cognitive skill at age 3 leads to a 0.123 standard deviation increase in maternal non-cognitive skill when the child is 11. For fathers, the effect is only 0.056. There are few differences between the main specification and the no contamination models since the parental measures themselves are not distorted and the self-productivity effects tend to dominate.

6.2 Evaluating the Impact of Child Care Policies

The impulse response functions studied in the previous section show how accounting for contamination in the measures of child non-cognitive skills influences our predictions for child and parental skill evolution. Another way to illustrate the importance of contaminated measures is to examine how the estimated impact of policies aimed at fostering child development are affected by reporting bias. Any child-centered policy that either directly or indirectly influences parents can generate spurious impacts on child non-cognitive skill measures. For example, a number of papers estimate the impact universal child care programs have on child non-cognitive skills and/or maternal non-cognitive skills.³³ As our results illustrate, part of the estimated impact of universal child care on child non-cognitive skills is most likely due to the contamination driven by changes in maternal skill.

Using our model we can quantify the effect of contamination in parent-reported measures of child non-cognitive skills under various policy scenarios. For the sake of brevity, we consider two simple policy experiments. First, we consider a policy that shifts maternal

 $^{^{33}}$ See Yamaguchi et al. (2018), Haeck et al. (2019), Baker et al. (2008) and Gupta & Simonsen (2010) among others.

non-cognitive skills by one standard deviation in the first period, holding fixed the initial level of all other child and parental skills. This policy will have a real impact on child non-cognitive skills in all subsequent periods through the skills transition function (the indirect effect). However, the change in maternal non-cognitive skills will also impact *measures* of child non-cognitive skills (the direct effect). Second, we consider a policy where we increase both maternal and child non-cognitive skills in the first period by 20% of a standard deviation.³⁴ We then ask how much of the *measured* increase in child non-cognitive skills is the result of contamination, i.e. the direct effect of a change in maternal non-cognitive skills.

Figures 1 and 2 (corresponding to policy experiments 1 and 2) illustrate that contamination in child non-cognitive skills measures can pose a serious threat to policy evaluations. Each panel in the figures represents a simulated SDQ measure of child non-cognitive skill, mimicking the exact measures we observe in the data. We focus on the SDQ measures because among our measures these are the most commonly utilized measures in the literature.³⁵ Variation along the x-axis of each panel reflects the different waves when child non-cognitive skill is measured. The dark vertical bars represent the 'real' effect of the policy in measured child non-cognitive skill once contamination has been netted out. The light grey bars show the overall effect of the policy over time, including the 'real' effect and the effect resulting from contamination.

For the policy experiment where only maternal skill is directly affected, the contamination effects are large. Figure 1 illustrates that the effect on child SDQ measures in the first period are on the order of 0.075 to 0.15 standard deviations. However, these effects are entirely spurious. Any increase in maternal non-cognitive skill can only influence child non-cognitive skill in subsequent periods by construction. Thus, starting in period 2 there will be a real effect that works through the production technology. For example,

 $^{^{34}}$ This is a large effect, but it is similar in magnitude to the estimates in Baker *et al.* (2008). Here the authors find that the introduction of universal child care subsidies in Quebec increased child care use and maternal labour supply, with negative impacts on maternal well-being and child emotional and social development.

³⁵In Appendix Tables 5 and 6, we present tabular versions of the estimates for all child non-cognitive skill measures.

in period 2 the SDQ emotional measure increases by 0.147 standard deviations, where approximately a third of the effect is real. In some cases it is not only the magnitude, but the dynamic patterns that are affected by contamination. For SDQ conduct, the evolution of the measure indicates that the policy effect fades out over time, but the real change is fairly constant. Ultimately, the purpose of this first policy simulation is to show the extent to which policy estimates can be biased as the result of contaminated child non-cognitive skill measures. A standard deviation increase in maternal non-cognitive skill can translate into a 0.15 standard deviation spurious increase in child non-cognitive skills.

Figure 2 instead illustrates the impact of a policy in which both maternal and child non-cognitive skills increase by 0.2 standard deviations in the first period. This policy is motivated by the findings in Baker *et al.* (2008), who found similar sized effects on maternal and child non-cognitive skills.³⁶ Looking again at SDQ emotional, we see that in the first period the measure increases by approximately 0.12 standard deviations.³⁷ However, the dark grey bar indicates that almost a quarter of this effect is the result of contamination. Across all periods and measures, we find that the size of the distortion ranges between 3% and 24%. More generally, the direction of the distortion will depend on whether the policy under consideration affects mothers and children in the same direction. If for example, a universal child care policy negatively affects the non-cognitive skill of mothers, but positively affects those of the children, the distortion would lead to an underestimate of the policy on child non-cognitive skill.

These simulations suggest that relying on parental measures of child non-cognitive skill to evaluate programs or policies that also influence parental non-cognitive skills is highly problematic. We are not the first to point this out, as Baker *et al.* (2008) explicitly recognize this as a potential threat to their findings. Are there clear solutions to this issue? First, it appears that having multiple assessors of the child is useful. This is because while each assessor is potentially biased, when the distortions are unrelated across assessors we

 $^{^{36}}$ Baker *et al.* (2008) find that the policy had negative impacts on both mothers and children. For illustrative purposes we find it convenient to reverse the sign of the impacts.

³⁷The increase is less than 0.2 due to measurement error, as the SDQ emotional is a noisy measure of the child's true underlying skill. Evaluating any policy, including those that do not directly affect maternal non-cognitive skill, will suffer from this measurement problem.

can obtain a clearer signal of child non-cognitive skills. Second, objective measures of child behavior, such as the number of emotional outbursts in the past week, might be less sensitive to parental distortion. Additional research is needed to determine whether this is a reasonable alternative.

7 Conclusion

Researchers are forced to rely upon externally reported measures of child non-cognitive skill when studying skill formation since small children are not capable of assessing their own behaviors and emotional well-being. However, external evaluators bring their own skills and traits to these evaluations, potentially contaminating measures of child non-cognitive skill. In this paper we show that contamination in measures of child non-cognitive skill can significantly affect our basic understanding of child skill dynamics. Additionally, when parental skills contaminate measures of child non-cognitive skill, it is difficult to evaluate the effect of policies on child non-cognitive skill when parental skills are also impacted.

A key finding of the current paper is that having multiple evaluators is critical to mitigating contamination issues. Going forward, data collection efforts that seek to study child development should attempt to include evaluations of the child beyond just parents. Alternatively, surveys could collect more objective measures of child non-cognitive skill related directly to observed behaviors.

The measurement issues we highlight are not necessarily specific to child non-cognitive skill. We assume that adult reports of their own non-cognitive skill are uncontaminated, but it would be interesting to obtain external evaluations or direct behaviors that could speak to this. Other literatures have faced similar issues. As an example, self-reported health status is likely impacted by individual interpretations of what constitutes excellent health or by a reference-level of health.³⁸ Researchers attempt to deal with these measurement concerns by using fixed-effect type models or incorporating objective health measures, such as blood pressure or cholesterol. Interest in the development of human capital is unlikely

³⁸See for example Michael Baker & Deri (2004), Bound (1991).

to wane and developing new techniques to address measurement concerns is a fruitful area for additional research.

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Table 1: Descripti	ve Statistics
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	All	families	Two-pa	rent families	Two-pai	rent families	Two-par	rent families
	t = 0) or $t = 1$ $t = 0$ or $t = 1$		t	t = 0 or t = 1		t = 1 or later	
	mean	std. dev. (1)	mean	std. dev. (2)	mean	std. dev. (3)	mean	std. dev. (4)
% Male % White % Single mothers % First born child Child's age Number of siblings Mother's age at birth Father's age at birth Mother's years of schooling Father's years of schooling England Wales Scotland	$\begin{array}{c} 0.51 \\ 0.87 \\ 0.15 \\ 0.42 \\ 0.93 \\ 0.90 \\ 28.89 \\ 32.09 \\ 12.06 \\ 12.13 \\ 0.82 \\ 0.05 \\ 0.09 \end{array}$	0.52 1.02 5.84 5.90 1.79 1.90	$\begin{array}{c} 0.51 \\ 0.88 \\ 0.00 \\ 0.41 \\ 0.91 \\ 0.91 \\ 29.83 \\ 32.34 \\ 12.28 \\ 12.18 \\ 0.83 \\ 0.05 \\ 0.09 \end{array}$	$\begin{array}{c} 0.48 \\ 1.00 \\ 5.35 \\ 5.75 \\ 1.82 \\ 1.91 \end{array}$	$\begin{array}{c} 0.51 \\ 0.88 \\ 0.00 \\ 0.41 \\ 0.91 \\ 0.91 \\ 29.84 \\ 32.35 \\ 12.28 \\ 12.18 \\ 0.83 \\ 0.05 \\ 0.09 \end{array}$	$\begin{array}{c} 0.48\\ 1.00\\ 5.35\\ 5.74\\ 1.82\\ 1.91\end{array}$	$\begin{array}{c} 0.51 \\ 0.88 \\ 0.00 \\ 0.40 \\ 3.33 \\ 1.22 \\ 30.05 \\ 32.48 \\ 12.35 \\ 12.22 \\ 0.83 \\ 0.05 \\ 0.09 \end{array}$	$\begin{array}{c} 0.90\\ 0.99\\ 5.23\\ 5.63\\ 1.83\\ 1.91\end{array}$
Northern Ireland	0.03		0.03 0.03		0.03		0.03	
Ν.	19,048		14,648		14,598		12,530	

Notes: UK Millennium Cohort Study. Sampling weights used throughout. Data on father's schooling only available for fathers in two-parent families. Column (1) is based on the whole sample of children who enter the study for the first time either in t = 0 or t = 1 (these correspond to the first and second wave of the study, when children were 9 months and 3 years old, respectively); column (2) restricts to the sub-sample of two-parent families; column (3) restricts further by eliminating observations with missing values on the demographics; column (4) shows our final estimation sample, which includes only two-parent families observed from t = 1 onwards.

		t = 2(age 5)		
Administered by interviewer Bracken School Readiness	\checkmark			
BAS Naming Vocabulary BAS Picture Similarities	\checkmark	\checkmark		
BAS Patterns Comprehension BAS Word Reading		\checkmark	\checkmark	
NFER Progress in Maths BAS Verbal Similarities			\checkmark	\checkmark
Cambridge Gambling Task: quality Spatial Working Memory Task: strategy				\checkmark
Spatial Working Memory Task: total errors				\checkmark
Assessed by teacher FSP (Reading, writing, calculating etc.)		\checkmark	/	1
Subject assessment (English, Maths and Science)			✓	✓

Table 2: Child Cognitive Measures

Notes: UK Millennium Cohort Study.

	$\begin{array}{c c} t = 1 \\ (age 3) \end{array}$	t = 2 (age 5)	t = 3 (age 7)	t = 4 (age 11)
Reported by the mother	<i>,</i>	,	,	ý
SDQ (hyperactivity, conduct, emotional, peer)	\checkmark	\checkmark	\checkmark	\checkmark
CSB (independence, emotional, cooperation)	✓	\checkmark	\checkmark	
Reported by interviewer				
Behavior score (extreme behaviour, attention, cooperation)	\checkmark			
Reported by teacher				
FSP (dispositions, social, emotional)		\checkmark		
SDQ (hyperactivity, conduct, emotional, peer)			\checkmark	\checkmark
- , , , , , , , , , , , , , , , , , , ,				

Notes: UK Millennium Cohort Study.

	Child Non-Cognitive $Skill_t$						
	Mother	Reported	Teach	er/Interv. Reported			
	(1)	(2)	(3)	(4)			
Child Cognitive $Skill_t$	0.206**	0.191**	0.291**	0.270**			
	(0.007)	(0.007)	(0.007)	(0.007)			
Mother Cognitive $Skill_t$	0.117^{**}	0.094^{**}	0.021**	0.019^{*}			
	(0.007)	(0.008)	(0.006)	(0.006)			
Mother Non-Cognitive $Skill_t$	0.299^{**}	0.291^{**}	0.071**	0.068**			
	(0.007)	(0.007)	(0.007)	(0.007)			
Missing Indicators	Y	Y	Y	Y			
Demographics	Ν	Υ	N	Y			
N	$33,\!905$	$33,\!905$	26,818	26,818			
R^2	0.182	0.208	0.1106	0.138			

Table 4: Evidence	of Distortions in	Child Non-Cognitive	Skill Measures
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Notes: UK Millennium Cohort Study, data from t = 1 to t = 4 (child age 3, 5, 7 and 11). Estimates are obtained using linear regressions. Variables representing child skills and parental skills are transformed by principal component analysis into factors with mean 0 and standard deviation 1. Demographic variables include: gender of the child, ethnicity, age of the child (in months) and its square, maternal age (in years) and its square, number of siblings, weekly family income and region of birth. Standard errors are clustered at the individual level. + p < 0.1, * p < 0.05, ** p < 0.01

	Chi	ld Non-Cognitive Sk	xill _t
	Mother Reported	Teacher Reported	Father Reported
	(1)	(2)	(3)
Child Cognitive Skill	0.186^{**}	0.267^{**}	0.152^{**}
	(0.007)	(0.007)	(0.046)
Mother Cognitive Skill	0.080^{**}	0.013^{+}	0.033
	(0.009)	(0.007)	(0.057)
Mother Non-Cognitive Skill	0.281^{**}	0.058^{**}	0.155^{*}
	(0.007)	(0.007)	(0.053)
Father Cognitive Skill	0.034^{**}	0.011	0.068
	(0.009)	(0.007)	(0.051)
Father Non-Cognitive Skill	0.044^{**}	0.040**	0.225^{**}
	(0.007)	(0.007)	(0.046)
Missing Indicators	Y	Y	Y
Demographics	Y	Υ	Υ
N	$33,\!905$	$26,\!818$	443
R^2	0.207	0.138	0.304

Table 5: Additional Evidence of Distortions in Child Non-Cognitive Skill Measures

Notes: UK Millennium Cohort Study, data from t = 1 to t = 4 (child age 3, 5, 7 and 11). Estimates are obtained using linear regressions. See footnote to Table 4 for the definition of variables. Standard errors are clustered at the individual level.⁺ p < 0.1, * p < 0.05, ** p < 0.01

	Child Non-Cognitive $Skill_{t+1}$					
	Mother	Reported	Teacher	Reported		
	(1)	(2)	(3)	(4)		
Child Cognitive $Skill_t$	0.057**	0.039**	0.156^{**}	0.134^{**}		
	(0.006)	(0.006)	(0.010)	(0.010)		
Mother Reported Child Non-Cognitive $Skill_t$	0.617^{**}	0.604^{**}		0.134^{**}		
	(0.007)	(0.007)		(0.010)		
Teacher Reported Child Non-Cognitive Skill_t		0.064^{**}	0.221^{**}	0.200^{**}		
		(0.007)	(0.011)	(0.012)		
Mother Cognitive Skill	0.016^{**}	0.018 **	0.035^{**}	0.023^{*}		
	(0.006)	(0.006)	(0.009)	(0.009)		
Mother Non-Cognitive $Skill_t$	0.070^{**}	0.072^{**}	0.059^{**}	0.027^{**}		
	(0.006)	(0.007)	(0.009)	(0.010)		
Father Cognitive Skill	0.023^{**}	0.025^{*}	0.033**	0.032^{**}		
	(0.006)	(0.006)	(0.009)	(0.010)		
Father Non-Cognitive $Skill_t$	0.024^{**}	0.024^{**}	0.031^{**}	0.025^{*}		
	(0.006)	(0.006)	(0.009)	(0.009)		
N	22,371	18,271	13,648	13,197		
R^2	0.466	0.465	0.166	0.180		

Table 6: Skill Technology Distortions

UK Millennium Cohort Study, data from t = 1 to t = 4 (child age 3, 5, 7 and 11). Estimates are obtained using linear regressions. Definition of variables and demographic controls as in footnote to Table 4. Standard errors are clustered at the individual level.⁺ p < 0.1, * p < 0.05, ** p < 0.01

	<i>t</i> =	$t = 1 \qquad t = 2$		<i>t</i> =	t = 3		= 4	
	signal	cont.	signal	cont.	signal	cont.	signal	cont.
Child Cognitive - Test Based								
Braken	58.1%	-						
BAS Naming Vocabulary	41.9%	-	36.7%	-				
BAS picture similarity			21.5%	-				
BAS pattern compr.			32.5%	-	33.3%	-		
BAS Word Recognition					52.1%	-		
NFER in Math					53.0%	-		
BAS verbal							25.7%	-
GTC quality							6.9%	-
CANTAB swm strat							12.4%	-
CANTAB swm err							24.1%	-
Child Cognitive - Teacher Re	eported							
FSP			31.8%	24.9%				
Teacher assessment					61.1%	5.7%	70.7%	3.8%
Child Non-Cognitive - Paren	t Report	ed						
SDQ emotional	20.9%	5.9%	28.6%	8.1%	30.3%	8.4%	34.0%	6.3%
SDQ conduct	35.3%	9.9%	34.6%	12.8%	35.5%	13.7%	29.7%	10.3%
SDQ hyperactivity	33.3%	12.4%	38.1%	18.0%	40.4%	18.6%	41.1%	14.6%
SDQ peer	20.0%	5.6%	33.7%	8.3%	35.5%	7.8%	49.4%	4.8%
Q. Independence	6.5%	0.1%	18.0%	1.0%	26.9%	1.7%		
Q. Emotional	36.4%	10.7%	40.1%	14.2%	40.6%	15.2%		
Q. Cooperation					32.0%	6.8%		
Child Non-Cognitive - Intver	viewer/	Teacher	Reported					
Focus	5.7%	94.1%						
Cooperation	2.7%	37.9%						
Extreme behavior	4.4%	11.4%						
FSP, personal			8.9%	44.9%				
FSP, social			8.5%	65.3%				
FSP, emotional			9.3%	70.9%				
SDQ emotional					6.9%	3.5%	15.6%	3.4%
SDQ conduct					7.8%	37.1%	12.7%	32.6%
SDQ hyperactivity					14.9%	53.6%	18.2%	42.1%
SDQ peer					11.8%	13.6%	17.6%	7.6%

Table 7: Contamination in measurements

UK Millennium Cohort Study. Estimates are obtained using the model and estimation method outlined in sections 4 and 5. Each entry in this table represents the fraction of the variance of a given measurement that is explained either by the true skill (signal) or by the contamination. Consider for example a parent reported measure of the child non-cognitive skills. This measure can be written as

$$M_{Pijt}^N = \alpha_{P,1jt}^N N_{it} + \alpha_{P,2jt}^N C_i^P + \alpha_{P,3jt}^N N_{it}^P + \alpha_{P,4jt}^N \theta_i + \epsilon_{P,ijt}^N \theta_i$$

The signal corresponds to the fraction of the total variance that is explained by the variance of $\alpha_{P,1jt}^N N_{it}$. The contamination corresponds to the fraction of the variance that is explained by the variance of $\alpha_{P,2jt}^N C_i^P + \alpha_{P,3jt}^N N_{it}^P + \alpha_{P,4jt}^N \theta_i$.

	Main Specification		С	Inly	All m	easures
			Parental	l measures	(No dis	tortions)
	C_4	N_4	C_4	N_4	C_4	N_4
$+1sd$ in C_1	0.578^{*} (0.018)	0.110^{*} (0.016)	0.593	0.115	0.633	0.072
$+1sd$ in N_1	0.088^{*} (0.019)		0.074	0.533	0.009	0.488
$+1sd$ in C_1^M	0.074^{*} (0.016)	0.008 (0.028)	0.077	0.001	0.069	0.010
$+1sd$ in N_1^M	$0.005 \\ (0.018)$	0.102^{*} (0.031)	-0.006	0.051	0.003	0.049
$+1sd$ in C_1^F	0.116^{*} (0.013)	0.044^{*} (0.012)	0.110	0.050	0.099	0.059
$+1sd$ in N_1^F	0.023 (0.016)		0.026	0.025	0.015	0.034

Table 8: Impulse Response: Child Skills

UK Millennium Cohort Study. Estimates are obtained using the model and estimation method outlined in sections 4 and 5. In each row we report the impact on children skills at age 11 when different skills are shocked by 1sd when the children are 3 years old. The first two columns refer to our main specification. The second two at a model that does not correct for the contamination and utilizes only parental reported children non-cognitive measures. The last two columns finally refers to a model that does not correct for the contamination and utilizes all available measures. Standard errors are obtained bootstrapping 30 times the original data set at the individual level and are available only for the main specification. * p < 0.05

	Main Spe	ecification	(Dnly	All measures	
			Parenta	l measures	(No dist	tortions)
	N_4^M	N_4^F	N_4^M	N_4^F	N_4^M	N_4^F
$+1sd$ in C_1	0.002 (0.011)	0.022 (0.012)	0.013	0.023	-0.005	0.014
$+1sd$ in N_1	0.123^{*} (0.012)	0.056^{*} (0.010)	0.116	0.061	0.115	0.065
$+1sd$ in N_1^M	0.309^{*} (0.014)	0.077^{*} (0.012)	0.286	0.072	0.291	0.074
$+1sd$ in N_1^F	0.066^{*} (0.014)	0.359^{*} (0.019)	0.072	0.354	0.080	0.363

Table 9: Impulse Response: Parental Skills

UK Millennium Cohort Study. Estimates are obtained using the model and estimation method outlined in sections 4 and 5. In each row we shock a different skill by 1sd when the child is 3 years old, and report the impact on parental skills when the child is 11. The first two columns refer to our main specification. The second two at a model that does not correct for the contamination and utilizes only parental reported children non-cognitive measures. The last two columns finally refers to a model that does not correct for the contamination and utilizes all available measures. Standard errors are obtained bootstrapping 30 times the original data set at the individual level and are available only for the main specification.* p < 0.05

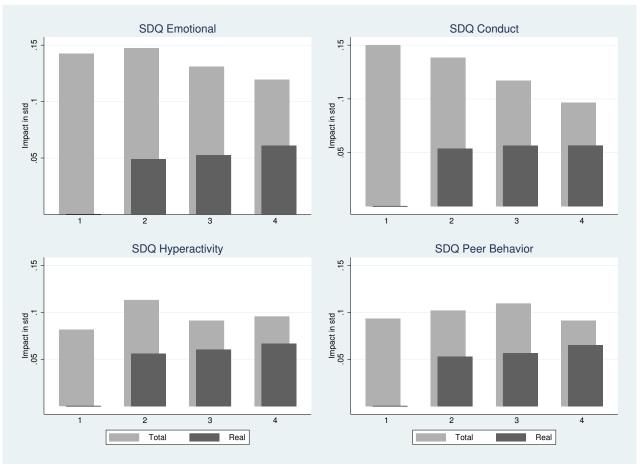


Figure 1: Policy evaluation with contamination: change in maternal skills only

Notes: Estimates are obtained using the model and estimation method outlined in sections 4 and 5. The light grey bars indicate the increase in each SDQ measure in different periods after we increase maternal non-cognitive skills by 1sd in period 1. The dark bars indicate the same impact after we have removed the contamination from those measures.

Source: UK Millennium Cohort Study.

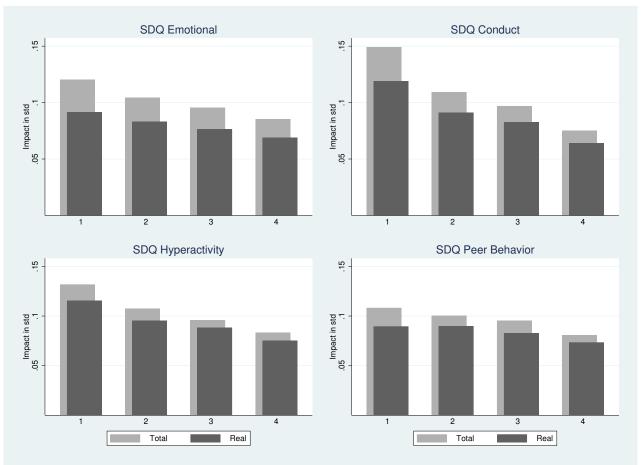


Figure 2: Policy evaluation with contamination: change in maternal and child skills



Notes: Estimates are obtained using the model and estimation method outlined in sections 4 and 5. The light grey bars indicate the increase in each SDQ measure in different periods after we increase maternal non-cognitive skills and children non cognitive skills by 0.2sd in period 1. The dark bars indicate the same impact after we have removed the contamination from those measures.

A Identification

In this section, we consider the identification of f_t , F_t , and all the parameters of the measurement equations. We proceed in two steps. First, we show how to identify the first and second moments of the joint density of S_i (along with the measurement equations). Next, we show that the density of S_i is non-parametrically identified. Once the joint density of S_i is identified, we can identify the law of motion of skills, or production function, as the expectation of one skill conditional on past skills. For skill Y_{it+1} for $Y \in \{C, N, N^M, N^F\}$ define:

$$f_{t+1}^Y(Y_{it}) \equiv E(Y_{it+1}|S_{it})$$

where the mean of v_{it+1} is normalized to zero. We can then recover $v_{it+1}^Y = Y_{it+1} - E(Y_{it+1}|S_{it})$ and identify F_t using the distribution of v_{it+1}^Y .

Although it is clear that f_t and F_t can be identified when the joint distribution of S_i is known, S_i is unobservable.³⁹ To identify the joint distribution of S_i , we turn to the measurement model (see equations 3 to 6 for the definition of the loading factors and their subscripts). Consider first the identification of the second moments of the joint density of S_i . Following Cunha et al. (2010), we normalize $\alpha_{1t}^Y = 1$ for all skills (normalize $\alpha_{T,11t}^N = 1$ for child non-cognitive skill). Also, normalize $\alpha_{P,411}^N = \alpha_{T,21t}^N = 1$ to set the scale of the random variables θ_i and T_i respectively. Finally, we normalize the means of S and ϵ to be equal to zero. As mentioned in the paper, all measures can be purged of the effect of observables in a first stage, i.e. we can identify the impact of observables looking at the conditional means of the measures. This is consistent with a model in which observables may affect directly the measure but also affect the production function of skills, although in a linearly separable fashion.

³⁹As discussed in the text, we treat parental cognitive skills as if they are observed since we lack multiple reliable measures. The observability of these skills does not affect our proof of identification and we exclude them from the discussion. The alternative would be to assume multiple perfect measures for parental cognitive skill and include them in the vector of available measures. Note that if multiple noisy measures of parental cognitive skills were available, it would be straightforward to show that the joint distribution of S_i including unobserved parental cognitive skills is identified.

Using the measurement equations of Equation (5), consider the following covariances:

$$\begin{aligned} Cov_1 &= Cov(M_{i1t}^{N^P}, M_{i1\tau}^{N^P}) = \sigma_{t,\tau}^{N^P} \quad \text{for } t \neq \tau \\ Cov_2 &= Cov(M_{ijt}^{N^P}, M_{i1\tau}^{N^P}) = \alpha_{jt}^{N^P} \sigma_{t,\tau}^{N^P} \quad \text{for } t \neq \tau \\ Cov_3 &= Cov(M_{ijt}^{N^P}, M_{i1t}^{N^P}) = \alpha_{jt}^{N^P} \sigma_t^{N^P} \\ Cov_4 &= Cov(M_{i1t}^{N^P}, C_i^P) = \sigma_t^{N^P C^P} \\ Cov_5 &= Cov(C_i^P, C_i^P) = \sigma_{t,\tau}^{C} \quad \text{for } t \neq \tau \\ Cov_7 &= Cov(M_{ijt}^C, M_{i1\tau}^C) = \alpha_{jt}^C \sigma_{t,\tau}^C \quad \text{for } t \neq \tau \\ Cov_8 &= Cov(M_{ijt}^C, M_{i1\tau}^C) = \alpha_{jt}^C \sigma_t^C \\ Cov_9 &= Cov(M_{i1t}^C, M_{i1\tau}^P) = \sigma_{t,\tau}^{CN^P} \\ Cov_{10} &= Cov(M_{i1t}^C, M_{i1\tau}^P) = \sigma_t^{CN^P} \\ Cov_{11} &= Cov(M_{i1t}^{N^P}, M_{i1\tau}^{N^P}) = \sigma_t^{N^P 1N^{P_2}} \\ Cov_{12} &= Cov(M_{i1t}^{N^P}, C_i^P) = \sigma_t^{N^P 1N^{P_2}} \\ Cov_{13} &= Cov(M_{i1t}^{N^P}, C^{P_2}) = \sigma_t^{N^P 1C^{P_2}} \end{aligned}$$

where $j \neq 1$, and $P_2 = M$ if $P_1 = F$ or $P_2 = F$ if $P_1 = M$. Cov_1 directly identifies $\sigma_{t,\tau}^{N^P}$. From the ratio of Cov_2 and Cov_1 we can identify $\alpha_{jt}^{N^P}$. With this knowledge, we can use Cov_3 to identify $\sigma_t^{N^P}$. Cov_4 directly identifies $\sigma_t^{N^P C^P}$. Cov_5 , which truly is a variance, identifies σ_{c}^{C} (remember our assumption that parental cognitive skills are observed). Cov_6 directly identifies $\sigma_{t,\tau}^C$. From the ratio of Cov_7 and Cov_6 we can identify α_{jt}^C which can then be used in Cov_8 to identify σ_t^C . Finally using Cov_9 to Cov_{13} , all second moments relative to C_t , C^P and N_t^P can be identified. Using the variance of a measurement M_{ijt}^Y for $Y \in \{C, N^M, N^F\}$ will identify the variance of the measurement error ϵ_{ijt}^Y .

To identify the second moments related to child non-cognitive skill, consider first the

teacher reported measures and the following covariances:

$$Cov(M_{T,i1t}^N, M_{T,i1\tau}^N) = \sigma_{t,\tau}^N \quad \text{for } t \neq \tau$$
$$Cov(M_{T,ijt}^N, M_{T,i1\tau}^N) = \alpha_{T,1jt}^N \sigma_{t,\tau}^N \quad \text{for } t \neq \tau$$

These two observable covariances identify the covariance of non-cognitive skills across different time periods and the loading factor relative to the teacher measure. Using the covariance of M_{Ti1t}^N with the child cognitive and parental cognitive and non-cognitive measures utilized previously, we can also identify the covariances between child non-cognitive skills and these other skills. Unfortunately, using just teacher reported measures of child non-cognitive skill we cannot identify $\sigma_{t,t}^N$, $\alpha_{T,2jt}^N$, and the variance of T_{it} . Consider now the parent reported measures of child non-cognitive skill and the following covariances

$$Cov(M_{P,ijt}^{N}, M_{i1t}^{C}) = \alpha_{P,1jt}^{N} \sigma_{t,t}^{NC} + \alpha_{P,2jt}^{N} \sigma_{t}^{C^{P}C} + \alpha_{P,3jt}^{N} \sigma_{t,t}^{N^{P}C}$$
$$Cov(M_{P,ijt}^{N}, C_{i}^{P}) = \alpha_{P,1jt}^{N} \sigma_{t}^{NC^{P}} + \alpha_{P,2jt}^{N} \sigma^{C^{P}} + \alpha_{P,3jt}^{N} \sigma_{t}^{N^{P}C^{P}}$$
$$Cov(M_{P,ijt}^{N}, M_{i1t}^{N^{P}}) = \alpha_{P,1jt}^{N} \sigma_{t,t}^{NN^{P}} + \alpha_{P,2jt}^{N} \sigma_{t}^{C^{P}N^{P}} + \alpha_{P,3jt}^{N} \sigma_{tt}^{N^{P}}.$$

All of the covariances in the above system have been previously identified. Thus, as long as the matrix

$$\begin{bmatrix} \sigma_{t,t}^{NC} & \sigma_t^{C^PC} & \sigma_{t,t}^{N^PC} \\ \sigma_t^{NC^P} & \sigma^{C^P} & \sigma_t^{N^PC^P} \\ \sigma_{t,t}^{NN^P} & \sigma_t^{C^PN^P} & \sigma_{tt}^{N^P} \end{bmatrix}$$

has full rank, we can also identify the loading factors in the parental measures of child non-cognitive skill related to the child and parental skills. The covariance

$$Cov(M_{T,i1t}^N, M_{P,ijt}^N) = \alpha_{P,1jt}^N \sigma_{t,t}^N + \alpha_{P,2jt}^N \sigma_t^{C^P N} + \alpha_{P,3jt}^N \sigma_{t,t}^{N^P N}$$

then identifies $\sigma_{t,t}^N$. Assuming we have at least 3 teacher reported measures, we can identify the parameters relative to the teacher random effects and the measurement errors for these measures using

$$Cov(M_{T,ijt}^{N}, M_{T,i1t}^{N}) - \alpha_{T,1jt}^{N}\sigma_{t,t}^{N} = \alpha_{T,2jt}^{N}\sigma_{t,t}^{T}$$
$$Cov(M_{T,ijt}^{N}, M_{T,ikt}^{N}) - \alpha_{T,1jt}^{N}\alpha_{T,1kt}^{N}\sigma_{t,t}^{N} = \alpha_{T,2kt}^{N}\alpha_{T,2jt}^{N}\sigma_{t,t}^{T}$$

The ratio of the first and second equations identifies $\alpha_{T,2kt}^N$, which in turns leads to the identification of $\sigma_{t,t}^T$. The variance of the measures will also identify the variance of $\epsilon_{T,ijt}^N$. An analogous procedure will also identify $\alpha_{P,4jt}^N$, $\sigma_{t,t}^\theta$, and the variance of $\epsilon_{P,ijt}^N$, again assuming at least three measures are available.

The first two moments of the joint density of S_i and all parameters related to the measurement equations are now identified. However, unless we want to assume S_i is jointly normal, we have not identified the full distribution of S_i . It is important to go beyond normality if we want to allow for non-linearity in the production function f_t . For this part of the identification, we rely on Theorem 1 of Cunha et al. (2010). Consider first W_j and ω_j for $j = \{1, 2\}$ where:

$$\tilde{W}_{j} = \left(\left\{ \frac{M_{jt}^{C}}{\alpha_{jt}^{C}} \right\}_{t=1}^{T}, C_{i}^{M}, \left\{ \frac{M_{jt}^{N^{M}}}{\alpha_{jt}^{N^{M}}} \right\}_{t=1}^{T}, C_{i}^{F}, \left\{ \frac{M_{jt}^{N^{F}}}{\alpha_{jt}^{N^{F}}} \right\}_{t=1}^{T}, \left\{ M_{P,1t}^{N} \right\}_{t=1}^{T} \right)$$
$$\tilde{\omega}_{j} = \left(\left\{ \frac{\epsilon_{jt}^{C}}{\alpha_{jt}^{C}} \right\}_{t=1}^{T}, 0, \left\{ \frac{\epsilon_{jt}^{N^{M}}}{\alpha_{jt}^{N^{M}}} \right\}_{t=1}^{T}, 0, \left\{ \frac{\epsilon_{jt}^{N^{F}}}{\alpha_{jt}^{N^{F}}} \right\}_{t=1}^{T}, \left\{ 0 \right\}_{t=1}^{T} \right).$$

Notice that the second, foruth and last elements of \tilde{W}_j do not have a j subscript (the last element is the first measurement of the parent reported children non-cognitive skills), which means that we will consider the same vector of measurements for j = 1 and j = 2. The reason for including $M_{P,1t}^N$ in \tilde{W}_j will be clear in a moment. C_i^P are observed and therefore we can think of $C_i^P + 0$ as the measurement. Trivially the error term for these two measurements will be independent between j = 1 and j = 2. Using these definitions we can write the system of measures as

$$\tilde{W}_j = \tilde{S} + \tilde{\omega}_j$$

where \tilde{S} is a vector of unobserved child cognitive skills, parental skills, and the components of the kth parental measure of child non-cognitive skill. Under the assumption that $E[\tilde{\omega}_1|\tilde{u},\tilde{\omega}_2] = 0$, Theorem 1 of Cunha et al. (2010) shows that with the knowledge of the distribution of \tilde{W}_j for j = 1 and j = 2, we can identify the joint density of \tilde{S} . Of course the joint density of \tilde{S} is not equivalent to the joint density of S since the former does not contain the child's non-cognitive skill. More specifically we have just shown identification of the joint density of children cognitive skills C, maternal skills C^M and N^M , paternal skills C^F and N^F , and the first measurement of the parent reported children non-cognitive skills $M_{P,1}^N$.

Using the fact that we have different people reporting child non-cognitive skill we can adapt Theorem 1 to show identification of the joint density including child non-cognitive skill. For illustrative purposes, assume for a moment that $\alpha_{P,21t}^N = \alpha_{P,31t}^N = 0$ so that

$$M_{P,i1t}^{N} = \alpha_{P,11t}^{N} N_{it} + \alpha_{P,41t}^{N} \theta_{i} + \epsilon_{P,i1t}^{N}$$
$$M_{T,i1t}^{N} = N_{it} + \alpha_{T,21t}^{N} T_{it} + \epsilon_{T,i1t}^{N}.$$

Notice how the measurement errors, $\alpha_{P,41t}^N \theta_i + \epsilon_{P,i1t}^N$ and $\alpha_{T,21t}^N T_{it} + \epsilon_{T,i1t}^N$, are independent of each other. If we define

$$W_{1} = \left(\left\{ \frac{M_{1t}^{C}}{\alpha_{1t}^{C}} \right\}_{t=1}^{T}, C_{i}^{M}, \left\{ \frac{M_{1t}^{N^{M}}}{\alpha_{1t}^{N^{M}}} \right\}_{t=1}^{T}, C_{i}^{F}, \left\{ \frac{M_{1t}^{N^{F}}}{\alpha_{1t}^{N^{F}}} \right\}_{t=1}^{T}, \left\{ M_{T,1t}^{N} \right\}_{t=1}^{T} \right)$$
$$W_{2} = \left(\left\{ \frac{M_{2t}^{C}}{\alpha_{2t}^{C}} \right\}_{t=1}^{T}, 0, \left\{ \frac{M_{2t}^{N^{M}}}{\alpha_{2t}^{N^{M}}} \right\}_{t=1}^{T}, 0, \left\{ \frac{M_{2t}^{N^{F}}}{\alpha_{2t}^{N^{F}}} \right\}_{t=1}^{T}, \left\{ \frac{M_{P,1t}^{N}}{\alpha_{P,11t}^{N}} \right\}_{t=1}^{T} \right).$$

and

$$\omega_{1} = \left(\left\{ \frac{\epsilon_{1t}^{C}}{\alpha_{1t}^{C}} \right\}_{t=1}^{T}, \frac{\epsilon_{1t}^{C^{M}}}{\alpha_{1t}^{C^{M}}}, \left\{ \frac{\epsilon_{1t}^{N^{M}}}{\alpha_{1t}^{N^{M}}} \right\}_{t=1}^{T}, \frac{\epsilon_{1t}^{C^{F}}}{\alpha_{1t}^{C^{F}}}, \left\{ \frac{\epsilon_{1t}^{N^{F}}}{\alpha_{1t}^{N^{F}}} \right\}_{t=1}^{T}, \left\{ \alpha_{T,21t}^{N}\theta + \epsilon_{T,1t}^{N} \right\}_{t=1}^{T} \right)$$
$$\omega_{2} = \left(\left\{ \frac{\epsilon_{2t}^{C}}{\alpha_{2t}^{C}} \right\}_{t=1}^{T}, \frac{\epsilon_{2t}^{C^{M}}}{\alpha_{2t}^{C^{M}}}, \left\{ \frac{\epsilon_{2t}^{N^{M}}}{\alpha_{2t}^{N^{M}}} \right\}_{t=1}^{T}, \frac{\epsilon_{2t}^{C^{F}}}{\alpha_{2t}^{C^{F}}}, \left\{ \frac{\epsilon_{2t}^{N^{F}}}{\alpha_{2t}^{N^{F}}} \right\}_{t=1}^{T}, \left\{ \frac{\alpha_{P,41t}^{N}}{\alpha_{P,11t}^{N}} T_{t} + \frac{\epsilon_{P,1t}^{N}}{\alpha_{P,11t}^{N}} \right\}_{t=1}^{T} \right).$$

we can again apply Theorem 1 of Cunha et al. (2010), where $W_j = u + \omega_j$. Under the

assumption that $E[\omega_1|u, \omega_2] = 0$, Theorem 1 shows that we can identify the joint density of S.

The issue with the above derivation is that α_{P21t}^N and α_{P31t}^N are not equal to zero. In order to recover the distribution of S, the final term of W_2 should be

$$\frac{M_{P,1t}^N}{\alpha_{P11t}^N} - \frac{\alpha_{P,2jt}^N}{\alpha_{P,11t}^N} C^P - \frac{\alpha_{P,3jt}^N}{\alpha_{P,11t}^N} N_t^P.$$

While we do not observe C^P and N^P , we have previously identified the joint density of $M_{P,i1t}^N$ with these variables and all other unobserved skills (other than the child noncognitive). In order to apply the theorem, we do not actually know need to observe each element of W_j . we need instead to be able recover the distribution of W_1 and W_2 , which in general is directly derived from the data. While for W_1 this is true, we can derive the corrected W_2 from the incorrect W_2 , after we modify it using our knowledge of the joint density of its last element with all other skills, i.e. the distribution of W_2 . After this correction, we can then apply Theorem 1.

	t > 2 SDQ total		t > 2 SD	Q Intern.	t > 2 SDQ Extern.		
N_t reported by:	Mother	Teacher	Mother	Teacher	Mother	Teacher	
	(1)	(2)	(3)	(4)	(5)	(6)	
Child Cognitive $Skill_t$	0.191**	0.228**	0.124**	0.148**	0.196**	0.224**	
	(0.009)	(0.012)	(0.009)	(0.012)	(0.009)	(0.011)	
Mother Cognitive $Skill_t$	0.065^{**}	0.019^{+}	0.041**	0.008	0.068^{**}	0.023^{*}	
	(0.009)	(0.011)	(0.010)	(0.012)	(0.010)	(0.011)	
Mother Non-Cognitive $Skill_t$	0.312^{**}	0.108^{**}	0.276**	0.092^{**}	0.253^{**}	0.088^{**}	
	(0.010)	(0.012)	(0.010)	(0.012)	(0.010)	(0.012)	
Missing Indicators	Y	Y	Y	Y	Y	Y	
Demographics	Υ	Υ	Y	Υ	Y	Υ	
N	$15,\!017$	$9,\!697$	$15,\!056$	9,701	$15,\!045$	$9,\!699$	
R^2	0.207	0.120	0.138	0.047	0.176	0.144	

Appendix Table 1: Definition of Non-Cognitive skills

UK Millennium Cohort Study, data from t = 3 and t = 4 (child age 7 and 11). Estimates are obtained using linear regressions. The dependent variable is obtained by combining all 4 sub-scales of the SDQ into a common factor in columns 1 and 2; columns 3 and 4 combine the SDQ sub-scores on 'emotional symptoms' and 'peer problems' into a measure of *internalizing* behaviour, and columns 5 and 6 use the SDQ sub-scores on 'hyperactivity/inattention' and 'conduct problems' to derive a measure of *externalizing* behaviour. Demographic controls as in footnote to Table 4. Standard errors are clustered at the individual level. + p < 0.1, * p < 0.05, ** p < 0.01

	ī	t = 1		t > 1
N_t reported by:	Mother	Interviewer	Mother	Teacher
	(1)	(2)	(3)	(4)
Child Cognitive $Skill_t$	0.188**	0.309**	0.194**	0.264**
	(0.011)	(0.013)	(0.008)	(0.009)
Mother Cognitive $Skill_t$	0.132^{**}	-0.018	0.078**	0.036^{**}
	(0.010)	(0.010)	(0.008)	(0.008)
Mother Non-Cognitive $Skill_t$	0.268^{**}	0.021	0.299**	0.091^{**}
	(0.011)	(0.010)	(0.008)	(0.008)
Missing Indicators	Y	Y	Y	Y
Demographics	Υ	Υ	Y	Υ
N	$9,\!391$	9,343	24,514	$17,\!475$
R^2	0.218	0.130	0.207	0.158

Appendix Table 2: Teachers versus Interviewers

UK Millennium Cohort Study, data from t = 1 (child age 3) in column 1, and for t = 2 to t = 4 (child age 5, 7 and 11). Estimates are obtained using linear regressions. Variables representing child skills and parental skills are transformed by principal component analysis into factors with mean 0 and standard deviation 1. Demographic controls as in footnote to Table 4. Standard errors are clustered at the individual level. + p < 0.1, * p < 0.05, ** p < 0.01

	Teacher Reported Measures, $t = 3$							
	Baseline	Teacher controls	School FE	Teacher FE				
	(1)	(2)	(3)	(4)				
Child Cognitive $Skill_t$	0.268**	0.262**	0.276**	0.281^{**}				
	(0.015)	(0.016)	(0.029)	(0.043)				
Mother Cognitive $Skill_t$	-0.003	0.003	0.020	0.021				
	(0.013)	(0.015)	(0.029)	(0.040)				
Mother Non-Cognitive $Skill_t$	0.094^{**}	0.088^{**}	0.095^{*}	0.101^{**}				
	(0.014)	(0.015)	(0.026)	(0.035)				
Missing Indicators	Y	Y	Y	Y				
Demographics	Υ	Υ	Υ	Υ				
N	$5,\!536$	4,725	$5,\!536$	$5,\!536$				
R^2	0.128	0.127	0.646	0.800				
H_0 : Teacher Controls = 0								
P-value:		0.835						
UK Millennium Cohort Study, data from $t = 3$ (child age 7). Estimates are obtained using lin-								

Appendix Table 3: Teacher and School Controls

UK Millennium Cohort Study, data from t = 3 (child age 7). Estimates are obtained using linear regressions. Variables representing child skills and parental skills are transformed by principal component analysis into factors with mean 0 and standard deviation 1. Demographic controls as in footnote to Table 4. Teachers' controls include gender, role (class teacher, or other e.g. TA), qualification and experience (in years). Standard errors are clustered at the individual level in column 1, at the school-level in column 3, and at the teacher level in columns 2 and 4. + p < 0.1, * p < 0.05, ** p < 0.01.

		t =	= 2			t = 3				t = 4			
	C_{t+1}	N_{t+1}	N_{t+1}^M	N_{t+1}^F	C_{t+1}	N_{t+1}	N_{t+1}^M	N_{t+1}^F	C_{t+1}	N_{t+1}	N_{t+1}^M	N_{t+1}^F	
C_t	0.546^{*}	0.020*	0.017	-0.008	0.927*	0.020*	-0.022	0.043	0.774*	0.054^{*}	-0.025	0.020	
	(0.018)	(0.005)	(0.014)	(0.017)	(0.030)	(0.010)	(0.029)	(0.037)	(0.033)	(0.013)	(0.032)	(0.021)	
N_t	0.249^{*}	0.951^{*}	0.170^{*}	0.131	0.019	1.139^{*}	0.305^{*}	0.041	0.022	0.721^{*}	0.300^{*}	0.119^{*}	
	(0.066)	(0.099)	(0.069)	(0.068)	(0.035)	(0.058)	(0.066)	(0.040)	(0.021)	(0.070)	(0.040)	(0.040)	
C^M	0.013^{*}	0.039^{*}	0.012	0.008	0.001	-0.055^{*}	-0.001	-0.001	0.028^{*}	0.009	0.011	0.003	
	(0.007)	(0.013)	(0.009)	(0.008)	(0.006)	(0.014)	(0.009)	(0.009)	(0.006)	(0.009)	(0.011)	(0.009)	
N_t^M	0.004	0.027^{*}	0.685^{*}	0.037^{*}	0.002	0.009	0.679^{*}	0.062^{*}	-0.006	0.021	0.627^{*}	0.030	
	(0.012)	(0.014)	(0.019)	(0.016)	(0.007)	(0.014)	(0.018)	(0.016)	(0.007)	(0.012)	(0.021)	(0.018)	
C^F	0.033^{*}	0.005	0.011	0.038^{*}	0.019^{*}	0.002	0.005	-0.002	0.021^{*}	0.006^{*}	0.034^{*}	0.014	
	(0.007)	(0.004)	(0.009)	(0.010)	(0.008)	(0.004)	(0.011)	(0.010)	(0.006)	(0.003)	(0.014)	(0.010)	
N_t^F	0.008	0.007	0.012	0.722^{*}	0.012	-0.002	0.036^{*}	0.707^{*}	-0.001	0.002	0.067^{*}	0.704^{*}	
	(0.010)	(0.004)	(0.013)	(0.020)	(0.009)	(0.006)	(0.015)	(0.020)	(0.006)	(0.006)	(0.019)	(0.019)	
$Own \times C_t$	-0.012^{*}	-0.021^{*}	0.004	0.024^{*}	0.018^{*}	-0.008	0.026^{*}	0.017	-0.003	-0.027^{*}	0.011	0.022	
	(0.003)	(0.007)	(0.010)	(0.008)	(0.007)	(0.008)	(0.010)	(0.010)	(0.003)	(0.008)	(0.012)	(0.013)	
$Own \times N_t$	-0.074^{*}	-0.093*	0.077^{*}	0.016	0.027^{*}	0.010	0.091^{*}	0.107^{*}	0.015	-0.002	0.111^{*}	0.068^{*}	
	(0.020)	(0.035)	(0.031)	(0.021)	(0.014)	(0.022)	(0.020)	(0.018)	(0.011)	(0.007)	(0.015)	(0.016)	
$\operatorname{Own}\times C^M$	-0.002	0.008	-0.008	0.006	0.001	-0.003	-0.015^{*}	-0.010	-0.001	0.010^{*}	-0.010^{*}	0.004	
	(0.003)	(0.006)	(0.006)	(0.006)	(0.003)	(0.006)	(0.005)	(0.006)	(0.003)	(0.004)	(0.005)	(0.006)	
$\operatorname{Own}\times N^M_t$	-0.040^{*}	-0.009	-0.014^{*}	0.061^{*}	0.041^{*}	0.017^{*}	0.003	0.064^{*}	0.003	0.007	0.005	0.055^{*}	
_	(0.006)	(0.006)	(0.004)	(0.005)	(0.004)	(0.005)	(0.004)	(0.005)	(0.005)	(0.004)	(0.004)	(0.005)	
$\operatorname{Own} \times C^F$	-0.005^{*}	-0.009	0.003	-0.016^{*}	0.003	0.008	-0.001	0.003	0.003	0.003	-0.003	-0.015^{*}	
_	(0.002)	(0.005)	(0.006)	(0.006)	(0.004)	(0.005)	(0.006)	(0.006)	(0.003)	(0.005)	(0.010)	(0.005)	
$\operatorname{Own}\times N^F_t$	-0.034^{*}	0.004	0.070^{*}	0.001	0.036^{*}	0.021^{*}	0.065^{*}	0.006^{*}	0.005	0.008^{*}	0.065^{*}	0.010^{*}	
	(0.005)	(0.005)	(0.005)	(0.005)	(0.004)	(0.005)	(0.004)	(0.003)	(0.005)	(0.004)	(0.005)	(0.005)	
R^2	0.649	0.609	0.463	0.478	0.836	0.730	0.459	0.484	0.846	0.713	0.442	0.492	

Appendix Table 4: The Production Function of Skills

UK Millennium Cohort Study. Estimates are obtained using the model and estimation method outlined in sections 4 and 5. Standard errors are obtained bootstrapping 30 times the original data set at the individual level. t = 2 refers to the production function of skills from the first to the second period in our data set, i.e. from 3 to 5 years old. t = 3 and t = 4 are derived similarly. Each column corresponds to a different production function, defined in equation 15. * p < 0.05.

	SDQ emot.	SDQ conduct	SDQ hyp.	SDQ peer	Q. Indep.	Q. Emot.	Q. Coop.			
			+1sd in .	1111						
Average Increase in Child Non-Cognitive Skill Measures										
t = 1	0.142	0.150	0.081	0.093	0.023	0.090				
	(0.032)	(0.032)	(0.031)	(0.031)	(0.022)	(0.030)				
t = 2	0.147	0.138	0.113	0.102	0.065	0.130				
	(0.024)	(0.023)	(0.025)	(0.027)	(0.019)	(0.027)				
t = 3	0.131	0.117	0.091	0.109	0.059	0.115	0.084			
	(0.022)	(0.021)	(0.026)	(0.024)	(0.021)	(0.026)	(0.022)			
t = 4	0.119	0.096	0.096	0.091						
	(0.023)	(0.017)	(0.021)	(0.021)						

Appendix Table 5: Policy Evaluation in the Presence of Distortions

Percent due to Contamination

t = 1	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	
t = 2	66.9% (15.2%)	61.3% (17.4%)	50.6% (20.0%)	48.2% (20.3%)	41.3% (23.8%)	55.5% (19.3%)	
t = 3	$\begin{array}{c} 60.1\% \ (11.4\%) \end{array}$	51.7% (12.0%)	33.7% (11.8%)	48.3% (11.8%)	$16.3\% \ (15.5\%)$	47.7% (11.8%)	36.4% (12.3%)
t = 4	49.0% (8.3%)	41.1% (10.0%)	30.4% (9.8%)	28.5% (8.7%)			

UK Millennium Cohort Study. Estimates are obtained using the model and estimation method outlined in sections 4 and 5. In the first panel, we report the impact on the SDQ measures of children non cognitive skills at different ages when maternal non-cognitive skills are increased by 1sd when the children are 3 years old. In the second panel we report the fraction of the impact that is due to contamination of the measures. Standard errors are obtained bootstrapping 30 times the original data set at the individual level.

	SDQ emot.	SDQ conduct	SDQ hyp.	SDQ peer	Q. Indep.	Q. Emot.	Q. Coop.			
		+	$0.2sd$ in N_1^N	l and N_{1}						
Average Increase in Child Non-Cognitive Skill Measures										
t = 1	0.120	0.149	0.132	0.108	0.055	0.139				
	(0.006)	(0.007)	(0.007)	(0.007)	(0.004)	(0.007)				
	0.104	0.100	0.105	0.100	0.071	0 1 1 4				
t = 2	0.104	0.109	0.107	0.100	0.071	0.114				
	(0.006)	(0.006)	(0.006)	(0.006)	(0.004)	(0.007)				
t = 3	0.095	0.097	0.096	0.095	0.074	0.101	0.086			
0	(0.005)	(0.005)	(0.007)	(0.005)	(0.005)	(0.007)	(0.005)			
	(0.000)	(0.000)	(0.001)	(0.000)	(0.000)	(0.001)	(01000)			
t = 4	0.085	0.075	0.083	0.081						
	(0.005)	(0.004)	(0.005)	(0.005)						
		с у	1	· .						
		%	due to conta	amination						
t = 1	23.7%	20.1%	12.4%	17.2%	8.4%	13.0%				
U 1	(4.4%)	(3.6%)	(4.2%)	(5.3%)	(7.7%)	(3.9%)				
	()	(010,0)	(, 0)	(0.0,0)	(, .)	(010,0)				
t = 2	20.3%	16.7%	11.4%	10.5%	8.2%	13.6%				
	(3.8%)	(3.9%)	(3.4%)	(3.1%)	(3.0%)	(4.1%)				
t = 3	19.8%	15.0%	7.7%	13.3%	3.1%	13.1%	8.6%			
	(3.3%)	(2.9%)	(2.5%)	(2.7%)	(2.7%)	(2.8%)	(2.2%)			
, ,	10.907	14.007	0.007	0.107						
t = 4										
	(2.5%)	(3.0%)	(2.9%)	(2.0%)						
t = 4	$19.3\% \\ (2.5\%)$	$14.8\% \\ (3.0\%)$	$9.8\%\ (2.9\%)$	9.1% (2.0%)						

Appendix Table 6: Policy Evaluation Motivated by Baker et al. (2008)

UK Millennium Cohort Study. Estimates are obtained using the model and estimation method outlined in sections 4 and 5. In the first panel, we report the impact on the SDQ measures of children non cognitive skills at different ages when maternal non-cognitive skills and children non-cognitive skills are both increased by 0.2sd when the children are 3 years old. In the second panel we report the fraction of the impact that is due to contamination of the measures. Standard errors are obtained bootstrapping 30 times the original data set at the individual level.