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Asset Prices and Unemployment Fluctuations

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ABSTRACT

Two recent critiques have shown that existing attempts to account for the unemployment volatility puzzle of search models are inconsistent with two features of the data: the measured cyclicality of the opportunity cost of employment and the volatility of profit flows. We propose a model that is immune to these two critiques and solves the unemployment volatility puzzle by allowing for preferences that generate *time-varying risk* over the cycle, and so account for observed asset pricing fluctuations, and *human capital accumulation on the job*, which is in line with documented wage growth with experience in the labor market. The key intuition for the ability of our model to reproduce the observed fluctuations in unemployment is that hiring a worker is a risky investment. Due to our asset pricing preferences, the price of risk rises sharply in downturns. Due to human capital accumulation on the job, the surplus flows to matches between workers and firms have long durations and thus are sensitive to variation in the price of risk. As a result, the benefits to creating matches sharply drops in downturns, firms substantially reduce the number of job vacancies they create, and unemployment greatly rises, as observed in the data.

The most important theoretical contribution of search models of the labor market to the study of business cycles is that they give rise to involuntary unemployment as an equilibrium phenomenon. The key insight of these models is that involuntary unemployment can occur even without any assumed inefficiencies in contracting, such as sticky wages. Despite its great promise, however, Shimer (2005) showed that the textbook search and matching model cannot generate anywhere near the observed magnitude of the fluctuations in the job-finding rate and in the unemployment rate in response to shocks of plausible magnitudes.

In the past decade, there have been numerous attempts to address the *unemployment volatility puzzle* of Shimer (2005). Prominent examples of these attempts include, among others, Hagedorn and Manovskii (2008), Hall and Milgrom (2008), and Pissarides (2009). Ljungqvist and Sargent (2017) review all of these attempts and show that they share a similar mechanism that features an *acyclical* opportunity cost of employment. In a recent paper, though, Chodorow-Reich and Karabarbounis (2016) critique this literature and argue that none of these attempts are consistent with the data. Specifically, these authors show that in the data the opportunity cost of employment is procyclical with an elasticity close to one rather than zero, as is assumed in the literature. These authors further demonstrate that once these models are made consistent with this aspect of the data, they are incapable of producing volatile unemployment. In this sense, none of these attempts has solved the unemployment volatility puzzle.

Our paper proposes a model that solves it. Critically, we develop a framework that respects the original promise of search models, that is, to generate involuntary equilibrium unemployment without relying on inefficient wage contracting, and is robust to the Chodorow-Reich and Karabarbounis (2016) critique. Thus, our model relies on a fundamentally different mechanism than the one isolated by Ljungqvist and Sargent (2017). (See the Appendix for a detailed discussion of this point.)

Recently, Borovicka and Borovickova (2019) pointed out a second critique of the literature that attempts to solve the Shimer puzzle, namely, this literature is grossly at odds with asset pricing facts. In particular, these authors show that the attempts discussed above as well as that by Hall (2017), which we discuss later, imply a volatility of either profit flows or risk-free rates that is inconsistent with the data. In contrast to existing work, our model not only reproduces the observed fluctuations in profits and risk-free interest rates, but also generates equity flows and stock market prices that are in line with the data. In this sense, our model overcomes this latter critique as well.

To generate involuntary unemployment without exploiting inefficiencies in wage contracting, we focus on labor market outcomes generated by a competitive search equilibrium. We find this concept appealing relative to common bargaining concepts such as Nash bargaining or alternating offer bargaining, since all these bargaining schemes introduce free parameters that give rise to inefficiencies in wage setting, unless chosen appropriately. For instance, as is well-known, when the Hosios (1990) condition holds, equilibrium wage setting under Nash bargaining is efficient and hence leads to the same outcomes reached under competitive search. Similarly, we show that under alternating offer bargaining, wage setting is also efficient when two conditions hold, namely, when the exogenous rate of breakdown of bargaining converges to one and when the probability that a worker makes the first offer when bargaining with a firm equals the elasticity of the matching function with respect to the measure of the unemployed. In light of these results, we can interpret our work as focused on economies with efficient wage setting, which can be achieved under any of the three most popular wage determination schemes, that is, competitive search and, as long as suitably parametrized, Nash bargaining or alternating offer bargaining.

Our model adds the textbook model two simple ingredients that are missing from it, so as to make it consistent with two well-known features of the data. In the data, asset prices fluctuate over the cycle and wages increase with workers' labor market experience. To accommodate the first feature, we augment the model with preferences that generate time-varying risk. To accommodate the second feature, we introduce human capital accumulation on the job and depreciation off the job. We choose parameters of preferences and technology that are consistent with key observed properties of asset prices and wage-experience profiles, and show that the resulting equilibrium allocations display fluctuations in unemployment similar to those observed in the data.

The main idea of our model is that hiring a worker is akin to investing in an asset with risky dividend flows that have long durations. To elaborate, due to our asset pricing preferences, the price of risk rises sharply in downturns. Due to human capital accumulation on the job, the surplus flows to matches between workers and firms have long durations and thus are sensitive to any variation in the price of risk. Once combined, these two features imply that the benefits to creating matches drop sharply in downturns. As a result, firms reduce substantially the number of job vacancies they create and, correspondingly, unemployment rises greatly as in the data.

We parameterize our model to be consistent with key aspects of the data. The new parameters we introduce are those for the process of human capital accumulation and for the preferences underlying asset pricing. To discipline human capital accumulation, we choose parameters to reproduce wage growth with experience and wage losses after spells of unemployment. To discipline asset pricing, we use moments related to the time variation of the price of risk, as captured by the mean maximum Sharpe ratio, and the fluctuations of the risk-free rate.

We show that both of our two simple ingredients are necessary to account for the observed volatility of unemployment. If we retain human capital accumulation but replace our asset pricing preferences with standard constant relative risk aversion preferences, then the model generates exactly no fluctuation in unemployment. If we retain our asset pricing preferences but abstract from human capital accumulation, then the model generates almost no fluctuation in unemployment regardless of the degree of time-varying risk.

We turn to providing further details about our two additional ingredients. Consider first human capital accumulation. For simplicity, we assume that human capital grows at a constant rate during employment and depreciates at a constant rate during unemployment and that market production, home production, and the cost of posting job vacancies are all proportional to human capital. This formulation is particularly convenient because it implies that only the aggregate amounts of human capital of employed and unemployed workers, rather than their distributions, need to be recorded as state variables.[1]

Consider next preferences. The asset pricing literature has developed several classes of preferences and stochastic processes for aggregates that give rise to large increases in the

[1] In the appendix, we consider a more general formulation of the human capital process in which the rates of human capital accumulation and depreciation can vary with the level of acquired human capital and be stochastic. This richer version yields similar results but is not amenable to aggregation.

price of risk in downturns and, hence, reproduce key features of the fluctuations of asset prices. As Cochrane (2011) emphasizes, all of these preference and stochastic structures have in common that most of the variation in asset prices they generate arises, as in the data, from the time variation in risk premia. To emphasize that our results are robust to the specific details of the preference and stochastic structures that achieve this time variation, we show that quantitatively our results hold for a wide range of the most popular specifications.

Specifically, we begin with the original Campbell and Cochrane (1999) preferences with external *habit* in consumption. We find these preferences appealing because they incorporate the idea that the price of risk rises in recessions in a transparent and intuitive way. An undesirable feature of these preferences, however, is that they give rise to a consumption externality. Hence, the resulting allocations are not fully efficient. Nonetheless, as we argue, since competitive labor market search implies efficient wage setting, allocations in this economy satisfy a constrained efficiency property in that they solve a restricted planning problem. We then consider a version of Campbell and Cochrane preferences that eliminates this externality by making the consumption habit function of exogenous shocks, referred to as *efficient Campbell and Cochrane preferences*, but has nearly identical implications for asset prices and unemployment fluctuations. We use this version as a benchmark for later comparisons.

We next examine two versions of Epstein and Zin (1989) preferences. Namely, we first consider a version of the original Bansal and Yaron (2004)' long-run risk setup modified along the lines suggested by Albuquerque, Eichenbaum, and Rebelo (2016) and Schorfheide, Song, and Yaron (2018) with Epstein and Zin (1989) preferences. Albuquerque et al. (2016) show that by coupling long-run risk with a shock to preferences that they term a *demand shock*, the model better replicates observed features of asset prices. We then consider Epstein and Zin (1989) preferences with a time-varying risk of *disasters*, defined as episodes of unusually large decreases in aggregate consumption driven by large decreases in productivity, and follow the setup of Wachter (2013).

Finally, there is a large class of reduced-form asset pricing models that simply specify a discount factor as a function of shocks. We explore a version of the affine discount factor model of Ang and Piazzesi (2003) as a representative model of this class.

After presenting the quantitative results implied by each of these preferences, we turn to inspecting the mechanism generating them. This analysis also helps to understand both the robustness of our results to a wide range of preference specifications and the role of human capital accumulation in our model. We first prove that the job-finding rate is proportional to the present value of the surplus flows from the match between a worker and a firm. This present value, in turn, can be expressed as a weighted average of the prices of claims proportional to aggregate productivity at each time horizon, in which the *weights* are determined by the degree of human capital accumulation, as implied by the search side of the model, and the *prices* of these claims are determined by the chosen preference and stochastic structure, as implied by the asset pricing side of the model. We refer to the prices of such claims as the prices of *strips*. Intuitively, the greater is the amount of human capital accumulation, the slower is the decay of the surplus flows to matches between firms and workers, and, hence, the slower is the decay of the weights attached to these strips.

Based on this analytical characterization of the job-finding rate, we show that its volatility can be well approximated by a single sufficient statistic: *a weighted average over different time horizons of the elasticity of the price of a strip with respect to productivity shocks at each horizon, multiplied by the volatility of the state.*[2] In this precise sense, then, although the five asset pricing models we consider may have very different implications for various asset pricing moments, their implications for the volatility of unemployment only depend on a single statistic, which naturally captures the persistence of the returns to hiring workers and the volatility of the economy.

In all of the preferences we embed, long-horizon claims to productivity are more sensitive to the state than short-horizon ones. In particular, in all of them, the elasticity of the price of strips (per unit of productivity) with respect to the underlying state starts at zero for an instantaneous claim and increases monotonically with the horizon of the claim. When any of the preference and shock structures we examine is made consistent with the observed time variation in the risk of asset prices, they yield similar values for our sufficient statistic for the volatility of the job-finding rate. This result thus explains why all of these structures

[2] With Campbell and Cochrane preferences, the state is the surplus consumption ratio, with Epstein and Zin preferences with long-run risk, the state is the long-run risk factor, with Epstein and Zin preferences with time-varying disaster risk, the state is the probability of a disaster, and with the Ang and Piazzesi discount factor, the state is simply an abstract one.

generate similar results for unemployment volatility.

The sufficient statistic we identify—a weighted average at different time horizons of monotonically increasing elasticities multiplied by the volatility of the state—further allows us to characterize analytically the roles of human capital and time-varying risk for our results. First, we show that when there is little or no human capital accumulation on the job, the weights are nearly all concentrated on short-horizon claims that display little volatility under all of our asset pricing specifications. Hence, absent human capital accumulation, the model cannot generate much volatility in the job-finding rate. Second, we show that when there is little time-varying risk, the elasticity of the price of strips with respect to the state is small regardless of the horizon. Therefore, the model cannot generate much volatility in the job-finding rate in this case either. It is only when both features are present that our model can produce sizable volatility in the job-finding rate and so in the unemployment rate: human capital accumulation, which places large weight on long-horizon claims, and time variation in the price of risk, which generates large elasticities of these claims.

To help illustrate our mechanism in the most transparent way, we abstract from physical capital throughout most of the paper. In an extension of our baseline model, though, we introduce physical capital accumulation and show that in the presence of physical capital, our model implies quantitative results that are essentially identical to those implied by the model in the absence of physical capital. Thus, our mechanism extends to this case as well.

Related Literature. A closely related paper is the important contribution of Hall (2017), which is also robust to the Chodorow-Reich and Karabarbounis (2016)' critique. Hall (2017) accounts for the observed volatility of unemployment within a model that features alternating wage offer bargaining, a reduced-form discount factor, and no human capital accumulation. The model, however, generates a volatility of the risk-free rate that is an order of magnitude larger than that in the data and relies on a parameterization of wage setting that yields highly inefficient outcomes, associated with counterfactually rigid wages.

As mentioned, we show that the alternating offer bargaining scheme generates efficient outcomes only if the exogenous breakdown rate of bargaining is close to one. We find it convenient to translate this breakdown rate into a mean duration and refer to it as the *duration of a job opportunity.*[3] In a monthly model, this duration must be one month to achieve efficient wage setting. In contrast, Hall posits an extremely low breakdown rate, which yields a duration of a job opportunity of more than six years.

In the paper, we first show that if we eliminate Hall's risk-free rate puzzle by embedding his mechanism in a standard model of asset pricing, it can generate sizable employment fluctuations *only* if the duration of a job opportunity is indeed long, of the order of six years. If, instead, we follow Christiano, Eichenbaum, and Trabandt (2016), who argue that the longest reasonable duration of a job opportunity is three months, then the model generates negligible fluctuations in the job-finding rate. In this sense, Hall's mechanism depends critically on very inefficient wage setting, which amounts to a type of real wage stickiness.

Next, we nest our and Hall's models by considering an integrated model with alternating offer bargaining and human capital accumulation, to shed light on the most promising combination of mechanisms. In particular, when the duration of a job opportunity is one month, the equilibrium is efficient and the model generates our outcomes, whereas when human capital accumulation is muted and the duration of a job opportunity is six years, the model generates Hall's outcomes. We choose the duration of a job opportunity and the parameters of the human capital process to best reproduce two statistics on the observed dynamics of wages: the cross-sectional wage growth with experience estimated by Elsby and Shapiro (2012) and the cyclicality of the user cost of labor estimated by Kudlyak (2014), which measures the variability of the present value of wages associated with matches formed at different points in time. When we do so, we find that the best fit occurs for a rate of human capital accumulation on the job comparable to ours and a short duration of a job opportunity, slightly longer than one month. In this precise sense, the data favors our mechanism over Hall's mechanism.

Also closely related to ours is the work of Kilic and Wachter (2018). These authors embed a reduced-form version of the mechanism in Hall (2017) within a model with Epstein-Zin preferences and variable disaster risk. Like in Hall (2017), their results rely heavily on a form of inefficient real wage stickiness. In contrast, we show that variable disaster risk can

[3] Although in equilibrium bargaining is always concluded immediately, the duration of a job opportunity greatly affects the equilibrium bargained wage.

generate realistic fluctuations in the job-finding rate under efficient wage setting, provided human capital accumulation is allowed for.

The vast bulk of the search and matching literature that addresses the unemployment volatility puzzle of Shimer (2005), including many studies discussed by Ljungqvist and Sargent (2017), consider either linear preferences or constant relative risk aversion preferences. In the Appendix, we revisit this literature but modify the models to be consistent with the critique of Chodorow-Reich and Karabarbounis (2016) on the opportunity cost of employment, together with the insights of Shimer (2010) on recruiting costs. Specifically, Shimer (2010) argues that if recruiting workers or posting vacancies takes time away from production, then the cost of doing so is proportional to the opportunity cost of a worker's time in production. Under these assumptions on the opportunity costs of employment and recruiting, we prove that with constant relative risk aversion preferences, including linear preferences, job-finding rates and unemployment rates are exactly constant. Nonetheless, for the same specification of these opportunity costs, our model generates fluctuations in job-finding rates and unemployment rates of the same magnitudes as in the data. In specific sense, then, our mechanism is complementary to this large literature.

1. Economy

We embed a Diamond-Mortenson-Pissarides (DMP) model of the labor market with competitive search in a general equilibrium model of an economy in which households are composed of employed and unemployed workers and own firms. The economy is subject to aggregate shocks including productivity shocks as well as idiosyncratic shocks.

We extend the DMP model to include two key extra ingredients, a type of asset-pricing preferences that generate time-varying risk and human capital accumulation. As discussed, we consider some of the most popular classes of asset pricing preferences. For concreteness only, we begin with Campbell-Cochrane preferences. The economy consists of a continuum of firms and consumers. Each consumer survives from one period to the next with probability ϕ. In each period, a measure $1 - \phi$ of new consumers is born, so that there is a constant measure one of consumers. Individual consumers accumulate human capital and are subject to idiosyncratic shocks. Firms post vacancies in markets indexed by a consumer's general human capital. Each consumer belongs to one of a large number of families that own firms and insure against idiosyncratic risks.

A. Technologies and Resource Constraints

Throughout the paper we consider several equilibrium concepts and several specifications of preferences. Here we lay out the technologies, transition laws, and resource constraints that will be common to all versions.

Consumers are indexed by a state variable that summarizes their ability to produce output. The variable z_t, referred to as *human capital*, captures returns to experience and stays with the consumer even after a job spell ends. A consumer with state variable z_t produces $A_t z_t$ when employed and $bA_t z_t$ when not employed. Here A_t is aggregate productivity which follows a random walk with drift g_a where

$$(1) \quad \log A_{t+1} = g_a + \log A_t + \varepsilon_{at+1}.$$

where $\varepsilon_{at+1} \sim N\left(0, \sigma_\varepsilon^2\right)$.

Newly born consumers draw their initial human capital from a distribution $\nu(z)$ with mean 1. After that, when a consumer is employed, human capital evolves according to

$$(2) \quad z_{t+1} = (1 + g_e) z_t,$$

and when the consumer is not employed it evolves according to

$$(3) \quad z_{t+1} = (1 + g_u) z_t,$$

where $g_e \geq 0$ and $g_u \leq 0$ are constant rates of human capital accumulation and decumulation.

An unemployed consumer with human capital z_t produces $bA_t z_t$ and it costs $\kappa A_t z_t$ of goods to produce one vacancy directed at a consumer with human capital z_t. Here the opportunity cost of employment corresponds to the value of working at home $bA_t z_t$ and, consistent with the findings of Chodorow-Reich and Karabarbounis (2016), we set the elasticity of this cost to productivity to one. Our specification of the cost of posting vacancies is consistent with Shimer's argument that the cost to the firm of spending time recruiting a new worker is the opportunity cost of using that time to produce goods (see Shimer 2010). Under this view, the cost of recruiting new workers moves one for one with the productivity of a worker in market production. Finally, note that scaling home production and vacancy costs by z is convenient because, as we show later, this specification implies that all value functions are linear in z, but it is not necessary.

Here the realization of the productivity innovation ε_t is the aggregate event. Let $\varepsilon^t = (\varepsilon_0, \ldots, \varepsilon_t)$ be the history of aggregate events at time t. An allocation is a set of stochastic processes for consumption $\{C(\varepsilon^t)\}$ and for each type z, measures of employed consumers, unemployed consumers, and vacancies posted, $\{e(z,\varepsilon^t), u(z,\varepsilon^t), v(z,\varepsilon^t)\}$. For notational simplicity, from now on we suppress explicit dependence on ε^t and write these allocations in shorthand notation as $\{C_t, e_t(z), u_t(z), v_t(z)\}$.

The measures of employed and unemployed consumers satisfy

$$(4) \qquad \int [e_t(z) + u_t(z)]\, dz = 1.$$

At the beginning of period t, current productivity A_t is realized. At that time the unemployed from the end of period $t-1$ search for new matches and firms post vacancies. New matches with consumers are formed and these consumers immediately begin to work. Fraction $1-\sigma$ of the employed at the end of period $t-1$ keeps their jobs and fraction σ separates and enters the unemployment pool in period t.

Consider the unemployed consumers searching for a job at the beginning of period t that have current human capital z, denoted $u_{bt}(z)$. These consumers were unemployed at the end of period $t-1$, had human capital $z/(1+g_u)$ in $t-1$ that grew at rate $1+g_u$ from $t-1$ to t (so that it was updated to z at the beginning of period t), and survived, so that

$$(5) \qquad u_{bt}(z) \equiv \phi u_{t-1}\left(\frac{z}{1+g_u}\right).$$

At the beginning of period t firms post a measure of vacancies $v_t(z)$ that targets consumers with human capital z and creates a measure $m_t(z) = m(u_{bt}(z), v_t(z))$ of matches, where m is a constant returns to scale matching function that is increasing in both arguments.

After matches are formed, there are new entrants into the pool of unemployed for period t with human capital z. These include the measure $\phi\sigma e_{t-1}(z/(1+g_e))$ of consumers who had $z/(1+g_e)$ units of human capital in $t-1$ and z units of human capital in t, worked in period $t-1$, separated from their firms with probability σ, and survived with probability ϕ. Hence, the transition law for employed workers is

$$(6) \quad e_t(z) = \phi(1-\sigma)(1+g_e)e_{t-1}\left(\frac{z}{1+g_e}\right) + m_t(z).$$

The transition law for the unemployed is

$$(7) \quad u_t(z) = u_{bt}(z) - m_t(z) + \phi(1+g_e)\sigma e_{t-1}\left(\frac{z}{1+g_e}\right) + (1-\phi)\nu(z).$$

For later use, we define the *job-finding rate* of an unemployed worker of type z to be $\lambda_{wt}(z) = m_t(z)/u_{bt}(z)$, the *job-filling rate* for a firm that posts a vacancy for type z as $\lambda_{ft}(z) = m_t(z)/v_t(z)$, and the *market tightness* for workers of type z to be $\theta_t(z) = v_t(z)/u_{bt}(z)$. It follows that $\lambda_w(\theta_t) = \theta_t\lambda_f(\theta_t)$. Finally, it is convenient to define the elasticity of the job-filling rate with respect to θ_t as $\eta_t = -\theta_t\lambda'_{ft}(\theta_t)/\lambda_{ft}(\theta_t)$ and so $1-\eta_t = \theta_t\lambda'_{wt}(\theta_t)/\lambda_{wt}(\theta_t)$. Note that when we later assume a Cobb-Douglas matching function, the elasticity η_t is a constant.

The aggregate resource constraint in period t is

$$(8) \quad C_t \leq A_t\int ze_t(z)dz + bA_t\int zu_t(z)dz - \kappa A_t\int zv_t(z)dz,$$

where the three terms on the right of (8) are the total output of the employed, the total output of the unemployed, and the total cost of posting vacancies.

B. A Family's Problem

We represent the insurance arrangements in the economy by imagining that each consumer belongs to one of a large number of identical families, each of which has a continuum of household members. Risk sharing within a family implies that at date t each household member consumes the same amount of goods, C_t, regardless of the idiosyncratic shocks that such a member experiences. (This type of risk-sharing arrangement is familiar from the work of Merz (1995) and Andolfatto (1996).)

Given this setup, we can separate the problem of a family into two parts. The first part is at the level of the family and determines the family's choice of assets and the common consumption level of family members. The second part is at the individual consumer and firm levels in the family. The individual consumer problem determines the employment and unemployment status of each consumer whereas the individual firm problem determines the vacancies created and the matches formed.

Each consumer has Campbell-Cochrane preferences of the form

$$(9) \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\left(C_t - \bar{X}_t\right)^{1-\alpha}}{1-\alpha}$$

where C_t is the common consumption level in a family and $\bar{X}_t$ is an external habit specified below—throughout we use the notation $\overline{Y}$ to distinguish an aggregate variable, $\overline{Y}$, from an individual or family variable, Y. We assume $\beta < 1$ and $\alpha > 1$. Since habit is external, marginal utility is

$$(10) \quad \beta^t (C_t - \bar{X}_t)^{-\alpha}.$$

In a symmetric equilibrium each consumer's consumption C_t equals aggregate consumption $\bar{C}_t$ and we can define the *aggregate surplus consumption ratio* as $\bar{S}_t = \left(\bar{C}_t - \bar{X}_t\right)/\bar{C}_t$, so that aggregate marginal utility is

$$(11) \quad \beta^t (\bar{C}_t - \bar{X}_t)^{-\alpha} = \beta^t \bar{C}^{-\alpha} \bar{S}_t^{-\alpha}$$

and the intertemporal marginal rate of substitution for every family is thus

$$(12) \quad Q_{t,t+1} = \beta \left(\frac{\bar{S}_{t+1}}{\bar{S}_t} \frac{\bar{C}_{t+1}}{\bar{C}_t} \right)^{-\alpha}$$

and let $Q_{t,r} = \beta^{r-t} \left(\frac{\bar{S}_r}{\bar{S}_t} \frac{\bar{C}_r}{\bar{C}_t} \right)^{-\alpha}$ be the discount factor for period r in units of period t for any period $r > t+1$.

Each family has access to complete one-period contingent claims against aggregate risk. In equilibrium, since each family is identical, the prices of contingent claims are related in the usual fashion to the marginal rate of substitution in (12). For notational simplicity, we do not explicitly include them in the budget constraint of a family which can be written as

$$(13)\quad C_t + I_t = W_t + H_t + \Pi_t,$$

where I_t denotes the total resources invested in new vacancies, W_t denotes the total wages of employed workers of the family, H_t is the total home production of unemployed members of the family, and Π_t is the flow of profits from firms the family owns. (Of course, in equilibrium, $I_t = \kappa A_t \int z v_t(z)dz$, $H_t = bA_t \int z u_t(z)dz$, and $W_t + \Pi_t = A_t \int z e_t(z)dz$, and the particular equilibrium concept used will determine the division of any amount of output produced by employed workers into wages and profits.)

We refer to $Q_{t,t+1}$ as the *pricing kernel* for the economy. This kernel determines the intertemporal price of consumption goods in the economy and is the relevant discount factor used by individual consumers and individual firms that are part of a same family.

Next, we follow Campbell and Cochrane (1999) and rather than directly specify a law of motion for habit $\bar{X}_t$, we specify a law of motion for the *aggregate surplus consumption ratio* $\bar{S}_t = (\bar{C}_t - \bar{X}_t)/\bar{C}_t$ as

$$(14)\quad \bar{s}_{t+1} = (1-\rho_s)\, s + \rho_s \bar{s}_t + \lambda\,(s_t)\,(\Delta\bar{c}_{t+1} - \mathbb{E}_t\Delta\bar{c}_{t+1}),$$

where lower case letters denote logs and s denotes the mean of $\bar{s}_t$. The *sensitivity function* $\lambda_t(\bar{s}_t)$ is given by

$$(15)\quad \lambda_t(\bar{s}_t) = \frac{\sigma(\varepsilon_{ct+1})}{\sigma_t(\varepsilon_{ct+1})}\frac{1}{\bar{S}}\left[1-2\,(\bar{s}_t - \bar{s})\right]^{1/2} - 1$$

as long as $\lambda_t(\bar{s}_t)$ is nonnegative and is zero otherwise. Here $\sigma(\varepsilon_{ct+1})$ and $\sigma_t(\varepsilon_{ct+1})$ are the unconditional and conditional standard deviations of the innovations to consumption $\varepsilon_{ct+1} = \bar{c}_{t+1} - \mathbb{E}_t\bar{c}_{t+1}$.

Both Campbell and Cochrane (1999) and Wachter (2006) consider economies in which consumption is exogenous and has a constant conditional variance such that $\sigma(\varepsilon_{ct+1})/$-

$\sigma_t(\varepsilon_{ct+1}) = 1$, in which case our sensitivity function reduces to theirs. Recall how the original Campbell and Cochrane (1999) model generates constant interest rates. In a recession, surplus consumption falls and is expected to increase slowly back to its mean. Since consumption is driven closer to habit, marginal utility rises and consumers would like borrow to increase current consumption. By itself, this intertemporal substitution motive would lead interest rates to rise. At the same time, however, the drop in surplus consumption makes consumers more risk averse and, hence, want to save more. This precautionary savings motive, by itself would lead interest rates to fall. When consumption has constant conditional variance, so that $\sigma(\varepsilon_{ct+1})/\sigma_t(\varepsilon_{ct+1}) = 1$, the functional form of the sensitivity function (15) ensures that the intertemporal substitution and precautionary savings motives exactly offset each other, leading to constant risk free interest rates.

Our production economy instead has endogenous consumption with time-varying conditional volatility. Here the term $\sigma(\varepsilon_{ct+1})/\sigma_t(\varepsilon_{ct+1})$ adjusts for this time-varying conditional volatility and helps the model generate stable interest rates over time by ensuring the precautionary saving motives and intertemporal substitution motives almost offset each other so as to replicate the observed volatility of interest rates.

Note for later that the risk-free rate in this economy $R_{ft} = 1 + r_{ft}$, namely the return on a claim bought at t to one unit of consumption in all states at $t+1$, is

$$(16) \quad R_{ft} = \frac{1}{\mathbb{E}_t Q_{t,t+1}}$$

More generally, in this competitive equilibrium, the return R_{t+1} on any asset in period $t+1$ must satisfy the first order condition $1 = \mathbb{E}_t Q_{t,t+1} R_{t+1}$. By a standard argument (see Hansen and Jagannathan, 1991), this fact implies that the *Sharpe ratio* of any asset, defined as the ratio of the conditional mean of the log excess return on the asset, $\log \mathbb{E}_t(R_{t+1}/R_{ft})$, to the conditional standard deviation $\sigma_t(\log(R_{t+1}))$, must satisfy the following inequality

$$(17) \quad \left| \frac{\log\left(\mathbb{E}_t(R_{t+1}/R_{ft}\right)}{\sigma_t(\log(R_{t+1}))} \right| \leq \; \sigma_t(\log Q_{t,t+1}) = \alpha[1 + \lambda(\bar{s}_t)]\sigma_t(\Delta \bar{c}_{t+1}).$$

assuming that returns are log-normally distributed. The right side of this Hansen-Jagannathan bound, namely $\alpha[1 + \lambda(\bar{s}_t)]\sigma_t(\Delta \bar{c}_{t+1})$, is the highest possible Sharpe ratio in this economy, the *maximal Sharpe ratio.* It is a common measure of the *price of risk.* As we will show later, a critical feature of these preferences is that the price of risk varies with surplus consumption, so in particular, when surplus consumption is low, the price of risk is high, and risky investments are not very attractive.

C. Competitive Search Equilibrium

We set up a competitive search equilibrium of Moen (1997) with the two stage timing of Burdett, Shi, and Wright (2001). In each period t, there are two stages. Let Z_t be the set of levels of human capital in period t. In stage 1, each firm commits to contingent offers for the present value of payments to consumers for each level of human capital $z \in Z_t$, say $\{W_t(z)\}$. In stage 2, having seen all the offers by firms in stage 1, workers of type z choose which market to search in, where each wage offer $W_t(z)$ defines a market together with z, and firms choose which market to enter and whether to post a vacancy. These stages should be thought of as occurring right at the beginning of the period t. Then, output is produced during the period and, at the end of the period, consumption takes place. (In a monthly model, one might think of these stages as all occurring early on the morning of the first day of the month. Then, immediately after that, on the same day consumers and firms match and produce that day and for the rest of the month.)

The interested reader will notice below that our two-stage timing assumption makes the analysis much simpler than the original one stage-timing assumption used by Moen (1997) and others. Briefly, our equilibrium natural results from simple subgame perfection because of the sequential nature of firm and worker decisions.

Consumers Choose Market to Search and Firms Post Vacancies

Here we set up a symmetric equilibrium. It is convenient to work backwards from stage 2 in period t. Consider the offers for consumers with human capital z. We suppose that all firms but one that end up making offers to workers of human capital z make an offer of $W_t(z)$, referred to as the *common market offer* in the common market, and that one firm makes a *deviation offer* $W_t^d(z)$ in the deviation market. We do so in order to construct the value of choosing $W_t^d(z) \neq W_t(z)$ and hence allow each firm to deviate at stage 1. We then show that no firm has a desire to do so and, hence, the equilibrium is symmetric.[4]

By a standard result, it is sufficient to consider one-shot deviations by the firm for each period. The deviation we consider is a one-shot deviation for period t only in that, after period t, the worker envisions that all firms will follow the prescriptions of the symmetric equilibrium, which imply a present value of unemployment of $U_t(z)$ at t. Regardless of whether the consumer accepts the symmetric offer $W_t(z)$ or the deviation offer $W_t^d(z)$, the consumer takes as given the same set of present values $\{U_r(z)\}_{r=t+1}^{\infty}$ that will be received at any period $r \geq t+1$ from a combination of future home production and future employment spells after separation with either firm occurs.

We refer to the present value of all payments to the worker from future home production or future employment spells made after the match made at t dissolves at *any* future date as the *post match value* at t and denote it by $P_t(z)$. In recursive form, we have

$$(18) \quad P_t(z) = \sigma \mathbb{E}_t Q_{t,t+1} U_{t+1}((1+g_e)z) + (1-\sigma)\sigma \mathbb{E}_t Q_{t,t+1} P_{t+1}((1+g_e)z).$$

Of course, the (total) value to a worker of a new match is $W_t + P_t$ since the current match pays W_t over the course of the match and the post match value of the worker is P_t. Here $U_t(z)$ is given by

$$(19) \quad bA_t z + \mathbb{E}_t Q_{t,t+1}\{\lambda_{wt+1}(\theta_{t+1}(z'))[W_{t+1}(z') + P_{t+1}(z')] + [1 - \lambda_{wt+1}(\theta_{t+1}(z'))]U_{t+1}(z')\}$$

with $z' = (1+g_u)z$.

A worker chooses to search in the common market or in the deviant market. If a worker searches in the common market the value of searching is

$$(20) \quad V_{St}(z) = \lambda_{wt}(\theta_t(z))[W_t(z) + P_t(z)] + [1 - \lambda_{wt}(\theta_t(z))]U_t(z),$$

[4] Note that this problem is a standard big K little k problem, with $W_t(z)$ playing the role of big K and $W_t^d(z)$ playing the role of little k.

whereas if the worker searches in the deviant market, the value of searching is

$$(21) \quad V_{St}^{d}(z) = \lambda_{wt}(\theta_{t}^{d}(z))[W_{t}^{d}(z) + P_{t}(z)] + [1 - \lambda_{wt}(\theta_{t}^{d}(z))]U_{t}(z).$$

Here, by construction of the symmetric equilibrium and our use of the one-shot deviation principle, regardless of which market the worker searches in at t, if the worker is unsuccessful in finding a job, the worker returns to the common market for the worker's level of human capital with a common present value of wages.

Optimal search behavior will imply that two outcomes are possible. First, if the wage offer in the deviant market for type z workers is sufficiently attractive, then workers will flow to that market until the value of searching in the two markets is equated, $V_{St}(z) = V_{St}^{d}(z)$. Second, if the wage offer in the deviant market for type z workers is sufficiently unattractive, then the value of searching in the deviant market is strictly lower than that of searching in the common market even if a worker who searches there finds a job with probability 1. This situation occurs when the deviation offer $W_{t}^{d}(z)$ for such workers is sufficiently low that

$$V_{St}(z) > W_{t}^{d}(z) + P_{t}(z).$$

Clearly, it is pointless for a firm to make an offer that attracts no workers. Thus, we restrict attention to *serious offers*, namely those that satisfy

$$(22) \quad V_{St}(z) \leq W_{t}^{d}(z) + P_{t}(z).$$

For such serious offers, a deviating firm anticipates that its offer leads to a market tightness $\theta_{t}^{d}(z)$ such that $\lambda_{wt}(\theta_{t}^{d}(z))$ satisfies the *worker participation constraint* in that $V_{St}(z) = V_{St}^{d}(z)$ or, equivalently,

$$(23) \quad V_{St}(z) = \lambda_{wt}(\theta_{t}^{d}(z))[W_{t}^{d}(z) + P_{t}(z)] + [1 - \lambda_{wt}(\theta_{t}^{d}(z))]U_{t}(z).$$

Any firm that desires to target a worker of type $z \in Z_t$ pays $\kappa A_t z$ to post a vacancy. We let $Y_t(z)$ denote the present value of output produced by the match of firm with a worker of type z. If a firm has made the common market offer $W_t(z)$ to workers of type z, then the value of a vacancy is

$$(24)\quad V_t(z) = -\kappa A_t z + \lambda_f(\theta_t(z))[Y_t(z) - W_t(z)].$$

Note that it is free for the firm to commit to a set of offers in stage 1 and it is only after the firm chooses to post a vacancy for workers of type z that the firm pays the cost $\kappa A_t z$ for that vacancy. Free entry into each such market implies for each z, $V_t(z) = 0$ so that

$$(25)\quad \kappa A_t z = \lambda_f(\theta_t(z))[Y_t(z) - W_t(z)].$$

Here we do not impose free entry into the deviation market, but instead show below that if all other firms are choosing the common market offer, the solution to the deviant's problem is also to make the common market offer.

At the end of stage 2 in period t, after consumers and firms have made their decisions, each family consumes C_t.

Firms Choose Contingent Offers

We now set up the problems of firms at stage 1 of period t with state ε^t and current productivity $A_t = A(\varepsilon^t)$. Since a match dissolves with exogenous probability σ, that present value can be written recursively as

$$(26)\quad Y_t(z) = A_t z + (1-\sigma)\mathbb{E}_t Q_{t,t+1} Y_{t+1}(z')$$

with $z' = (1+g_e)z$. If a firm makes the common market offer $W_t(z)$, the value of a vacancy is (24), where $\lambda_f(\theta_t(z))$ is the job-filling probability in the common market.

In the deviation market, any serious offer $W_t^d(z)$ leads to a value

$$(27)\quad V_t^d(z) = -\kappa A_t z + \lambda_f(\theta_t^d)[Y_t(z) - W_t^d(z)],$$

where, critically, the market tightness $\theta_t^d(z)$ that corresponds to the offer $W_t^d(z)$ is determined by the worker participation constraint (23). The problem of the deviator in market z is to solve

$$\max_{\left\{W_t^d(z),\theta_t^d(z)\right\}} V_t^d(z) \tag{28}$$

subject to the serious offer constraint (22) and the participation constraint (23). Notice that if the deviator chooses to offer $W_t^d(z) = W_t(z)$ then the participation constraint implies that corresponding market tightness is $\theta_t^d(z) = \theta_t(z)$.

Notice that the problem of a deviator is simply the problem of every firm in the common market, that is, every firm contemplates a deviation. That is, in a symmetric equilibrium, each firm solves for its profit maximizing wage offer taking as given that all other firms are making the common market offer and thus solves (28). Hence, (28) is simply the *firm problem* and we will refer to it as such hereafter. Note for later that since all firms are solving the same problem, it will be immediate that in equilibrium, $W_t^d(z) = W_t(z)$.

D. Equilibrium: Definition and Characterization

Given an initial condition e_0, we define a *competitive search equilibrium* is a collection of state-contingent sequences $\{C_t, Q_{t,t+1}, S_t\}_{t=0}^{\infty}$ and state- and z-contingent sequences $\left\{W_t(z), W_t^d(z), V_{St}(z), V_{St}^d(z), P_t(z), U_t(z), Y_t(z), \theta_t(z), \theta_t^d(z), v_t(z), e_t(z)\right\}_{t=0}^{\infty}$ such that: *i)* for each t, taking as given $U_t(z), Y_t(z), W_t(z), P_t(z), V_{St}(z), Q_{t,t+1}$, the wage payment $W_t^d(z)$ and market tightness $\theta_t^d(z)$ solve the firm's problem (28), *ii)* the collection of state-contingent sequences $\{P_t(z), U_t(z), V_t(z)\}_{t=0}^{\infty}$ satisfy the valuation equations in the common market, (18), (19), and (24), *iii)* the value $V_{St}(z)$ and $V_{St}^d(z)$ satisfy (20) and (21), *iv)* $W_t^d(z) = W_t(z)$ and $\theta_t^d(z) = \theta_t(z)$, *v)* the law of motions for employment and unemployment satisfy (6) and (7), *vi)* the resource constraint (8) holds, and *vii)* $Q_{t,t+1}$ satisfies (12).

We turn now to a characterization of the competitive search equilibrium. We first show that because market production, home production, and the cost of posting vacancies all scale with z, the resulting equilibrium value functions are all linear in z, and market tightness, job-finding rates, and job-filling rates are independent of z. To set up this lemma, let Y_t denote $Y_t(1)$, and use similar notation for other values.

Lemma 1 (Linearity of Competitive Search Equilibrium)**.** *In the competitive search equilib-*

rium, labor market tightness is independent of z, $\theta_t(z) = \theta_t$, and the values are linear in z, $Y_t(z) = Y_t z$, $U_t(z) = U_t z$, $P_t(z) = P_t z$, and $W_t(z) = W_t$.

We prove this result in the Appendix using a simple guess and verify strategy. This result immediately implies that to solve for the valuations, we do not need to record the distributions of $e_t(z)$ and $u_t(z)$ but rather only need to record the aggregate human capital of the employed and the unemployed, given by $Z_{et} = \int z e_t(z) \mathrm{d}z$ and $Z_{ut} = \int z u_t(z) \mathrm{d}z$. Integrating (6) and (7) gives that the transitions laws for the aggregate human capital of the employed and the unemployed satisfy

(29) $Z_{et} = \phi(1-\sigma)(1+g_e) Z_{et-1} + \phi\lambda_{wt}(1+g_u) Z_{ut-1}$,

(30) $Z_{ut} = 1 - \phi + \phi(1-\lambda_{wt})(1+g_u) Z_{ut-1} + \phi\sigma(1+g_e) Z_{et-1}$,

and that the aggregate resource constraint can be written as

(31) $C_t \leq A_t Z_{et} + bA_t Z_{ut} - (1+g_u)\kappa A_t \theta_t Z_{ut-1}$,

where we have used that $Z_{vt} = \int z v_t(z) \mathrm{d}z = \theta_t(1+g_u) Z_{ut-1}$ are aggregate vacancy costs.

We now show that, even with the consumption externality emanating from the external habit $\bar{X}_t$ in Campbell and Cochrane preferences, the allocations in the competitive search equilibrium are constrained efficient in that they solve a *restricted planning problem.*

Proposition 1. *Taking as given the process for the date zero discount factors $\{Q_{0,t}\}$ and the initial conditions on aggregate human capital $Z_{e,-1}$ and $Z_{u,-1}$, the allocations $\{C_t, Z_{et}, Z_{ut}, \theta_t\}$ maximize $E_0 \sum_{t=0}^{\infty} Q_{0,t} C_t$ subject to (29) to (31).*

The idea is that since the competitive search wage setting mechanism leads to an efficient labor market equilibrium, the equilibrium is efficient conditional on the consumption process. Consider now the first order conditions for the restricted planning problem given by

(32) $\mu_{et} = A_t + \phi(1+g_e)\mathbb{E}_t Q_{t,t+1}\left[(1-\sigma)\mu_{et+1} + \sigma\mu_{ut+1}\right]$,

(33) $\mu_{ut} = bA_t + \phi(1+g_u)\mathbb{E}_t Q_{t,t+1}\left[\eta_{t+1}\lambda_{wt+1}\mu_{et+1} + \left(1 - \eta_{t+1}\lambda_{wt+1}\right)\mu_{ut+1}\right]$,

$$(34) \quad \kappa A_t = (1-\eta_t)\lambda_{ft}(\mu_{et} - \mu_{ut}),$$

where μ_{et} and μ_{ut} are the multipliers on the transitions laws for the aggregate human capital of the employed and the unemployed (29) and (30). Here μ_{et} and μ_{ut} are, respectively, the shadow values of increasing the stocks of employed human capital and unemployed human capital by one unit. Notice the similarity of these three equations with those that arise in a random search model. In particular, equation (32) is analogous to the sum of the value of an employed worker and the value of an employing firm, (33) is analogous to sum of the value of an unemployed worker and the value of a firm without a worker, and (34) is analogous the free entry condition. The key difference here is that the planner internalizes the link between vacancy creation and the job-finding and job-filling rates, and, hence, internalizes the search externality from posting vacancies.

The dynamic system governing the search equilibrium is given by (32), (33), (34), along with the system governing the evolution of the aggregate human capital of the employed and the unemployed in (29) and (30), as well as the evolution of $Q_{t,t+1}$, namely, (12), (14), and the resource constraint.

To develop intuition for a solution to (32), (33), and (34), we consider an approximation in which we ignore the variation in the future job-finding rates ($\lambda_{ws} = \lambda_w$ for $s > t$) in the difference equations for the values of employment μ_{et} and unemployment μ_{ut} in (32) and (33), solve the dynamic system forward, and after imposing the appropriate limiting condition, find that

$$(35) \quad \begin{bmatrix} \mu_{et} \\ \mu_{ut} \end{bmatrix} = \sum_{n=0}^{\infty} \phi^n \begin{bmatrix} (1+g_e)(1-\sigma) & (1+g_e)\sigma \\ (1+g_u)\eta\lambda_w & (1+g_u)(1-\eta\lambda_w) \end{bmatrix}^n \begin{bmatrix} 1 \\ b \end{bmatrix} \mathbb{E}_t Q_{t,t+n} A_{t+n}.$$

We also impose the Cobb-Douglas matching function $m(u,v) = Bu^{\eta}v^{1-\eta}$ that we use in our quantitative analysis. This matching function implies that the job-filling rate λ_{ft} and the job-finding rate λ_{wt} satisfy $\lambda_{ft}^{1-\eta} = B/\lambda_{wt}^{\eta}$, and, hence, we can rewrite the free entry condition as

$$(36) \quad \log(\lambda_{wt}) = \chi + \frac{1-\eta}{\eta} \log\left(\frac{\mu_{et} - \mu_{ut}}{A_t}\right).$$

The dynamic system (35) makes it clear that the value of hiring a worker $\mu_{et} - \mu_{ut}$ depends on the future discounted values of productivity.

To interpret (36), it is useful to write the value of hiring a worker as the present value of the flows from a match, namely

$$(37) \quad \mu_{et} - \mu_{ut} = \sum_{n=0}^{\infty} \mathbb{E}_t Q_{t,t+n} v_{t+n}$$

where $v_{t+n} = (c_\ell \delta_\ell^n + c_s \delta_s^n) A_{t+n}$ is the surplus flow in period $t+n$ from a match formed in period t, δ_ℓ and δ_s are the large and small roots of the vector difference equation (35) with corresponding weights c_ℓ and c_s, discussed below. To interpret this flow, recall that $\mu_{et} - \mu_{ut}$ is the difference in the value of employment and unemployment, or the present value of the *match flows* from a match formed at t. Clearly, the match flow in a period is proportional to productivity in that period. The present value of these flows decays with the length of time since the match was formed because an unemployed worker can find a job and an employed worker can lose one. Critically, as we elaborate on below, the present value of these flows decay more slowly the larger is the growth in human capital when a worker is employed and the larger is the decay in human capital when a worker is unemployed.

2. Quantification

We next describe how we choose parameters for our quantitative analysis and the model's steady-state implications.

A. Parameters

The model is monthly. Table 1 lists the parameters. We have eight assigned parameters $\{b, \sigma, \eta, \gamma, \phi, g_e, g_u, \rho_s\}$ and choose the value of the remaining seven parameters $\{\kappa, B, \alpha, \bar{S}, g_a, \sigma_a, \beta\}$ to match seven moments of the data.

We follow Ljungqvist and Sargent (2017) and set the home production parameter $b = 0.6$ and the matching function elasticity η equal to 1/2. The separation rate σ is set to 2.8% per month, which is in the range of separation rates reported in Krusell et al. (2011) for prime-age males aged 21 to 65.[5] We choose the survival probability ϕ to be consistent

[5] We reproduced this separation rate using the Current Population Survey (CPS) data and the seasonal

with an average working life of 40 years. We set the growth rate of human capital when employed $g_e = 2.5\%$ per year, which is in the range estimated by Altonji and Shakotko (1987) and Buchinsky et al. (2010) for returns to experience. Here we choose a conservative estimate of human capital depreciation of $g_u = -5.7\%$ per year and later show how our results are strengthened if we use less conservative estimates.[6] We calibrate the parameters of the exogenous productivity process g_a and σ_a to equal the mean of labor productivity growth and the standard deviation of labor productivity from the Bureau of Labor Statistics.[7] We later show the robustness of our results to these parameters.

To pin down the parameter κ for the vacancy posting cost and the match efficiency parameter B, we normalize the mean value of θ to 1, as does Shimer (2005), then choose κ and B to reproduce two moments of the data: a mean job-finding rate λ_w of .455 from Shimer (2012) and a mean unemployment rate of 5.9% consistent with the 1951-2015 data from BLS.

Consider next the preference parameters. We set habit persistence ρ_s to 0.89 at an annual frequency, which Wachter (2006) argues is consistent with the autocorrelation of several financial yields in the data. Moreover, given the rest of the chosen parameters, including those of the productivity process, it turns out that this parameter implies a volatility of our endogenous consumption process close to the observed one (93% annualized in the model and 1.07% annualized in the data). We follow Campbell and Cochrane (1999) and Wachter (2006) in choosing the inverse elasticity of intertemporal substitution α so that the maximum Sharpe ratio in the model, given by the right side of (17), equals the Sharpe ratio of the aggregate stock market return measured from the CRSP value-weighted stock index

adjustments and classification error corrections for reported employment status adopted by Krusell et al. (2011), who used data from 1994 to 2007. We also updated the series using data from 1978 to 2012 and, using the same corrections, replicated the total separation rate of 2.8% per month estimated by Krusell et al. (2017). We note that this updated number is close to the total separation rate of 2.66% from Krusell et al. (2011) and thus close to our number as well. This figure is lower than the 3.6% used by Shimer (2005) because Shimer includes job-to-job transitions, whereas we focus solely on employment-to-nonemployment transitions. We also experimented with a recalibration in which we used the higher separation rate from Shimer and found very similar results. As will become evident later, employment responses in our model are determined by the duration of the benefit flows from a match, which is primarily influenced by the amount of general human capital accumulation, rather than by the length of time a worker spends in any given match.

[6] This estimate, based on the Panel Study of Income Dynamics 1975-1992, is the average wage change for workers with fewer than 35 years of labor market experience who experience up to one year of nonemployment based on the same sample used by Buchinsky et al. (2010).

[7] We use the variable "Nonfarm Business Sector: Real Output Per Hour of All Persons."

covering all firms continuously listed on NYSE, AMEX, and NASDAQ. (As shown by Larrain and Yogo (2008), the returns measured from CRSP are highly correlated with returns on the aggregate stock market measured from Flow of Fundsdata; in our sample, this correlation is of 0.97.)

We choose the remaining preference parameters—the rate of time preference β and mean surplus consumption $\bar{S}$—to match the mean and the standard deviation of the real risk-free rate $r_{ft} = \log R_{ft}$ that we construct as $i_t - \mathbb{E}_t\pi_{t+1}$, where i_t is the 1-month Tbill rate from an updated version of the Fama and French (1993) data on Kenneth French's website and $\mathbb{E}_t\pi_{t+1}$ is proxied as the projection of monthly CPI inflation on 12 of its lags.

As for the calculation of $\mathbb{E}_t\pi_{t+1}$, note using $i_t - \mathbb{E}_t\pi_{t+1}$ as a proxy for the real risk-free rate is natural. Fitting a univariate AR(1) specification with lags up to one year is standard. See, for instance, Hur, Kondo, and Perri (2019).

To gain some intuition for how we can choose the mean surplus consumption $\bar{S}$ to generate modest volatility in risk free rates r_{ft}, note that if consumption is conditionally log normal then the real rate r_{ft} is given by

$$(38)\quad r_{ft} = -\log(\beta) - \frac{\alpha(1-\rho_s - d/\alpha)}{2} + \alpha\mathbb{E}_t\Delta c_{t+1} - d(\bar{s}_t - \bar{s}),$$

where d is implicitly determined by

$$\bar{S} = \sigma(\varepsilon_c)\left(\frac{\alpha}{1-\rho_s - d/\alpha}\right)^{1/2}.$$

Clearly, choosing $\bar{S}$ is equivalent to choosing d, which from (38) can be pinned down by matching the observed volatility in risk free rates, given that the other parameters are fixed.

B. Computational Algorithm

Asset prices in habit models are highly nonlinear and we solve for them by a global numerical strategy. In particular, we solve the model numerically by projecting the global solution of our model onto the space spanned by a basis of Chebyshev polynomials of up to degree twenty, and evaluate expectations by 100-point Gauss-Hermite quadrature. As shown by Wachter (2005), the best practice in solving models with Campbell-Cochrane habit is to consider a large and fine grid over the surplus consumption space with many grid points close to zero. In practice, we refine the grid and widen it progressively until results no longer change. (For details see the Appendix.)

3. Findings

We start by noting that Shimer (2012) has argued that in the data the variations in job-finding rates account for over two-thirds of the fluctuations in the unemployment rate and that the key problem for existing search models is that they generate much too small variations in the job-finding rate. Our study is focused solely on a mechanism that increases the volatility of job-finding rates and we purposely abstract from fluctuations in job separation rates. Thus, the most obvious statistics to compare in the model and the data are those on job-finding rates.

A. Baseline Findings

In Table 1, we see that our model produces a volatility of the job-finding rate very similar and slightly higher than that in the data, namely, 6.73 in the model and 6.68 in the data (see Shimer, 2012). The serial correlation in the job-finding rate in the model is similar and slightly higher than that in the data. Based on these two statistics, we argue that our model solves the Shimer puzzle. Note that, by construction, our model is consistent with the Chodorow-Reich and Karabarbounis (2016) critique. We show later that it is also consistent with the Borovicka and Borovickova (2019) critique.

Note now that even if our model reproduced exactly the observed time series for the job-finding rate, it would not produce the observed time series for the unemployment rate because in the data the separation rate varies whereas in our model it is constant. To translate the implications of our model into the relevant ones for the unemployment rate, we construct a *constant separation unemployment rate* series from the data on employment and separations, $\{\bar{u}_t\}$, as

$$(39) \quad \bar{u}_{t+1} = 1 - \phi + \phi\sigma(1 - \bar{u}_t) + \phi(1 - \lambda_{wt}^{data})\bar{u}_t,$$

where we choose ϕ and σ as in our baseline and feed in the observed λ_{wt} from the data. We start from the initial condition u_0^{data} given by the initial unemployment rate at the beginning of our sample, namely 1948 Q1. Simply put, $\{\bar{u}_t\}$ is the unemployment series generated by the observed job-finding rate in the data given our assumed constant separation rates and birth and death rates. Notice that this series really contains exactly the same information as λ_{wt}^{data} because if our model generated a λ_{wt} series that matches λ_{wt}^{data}, then by construction it would also generate an unemployment series u_t that matches $\bar{u}_t$. In Table 1 we see that our model also produces a volatility and serial correlation for this constant separation unemployment rate similar to that in data. For brevity, both in this table and hereafter we refer to this series as simply the *unemployment rate*.

The two panels in Figure 1 plot the impulse responses of the job-finding rate and the unemployment rate to a one-percent increase in productivity starting from the ergodic mean of the state variables S_t, Z_{ut}, and Z_{et}.[8]

We argue that both some properties of our preferences and human capital are critical to generating large movements in the job-finding rate and, hence, solving the employment volatility puzzle. Here we study both forces in turn.

B. Implications for Wage Rigidity

In a search model, hiring a work is akin to acquiring a long-term asset subject to adjustment costs. Hence, a more accurate measure of the cost of employing a work than the current wage is the *user cost of labor*, defined as the difference in the present value of wages from a match that starts at time t and one that starts at time $t+1$. Kudlyak (2014) formalizes this argument and documents that the user cost of labor is highly cyclical. In particular, based on NLSY data, she estimates that a one percentage point increase in unemployment is associated with more than a 5% drop in the user cost of labor, which is substantially larger than the drop in the wages of newly hired workers (3%) or average wages (1.8%). Basu and House (2016) extend her analysis and estimate a even higher cyclicality for the user cost of labor.

In assessing the ability of our model to account for the observed volatility of unem-

[8] Note that since the model is nonlinear, the response to a shock depends on the levels of the state variables and the size of the shock. As is standard, we compute the impulse response for, say, job-finding rates λ_{wt} as $E_t(\lambda_{wt+n}|\varepsilon_t = \Delta, S_t, Z_{ut}, Z_{et}) - E_t(\varepsilon_t = \Delta, S_t, Z_{ut}, Z_{et})$ with S_t, Z_{ut}, and Z_{et} all set to their ergodic means.

ployment in Table 1, we compare the cyclicality of the user cost of labor implied by our model with that estimated by Kudlyak (2014) and find that our model is close to it. Hence, unlike most of the literature, our mechanism *does not* rely on wage rigidity. We also show below that when this cyclicality, as well as wage growth with experience, are explicitly targeted, our model successfully matches both statistics.

4. The Critical Ingredients: Preferences and Human Capital

Here we demonstrate the critical role played by preferences, through time varying risk, and human capital. We show that the presence of both are critical: without either time-varying risk or human capital accumulation, the model does not generate volatile job-finding rates or unemployment.

In terms of preferences, we begin by showing that with standard constant relative risk aversion preferences, the model generates zero volatility in job finding rates and employment. We then show that our results holds for a variety of specifications popular in the asset pricing literature. In terms of human capital accumulation, we also demonstrate that even if we allow for both human capital accumulation on the job and depreciation off the job, the main quantitative force comes from the accumulation on the job. In the next section, we identify a sufficient statistic for the volatility of the job-finding rate and show that all the preferences we consider give rise to the same value of this statistic and so to a very similar degree of volatility of unemployment. See Table 9 for the parametrization of all models considered.

A. Role of Preferences

Here we investigate the role of preferences in generating our results. To highlight the importance of time-varying risk, we begin with contrasting our results to those from a model with constant relative risk aversion (CRRA) preferences. To show that our results do not depend on having a consumption externality, we consider a version of Campbell and Cochrane preferences in which we reinterpret the habit as an exogenous shock to preferences, engineered so as to reproduce the same asset pricing properties as those produced by the version of Campbell and Cochrane preferences with external habit. To show that our results hold for the other most popular asset-pricing preferences, we consider two version of Epstein-Zin preferences, one with long-run risk shocks and one with variable disaster risk and, finally, consider an affine pricing kernel.

Constant Relative Risk Aversion Preferences

In Table 2 we compare our baseline model to the *CRRA model* in which we keep all parameters the same except that we set habit $\bar{X}_t$ to zero. Thus, preferences take the standard constant relative risk aversion form

$$(40)\quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\alpha}}{1-\alpha}.$$

In Table 2, we see that the resulting fluctuations in both the job-finding rate and the unemployment rate are identically zero.

When interpreting this result, it is important to note that, by construction, we have abstracted from the standard *differential productivity across sectors* mechanism in search models, which implies that an increase in productivity raises the productivity in the market but leaves both the productivity in home production and the cost of posting vacancies unaffected. Here, instead, an increase in A_t increases equally a worker's productivity in the market, in home production, and the cost of posting vacancies. In particular, a consumer with state variable z produces $A_t z$ when employed and $bA_t z$ when not employed, and it costs a firm $\kappa A_t z$ to post a vacancy for such a consumer.

Importantly, the only effect of a change in productivity in our model is that it changes the expected discounted value of the surplus from a match scaled by current productivity. To understand this better, divide both sides of the free-entry condition in market z in (24) by $A_t z$ and rewrite it as

$$(41)\quad \kappa = (1-\eta_t)\lambda_{ft} \frac{Y_t + P_t - U_t}{A_t},$$

where $(Y_t + P_t - U_t)/A_t$ is the *scaled net value of a match* at the margin. We can prove the following result:

Proposition 2. *Starting from the steady state values of the total human capital of the employed and the unemployed Z_e and Z_u, with preferences of the form (40), both the job-finding rate and the unemployment rate are constant.*

The proof proceeds by guessing that such an equilibrium exists and then verifying that it does by showing that, under that conjecture, the equations that define an equilibrium are satisfied. To understand the logic of this argument, note that under this conjecture scaled consumption and scaled net value $\tilde{C}_t \equiv C_t/A_t$ and $\tilde{Y}_t + \tilde{P}_t - \tilde{U}_t \equiv (Y_t + P_t - U_t)/A_t$ are constant, so that the present value of scaled surplus satisfies

$$(42) \quad \tilde{Y} + \tilde{P} - \tilde{U} = 1 - b + \phi(1 - \sigma - \eta\lambda_w)\delta(\tilde{Y} + \tilde{P} - \tilde{U}),$$

where $\delta = \beta e^{(1-\alpha)g_a + (1-\alpha)^2\sigma_a^2/2}$. Note that here the curvature in the CRRA utility function comes into the constant effective discount rate δ, which increases the mean risk premium. Critically, however, under this utility function, the present value of investing in workers by posting vacancies does not fluctuate with productivity. Since, under this conjecture, market tightness θ as well as the total human of the employed and unemployed, Z_e and Z_u, are constant, from the resource constraint (31), scaled consumption $\tilde{C}$ satisfies

$$(43) \quad \tilde{C} = Z_e + (b - \kappa\theta)Z_u,$$

and, hence, is constant as well. Using similar logic with the other equations verifies our conjecture.[9]

Observe that, in contrast, in our baseline model with habit, the time varying price of risk means that the risk of investing in workers does indeed vary with productivity: when surplus is low today, agents are particularly averse to taking on a risky investment, such as investing in vacancies. So this direct effect leads investment in vacancies to fall in recessions. There is a second effect, intertemporal substitution, that reinforces this direct effect. Here, because surplus consumption is mean reverting, after a bad shock agents expect surplus consumption states to revert back to the mean in the future as their habit slowly adapts by the habit mechanism in (11). Hence, they are less willing to invest today because the marginal utility is high relative to the future; agents have an intertemporal substitution motive.

[9] Note that we posit that the states Z_e and Z_u are initially at their steady-state values because if these variables start away from their steady states, they will deterministically drift towards the steady state and job finding rates and unemployment rates will move a bit because of this drift and not because of technology shocks. For details of the proof see the Appendix.

Overall, without habit neither the time varying price of risk force nor the intertemporal substitution force are present. Moreover, as noted before we have abstracted from the differential productivity across sectors forces that are standard in search models, namely that when productivity is high, the productivity of working in the market is high relative to that of working at home, and that hiring workers is relatively cheap. Proposition 2 shows that without either habits or these standard forces, labor is exactly constant.

Efficient Campbell and Cochrane Preferences

Here we consider how to rewrite Campbell-Cochrane preferences to obtain nearly identical quantitative results but without a consumption externality, so that the resulting allocations are not just constrained efficient but fully efficient. To see how, recall that our baseline specification of preferences

$$(44) \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\left(C_t - \bar{X}_t\right)^{1-\alpha}}{1-\alpha}$$

have a consumption externality because the marginal utility as in (10) depends on aggregate consumption rather than on the family's own consumption. Of course, for the rest of the model, all that we use these preferences for is to obtain the pricing kernel, namely $Q_{t,t+1}$ defined as before in (12).

There is a simple way to change the model slightly to get rid of the consumption externality but to keep all the asset pricing properties of the original specification. Before we do so, we note that if we rewrite preferences in (44) as

$$(45) \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \delta_t \frac{C_t^{1-\alpha}}{1-\alpha}$$

with $\delta_t = \beta^t \bar{S}_t^{-\alpha}$, where $\bar{S}_t$ is external to the family, we find that the aggregate marginal utility is again equal to $\beta^t \bar{S}_t^{-\alpha} \bar{C}_t^{-\alpha}$ and, hence, so the stochastic discount factor is also (12). So far this model is exactly the same as that in Campbell-Cochrane just re-expressed a bit. Here we are motivated in part by the observation in Cochrane (2017, p. 948) that nearly all the new approaches in macro-finance can be cast in the form (45) in which the new variable

$\bar{S}_t$ is added to the pricing kernel and where this new variable "does most of the work".

Now, in the original Campbell-Cochrane specification, all innovations to aggregate consumption are driven by innovations in aggregate productivity shocks. In our alteration of the model we exploit that, quantitatively, innovations in aggregate consumption are approximately proportional to those in productivity shocks. Motivated by this quantitative result we define the *efficient Campbell-Cochrane model* to be one with preferences of the form (45), S_{at} defined in log form by

$$(46) \quad \lambda_t(\bar{s}_t) = \frac{\sigma(\varepsilon_{ct+1})}{\sigma_t(\varepsilon_{ct+1})} \frac{1}{\bar{S}} \left[1 - 2\left(\bar{s}_t - \bar{s}\right)\right]^{1/2} - 1,$$

and

$$(47) \quad s_{at+1} = (1 - \rho_s)\, s_a + \rho_s s_{at} + \lambda_a\left(s_{at}\right)\left(\Delta a_{t+1} - \mathbb{E}_t \Delta a_{t+1}\right).$$

It is then straightforward to show the following proposition.

Proposition 3. *In the efficient Campbell-Cochrane model with preferences given by (45) and a process for s_{at} given by (47), the competitive search allocations for this economy are efficient.*

The reason is straightforward: with efficient Campbell and Cochrane preferences, the habit is driven by exogenous shocks so there are no consumption externalities. Hence, the competitive search equilibrium in this new economy generates efficient allocations for standard reasons. Table 2 confirms that the efficient Campbell-Cochrane model produces nearly identical results as the baseline model.

Epstein-Zin Preferences with Long-Run Risk

We consider next a model with Epstein-Zin preferences, a slow-moving predictable component in productivity as in Bansal and Yaron (2004), and demand shocks as in Albuquerque, Eichenbaum, Luo, and Rebelo (2016) and Schorfheide, Song, and Yaron (2018). In particular, the specification of preferences is now

$$V_t = \left[(1-\beta)\lambda_t C_t^{1-\rho} + \beta\left(\mathbb{E}_t V_{t+1}^{1-\alpha}\right)^{\frac{1-\rho}{1-\alpha}}\right]^{\frac{1}{1-\rho}},$$

where α is the coefficient of relative risk aversion and ρ is the inverse elasticity of intertemporal substitution. Productivity growth now has predictable component s_t, where

(48) $\Delta a_{t+1} = g_a + s_t + \sigma_a \varepsilon_{at+1},$

(49) $s_{t+1} = \rho_s s_t + \phi_s \sigma_a \varepsilon_{st+1},$

and the shocks ε_{at} and ε_{st} are standard normal i.i.d. and orthogonal. Demand shocks λ_t follow an autoregressive process in growth rates

(50) $\log(\lambda_{t+1}/\lambda_t) = \rho_\lambda \log(\lambda_t/\lambda_{t-1}) + \phi_\lambda \sigma_a \varepsilon_{\lambda t+1},$

where the shock $\varepsilon_{\lambda t}$ is a standard normal i.i.d. process orthogonal to the other shocks.

We set the model's parameters as follows. We parametrize the predictable component of productivity growth to have the same share of volatility $\phi_s^2/(1-\rho_s^2) = 0.047$ of the productivity growth process as that chosen by Bansal and Yaron (2004) for consumption growth. We use a relatively high persistence parameter $\rho_s = 0.96^{1/12}$ that generates an autocorrelation of productivity growth at an annual frequency a bit higher than but close to the observed one but is closer to matching the observed volatility of the job-finding rate. We choose $\sigma_a = 0.0051$ so that the volatility of the productivity process $\sigma_a^2 + \phi_s^2\sigma_a^2/(1-\rho_s^2)$ is the same as in the data. We choose the demand shock process to have the same persistence we used in the case with habit and a volatility that reproduces the standard deviation of the risk-free rate.

We choose a risk aversion coefficient of 4 to hit a Sharpe ratio of 0.45 for the consumption portfolio, and we pick a large elasticity of intertemporal substitution of 10. To understand this choice, note first that with an elasticity of intertemporal substitution equal to one, the volatility of the job-finding rate is exactly zero—see the Appendix for a proof of this claim. Note next that a large elasticity parameter is not necessarily inconsistent with the available evidence of a low elasticity of intertemporal substitution because of the presence of demand shocks. Indeed, when we estimate the contemporaneous elasticity of consumption growth with respect to interest rates on simulated data using powers of s_t and lagged consumption growth as instrumental variables, we find a coefficient around 0.2, which is consistent with estimates in the literature (see, for instance, Hall (1988) and Beeler and Campbell (2012)). As in the case with habit, we pick the coefficient of relative risk aversion α so that a consumption claim has the same Sharpe ratio as in observed stock market data.

In Table 2, we show that with these preferences and human capital accumulation, the model can produce around 89% of the volatility of unemployment (0.67 in the model versus 0.76 in the data).

Epstein-Zin Preferences with Variable Disaster Risk

We adopt a discrete-time version of the model of Wachter (2013) with Epstein-Zin preferences and a slow-moving probability of rare disasters. In particular, the specification of preferences becomes

$$V_t = \left[(1-\beta)C_t^{1-\rho} + \beta\left(\mathbb{E}_t V_{t+1}^{1-\alpha}\right)^{\frac{1-\rho}{1-\alpha}}\right]^{\frac{1}{1-\rho}},$$

where α is the coefficient of relative risk aversion and ρ is the inverse elasticity of intertemporal substitution, and productivity growth is now driven by a discrete-valued jump component j_{t+1} as

$$\Delta a_{t+1} = g_a + \sigma_a \varepsilon_{at+1} - \theta j_{t+1}.$$

We assume that the disaster component is distributed as a Poisson random variable $j_{t+1} \sim Poisson(s_t)$, with a nonnegative slow-moving disaster intensity s_t that evolves as

$$(51) \quad s_{t+1} = (1-\rho_s)s + \rho_s s_t + \sqrt{s_t}\sigma_s \varepsilon_{st+1}.$$

We choose value for the parameters to generate an ergodic mean disaster probability s of 3.55% per year as in Wachter (2013), a disaster impact θ of 0.26 as in Wachter (2013), a persistence ρ_s of $0.96^{1/12}$, a volatility σ_s of 0.0131 to match the standard deviation of the risk-free rate, and a risk aversion coefficient of 2 to target a Sharpe ratio of 0.45 for the consumption portfolio. We set the elasticity of intertemporal substitution to 10. As mentioned, we show in the Appendix that with an elasticity of intertemporal substitution equal to one, the volatility of the job-finding rate is exactly zero. As noted by Kilic and Wachter (2018), a large elasticity parameter is not necessarily inconsistent with the available evidence of a low elasticity of intertemporal substitution that reflects the weak relation between consumption growth and interest rates. In particular, when we run estimates of the contemporaneous elasticity of consumption growth to interest rates on simulated data using powers of s_t and lagged consumption growth as instrumental variables, we find estimates between 0.01 and 0.5, despite the assumption of $1/\rho = 10$.

In Table 2, we show that these preferences and human capital accumulation can produce around 120% of the volatility of unemployment in normal times, that is, in times without a disaster (0.92 in the model versus 0.76 in the data). Importantly, we produce our results under competitive search, and do not rely on either inefficient real wage stickiness or on exogenous movements in scaled hiring costs as in Kilic and Wachter (2018).

An Affine Discount Factor

So far we have shown our results for a search model with Campbell and Cochrane preferences. We argue that our results do not critically depend on this particular utility function. To make this point in a simple fashion, we consider an affine discount factor motivated by the work of Ang and Piazzesi (2003) among others.

We note that this approach is reduced form in that we do not specify a utility function and then derive the consumption-based discount factor $Q_{t,t+1}$, but rather we specify directly the stochastic discount factor as a function of an exogenous state, the innovations of which are also the innovations to productivity. We note that this approach is similar to that in Hall (2017), who also specifies a reduced-form discount factor not directly linked to a utility function.

Specifically, we assume the following affine specification for the discount factor

$$\log Q_{t,t+1} = -r_{ft} - \frac{1}{2}\sigma_t(\log Q_{t,t+1})^2 - \sigma_t(\log Q_{t,t+1})\varepsilon_{at+1},$$

where the exogenous state s follows the autoregressive process

(52) $s_{t+1} = \rho_s s_t + \sigma_a \varepsilon_{at+1}$

and is driven by fluctuations in productivity ε_{at+1}, where productivity still follows a random walk as in $\Delta a_{t+1} = g_a + \sigma_a \varepsilon_{a,t+1}$with ε_{at+1} is $N(0,1)$. This discount factor is termed *affine* because it implies that both the risk-free rate r_{ft} and the standard deviation of the log of the pricing kernel $\sigma_t(\log Q_{t,t+1})$ are linear in the state s_t. In particular,

(53) $r_{ft} = \mu_0 - \mu_1 s_t$, and $\sigma_t(\log Q_{t,t+1}) = (\gamma_0 - \gamma_1 s_t)\sigma_a$.

Note that $\sigma_t(\log Q_{t,t+1})$ is also the *maximum Sharpe ratio* for continuously compounded and log-normally distributed returns. Here the parameters μ_0 and μ_1 control the mean and volatility of the risk-free rate, whereas γ_0 and γ_1 control mean and volatility of the maximum Sharpe ratio.

We investigate the quantitative properties of this affine discount factor model for the volatility of unemployment by keeping the persistence of the state, ρ_s, and the parameters for productivity, g_a and σ_a, as before and choosing the four parameters $(\mu_0, \mu_1, \gamma_0, \gamma_1)$ to reproduce the mean and standard deviation of the risk-free rate and to ensure that the maximum Sharpe ratio and the risk premium of the consumption portfolio in the model equal the same Sharpe ratio and risk premium generated by the baseline model.

In Table 2 we show that when we choose the parameters of this discount factor to generate the same properties of asset prices as we did with the habit preferences, we find that with this discount factor and human capital accumulation the model produces about 92% of the volatility of unemployment as in the data.

B. Role of Human Capital

Consider next the role of human capital in generating large fluctuations in the job-finding rate and in the unemployment rate. We begin with some quantitative results and then develop some intuition by developing some analytical results. Specifically, we characterize the dependence of the elasticity of the job-finding rate on productivity and time-varying risk, we show analytically how this dependence is affected by human capital acquisition, and illustrate by way of examples the amplification effect of human capital.

In Table 2, we compare our baseline model to one in which we set $g_e = g_u = 0$. We refer to this latter model as the *DMP with habit* model. In this latter model, as well as in the other variations that we will consider in this section, we maintain the same parametrization as in the baseline model with the exception of the hiring cost parameter κ and mean state $\bar{S}$. These parameters are chosen to ensure that the model exactly reproduces the mean unemployment rate and the standard deviation of the risk-free rate in panel B of Table 1. We see that the volatility of the job-finding rate drops 97% and likewise for the unemployment rate. Thus, absent human capital both barely move.

In the third column we consider the *baseline model with* $g_e = g_u = 2.5$ so that human capital grows the same regardless of whether the consumer is employed or unemployed. We see that here also the volatility of the job-finding rate drops 98%. This finding makes clear that it is not the presence of human capital in and of itself that is important for our result, but rather the differential rate of growth between human capital accumulation on the job and off the job. It is this differential that makes hiring a worker an investment with long duration payoffs.

The two panels of Figure 1 show that the impulse responses of the job-finding rate and the unemployment rate to an increase in productivity are much larger in our baseline model with human capital accumulation than in the DMP model without it.

Next we show that the model's implications for the fluctuations of the job-finding rate are relatively insensitive to the rate of depreciation when unemployed. In Figure 2, we let g_u vary from 0 to more than 40% in absolute value per year, keeping all other parameters except κ at their baseline values, including $g_e = 2.5\%$, and vary κ so as to keep the mean unemployment rate as in the baseline model. The figure shows that the standard deviation of the job-finding rate varies from only between 5% to 7.5% even as the depreciation rate of human capital ranges from 0 to -40%.

5. Inspecting the Mechanism

Here with inspect the details of our mechanism using closed form solutions that result from simple approximations.

A. The Mechanism with Baseline Preferences

We now turn to providing some intuition for how our mechanism works in the baseline model using the approximation in (35) for the nonlinear dynamical system. First, to help with interpretation, rewrite the expected discounted value of the match flow $v_{t+n} = (c_\ell \delta_\ell^n + c_s \delta_s^n) A_{t+n}$ in the n-th period after the match is formed, namely $\mathbb{E}_t Q_{t,t+n} v_{t+n}$, as

$$(54)\quad (c_\ell \delta_\ell^n + c_s \delta_s^n) P_{nt},$$

where $P_{nt} \equiv \mathbb{E}_t Q_{t,t+n} A_{t+n}$ is the price of a claim to an asset that pays a one-time dividend of A_{t+n} in period $t+n$. We refer to this asset as a claim to productivity in n periods, or simply a *productivity strip.* Next, consider the roots and the weights associated with the solution to the dynamic system for μ_{et} and μ_{ut} in (35). To keep the algebra simple set $\phi = 1$, so agents do not die, and $g_u = 0$, so that human capital is constant during unemployment. Then the large root, $\delta_\ell > 1$, and the small root, $\delta_s < 1$, are given by

$$\delta_\ell = 1 + \frac{1}{2}\left[\sqrt{(1-\lambda)^2 + 4\eta\lambda_w g_e} - \sqrt{(1-\lambda)^2}\right] \text{ and } \delta_s = \lambda - \frac{1}{2}\left[\sqrt{(1-\lambda)^2 + 4\eta\lambda_w g_e} - \sqrt{(1-\lambda)^2}\right]$$

and the corresponding weights on these roots are given by

$$c_\ell = \left[(1-b)(\lambda - \delta_s) + b g_e\right] / (\delta_\ell - \delta_s) \text{ and } c_s = 1 - b - c_\ell,$$

where $\lambda \equiv (1-\sigma)(1+g_e) - \eta\lambda_w < 1$. We provide formulas for the general case in the Appendix. Note that these roots and weights do not depend on the utility function or on the process for technology. Combining these formulas and (36) we then have:

Proposition 4. *The job-finding rate approximately satisfies*

$$(55)\quad \log(\lambda_{wt}) = \chi + \frac{1-\eta}{\eta} \log\left[\sum_{n=0}^{\infty} (c_\ell \delta_\ell^n + c_s \delta_s^n) \frac{P_{nt}}{A_t}\right],$$

where δ_ℓ, δ_s, c_ℓ, and c_s are given above and χ is a constant.

As we will show, this characterization result applies to *all* preferences considered here. The proposition shows that the job-finding rate is a weighted average of the prices of claims to future productivity. Hence, all movements in job-finding rates come only through movements in the prices of these claim. To further understand the implications of this expression for how job-finding rates move with productivity shocks, we derive an approximate expression for the prices of productivity strips. To do, so we make an assumption that simplifies the calculation of the terms P_{nt} in (55), namely that in calculating this present value, the growth rate in consumption is taken to be approximately equal to the growth rate in productivity, $\Delta\bar{c}_{t+1} \approx \Delta a_{t+1}$.

Under this approximation, the pricing kernel becomes

$$Q_{t,t+1} = \beta \left(\frac{\bar{S}_{t+1}}{\bar{S}_t} \frac{A_{t+1}}{A_t} \right)^{-\alpha}.$$

In the next lemma, we consider a risk-adjusted linear approximation to the price of strips.

Lemma 2. *The price of a claim to productivity in n periods approximately satisfies*

$$(56) \quad \log\left(\frac{P_{nt}}{A_t}\right) = a_n + b_n(\bar{s}_t - s),$$

where the constants satisfy

$$(57) \quad a_n = \ln(\beta) + (1-\alpha)g_a + a_{n-1} + \left(1 - b_{n-1} - \frac{\alpha - b_{n-1}}{S}\right)^2 \frac{\sigma_a^2}{2}$$

and the elasticities of these prices with respect to surplus consumption satisfy

$$(58) \quad b_n = \alpha(1-\rho_s) + \rho_s b_{n-1} + \left(1 - b_{n-1} - \frac{\alpha - b_{n-1}}{S}\right) \frac{\alpha - b_{n-1}}{S} \sigma_a^2$$

with $a_0 = b_0 = 0$.

Note that the price $P_{nt} \equiv \mathbb{E}_t Q_{t,t+n} A_{t+n}$ grows over time since A_t follows a random walk process with drift and, hence, is nonstationary, whereas the *scaled price* $P_{nt}/A_t \equiv$

$\mathbb{E}_t Q_{t,t+n} A_{t+n}/A_t$, which is the price of claim to the growth rate of productivity A_{t+n}/A_t in period $t+n$, is stationary. Here the constants a_n are the discount factors adjusted for risk and growth at the mean $s_t = s$. The elasticities b_n capture how these prices move with surplus consumption s_t as it deviates from its mean s. Note that the constants a_n decrease with n as long as the drift rate g_a is not too large. The elasticities increase monotonically from 0 to α under the assumption that $1 - \rho_s + (1 - \alpha/S)\sigma_a^2/S > 0$.[10]

There are two useful features of these prices that we exploit later. First, these prices depend only on the utility function and the productivity process and not on the search side of the model, such as the human capital accumulation parameters, separation probabilities, and so on. Second, and most importantly, since these elasticities b_n increase with the maturity n, the longer the maturity of a claim, the more sensitive the scaled price is to surplus consumption s_t in n. Thus, the larger are the weights on the long maturity claims, the larger is the response of the job-finding rate to a given technology shock.

We then have the following characterization of how the job-finding rate moves with changes in surplus consumption.

Proposition 5. *Under our approximations, the change in the job-finding rate with respect to a change in surplus consumption around a risky steady state is given by*

$$(59) \quad \frac{d\log(\lambda_{wt})}{ds_t} = \frac{1-\eta}{\eta}\sum_{n=0}^{\infty}\omega_n b_n \quad \textit{with} \quad \omega_n = \frac{e^{a_n}(c_\ell \delta_\ell^n + c_s \delta_s^n)}{\sum_{n=0}^{\infty} e^{a_n}(c_\ell \delta_\ell^n + c_s \delta_s^n)},$$

where a_n and b_n are given by (57) and (58) and the standard deviation of the job-finding rate satisfies

$$(60) \quad \sigma(\lambda_{wt}) = \frac{d\log(\lambda_{wt})}{ds_t}\sigma(\bar{s}_t),$$

where $\sigma(s_t)$ is the standard deviation of the state.

Since the elasticities of the claims to productivity, b_n, increase with the horizon of the claim, the model implies that a change in the state s_t leads to a large change in the job-finding

[10] Note that this last assumption is easily satisfied in practice for any reasonable parameterization of our preferences, since the variance of an innovation to productivity σ_a^2 is only about .00003.

rate only if the weights ω_n placed on long-term productivity claims are large. Notice that the term $\sum_{n=0}^{\infty} \omega_n b_n$ is a type of alternative Macaulay duration to the standard one, $\sum_{n=0}^{\infty} \omega_n n$, where instead of weighting the horizon length n by the fraction of the present value of the surplus ω_n at that horizon, we weight it by the elasticity of the price of claim to productivity at horizon n to surplus consumption, namely, b_n.

In Figure 3 we graph the exact prices of productivity strips against the state, that is, using neither the assumption that $\Delta \bar{c}_{t+1} \approx \Delta a_{t+1}$ nor the risk-adjusted linear approximation. Note that the prices of the longer maturity strips are much more sensitive to variations in surplus consumption than shorter maturity strips. Moreover, as the figure makes clear, the log of these prices are indeed approximately linear in the state. In Figure 4A, we show the impulse responses of these strips to a negative productivity shock. Clearly, the short-horizon strips fall little with this shock, but the long-horizon strips fall greatly. Thus, from these two figures it follows that our model will generate large variations in the job-finding rate only if the weights ω_n are sufficiently large for large n.

We begin by showing that without human capital accumulation, these weights decay very quickly and the larger the gains to human capital from working relative to not working, as determined by $g_e - g_u$, the slower these weights decay.

DMP Model with Habit. Consider first the DMP case with Campbell and Cochrane preferences and $g_e = g_u = 0$. Here the constant c_ℓ on the large root is zero and the small root, referred to as the *DMP root*, is given $\delta_{DMP} = 1 - \sigma - \eta\lambda_w$ where σ is the match destruction rate, η is the elasticity of the matching function, and λ_w is the worker's job-finding rate. Thus, in the DMP version of our model, the match flows from a match n periods after it is formed follow a first-order difference equation with the match flow at $n+1$ proportional to $\delta_{DMP}^n A_{t+n}$. Hence, the weight

$$\omega_n = \frac{e^{a_n}\delta_{DMP}^n}{\sum_{n=0}^{\infty} e^{a_n}\delta_{DMP}^n}$$

in the analogous expression for $d\log(\lambda_{wt})/ds_t$. For standard parametrizations, the DMP root is substantially smaller than one, implying that the match flows decay quickly—at a rate of about 25% per month. More precisely, using $\delta_{DMP} = 1 - \sigma - \eta\lambda_w$ with $\sigma = 2.8\%$, $\eta = .5$, and $\lambda_w = 45.5\%$, we have $\delta_{DMP} = 74.4\%$. In particular, after only two years $(\delta_{DMP})^{24}$ is only 0.08%, so the weights put on long maturity productivity strips are essentially zero.

Baseline Model. Consider next the model with Campbell and Cochrane preferences, human capital accumulation on the job, $g_e > 0$ and, for simplicity, human capital constant when a worker is unemployed, $g_u = 0$. Then, the match flows follow a second-order difference equation with a solution such that that n-th flow is proportional to $(c_\ell \delta_\ell^n + c_s \delta_s^n) A_{t+n}$. Note from our formula for the roots above, the large root δ_ℓ is bigger than one and the weight on this root c_ℓ is positive so that the discounted value of match flows dies out slowly over time. In turn, this fact implies that the job-finding rate in (55) puts sizable weights on long-maturity productivity strips, which fluctuate a lot with surplus consumption and, hence, with current productivity shocks.

In Figure 4B we plot the cumulative weights in the DMP model with habit and the baseline model, without making any approximation. Clearly, the weights in the DMP model with habit die out very quickly relative to those in the baseline model. For a sense of the rate at which these weights decay with the horizon, define the (Macaulay) *duration* of these weights as $\sum_{n=0}^{\infty} \omega_n n$ and note that the duration of weights in the DMP models is 3.6 months whereas in the baseline model it is 11 years.

Interestingly, the expression in (59) implies that a more relevant measure of duration is the elasticity of the job-finding rate with respect to surplus consumption defined by $\sum_{n=0}^{\infty} \omega_n b_n$, which can be interpreted as a modified duration. For the DMP with habit model, this modified duration is 0.03 and for the baseline model it is 0.89.

B. The Mechanism for Other Preferences

Note first that Proposition 4 holds as stated for our specifications with efficient Campbell and Cochrane preferences, Epstein and Zin preferences with long run risks, Epstein and Zin preferences with variable disasters, and the affine discount factor. The reason is simply that this result depends only on the search side of the model and not on the specific discount factor.

It turns out that an analogue of Lemma 2 holds for each of these preferences as well.

For efficient Campbell and Cochrane preferences, the log-linear approximation in (56) holds with constants given by (57) and (58) except that in b_n we replace the constant S by the adjusted constant S_a. Proposition 5 then applies as stated.

For Epstein-Zin with long run risk, the analogue of Lemma 2 is

$$\log\left(\frac{P_{nt}}{A_t}\right) = a_n + b_n s_t + c_n \log(\lambda_t/\lambda_{t-1}),$$

with

$$b_n = (1-\rho)\frac{1-\rho_s^n}{1-\rho_s} \text{ and } c_n = \rho_\lambda \frac{1-\rho_\lambda^n}{1-\rho_\lambda},$$

where here s_t and $g_{\lambda t} \equiv \log(\lambda_t/\lambda_{t-1})$ evolve according to (49) and (50); the constants a_n given in the Appendix. For these preferences, the analog of Proposition 5 is that

$$\frac{d\log(\lambda_{wt})}{ds_t} = \frac{1-\eta}{\eta}\sum_{n=0}^{\infty}\omega_n b_n \text{ and } \frac{d\log(\lambda_{wt})}{dg_{\lambda_t}} = \frac{1-\eta}{\eta}\sum_{n=0}^{\infty}\omega_n c_n,$$

with the same form for ω_n. Since there are two shocks it is useful to observe that, approximately,

$$Var(\lambda_{wt}) = \left[\frac{d\log(\lambda_{wt})}{ds_t}\right]^2 Var(s_t) + \left[\frac{d\log(\lambda_{wt})}{dg_{\lambda_t}}\right]^2 Var(g_{\lambda_t}),$$

so that the overall fluctuations in the job-finding rate depend on the variance of the two sources of shocks: productivity shocks and taste shocks.

For Epstein-Zin with variable disasters, Lemma 2 and Proposition 5 hold as stated except the elasticities are now given by

$$b_n = b_{n-1}\rho_s - (1-\rho)\frac{e^{(\gamma-1)\theta+\frac{1}{2}(1-\gamma)^2\theta^2\tau^2}-1}{\gamma-1} + \frac{(\gamma-\rho)(1-\gamma)}{2}\psi_{vs}^2\phi_s^2\sigma_a^2 + \frac{[(\rho-\gamma)\psi_{vs}+b_{n-1}]^2\phi_s^2}{2}\sigma_a^2$$

where ψ_{vs} and ϕ_s are constants given in the Appendix and s_t evolves according to (51) and the constants a_n are given in the Appendix

For the affine discount factor, Lemma 2 and Proposition 5 hold as stated except the elasticities are now given by

$$(61) \quad b_n = (\mu_1 + \gamma_1 \sigma_a^2) \cdot \frac{1 - (\rho_s + \gamma_1 \sigma_a^2)^n}{1 - (\rho_s + \gamma_1 \sigma_a^2)},$$

and the evolution of the state s_t is given by (52) with the constants a_n described in the Appendix.

In order to provide some intuition as to how these elasticities and the weights vary across models, in Figure 5 we graph $b_n \sigma(s_t)$ and $\sum_{j=1}^{n} \omega_j$ for them (as well as $c_n / \sigma(g_{\lambda t})$ for the long run risk model which has the additional λ_t shock). Notice that in all of these models the b_n (and c_n) are increasing in the horizon n. Hence, for all of these models the intuition for the role of human capital is the same: the greater the degree of human capital accumulation, the larger the weights on the long-horizon claims, which are relatively more sensitive to changes in the state. This shows that precise sense in which as far as the volatility of the job-finding rate is concerned, all of these models work in the same way.

6. Implications for Stock Market Returns

In our baseline model we chose the parameters governing our stochastic discount factor using the strategy of Wachter (2006). Accordingly, we have shown that in doing so, the model generates patterns for risk-free rates and the maximum Sharpe ratio similar to those in the data.

We purposely proceeded as we did because we wanted to make clear that we could simply borrow preferences popular in the asset pricing literature and work out their implications for the job-finding rate in our model without the need to take a stand on how payoff flows in the model map into payments flows to equity holders for U.S. firms.

The reason for such an approach is that in the data the flows of payments to equity holders, of course, are in large part payments to physical and intangible capital and depend on firm leverage. Our simple model has none of these payments to capital nor does it feature leverage. Indeed, as the free-entry condition makes clear, equity flows are simply payments for the up-front vacancy costs needed to start a match. Moreover, as we discuss below, the competitive search equilibrium only determines the present value of wages in a match, not the wage paid each period of a match and, hence, is silent about the time profile of firm profits.

For all of the reasons given, we think that is well beyond the scope of our exercise to take seriously a quantitative comparison of the equity flows in our simple model to those in the data. Nonetheless, for the sake of completeness and with the obvious caveats in mind, here we evaluate the ability of our model to match salient features of stock market returns under two simple approaches, which correspond to two different interpretations of the flows of payments to equity holders in our model.

In the first case, we follow a simple approach used in the asset pricing literature—see, for instance, Campbell and Cochrane (1999) and Wachter (2006)—that dates back to at least Mehra and Prescott (1985), which interprets stocks as claims to streams of aggregate consumption. Following this approach, we price claims to streams of aggregate consumption in the model and comparing them to statistics on stock prices in the data. In Table 4, we compare the statistics on excess returns, equity payouts, and price-dividend ratios from the data, computed from the Flow of Funds, to those on consumption claims in the model, and verify that they are indeed close.

In the second case, we continue to abstract from both physical and intangible capital in the model but add leverage and compare profit flows and equity flows in the model and the data.

One component of both profit flows and equity flows are wages in each period. Our competitive search model pins down the present value of payments when a consumer is first hired by a firm but, without further assumptions, does not pin down the period-by-period wages. In particular, if the firm can pay both wages and severance payments that are contingent on how long the consumer has worked at the firm, then it should be clear there are a continuum of combinations that does so. At one extreme, given there is commitment on behalf of both the firm and the consumer, the firm could pay the entire present value of wages when the consumer is first hired and none thereafter, with the understanding that the consumer will separate only when the exogenous shock occurs (or post a bond with appropriately chosen contingencies if the consumer otherwise separates).

At another extreme, the firm could pay zero wages and only pay appropriately chosen severance payments when the consumer separates. In either case, the consumer can use the complete asset markets to convert these contracted payments to whatever period by period payments the consumer desires. Of course, the firm could pay the worker the same present value in a continuum of ways by using a combination of these extremes along with a somewhat arbitrary flow of period-by-period wages. More generally, the firm could make these contingent payments in almost an arbitrary way across states in a way that generates the same present value and any family would be indifferent between them.

One way to resolve this indeterminacy is to assume that there are no severance payments and to make the Markovian assumption that, regardless of when a consumer was hired, two workers earn the same wages per unit of human capital in the same aggregate state s^t. Under these assumptions, since the present value of payments per unit of human capital for a worker hired in state ε^t is $W_t(\varepsilon^t)$ and that for a worker hired in state ε^{t+1} is $W_{t+1}(\varepsilon^{t+1})$, we have that the wage per unit of human capital in ε^t is uniquely determined by

$$(62)\quad W_t(\varepsilon^t) = w_t(\varepsilon^t) + (1 + g_e) \int W_{t+1}(\varepsilon^t, \varepsilon_{t+1}) d\pi(\varepsilon_{t+1}|\varepsilon^t),$$

where $\pi(\varepsilon_{t+1}|\varepsilon^t)$ is the density of ε_{t+1} given ε^t induced by the unconditional densities $\pi(\varepsilon^{t+1})$ and $\pi(\varepsilon^t)$. Note under this resolution of indeterminacy, the period wages also equal those from Nash bargaining under the Hosios condition.

Next, we choose a process for leverage so that the model's process for leverage looks similar to that in the data, and define a claim to an equity index in the model as consisting of the payouts of all operating firms in the economy, as described below. In Table 3, we see that the statistics of the excess returns and price-dividend ratios of the resulting equity index in the model are also close to those in the data.

We also note that the implied process for profits are relatively stable. In other words, our explanation for the unemployment volatility puzzle does *not* rely on extremely volatile profits. Hence, our model is not subject to the Borovicka and Borovickova (2018) critique.

Let us turn to the details of the leverage process. We interpret the life cycle of an individual firm as follows. A family pays vacancy costs to acquire private ownership of a firm that attempts to hire a worker of type z. If a match occurs, then a firm becomes publicly traded and each period has operating profits $\Pi_t(z) \equiv (1 - \omega_t)A_t z$, where z grows at rate

$1 + g_e$ and the firm survives each period with probability $\phi(1-\sigma)$. The firm is financed by equity and debt in our Modigliani-Miller context, so that the timing of debt does not affect its present value of the value of the firm but does affect the timing of flows to equity holders. If a firm survives, then debt pays out a net return $\tilde{r}_t$ such that $1 = \phi(1-\sigma)\mathbb{E}_t Q_{t,t+1} e^{\tilde{r}_t}$ and hence $e^{\tilde{r}_t} = e^{r_{ft}}/\phi(1-\sigma)$, since this risk is idiosyncratic. Net equity payouts are profits net of debt payouts,

$$(63) \quad D_t(z) = \Pi_t(z) + B_t(z) - \frac{e^{r_{ft-1}}}{\phi(1-\sigma)} B_{t-1}\left(\frac{z}{1+g_e}\right)$$

with $B_t(z) = B_t z$, and are distributed to the household. Clearly, the split of net equity payouts between dividends and net equity issuance is indeterminate, so we just focus on their sum and note that these equity payouts are leveraged profit flows.

To define the equity index, we assume that firms are traded in the market, and hence enter the index, once they have made a successful match with a worker and started to produce, and that the equity index is a claim to equity payments of all such operating firms. For each firm, we follow the strategy of Belo, Collin-Dufresne, and Goldstein (2015) of augmenting our model of firms with a stationary process for leverage that generates smooth debt dynamics. Here leverage L_t is defined as the ratio of debt B_t to the value of a firm J_t net of current flow profits Π_t, where B_t is the debt. We specify the dynamics of debt indirectly by the law of motion of leverage $L_t \equiv B_t/(J_t - \Pi_t)$, where $J_t(z) = J_t z$ is the value of the firm after the current flow of profits Π_t is paid.[11] Specifically, as in Belo et al. (2015), we assume that log leverage is given by

$$\log(L_t) = (1-\rho_\ell)\ell + \rho_\ell \log(L_{t-1}) + \rho_{\ell z}(\sqrt{1-2(s_t - s)} - 1) - (\mathbb{E}_{t+1} - \mathbb{E}_t)\log(J_{t+1} - \Pi_{t+1}),$$

where $(\mathbb{E}_{t+1} - \mathbb{E}_t)\log(J_{t+1} - \Pi_{t+1}) \equiv \log(J_{t+1} - \Pi_{t+1}) - \mathbb{E}_t \log(J_{t+1} - \Pi_{t+1})$.Note that this law of motion, and in particular the assumption about innovations to leverage, implies smooth dynamics for corporate debt, namely that $\log B_{t+1} = \mathbb{E}_t \log B_{t+1}$. This process for leverage implies that, in the face of an drop in the value of the firm that decreases earnings, to smooth

[11] Here in defining the value of the firm as $J_t - \Pi_t$, we follow the standard convention adopted by Belo et. al (2015) and others that this value is net of current profits Π_t.

leverage the firm must decrease its debt, and, hence decrease its equity payouts even more than earnings. This process for leverage thus makes equity payouts riskier than earnings in the short run. In the long run, however, the stationarity of the leverage process implies that equity payouts and earnings are cointegrated, and hence have the same riskiness in the long run. As in Belo et al. (2015), we choose the three parameters of the leverage process to match mean, standard deviation, and autocorrelation of the log leverage process at annual frequency. The top panel of Table 4 shows that the resulting process matches these three moments closely.

In the bottom panel of Table 4, we remind the reader how different are earnings from equity payouts. In particular, we see that the volatility of earnings in the data is much smaller than the volatility of equity payouts, so that claims to equity payouts are quite different from claims to earnings themselves. The bottom panel of Table 4 also shows that once we augment our model with a process for leverage that mimics that in the data, the volatility of equity payouts in the augmented model is much greater than the volatility of earnings in the model and similar to the volatility of equity payouts in the data.

Recently, the work of Binsbergen, Brandt, and Koijen (2012) considers the excess return on claims to dividends at specific horizons between 6 months and 24 months. They show that excess returns on these claims are higher than the excess returns on the sum of the dividends—the latter is a weighted average of the claims to dividends at *all* horizons. They argue that this feature is inconsistent with most standard asset pricing models. Here we show that once we include leverage as discussed above, our model is consistent with this evidence. Specifically, in Figure 6, we graph that model's implications for the excess returns on dividend strips. Here a *dividend strip* is a claim to an asset that pays a one-time dividend of D_{t+n} in period $t+n$ where dividends in the model are defined by (63). Clearly, $P^D_{nt} \equiv \mathbb{E}_t Q_{t,t+n} D_{t+n}$ is the price of such a dividend strip. We follow Lopez (2016) in extracting the prices of dividend strips using data on put and call European options on the S&P 500 index.[12] The vertical

[12] This approach exploits put-call parity no-arbitrage relations and relies on a strategy that uses options-implied interest rates. Observations that violate the put-call parity relation are excluded so as to mitigate the impact of measurement error. Intuitively, put-call parity summarizes a forward claim to the S&P500, which coincides, by the law of one price, with a claim to today's index less a claim to all dividends paid until the maturity of the forward claim, that is, a dividend strip that is observable as long as European puts and calls are traded.

dashed lines indicate the 90% bootstrapped confidence bands on the dividend strips for the S&P 500 for the period between 1994 to 2015.

7. Adding Physical Capital

Here we add physical capital to our economy. The production functions for a consumer with human capital z when paired with physical capital depend on the nature of the consumer's production activities, that is, producing goods in the market, producing vacancies in the market, or producing goods at home. For example, a consumer with human capital z paired with physical capital $K_{et}(z)$ produces $(A_t z)^{1-\alpha} K_{et}(z)^{\alpha}$ of goods when employed in the market producing goods. Likewise, such a consumer produces $\kappa(A_t z)^{1-\alpha} K_{vt}(z)^{\alpha}$ units of vacancies in the market and $b(A_t z)^{1-\alpha} K_{ut}(z)^{\alpha}$ units of goods at home if paired with $K_{vt}(z)$ and $K_{ut}(z)$ units of physical capital, respectively. Notice that, for simplicity, all three production functions have the same Cobb-Douglas shares which will imply that, in equilibrium, all will have the same capital-labor ratios. There are costs of adjustment to the aggregate capital stock, but for a given level of the aggregate capital stock, this capital can be costlessly moved between use in the market production of goods, market production of vacancies, and home production of goods after the aggregate shock at time t is realized.

We consider efficient Campbell and Cochrane preferences and examine the efficient allocations that are solution to the corresponding planning problem. As we show in the Appendix, it is immediate that it is optimal to allocate capital so that the units of physical capital per units of human capital are equated for any level of human capital z in all three activities in that

$$\frac{K_{et}(z)}{z} = \frac{K_{vt}(z)}{z} = \frac{K_{ut}(z)}{z} \text{ for all } z.$$

Thus, it is immediate that the economy aggregates, like in our baseline model, in that the aggregate resource constraint can be written as

$$C_t + I_t \leq \left(A_t Z_{et}\right)^{1-\alpha} K_{et}^{\alpha} + \left(bA_t Z_{ut}\right)^{1-\alpha} K_{ut}^{\alpha} - \left(\kappa A_t Z_{vt}\right)^{1-\alpha} K_{vt}^{\alpha},$$

where $K_{et} = \int K_{et}(z)\mathrm{d}z$ is the measure of physical capital used by the employed—we use a similar notation for K_{ut} and K_{vt}. The aggregate capital stock is subject to adjustment costs and follows the accumulation law

$$K_{t+1} = (1-\delta)K_t + \Phi\left(\frac{I_t}{K_t}\right)K_t,$$

where $K_{et} + K_{ut} + K_{vt} \leq K_t$. The aggregate investment decision is made at the end of period $t-1$ and the aggregate capital stock K_t that enters period t is divided between its three uses after the time t aggregate shocks have been realized. We choose

$$\Phi(I/K) = \frac{\delta}{1-\frac{1}{\xi}}\left[\left(\frac{I}{\delta K}\right)^{1-\frac{1}{\xi}} - 1\right],$$

as in Jermann (1998). We choose $\alpha = 1/4$, $\delta = 0.1/12$, and the curvature parameter of the adjustment cost function ξ so that the model produces a similar standard deviation of HP filtered investment to HP filtered output as in the data.

We turn now to the results, reported in Table 5. Note that although they face the same amount of risk in productivity, agents in this model have another way to smooth consumption risk, namely, by decreasing investment in physical capital in downturns and increasing it in upturns. Doing so decreases the risk in consumption and, therefore, dampens a bit the fluctuations in the price of risk, which in turn reduces a bit the fluctuations in the present value of surplus flows and so the fluctuations in vacancy creation. Overall, though, our results are robust to the inclusion of physical capital. In particular, this augmented model produces a standard deviation of unemployment that is 89% (0.68/0.76) of that in the data.

8. Comparison with Various Bargaining Schemes

Here we compare our model with competitive search to existing models that feature Nash bargaining and two types of alternating offer bargaining. We use the model with Campbell and Cochrane preferences but no externality as baseline, that is, the model with efficient Campbell and Cochrane preferences described above. Moreover, we focus on the case without human capital accumulation. As we have discussed, without human capital accumulation, the efficient allocations are characterized by negligible fluctuations in the job-finding rate. We study this case, however, because we are interested in analyzing how alternating offer bargaining in an economy similar to Hall (2017) can produce large fluctuations in job-finding rates even without human capital acquisition.

We argue that this is the case not because alternating offer bargaining is fundamentally different from competitive search or Nash bargaining—both of these bargaining schemes can support the same efficient allocations that competitive search supports as long as the parameters governing them are suitably chosen. Rather, the bargaining schemes considered here generate large fluctuations in the job-finding rate only when parameters are chosen to be far from the values that support efficient allocations and, hence, generate a type of *real wage rigidity*. Indeed, as we show below, even Nash bargaining can generate sizable volatility in the job-finding rate if we select a parameterization that yields very inefficient outcomes.

It has long been known that one way to generate plausible fluctuations in unemployment is to impose a form of sticky wages. Intuitively, if the cost of employing a worker does not decrease much in downturns following a drop in productivity, then firms' incentives to hire workers are greatly reduced and unemployment becomes much more cyclical than when wages are efficiently set.

Recent evidence on the extent of actual wage rigidity, though, has challenged the relevance of this mechanism. For example, Beraja et al. (2019) show that wages are rather flexible in the cross-section of U.S. states. Kudlyak (2014) and Basu and House (2016) document that the present values of wages, as measured by the user cost of labor, is highly cyclical. Here, we argue that our mechanism matches the observed cyclicality of the user cost of labor and is consistent with common estimates of cross-sectional wage growth with experience. In particular, we show that a model that integrates both our mechanism and Hall (2017)'s mechanism, once made consistent with the cyclicality of wages and their dynamics with experience, provides strong support for our mechanism.

A. Alternating Offer Bargaining

Several popular bargaining schemes involve alternating offer bargaining. Hall and Milgrom (2008) and Hall (2017) use *across-period* alternating offer bargaining, in which each bargaining round takes a full period of time in the model whereas Christiano, Eichenbaum, and Trabandt (2016) use *within-period* alternating offer bargaining, in which there are many rounds of bargaining within one period of time in the model. As we discuss later, Hall's interpretation of alternating offer bargaining as taking place over many periods leads to drastically different outcomes than does that of Christiano, Eichenbaum, and Trabandt (2016), who interpret all offers as taking place within a single period, say, a month.

Now, just as with Nash bargaining, both of the alternating offer bargaining schemes we examine will decentralize the efficient allocations only when the bargaining parameters are suitably chosen. The works by Hall and Milgrom (2008), Hall (2017) and Christiano, Eichenbaum, and Trabandt (2016) use these schemes not to decentralize the efficient allocations but rather to select particularly inefficient allocations that the authors found interesting.

Across-Period Alternating Offer Bargaining

We begin by analyzing the across-period alternating offer bargaining scheme. We give conditions under which it decentralizes the efficient allocations and then analyze the behavior of various inefficient allocations that, as in Hall (2017), yield large fluctuations in job-finding rates. In Hall's interpretation, the time between each offer is a period in the model, here a month. We briefly lay out the alternating offer bargaining equilibrium for our environment. The only difference between this model and our competitive search model is that it posits an alternative way the wages are determined.

In particular, in this equilibrium notion, the formulas for the values for post-match value P_t, unemployment U_t, the value of a vacancy V_t, and the present value of output in a match Y_t are identical to those in the competitive search model and given by (18), (19), (24), and (26). Likewise, the free-entry condition (25) and the resource constraint (31) are the same. The only difference is that here wages are set in an imperfectly competitive way rather than a competitive way. Here we abstract from human capital accumulation by setting $g_e = g_u = 0$ to make our model more similar to that in Hall.

The worker makes the first offer with probability ξ and the firm makes the first offer with probability $1 - \xi$. In each subsequent period, firms and workers deterministically alternate making offers each period, at least if bargaining has not broken down. If period t is one in which the firm makes the offer, we denote the offer by W_{ft}, whereas if period t is one in which the worker makes the offer at t, we denote it by W_{wt}. (Here we note that these offers are contingent on the exogenous state ε^t, namely $W_f(\varepsilon^t)$ and $W_w(\varepsilon^t)$, but we have suppressed this explicit dependence.) In each period of bargaining, with probability δ bargaining exogenously breaks down, in which case the firm returns to the market with an unfilled vacancy and the workers enters unemployment. If the firm makes offer W_{ft} in period t, the worker can either accept it, reject it and plan to make a counteroffer W_{wt+1} in period $t+1$ if no exogenous breakdown occurs, or abandon negotiations and immediately return to unemployment. The firm has symmetric options if it is the worker that makes the offer at t.

There are costs to bargaining. In particular, it costs the firm ψA_t to make a counteroffer to the worker at t where we refer to ψ as a *haggling cost*. The cost to the worker of bargaining is that during bargaining the worker only receives the value of home production bA_t rather than a wage if the worker accepts, so the implicit delay cost is the difference between foregone wages and home production. Now, Hall and Milgrom (2008) argue that it makes little sense for the probability of an exogenous breakdown of bargaining to be lower than the separation rate, that is, they argue that $\delta \geq \sigma$ (Hall and Milgrom, 2008). Nonetheless, Hall (2017) allows δ to be less than σ and thus so shall we, in order to highlight the importance of this parameter. Note here that the three new parameters introduced a part of this bargaining scheme are (ξ, δ, ψ).

As explained in detail in Hall and Milgrom, standard recursive logic implies that the firm will make the best possible offer for itself such that the worker will prefer to accept it rather than making a counteroffer in the event of no exogenous breakdown or abandoning negotiations. Thus, the firm's offer W_{ft} satisfies

$$(64) \quad W_{ft} + P_t = \max\left\{bA_t + (1-\delta)\phi\mathbb{E}_t Q_{t,t+1}(W_{wt+1} + P_{t+1}) + \delta\phi\mathbb{E}_t Q_{t,t+1}U_{t+1}, U_t\right\},$$

where the maximum ensures that the worker does not strictly prefer unemployment today to accepting the offer. Of course, the firm's offer W_{ft} must be such that, if accepted, it is smaller than the discounted value of output from the match Y_t, or else the firm would prefer to stay idle, so $W_{ft} \leq Y_t$. In turn, the worker will make the best possible offer from the worker's perspective, such that the firm will prefer to accept it rather than making a counteroffer in the event of no breakdown or abandoning negotiations. Thus, the worker's offer satisfies,

$$(65) \quad Y_t - W_{wt} = \max\{-\psi A_t + \phi(1-\delta)\mathbb{E}_t Q_{t,t+1}(Y_{t+1} - W_{ft+1}), 0\}.$$

where the maximum ensures that the firm does not strictly prefer abandoning negotiations to accepting the offer. Clearly, the worker will only make offers such that, if accepted, the worker will prefer it to unemployment, that is, $W_{wt} + P_t \geq U_t$ must hold.

Since the family consists of a large number of workers, the value to the family of all the workers' wages who are bargaining at t is

$$(66) \quad W_t = \xi W_{wt} + (1-\xi)W_{ft}.$$

Likewise, the value to the firm of the present value of wages from bargaining is also W_t.

We first ask, can this bargaining scheme decentralize the efficient allocations? The answer is that if we choose the parameters that define this bargaining scheme appropriately it can.

Proposition 6. *When the probability that the worker makes the first offer ξ equals the elasticity of the matching function η with respect to the measure of the unemployed, then the allocations in a sequence of bargaining games indexed by the breakdown probabilities $\{\delta_n\}_{n=1}^{\infty}$ converge to the efficient allocations as δ_n converges to one.*

Proof: For each δ_n, the associated sequences of values, indexed by n, satisfy

$$(67) \quad W_{ft}^n + P_t^n = \max\left\{bA_t + (1-\delta_n)\phi\mathbb{E}_t Q_{t,t+1}^n(W_{wt+1}^n + P_{t+1}^n) + \delta_n\phi\mathbb{E}_t Q_{t,t+1}^n U_{t+1}^n, U_t^n\right\},$$

$$(68) \quad Y_t - W_{wt}^n = \max\{-\psi A_t + \phi(1-\delta_n)\mathbb{E}_t Q_{t,t+1}^n(Y_{t+1} - W_{ft+1}^n), 0\}.$$

Clearly, all of these sequences are continuous in n and, taking limit of both sides of these equations as n diverges to infinity yields,

$$(69) \quad W_{ft} + P_t = U_t$$

(70) $Y_t - W_{wt} = 0$

where in (69) we used that for δ_n sufficiently close to 1, the two terms in the maximum in (67) converge and in (70) the first term in the maximum in (68) is strictly negative. By continuity, the participation constraints $W_{ft} \leq Y_t$ and $W_{wt}+P_t \geq U_t$ also clearly hold. Hence, substituting for W_{ft} and W_{wt} from (69) and (70) into (66) and using that $\xi = \eta$ implies

(71) $W_t = (1-\eta)(U_t - P_t) + \eta Y_t.$

Adding $P_t - U_t$ to both sides and collecting terms gives that

(72) $W_t + P_t - U_t = \eta(Y_t + P_t - U_t),$

that is, workers receives a share η of the surplus and, hence, firms receive a share $1-\eta$ of the surplus. But this splitting of the surplus are the conditions for efficiency of Nash bargaining under the Hosios condition. Hence, the allocations are efficient. □

The intuition for this is simple. As the breakdown rate gets very high, the bargaining scheme gets close to one in which first a weighted coin is flipped so that with probability ξ the worker makes a take it or leave it offer and with probability $1-\xi$ the firm makes a take it or leave it offer.

Likewise, this proposition implies that the only way that an alternating offer bargaining equilibrium can lead to allocations that differ greatly from the efficient ones is that the parameters of the bargaining process differ greatly from those in the proposition. It turns out that as we lower the probability of a breakdown towards zero, the economy gets more and more inefficient and the volatility of job-finding rates and unemployment increase.

In Table 6, we contrast the competitive search allocations in an economy with efficient Campbell and Cochrane preferences with no human capital accumulation to that of various bargaining schemes. As we have emphasized, a key part of our mechanism is human capital accumulation so that in its absence, the efficient allocations produced by competitive search show little variation in the job-finding rate and unemployment.

To warm up, consider Nash bargaining and recall that if the bargaining weight of consumers γ is chosen to equal to the elasticity of the matching function η, Nash bargaining also produces the efficient allocations. As the table shows, of course, under this Hosios condition the economy without human capital shows little variation. More importantly, this table shows that even with Nash bargaining, if we set this bargaining weight to pick out a very inefficient allocation, say by choosing the worker bargaining weight $\gamma = 0$ so that the firm gets all the match surplus, we can increase the volatility of job-finding rates by a factor of around 20 relative to the efficient allocations (from 0.15 to 3.33), and thus generate over half of the volatility in the data. In the table we see a similar increase in the volatility of unemployment rates. We think of this exercise as simply imposing an inefficient wage setting scheme that if agents met behind the veil of ignorance, they would never agree to. In this sense, we view this exercise as akin to simply imposing one particular type of sticky wages from outside of the model.

It turns out that, with alternating offer bargaining, as we move further and further away from the efficient allocations by making the breakdown probability closer and closer to zero, we greatly increase volatility. It is not easy to interpret this exogenous breakdown probability based on actual bargaining behavior because in equilibrium the first offer is accepted regardless of the value of δ. For what it is worth, we can at least put this parameter in perspective by translating δ into units of time by asking what would be the (mean) duration of the opportunity to negotiate to create a job, if bargaining continued until it exogenously broke down. We refer to this duration as the *duration of a job opportunity.*

In Table 6, we see that if $\delta = 1$ so that bargaining will breakdown for sure if no agreement is reached on the first offer, then this model produces identical implications to those produced by the competitive search model or the Nash bargaining model under the Hosios condition. Next, if $\delta = 1/3$, so that the average duration of the opportunity to negotiate is 3 months, and we see that the volatility of the job-finding rate is only 7% of that in the data (.46/6.68). It turns out that if we set this breakdown rate sufficiently small, say, to the value $\delta = 0.013$ used by Hall (2017), then the average duration of a job opportunity is over 6 years (74 months) and the model can produce the volatility of the job-finding rate in the data. For brevity, we refer to this model with efficient Campbell and Cochrane preferences, no human capital accumulation, and alternating offer bargaining with $\delta = 0.0013$ as *Hall's* *model.*

An alternative and, perhaps more useful, way to interpret δ is to translate it into a wedge that distorts the efficient allocations. To do so, we introduce a time-varying distortion into the competitive search equilibrium so that it generates the same allocation as in the extreme alternating offer bargaining equilibrium, with a duration of a job opportunity of 74 months. Specifically, suppose that in the competitive search equilibrium, we replace the cost of posting of a vacancy κA_t per unit of z with $(1-\tau_t)\kappa A_t$, so that τ_t is a percentage subsidy on vacancy costs. We can think of the government as levying this subsidy and then paying for it with lump-sum taxes. We then solve for the subsidy process τ_t so that the allocations from the distorted planning problem coincide with those in the alternating offer bargaining equilibrium.

In Figure 7A, we plot the subsidy against the state S_t that drives the time-varying discount rates. We see that when the state is low, which occurs in recessions, the subsidy is negative and when the state is high, which occurs in booms, the subsidy is positive. That is, the government increases the volatility of job-finding rates and unemployment from its efficient level by taxing vacancies in recessions, thus decreasing them by more than is efficient, and subsidizing them in booms, thus increasing them by more than is efficient.

In Figure 7B, we plot a realization of the job-finding rates from the extreme bargaining economy with $\delta = 0.013$, the associated subsidies, and the job-finding rates in the efficient allocations, shifted to have the same overall mean as in the extreme bargaining economy. We see that at the deepest part of the downturn there is a tax on posting vacancies of 150%. Note that the correlation of the job-finding rate and the subsidy is .94. Thus, nearly all of the movement in the job-finding rate is coming from the distortions introduced by the inefficient bargaining.

An alternative way to quantify the difference between this type of bargaining and efficient bargaining is to solve for what the time-varying Nash bargaining weight of the workers and firms would have to be to produce the job-finding rates in the extreme bargaining economy. In Panel B of Figure 8, we see that these Nash bargaining weight for firms fluctuate from about .13 in the deep downturn to about 1/2 in the upturn. That is, in a deep downturn workers experience an increase in their bargaining power from 0.5 to 0.87. Firms understand this and drastically cut their vacancies.

Within-Period Alternating Offer Bargaining

Here we consider a variant of the alternating offer bargaining game above used by Christiano, Eichenbaum, and Trabandt (2016). It differs from that used by Hall (2017) in that all the bargaining takes place within one period, here one month. Specifically, this setup imposes that if the worker and firm meet at the beginning of one month, the bargaining process can last at most one month. In this sense, the mean duration of negotiations until exogenous breakdown is an order of magnitude shorter than that in our version of Hall (2017), in which the duration until exogenous breakdown is 6.2 years.

To elaborate, in this setup a period t is broken into M equal-sized subperiods with M even. In subperiod 1, the firm makes an offer to the worker. The worker can accept the offer or reject it. If the worker rejects the offer, with probability δ bargaining breaks down and the worker simply collects unemployment benefits for the remaining subperiods, whereas the firm obtains zero. With probability $1-\delta$, the worker makes an offer in subperiod 2. If the firm rejects this offer, with probability $1-\delta$ bargaining continues and the firm has to pay a cost ψA_t to make a counteroffer in the next subperiod and with probability δ bargaining ends. This alternating process with stochastic breakdowns continues at most to period M, at which time the worker makes the final offer. If the firm rejects that final offer, then bargaining is over and at the beginning of period $t+1$, a new aggregate shock is drawn, the worker starts as unemployed, and the firm starts with no workers.

If an agreement is reached in subperiod $M-j$, then the match will produce the pro-rated output $(j+1)A_t/M$ for the remaining subperiods whereas if no agreement is reached in subperiod $M-j$, the worker receives the pro-rated benefits bA_t/M and the firm receives 0. If , instead, bargaining breaks down in subperiod $M-j$, then the worker gets the pro-rated benefits $b(j+1)A_t/M$ for the remaining subperiods and the firm receives 0. In this bargaining game, each player takes as given the values that will be negotiated in future periods. We formally analyze this equilibrium in the Appendix and establish the following proposition.

Proposition 7. *The allocations in a sequence of within-period alternating offer bargaining games with $\psi = 0$, $\delta = (1-2\eta)/(1-\eta)$ for $0 \leq \eta \leq 1/2$, and indexed by the exogenous* *number of maximal rounds $\{M_n\}$ converge to the efficient allocations as the number of rounds converges to infinity.*

In Table 6, we also analyze the properties of our model when it features within-period alternating offer bargaining. To understand the first of these columns, note that $\eta = 1/2$ in the baseline model so that to be consistent with Proposition 7, the exogenous breakdown rate is $\delta = 0$ and we set $\psi = 0$. Hence, that version of the model decentralizes the efficient allocations. In the second column, we use the parameterization in Christiano, Eichenbaum, and Trabandt (2016). For this parametrization, we see that the model produces even less variation in employment than under the efficient allocation, that is, less than 2% (0.01/0.76) of the observed fluctuations in job-finding rates.

B. Using Data to Distinguish Our Mechanism from Hall's Mechanism

Here we consider an integrated model that nests our model with human capital accumulation and Hall's model with alternating offer bargaining. Both models feature efficient Campbell and Cochrane preferences. To see that this integrated model nests our model, note from Proposition 6 that if we let the exogenous breakdown rate δ converge to one and let the probability ξ that the worker makes the first offer equal the elasticity of the matching function η with respect to the measure of the unemployed, then the allocations converge to the efficient ones from our model. It is immediate that this model nests Hall's model when we set $g_e = g_u = 0$, so that human capital accumulation is muted.

These two models emphasize distinct mechanisms that generate volatile unemployment by generating volatility in the job-finding rate. The key mechanism in our model relies on the interaction between human capital accumulation and time-varying risk to generate volatility in the job-finding rate. In contrast, the key mechanism in Hall (2017)'s model relies on the interaction between inefficiency in wage setting and time-varying risk to generate this volatility.

We use two statistics on wages to distinguish between these two mechanisms. The first statistic is the cross-sectional wage growth with experience estimated by Elsby and Shapiro (2012), specifically, the difference in the (log) real wages of workers with 30 years of experience and those with 1 year of experience. This statistic is a measure of the magnitude of human capital accumulation in the data. The second statistic is the cyclicality of the user cost of labor estimated by Kudlyak (2014), namely, the semi-elasticity of the user cost of labor to unemployment, where the user cost of labor is defined as the difference between the present value of wages from a new match formed in period t and that from a new match formed at $t+1$, with wages discounted at a fixed rate. Kudlyak (2014) argues that this semi-elasticity is the appropriate measure of wage rigidity in search models.

Table 7 shows the estimated version of the model with human capital accumulation and alternating offer bargaining. We first note that the estimated model matches the cross-sectional wage growth with experience estimated by Elsby and Shapiro (2012). Interestingly, to match the cyclicality of the user cost of labor estimated by Kudlyak (2014), a bargaining breakdown rate corresponding to a duration of a job opportunity of just 1.3 months is sufficient. In Table 8, we compare the implications of the nested model for these two statistics when we vary the duration of a job opportunity from 1 month up to 6 months. Note that if we increase the duration of a job opportunity even just to 6 months, the cyclicality of the user cost of labor generated by the model becomes much too small, which implies that the model leads to much more wage rigidity than in the data.

Thus, the cyclicality of wages leaves a limited role for Hall's mechanism. In this sense, the data clearly favors our mechanism over Hall's mechanism.

9. Conclusion

We propose a mechanism for search models to reproduce the observed fluctuations in the job-finding rate and in the unemployment rate at business cycle frequencies. Our model not only solves the unemployment volatility puzzle of Shimer (2005) but also is immune to the critiques of Chodorow-Reich and Karabarbounis (2016) and Borovicka and Borovickova (2019) of existing mechanisms that address the Shimer puzzle. To this purpose, we augment the textbook search model with two features: preferences from the macro-finance literature that match the observed variation in asset prices, and degrees of human capital accumulation on and off the job that are consistent with the micro evidence on wage growth with experience and wage losses after spells of unemployment. In such a framework, investing in hiring workers becomes a risky endeavor with long-duration flows of the surplus from a match between a firm and a worker. Hence, cyclical movements in productivity make the present value of these surplus flows fluctuate sharply over the cycle. In turn, fluctuations of these present values imply that investments in hiring workers are highly cyclical and, hence, that job-finding rates and unemployment rates are highly volatile.

We show that both of the new features we introduce play a critical role, namely, if we abstract from either preferences that generate time-varying risk or human capital accumulation, the model generates only negligible movements in unemployment. Moreover, since our model leads to efficient allocations, the particular equilibrium decentralization we focus on is in no way central to our results.

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10. Appendix: Comparison to Work on the Fundamental Surplus

Here we discuss the relation of our work to the literature on solving the unemployment volatility puzzle and analyzed in detail in Ljungqvist and Sargent (2017). These authors summarize the existing literature as follows: "To generate big responses of unemployment to productivity changes, researchers have reconfigured matching models in various ways: by elevating the utility of leisure, by making wages sticky, by assuming alternating-offer wage bargaining, by introducing costly acquisition of credit, by assuming fixed matching costs, or by positing government-mandated unemployment compensation and layoff costs. All of these redesigned matching models increase responses of unemployment to movements in productivity by diminishing the fundamental surplus fraction, an upper bound on the fraction of a job's output that the invisible hand can allocate to vacancy creation. Business cycles and welfare state dynamics of an entire class of reconfigured matching models all operate through this common channel" (Ljungqvist and Sargent 2017, p. 2630).

Here we show that the mechanism in our model works quite differently from those studied by Ljungqvist and Sargent (2017). Recall that these authors compute the steady state response of the job-finding rate and, hence, unemployment to a steady state change in the level of productivity. We establish two results. We first show that if in our model we perform the same steady state experiment of changing productivity we get no change in the job-finding rate.

We then re-examine the results in Ljungqvist and Sargent (2017) if we change their models so that productivity enters their models as it does in ours. In their models an increase in productivity only increases the productivity of working in the market. In our model an increase in productivity also increases the productivity of working at home and the cost of posting vacancies.

The first part of this assumption is consistent with the work of Chodorow-Reich and

Karabarbounis (2016) who argue that the elasticity of the opportunity cost of employment with respect to productivity is approximately one.

The second part is that, as we argued earlier, if we think it takes a worker a fixed amount of time to post a vacancy that could otherwise be devoted to producing goods, then a doubling of this worker's productivity in the market also doubles the cost to a firm of spending the same amount of time posting a vacancy. This assumption is consistent with the model in Shimer (2010). Once we make this change, we establish our second result: in both the basic matching model and in the alternating offer bargaining model, a change in steady state productivity has no effect on the job-finding rate.

Taken together these results clarify that the mechanism in our model is fundamentally different than those in the literature surveyed by Ljungqvist and Sargent (2017).

A Steady-State Change in A in our Baseline Model

We consider the experiment conducted in Ljungqvist and Sargent (2017) in our model, namely a steady state increase in A. We show that this has no effect on the job-finding rate and, hence, on unemployment. The following proposition is immediate.

Proposition 8. *In our baseline model the steady state levels of the job-finding rate and the unemployment rate are independent of the productivity parameter A.*

Proof: In a steady state with $S_t = S$, $C_t = C$, so that from (12), $Q_{t,t+1} = \beta$. Evaluating the free-entry condition job-finding rate (36) at a steady state gives

$$(73) \quad \log(\lambda_w) = \chi + \frac{1-\eta}{\eta} \log\left(\frac{\mu_e - \mu_u}{A}\right).$$

To evaluate μ_e and μ_e, consider the steady state versions of (32) and (33), namely

$$(74) \quad \mu_e = A + \phi(1+g_e)\beta\left[(1-\sigma)\mu_e + \sigma\mu_u\right],$$

$$(75) \quad \mu_u = bA + \phi(1+g_u)\beta\left[\eta\lambda_w\mu_e + (1-\eta\lambda_w)\,\mu_u\right].$$

From inspection of these equations it is immediate that μ_e and μ_u are proportional to A so $(\mu_e - \mu_u)/A$ is independent of A. Using this result in (73) proves that the job-finding rate is independent of A. $\square$

Notice that the key to this result is that the steady state value of the discount factor Q does not vary with the steady state value of A. Since this same property holds for a broad class of consumption based discount factors, including Epstein-Zin, CRRA, and so on, all of these discount factors will be consistent with Proposition 8.

Basic Matching Model

Consider the *basic matching model* in Ljungqvist and Sargent (2017) and use notation similar to ours. We follow Ljungqvist and Sargent (2017) in restricting attention to steady states and drop all time subscripts. Workers are risk neutral with discount factor $\beta = 1/(1+\delta)$. A consumer makes A units of output when employed and b units of output when not employed. The cost of posting a vacancy is κ, the exogenous separation rate is σ, the worker's bargaining share is γ, and the job-filling rate for a firm is $\lambda_f(\theta)$. As Ljungqvist and Sargent (2017, p. 2635) equation (12) show the equilibrium value of market tightness θ is determined by the free entry condition, which we rearrange and express in our notation as

$$(76) \quad \kappa = (1-\gamma)\lambda_f(\theta)\frac{\beta(A-b)}{1-\beta\left(1-\sigma-\gamma\theta\lambda_f(\theta)\right)}$$

They then differentiate this equation to derive $d\theta/dA$ and explain how their measure of fundamental surplus is critical for understanding how large is this derivative. Now, in our model we assume that output when not employed and the cost of posting a vacancy are proportional to productivity A, so that b and κ are replaced by bA and κA respectively, so that our analogous free entry is

$$(77) \quad \kappa A = (1-\gamma)\lambda_f(\theta)\frac{\beta(1-b)A}{1-\beta\left(1-\sigma-\gamma\theta\lambda_f(\theta)\right)}.$$

Since A cancels on both sides, θ is constant, so $d\log(\theta)/d\log(A) = 0$.

Proposition 9. *In the basic matching model, if output when not employed and the cost of posting a vacancy are proportional to productivity then the change in unemployment with respect to a change in productivity is zero regardless of all other parameters.*

Note that this constancy of unemployment in the face of different levels of productivity

A holds regardless of the size of the home production parameter b, which plays an important role in the Hagedorn and Manovskii (2008) and Shimer (2005) debate. More generally, this property holds regardless of the size of the fundamental surplus, which plays a key role in the Ljungqvist and Sargent (2017) analysis.

Consider briefly our proportionality assumptions. First, scaling κ by A_t is equivalent to making the cost of posting vacancies take a fixed unit of workers' time in recruiting (as in Shimer (2010)) that could be otherwise devoted to the production of goods. Second, scaling home production b by A_t assumes that new technological changes that improve the technology in the market also improve the technology at home. Third, in our baseline model, productivity follows a random walk with positive drift. Hence, it would not make sense to assume that b and κ are constants because then the ratio of home production to market production, b/A_t, and the ratio of vacancy costs to market production κ/A_t, would (in a precise stochastic sense) go to zero and all agents would (in the same sense) always work.

Pissarides (2009): The Role of Training Costs

Pissarides (2009) shows how the presence of fixed training costs that occur after bargaining has been completed can make unemployment more responsive to productivity changes. We show a similar result applies here. In this set up, firms pay a cost κ to post a vacancy and, when a match with a worker occurs, they pay a fixed cost h to train the worker for the job. It follows that the value of match surplus is reduced by the fixed cost, and hence free entry condition (76) becomes

$$(78)\quad \kappa = (1-\gamma)\lambda_f(\theta)\beta\left[\frac{A-b}{1-\beta\left(1-\sigma-\gamma\theta\lambda_f(\theta)\right)} - h\right].$$

Our proportionality assumption in this context implies that a doubling of productivity also doubles the training cost, so that h is replaced by hA. It follows that our analogous free-entry condition is

$$(79)\quad A\kappa = (1-\gamma)\lambda_f(\theta)\beta\left[\frac{(1-b)}{1-\beta\left(1-\sigma-\gamma\theta\lambda_f(\theta)\right)} - h\right]A.$$

Since A cancels on both sides, θ is constant, so $d\log(\theta)/d\log(A) = 0$. We summarize this result as:

Proposition 10. *In the matching model with fixed training costs, if output when not employed, the cost of posting a vacancy, and training costs are proportional to productivity then the change in unemployment with respect to a change in productivity is zero regardless of all other parameters.*

Hall and Milgrom (2008): Alternating Offer Bargaining Model

A similar result also applies to alternating offer bargaining models. Consider the exposition in Ljungqvist and Sargent (2017) of the alternating offer bargaining model of Hall and Milgrom (2008). In it firms and workers make alternating offers and after each unsuccessful bargaining round, the firm incurs a cost of ψ of making a new offer while the worker receives b. We refer to ψ as a *haggling cost.* There is a probability δ that the job opportunity is exogenously destroyed between bargaining rounds and the worker reenters unemployment. Ljungqvist and Sargent (2017) assume that $\delta = \sigma$ so the probability of the job opportunity being destroyed between rounds equals the exogenous separation probability. Under this assumption, the free entry condition (equation 36, p. 2648 of Ljungqvist and Sargent, 2017), can be rearranged to be

$$(80) \quad \kappa = \frac{\lambda_f(\theta)\beta}{1-\beta(1-\sigma)}\left[A - \frac{b+\beta(1-\sigma)(A+\psi)}{1+\beta(1-\sigma)}\right]$$

Now, suppose we extend the earlier idea that recruiting new workers takes a fixed amount of an existing worker's time to the idea that each round of bargaining also uses a fixed amount of an existing worker's time in haggling. With this interpretation it is natural to scale both κ and ψ by A since it reflects the foregone opportunity of direct production by a worker that is diverted to recruiting or bargaining. Hence, (80) becomes

$$(81) \quad \kappa A = \frac{\lambda_f(\theta)\beta}{1-\beta(1-\sigma)}\left[1 - \frac{b+\beta(1-\sigma)(1+\psi)}{1+\beta(1-\sigma)}\right] A.$$

Since A cancels on both sides, θ is constant, so $d\log(\theta)/d\log(A) = 0$.

Proposition 11. *In the alternating offer bargaining model, if output when not employed,* *the cost of posting a vacancy, and the haggling cost are proportional to productivity, then the change in unemployment with respect to a change in productivity is zero regardless of all other parameters, including the size of the fundamental surplus.*

Note this same result holds even if δ does not equal σ because all value functions are proportional to A. In sum, our model does indeed produce big movements in response to productivity changes but our model works differently to those analyzed by Ljungqvist and Sargent (2017) in their excellent synthesis of work on the Shimer puzzle. In short, our model seems to be a counterexample to the claim that in matching models, "the fundamental surplus is the single intermediate channel through which economic forces generating a high elasticity of market tightness with respect to productivity must operate" (Ljungqvist and Sargent 2017, p. 2663).

Table 1: Parametrization and results of the model with Campbell-Cochrane habits

Panel A: Parameters		Panel B: Moments	Data	Model
Endogenously chosen		*Targeted*	Data	Model
g_a, mean productivity growth (%p.a.)	2.22	Mean productivity growth (%p.a.)	2.22	2.22
σ_a, s.d. productivity growth (%p.a.)	1.79	S.d. productivity growth (%p.a.)	1.79	1.79
β, time preference factor	0.9956	Mean risk-free rate (%p.a.)	0.63	0.63
$\bar{S}$, mean surplus consumption	0.1059	S.d. risk-free rate (%p.a.)	2.34	2.34
α, inverse EIS	2.5	Maximum Sharpe ratio (p.a.)	0.45	0.45
B, efficiency of matching technology	0.455	Mean job-finding rate	0.46	0.46
κ, hiring cost	1.575	Mean unemployment rate	5.9	5.9
Assigned		*Results*		
b, home production parameter	0.6	S.d. job-finding rate	6.68	6.73
σ, probability of separation	0.028	Autocorrelation job-finding rate	0.94	0.98
η, matching function elasticity	0.5	S.d. unemployment rate	0.76	0.86
ϕ, survival probability	0.9979	Autocorrelation unemployment rate	0.97	0.99
ρ_s, habit persistence	0.9903	Correlation unemployment, job-finding rate	-0.97	-0.96
g_e, human capital growth when employed (%p.a.)	2.5	Elasticity user cost labor to u (Kudlyak)	-5.2	-3.6
g_u, human capital growth when unemployed (%p.a.)	-5.7			

Notes: Data for labor productivity is the nonfarm business sector real output per hour from BLS. The maximum Sharpe ratio in the data is the Sharpe ratio of the aggregate stock market return measured from the CRSP value-weighted stock index covering all firms continuously listed on NYSE, AMEX, and NASDAQ . Here and throughout, the constant separation unemployment rate is calculated as the unemployment rate that would occur with constant separations. (See text for details.) The model is simulated at a monthly frequency; statistics are calculated from artificial time-averaged data at an annual frequency.

Table 2: Role of human capital accumulation and preferences

			alt. preferences					alt. human capital	
	Data	Baseline	CRRA	Efficient CC	EZ w/ LRR	EZ w/ disasters	Affine SDF	$g_e = 0$ $g_u = 0$	$g_e = .025$ $g_u = .025$
S.d. job-finding rate	6.68	6.73	0.00	6.51	5.94	5.46	6.67	0.15	0.15
Autocorr. job-finding rate	0.94	0.98	—	0.98	0.99	0.99	0.98	0.98	0.98
S.d. unemployment rate	0.76	0.86	0.00	0.83	0.67	0.92	0.70	0.02	0.02
Autocorr. unemployment rate	0.97	0.99	—	0.99	0.99	0.99	0.99	0.99	0.99
Corr. unemp., job-finding rate	-0.97	-0.96	—	-0.98	-0.98	-0.98	-0.98	-0.99	-0.99

Notes: We adjust the values of parameter κ to maintain the same target value for mean unemployment.

Table 3: Implications of the baseline model for stock prices

	Data		Claims	
Statistics	CRSP	Flow of funds	Consumption	Index of equity
Mean excess return (%p.a.)	7.64	9.20	7.80	8.70
S.d. excess return (%p.a.)	17.1	16.9	17.2	18.6
Sharpe ratio (p.a.)	0.45	0.54	0.45	0.47
Mean price-dividend ratio	34.8	31.9	23.9	21.7
S.d. log price-dividend ratio	0.43	0.48	0.36	0.36

Notes: The model is simulated at a monthly frequency; statistics are calculated from time-averaged data at an annual frequency. The data refers to statistics for the CRSP value-weighted stock index covering all firms continuously listed on NYSE, AMEX and NASDAQ, and for the market value of outstanding equities and net equity payouts from the Flow of Funds as discussed in the text. The consumption claim is a claim to the aggregate consumption process. The equity index claim is a claim to aggregate net equity payouts as discussed in the text.

Table 4: Processes for leverage, earnings, and equity payouts

Log leverage process	Mean	Standard deviation	Autocorrelation
Data	-.73	.186	.90
Baseline model	-.72	.187	.90

Growth rate volatility	1 year	2 years	4 years	6 years	8 years	10 years
Earnings, data	9.91	9.80	8.72	7.20	5.85	4.90
Earnings, baseline model	5.02	5.21	5.11	4.91	4.70	4.51
Equity payouts, data	30.0	29.8	24.9	22.9	19.0	16.6
Equity payouts, baseline model	29.5	28.5	24.3	20.7	18.0	16.1

Notes: Earnings in the data is defined as net operating surplus from the National Income and Products Account. Earnings in the model are output less wages. Equity payouts in the data are defined as dividends net of share repurchases from the Flow of funds. The parametrization of the log leverage process is $\ell = \ln(.46)$, $\rho_\ell = .97$, and $\rho_{\ell z} = -.02$. The model is simulated at a monthly frequency; statistics are calculated from time-averaged data at an annual frequency. Growth rate volatility at different horizons for the a cashflow process C is defined as $std(\log(C_{t+n}/C_t))/\sqrt{n}$.

Table 5: Parametrization and results of the model with physical capital and efficient Campbell-Cochrane preferences

Panel A: Parameters

Endogenously chosen	
g_a, mean productivity growth (%p.a.)	2.22
σ_a, s.d. productivity growth (%p.a.)	1.79
β, time preference factor	0.9988
$\bar{S}$, mean surplus consumption	0.1262
γ, inverse EIS	2.5
B, efficiency of matching technology	0.455
κ, hiring cost	1.9
α, curvature production function	0.25
ξ, curvature adjustment costs	0.40
Assigned	
b, home production parameter	0.6
σ, probability of separation	0.028
η, matching function elasticity	0.5
ϕ, survival probability	0.9979
δ, physical capital depreciation rate	0.1/12
g_e, human capital growth when employed (%p.a.)	2.5
g_u, human capital growth when unemployed (%p.a.)	-5.7
ρ_s, habit persistence	0.9903

Panel B: Moments

Targeted	Data	Model
Mean productivity growth (%p.a.)	2.22	2.22
S.d. productivity growth (%p.a.)	1.79	1.79
Mean risk-free rate (%p.a.)	0.63	0.63
S.d. risk-free rate (%p.a.)	2.34	2.34
Maximum Sharpe ratio (p.a.)	0.45	0.45
Mean job-finding rate	0.46	0.46
Mean unemployment rate	5.9	5.9
Mean labor share of output	0.64	0.64
Ratio s.d. hp-filtered investment/GDP	2.8	2.4
Results		
S.d. job-finding rate	6.68	5.24
Autocorrelation job-finding rate	0.94	0.98
S.d. unemployment rate	0.76	0.68
Autocorrelation unemployment rate	0.97	0.99
Correlation unemployment, job-finding rate	-0.97	-0.96

Notes: Data for labor productivity is the nonfarm business sector real output per hour from BLS. The maximum Sharpe ratio in the data is the Sharpe ratio of the aggregate stock market return measured from the CRSP value-weighted stock index covering all firms continuously listed on NYSE, AMEX, and NASDAQ. Here and throughout, the constant separation unemployment rate is calculated as the unemployment rate that would occur with constant separations. (See text for details.) The model is simulated at a monthly frequency; statistics are calculated from artificial time-averaged data at an annual frequency.

Table 6: Model with efficient Campbell-Cochrane preferences and no human capital accumulation under different equilibria

	Data	Comp. search	Nash barg.		Across-period AOB			Within-period AOB	
Calibration of key parameters			Efficient	$\gamma \downarrow 0$	Efficient		Hall	Efficient	Christiano et al.
Avg duration of job opportunity during bargaining (in months)	–	–	–	–	1	3	74	1	.95
Per-round prob. barg. ends, δ	–	–	–	–	$\uparrow 1$	1/3	.013	$\downarrow 0$	.0019
Bargaining delay cost, ψ	–	–	–	–	0	.355	.352	0	.0098
S.d. job-finding rate	6.68	.15	.15	3.33	.15	.46	6.80	.15	.11
S.d. unemployment rate	.76	.02	.02	.41	.02	.06	0.98	.02	.01

Notes: The probability that a job opportunity breaks down after n rounds of bargaining is $\delta(1-\delta)^n$, so the expected duration of the job opportunity during bargaining is $\delta + 2\delta(1-\delta) + ... + n\delta(1-\delta)^{n-1} + ... = 1/\delta$ rounds, which occur within 1 month in within-period AOB and over $1/\delta$ months in across-period AOB. Expected duration in the within-period AOB is calculated as $(\delta \sum_{n=1}^{M-1} n(1-\delta)^{n-1} + Mp^*)/M$, with $p^* = 1 - \delta \sum_{n=1}^{M-1}(1-\delta)^{n-1}$. The equilibrium duration of negotiations is 1 round in both AOB schemes, as the first offer is accepted. The underlying parameters of the across-period AOB model are the same as in the model with competitive search and no human capital accumulation; the delay cost parameter ψ is chosen to bring the mean unemployment rate under AOB back to baseline.

Table 7: Parametrization and results of the model with efficient Campbell-Cochrane habits and alternating-offer bargaining

Panel A: Parameters		Panel B: Moments	Data	Model
Endogenously chosen		*Targeted*	Data	Model
g_a, mean productivity growth (%p.a.)	2.22	Mean productivity growth (%p.a.)	2.22	2.22
σ_a, s.d. productivity growth (%p.a.)	1.79	S.d. productivity growth (%p.a.)	1.79	1.79
β, time preference factor	0.9998	Mean risk-free rate (%p.a.)	0.63	0.63
$\bar{S}$, mean surplus consumption	0.178	S.d. risk-free rate (%p.a.)	2.34	2.34
α, inverse EIS	4.1	Maximum Sharpe ratio (p.a.)	0.45	0.45
B, efficiency of matching technology	0.455	Mean job-finding rate	0.46	0.46
κ, hiring cost	1.0	Mean unemployment rate	5.9	5.9
ψ, haggling cost	1.3			
ρ_s, habit persistence	0.9944	Autocorrelation price-dividend ratios	0.96	0.96
g_e, human capital growth when employed (%p.a.)	4.4	30y wage-experience profile (Elsby-Shapiro)	1.21	1.21
δ, haggling breakdown probability	0.75	Elasticity user cost labor to u (Kudlyak)	-5.2	-5.2
Assigned		*Results*		
b, home production parameter	0.6	S.d. job-finding rate	6.68	7.42
σ, probability of separation	0.028	Autocorrelation job-finding rate	0.94	0.98
η, matching function elasticity	0.5	S.d. unemployment rate	0.76	0.81
ϕ, survival probability	0.9958	Autocorrelation unemployment rate	0.97	0.99
g_u, human capital growth when unemployed (%p.a.)	0	Correlation unemployment, job-finding rate	-0.97	-0.96

Notes: Data for labor productivity is the nonfarm business sector real output per hour from BLS. The maximum Sharpe ratio and the price-dividend ratio in the data is the Sharpe ratio of the aggregate stock market return measured from the CRSP value-weighted stock index covering all firms continuously listed on NYSE, AMEX, and NASDAQ. The corresponding moments in the model are those of a consumption claim. Here and throughout, the constant separation unemployment rate is calculated as the unemployment rate that would occur with constant separations. (See text for details.) The model is simulated at a monthly frequency; statistics are calculated from artificial time-averaged data at an annual frequency.

Table 8: Model with efficient Campbell-Cochrane habits and alternating-offer bargaining as δ varies

	Data	(ours)	Across-period AOB		
Duration job opportunity $1/\delta$ (months)	–	1	2	3	6
Std. dev. λ_w	6.68	6.58	14.3	19.9	22.0
Std. dev. u	0.76	0.71	1.51	2.08	2.61
Wage-experience profile	1.2	1.2	1.2	1.2	1.2
Elasticity user cost labor to u	-5.2	-6.1	-2.1	-1.2	-0.44

Notes: $1/\delta$ denotes the average duration of the job opportunity during bargaining in the across-period alternating-offer bargaining scheme. Across the columns we adjust the haggling cost parameter ψ to maintain the same average unemployment rate.

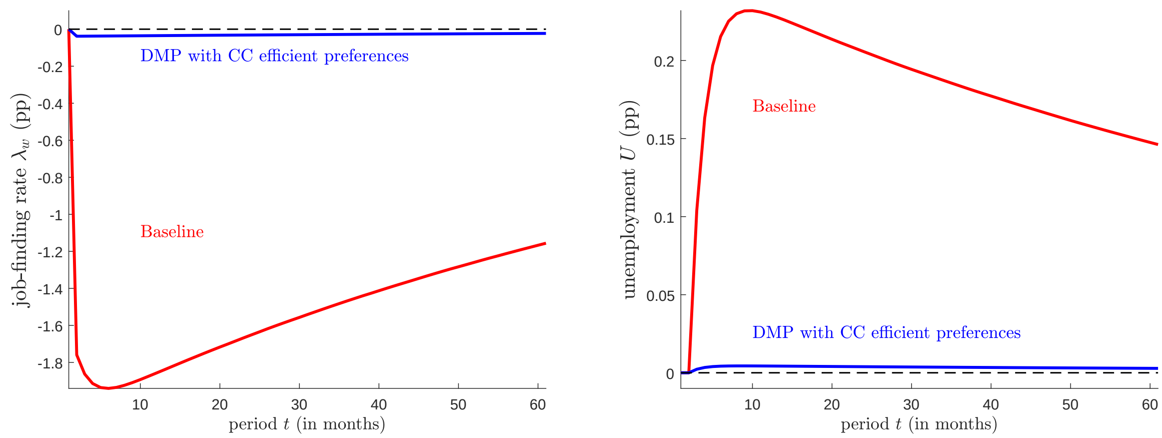
Table 9: Parametrization and results across models

Endogenously chosen parameters	Data	Baseline	Efficient CC	EZ w/ LRR	EZ w/ disasters	Affine SDF
g_a, mean productivity growth (%p.a.)		2.22				
σ_a, s.d. productivity growth (%p.a.)		1.79				
β, time preference factor		0.9956	0.9909	0.9982	0.9974	—
$\bar{S}$, mean surplus consumption		0.1059	0.0998	—	—	—
Inverse EIS (α w/ CC, ρ w/ EZ)		2.5	2.5	0.1	0.1	—
Risk aversion coefficient (α w/ EZ)		—	—	4	2	—
μ_0, mean risk-free (affine SDF)		—	—	—	—	.00058
μ_1, elasticity risk-free (affine SDF)		—	—	—	—	-.00028
$\gamma_0\sigma_a$, mean price of risk (affine SDF)		—	—	—	—	27.7
$\gamma_1\sigma_a$, elasticity price of risk (affine SDF)		—	—	—	—	1.24
B, efficiency of matching technology		0.46	0.46	0.46	0.46	0.46
κ, hiring cost		1.57	1.55	2.75	2.00	1.52
Targeted moments						
Mean productivity growth (%p.a.)	2.22	2.22	2.22	2.22	2.22	2.22
S.d. productivity growth (%p.a.)	1.79	1.79	1.79	1.79	1.79	1.79
Mean risk-free rate (%p.a.)	0.63	0.63	0.63	0.63	0.63	0.63
S.d. risk-free rate (%p.a.)	2.34	2.34	2.34	2.34	2.34	2.34
Maximum Sharpe ratio (p.a.)	0.45	0.45	0.45	0.45	0.45	0.45
S.d. excess return (%p.a.)	17.8	17.2*	17.3*	10.1*	13.4*	17.8
Mean job-finding rate	0.46	0.46	0.46	0.46	0.46	0.46
Mean unemployment rate	5.9	5.9	5.9	5.9	5.9	5.9
Results						
S.d. job-finding rate	6.68	6.73	6.51	5.94	5.46	6.67
Autocorrelation job-finding rate	0.94	0.98	0.98	0.99	0.99	0.98
S.d. unemployment rate	0.76	0.86	0.83	0.67	0.92	0.70
Autocorrelation unemployment rate	0.97	0.99	0.99	0.99	0.99	0.99
Correlation unemployment, job-finding rate	-0.97	-0.98	-0.96	-0.98	-0.98	-0.98

Notes: Data for labor productivity is the nonfarm business sector real output per hour from BLS. The maximum Sharpe ratio and the excess return in the data are the Sharpe ratio and excess return of the CRSP value-weighted stock index covering all firms continuously listed on NYSE, AMEX, and NASDAQ. Here and throughout, the constant separation unemployment rate is calculated as the unemployment rate that would occur with constant separations. (See text for details.) The model is simulated at a monthly frequency; statistics are calculated from artificial time-averaged data at an annual frequency. * denotes a moment not targeted.

Figure 1: Responses to a productivity shock with Campbell-Cochrane preferences

(A) Job-finding rate (B) Unemployment



Notes: Impulse responses of the job-finding rate and unemployment to a -1% permanent productivity shock. Generalized impulse response functions are based on 10,000 simulations.

Figure 2: Sensitivity of the standard deviation of the job-finding rate to human capital depreciation g_u

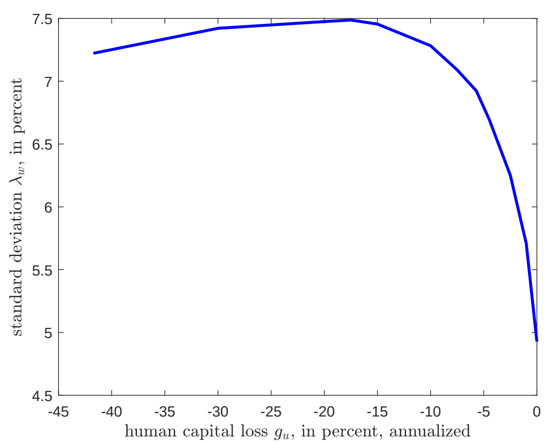
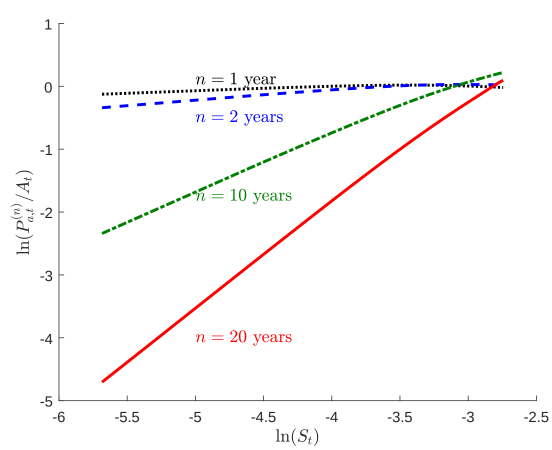


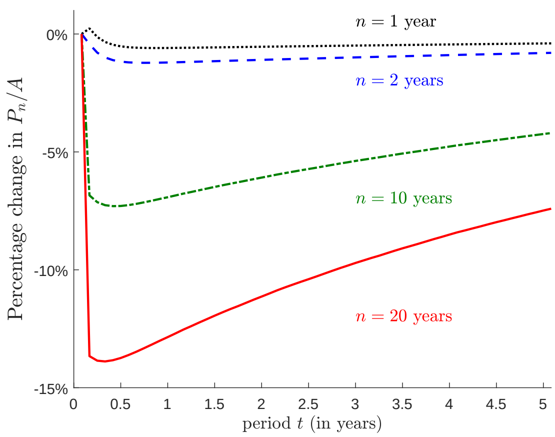
Figure 3: Prices of productivity strips with Campbell-Cochrane preferences



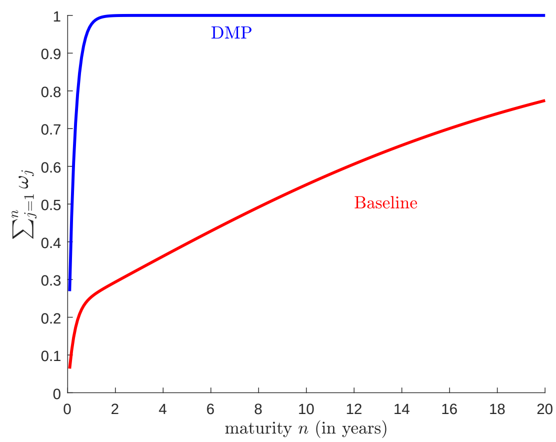
Notes: Price of underlying claims to productivity at different horizons as a function of surplus consumption.

Figure 4: Responses to a productivity shock with Campbell-Cochrane preferences

(A) Prices of productivity strips by maturity

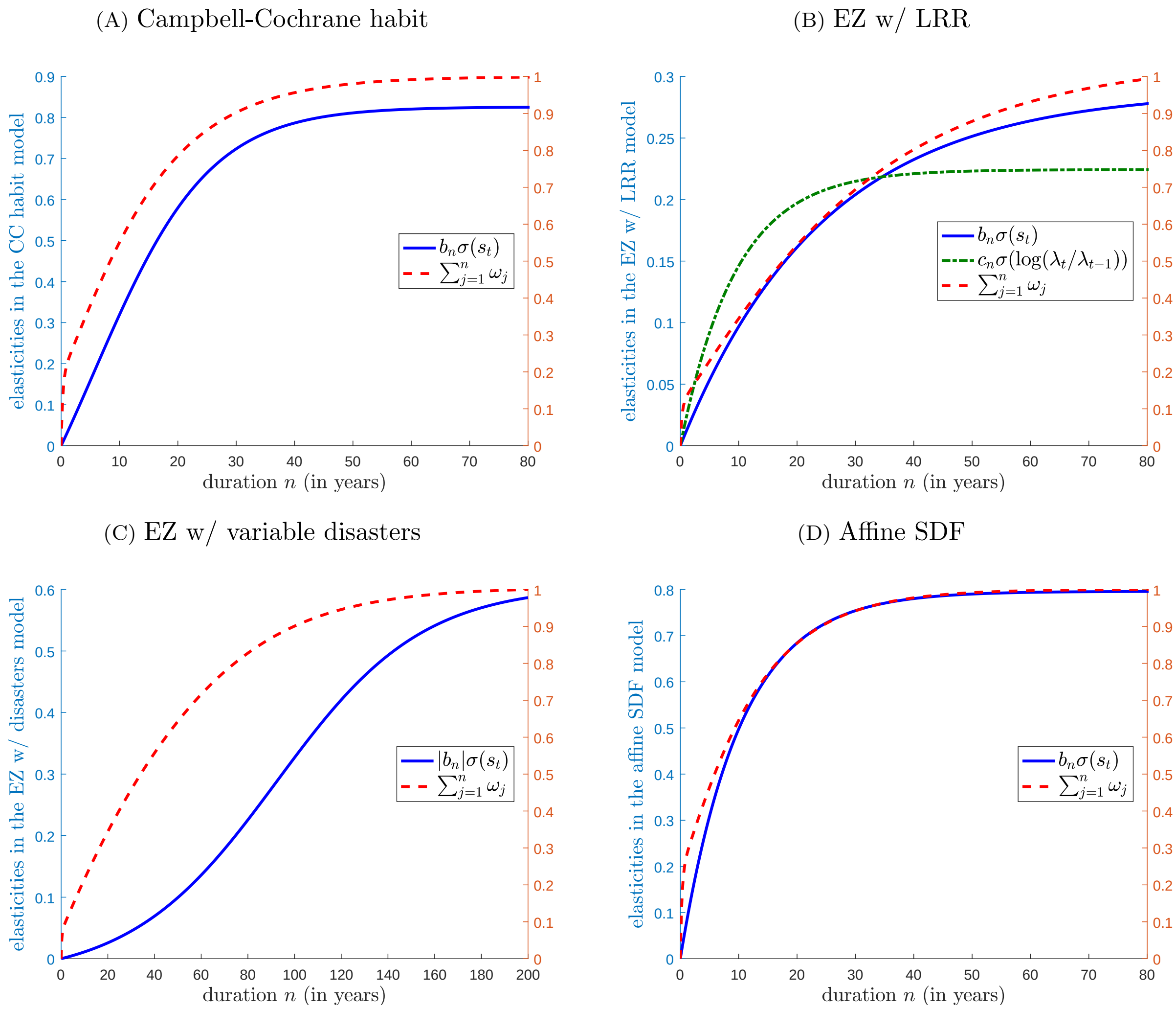


(B) Cumulative weights by maturity



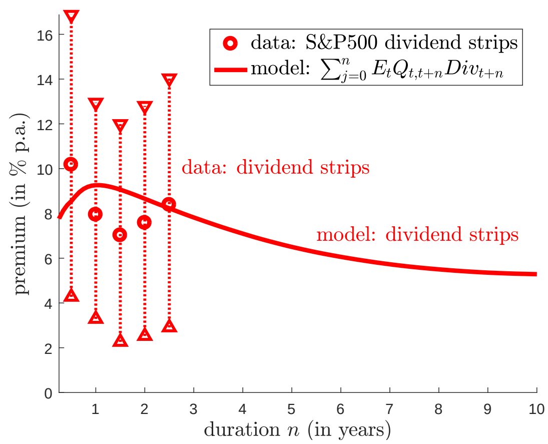
Notes: Impulse responses of the prices of productivity strips to a -1% permanent productivity shock, and the weights on these claims in the job-finding rate formula. Generalized impulse response functions are based on 10,000 simulations.

Figure 5: Determinants of the volatility of the job-finding rate λ_{wt}



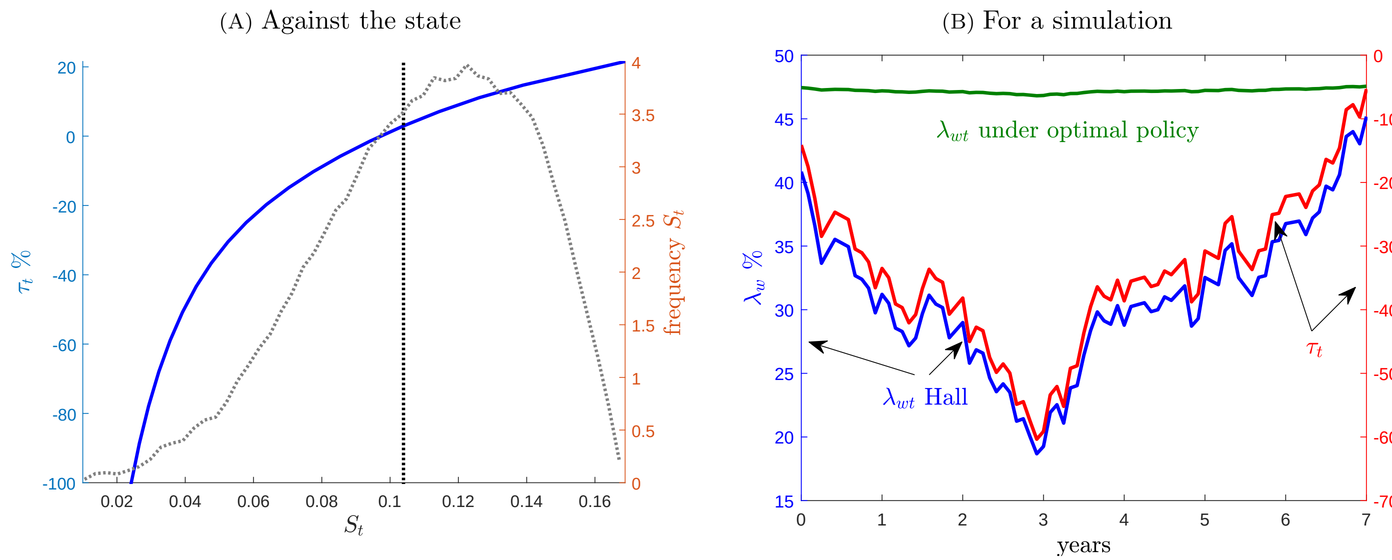
Notes: $\sigma(\lambda_{wt}) = |\sum_{n=1}^{\infty}\omega_n b_n|\,\sigma(s_t)$ under Campbell-Cochrane, affine stochastic discount factor, and Epstein-Zin with variable disasters, and $\sigma(\lambda_{wt}) = |\sum_{n=1}^{\infty}\omega_n b_n|\,\sigma(s_t) + |\sum_{n=1}^{\infty}\omega_n c_n|\,\sigma(\log(\lambda_t/\lambda_{t-1}))$ under Epstein-Zin with long-run risks.

Figure 6: Term structures of excess returns on corporate profits (EBIT) and dividend strips



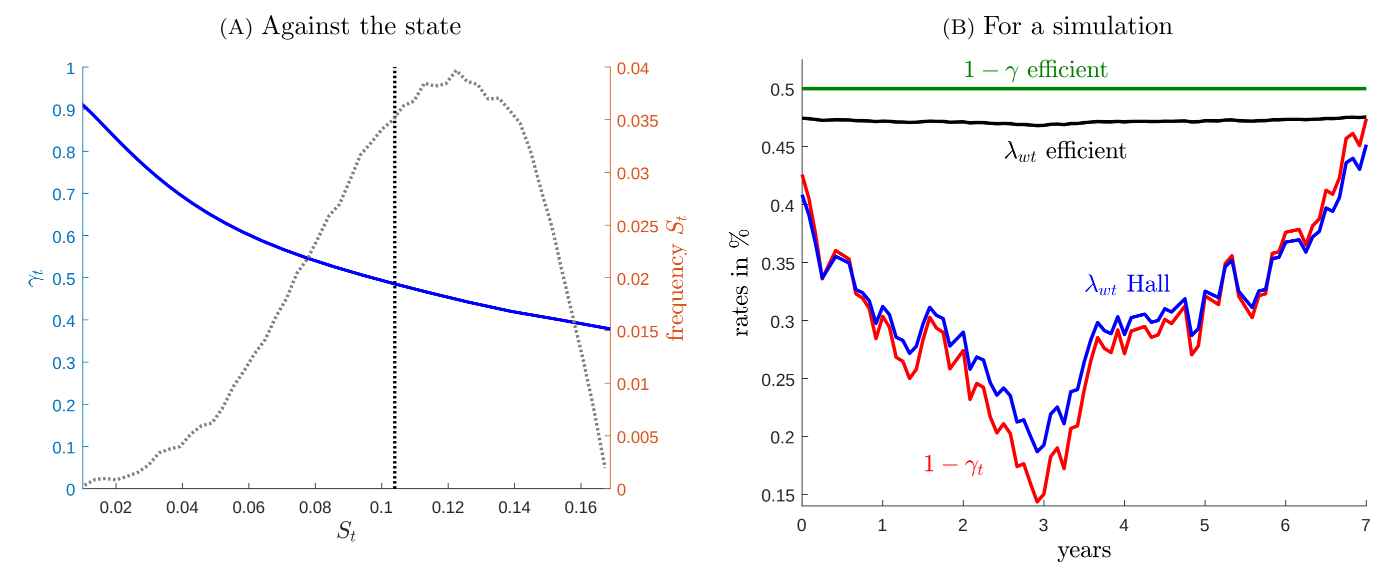
Notes: Evidence about dividend strips are from Lopez (2016) and are extracted from options on the S&P500 index, with block-bootstrapped 90% confidence intervals.

Figure 7: Across-period AOB ($\delta = .013$): Vacancy subsidy τ



Notes: Policy function and sample simulation. The subsidy on vacancy posting is applied to the decentralized competitive search allocation to lead it to the allocation under across-period AOB with $\delta = .013$. The ergodic distribution of the state (surplus consumption) is based on 120,000 months of simulations.

Figure 8: Across-period AOB ($\delta = .013$): Time-varying worker bargaining power



Notes: Policy function and sample simulation. The time-varying worker bargaining power γ_t is the Nash bargainining weight that mimics the AOB solution under the parametrization $\delta = .013$. The ergodic distribution of the state is based on 120,000 months of simulations.