OLG with search

Consider an overlapping generations economy in which a household is born in each generation t, $t = 1, 2, 3, \ldots$, and lives for three periods.

In the first period of life, the household consumes, $c_t^t \geq 0$, searches for a job with effort $e_t^t \geq 0$, and receives an exogenous endowment $y_t^t \geq 0$. The household gets a job offer with probability $h\left(e_t^t\right) \in (0,1)$, where $h' > 0$ and $h'' < 0$ for all $e_t^t \geq 0$. If the household gets a job offer, the wage offer will be w_t^t, where w_t^t comes from a distribution $\Phi\left(w_t^t\right)$. If the household accepts the offer, she will work in period 1 for one unit of time (you can assume the effort is undertaken at the start of period 1) and work in period 2, also one unit of time, at the same wage w_t^t (the household will still receive the exogenous endowment $y_t^t \geq 0$). Otherwise, the household remains unemployed and her only income is the exogenous endowment $y_t^t \geq 0$. The household can also save a quantity a_t^t at an equilibrium interest rate r_t^t.

In the second period of life, the household consumes $c_{t+1}^t \geq 0$, works at wage w_t^t if it accepted a job offer in period t or, if the household did not receive an offer or did not accept an offer in the period t, search with effort $e_{t+1}^t \geq 0$ and get a job offer with probability $h\left(e_{t+1}^t\right) \in (0,1)$, where $h' > 0$ and $h'' < 0$ for all $e_{t+1}^t \geq 0$. If the household gets a job offer, it will come with a wage w_{t+1}^t where w_{t+1}^t is from a distribution $\Phi\left(w_{t+1}^t\right)$. If the household accepts the offer, she will work in period 2 for one unit of time (you can assume the effort is undertaken at the start of period 2) at wage w_{t+1}^t. Regardless of whether the household works or not (either because it received and accepted an offer in period 1 or received and accepted an offer in period 2), the household receives an exogenous endowment $y_{t+1}^t \geq 0$. The household can also save a quantity a_{t+1}^t at an equilibrium interest rate r_{t+1}^t.

In the third period of life, the household consumes $c_{t+2}^t \geq 0$, financed from her assets $\left(1 + r_{t+1}^t\right) a_{t+1}^t$. However, the household does work in this last period of life.

The household's preferences are

$$\ln c_t^t - \frac{\left(e_t^t\right)^2}{2} + \beta\left(\ln c_{t+1}^t - \frac{\left(e_{t+1}^t\right)^2}{2}\right) + \beta^2 \ln c_{t+2}^t$$

In addition there are two initial old generations with utility functions

$$\ln c_1^0 + \beta^2 \ln c_2^0$$

and

$$\ln c_1^{-1}$$

where generation 0 has a job at wage w_1^0. There is no money is this economy.

1. Write the budget constraints of the household period by period.

2. Define an Arrow-Debreu equilibrium for this economy.

3. Define a sequential markets equilibrium for this economy.

4. Are the allocations of 2. and 3. the same?

5. Describe the search strategy of the household in period 1 and 2. Does the household have a reservation wage strategy? How does the fact that we deal with an OLG model change the

search problem of the household with respect to a standard infinite horizon search problem? Go as far as you can.

6. Compute the Arrow-Debreu equilibrium. Go as far as you can.
7. Define a Pareto optimal allocation for this economy.
8. Is the Arrow-Debreu equilibrium you computed in 6. Pareto optimal?