Microeconomic Theory II Preliminary Examination June 4, 2018

The exam is worth 120 points in total.

There are **3** questions. Do all questions. Start each question in a new book, clearly labeled. **Fully justify** all answers and show all work (in particular, describing an equilibrium means providing a **full description of the strategy profile** and **proving** that it has the desired properties). Label all diagrams clearly. Write legibly. If you need to make additional assumptions, state them clearly.

Good luck!

1. **[35 points]** Consider the following game between players *I* (the row player) and *II* (the column player):

	L	С	R
U	5,5	3,9	0, -1
D	3,3	5,1	0, -1

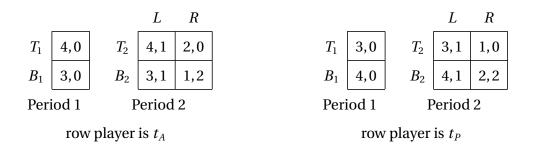
(a) What is the unique Nash equilibrium of this game?

[10 points]

- (b) Now suppose that player *I* has the option of either publicly choosing his action of *U* or *D* before *II* chooses, or of making his choice in secret (so that *II* does not know *I*'s choice when choosing). The cost to *I* of publicly choosing before *II* is 1 util.
 - i) Describe an extensive form fitting this description in which player *I* publicly choosing *U* and player *II* responding with *C* is a subgame perfect equilibrium outcome. [10 points]
 - ii) Describe an extensive form with the same reduced normal form as the game in part i), but in which player *I* publicly choosing *U* and player *II* responding with *C* is *not* a subgame perfect equilibrium outcome. [15 points]

[Question 2 is on the next page.]

2. **[45 points]** Consider the following version of a reputation game: There are two periods and two players: a "long-lived" row player (he chooses in both periods and total payoffs are the sum of payoffs from each period) and a column player choosing in the second period. The row player is either an aggressive type (denoted t_A), in which case payoffs are as described in the first pair of payoff matrices, or a passive type (t_P), in which case payoffs are as described in the second pair of payoff matrices (only the row player's payoffs differ by type). The column player receives a payoff of 0 in the first period, irrespective of the choice of the row player.



The prior probability that the column player assigns to the row player being aggressive is p. Assume $p \in (\frac{1}{2}, 1)$. In the second period, the column player chooses her action simultaneously with the row player, not knowing his type, but having observed the choice of the row player in period 1.

- (a) What restrictions on second-period behavior of the row player are implied by sequential rationality (Hint: These restrictions are also implied by perfect Bayes and sequential equilibrium)? Describe the signaling game thus induced. [10 points]
- (b) Prove that there is no separating perfect Bayesian equilibrium of the induced signaling game. [10 points]
- (c) Describe the two pooling perfect Bayesian equilibria of the induced signaling game in which both types of row player choose the same action in period 1. [15 points]
- (d) One of the pooling equilibria of the signaling game is more plausible than the other. Which one and why? [10 points]

[Question 3 is on the next page.]

3. **[40 points]** A seller will run an auction to sell a one-of-a-kind replica of the Sydney Harbour Bridge. There are two potential buyers, with buyer *i*'s value for the replica, θ_i , distributed on $[0, \bar{\theta}_i]$, independently of buyer $j \neq i$. As usual, outcomes are a function of type profiles denoted by

$$(\rho, t): [0, \overline{\theta}_1] \times [0, \overline{\theta}_2] \rightarrow \Delta(\{0, 1, 2\}) \times \mathbb{R}^2$$

where $\rho(\theta)$ is the probability distribution over who obtains the object (with 0 meaning the seller retains the good), and $t_i(\theta)$ is the transfer from buyer *i* to the seller. Define (where $j \neq i \in \{1,2\}$)

$$p_i(\theta_i) := \int_{\theta_j} \rho_i(\theta_i, \theta_j) dF_j(\theta_j) \text{ and } T_i(\theta_i) := \int_{\theta_j} t_i(\theta_i, \theta_j) dF_j(\theta_j),$$

where F_j is the distribution function of θ_j . Each F_j has a strictly positive density f_j on its support $[0, \overline{\theta}_i]$.

- (a) By the Bayesian revelation principle, the seller can restrict attention to incentivecompatible individually-rational direct mechanisms. Carefully define the terms "direct mechanism," "incentive compatibility," and "individual rationality." [10 points]
- (b) Suppose (ρ, t) is an incentive-compatible individually-rational direct mechanism. Then p_i is nondecreasing and

$$T_i(\theta_i) = -k_i + p_i(\theta_i)\theta_i - \int_0^{\theta_i} p_i(\tilde{\theta}_i)d\,\tilde{\theta}_i,$$

for some $k_i \ge 0$. Using this fact, prove that buyer *i*'s expected payment in the direct mechanism is given by

$$\int_0^{\theta_i} \left[\theta_i - \frac{1 - F_i(\theta_i)}{f_i(\theta_i)} \right] p_i(\theta_i) \, dF_i(\theta_i) - k_i,$$

where f_i is the density of F_i .

- (c) Suppose $\bar{\theta}_1 = 1$, $\bar{\theta}_2 = 2$, and both buyers' valuations are uniformly distributed. Using part (b), derive the allocation rule in the revenue-maximizing incentive-compatible individually-rational direct mechanism. Is the allocation efficient? [10 points]
- (d) Suppose the seller values the replica at $v_s \in (0,2)$, so that the seller's expost payoff from the direct mechanism (ρ, t) is

$$\rho_0(\theta_1, \theta_2)\nu_s + t_1(\theta_1, \theta_2) + t_2(\theta_1, \theta_2) = (1 - \rho_1(\theta_1, \theta_2) - \rho_2(\theta_1, \theta_2))\nu_s + t_1(\theta_1, \theta_2) + t_2(\theta_1, \theta_2).$$

What is the seller's optimal incentive-compatible individually-rational direct mechanism? [10 points]

[10 points]