

Using Close Bids to Detect Cartels in Procurement Auctions

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Abstract

Market division and bid rotation are two of the most commonly employed ways of allocating markets under collusion. However, establishing a tight link between these allocation patterns and firm conduct has been difficult because there exist cost-based explanations that can generate these allocation patterns under competition. Focusing on the set of auctions in which the winning bid and the losing bids are very close, we use ideas similar to regression discontinuity design to distinguish between allocation patterns that simply reflect cost differences across firms and those that are indicative of collusion. We derive conditions under which our test has correct size under the null of competition. Applying our test to the sample of municipal auctions in Japan, we find evidence of collusion among the set of procurement auctions whose winning bid is relatively high.

KEYWORDS: Procurement, Collusion, Backlog, Incumbency, Regression Discontinuity, Regulation.

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The ability of competition authorities to proactively detect and punish collusion is crucial for achieving the goal of promoting and maintaining competition. Not only do the possibility of detection and prosecution serve as strong deterrents against collusion, they also affect the incentives of firms in existing cartels to apply for leniency programs. Successful identification of cartels thus deters collusive activity and complements the effectiveness of leniency programs.

In the absence of concrete leads, using screens to flag suspicious firm conduct can be useful for regulators as a first step in identifying collusion. While screens cannot substitute for direct evidence of collusion such as testimonies and records of communication, they can provide guidance on which markets or firms to focus investigation. A growing number of agencies are adopting screens that use algorithms to flag suspicious behavior using bidding data from public procurement auctions.¹ In fact, there are a number of cases in which investigation was initiated on the basis of screens alone, and, ultimately, resulted in successful cartel prosecution.² The results from screens can also be used in court to obtain warrants or authorization for a more intrusive investigation. They can be used in court for civil antitrust litigation and private litigation as well.³

Screening of cartels can also be useful to those outside of antitrust authorities. For example, screening can help procurement offices counter suspected bidding rings by more aggressively soliciting new bidders or adopting auction mechanisms that are less susceptible to collusion. Screening may also be helpful for internal auditors and compliance officers of complicit firms to identify collusion and help contain potential exposure from it.

In this paper, we propose a way to test for collusion using one of the most commonly

¹Competition authorities that use statistical analysis or algorithms to screen for collusion include those in Brazil, South Korea, Switzerland and United Kingdom. A report by the OECD (OECD, 2018) gives a brief description of the screening programs used in Brazil, Switzerland and the U.K. A document titled “Cartel Enforcement Regime of Korea and Its Recent Development” maintained by the Fair Trade Commission of Korea describes South Korea’s bid screening program.

²See e.g., Imhof et al. (2018)

³Baker and Rubinfeld (1999) give an overview on the use of statistical evidence in court for antitrust litigation. Civil cases involving collusion include civil damages claims on behalf of the government. See e.g., Clark (1985) for a brief summary of civil non-merger cases handled by the U.S. Department of Justice.

mentioned ideas to screen for collusion: using patterns of bid rotation and incumbency advantage to screen for collusion. Because bidding rings often adopt rotation schemes or give priority to incumbents in project allocation, bid rotation and incumbency advantage are very often suggested as indicators of collusion. However, as is well known, there are non-collusive cost-based explanations for these allocation patterns. In particular, bid rotation patterns can arise under competition with increasing marginal costs. Incumbency advantage can be explained by cost asymmetries among competitive firms. Hence, establishing a tight link between these bidding patterns and collusion has been difficult. As Porter (2005) describes, “*An empirical challenge is to develop tests that can discriminate between collusive and non-cooperative explanations for rotation or incumbency patterns.*”

In this paper, we propose a way to use rotation and incumbency patterns that allows us to discriminate between competition and collusion by applying ideas from regression discontinuity design (Thistlethwaite and Campbell, 1960). In particular, we compare the backlog and incumbency status of a bidder who wins the auction by a small margin to those of a bidder who loses by a small margin. Under fairly mild assumptions, we show that the probability that a given bidder wins or loses an auction conditional on close bids is 0.5 regardless of the bidders’ characteristics (e.g., the size of backlog, incumbency status, etc.) under the null of competition. This implies that, under competition, even if backlog or incumbency status affect bidder costs, the differences in these variables between the winner and the loser should vanish as the bid difference between them approaches zero. If, on the other hand, bids are generated by collusive bidding, the differences in these variables between the winner and the loser may not disappear depending on the manner in which the bidding ring allocates projects. For example, if the bidding ring always allocates projects to the incumbent bidder, there will necessarily be a stark difference in the extent to which the winner is an incumbent even conditional on auctions in which the winner and the loser bid very close to each other. We use these results to construct our empirical test.

We apply our test to a dataset of public procurement auctions from the Tohoku region

of Japan. Our baseline sample consists of about 18,000 auctions from 22 municipalities between 2004 to 2017. The format of the auctions is first-price sealed bid. While none of the firms in our data have been implicated by the antitrust authorities, there is reason to suspect that bidding rings may have been active in the municipalities that we study. In one of our previous work (Kawai and Nakabayashi, 2018), we find evidence of collusive bidding in public works auctions let by the Ministry of Land Infrastructure and Transportation. Some of the bidders that we found to be bidding non-competitively in our earlier work are also active participants of the municipal auctions that we study in this paper.

We find that there are significant differences in the backlog and incumbency status of marginal winners and those of marginal losers when we focus on the subset of auctions in which the winning bid, as measured by the fraction of the reserve price, is above the median of the sample. We do not find statistically significant differences among the subset of auctions in which the winning bid is below the median. Because collusive bidding tends to elevate prices, the fact that we find significant differences for auctions with high winning bids but not for auctions with low winning bids suggests that the tests based on backlog and incumbency have reasonable size and power in practice.

In order to explore the ability of our test to screen for collusion at a more granular level, we next use a clustering algorithm to partition bidders into disjoint sets based on auction participation patterns of the bidders. The clustering algorithm places bidders that tend to participate together into the same group. We then apply our test on groups of firms. We find that, out of 30 bidder groups that participate in the most number of auctions, our test rejects the null of competitive bidding for 6 groups at the 5% level, including a group that has 6 bidders. These results suggest that our tests have power even for samples with moderate sample size.

The test that we propose in our paper has several attractive features. First, our test is based on simple and intuitive ideas that are mentioned very often by antitrust agencies. The test is also easy to implement and requires no sophisticated programming. Moreover, our

test does not require detailed data on project or bidder characteristics because the regression discontinuity design makes it less important to control for auction and bidder heterogeneity. Another attractive feature of our test is that it is valid under relatively mild assumptions on the smoothness of the bid distribution. In particular, the validity of our tests do not depend on independent bidder signals, private values, or risk neutrality.

More broadly, our idea of focusing on the characteristics of marginal winners and marginal losers can provide a useful way of turning many existing ideas on screening for collusion into formal tests of competition. For example, geographic segmentation is often considered to be a common way for bidding rings to allocate projects.⁴ With data on the location of firms and the project site, one can construct a test of collusion that compares whether or not marginal winners are more closely located to the project site than marginal losers. Other ideas for screens include the extent of subcontracting and joint bidding.⁵ Given that some procurement agencies require the list of subcontractors to be specified at the time of the bid, one can test whether or not marginal winners have more subcontractors than marginal losers.⁶ Similarly, it would be straightforward to construct a test that compares the extent of joint bidding between the marginal winner and the marginal losers. In a related paper, Nakabayashi et. al. (20xx), we show that similar ideas can also be applied to screen for collusion in scoring auctions.

⁴For example, Pesendorfer (2000) documents evidence of market division among school milk providers in Texas.

⁵For example, the Department of Justice maintains a document called “Price Fixing, Bid Rigging, and Market Allocation Schemes: What They Are and What to Look For”, in which they state “Subcontracting arrangements are often part of a bid-rigging scheme.” Similar statements are found in a report by the OECD (2013). See also Conley and Decarolis (2016) for a discussion that links subcontracting to collusion.

⁶For example, “Subletting and Subcontracting Fair Practices Act” (Public Contract Code 4100 et seq.) of California requires that “any person making a bid or offer to perform the work, shall, in his or her bid or offer, set forth ... (T)he name, the location of the place of business, ... of each subcontractor who will perform work or labor or render service to the prime contractor.” As another example, the state of Hawaii’s public procurement code includes a section called “Construction contracts; requirement to list subcontractors”, where it is stated that, “If the invitation for bids is for construction, the invitation shall specify that all bids include the name of each person/firm to be engaged by the bidder as a joint contractor or subcontractor in the performance of the contract and the nature and scope of the work to be performed by each.”

Literature Our work is most closely related to those that propose ways to screen for collusion in auctions. Some of the pioneering work include Hendricks and Porter (1988), Baldwin et al. (1997), Porter and Zona (1993, 1999). The paper that is closest to ours is Porter and Zona (1993) who study how cost shifters such as backlog and proximity to construction sites affect the level of bids and the rank order of bidders in auctions for road pavement projects. They find that the losing bids of suspected ring members do not respond to cost shifters which suggests that those bids are likely to be phantom bids. The obvious similarity between Porter and Zona (1993) and our paper is that they both study the relationship between the rank order of bids and cost shifters to screen for collusion. There is a difference in the underlying ideas behind the two papers, however. Porter and Zona (1993) focus on the lack of incentives among the losing cartel bidders to bid in ways that reflect their true costs. Hence, their primary focus is on the rank order among losing bidders.⁷ The tests that we propose in our paper is based on the idea that, under collusion, allocation is based on rotation or incumbency. Hence, our focus is on the difference between the winner and the losers.

Some of the more recent papers that distinguish between competition and collusion include Bajari and Ye (2003), Ishii (2009), Athey et al. (2011), Conley and Decarolis (2016), Andreyanov (2017), Schurter (2017), and Kawai and Nakabayashi (2018). Other related work that study collusion in auctions include Pesendorfer (2000), who studies bidding rings with and without side-payments and Asker (2010) who studies knockout auctions among members of a bidding ring. Ohashi (2009) and Chassang and Ortner (forthcoming) document how changes in the details of the auction can affect the ability of bidders to maintain collusion. Clark et al. (2018) analyze the breakdown of a cartel and its implications on prices.⁸

⁷Porter and Zona (1993) describe their tests as follows: "... our rank-based test is designed to detect differences in the ordering of higher bids, as opposed to the determinants of the probability of being the lowest bid ...", although parts of their paper analyze the determinants of the winner.

⁸For a survey of the literature up to the mid 2000s, see Harrington (2008) and Porter (2005).

1 Theoretical Foundations

In this section we provide theoretical foundations for the validity of regression discontinuity analysis based on close bids. Specifically we establish (Proposition 2 and Corollary 1) that under a class of equilibria satisfying plausible competitive requirements, the probability of winning an auction conditional on close bids is independent of other characteristics. Equivalently, conditional on close bids, bidder characteristics are independent of whether the bidder wins or loses.

1.1 Framework

Players, actions, and payoffs. We study a dynamic game in which, at each period $t \in \mathbb{N}$, a buyer procures a single item. The procurement contract is allocated through a sealed-bid first-price auction with reserve price r , which we normalize to 1.

Let N be the set of all bidders. At each time t , each firm $i \in N$ can deliver the good at cost $c_{i,t}$. Each bidder $i \in N$ submits a bid $b_{i,t} \in [0, 1] \cup \emptyset$, where bid \emptyset denotes not participating. We assume that bidders incur a cost $c_b \geq 0$ from submitting a bid in $[0, 1]$. Profiles of bids and costs are denoted by $\mathbf{b}_t = (b_{i,t})_{i \in N}$ and $\mathbf{c}_t = (c_{i,t})_{i \in N}$, respectively. We let $\mathbf{b}_{-i,t} \equiv (b_{j,t})_{j \neq i}$ denote bids from firms other than firm i , and define $\wedge \mathbf{b}_{-i,t} \equiv \min_{j \neq i} b_{j,t}$. The procurement contract is allocated to the firm who submits the lowest bid.⁹ We use the notation $\wedge \mathbf{b}_{-i} \succ b_i$ whenever b_i is the lowest bid and possible ties are broken in favor of bidder i . Bids are publicly revealed at the end of each period.

Let $x_{i,t} \in \{0, 1\}$ denote whether firm i wins the contract at time t . Firm i 's payoff at time t is $x_{i,t}(b_{i,t} - c_{i,t}) - c_b$ if $b_{i,t} \in [0, 1]$, and zero otherwise. Bidders discount future payoffs

⁹In the case of ties, we follow Athey and Bagwell (2001) and Chassang and Ortner (forthcoming) and let the bidders jointly determine the allocation. We allow bidders to simultaneously pick numbers $\gamma_t = (\gamma_{i,t})_{i \in N}$ with $\gamma_{i,t} \in [0, 1]$ for all i, t . When lowest bids are tied, the allocation to a lowest bidder i is

$$x_{i,t} = \frac{\gamma_{i,t}}{\sum_{\{j \in N \text{ s.t. } b_{j,t} = \min_k b_{k,t}\}} \gamma_{j,t}}.$$

using common discount factor $\delta < 1$.

States and information. Time t procurement costs \mathbf{c}_t are assumed to be independent conditional on a state variable θ_t , which evolves over time as an endogenous Markov chain: the distribution of θ_{t+1} depends on θ_t and the allocation at period t (i.e., on the winner's identity at t). We assume that θ_t is publicly revealed to bidders at the beginning of period t . After θ_t is realized, each bidder $i \in N$ privately observes a signal $z_{i,t}$ that is conditionally i.i.d. given (θ_t, \mathbf{c}_t) . Bidders place their bids after observing their signals.

Our model nests many informational environments, including asymmetric information private value auctions, as well as complete information. The endogenous evolution of θ_t captures settings in which bidder's procurement costs may depend on backlog or incumbency status. Since θ_t is unobservable to the econometrician, our model allows for unobserved heterogeneity across auctions.

Strategies and solution concepts. The public history h_t at time t takes the form $h_t = (\theta_{s-1}, \mathbf{b}_{s-1})_{s \leq t}$. A public strategy σ_i of player i maps public histories h_t , current state θ_t and current signal $z_{i,t}$ to bids in $[0, 1] \cup \emptyset$. Our solution concept is public perfect bayesian equilibrium; i.e., perfect bayesian equilibria in which firms use public strategies.¹⁰

Definition 1. *We say that a public perfect bayesian equilibrium $\sigma = (\sigma_i)_{i \in N}$ is Markov Perfect if and only if, for each $i \in N$, bidder i 's strategy at each time t depends only on the realization of the state variable θ_t and her current signal $z_{i,t}$.*

Under a Markov Perfect Equilibrium (MPE), players' strategies are measurable with respect to a coarse partition of public histories. Hence, the range of collusive arrangements that are sustainable under a MPE is limited.¹¹

¹⁰Since state θ_t is revealed to bidders at the start of each period, past play conveys no information about the private types of other players. As a result, we do not need to specify out-of-equilibrium beliefs.

¹¹Since the evolution of θ_t depends on the identity of the winner, MPE may still allow for collusive strategies. For instance, if state θ_t records the identity of all past winners, bidders may be able to sustain bid rotation under a MPE.

Fix a public perfect equilibrium σ . For all histories $h_{i,t} = h_t \sqcup (\theta_t, z_{i,t})$, bidder i 's *counterfactual demand* from placing bid b is

$$D_i(b|h_{i,t}) \equiv \text{prob}_\sigma(i \text{ wins if } b_{i,t} = b|h_{i,t}).$$

1.2 The case of smooth demand

Regression discontinuity relies on the intuitive idea that competitive bidders that bid similarly must have statistically similar characteristics regardless of whether they win or lose. Equivalently, conditional on close bids, a bidder must win with probability that is independent of her other characteristics.

This result would follow immediately under the assumption that players' equilibrium demand functions $D_i(\cdot|h_{i,t})$ are smooth at all histories. Indeed, observe that

$$\text{prob}(i \text{ wins} | h_{i,t} \ \& \ |b_{i,t} - \wedge \mathbf{b}_{-i,t}| \leq \epsilon) = \frac{D_i(b_{i,t}|h_{i,t}) - D_i(b_{i,t} + \epsilon|h_{i,t})}{D_i(b_{i,t} - \epsilon|h_{i,t}) - D_i(b_{i,t} + \epsilon|h_{i,t})}.$$

Whenever D_i is smooth, then for ϵ small, the probability of winning conditional on close bids is approximately 1/2, regardless of other covariates.

Lemma 1 (smooth demand). *Assumption that there exist $k, M \in \mathbb{R}^+$ such that for all histories $h_{i,t}$, $D_i(\cdot|h_{i,t})$ is twice differentiable, with $D_i'(b_i|h_{i,t}) \geq k > 0$ and $|D_i''(b_i|h_{i,t})| \leq M$. For all $\eta > 0$, there exists $\epsilon > 0$ small enough such that for all histories $h_{i,t}$,*

$$\left| \text{prob}(i \text{ wins} | h_{i,t} \text{ and } |b_{i,t} - \wedge \mathbf{b}_{-i,t}| \leq \epsilon) - \frac{1}{2} \right| \leq \eta.$$

Proof. Observe that for ϵ small

$$\begin{aligned} \text{prob}(i \text{ wins} \mid h_{i,t} \text{ and } |b_{i,t} - \wedge \mathbf{b}_{-i,t}| \leq \epsilon) &= \frac{D_i(b_{i,t} | h_{i,t}) - D_i(b_{i,t} + \epsilon | h_{i,t})}{D_i(b_{i,t} - \epsilon | h_{i,t}) - D_i(b_{i,t} + \epsilon | h_{i,t})} \\ &\sim \frac{\epsilon D'_i(b_{i,t} | h_{i,t})}{2\epsilon D'_i(b_{i,t} | h_{i,t})} = \frac{1}{2}. \end{aligned}$$

□

Using Bayes rule, the fact that the probability of winning conditional on close bids is independent of other aspects of history $h_{i,t}$ implies that characteristics of the bidder are independent of whether she wins or loses conditional on close bids.

Why equilibrium demand need not be smooth. The issue is that even if underlying costs are smoothly distributed, the residual demand faced by bidders need not be smooth. The reason for this is that bids are endogenous. Consider for simplicity a complete information auction with an incumbent I and an entrant E with respective known costs $c_I < c_E$. Suppose that bidding cost c_b is zero.

Lemma 2 (non-smooth demand). *In any efficient equilibrium in weakly undominated strategies, the entrant wins with bid c_E with probability 1. The density of entrant bids below c_E is 0. The density of entrant bids above c_E is strictly positive and bounded away from 0. Specifically, for all $\epsilon > 0$, the incumbent's demand D_I satisfies $\frac{D_I(c_E + \epsilon) - 1}{\epsilon} \leq -\frac{1}{c_E + \epsilon - c_I}$.*

Proof. In an efficient equilibrium in weakly undominated strategies, the incumbent cannot bid above c_E with positive probability: the entrant's optimal bid would win with positive probability.

In turn, the entrant cannot bid below c_E . This implies that the incumbent's optimal bid is c_E .

Optimality of c_E implies that for any $\epsilon > 0$,

$$D_I(c_E + \epsilon)(c_E + \epsilon - c_I) \leq D_I(c_E)(c_E - c_I) = c_E - c_I \iff \frac{D_I(c_E + \epsilon) - 1}{\epsilon} \leq -\frac{1}{c_E + \epsilon - c_I}.$$

□

In this example, the demand faced by either bidder is not smooth: it involves either a kink or a discontinuity. And indeed, the probability of winning is not independent of bidder characteristics conditional on close bids: the incumbent wins with probability 1, while the entrant wins with probability 0.

In Section 1.3 we provide sufficient conditions under which regression discontinuity on the basis of close bids is valid when maintaining the assumption that bids are fully endogenous. An alternative way of justifying regression discontinuity designs would be to assume that players add a smooth trembling component to their bid, which could possibly depend on the Markov state θ and their signal z .¹² The assumption that bids are subject to noise is not be unrealistic. Indeed, Dyer and Kagel (1996) and Ahmad and Minkarah (1988) provide evidence suggesting that the bidding process for construction projects is typically affected by a variety of seemingly random factors.¹³

1.3 Equilibrium beliefs conditional on close bids

We now provide sufficient conditions under which regression discontinuity on the basis of close bids is valid under Markov perfect equilibria. We start by making the following assumption.

Assumption 1 (non-zero profits). *Bidders incur a strictly positive cost of bidding $c_b > 0$.*

Assumption 1 implies that, under a MPE, firms will only submit a bid if they expect to obtain strictly positive profits. This rules out the types of discontinuities in the bid

¹²Allowing trembles to depend on *theta* and *z* can be thought of as a reduced form for Quantal Response Equilibria (e.g. McKelvey and Palfrey, 1995, 1998).

¹³See Kawai and Nakabayashi (2018) for a detailed discussion of these two papers.

distribution in Lemma 2.¹⁴

Fix a MPE σ , and a history $h_{i,t}$ for firm i . For each bid $b \in [0, 1]$, let $U_i^\sigma(b, h_{i,t})$ denote firm i 's expected discounted payoff from placing bid b at history $h_{i,t}$ under σ :

$$\begin{aligned} U_i^\sigma(b|h_{i,t}) &= \mathbb{E}^\sigma[(b - c_{i,t} + \delta V_i^\sigma(i, h_{i,t}))\mathbf{1}_{\wedge \mathbf{b}_{-i,t} > b} + (1 - \mathbf{1}_{\wedge \mathbf{b}_{-i,t} > b})V_i^\sigma(-i, \wedge \mathbf{b}_{-i,t}, h_{i,t})|h_{i,t}] - c_b \\ &= D_i(b|h_{i,t})(b - c_{i,t} - \delta(V_i^\sigma(i, h_{i,t}) - \hat{V}_i(b, h_{i,t}))) + \mathbb{E}^\sigma[V_i^\sigma(-i, \wedge \mathbf{b}_{-i,t}, h_{i,t})|h_{i,t}] - c_b, \end{aligned}$$

where $V_i^\sigma(i, h_{i,t})$ is firm i 's expected continuation value if she wins the auction at t , $V_i^\sigma(-i, \hat{b}, h_{i,t})$ is firm i 's expected continuation value if she losses the auction at t and the winning bid is \hat{b} ,¹⁵ and

$$\hat{V}_i(b, h_{i,t}) = \mathbb{E}^\sigma[V_i^\sigma(-i, \wedge \mathbf{b}_{-i,t}, h_{i,t})|h_{i,t}, \wedge \mathbf{b}_{-i,t} \succ b].$$

Definition 2. *We say that MPE σ is smooth if there exists $M, M' > 0$ such that, for all histories $h_{i,t}$, $\hat{V}_i(\cdot, h_{i,t})$ is twice differentiable, with $|\hat{V}_i'(b, h_{i,t})| \leq M$ and $|\hat{V}_i''(b, h_{i,t})| \leq M'$.*

In words, under a smooth MPE, firms' continuation value $V_i(-i, \wedge \mathbf{b}_{-i,t}, h_{i,t})$ conditional on losing the auction depends smoothly on the lowest bid among their opponents.

Remark 1. *We note that any MPE is smooth whenever the evolution of state θ_t evolves exogenously. More broadly, an MPE is smooth whenever firms' continuation value depends solely on whether they win or lose the current auction; i.e., when $V_i^\sigma(-i, \hat{b}, h_{i,t})$ does not vary with \hat{b} . This would be true when firms are symmetric, and their cost distribution at any given period depends on whether they won or lost last period's auction.*

The following result holds.

Proposition 1 (equilibrium beliefs conditional on close bids). *Let σ be a smooth MPE. Then, for all $\eta > 0$ there exists $\bar{\epsilon} > 0$ such that, for all $\epsilon \in (0, \bar{\epsilon})$ and for all histories $h_{i,t}$*

¹⁴Alternatively, we could assume that each bidder expect that, with small but positive probability, she will be the sole participant in the auction.

¹⁵In general, firm i 's expected continuation value when losing the auction will depend on the winning bid, since this bid determines the relatively likelihood with which each of firm i 's opponents wins when firm i losses.

with $\sigma_i(h_{i,t}) = b_{i,t} \leq 1$,

$$\text{prob}(i \text{ wins} \mid h_{i,t} \text{ and } |b_{i,t} - \wedge \mathbf{b}_{-i,t}| < \epsilon) \geq 1/2 - \eta. \quad (1)$$

Proof heuristic: The full proof of Proposition 1 is in Appendix B. Here, we provide a heuristic proof under the assumption that, for all $i \in N$ and all histories $h_{i,t}$, firm i 's continuation payoff depends only on whether i wins or losses the auction; that is, for all b , $V_i^\sigma(-i, b, h_{i,t}) = \tilde{V}_i^\sigma(-i, h_{i,t})$.

Fix a competitive equilibrium σ and a history $h_{i,t} = (h_t, \theta_t, z_{i,t})$, and let $\sigma_i(h_{i,t}) = b_{i,t}$ denote the equilibrium bid of firm i at $h_{i,t}$. Bidder i 's payoff from bidding some bid $b \leq 1$ at history $h_{i,t}$ is

$$\begin{aligned} & D_i(b|h_{i,t})(b - c_{i,t} + \delta V_i^\sigma(i, h_{i,t})) + (1 - D_i(b|h_{i,t}))\delta \hat{V}_i^\sigma(-i, h_{i,t}) - c_b \\ & = D_i(b|h_{i,t})(b - \kappa_{i,t}) + \delta \tilde{V}_i^\sigma(-i|h_{i,t}) - c_b, \end{aligned}$$

where $\kappa_{i,t} = c_{i,t} - \delta(V_i^\sigma(i|h_{i,t}) - \tilde{V}_i^\sigma(-i|h_{i,t}))$ is the net cost of winning the auction. Note that firm i would obtain a payoff of $\delta \tilde{V}_i^\sigma(-i|h_{i,t})$ if she didn't submit a bid. Hence, $D_i(b_{i,t}|h_{i,t})(b_{i,t} - \kappa_{i,t}) \geq c_b > 0$, and so $b_{i,t} - \kappa_{i,t} \geq c_b$.

Since bid $b_{i,t}$ is optimal, for all $\epsilon > 0$ it must be that

$$D_i(b_{i,t} + \epsilon|h_{i,t}) \leq D_i(b_{i,t}|h_{i,t}) \frac{b_{i,t} - \kappa_{i,t}}{b_{i,t} + \epsilon - \kappa_{i,t}} \quad (2)$$

$$D_i(b_{i,t} - \epsilon|h_{i,t}) \leq D_i(b_{i,t}|h_{i,t}) \frac{b_{i,t} - \kappa_{i,t}}{b_{i,t} - \epsilon - \kappa_{i,t}} \quad (3)$$

Then,

$$\begin{aligned}
\text{prob}(i \text{ wins} \mid h_{i,t} \text{ and } |b_{i,t} - \wedge \mathbf{b}_{-i,t}| < \epsilon) &= \frac{D_i(b_{i,t} \mid h_{i,t}) - D_i(b_{i,t} + \epsilon \mid h_{i,t})}{D_i(b_{i,t} - \epsilon \mid h_{i,t}) - D_i(b_{i,t} + \epsilon \mid h_{i,t})} \\
&\geq \frac{1 - \frac{b_{i,t} - \kappa_{i,t}}{b_{i,t} + \epsilon - \kappa_{i,t}}}{\frac{b_{i,t} - \kappa_{i,t}}{b_{i,t} - \epsilon - \kappa_{i,t}} - \frac{b_{i,t} - \kappa_{i,t}}{b_{i,t} + \epsilon - \kappa_{i,t}}} \\
&\geq \frac{1}{2} \frac{c_b - \epsilon}{c_b} \rightarrow \frac{1}{2} \text{ as } \epsilon \searrow 0,
\end{aligned}$$

where the first inequality uses equations (2) and (3), and the second inequality uses $b - \kappa_{i,t} \geq c_b$. ■

1.4 Sample implications conditional on close bids

Proposition 1 provides only a lower bound on firms' subjective beliefs at any given history. We now show that this lower bound on beliefs, together with symmetry and a law of large numbers for martingale increments, implies that a sample counterpart of (1) holds with equality.

Let A denote an unselected sample of auctions, and let $B = \{(b_{i,a}, x_{i,a}) \mid i \in N, a \in A\}$ denote a corresponding sample of bids, and bidder characteristics, $(b_i, x_i)_{i \in N}$ generated under a smooth competitive MPE. Characteristics $x_i \in X$ are finite valued and correspond to the subset of bidder i 's information also observed by the econometrician. We denote by $\widehat{\text{prob}}$ the sample measure over auction bids and characteristics $(b_{i,a}, x_{i,a})_{i \in N}$ defined by B .

Given $\epsilon > 0$ and $x \in X$, we define $B_{x,\epsilon} \equiv \{(i, a) \in N \times A \text{ s.t. } x_{i,a} = x, |b_{i,a} - \wedge b_{-i,a}| \leq \epsilon\}$ the subsample of close bids such that the bidders characteristics x_i are equal to x . We denote by $B_\epsilon \equiv \{(i, a) \in N \times A \text{ s.t. } |b_{i,a} - \wedge b_{-i,a}| \leq \epsilon\}$ the sample of close bids. A bidder's sample probability of winning conditional on close bids and type x is denoted by $\widehat{P}_{x,\epsilon}$. Formally, we

have,

$$\begin{aligned}\widehat{P}_{x,\epsilon} &\equiv \widehat{\text{prob}}(i \text{ wins} \mid x_i = x, |b_i - \wedge b_{-i}| \leq \epsilon) \\ &= \frac{|\{(i, a) \in B_{x,\epsilon} \text{ s.t. } b_{i,a} \prec \wedge b_{-i,a}\}|}{|B_{x,\epsilon}|}\end{aligned}\tag{4}$$

We make the following assumption about data.

Assumption 2. *There exists λ such that for all datasets of interest B , and all $x \in X$,*

$$\frac{\sum_{x' \in X \setminus x} |B_{x',\epsilon}|}{|B_{x,\epsilon}|} \leq \lambda$$

[We might have to formulate this as a property of sequences of datasets.]

The following result holds:

Proposition 2 (winning is independent of bidder characteristics). *For all $\eta > 0$, there exists $\epsilon > 0$ small enough such that with probability approaching 1 as $|B_\epsilon|$ goes to infinity,*

$$\forall x \in X, \quad \left| \widehat{P}_{x,\epsilon} - \frac{1}{2} \right| \leq \eta.$$

A corollary of Proposition 2 is that, under our assumptions, regression discontinuity

Corollary 1 (close winners and losers have similar characteristics). *For all $\eta > 0$, there exists $\epsilon > 0$ small enough such that with probability approaching 1 as $|B_\epsilon|$ goes to infinity,*

$$\forall x \in X, \quad \left| \frac{\widehat{\text{prob}}(x_i = x \mid i \text{ wins}, |b_i - \wedge b_{-i}| \leq \epsilon)}{\widehat{\text{prob}}(x_i = x \mid |b_i - \wedge b_{-i}| \leq \epsilon)} - 1 \right| \leq \eta.$$

2 Empirical Analysis

2.1 Data and Institutional Background

Our baseline analysis focuses on auctions for construction projects let by 22 municipalities from the Tohoku region of Japan.¹⁶ There are a total of about 18,000 procurement auctions in our baseline sample, from 2004 to 2018. The total award amount of the auctions is about \$3.8 billion U.S. dollars.

The format of the auctions is first-price sealed bid and the lowest bidder is awarded the project subject to the reserve price. Some of the municipalities use public reserve prices and others use secret reserve prices. For example, in 2013, 8 municipalities used public reserve prices, 13 municipalities used secret reserve prices, and 1 municipality used both. The low bid was rejected in about 9% of our sample.¹⁷

We have data on all of the bids, the identity of the bidders, and a brief description of the construction project. Column (1) of Table 1 reports summary statistics of the auctions. On average, the reserve price is about 23.55 million yen, or about \$230,000. The average winning bid is about 21.84 million yen, which implies that the average ratio of the winning bid to the reserve is about 92.6%. There are about 8.67 bidders on average. Column (2) reports summary statistics of the bidders in our sample. The average bidder in our sample participates 33.57 times and wins about 3.84 times.

The table also reports summary statistics on incumbents and the amount of backlog of the bidders. We discuss how we define these variables next.

¹⁶We focus on auctions from these municipalities because for this sample, we do not find evidence of obvious manipulation of the bids that we document in an earlier paper (Chassang et al., 2019). In the Online Appendix, we analyze auctions from all of the municipalities from which we have obtained data.

¹⁷The fraction of auctions in which the low bid is rejected is comparable to other settings with a secret reserve. For example, in their study of federal offshore oil and gas drainage lease sales, Hendricks and Porter (1988) report that the most competitive bid was rejected on 7 percent of the wildcat tracts, and on 15 percent of the drainage tracts. In Section 2.2.1, we present the results when we restrict the sample of auctions to only those with a public reserve price.

	(1)		(2)	
	By Auctions		By Bidders	
	Mean	Std. Dev.	Mean	Std. Dev.
Reserve (Mil. Yen)	23.55	89.58		
Winning Bid (Mil. Yen)	21.84	82.65		
Win Bid/Reserve	0.926	0.078		
# of Bidders	8.67	5.62		
Incumbent	0.035	0.185		
# of Participation			33.57	57.14
# of Wins			3.84	7.01
Raw Backlog (90)			3.52	11.96
Raw Backlog (180)			6.14	18.63
Obs.		17,724		4,516

Table 1: Summary Statistics of Auctions and Bidders.

Backlog One of the key variables that we use in our analysis is firms' backlog. We consider both raw backlog and standardized backlog. We define the raw backlog of bidder i at auction t as either the 90-day or 180-day cumulative award amount of the projects won by bidder i . Denoting the 90-day and 180-day backlog as $x_{i,t}^{B_{90}}$ and $x_{i,t}^{B_{180}}$ respectively, they are expressed as follows:

$$x_{i,t}^{B_k} = \sum_{\tau \in T_t^k} b_{i,\tau} \mathbf{1}_{\wedge b_{-i,\tau} > b_{i,\tau}},$$

where T_t^k denotes the set of auctions in our sample that take place in the k days prior to auction t . Although the raw backlog is a natural metric for capturing the amount of work recently awarded to a firm, variation in raw backlog captures both intertemporal change in backlog as well as across-firm heterogeneity in firm size. In order to construct a measure of backlog that only captures the intertemporal variation, we construct a second measure of backlog by standardizing the raw backlog by the within-firm mean and standard deviation. In particular, we define the 90-day and 180-day standardized backlog, $\bar{x}_{i,t}^{B_{90}}$ and $\bar{x}_{i,t}^{B_{180}}$, as

follows:

$$x_{i,t}^{\overline{B}_k} = \frac{x_{i,t}^{B_k} - \mu_{x_i^{B_k}}}{\sigma_{x_i^{B_k}}},$$

where $\mu_{x_i^{B_k}}$ is the within-firm mean of $x_{i,t}^{B_k}$ and $\sigma_{x_i^{B_k}}$ is the within-firm standard deviation of $x_{i,t}^{B_k}$. Because our second measure of backlog is defined relative to the firm’s own historical average, $x_{i,t}^{\overline{B}_k}$ is zero if firm i ’s raw backlog is equal to its time-series average at the time of auction t .

Note that our backlog measures can be, but need not be relevant for firms’ actual costs. Given that the length of days that we use to define our backlog measure (i.e., 90 days or 180 days) is arbitrary, and given that most firms work on projects that are not included in our sample, our backlog measures are, at best, only weakly correlated with the one that is relevant for firms’ actual costs.¹⁸ This is not a problem for our purposes. Under competition, marginal winners and marginal losers should have similar amounts of backlog regardless of whether or not those measures of backlog are related to the firms’ true costs.¹⁹ What is important for our purposes is that our measures of backlog be correlated with factors that cartels use to allocate projects. Column (2) of Table 1 reports summary statistics of raw backlog. Note that standardized backlog averages to zero for each firm by construction.

Incumbency The second key variable that we use in our analysis is whether or not a given firm is an incumbent for a given project. We define a firm to be an incumbent firm if it is the winner of the previous auction with the same project name let by the same municipality. To give an example, the city of Miyako in Iwate prefecture held procurement auctions with the project name “Restoration of Yagisawa public housing complex ” on 3 occasions, Nov. 22, 2011, Sept. 19, 2012, and Dec. 16, 2014. A firm named Kikuchi Painting won each

¹⁸Many bidders who participate in auctions let by municipal governments also participate in auctions that are let by the Ministry of Land Infrastructure and Transportation and prefectural governments. Some firms may also do work for private firms.

¹⁹If anything, the less our measures of backlog are related to firms’ true costs, the more plausible it is that differences in backlog between winners and losers suggests collusion.

time. We define this firm to be the incumbent in the second and third auctions. We define all of the bidders in the first auction to be a non-incumbent. Column (1) of Table 1 reports summary statistics of incumbency status. There is an incumbent in 3.5% of the auctions in our sample.

Running Variable We define the running variable as the difference between bidder i 's bid and its most competitive rival bid normalized by the reserve price:

$$\Delta_{i,t}^1 = \frac{b_{i,t} - \wedge \mathbf{b}_{-i,t}}{r_t},$$

where r_t is the reserve price of auction t . If firm i is the lowest bidder in auction t , $\Delta_{i,t}^1$ is negative, and it is the difference between the second lowest bid and the lowest bid. If firm i is not the lowest bidder, $\Delta_{i,t}^1$ is positive and it is the difference between bidder i 's bid and the lowest bid. We normalize the bid difference by the reserve price because auctions with relatively small reserve prices have smaller bid differences, on average. Unless we normalize the bids by the reserve price, the regression discontinuity results will be driven by very small auctions. The left panel of Figure 1 is the histogram of $\Delta_{i,t}^1$. There is less mass to the left of zero and more mass to the right of zero because the number of bidders is about 8.67. Note that $\Delta_{i,t}^1$ is negative for only one bidder per auction, and it is positive for all of the losing bidders. Because we report our regression discontinuity results separately for the set of auctions in which the winning bid is above and below the median, the next two panels of Figure 1 plot the histogram of $\Delta_{i,t}^1$ separately for the two sets of auctions.²⁰ The middle panel corresponds to the sample in which the winning bid is below the median and the right panel corresponds to those in which the winning bid is above the median.

²⁰More precisely, we take the median of the winning bid in each municipality and partition the auctions according to whether or not the winning bid is above or below the municipality median. Hence, half of the auctions in each municipality is in one set and the other half is in the other set.

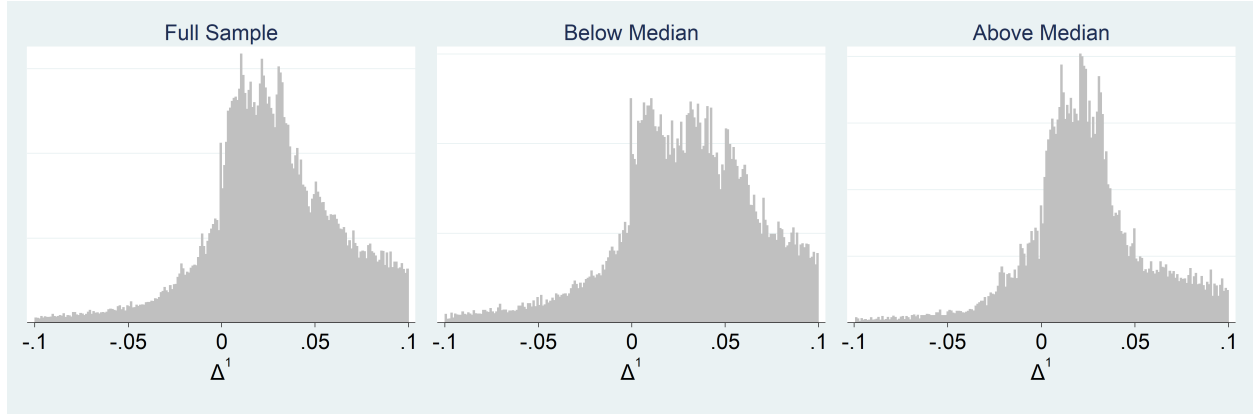


Figure 1: Histogram of $\Delta_{i,t}^1$. The left panel corresponds to the histogram for the full sample. The middle panel corresponds to the sample in which the winning bid is below the median. The right panel corresponds to the sample in which the winning bid is above the median.

Regression Discontinuity The regression discontinuity in the variable of interest that we wish to estimate is the following:

$$\beta = \lim_{\Delta_{i,t}^1 \searrow +0} \mathbf{E}[x_{i,t} | \Delta_{i,t}^1] - \lim_{\Delta_{i,t}^1 \nearrow -0} \mathbf{E}[x_{i,t} | \Delta_{i,t}^1].$$

The variable $x_{i,t}$ is one of our measures of backlog or a dummy variable for incumbency. The first term of this expression is the expected value of $x_{i,t}$ conditional on the set of marginal losers and the second term is the expected value of $x_{i,t}$ conditional on the set of marginal winners.

We estimate β using a local linear regression as follows:

$$\begin{aligned} \hat{\beta} &= \hat{b}_0^+ - \hat{b}_0^-, \\ (\hat{b}_0^+, \hat{b}_1^+) &= \arg \min \sum_{i,t}^T 1\{\Delta_{i,t}^1 > 0\} (X_{i,t} - b_0^+ - b_1^+ \Delta_{i,t}^1)^2 K\left(\frac{\Delta_{i,t}^1}{h_n}\right) \\ (\hat{b}_0^-, \hat{b}_1^-) &= \arg \min \sum_{i,t}^T 1\{\Delta_{i,t}^1 < 0\} (X_{i,t} - b_0^- - b_1^- \Delta_{i,t}^1)^2 K\left(\frac{\Delta_{i,t}^1}{h_n}\right), \end{aligned}$$

where h_n is the bandwidth and $K(\cdot)$ is the kernel. For our baseline estimates, we use a coverage error rate optimal bandwidth and a triangular kernel with a bias correction

procedure as proposed in Calonico et al. (2014).²¹ The standard errors are clustered at the auction level.

2.2 Results

Table 2 reports the regression discontinuity estimates. We report the results separately for the set of auctions in which the winning bid is above the median (Panel (A)) and below the median (Panel (B)). We expect the former set to have more collusive bidding and the latter set to have less collusion.

Focusing on Panel (A), Column (1) of Table 2 reports the regression discontinuity estimates for the 90-day backlog measured in millions of yen. We find that the estimate is 6.776, which implies that marginal losers have, on average, about 6.776 million yen more in terms of 90-day backlog relative to marginal winners. The estimate is statistically significant at the 5% confidence level, implying that we can reject the null that marginal winners and marginal losers have the same amount of backlog on average. The coverage error rate optimal bandwidth that we use is 0.011, or about 1.1% of the reserve price. In column (2), we report the corresponding estimate for the 90-day standardized backlog. We find that the estimate is 0.230, which implies that the average backlog of marginal losers is about 0.230 standard deviations higher than marginal winners. The estimate is statistically significant at the 1% confidence level.²² Columns (3) and (4) report our results for the 180-day backlog. We find that the regression discontinuity estimates for raw backlog and standardized backlog are 13.855 and 0.232 respectively, implying that the average backlog of marginal losers is about 13.855 million yen and 0.232 standard deviations higher than that of marginal winners.²³

²¹We also restrict our sample to those in which $|\Delta^1|$ is less than 20% of the reserve price, because bids that are more than 20% lower than the second lowest bid and bids that are more than 20% higher than the lowest bid are likely to be misrecorded.

²²Note that the sample sizes for Columns (1) and (2) are slightly different. This reflects the fact that we can define the standardized backlog only for firms that win at least once in our sample. For firms that never win any contracts, within-firm standard deviation of backlog will be zero, and $x_{i,t}^{\bar{B}^k}$ is undefined.

²³The reason for why the sample size in Column (4) is larger than in Column (2) is as follows. Suppose that a firm participates twice in the sample, say, Jan. 1, 2015 and May 1, 2015. Suppose that the firms

Column (5) reports our estimates for incumbency. We find that marginal losers are about 26.3 percentage points less likely to be an incumbent than marginal winners. For Column (5), we only use the set of auctions in which there is an incumbent. On the basis of these five regression results, we reject the null of competition for this sample.

Panel (B) reports the results for auctions in which the winning bid is below the median. Unlike for Panel (A), we find that the results are not statistically significant for any of the outcome variables. Hence, we cannot reject the hypothesis that data for Panel (B) are consistent with competition. Note that the number of observations are different than in Panel (A) because the average number of bidders is higher in Panel (B).

Figure 2 shows the binned scatter plots that correspond to the regression results in Columns (1) and (2) of Table 2. The top two panels correspond to Panel (A) and the bottom two panels correspond to Panel (B). The left two panels correspond to the raw 90-day backlog and the right two panels correspond to the standardized 90-day backlog. The horizontal axis in each panel is Δ^1 . The length of each bin is half of the coverage rate optimal bandwidth reported in Table 2. Hence, only the sample of bids in the four bins around zero (i.e., two bins on either side of zero) are used to estimate $\hat{\beta}$. The dots in the panels correspond to the bin averages and the vertical bars correspond to the confidence intervals of the averages. Because there are more bids to the right of zero than to the left of zero, the averages are more precisely estimated to the right of zero.²⁴

We find that there is a modest discontinuity in the binned averages for the raw backlog in the top left panel, which corresponds to Column (1) of Panel (A). The discontinuity in the binned averages for the standardized backlog is more visible in the top right panel, which

wins the first auction. According to our 90-day backlog measure, the firm's backlog would be zero for both auctions. Hence, we cannot define the standardized backlog for this firm. However, according to our 180-day backlog measure, the firm has positive backlog in the second auction.

²⁴Because the average number of bidders is about 8.67, there are about 7.67 times more bids to the right of zero.

	(1)	(2)	(3)	(4)	(5)
	90-Day Backlog		180-Day Backlog		Incumbent
	Raw	Standardized	Raw	Standardized	
Panel (A) : Above Median					
$\hat{\beta}$	6.776** (2.986)	0.230*** (0.044)	13.855*** (4.639)	0.232*** (0.049)	-0.263*** (0.105)
h	0.011	0.018	0.009	0.015	0.023
Obs.	47,263	42,683	47,263	43,390	1,319
Panel (B) : Below Median					
$\hat{\beta}$	-1.636 (1.262)	0.018 (0.042)	-2.271 (2.033)	0.052 (0.050)	-0.114 (0.154)
h	0.035	0.021	0.024	0.014	0.028
Obs.	55,848	50,222	55,848	51,018	1,405

Panel (A) corresponds to the sample of auctions in which the winning bid is above the median. Panel (B) corresponds to the sample of auctions in which the winning bid is below the median. Standard errors are clustered at the auction level and reported in parenthesis. The table also reports the bandwidth used for the estimation. * denotes significance at 10%, ** denotes significance at 5%, and *** denotes significance at 1%.

Table 2: Estimation Results

corresponds to Column (2) of Panel (A).

Figure 3 shows the binned scatter plots that correspond to the regression results in Columns (3) and (4) of Table 2. The top two panels correspond to Panel (A) and the bottom two panels correspond to Panel (B). As before, we find that the discontinuity is quite visible in the top right panel while it is somewhat modest for the top left panel.

Lastly, Figure 4 corresponds to the binned scatter plots for Column (5) of Table 2. The discontinuity in the binned averages is visible in the top panel.

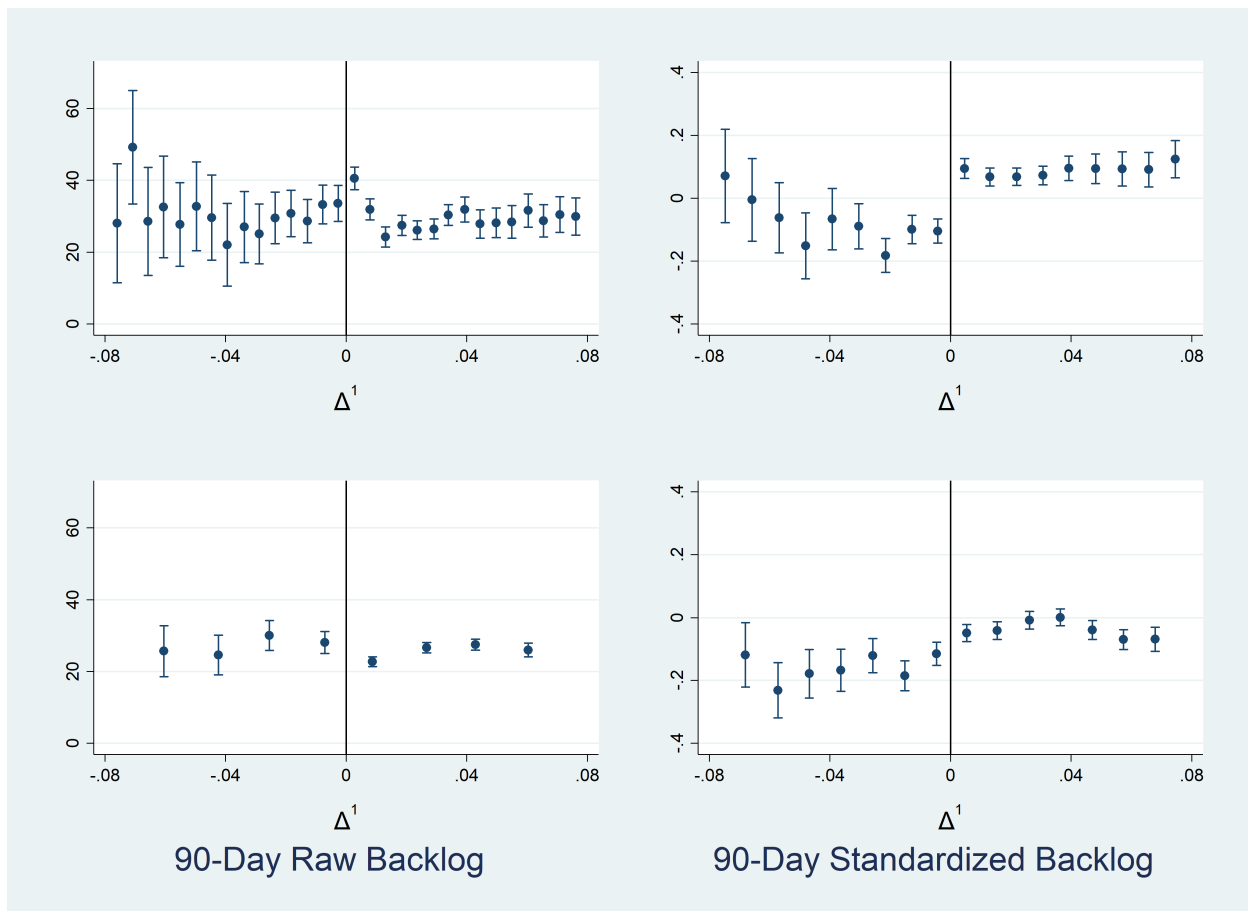


Figure 2: Binned Scatter Plot for 90-Day Raw Backlog and 90-Day Standardized Backlog. Top panels correspond to Panel (A) of Table 2 and bottom panels correspond to Panel (B) of Table 2. Left panels correspond to 90-day Backlog and the right panels correspond to 90-day standardized backlog. Bin size is half of the coverage rate optimal bandwidth used for estimating $\hat{\beta}$ in Table 2.

2.2.1 Robustness

In this section, we explore the robustness of our results. First, we consider regression discontinuity of backlog and incumbency with respect to the second lowest bidder among the rivals. In other words, we compare the backlog and incumbency status of bidders who almost tie for second place. Because the precise order of the losing bids is unimportant for allocation, it seems plausible that bidding rings would not have specific rules for determining which bidder should bid the second lowest. If this is the case, we should expect no significant differences

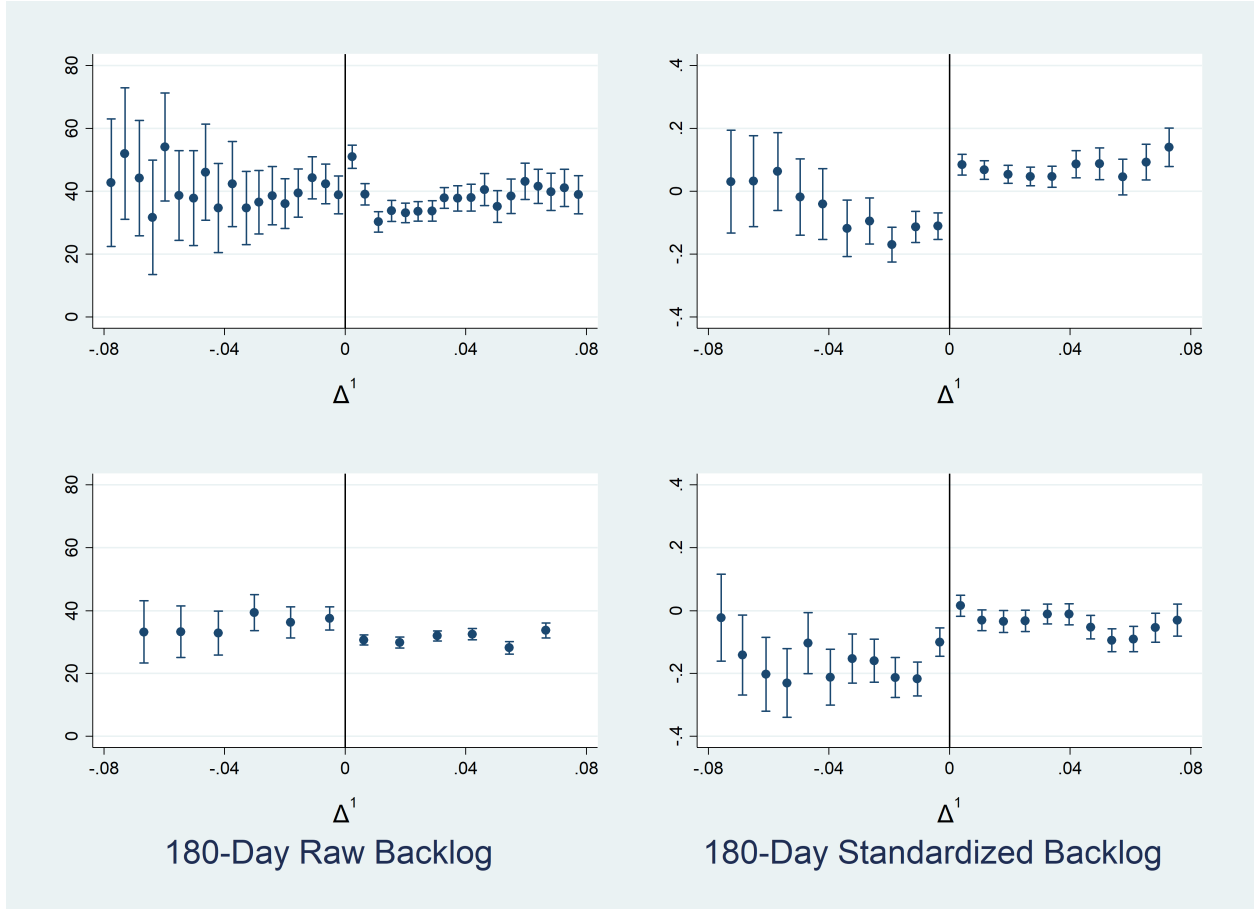


Figure 3: Binned Scatter Plot for 180-Day Raw Backlog and 180-Day Standardized Backlog. Top panels correspond to Panel (A) of Table 2 and bottom panels correspond to Panel (B) of Table 2. Left panels correspond to 180-day Backlog and the right panels correspond to 180-day standardized backlog. Bin size is half of the coverage rate optimal bandwidth used for estimating $\hat{\beta}$ in Table 2.

in backlog and incumbency between marginally second and marginally third place bidders for both competitive and noncompetitive auctions.

Second, we consider the robustness of our results to whether or not the reserve price is public. In particular, we rerun our analysis focusing only on the subset of auctions with a public reserve price to examine the robustness of our results.

Regression Discontinuity with respect to Second Lowest Bidder Among Rivals We consider regressions discontinuity of backlog and incumbency with respect to the

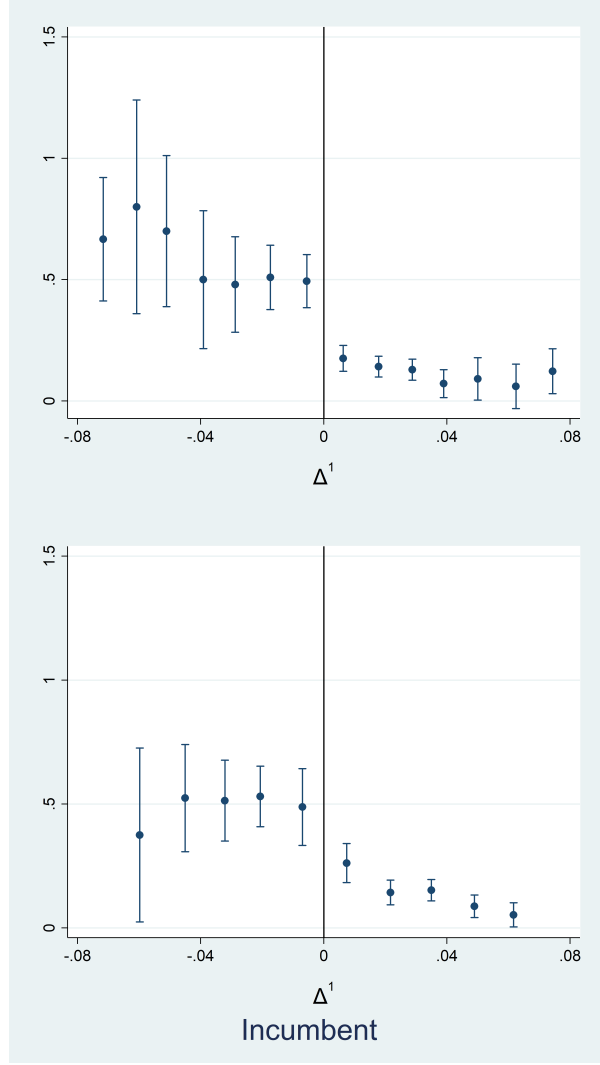


Figure 4: Binned Scatter Plot for Incumbency. Top panels correspond to Panel (A) of Table 2 and bottom panels correspond to Panel (B) of Table 2. Bin size is half of the coverage rate optimal bandwidth used for estimating $\hat{\beta}$ in Table 2.

difference between own bid and the second lowest bid among its rivals. In particular, define $\Delta_{i,t}^2$ as follows:

$$\Delta_{i,t}^2 = \begin{cases} \frac{(b_{i,t} - b_{(3),t})}{r_t} & \text{if } i \text{ is lowest or second lowest} \\ \frac{(b_{i,t} - b_{(2),t})}{r_t} & \text{if } i \text{ is 3rd lowest or higher,} \end{cases}$$

	(1)	(2)	(3)	(4)	(5)
	90-Day Backlog		180-Day Backlog		Incumbent
	Raw	Standardized	Raw	Standardized	
Panel (A) : Above Median					
$\hat{\beta}$	-1.421 (1.257)	-0.005 (0.038)	-2.819 (1.887)	-0.027 (0.040)	-0.069 (0.071)
h	0.017	0.013	0.015	0.011	0.019
Obs.	44,885	40,508	44,885	41,166	1,294
Panel (B) : Below Median					
$\hat{\beta}$	0.265 (0.701)	0.025 (0.033)	-0.196 (1.018)	0.030 (0.035)	0.063 (0.104)
h	0.021	0.018	0.020	0.017	0.019
Obs.	57,314	51,382	57,314	52,185	1,449

Panel (A) corresponds to the sample of auctions in which the winning bid is above the median. Panel (B) corresponds to the sample of auctions in which the winning bid is below the median. Standard errors are clustered at the auction level and reported in parenthesis. The forcing variable is Δ^2 . The table also reports the bandwidth used for the estimation. * denotes significance at 10%, ** denotes significance at 5%, and *** denotes significance at 1%.

Table 3: Estimation Results: Regression Discontinuity with respect to Δ^2

where r_t is the reserve price of auction t and $b_{(2),t}$ and $b_{(3),t}$ denote second and third lowest bids of auction t respectively. The value of $\Delta_{i,t}^2$ is negative for the lowest and the second lowest bidder while it is positive for the rest. Small negative values of $\Delta_{i,t}^2$ correspond to bids that barely came in second, in the sense that the third lowest bid is very close. Small positive values of $\Delta_{i,t}^2$ correspond to bids that barely missed second place. A regression discontinuity estimates of backlog and incumbency with respect to Δ^2 capture any differences in these outcome variables between bidders who barely came in second place and barely missed second place.

Table 3 reports the regression discontinuity estimates. The top panel corresponds to

the estimates for auctions in which the winning bid is above the median and the bottom panel corresponds to those for auctions in which the winning bid is below the median. We report the regression discontinuity estimates in the first row and the bandwidth used for the estimation in the second row.

Unlike in Panel (A) of Table 2, we find that none of the regression discontinuity estimates are statistically significant in Panel (A) of Table 3. This implies that there are no significant differences between marginal second place bidders and marginal third place bidders among the set of auctions in which the winning bid is above the median. Together with the results in Table 2, our results suggest that bidding rings use backlog and incumbency for determining the lowest bidder, but not for determining the order of the losing bids. Panel (B) of Table 2 reports regression estimates for the sample of auctions in which the winning bid is below the median. Perhaps not surprisingly, the regression discontinuity estimates are not statistically different from zero at the 5% significance level. The binned scatter plots that correspond to these estimates are given in Online Appendix A.

Public Reserve Prices We now consider whether or not our results are robust to removing auctions with secret reserve prices. Table 4 reports the regression discontinuity estimates for the subset of auctions with public reserve prices. Similar as before, Panel (A) corresponds to the set of auctions with a winning bid above the median. We find that the regression discontinuity estimates are positive and statistically significant in Columns (1) through (4), although the coefficient for Incumbent in Column (5) is not statistically significant. The reason for statistical insignificance in Column (5) is likely to be because of small sample size. The number of observations (bids) is 210 and the number of auctions is 33. Panel (B) reports the results for the set of auctions with a winning bid below the reserve price. We find that none of the regression coefficients are statistically significant. Overall, our findings suggest that the validity of our approach is not dependent on the whether or

	(1)	(2)	(3)	(4)	(5)
	90-Day Backlog		180-Day Backlog		Incumbent
	Raw	Standardized	Raw	Standardized	
Panel (A) : Above Median					
$\hat{\beta}$	11.837* (6.8911)	0.269*** (0.099)	22.909** (9.410)	0.236** (0.099)	-0.042 (0.285)
h	0.010	0.009	0.009	0.010	0.007
Obs.	5,782	5,190	5,782	5,292	210
Panel (B) : Below Median					
$\hat{\beta}$	-2.978 (5.172)	0.134 (0.112)	-5.651 (7.301)	0.101 (0.116)	-0.165 (0.336)
h	0.024	0.023	0.021	0.018	0.017
Obs.	8,533	7,746	8,533	7,865	315

The sample of auctions include only those with public reserve prices. Panel (A) corresponds to the sample of auctions in which the winning bid is above the median. Panel (B) corresponds to the sample of auctions in which the winning bid is below the median. Standard errors are clustered at the auction level and reported in parenthesis. The forcing variable is Δ^1 . The table also reports the bandwidth used for the estimation. * denotes significance at 10%, ** denotes significance at 5%, and *** denotes significance at 1%.

Table 4: Regression Discontinuity Estimates with respect to Δ^1 : Auctions with Public Reserve Prices.

not the reserve price is public.

2.2.2 Screening for Collusion

In order to explore the ability of our test to detect collusion at a more granular level, we partition bidders into groups based on bidder participation patterns and test for collusion group by group. In particular, we first construct a square matrix with number of rows and columns equal to the number of bidders in which the (i, j) element corresponds to the fraction of time bidder i bids with bidder j . Because we treat a firm bidding on auctions in

two different cities as different firms, the matrix is block diagonal. We then use an average-linkage clustering algorithm on this matrix to partition bidders into disjoint groups. Because there are 13 major project categories in our sample, we consider an iterative procedure that partitions the bidders based on average linkage until we have 13 distinct groups per city.²⁵ The resulting groups tend to cluster firms that regularly participate together in the same group.

Table 5 reports the regression discontinuity results with respect to the 90-day standardized backlog for the 30 groups with the most number of observations. Column (1) reports the regression discontinuity estimates, Column (2) reports the standard errors, and Column (3) reports the bandwidths. As before, we use a coverage error rate optimal bandwidth with a bias correction procedure proposed in Calonico et al. (2014). Column (4) reports the number of firms that are in each group and Column (5) reports the number of bids. Out of the 30 largest groups, we find that there are 6 groups with a positive and statistically significant estimate at the 5% confidence level. Some of these groups consist of relatively small number of bidders. For example, Group 11 has 22 firms, Group 21 has 8 firms and Group 26 has 18 firms. These findings suggest that our approach can be quite useful for screening for collusion at a relatively granular level.

In column (3) and (4), we compare the backlog of marginal second place bidders to those who are ranked lower than second. Unlike between winners and non-winners, there are no compelling reasons to expect bidder backlog to differ between marginal second place bidders and those who are lower ranked even under collusion. For example, if the losing bids are submitted randomly, then the differences in backlog between marginal second place bidders and those who are lower ranked should be zero. The coefficients reported in columns (3) and

²⁵In principle, we can group the firms based on the project categories of the auctions on which bidders bid. However, the project category data are missing in many auctions. It is available in about x% of the sample.

	(1)	(2)	(3)	(4)	(5)
Rank	Estimate	Std. Error	Bandwidth	Firms	Obs.
1	-0.227	0.159	0.027	118	7,408
2	0.280**	0.136	0.032	252	6,906
3	-0.493	0.425	0.009	94	5,778
4	-0.012	0.098	0.030	73	4,057
5	0.323**	0.138	0.011	153	3,404
6	0.066	0.182	0.020	62	1,816
7	-0.018	0.185	0.024	255	1,544
8	-0.576	0.391	0.024	32	1,155
9	-0.381	0.487	0.035	156	1,025
10	0.291	0.327	0.056	59	869
11	0.814***	0.243	0.012	22	864
12	0.028	0.513	0.023	10	781
13	0.201	0.439	0.017	39	687
14	0.047	0.480	0.010	13	665
15	-0.076	0.378	0.036	290	641
16	0.385	0.290	0.019	7	593
17	0.522	0.385	0.019	9	555
18	-0.351	0.328	0.016	30	512
19	-1.389***	0.363	0.018	22	504
20	0.470	0.318	0.031	12	503
21	0.438**	0.201	0.030	8	478
22	0.290*	0.163	0.012	13	452
23	0.165	0.326	0.001	6	374
24	-0.134	0.379	0.039	10	353
25	0.085	0.184	0.013	39	309
26	1.342**	0.635	0.021	18	278
27	0.278	0.474	0.005	6	272
28	0.093	0.385	0.022	12	270
29	-0.062	0.487	0.007	11	268
30	1.382**	0.668	0.031	192	227

Table 5: Regression Discontinuity Estimates for Top 30 Groups.

(4) are both statistically indistinguishable from zero, suggesting that the ranking of losing bids are not systematically related to backlog. The binplots that correspond to the results in columns (3) and (4) are illustrated in the right panels of Figure 1.

While colluding firms may coordinate not to participate in the same auction, the nature

of our test does not have power in detecting cartels that avoid facing each other in auctions. testing for collusion among groups of firms that frequently bid together is

Firm Response to Screening Almost any screening, if known to colluders, can be countered, but some screens impose more or less cost on the cartels for avoiding detection. some screens impose than others to

and x_t are auction covariates. In practice, we include only the reserve price as a covariate in the first regression. Relatively large firms have high backlog, and they also participate in large auctions. This induces a strong positive correlation between the reserve price and backlog. Hence, controlling for x_t can reduce the standard error of the estimates.

The costs of the public works projects range from a low of less than \$10,000 to a high of more than \$10 million. The median estimated cost is about \$74,000.

Our baseline sample consists of about 35,000 auctions from 10 municipalities between 2004 to 2014.

well established already already use screens Because limited regulatory resources preclude conducting a thorough scrutiny of all firms in most cases, that identify collusion, albeit imperfectly, can be useful to regulators as a guide on where to focus their resources.

Detection that does not rely on There is some evidence that competition policy has a positive and significant effect on total factor productivity growth (Buccirossi, Ciari, Duso, Spagnolo, Vitale [2013]).

While much of the collusion cases come about through leniency programs in many countries,

These ideas have been incorporated as part of the screening strategy of some regulators and training of procurement officials in certain countries. See OECD report, Ex officio cartel investigations and the use of screens to detect cartels (2013), in particular, India and Lithuania.

	(1)	(2)	(3)	(4)	(5)
	90-Day Backlog		180-Day Backlog		Incumbent
	Raw	Standardized	Raw	Standardized	
Panel (A) : Above Median					
$\hat{\beta}$	3.570*** (1.226)	0.145*** (0.020)	6.010*** (1.994)	0.158*** (0.020)	-0.329*** (0.051)
h	0.013	0.011	0.015	0.012	0.019
Obs.	365,396	342,296	365,396	345,362	1,294
Panel (B) : Below Median					
$\hat{\beta}$	1.449** (.606)	0.038** (0.017)	1.785* (1.032)	0.041** (0.018)	-0.096*** (0.035)
h	0.017	0.01	0.016	0.010	0.012
Obs.	380,856	350,068	380,856	353,819	13,869

Panel (A) corresponds to the sample of auctions in which the winning bid is above the median. Panel (B) corresponds to the sample of auctions in which the winning bid is below the median. Standard errors are clustered at the auction level and reported in parenthesis. The forcing variable is Δ^2 . The table also reports the bandwidth used for the estimation. * denotes significance at 10%, ** denotes significance at 5%, and *** denotes significance at 1%.

Table A.1: Estimation Results: Regression Discontinuity with respect to Δ^2

Appendix

A Further Empirics

In this section, we first show the binned scatter plots that correspond to the regression results in Table 3 and Table 4 in Section 2.2.1. We then apply our tests to all of the municipal auctions that we have data on.

Third, we consider the robustness of our results to sample selection. As we discussed earlier, our baseline sample consists of 22 municipalities which we select based on whether or not there is obvious manipulation of the running variable. In order to show that our results are not driven by sample selection, we present our analysis using auctions from all of the municipalities for which we have data.

B Proofs

Proof of Proposition 1. Let σ be a smooth MPE, and fix a history $h_{i,t}$ with $\sigma_i(h_{i,t}) = b_{i,t} \leq 1$. The payoff firm i obtains from placing bid $b \leq 1$ at history $h_{i,t}$ can be written as

$$U_i^\sigma(b|h_{i,t}) = D_i(b|h_{i,t})(b - \kappa_i(b|h_{i,t})) + \delta \mathbb{E}^\sigma[V_i^\sigma(-i, \wedge \mathbf{b}_{-i,t}, h_{i,t})|h_{i,t}] - c_b,$$

where

$$\kappa_i(b|h_{i,t}) = c_{i,t} - \delta(V_i(i, h_{i,t}) - \hat{V}_i(b, h_{i,t})).$$

Note that firm i would obtain a payoff of $\delta \mathbb{E}^\sigma[V_i^\sigma(-i, \wedge \mathbf{b}_{-i,t}, h_{i,t})|h_{i,t}]$ if she didn't submit a bid. Hence, $b_{i,t} - \kappa_i(b_{i,t}|h_{i,t}) \geq c_b > 0$. Since σ is smooth, $b - \kappa_i(b|h_{i,t}) > 0$ for all b close to $b_{i,t}$.

Since bid $b_{i,t}$ is optimal, for all $\epsilon > 0$ small is must be that

$$D_i(b_{i,t} + \epsilon|h_{i,t}) \leq D_i(b_{i,t}|h_{i,t}) \frac{b_{i,t} - \kappa_i(b_{i,t}|h_{i,t})}{b_{i,t} + \epsilon - \kappa_i(b_{i,t} + \epsilon|h_{i,t})} \quad (5)$$

$$D_i(b_{i,t} - \epsilon|h_{i,t}) \leq D_i(b_{i,t}|h_{i,t}) \frac{b_{i,t} - \kappa_i(b_{i,t}|h_{i,t})}{b_{i,t} - \epsilon - \kappa_i(b_{i,t} - \epsilon|h_{i,t})} \quad (6)$$

Then,

$$\begin{aligned} \text{prob}(i \text{ wins } | h_{i,t} \text{ and } |b_{i,t} - \wedge \mathbf{b}_{-i,t}| < \epsilon) &= \frac{D_i(b_{i,t}|h_{i,t}) - D_i(b_{i,t} + \epsilon|h_{i,t})}{D_i(b_{i,t} - \epsilon|h_{i,t}) - D_i(b_{i,t} + \epsilon|h_{i,t})} \\ &\geq \frac{1 - \frac{b_{i,t} - \kappa_i(b_{i,t}|h_{i,t})}{b_{i,t} + \epsilon - \kappa_i(b_{i,t} + \epsilon|h_{i,t})}}{\frac{b_{i,t} - \kappa_i(b_{i,t}|h_{i,t})}{b_{i,t} - \epsilon - \kappa_i(b_{i,t} - \epsilon|h_{i,t})} - \frac{b_{i,t} - \kappa_i(b_{i,t}|h_{i,t})}{b_{i,t} + \epsilon - \kappa_i(b_{i,t} + \epsilon|h_{i,t})}} \\ &= \left(\frac{b_{i,t} - \epsilon - \kappa_i(b_{i,t} - \epsilon|h_{i,t})}{b_{i,t} - \kappa_i(b_{i,t}|h_{i,t})} \right) \left(\frac{1 - \frac{\kappa_i(b_{i,t} + \epsilon|h_{i,t}) - \kappa_i(b_{i,t}|h_{i,t})}{\epsilon}}{2 - \frac{\kappa_i(b_{i,t} + \epsilon|h_{i,t}) - \kappa_i(b_{i,t} - \epsilon|h_{i,t})}{\epsilon}} \right), \end{aligned}$$

where the inequality uses (5) and (6). Since σ is smooth, and since $b_{i,t} - \kappa_i(b_{i,t}|h_{i,t}) \geq c_b > 0$,

it follows that

$$\text{prob}(i \text{ wins} \mid h_{i,t} \text{ and } |b_{i,t} - \wedge b_{-i,t}| < \epsilon) \geq \frac{c_b - \epsilon - \epsilon M}{c_b} \frac{1 - M}{2 - 2M - 2\epsilon M'} \rightarrow \frac{1}{2} \text{ as } \epsilon \searrow 0.$$

■

Proof of Proposition 2. Take $\eta' > 0$ as given. We know from Proposition 1 that for epsilon small enough, for all histories $h_{i,t}$, $D_i(b_{i,t} \mid h_{i,t}) \geq \frac{1}{2} - \eta'$.

We first show that for all $x \in X$, with probability approaching 1 as $|B_\epsilon|$ goes to infinity, $\widehat{P}_{x,\epsilon} \geq \frac{1}{2} - 2\eta'$. Observe first that when $|B_\epsilon|$ grows large, $|B_{x,\epsilon}|$ grows proportionally large:

$$\frac{|B_{x,\epsilon}|}{|B_\epsilon|} = 1 - \frac{\sum_{x' \in X \setminus x} |B_{x',\epsilon}|}{|B_{x,\epsilon}| + \sum_{x' \neq x} |B_{x',\epsilon}|} \geq 1 - \frac{\lambda}{1 + \lambda}.$$

We denote by $\{a_1, \dots, a_n\}$ auctions a such that $(i, a) \in B_{x,\epsilon}$, ordered according to the timing of the auction. Since the number N of bidders is finite, n grows large proportionally with $|B_{x,\epsilon}|$. We define $C_k = \{i \in N \text{ s.t. } (i, a_k) \in B_{x,\epsilon}\}$. In equilibrium,

$$H_K \equiv \sum_{k=1}^K \sum_{i \in C_k} \mathbf{1}_{b_{i,a_k} \prec \wedge b_{-i,a_k}} - \text{prob}_i(b_{i,a_k} \prec \wedge b_{-i,a_k} \mid i \in C_k)$$

is a martingale. Indeed note that given the information I_K available at the time of bidding

in auction K ,

$$\begin{aligned}
\mathbb{E} \left[\sum_{i \in C_K} \mathbf{1}_{b_{i,a_K} \prec \wedge b_{-i,a_K}} \middle| I_K \right] &= \mathbb{E} \left[\sum_{i \in N} \mathbf{1}_{i \in C_K} \mathbf{1}_{b_{i,a_K} \prec \wedge b_{-i,a_K}} \middle| I_K \right] \\
&= \mathbb{E} \left[\mathbb{E}_{C_K} \left[\sum_{i \in N} \mathbf{1}_{i \in C_K} \mathbf{1}_{b_{i,a_K} \prec \wedge b_{-i,a_K}} \middle| I_K \right] \right] \\
&= \mathbb{E} \left[\sum_{i \in N} \mathbf{1}_{i \in C_K} \text{prob}_i(\mathbf{1}_{b_{i,a_K} \prec \wedge b_{-i,a_K}} | i \in C_K) \middle| I_K \right] \\
&= \mathbb{E} \left[\sum_{i \in C_K} \text{prob}_i(\mathbf{1}_{b_{i,a_K} \prec \wedge b_{-i,a_K}} | i \in C_K) \middle| I_K \right].
\end{aligned}$$

This implies that

$$G_K \equiv \sum_{k=1}^K \sum_{i \in C_k} \mathbf{1}_{b_{i,a_k} \prec \wedge b_{-i,a_k}} - \frac{1}{2} + \eta'$$

is a submartingale with increments bounded by N (the maximum number of bidders in an auction). It follows for the Azuma-Hoeffding Theorem that as n grows large, with probability approaching 1, $G_n \geq -\eta'n$. Since $n \leq |B_{x,\epsilon}|$, this implies that with probability approaching 1,

$$\widehat{P}_{x,\epsilon} \equiv \frac{1}{|B_{x,\epsilon}|} \sum_{k=1}^n \sum_{i \in C_k} \mathbf{1}_{b_{i,a_k} \prec \wedge b_{-i,a_k}} \geq \frac{1}{2} - 2\eta'.$$

Since X is finite, with probability approaching 1 as $|B_\epsilon|$ becomes large, we have that for all $x \in X$, $\widehat{P}_{x,\epsilon} \geq \frac{1}{2} - 2\eta'$. In addition, since $\sum_{x' \in X} |B_{x',\epsilon}| \widehat{P}_{x',\epsilon} = |\{(i, a) \in B_\epsilon \text{ s.t. } i \text{ wins}\}|$, it follows that

$$\frac{\sum_{x' \in X} |B_{x',\epsilon}| \widehat{P}_{x',\epsilon}}{\sum_{x' \in X} |B_{x',\epsilon}|} \leq \frac{1}{2}.$$

Hence, with probability approaching 1, we have that

$$\begin{aligned} |B_{x,\epsilon}| \widehat{P}_{x,\epsilon} &= \frac{1}{2} |B_{x,\epsilon}| + \sum_{x' \in X \setminus x} |B_{x',\epsilon}| \left(\frac{1}{2} - \widehat{P}_{x',\epsilon} \right) \\ \Rightarrow \widehat{P}_{x,\epsilon} &\leq \frac{1}{2} + 2\eta' \frac{\sum_{x' \in X \setminus x} |B_{x',\epsilon}|}{|B_{x,\epsilon}|} \leq \frac{1}{2} + 2\eta'\lambda. \end{aligned}$$

Hence by selecting η' sufficiently small in the first place, it follows that for any $\eta > 0$, there exists ϵ such that as $|B_\epsilon|$ grows large, $|\widehat{P}_{x,\epsilon} - \frac{1}{2}| \leq \eta$ with probability 1. ■

Proof of Corollary 1. Observe that

$$\begin{aligned} \widehat{\text{prob}}(x_i = x \mid i \text{ wins}, |b_i - \wedge b_{-i}| \leq \epsilon) &= \frac{\widehat{\text{prob}}(x_i = x \text{ and } i \text{ wins} \mid |b_i - \wedge b_{-i}| \leq \epsilon)}{\widehat{\text{prob}}(i \text{ wins} \mid |b_i - \wedge b_{-i}| \leq \epsilon)} \\ &= \widehat{\text{prob}}(x_i = x \mid |b_i - \wedge b_{-i}| \leq \epsilon) \frac{\widehat{\text{prob}}(i \text{ wins} \mid x_i = x, |b_i - \wedge b_{-i}| \leq \epsilon)}{\widehat{\text{prob}}(i \text{ wins} \mid |b_i - \wedge b_{-i}| \leq \epsilon)}. \end{aligned}$$

It follows from Proposition 2 that for any $\eta' > 0$, there exists ϵ such that with probability 1 as $|B_\epsilon|$ grows large,

$$\frac{\widehat{\text{prob}}(i \text{ wins} \mid x_i = x, |b_i - \wedge b_{-i}| \leq \epsilon)}{\widehat{\text{prob}}(i \text{ wins} \mid |b_i - \wedge b_{-i}| \leq \epsilon)} = \frac{\widehat{P}_{x,\epsilon}}{\sum_{x' \in X} \frac{|B_{x,\epsilon}|}{|B_\epsilon|} \widehat{P}_{x',\epsilon}} \in \left[\frac{1/2 - \eta'}{1/2 + \eta'}, \frac{1/2 + \eta'}{1/2 - \eta'} \right].$$

By picking η' small enough, this implies that with probability approaching 1,

$$\left| \frac{\widehat{\text{prob}}(x_i = x \mid i \text{ wins}, |b_i - \wedge b_{-i}| \leq \epsilon)}{\widehat{\text{prob}}(x_i = x \mid |b_i - \wedge b_{-i}| \leq \epsilon)} - 1 \right| \leq \eta.$$

■

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