Economics 702, Dirk Krueger

1 A Pure Exchange Economy with Household Heterogeneity

Consider a stochastic pure exchange economy where the current state of the economy is described by $s_t \in S = \{s_n, s_r, s_d\}$. That is, the economy can either be in normal times, $s_t = s_n$, in a recession, $s_t = s_r$ or a depression, $s_t = s_d$. Event histories are denoted by s^t and the initial node s_0 is fixed. Probabilities of event histories are given by $\pi_t(s^t)$. There are N different types of households with equal mass normalized to $1/N$. Households potentially differ in their endowment stream $\{e_t^i(s^t)\}$. Preferences for each household over consumption allocations $c^i = \{c_t^i(s^t)\}$ are given by

$$u^i(c^i) = \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi_t(s^t) U(c_t^i(s^t)).$$

where $U(.)$ is strictly increasing and strictly concave.

1. Suppose that $\pi_t(s^t)$ is Markov with transition matrix

   $$\pi(s'|s) = \begin{pmatrix} \pi_n & 1-\pi_n & 0 \\ 1-\pi_d & 0 & \pi_d \\ 1 & 0 & 0 \end{pmatrix}$$

   where $\pi_n \in (0,1)$ is the conditional probability that the economy remains in normal times, and $\pi_d \in (0,1)$ is the conditional probability that the economy will fall into a depression once it enters a recession. Compute the invariant distribution(s) associated with π.

2. Define an Arrow-Debreu equilibrium.

3. Under the assumption that households can trade a full set of Arrow securities, define a recursive competitive equilibrium.

4. For the rest of this question, suppose the aggregate endowment $e_t(s^t) = \sum_{i=1}^{N} e_t^i(s^t)$ is simply a function of the current state, such that $e_t(s^t) =$

$e_t(s_t)$ and

$$\begin{aligned} e_t(s_n) &= 1 \\ e_t(s_r) &= e_r \\ e_t(s_d) &= e_d \end{aligned}$$

where $e_r < 1$ is the aggregate endowment in a recession, and $e_d \ll e_r$ is the aggregate endowment in a depression. Also assume that $U(c) = \log(c)$ for the next two questions. Households still can trade a full set of Arrow securities. Characterize as fully as possible the equilibrium prices of the Arrow securities $q_t(s^t, s_{t+1})$ in a sequential markets equilibrium.

5. State the price of a one period risk-free bond in terms of the prices of Arrow securities and use your answer from the previous question to characterize it as completely as possible. How is the risk-free interest rate in normal times ($s_t = s_n$) and in recessions ($s_t = s_r$) affected by the probability of a depression, π_d. Try to explain your result.

6. Now suppose that the utility function of each household is

$$U(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

with $\sigma > 0, \sigma \neq 1$, and that the aggregate endowment follows the same process as before, but in addition there is secular income growth, such that the new aggregate endowment process $\{\tilde{e}_t(s^t)\}$ satisfies

$$\tilde{e}_t(s^t) = e_t(s^t)(1+g)^t$$

with $g > 0$. Discuss how (σ, g) impact your answer to question 5.

7. Define a stock as an asset that pays the aggregate endowment in every event history. Characterize by how much the **cum-dividend** price of the stock falls as the economy turns from normal times ($s_t = s_n$) into a recession ($s_{t+1} = s_r$). You can assume that $\pi_d = 0, g = 0$ and $U(c) = \log(c)$ to simplify the calculation. Hint: conjecture and verify that the cum-dividend stock price P^{stock} only depends on the current shock realization, that is, $P^{stock}(s^t) = P^{stock}(s_t)$, then calculate $P^{stock}(s_n)$ and $P^{stock}(s_r)$ and finally characterize $\frac{P^{stock}(s_r)}{P^{stock}(s_n)}$.