A Fisherian Approach to Explaining & Preventing Financial Crises: Quantitative Implications from the Sudden Stops Literature*

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Abstract

A Sudden Stop is a financial crisis defined by a large, sudden reversal in the current account. Sudden Stops occur in both advanced and emerging economies, and result in deep recessions, collapsing asset prices and large real depreciations. They are preceded by economic expansions, current account deficits, credit booms, and appreciated asset prices and real exchange rates. Quantitative studies have shown that Fisherian models, namely models with credit constraints linked to market prices, can explain the main stylized facts of Sudden Stops as a result of financial amplification driven by Irving Fisher’s debt-deflation mechanism. These models also feature a pecuniary externality, which creates scope for macroprudential policy (MPP) aimed at reducing the magnitude and frequency of crises. Results show that optimal MPP is powerful but complex and time-inconsistent, and simple rules are much less effective. We review the stylized facts of Sudden Stops, the evidence on MPP use and effectiveness, and the positive and normative contributions of the quantitative literature on Fisherian models. We also conduct a new analysis examining the tradeoffs of MPP related to its adverse effects on capital accumulation and to the effectiveness of regulatory LTV ratios v. debt taxes. Simple rules have costly tradeoffs and while both regulatory LTVs and debt taxes reduce credit and distort investment, they are not equivalent instruments.

Keywords: Sudden Stops, Financial crises, Macroprudential policy, Time inconsistency

JEL Classifications: E31, E37, E52, F41

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“...debt happens as a result of actions occurring over time. Therefore, any debt involves a plot line: how you got into debt, what you did, said and thought while you were there, and then—depending on whether the ending is to be happy or sad—how you got out of debt, or else how you go further and further into it until you became overwhelmed by it, and sank from view.”


1 Introduction

The Mexican crisis of 1994 was the harbinger of a series of financial crises that have affected both emerging and advanced economies since then. The defining feature of these crises is a large, sudden reversal in the current account, which is a country’s broadest measure of net foreign financing. Because of the sudden loss of access to credit from international capital markets, these events came to be known as Sudden Stops.¹ As we document in the next section, by the end of 2016 there had been 58 Sudden Stop events worldwide, 35 in emerging markets and 23 in advanced economies.

Sudden Stops have been the focus of a large theoretical and empirical literature since the mid 1990s, and a growing literature has been studying macroprudential regulation as a way to avert financial crises since the aftermath of the 2008 Global Financial Crisis. In this article, we start by reviewing the stylized facts of Sudden Stops based on a new event analysis spanning the 1979-2016 period and including both advanced and emerging economies. Then we provide a short survey of the findings of the empirical literature on the use and effectiveness of macroprudential policies to date. As we document, results have been mixed at best, except for regulation on loan-to-value and debt-to-income ratios of borrowers, particularly for home mortgages. The paper then moves on to review the literature that examines the quantitative implications, both positive and normative, of a class of Sudden Stops models labeled Fisherian models. In these models, Sudden Stops are the result of a strong financial amplification mechanism characterized by the debt-deflation mechanism proposed in the seminal work on the Great Depression by Fisher (1933), and emphasized also in the classic studies by Minsky (1992) and Kiyotaki and Moore (1997).

The key feature of a Fisherian model is an occasionally binding credit constraint that limits borrowing capacity to a fraction of the market value of the goods or assets pledged as collateral. This constraint is the essential part of the mechanism by which the Fisherian debt-deflation mechanism operates: When the constraint binds, agents fire-sale the goods or assets that serve as collateral, and as they do they drive down the value of those goods or assets, which tightens

¹Oral tradition has it that this nickname originated in a commentary from the audience on a presentation by the late Rudi Dornbusch on the Mexican crisis, making the point that like in the familiar Douglas Adams quote, with the sharp current account reversals “it is not the fall that kills you, it is the sudden stop at the end.”
the constraint further and forces further fire-sales. The Fisherian constraint also introduces a
distortionary pecuniary externality that is the foundation of the normative implications derived
from these models, particularly for the analysis of macroprudential policy (MPP): Atomistic
borrowers do not internalize how their borrowing decisions made in “good times” affect the size
of the deflation in collateral values and the reduction in borrowing capacity during a Sudden
Stop. If borrowing during expansions induces larger crashes in collateral values during crises,
private agents undervalue the social marginal cost of borrowing and hence they “overborrow”
(Bianchi, 2011,Bianchi and Mendoza, 2018). Optimal MPP thus calls for tightening access to
credit in procyclical fashion.

In addition to reviewing the findings of the quantitative literature on Fisherian models, we
conduct a new analysis examining MPP tradeoffs in terms of capital accumulation and con-
sumption smoothing and comparing the implications of regulatory loan-to-value ratios (LTVs)
v. debt taxes. To this end, we modify a widely-used Fisherian two-sector model with tradable
and nontradable goods to introduce a representative firm that produces investment goods using
both tradables and nontradables as inputs. Households consume tradables and nontradables,
make investment decisions, and face a Fisherian constraint that limits their debt not to exceed a
fraction of the market value of the capital stock. In this setup, MPP instruments that raise the
cost of borrowing above the risk-free rate, distort not only intertemporal consumption but also
investment. To date, studies quantifying MPP tradeoffs have focused on the latter only.

We characterize the feasible set of output and crisis exposure that can be achieved with a
fixed tax on borrowing. A higher debt tax allows to reduce the magnitude and frequency of
crises, but this leads to a permanently lower level of output. In some states, reducing both
consumption and investment is efficient. While a fixed tax can deliver welfare gains on average,
they can also be welfare reducing in many states (Bianchi and Mendoza, 2018). There are also
important tradeoffs in implementing MPP with debt taxes v. regulatory LTVs. Both reduce
credit and distort investment, but LTVs adjust endogenously in response to credit conditions and
have different effects on asset prices. In particular, at the same levels of credit and consumption,
regulatory LTVs support higher asset prices than debt taxes. As result, regulatory LTVs and
debt-tax-like instruments are not equivalent.

The rest of the paper is organized as follows. Section 2 conducts an empirical analysis of
Sudden Stop events using a cross-country panel dataset and reviews the evidence on MPP use
and effectiveness. Section 3 reviews the main elements and findings of the quantitative literature
on Fisherian models of Sudden Stops, both positive and normative. Section 4 proposes the
model we use to analyze MPP tradeoffs vis-a-vis capital accumulation and in the use of LTV
regulation v. debt taxes. Section 5 examines the quantitative implications of the model. Section
6 concludes.
2 Empirical Evidence

In this Section of the paper we examine the stylized facts of Sudden Stops and provide a literature review of the empirical work on the use and effectiveness of macroprudential policies. The objective of the former is to provide a quantitative characterization of the movements in macroeconomic variables that define Sudden Stops. The goal of the latter is to briefly document the existing evidence on the performance of policies aimed at preventing credit crises, which have been Sudden Stops in the data studied in most of the literature.

2.1 Stylized Facts of Sudden Stops

Sudden Stops (SS) are economic fluctuations defined by a set of empirical regularities associated with a large, sudden reversal of capital inflows into the economy (i.e. a sudden “loss of access” to international financial markets). Empirical studies on this subject originated in the seminal contributions by Ferretti and Razin (2000) and Calvo et al. (2004) and Calvo (2006), which motivated several authors to conduct further studies, resulting in a large empirical literature that includes many well-known contributions (e.g. Edwards (2004), Rothenberg and Warnock (2006), Forbes and Warnock (2012), Calvo et al. (2013), Eichengreen et al. (2017)). Most empirical studies apply event analysis tools to cross-country panel datasets, using one or more filters to identify Sudden Stop events. A sufficiently large increase in the current account-GDP ratio \((ca/y)\) is widely used as the main identification filter, because the current account is the broadest measure of the flow of credit of an economy vis-a-vis the rest of the world, and hence a large increase in \(ca/y\) indicates a sharp contraction in credit from abroad (both private and public credit). This filter is often used together with a second filter that detects if the Sudden Stop is systemic across countries (e.g. using the EMBI+ index for emerging markets) and in some instances other filters are added, such as large output drops to capture Sudden Stops with particularly deep recessions (see, for example, Calvo et al. (2013)).

We re-visit here the analysis of the empirical regularities of Sudden Stops using a similar methodology but applying it to a panel dataset that includes data up to 2016, covering both emerging and advanced economies. We adopt the same event analysis specification as in the survey by Korinek and Mendoza (2014), which used data ending in 2012.\(^2\) This specification defines Sudden Stops as year-on-year increases in \(ca/y\) that are in the 95 percentile of the frequency distribution of annual changes in \(ca/y\) of a particular county. It also includes a filter for systemic Sudden Stops, which is the market-wide EMBI for emerging markets or the VIX index for advanced economies. We present the results using both current-account and systemic filters

\(^2\)The full details of the methodology are described in the data appendix of the Korinek-Mendoza paper available at: https://www.annualreviews.org/doi/suppl/10.1146/annurev-economics-080213-041005.
for consistency with the more recent literature, but the event plots characterizing the stylized facts of Sudden Stops in macro data are very similar if we use only the $ca/y$ filter. We use annual data for 35 emerging market (EM) economies and 23 advanced economies (AE) covering the 1979-2016 period, and construct five-year event windows centered on the date of Sudden Stops for the HP cyclical components of macroeconomic data. Each of the window plots shows the median for EMs and AEs as separate curves.

The event analysis yields five key stylized facts, which are consistent with the findings from the existing literature:

1. A typical SS event across all countries in the sample is defined as a current account reversal of 3.7 percentage points of GDP. The reversals are larger in EMs (4.4 percentage points) than in AEs (2.7 percentage points), as shown in panel (a) of Figure 1. Moreover, SS events are preceded by growing current account deficits and followed by persistent surpluses of about 0.7 percent of GDP two years later. Hence, the credit inflow from the rest of the world expands in the run-up to a Sudden Stop, then it halts and reverts sharply into an outflow when a Sudden Stop hits, and it remains an outflow afterwards.

2. SS events are infrequent, but they are twice as likely to occur in emerging than in advanced economies. We found 51 Sudden Stops in total (2.4 percent frequency), of which 36 occurred in EMs (2.9 percent frequency) v. 15 in AEs (1.7 percent frequency). Hence, Sudden Stops are rare events that co-exist with typical business cycles, in which current accounts move countercyclically but with much smaller increases than what we observe in Sudden Stops.

3. SS events are not randomly distributed over time, rather they are clustered around “big events.” As Figure 3 shows, there are several years in which no Sudden Stops occur, while we observe 14 Sudden Stops in 1982 and 1983 when the Sovereign Debt Crisis of the early 1980s exploded, 13 in 1998 and 1999 when the Asian crisis occurred, and 7 in 2009 with the Global Financial Crisis.

4. SS events are associated with sharp economic downturns, preceded by expansions and followed by protracted recessions (see panels (b) and (c) of Figure 1). For all countries combined, GDP and consumption are 2.5 and 1.6 percent below trend respectively. In EMs (AEs) they are 3.6 (1.1) and 1.5 (1.6) percent below trend respectively. Moreover, these downturns represent sharp reversals compared with the expansions that precede Sudden Stops. Relative to the year before a Sudden Stop hits, the deviations from trend in GDP and consumption for all countries fall by 4.4 and 4 percentage points respectively, and again the reversals are larger for EMs than AEs (but keep in mind that business cycles are also larger in EMs, so relative to the standard deviations of cyclical components they are comparable). Investment (panel (d) of Figure 1) shows a similar pattern, but with larger
changes since investment is also more volatile over the business cycle. For all countries, investment is nearly 11 percentage points below trend when a Sudden Stop hits, and this represents a reversal of nearly 19 percentage points relative to the year before. Two years after SS events, all three macro-aggregates remain significantly below trend. GDP and consumption are 1.5 to 2 percent below trend and investment 3.3 to 5.5 percent below trend across EMs and AEs. This result is in line with the findings from Reinhart and Rogoff (2009) indicating that recoveries from recessions triggered by financial crises are slow.

5. SS are associated with falling real equity prices and depreciated real exchange rates (again with larger declines in EMs), and they are preceded (followed) by higher (lower) equity prices and appreciated real exchange rates (see panels (e) and (f) of Figure 1). Hence, the sharp credit reversal reflected in the current account reversal coincides with deflation in relative prices, both equity prices and relative consumer prices. This is an important observation for it indicates that relative price deflation is an empirical regularity of Sudden Stops, and hence should be considered in the design of models aiming to explain this phenomenon.

It is also worth noting that studies conducting growth accounting of the recessions associated with Sudden Stops found that conventional measures of capital and labor account for a small fraction of the large declines observed in GDP (Mendoza (2010), Meza (2008), Calvo (2006)). Hence, the real effects of Sudden Stops would seem to be largely due to large, negative technology shocks as measured by standard Solow residuals. Those studies also showed, however, that sharp declines in factor utilization and large increases in relative prices of imported inputs, both of which cause bias in Solow residuals, can explain the large decline in Solow residuals.
Figure 1: Sudden Stops Dynamics

(a) Current Account

(b) GDP

(c) Consumption

(d) Investment

(e) Equity Prices

(f) Real Exchange Rate
Figure 2: Number of Sudden Stops per Year
2.2 Macroprudential Policy Use & Performance

The event analysis shows the macroeconomic effects of Sudden Stops, which are a type of financial crisis defined by a sudden loss of access to external credit, as measured by a large current account reversal. There is a related empirical literature documenting the deep recessions and price corrections that follow the collapse of credit booms (e.g. Mendoza and Terrones (2008), Mendoza and Terrones (2012), Schularick and Taylor (2009)) and the historical characteristics of financial crises (Reinhart and Rogoff (2009)). Together with the recent experience of the Global Financial Crisis (GFC) in many countries, the evidence documented in these studies highlights the importance of implementing policies aimed at reducing the frequency and magnitude of financial crises, which are now grouped under the label of macroprudential policy.

To be sure, financial policies that can be regarded as macroprudential are not new. There are examples of policies aimed at managing aggregate credit dating back to the 1930s, and also several emerging economies introduced financial regulation that is now considered macroprudential since the late 1990s, in the aftermath of the Emerging Markets Crisis. Yet, it was only in the aftermath of the 2008-2009 GFC that a wide consensus was reached on the desirability of using macroprudential policies. Several countries adopted new financial policies and institutional changes accordingly, international organizations endorsed their use and contributed to their implementation (e.g. the countercyclical capital buffer included in the BIS’s Basel III regulatory framework), and new institutions were created to implement them and coordinate them (e.g. the Financial Stability Board and the European Systemic Risk Board).

In the paragraphs that follow, we summarize the macroprudential policies that are more commonly used since the 1990s and the existing evidence on their effectiveness so far. The aim is to provide an empirical background for the normative analysis of the models of Sudden Stops that we conduct later in this paper. A number of comprehensive empirical studies have analyzed macroprudential policies and their effectiveness in emerging and advanced economies (e.g. Cerutti et al., 2015, European Systemic Risk Board [2018], Ahnert and Kakhbod, 2017, Cordella et al., 2014). We base the description provided in the rest of this Section on the work of Cerutti et al. (2015) and on the comprehensive survey by Galati and Moessner (2018).

Cerutti et al. (2015) constructed time-series, cross-sectional indexes of 12 macroprudential policy instruments using data from two IMF surveys. They grouped the instruments into two categories: instruments aimed at borrowers and those aimed at financial intermediaries. The former consist of loan-to-value (LTV) and debt-to-income (DTI) ratios on new loans, and the latter consist of ten instruments, including in particular countercyclical capital buffers, bank

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3The precise timing with which the macroprudential term was first used is unknown, but its first widely-cited use is generally attributed to Borio (2003) discussion of the need to revamp financial regulation and supervision with a macroprudential perspective motivated by the financial turbulence observed in emerging markets during the 1990s.
leverage ratios and limits on foreign currency loans (see Annex 1 in Cerutti et al. (2015) for full details). The overall macroprudential index (MPI) is defined as the sum of the scores on all twelve instruments in a given year and country (120 countries from 2000 to 2013), assigning a value of 1 to each instrument that a given country has, from the year the instrument was introduced to the year it was removed. Similar indexes are defined for the borrower-targeted (BTI) and financial institution-targeted (FIT) instruments.

The evolution of the MPI since 2000 highlights the growing importance of macroprudential policy. The average MPI across all countries rose from 1.1 in 2000 to nearly 2.5 in 2013. The increase has been common to advanced, emerging and developing economies, although emerging markets show higher average MPIs than advanced economies throughout the sample period (1.3 v. 0.8 in 2000, 2.6 v. 1.9 in 2013). By 2013, 90 percent of the 120 countries in the sample had implemented at least one macroprudential policy instrument. Instruments targeted to financial institutions are the most common, but borrowed-targeted instruments have gained relevance since the mid-2000s. By 2013, 35 percent of the countries had adopted borrower-oriented LTV and/or DTI regulations. As we review below, this is perhaps in response to the stronger empirical evidence on the effectiveness of borrower-oriented instruments v. financial institution-oriented instruments.

While the data studied by Cerutti et al. (2015) shows that macroprudential instruments have become widely used, the survey by Galati and Moessner (2018) documents that the empirical literature studying their effectiveness has produced mixed results. On one hand, there is strong evidence across several studies on the effectiveness of LTVs and DTIs for moderating credit and asset price growth, particularly in housing markets, in emerging and advanced economies and across exchange rate regimes (see, for example, Cerutti et al. (2015), Claessens and Kose (2013), Kuttner and Shim (2016)). On the other hand, evidence on the effectiveness of financial institutions-based instruments is less conclusive. For instance, Cerutti et al. (2015) find that, taken as a group, these policies reduce credit growth in emerging economies but in advanced economies the effect is not statistically significant. Similarly, a variety of studies looking at specific instruments using different methodologies and/or sample periods often find conflicting results. Moreover, they generally find that while in some cases the financial institution-targeted instruments are effective in expansions, they are much less helpful at dampening credit contractions in bad times. In particular, Dell’Ariccia et al. (2012) found that macroprudential instruments reduce both the overall frequency of credit booms and the frequency with which booms end up in crises, and Kuttner and Shim found that hiking housing taxes reduces the growth of house prices but a tax cut does not have a statistically significant effect. These results are interesting because, as we show later in the paper, the normative analysis of Sudden Stops models yields the result that macroprudential policy reduces the frequency and magnitude of financial crises, and their optimal use often calls for no intervention during crises and in early
stages of credit expansions (when the one-step ahead probability of a Sudden Stop is zero).

One of the most widely-studied financial institution-targeted macroprudential policy instruments are capital controls. This is natural because, as we documented earlier, financial crises characterized by large reversals in international credit flows (i.e. Sudden Stops) have affected a large number of countries since the 1980s. The large current account deficits that precede Sudden Stops indicate that a rapid expansion in foreign capital inflows plays an important role in these crises, and hence the interest from policymakers in using capital controls as a macroprudential policy instrument. Unfortunately, Galati and Moessner (2018) found that the empirical evidence on the effectiveness of macroprudential capital controls is also inconclusive and mixed.4 Ostry et al. (2012) find that they affect the composition of capital flows, while Beirne and Friedrich (2014) find only a limited effect on aggregate capital inflows. Forbes and Warnock (2012) find that macroprudential capital controls weaken some indicators of financial fragility, such as leverage, credit growth and gross inflows in the banking sector, but they do not have a significant effect on net capital inflows. These empirical findings are interesting in light of the findings of theoretical work showing that, in models of pecuniary externalities in the valuation of collateral, optimal macroprudential policy does not require discriminating foreign from domestic credit sources, and when it does it may even call for capital controls that make foreign inflows cheaper than domestic credit (see Mendoza and Rojas (2019)) or that are lower in expansions than in recessions (see Schmitt-Grohé and Uribe (2017)).

Looking forward, the countercyclical capital buffer (CCyB) introduced with the Basle III Global Regulatory Framework will also become a widely-studied macroprudential policy tool. The CCyB calls for an “add-on” capital buffer above standard regulation during large credit expansions, as defined by a “common reference guide” based on the deviation from the Hodrick-Prescott trend in aggregate credit to the private sector as a share of GDP.5 The CCyB is activated when this deviation from trend rises above a given activation threshold, then it is tightened progressively if credit continues to expand up to a maximum, and then it is removed progressively when the deviation from trend reverts to the activation threshold. The activation threshold and the maximum are left to the discretion of country regulators, but 2 and 10 percentage points are the Basle III recommendation. Similarly, the values that the add-on buffer can take and the rate at which they are tightened in the upswing and weakened in the downswing are left to the discretion of country authorities. Implementation by BIS member countries was set to be gradual starting in 2016 and completed by the end of 2018. As of February, 2018, nine countries had active CCyBs or announced the forthcoming activation of their CCyBs, including the Czech Republic, Denmark, Hong Kong, Iceland, Lithuania, Norway, the Slovak Republic, Sweden and

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4Not all forms of capital controls are macroprudential. As Galati and Moessner (2018) explain, capital controls policies are regarded as macroprudential only if their governance rules state the aim of reducing system-wide vulnerabilities in the financial system.

5See https://www.bis.org/bcbs/ccyb/ for full details.
the United Kingdom. As of now, we lack enough data to conduct a systematic evaluation of the CCyB’s effectiveness. Quantitative experiments of macroprudential policy rules linked to the credit-GDP ratio suggest, however, that while these rules contribute to dampen credit cycles they are much less effective than optimal macroprudential policy rules at reducing the magnitude and frequency of financial crises (see Bianchi and Mendoza (2018) and Hernandez and Mendoza (2017)).

In summary, empirical work on the effectiveness of macroprudential policy to date finds significantly stronger evidence in favor of borrower-targeted instruments (e.g. LTVs DTIs, housing taxes) than any other instrument. It finds also that macroprudential instruments are effective at hampering credit growth in booms but not so at containing credit collapse in bad times, and that macroprudential capital controls alter the composition of capital inflows and weaken some measures of banking fragility, but do not seem to affect aggregate and net capital inflows. In addition, one important caveat of the empirical literature from the perspective of theoretical work is that while it identifies the effects of financial regulation instruments used with a systemic focus (i.e. across the financial system as a whole), it does not identify as accurately the extent to which the policy instruments are used in a prudential form (i.e. in “ex ante” fashion relating the current value of the instrument to potential future outcomes). In the normative theory discussed later in the paper, macroprudential policy instruments are explicitly defined as instruments that make credit more expensive in “good times” so as to induce agents to internalize the social cost of future financial crises associated with increased borrowing in those good times.

3 Fisherian Theory of Sudden Stops

The Fisherian theory of Sudden Stops provides a quantitative framework for explaining the stylized facts documented in the event analysis of the previous Section and for designing and assessing macroprudential policies. This Section of the paper provides a brief, generic description of Fisherian models of Sudden Stops. The goal is to describe the essential elements that drive the transmission mechanism of financial crises in these models and their normative implications. The next Section will discuss in detail the specifics and quantitative implementation of a particular Fisherian model that aims to contribute to the literature by studying the tradeoffs of macroprudential policy in a Sudden Stops model with capital accumulation.

The defining element of Fisherian models of Sudden Stops is an occasionally-binding collateral constraint that limits borrowing capacity not to exceed a measure of pledgeable collateral that depends on market prices.\(^6\) The constraint is occasionally-binding because whether it binds or not is a state-contingent equilibrium outcome that depends on the optimal plans formu-

\(^6\)For a textbook presentation of these models see Chapter 12 of Uribe and Schmitt-Grohé (2017).
lated by the models’ agents, the realizations of exogenous shocks and the values of aggregate variables, particularly equilibrium prices. Pledgeable collateral is defined as a (potentially time-varying) fraction $\kappa_t$ of the market value of an agent’s income or assets. Most models assume a representative-agent small open economy in which debt takes the form of a negative position in an internationally-traded one-period bond $b_{t+1}$ sold at a world-determined (possibly stochastic) price $q_t^{b_t}$ (i.e. the gross real interest rate is $R_t \equiv 1/q_t^{b_t}$).\(^7\)

Most of the literature studies models in which the constraints are imposed directly on the optimization problems of agents, rather than modeled as an endogenous outcome of an explicit contractual relationship. This is common practice in a branch of the macro literature on financial frictions, as in the seminal studies by Kiyotaki and Moore (1997) and Aiyagari and Gertler (1999). There are, however, studies of Fisherian models in which the collateral constraint is derived from a contractual setup, typically as a result of a limited enforcement or costly state verification problem (e.g. Bianchi and Mendoza (2018), Mendoza and Quadrini (2010)). Moreover, both the financial amplification mechanism and the pecuniary externality argument underpinning the normative implications of these models apply to a wider class of financial frictions models in which market prices determine borrowing capacity. For instance, the classic Bernanke-Gertler financial accelerator model, in which an external financing premium as a function of net worth emerges endogenously as an outcome of an optimal contract, features a similar pecuniary externality, because net worth is valued at market prices and borrowers do not internalize the effect of their actions on those prices. Models like those studied by Lorenzoni (2008), Stein (2012) and Brunnermeier and Sannikov (2014) also feature credit frictions and related pecuniary externalities affecting the efficiency of competitive equilibria.

The majority of the existing research focuses on two types of constraints:

1. **Loan-to-value (LTV) or stock constraints**: Pledgeable collateral is an asset $k_t$ or $k_{t+1}$ (e.g. land, equity, housing, etc) that carries a market price $q_t$ and the borrowing constraints take this form:

$$q_t^{b_t} b_{t+1} \geq \kappa_t q_t k_{t+j}, \quad j = 0, 1.$$  \(1\)

These constraints are similar to those proposed by Kiyotaki and Moore (1997) and Aiyagari and Gertler (1999). The timing of the assets posted as collateral in the right-hand-side of the constraint depends on assumptions about the nature of credit contracts and their enforcement. For instance, Mendoza and Smith (2006) use $k_{t+1}$ based on the Aiyagari-Gertler setup that models margin loans in which the assets that an agent buys at date $t$ are used as collateral. Bianchi and Mendoza (2018) use $k_t$ based on the incentive-compatibility constraint of an optimal debt contract with limited enforcement that is set at the beginning of each date $t$ before the asset market opens (see Appendix A5 of Bianchi and Mendoza

\(^7\)See Mendoza and Quadrini (2010) for an example of a Sudden Stops model with heterogeneous agents.
(2018) for details). Constraints with either timing specification have been used extensively, including in models with capital accumulation and working capital financing added to the credit constraint (Mendoza (2010)), models of learning and financial innovation (Boz and Mendoza (2014)), studies on the effects of financial policies and financial integration (Durdu and Mendoza (2006), Mendoza and Smith (2014), Jeanne and Korinek (2018), Reyes-Heroles and Tenorio (2017)), and models of self-fulfilling Sudden Stops (Schmitt-Grohé and Uribe (2017)).

2. Debt-to-income (DTI) or flow constraints: Debt cannot exceed a given fraction of income. These constraints are typical in household financing (e.g. DTI’s on home mortgages required by government-sponsored enterprises like Fannie Mae or Freddie Mae or those used for credit scoring for auto and credit card loans). In the Sudden Stops literature, DTI constraints have been mainly used to study the feedback between real-exchange-rate movements and borrowing capacity, based on the following credit constraint proposed by Mendoza (2002):

\[ q_t b_{t+1} \geq \kappa_t \left[ y_t^T + p_t^N y_t^N \right]. \]  

Debt cannot exceed a fraction of total income, which includes income from the tradables sector \( T \) and the nontradables sector \( N \), denoted \( y^T \) and \( y^N \) respectively. Since the debt is an internationally-tradable bond (i.e. all net creditors of the economy are foreign), the relevant measure of income that creditors focus on is in units of tradables \( p_t^N y_t^N \), where \( p_t^N \) is the relative price of \( N \) goods in units of \( T \) goods (and since PPP is assumed to hold for \( T \) goods, this price also determines the real exchange rate). The literature has examined several variations of this setup, including models of the optimal precautionary accumulation of foreign reserves (Durdu et al. (2009)), models of macroprudential policy in which \( y_t^T \) and/or \( y_t^N \) are stochastic and/or endogenous (see Bianchi (2011), Benigno et al. (2013), Hernandez and Mendoza (2017), Schmitt-Grohé and Uribe (2018)), models with noisy news and regime-switching shocks (Bianchi et al. (2016), models with banks borrowing from abroad in \( T \) units to fund domestic loans in units of the domestic CPI (Mendoza and Rojas (2019)), models with regulated and unregulated borrowers (Bengui and Bianchi (2018)), and models about exchange-rate policy in the presence of nominal rigidities and credit frictions Ottonello (2015)).

3.1 Fisherian debt-deflation amplification mechanism

The models this paper focused on are labeled “Fisherian” because, when the collateral constraint binds, they display dynamics driven by the classic debt-deflation mechanism first proposed in work of Fisher (1933): Agents fire sale goods and/or assets in order to meet their obligations,
but as they do so the prices of those goods or assets fall, and since the Fisherian constraint links borrowing capacity to prices, the decline in prices tightens further the constraint forcing further fire sales. This feedback mechanism amplifies the effects of shocks relative to states of nature in which the credit constraint does not bind. These models differ from other models with credit constraints because while a binding credit constraint always implies a negative effect on aggregate demand, due to the direct (or balance sheet) effect of the constraint on demand for consumer goods, only models in which market prices enter in the credit constraint feature Fisherian amplification. Moreover, some Fisherian models also include adverse supply-side effects due to declining values of marginal products of inputs in response to price deflation (e.g. Durdu et al. (2009)), binding credit limits for working capital (e.g. Bianchi and Mendoza (2018)), and falling investment in response to collapsing equity prices (e.g. Mendoza (2010)). In addition, some models include international spillovers driven by international asset trading and short-selling constraints or mark-to-market capital requirements (e.g. Mendoza and Smith (2006), Mendoza and Quadrini (2010)).

A tractable analytic characterization of the Fisherian amplification mechanism is difficult to obtain in general, because of the lack of closed-form solutions typical of dynamic general equilibrium models and also because of the non-linearities implied by the occasionally-binding, state-contingent credit constraint. One exception is the perfect-foresight analysis of the DTI model with endowment incomes examined in Mendoza (2005) and extended to examine equilibrium multiplicity in Schmitt-Grohé and Uribe (2018) and intermediation of capital inflows into domestic debt by Mendoza and Rojas (2019). In these models, a representative agent consumes a CES composite commodity \( (c(c^T, c^N)) \) that combines tradables and nontradables consumption, and chooses consumption and optimal bond holdings so as to maximize a standard intertemporal utility function subject to the DTI collateral constraint and a budget constraint with a time-varying income of tradables and a fixed endowment of nontradables. In Mendoza (2005) formulation, the competitive equilibrium of the economy is given by sequences of allocations \( \{c_t^T, c_t^N, b_{t+1}\}_{t\geq 0} \), and prices \( \{p_t^N\}_{t\geq 0} \) that satisfy the following conditions:

\[
p_t^N = \left( \frac{1 - \omega}{\omega} \right) \left( \frac{c_t^T}{c_t^N} \right)^{\eta+1}
\]

\[
u_T(t) = \beta R \mathbb{E}_t[u_T(t+1)] + \mu_t \tag{4}
\]

\[
q^b b_{t+1} \geq -\kappa \left[ y_t^T + p_t^N \bar{y}^N \right], \quad \text{with equality if } \mu_t > 0,
\]

\[
c_t^N = \bar{y}^N \tag{5}
\]

\[
c_t^T = y_t^T - q^b b_{t+1} + b_t, \tag{6}
\]

where \( \mu_t \) is the non-negative Lagrange multiplier on the credit constraint, \( u_T(t) \equiv u'(c_t)\partial c_T/\partial c_t^T \),
and $\omega$ and $\eta$ are parameters of the CES aggregator $c(.)$ such that $\omega$ is the tradables share parameter and $1/(1 + \eta)$ is the elasticity of substitution between tradables and nontradables consumption.\footnote{The CES aggregator has the form $c_t = \left[ \omega (c_T^t)^{-\eta} + (1 - \omega) (c_N^t)^{-\eta} \right]^{-\frac{1}{\eta}}, \eta > -1, \omega \in (0, 1).$}

The above equilibrium conditions have two immediate implications that are important for the analysis that follows: First, (3) and (6) imply that at equilibrium the price of nontradables is an increasing, convex function of the allocation of tradables consumption, so that the equilibrium price can be denoted $p^N(C_t^T)$. Second, if the credit constraint binds, $C_t^T$ is in fact independent of the value of $\mu_t$ and is given by the solutions to the following nonlinear equation in $C_t^T$ formed by conditions (3), (5) holding with equality, (6) and (7):

$$C_t^T = (1 + \kappa) y_t^T + \kappa p^N(C_t^T) y^N + b_t. \quad (8)$$

Assuming the standard no-Ponzi-game condition, the intertemporal resource constraint for tradables is:

$$\sum_{t=0}^{\infty} R^{-t} c_t^T = \sum_{t=0}^{\infty} R^{t} y_t^T + b_0, \quad (9)$$

where $\sum_{t=0}^{\infty} R^{-t} y_t^T \equiv W_0$ is the tradables non-financial wealth of the economy. This condition, together with (3),(4), (5) and (6) characterize fully this model’s equilibrium. Following Mendoza (2005), we simplify the analysis by assuming that $\beta R^* = 1$, $b_0 < 0$ (i.e. the economy starts with some debt), initial tradables income is lower than in the future so that agents would want to set $b_1 < 0$, and we study wealth-neutral shocks such that $y_0^T$ falls but keeping $W_0$ constant. This induces agents to borrow more.\footnote{A wealth-neutral income shock at $t=0$ is defined by income levels $(y_0^T, y_1^T)$ such that $y_1^T - \bar{y}^T = R(\bar{y}^T - y_0^T)$ and $\bar{y}_t^T = \bar{y}^T$ for $t \geq 2$. Hence, wealth remains constant at $W_0 = (1 - \beta) \bar{y}^T$.} For a sufficiently large cut in $y_0^T$, the collateral constraint binds, but for smaller shocks it does not.

If the collateral constraint does not bind, and since $\beta R^* = 1$, we get the standard result that tradables consumption is a constant fraction of total wealth:

$$\bar{c}^T = (1 - \beta) (W_0 + b_0). \quad (10)$$

Moreover, since consumption of tradables and the nontradables endowment (and consumption) are constant, the equilibrium price of nontradables is also constant.

If the reduction in $y_0^T$ is sufficiently large to make the collateral constraint bind at $t = 0$, a Sudden Stop occurs. Condition (7) implies that $c_0^T$ falls, because access to debt for tradables consumption is insufficient to sustain $\bar{c}^T$. Then it follows from condition (3) that $p_0^N$ falls to clear the nontradables market. This triggers the Fisherian amplification mechanism: the endogenous
price drop generates a further tightening of the collateral constraint, because it reduces the value of collateral provided by the nontradables endowment in condition (5). Formally, the date-0 value of $c^T_0$ is now determined by condition (8).

Figure 3.1 illustrates the determination of unique constrained and unconstrained equilibria in a manner analogous to Figure 2 in Mendoza (2005). The multiple equilibria case is discussed later in this Section. The PP curve is the $p^N(C^T_t)$ function, which as we noted above is increasing and convex in $C^T$. The BB$^{SS}$ and BB$^{NB}$ curves plot values of $p^N$ that correspond to values of $c^T$ such that the collateral constraint holds with equality and the tradables resource constraint is satisfied (i.e. equation (8) solved for $p^N_0$ as a function of $c^T_0$) for different values of $y^T_0$. $^{10}$ BB$^{NB}$ uses the threshold value $\hat{y}^T_0$ such that the constraint is marginally binding, in the sense that a date-0 wealth-neutral shock of this magnitude sustains exactly the amount of debt that agents would want to have. $^{11}$ BB$^{NB}$ uses a value of $y^T_0$ lower than $\hat{y}^T_0$, so that the binding constraint does alter allocations and prices.

Figure 3: Sudden Stops under Perfect Foresight: Unique Equilibrium

The competitive equilibrium when the constraint is not binding (binding) is determined at points NB and SS respectively, which are the intersections of the PP curve with the BB$^{NB}$ and BB$^{SS}$ curves respectively. For any $y^T_0 \geq \hat{y}^T_0$ the constraint does not bind and the equilibrium remains at point NB with consumption at $\bar{c}^T$ and the nontradables price at $\bar{p}^N$. Income shocks

\footnote{Mendoza (2005) showed that the BB curves are increasing, linear functions of $c^T_0$ with an horizontal intercept given by $I^{SS} \equiv (1 + \kappa)y^T_0 + b_0$ and a slope of $m^{SS} \equiv 1/(\kappa\bar{p}^N)$.}

\footnote{$\hat{y}^T_0 = \frac{c^T_0 - b_0 - \kappa\bar{p}^N\bar{y}^N}{1+\kappa}$, where $c^T$ and $p^N$ are the unconstrained equilibrium outcomes.}
such that $y_0^T < y_0^T$ shift the BB curve to the left, triggering the credit constraint and causing a Sudden Stop. The credit constraint forces agents to reduce consumption to the value consistent with point A, which is lower than $\bar{c}^T$, but at that consumption level clearing the market of non-tradables would make the price fall to the one consistent with point B. But at this lower price the credit constraint tightens and forces consumption to fall to the value consistent with point C, and market-clearing then requires the price to fall to point D. This Fisherian debt-deflation feedback loop continues until the Sudden Stop equilibrium is reached at point SS. This point yields the consumption and price values that solve equation (8). Hence, this Figure highlights how the Fisherian mechanism amplifies the effects of income shocks causing a sharp drop in consumption and the price of nontradables (i.e. the real exchange rate), and implicit in the consumption reversal is a sharp reversal of the current account, implied by the sudden increase in the net foreign asset position forced by the binding credit constraint at the sharply lower nontradables price.

How do results change if instead of the Fisherian credit constraint we consider a credit constraint independent of market prices? For example, debt could be set not to exceed a constant value, or set to keep a debt-to-income cap in which nontradables are valued at a notional “book value.” In this case, the BB curves in Figure 3.1 become vertical lines at the level of tradables consumption that the fixed credit constraint supports. The Figure determines only the equilibrium price (at the intersection of the vertical BB curves with the PP curve), because consumption is exogenously determined by the credit constraint. The constraint still forces consumption to fall relative to the unconstrained outcome, but there is no Fisherian financial amplification mechanism, as the lower equilibrium price that results does not cause an endogenous tightening of the credit limit.

It is worth noting also that there is a second intersection of the BB and PP curves in Figure 3.1, which would be visible if we extended its domain far enough. This second intersection, however, is not an equilibrium, because $c_0^T$ would be higher than $\bar{c}^T$, and given the specification of the wealth-neutral shock the intertemporal resource constraint would imply $c_1^T < c_0^T$, which would imply a negative Lagrange multiplier ($\mu_0 = u'(c_0^T) - u'(c_1^T) < 0$). Moreover, the unconstrained equilibrium cannot co-exist with the unique Sudden Stops equilibria, because as Figure 3.1 shows, the value of $p_0^N$ at which the nontradables market would clear if tradables consumption is $\bar{c}^T$ is too low for the resource constraint to be satisfied with the credit constraint binding.

While the constrained and unconstrained equilibria illustrated in Figure 1 are unique, it is possible to obtain multiple equilibria in both DTI and LTV Sudden Stops models, as Schmitt-Grohé and Uribe (2017) and Schmitt-Grohé and Uribe (2018) showed. This opens up the possibility of Sudden Stops due to self-fulfilling expectations or “sunspots.” In the specific DTI example we are analyzing, Mendoza (2005) noted that a sufficient condition for uniqueness is that the BB curve be flatter than the PP curve around the unconstrained equilibrium (point NB). Formally,
the sufficiency condition for uniqueness can be expressed as the following upper-bound condition on $\kappa$: \(^{12}\)

$$\kappa \leq \hat{\kappa} \equiv \frac{\bar{c}^T}{(1 + \eta)\bar{p}^N\bar{y}^N}$$  \hspace{1cm} (11)

Note, however, that the condition involves not only the parameter $\kappa$ but also the preference parameters that determine $\bar{c}^T$ and $\bar{p}^N$, which include $\beta$, $\omega$ and $\eta$. Given these parameters, if $\kappa$ satisfies this condition, the constrained and unconstrained equilibria are unique. Intuitively, the condition states that the cap on debt relative to income cannot exceed the product of the elasticity of substitution times the ratio tradables-to-nontradables expenditures.

Failure of the above condition is necessary but not sufficient for multiplicity. Multiplicity requires also that $y_0^T$ be in a particular interval. To see this, Figure 3.1 shows the equilibrium determination assuming that the sufficiency condition for uniqueness fails. For simplicity, we also assume that the preference parameters are given and hence the condition is violated because $\kappa$ is “too large.” As the Figure shows, at $y_0^T = \hat{y}_0^T$ we now have two equilibria. One is the same unconstrained equilibrium as in Figure 1 (point NB), but now point A is also an equilibrium, and is one in which the constraint binds resulting in sharply lower consumption and prices. Here, Sudden Stops are the result of self-fulfilling expectations. For $y_0^T < \hat{y}_0^T$ the BB curve would shift to the left and there is a unique Sudden Stop equilibrium. But if BB shifts to the right slightly, namely if income rises a little above $\hat{y}_0^T$, there are now three equilibria: Two Sudden Stop equilibria at the two points were the BB curve intersects with the PP curve (points B and C), as well as the unconstrained equilibrium at NB (because with higher income than the threshold at which the constraint binds, the unconstrained outcome is also an equilibrium). BB keeps shifting rightward as income rises more and we keep finding three equilibria, but notice that when we reach the income level $\bar{y}_0^T$ such that BB is tangent to SS, we have only two equilibria (the tangency point D and the unconstrained equilibrium NB). For $y_0^T > \bar{y}_0^T$ multiplicity disappears and the unconstrained equilibrium is the unique equilibrium. Hence, multiplicity requires $\hat{y}_0^T \leq y_0^T \leq \bar{y}_0^T$.

The above analysis of multiplicity has an equivalent formulation consistent with the work of Schmitt-Grohé and Uribe (2018). They derived the same necessary condition on $\kappa$ required for multiplicity, but focused on how multiplicity emerges if initial bond holdings fall within an interval of relatively high values (i.e. multiplicity requires relative low initial debt), instead of focusing on income shocks. The value of $b_0$ (for $b_0 < 0$) must be high enough so that at the unconstrained equilibrium condition (11) holds, but not so high that the credit constraint does not bind (since higher $b_0$ implies also higher $b_1$ at the unconstrained equilibrium). Intuitively, the consistency of the two treatments follows from noticing that parametric differences in $b_0$ keeping $y_0^T$ constant can be alternatively represented as parametric differences in $y_0^T$ keeping $b_0$ constant.

---

\(^{12}\)This condition follows from noting that the slope of the BB curve is $1/(\kappa \bar{y}^N)$ and, since the PP curve is given by condition (3), its slope can be expressed as $\frac{(1 + \eta)\bar{p}^N}{\bar{c}^T}$, which at the unconstrained equilibrium equals $\frac{(1 + \eta)\bar{p}^N}{\bar{c}^T}$. 

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by capitalizing initial income differences into changes in bond holdings. Hence, an interval of relatively high $b_0$ (i.e. low debt) that sustains multiplicity is equivalent to an interval of relatively high income values.

The quantitative relevance of equilibrium multiplicity in Fisherian Sudden Stops models is an open question. It is possible to construct reasonably calibrated models where condition (11) can hold or fail (see, for example, Hernandez and Mendoza (2017) v. Schmitt-Grohé and Uribe (2018)). This depends on assumptions about the relevant values of $\kappa$ and parameters like the intertemporal elasticity of substitution in consumption, estimates of which are very noisy, as well as the size of debt ratios and the data frequency at which they are being targeted. Moreover, condition (11) is only a sufficiency condition. If it is violated, the likelihood of multiplicity then depends on the probability of observing income in the relevant interval of relatively high but not too high values (values above $\tilde{y}_T^0$ but below $\check{y}_T^0$). But this implies that Sudden Stops should be associated with relatively high income, which is not in line with the stylized facts documented earlier. Finally, both condition (11) and the multiplicity income interval are model-dependent. For instance, if banks intermediate capital inflows in units of $T$ goods to fund domestic loans in units of the domestic CPI, Mendoza and Rojas (2019) show that multiplicity requires $\kappa$ to be higher than in condition (11) and the multiplicity income interval narrows sharply. Fortunately, the literature has found that Fisherian amplification in models with unique equilibrium is quantitatively large, and hence the Fisherian framework can provide a quantitatively plausible theory of Sudden Stops even without multiplicity (see Mendoza (2010)). Moreover, Sudden Stops result from financial amplification of shocks of standard magnitudes included in the known set of shock realizations, so this theory of Sudden Stops does not rely on large, unexpected shocks.
3.2 Normative implications of Fisherian models

The remainder of this Section summarizes the market-failure argument that justifies macroprudential policy when Fisherian collateral constraints are present. The key to the argument is that these constraints produce a pecuniary externality because collateral is valued at market prices, and thus agents do not internalize the effects of their actions on collateral values. Pecuniary externalities generally do not distort allocations, but in models of this class they do because borrowing capacity can be altered by market prices. Of particular interest for macroprudential policy, given that it is a preemptive or ex-ante policy, is a state of nature in which the collateral constraint does not bind at date t but can bind with some probability at t+1. In this case, agents make borrowing decisions by equating the marginal cost and benefit of the debt they take on at date t, but in the marginal cost they do not internalize the response of collateral values at t+1 if the credit constraint becomes binding then. A social planner or financial regulator would factor this in, and hence private and social marginal costs of borrowing differ.

To formalize the above argument in a generic form that applies to both DTI and LTV models, consider that the equilibrium relative prices determining the market value of collateral correspond to marginal rates of substitution in consumption and/or marginal rates of technical substitution in production. Because these are general equilibrium outcomes, individual borrowers do not internalize the effects of their own borrowing decisions on the aggregate variables that pin down the value of collateral via these equilibrium conditions, but a social planner does, because the planner internalizes that prices are determined jointly with allocations at equilibrium. In DTI models, the relevant marginal rate of substitution for the value of collateral is that between $c_T$ and $c_N$, and in the widely-used case in which income from N goods is an exogenous endowment, we saw before that we can express the price of non-tradables as a function $p_N^T(C_T^t)$ of aggregate tradables consumption $C_T^t$. In the LTV models, the relevant marginal rate of substitution is the intertemporal one (i.e. the stochastic discount factor), so the equilibrium value of collateral can be expressed as $q_t(C_t, C_{t+1})$. Notice a key difference between the two cases: In the DTI model the price depends only on date-t aggregate variables, whereas in the LTV model it depends on date-t and date-t+1 variables. This difference has crucial implications for time-consistency of optimal macroprudential policy that we discuss later in this Section.

A property common to all Fisherian models of Sudden Stops is that, in a decentralized equilibrium without policy intervention, the households’ Euler equation for bond holdings is:

$$u'(t) = \beta R_t E_t [u'(t + 1)] + \mu_t$$  \hspace{1cm} (12)

Here, $u'(t)$ denotes the marginal utility of individual consumption, which as we saw before in the DTI models corresponds to the marginal utility of $T$ goods and in LTV models corresponds to the standard marginal utility of consumption. The non-negative Lagrange multiplier $\mu_t$ is again
the multiplier on the collateral constraint. Intuitively, when the constraint binds the marginal cost of borrowing rises because it is as if the effective interest rate rose above \( R_t \) by an amount that depends on the shadow value of the constraint.

Studies of optimal macroprudential policy yield properties that can differ widely depending on the particular structure of models, especially on whether collateral values depend on contemporaneous and/or future aggregate variables and whether there is scope for the social planner to alter allocations not just before the constraint becomes binding but also when it is already binding (i.e. whether they support ex-post intervention during a crisis). To characterize the macroprudential pecuniary externality, however, we abstract from both of these issues by focusing only on a state of nature in which the collateral constraint does not bind at date \( t \) (i.e. \( \mu_t = 0 \)). In this case, the planner’s Euler equation for bonds typically takes this form:

\[
u'(t) = \beta R_t E_t \left[ u'(t + 1) + \mu_{t+1}^* \kappa_{t+1} \psi_{t+1}^i \right], \quad i = DTI, LTV \tag{13}
\]

where \( \mu_{t+1}^* \) is the planner’s multiplier on the borrowing constraint and \( \psi_{t+1}^i \) measures the change in the market value of collateral at \( t + 1 \) as the bond holdings chosen by the planner at date \( t \) (i.e. \( B_{t+1} \)) change, which for the DTI and LTV models is given by these expressions:

\[
\psi_{t+1}^{DTI} = y_{t+1}^N (\partial p_{t+1}^N / \partial C_{t+1}^T) (\partial C_{t+1}^T / \partial B_{t+1}),
\]

\[
\psi_{t+1}^{LTV} = K_{t+1+j} (\partial q_{t+1} / \partial C_{t+1}) (\partial C_{t+1} / \partial B_{t+1}), \quad j = 0, 1
\]

The term \( \mu_{t+1}^* \kappa_{t+1} \psi_{t+1}^i \) represents an externality because it is the amount by which the marginal cost of borrowing faced by the social planner v. the one faced by private agents in the absence of regulation differ in states in which \( \mu_t = 0 \). It is only relevant if the constraint is expected to bind in at least some states of nature at \( t + 1 \). It is a pecuniary externality because it is caused by the price effects that are the aggregate result of individual choices and, as such, are not internalized by private agents. The externality is said to induce overborrowing (underborrowing) if \( \psi_{t+1}^i > 0 \) (\( \psi_{t+1}^i < 0 \)), because it implies that private agents undervalue (overvalue) the marginal cost of borrowing relative to what is socially optimal. In turn, the sign of \( \psi_{t+1}^i \) is determined by the sign of the derivative of the collateral price with respect to aggregate consumption: \( \partial p_{t+1}^N / \partial C_{t+1}^T \) and \( \partial q_{t+1} / \partial C_{t+1} \) for the DTI and LTV models respectively. \(^{13}\) These derivatives are an equilibrium object, so their sign should not be assumed, as in some of the existing literature (see Appendix K of Bianchi and Mendoza (2018) for a discussion of this issue).

In the simpler versions of DTI and LTV models, it is relatively straightforward to establish

\(^{13}\)Notice that the resource and budget constraints typically imply that \( \partial C_{t+1}^T / \partial B_{t+1} = 1 \), and standard non-negativity conditions apply to income and physical assets.
that indeed the price derivatives are positive, because of the concavity of utility functions. The equilibrium pricing function derivatives in these models are:

\[
\frac{\partial p_N^{t+1}}{\partial C^{t+1}} = -\frac{p_N^{t+1} u''(t+1)}{u'(t+1)} > 0, \quad \frac{\partial q_{t+1}}{\partial C_{t+1}} = -\frac{q_{t+1} u''(t+1)}{u'(t+1)} > 0.
\]

As before, the derivatives of \( u(.) \) for the DTI model are with respect to tradables consumption, with composite consumption given by a CES aggregator, and those for the LTV model are with respect to the standard consumption measure.

Since the pricing derivatives are positive, the pecuniary externality is positive and agents overborrow at date \( t \) when the constraint does not bind, because they fail to internalize that additional debt taken at \( t \) leads to a larger collapse in collateral values if the credit constraints bind at \( t + 1 \). The allocations of the social planner can therefore be decentralized by taxing debt at date \( t \) by the correct amount, rebating the revenue generated by this tax as a lump-sum transfer. The optimal debt tax is the one that leads the private marginal cost of borrowing in a decentralized equilibrium with regulation (i.e. with debt taxes) to equalize the social marginal cost of borrowing. This requires an optimal macroprudential debt tax given by:

\[
\tau_t = \frac{E_t \left[ \mu_{t+1}^{*} \kappa_{t+1}^{*} \psi_{t+1}^{*} \right]}{E_t \left[ u'(t+1) \right]} \quad i = DTI, LTV
\]

If the constraint is expected to bind in at least some states of nature at \( t + 1 \), this tax is strictly positive, because it inherits the sign of the pecuniary externality, which is given by the sign of the pricing derivatives determined above. On the other hand, if the constraint is not expected to bind in at least some states of nature at \( t + 1 \), the optimal macroprudential debt tax is zero. Moreover, everything else the same, the tax is higher when: (a) the constraint is more likely to bind at \( t + 1 \) (i.e. the larger is the set of states at \( t + 1 \) for which \( \mu_{t+1}^{*} > 0 \)), (b) prices fall more in response to the collapse in demand at \( t + 1 \) (i.e. the larger are the price derivatives), (c) the larger the value of pledgeable collateral at \( t + 1 \).

The above argument uses debt taxes to decentralize the optimal macroprudential policy because they are a natural way of doing so given that we are dealing with an externality. As documented in Section 1, however, standard tax instruments are not the most widely used macroprudential policy instrument, compared with rules for banks’ liquidity coverage or capital buffers with countercyclical elements, and more recently limits on loan-to-value and debt-to-income ratios on borrowers. Still, under some conditions, it is possible to implement the optimal macroprudential policy with these instruments (Bianchi, 2011), echoing results on the equivalence between price and quantity instruments. Below, however, we will argue that the exact equivalence between taxes and LTV do not hold when households pledge their assets as collateral because these instruments yield different effects on asset prices. In line with the properties of the optimal
debt tax, these policies would tighten as the probability of crisis rises, the price responses become steeper, or the value of pledgeable collateral rises, and they would be removed completely if the constraint has zero one-step-ahead conditional probability of becoming binding. An alternative macroprudential instrument studied in Arce et al. (2019) is reserve accumulation.

There are three additional considerations to the above discussion:

1. Depending on model structure, the social planner may have incentives to intervene not just with macroprudential or ex-ante policy when \( \mu_t^* = 0 \) and \( E_t[\mu_{t+1}^* > 0] \), but also with ex-post policy when \( \mu_t^* > 0 \). Benigno et al. (2013) study a DTI model in which tradables and nontradables are produced with labor, and wage income enters in the credit constraint. The planner intervenes ex-post to increase the relative price of nontradables by reallocating labor toward tradables, and ex-ante agents may underborrow if the goods are complements, because higher nontradables consumption implies higher tradable consumption, so that the planner desires higher borrowing ex-ante.\(^{14}\) Hernandez and Mendoza (2017) study a DTI model with sectoral production that uses imported inputs only. There is overborrowing only, but again the planner reallocates inputs from nontradables to tradables production ex-post, because this props up the value of collateral and weakens the constraint. In Bianchi (2016) model with firms facing collateral and dividend constraints, a pecuniary externality operating via wages reduces profits and investment when the dividend constraint binds. The planner uses bailouts to stabilize firms’ net-worth ex-post and taxes debt to hamper overborrowing ex-ante.

2. When collateral values at date \( t \) are determined jointly by date-t and date-t+1 equilibrium outcomes, the planner’s optimal plans become time-inconsistent under commitment. Time-inconsistency is a critical issue for the normative analysis of Sudden Stop models, because it undermines the credibility of macroprudential policy and raises the well-known problems of rules v. discretion examined in the broader literature on optimal policy. This issue was first raised by Bianchi and Mendoza (2018) in the context of an LTV model. Bianchi et al. (2019) conduct a detailed analysis of the value of commitment in the Bianchi-Mendoza setup.

Bianchi and Mendoza (2018) present explicit formulations of a planner’s optimal policy problems in an LTV model with and without commitment. They show that the time-inconsistency of the optimal plans under commitment emerges because the planner internalizes the Euler equation driving asset prices, which makes \( q_t \) increasing in \( c_t \) and decreasing in \( c_{t+1} \). Hence, the planner realizes that an increase in \( c_t \) weakens the credit

\(^{14}\)A drawback of this formulation as a platform for policy analysis is that it does poorly at matching the stylized facts of Sudden Stops. Since agents can borrow more by working harder, the model underestimates the magnitude of Sudden Stops significantly and predicts that tradables output rises during Sudden Stops.
constraint at \( t \) but tightens it at \( t - 1 \). Intuitively, when the LTV constraint binds at \( t \), the planner acting under commitment promises lower future consumption so as to prop up asset prices and borrowing capacity at \( t \), but ex post when \( t + 1 \) arrives it is suboptimal to keep this promise. In line with this result, they found in numerical examples that the planner with commitment supports higher asset prices and higher debt than in the unregulated competitive equilibrium, while the opposite occurs in the case of the planner without commitment. The latter was formulated as a Markov-stationary equilibrium in which the planner at date \( t \) internalizes the pecuniary externality and also takes into account the effects of its debt decisions on the prices and allocations chosen by future planners.

Bianchi and Mendoza also show that debt taxes can be used to decentralize the optimal policy of the planner with or without commitment. The debt tax can be separated into two components. One is macroprudential and again it tackles the pecuniary externality, and the other captures the effects of the incentives to intervene when the credit constraint binds. The macroprudential component is not, however, characterized by the same expressions with and without commitment. They are the same only the first time the constraint binds along a given equilibrium path (i.e. if the collateral constraint has never been binding up to some date \( t \) and is expected to bind with some probability at \( t + 1 \)). Otherwise, in future dates \( t + j \) in which the constraint does not bind but can bind at \( t + j + 1 \) the expressions determining optimal macroprudential debt taxes differ with and without commitment.

3. Quantitatively, optimal macroprudential policy (both under commitment in the absence of time-inconsistency or under discretion when time inconsistency is present) is very effective at reducing the magnitude and frequency of financial crises, but it requires complex, non-linear state-contingent schedules for the management of policy instruments. In the presence of constraints linked to the real exchange rate in an endowment economy model, Bianchi (2011) found that fixed taxes can achieve significant gains. In the presence of collateralized assets, however, Bianchi and Mendoza (2018) found that collection of simple rules, including constant-elasticity rules targeting financial variables akin to a Taylor rule, have turned to be significantly inferior to the optimal policy and can easily lead to welfare losses.

4 Macroprudential Policy Tradeoffs

The finding that optimal macroprudential policy in Fisherian models is very effective but also very complex, while at the same time (sub-optimal) simple rules are less effective and can be
welfare-reducing raises questions as to whether it is a policy that can be used successfully.\footnote{This is in sharp contrast with findings for monetary and fiscal policies. The Taylor rule is generally sub-optimal in the majority of New Keynesian DSGE models used in central banking, but quantitatively it is very effective at smoothing fluctuations and targeting inflation, and it is welfare-improving. Re-arrangements of the tax structure, even of simple constant taxes on factor incomes and consumption, produce large efficiency and welfare gains.} In fact, the quantitative assessments of simple rules conducted to date are likely to underestimate the limitations of these rules, because they have focused mainly on models in which the main tradeoff of sub-optimal credit regulation is hampering consumption smoothing and distorting precautionary savings. Over-regulation is present in the sense that borrowing may be over-taxed in some states in which the optimal policy would have allowed for more debt, and in that precautionary savings incentives are strengthened by the higher effective interest rate. In practice, however, over-regulation is likely to be more costly because it also affects capital accumulation, factor allocations and relative prices. The different performance of intermediary-based MPP instruments v. regulatory loan-to-value ratios also raises questions about the tradeoffs involved in choosing one class of instruments v. the others. In this Section, we shed some light on these issues by proposing a framework that combines elements of both LTV and DTI Fisherian models in a setup with endogenous capital accumulation.

4.1 A Two-Sector Model with Investment

**Investment goods production.** A representative firm produces investment goods \(i_t\) using as input a composite good \(x_t = x(x^T_t, x^N_t)\) formed by a constant-elasticity-of-substitution (CES) aggregator of inputs from the tradables and nontradable sectors \(x^T_t\) and \(x^N_t\):

\[
x_t = \left[ \pi \left( x^T_t \right)^{-\theta} + (1 - \pi) \left( x^N_t \right)^{-\theta} \right]^{-\frac{1}{\theta}}, \theta > -1, \pi \in (0, 1),
\]

where \(1/(1 + \theta)\) is the elasticity of substitution between \(x^T_t\) and \(x^N_t\).

This firm’s optimization problem is solved using standard two-stage budgeting and duality results. In the first stage, the producer minimizes the cost of inputs required to obtain a target amount of \(x\) denoted \(\bar{x}\):

\[
C(p^N_t, \bar{x}) = \min_{x^T_t, x^N_t} \left[ x^T_t + p^N_t x^N_t \right]
\]

s.t.

\[
x(x^T_t, x^N_t) = \bar{x}
\]

As in the DTI models reviewed earlier, tradable goods are the numeraire and \(p^N_t\) denotes the relative price of nontradables in units of tradables.
The first-order conditions of this problem imply that:

$$x_{xN}(x_t^T, x_t^N)/x_{xT}(x_t^T, x_t^N) = p_t^N$$ (18)

Then using the linear homogeneity of $x(x_t^T, x_t^N)$ we obtain a linearly-homogenous price index $P(p_t^N)$ in units of tradables such that the cost function of the producer is $C_t = P(p_t^N)x_t$. $P(p_t^N)$ is equal to the minimum expenditure needed to produce one unit of an investment good for a given $p_t^N$ (i.e, $C(p_t^N, 1) = P(p_t^N) = x_t^T + p_t^N x_t^N$). Given the CES structure of $x_t$, the price index of investment goods is:

$$P_t(p_t^N) = \left[ \pi^{\frac{1}{\sigma}} + (1 - \pi)^{\frac{1}{\sigma}} \left( p_t^N \right)^{\frac{\theta}{1+\theta}} \right]^{\frac{1+\theta}{\theta}}.\] (19)

In the second stage, the firm chooses the optimal $x_t$ so as to maximize profits of selling investment goods at a relative price $q_t$ subject to the investment goods production technology denoted $f(x)$. In order to obtain an equilibrium in which the price of investment goods $q_t$ is determined by the cost price index $P_t$ in “static” fashion (since $P_t$ is determined by the relative price of nontradable goods), we assume a simple linear technology $f(x_t) = z_t^x x_t$, where $z_t^x$ is a shock to the productivity of producing investment goods. The profit maximization problem is:

$$\max_{i_t, x_t} [q_t i_t - P_t x_t] \] (20)

s.t.

$$i_t = z_t^x x_t$$

The first-order condition implies that $q_t = P_t/z_t^x$. Hence, at equilibrium profits are zero and the price of investment goods is equal to the productivity-adjusted price index of the inputs used to produce them (which is a function of $p_t^N$).

**Final goods production** Households own the firms that produce final goods which operate a Neoclassical production technology that uses capital $k$ as the only input. Capital depreciates at a rate $(1 - \delta)$. The production function is $z_t k_t^\alpha$, where $z_t$ is a productivity shock to final goods production. Each period, this technology produces a flow of tradable (nontradable) goods equal to a fraction $a (1 - a)$ of $z_t k_t^\alpha$:

$$y_t^T = a z_t k_t^\alpha \] (21)

$$y_t^N = (1 - a) z_t k_t^\alpha \] (22)
Households. A representative household makes optimal consumption, saving and investment decisions so as to maximize standard expected lifetime utility:

$$\sum_{t=0}^{\infty} \beta^t u(c_t)$$  \hspace{1cm} (23)

where $c_t$ is a CES aggregator of tradables and nontradables consumption $c_t(c_t^T, c_t^N)$, which takes the same form as in Section 2. The household faces the following budget and credit constraints:

$$q_t [k_{t+1} - k_t (1 - \delta)] + p_t^N c_t^N + c_t^T + \frac{b_{t+1}}{R} = b_t + p_t^N (1 - a) z_t k_t^\alpha + a z_t k_t^\alpha$$  \hspace{1cm} (24)

$$\frac{b_{t+1}}{R} \geq -\kappa q_t k_{t+1}$$  \hspace{1cm} (25)

where in the left-hand-side of the budget constraint we used the law of motion of the capital stock to substitute for investment (i.e. purchases of investment goods are given by $q_t = q_t [k_{t+1} - k_t (1 - \delta)]$). As in the case of the producer of investment goods, the household’s total consumption expenditures can be expressed as $P_t c(c_t^T, c_t^N) = c_t^T + p_t^N c_t^N$ where $P_t^c$ is the CES price index for consumption expenditures (which is again a linear homogeneous function of $p_t^N$):

$$P_t^c = \left[ \omega^{\frac{1}{1+\eta}} + (1 - \omega)^{\frac{1}{1+\eta}} (p_t^N)^{\frac{\eta}{1+\eta}} \right]^\frac{1+\eta}{\eta}.$$  \hspace{1cm} (26)

Notice that, since $q$ is both the market price of new capital at which collateral is valued and the market price of investment goods and the latter is a function of $p_t^N$ at equilibrium, the model features financial amplification and externality effects similar to those resulting from having the price of non-tradables in the credit constraint of the standard DTI models. In addition, since the collateral constraint is of the LTV form, the Fisherian mechanism affecting excess returns and capital accumulation will also be at work.

Let $\mu$ denote the Lagrange multiplier on the collateral constraint. The first-order conditions of the household’s problem imply the following optimality conditions:

$$c_t^c(c_t^T, c_t^N)/c_t^c(c_t^T, c_t^N) = p_t^N$$  \hspace{1cm} (27)

$$u_{c,c}(t) - \mu_t = \beta R \mathbb{E}_t \{ u_{c,c}(t + 1) \}$$  \hspace{1cm} (28)

$$q_t [u_{c,c}(t) - \kappa \mu_t] = \beta \mathbb{E}_t \{ u_{c,c}(t + 1) [(1 - \delta) q_{t+1} + \alpha z_{t+1} k_{t+1}^\alpha ((1 - a) p_{t+1}^N + a)] \}$$  \hspace{1cm} (29)

The Euler equations for bonds and capital imply that the excess return on capital simplifies to:

$$\mathbb{E}_t (R_{t+1}^q) - R = \frac{-\text{cov}(\beta u_{c,c}(t + 1), R_{t+1}^q) + (1 - \kappa) \mu_t}{\beta \mathbb{E}_t[u_{c,c}(t + 1)]}$$  \hspace{1cm} (30)
where \( R_{t+1}^q \equiv [q_{t+1} - \delta q_{t+1} + \alpha z_{t+1} k_{t+1}^{\alpha-1}((1-a)p_{t+1}^N + a)]/q_t \) is the rate of return of capital. Hence, in the absence of uncertainty and if the credit constraint does not bind, the model yields the standard property of small open economy models equating the return on domestic capital to the world real interest rate. Uncertainty introduces an equity-risk-premium term, and the credit constraint adds an additional premium given by the fraction \( 1 - \kappa \) of the shadow value of the constraint.

**Market-clearing and unregulated equilibrium.** The market-clearing conditions in the markets for nontradable goods and investment goods are:

\[
x_t^N + c_t^N = y_t^N
\]

\[
k_{t+1} - k_t(1 - \delta) = z_t^x x_t(x_t^T, x_t^N)
\]

Using the above conditions together with \( q_t = P_t/z_t^x \), \( i_t = z_t^x x_t \) and \( P_t x_t = p_t^N y_t^N + x_t^T \), we obtain the following resource constraint for tradables:\(^{16}\)

\[
x_t^T + c_t^T = y_t^T + b_t - \frac{b_{t+1}}{R}
\]

The unregulated competitive equilibrium is defined by sequences of prices \( \{ p_t^N, q_t, P_t \}_{t=0}^\infty \) and allocations \( \{ c_t^T, c_t^N, b_{t+1}, k_{t+1}, x_t^T, x_t^N \}_{t=0}^\infty \) such that: (i) \( \{ c_t^T, c_t^N, b_{t+1}, k_{t+1} \}_{t=0}^\infty \) solve the household’s optimization problem; (ii) \( \{ x_t^T, x_t^N, i_t \}_{t=0}^\infty \) solve the optimization problem of investment goods producers; (iii) the market-clearing conditions for non-tradable goods and investment goods hold (i.e. equations (31) and (32) hold). The equilibrium without credit frictions is defined in the same way, except that there are no \( \mu \) terms in the household’s optimality conditions.

**Equilibrium with regulation.** The government implements simple macroprudential policy rules, which take the form of a tax on debt \( \tau_t \), a subsidy on household total income \( s_t^k \) and a regulatory LTV ratio \( \chi_t \) such that \( 0 \leq \chi_t \leq 1 \) (to reflect the assumption that regulation reduces borrowing capacity below what private markets can provide). When these policies are used, the

\[^{16}\]This constraint is derived as follows:

\[
q_t [k_{t+1} - k_t(1 - \delta)] + p_t^N c_t^N + c_t^T + \frac{b_{t+1}}{R} = b_t + p_t^N (1 - a) k_t^\alpha + ak_t^\alpha
\]

\[
P_t x_t + p_t^N (y_t^N - x_t^N) + c_t^T + \frac{b_{t+1}}{R} = b_t + p_t^N (1 - a) k_t^\alpha + ak_t^\alpha
\]

\[
(p_t^N x_t^N + x_t^T) + p_t^N (y_t^N - x_t^N) + c_t^T + \frac{b_{t+1}}{R} = b_t + p_t^N (1 - a) k_t^\alpha + ak_t^\alpha
\]

In the last expression, the terms with \( p_t^N x_t^N \) cancel out, and we arrive at the result in the text.
household’s budget and borrowing constraints are modified as follows:

\[ q_t k_{t+1} + p_t^N c_t^N + c_t^T + \frac{b_{t+1}}{R(1 + \tau_t)} = b_t + \left[ p_t^N (1 - a) z_t k_t^\alpha + a z_t k_t^\alpha + k_t(1 - \delta) \right] (1 + s_t^k) + T_t \]  

(34)

\[ \frac{b_{t+1}}{R} \geq -\kappa(1 - \chi_t) q_t k_{t+1} \]  

(35)

The government offsets the income effect of the taxes and subsidies with lump-sum transfers \( T_t \) such that

\[ T_t = -\tau_t \frac{b_{t+1}}{R(1 + \tau_t)} - s_t^k \left[ p_t^N (1 - a) z_t k_t^\alpha + a z_t k_t^\alpha + k_t(1 - \delta) \right] \].

The model’s equilibrium conditions are the same as before except for these key changes:

\[ u_{cT}(t) = \beta R(1 + \tau_t) \mathbb{E}_t \{ u_{cT}(t + 1) \} + \mu_t(1 + \tau_t) \]  

(36)

\[ q_t \{ u_{cT}(t) - \mu_t \kappa(1 - \chi_t) \} = \beta \mathbb{E}_t \{ u_{cT}(t + 1) \} \left[ (1 - \delta) q_{t+1} + \alpha z_{t+1} k_{t+1}^{\alpha-1} (1 - a) p_{t+1}^N + a \right] (1 + s_{t+1}^k) \} \]  

(37)

\[ \mathbb{E}_t \left( R_{t+1}^q \left( 1 + s_{t+1}^k \right) \right) = R(1 + \tau_t) \frac{-\text{cov} \left( \beta u_{cT}(t + 1), R_{t+1}^q \left( 1 + s_{t+1}^k \right) \right) + (1 - \kappa(1 - \chi_t)) \mu_t}{\beta \mathbb{E}_t \{ u_{cT}(t + 1) \} } \]  

(38)

Consider first the debt tax without LTV regulation and without subsidies. Condition (36) implies that the debt tax increases the effective cost of borrowing and thus reduces incentives to borrow. We assume that when the constraint binds \( \tau_t = 0 \), in line with the properties of the optimal debt taxes examined earlier. Hence, the debt tax aims to hamper incentives to borrow in non-crisis times. The debt tax, however, distorts investment decisions by requiring a higher marginal return on capital as eq. (38) shows, which reduces investment. In fact, the debt tax has an effect analogous to that of a capital income tax. This is clearer assuming perfect-foresight and no credit frictions: The debt tax rises the opportunity cost of accumulating capital and is equivalent to taxing capital returns at a rate equal to \( \tau_t/(1 + \tau_t) \). This is the standard efficiency distortion of capital income taxation. Under uncertainty, the debt tax has an additional effect operating in the same direction, because to the extent that it hampers consumption smoothing it makes the covariance between marginal utility and equity returns “more negative,” which also requires higher capital returns. Moreover, higher returns imply heavier discounting of dividends, and thus lower asset prices. Hence, the debt tax has potentially costly tradeoffs in terms of investment and asset prices.

With the credit constraint present, these tradeoffs interact with the dynamics of debt and borrowing capacity. On one hand, the debt tax makes it less likely that the constraint binds and induces lower debt levels, so that when it binds its shadow value is smaller, which in turn contributes to reduce capital returns when the constraint binds. On the other hand, the debt tax imposed when the constraint was not binding can also result in the economy attaining higher leverage ratios because of lower capital values. Thus, the debt tax distorts equilibrium outcomes.
both when the constraint is not binding and when it binds.

Assume now that the regulatory LTV is used instead of the debt tax, still without subsidies. The LTV regulation has qualitatively similar effects as the debt tax. Its intent is to make the constraint bind more often but with the aim of preventing debt from rising too much in “good times.” In line with this idea, we assume that $\chi_t = 0$ in states in which the constraint would have been binding in the absence of regulation (i.e. LTV regulation does not tighten credit market access in conditions in which it would have been constrained already without regulation). When the regulatory LTV binds, the shadow value of the binding credit constraint takes the place of the debt tax, increasing the effective marginal cost of borrowing in the right-hand-side of condition (36). In addition, the LTV regulation increases the marginal cost of accumulating capital in the left-hand-side of condition (37), which leads to higher marginal returns on capital and reduced investment in condition (38).

There are, however, two important differences between the debt tax and the regulatory LTV. First, while the direct effects of the debt tax on the marginal cost of borrowing and capital returns are exogenous (since $R(1 + \tau)$ is a product of exogenous variables), the effects of the LTV are partly endogenous, since they depend on the value of $\mu_t$. Hence, regulators have full control of the direct effect of the policy instrument on equity returns with the debt tax but not with the LTV regulation. Second, the regulatory LTV distorts the optimality conditions only when it binds while a debt tax governed by a simple rule may remain active even when debt is too low for taxing debt to be optimal for macroprudential reasons (i.e. when the credit constraint is not binding at $t$ and has zero probability of being binding at $t+1$). If in some of these low-debt states the regulatory LTV does not bind, the debt tax still has adverse distortionary effects on capital accumulation but the LTV does not, and it does not because of a self-adjusting mechanism that turns off the policy instrument when the LTV regulation is not binding. Moreover, in general it is not true that an equilibrium supported by a given debt tax policy $\tau_t$ can be supported by some LTV regulation $\chi_t$. As shown in the Appendix, LTV regulation aimed at matching the allocations of the debt-tax regime at a given date $t$ implies a higher price of capital than under debt taxes, and different allocations in previous periods because of the forward-looking nature of asset prices. Hence, qualitatively a debt tax and a regulatory LTV have similar effects in terms of distorting optimality conditions, but they are not equivalent instruments.

The role of the subsidy $s^k_t$ is to provide the regulator with an instrument that can help offset the distortionary investment effects of the debt tax or the regulatory LTV. The subsidy increases the marginal benefit of capital accumulation, as is evident from the right-hand-side of condition (37), which in turn contributes to increase the post-subsidy marginal returns on capital, as shown in the left-hand-side of condition (38). Hence, the subsidy can hamper the adverse direct and indirect effects of either the debt tax or the regulatory LTV that work to increase the effective opportunity cost of investing in physical capital. Given the equivalence of debt and capital
income taxes discussed earlier, it is straightforward to show that a subsidy on household income can fully remove the investment tradeoff of macroprudential policy in a simple perfect-foresight setup without credit constraints (the subsidy must be such that \( (1 + s^k)/(1 + \tau_t) = 1 \)). Under uncertainty and with the collateral constraint, however, debt and capital income taxes are not equivalent and hence the subsidy cannot fully offset the tradeoffs of the debt tax.

5 Quantitative Analysis

5.1 Calibration.

[PRELIMINARY]

Table 1: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source/Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest rate</td>
<td>( R - 1 = 0.02 )</td>
<td>Standard value</td>
</tr>
<tr>
<td>Depreciation rate</td>
<td>( \delta = 0.1 )</td>
<td>Standard value</td>
</tr>
<tr>
<td>Share of capital</td>
<td>( \alpha = 0.36 )</td>
<td>Standard value</td>
</tr>
<tr>
<td>Elasticity of substitution consumption</td>
<td>( \eta = 0.205 )</td>
<td>Bianchi (2011)</td>
</tr>
<tr>
<td>Preference weight ( c^T )</td>
<td>( \omega = 0.19 )</td>
<td>Lombardo and Ravenna (2012)</td>
</tr>
<tr>
<td>Production weight ( x^T )</td>
<td>( \pi = 0.19 )</td>
<td>Lombardo and Ravenna (2012)</td>
</tr>
<tr>
<td>Productivity parameter</td>
<td>( \bar{\varepsilon} = 1 )</td>
<td>Normalization</td>
</tr>
<tr>
<td>Risk aversion</td>
<td>( \sigma = 1 )</td>
<td>Standard value</td>
</tr>
<tr>
<td>Discount factor</td>
<td>( \beta = 0.985 )</td>
<td>Mean Capital-Output = 10</td>
</tr>
<tr>
<td>Productivity parameter</td>
<td>( \kappa_L = 0.16 )</td>
<td>leverage in determ. economy = average NFA</td>
</tr>
<tr>
<td>Productivity parameter</td>
<td>( \kappa_H = 0.2 )</td>
<td>leverage in determ. economy = min(NFA)</td>
</tr>
</tbody>
</table>

Parameters set by simulation Value Target

Elasticity of substitution production \( \theta = \) Standard deviation of investment

TFP process \((\sigma, \rho)\) Output process

Markov chain \( P(\kappa) \) Probability of crisis

We assume a first-order autoregressive process for TFP: \( \ln z_t = \rho \ln z + \varepsilon_t \). We allow for financial shocks to induce fluctuations in \( \kappa \) following a two-state Markov chain.

We calibrate the model using data for Mexico. A period represents a year. We split parameters in three sets. A first set is calibrated directly \( \{\delta, \alpha, \eta, \sigma, \pi\} \). These are the depre-
The values of the first four parameters are set to \( \delta = 0.1, 1/(1 + \eta) = 0.82, \sigma = 2 \). Regarding the weight of tradables in investment we set \( \pi, \omega \), based on Lombardo and Ravenna (2012).

A second set of parameters are set to match moments in a deterministic version of the model \( \{\beta, \omega, \kappa^L, \kappa^H\} \). The discount factor and \( \beta \) and the preference weight \( \omega \) are set to jointly match the share of non-tradable to tradable output and the value of the capital output ratio. We estimate these two moments to be 50% and 2.5 in the data. These equations and four unknowns \( (p^N, q, k, \omega_c, x^N, x^T) \) determine these moments in the model

\[
\begin{align*}
\frac{p^N(1-a)(ak^\alpha - x^T)}{a(ak^\alpha - x^N)} &= 0.5 \\
\frac{qk}{ak^\alpha} &= 2.5 \\
p^N &= \frac{1 - \omega}{\omega} \left( \frac{ak^\alpha - x^T + (1 - \delta)k}{(1-a)k^\alpha - x^N} \right)^{\eta+1} \\
q_t &= \left[ \omega \frac{1}{\eta} + (1 - \omega) \frac{1}{\eta} \left( p_N^\eta \right)^{\frac{1}{\eta}} \right]^{\frac{1}{1+\eta}} \\
1 &= \beta[(1 - \delta) + \frac{\alpha}{q}k^{\alpha-1}(1 - a)p^N + a] \\
x_N &= \frac{\delta k}{(\pi x_T^{1-\theta} + 1 - \pi)^{-1/\theta}} \\
x_{TN} &= \left( \frac{p_N}{\phi} \right)^{1/(1+\theta)}
\end{align*}
\]

We calibrate \( \kappa^L \) so that the level of debt in a deterministic economy is equal to the average NFA position for Mexico, which is about \(-40\%\). Absent uncertainty, we have that

\[
\frac{b}{R} = -\kappa^L qk'
\]

Given the calibrated values above, this yields \( \kappa^L = 0.16 \). Similarly, we set \( \kappa^H \) so that the minimum NFA is consistent with \( \kappa^H \), which yields \( \kappa = 0.2 \).

A third set of parameters are set to match moments after simulating the model \( \{\theta, \sigma, \rho, P_\kappa\} \). The four moments are: \( \rho \) and \( \sigma \) to match the standard deviation and autocorrelation of the HP-filtered GDP for Mexico, \( \theta \) to match the volatility of investment and \( P_\kappa \) to match the probability of a financial crisis, defined as a current account of more than two standard deviation.
5.2 Results

The results below are based on a preliminary calibration with the following differences relative to what we described in the previous paragraphs: We set $\theta = -0.25$, replace the $\kappa$ shocks with a single leverage parameter $\kappa = 0.12$, and use a two-state Markov chain for the shocks on productivity of final goods production with a 2 percent standard deviation and autocorrelation of 0.80 (there are no shocks to the productivity of investment goods production).

5.2.1 Financial Crisis Events

We examine first the characteristics of financial crises in the unregulated competitive equilibrium. Column (1) of Table 2 shows key statistical moments. The top panel shows the long-run average of the debt ratio, the probability of Sudden Stops and the probability that the credit constraint binds. The bottom panel shows the average changes in macro variables displayed in Sudden Stop events identified using a long time-series simulation of the model. Sudden Stops are defined as in the related literature: States in which the collateral constraint binds and there is a reversal in the current account-output ratio larger than two standard deviations.

The probability of Sudden Stops is 3.25 percent, which is a notch higher than the observed frequency of Sudden Stops in emerging economies. The event windows show that, qualitatively, the model matches previous findings in showing that financial amplification via Fisherian deflation produces dynamics consistent with those observed during Sudden Stops. Quantitatively, however, the mean Sudden Stop changes in the model’s endogenous variables reported in Column (1) of Table 2 show that this preliminary calibration yields Sudden Stop generally smaller than those observed in the data and reported in the studies reviewed in the previous Section.

The event plots also illustrate that this model has a mechanism that delays the decline in consumption of tradables and hence contributes to make the fluctuations in total consumption smoother. While GDP and investment fall sharply when Sudden Stops occur, consumption falls much less, and this is largely because nontradables consumption does not fall sharply at the same time. On impact, GDP falls only because of the adverse productivity shock (since capital was determined a period earlier), while investment declines sharply as a result of the Fisherian deflation. The price of nontradables falls because demand for tradables drops due to the Sudden Stop. With the calibrated value of $\theta$, tradables and nontradables are gross complements in production of investment goods, and hence the use of both inputs falls sharply. In contrast, with the calibrated value of $\eta$, tradables and nontradables are gross substitutes in consumption, so that in fact there is a small increase in $c^N$ at $t = 0$. For similar reasons, when the price of nontradables rises at $t = 1$ we observe a sharp decline in nontradables consumption: The drop in investment at $t = 0$ reduces capital used in production at $t = 1$ which hampers the supply of nontradables. The price of nontradables recovers as the credit constraint weakens, and hence the
recovery of investment drives up demand for $x^T$ and $x^N$. Then, since supply of nontradables is weak but $x^N$ is bouncing back, market clearing requires $c^N$ to drop, which is again implied by the fact that $c^T$ and $c^N$ are gross substitutes. Effectively, the increase in investment that occurs after the Sudden Stop is attained partly by reallocating a lackluster supply of nontradables resources from $c^N$ to $x^N$.

Table 2: Long-Run and Sudden Stop Moments

<table>
<thead>
<tr>
<th>Long-run Moments$^1$</th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CT</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average $B/Y$ %</td>
<td>-28.25</td>
<td>-27.71</td>
</tr>
<tr>
<td>Welfare Gain$^2$%</td>
<td>n/a</td>
<td>0.006</td>
</tr>
<tr>
<td>Prob. of Sudden Stops$^3$%</td>
<td>3.246</td>
<td>1.083</td>
</tr>
<tr>
<td>Prob($\mu_t &gt; 0$) %</td>
<td>34.95</td>
<td>10.03</td>
</tr>
<tr>
<td>Debt Tax Rate $\tau^R$ %</td>
<td>n/a</td>
<td>1.57</td>
</tr>
</tbody>
</table>

**Average change in Sudden Stops (%)**

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>-0.46</td>
<td>-0.18</td>
</tr>
<tr>
<td>$I$</td>
<td>-9.42</td>
<td>-8.18</td>
</tr>
<tr>
<td>$p^N$</td>
<td>-3.63</td>
<td>-2.61</td>
</tr>
<tr>
<td>$q^K$</td>
<td>-2.54</td>
<td>-1.82</td>
</tr>
</tbody>
</table>

$^1$ DE denotes the decentralized unregulated economy and CT an economy with a welfare-maximizing constant debt tax.

$^2$ Welfare gains are computed as compensating variations in the argument of period utility constant across dates and states that equate welfare in the economy with regulation with that in the unregulated decentralized equilibrium. The welfare gain $W$ at state $(b,k,s)$ is given by $(1 + W(b,k,s))^{1-\sigma}V^{DE}(b,k,s) = V^i(b,k,s)$. The unconditional (long-run) average is computed using the ergodic distribution of the unregulated economy.

$^3$ A Sudden Stop is defined as a period in which the constraint binds and the current account-GDP ratio raises by more than two standard deviations in the ergodic distribution of the decentralized economy.
Figure 5: Sudden Stop Events

Note: All variables except those measured as output ratios are shown as percent deviations from their long-run averages. Variables measured as output ratios are shown as differences relative to their long-run averages and expressed in percent.
5.3 Growth v. Financial Stability

Next we use the quantitative results to explore the tradeoff of MPP induced by the efficiency losses due to the distortion on capital accumulation. The results show that there is a potentially significant tradeoff between losses in output and investment, on one hand, and financial stability on the other. We examine features of this tradeoff both in the long-run (using averages from the long-run distributions) as well as in the short-run (conditional on particular states of nature). The results show that higher taxes on debt or tighter regulatory LTVs generate more financial stability but also lower levels of output relative to the unregulated economy. To illustrate this point, we introduce a “growth-financial stability frontier”, which plots the values of the probability of Sudden Stops and output losses (vis-a-vis the unregulated economy) that are attained for different values of the policy instruments. Hence, this frontier indicates the amount of output that is sacrificed in order to achieve higher levels of financial stability.

Unconditional frontier. We first show in Figure 6 the growth-financial stability frontier that emerges from varying a constant debt tax from zero to 3 percent. We assume that the tax is constant across all states in which the collateral constraint does not bind and zero otherwise. In the figure, the y-axis shows the unconditional (long-run) probability of a financial crisis and the x-axis shows the associated output loss as the percent difference in the long-run average of output relative to that in the unregulated economy. The probability of a crisis corresponds to the probability of observing in the stationary equilibrium of the regulated economy a current account-GDP-ratio reversal of 1 percentage point, which is about two-standard deviations of the current account in the economy without regulation.

The Figure shows that there is an important trade-off between output and financial stability. To reduce the probability of crisis down to half of what it is without regulation (1.63 v 3.25
percent), the economy needs to give up roughly half of a percentage point of (stochastic) steady-state output! This outcome is attained with a debt tax of just above 1 percent. Reducing the crisis probability to zero requires a sacrifice of three-quarters of a percent of steady-state output. These efficiency losses are due to the distortionary effect of the debt tax on the no-arbitrage condition between capital and bonds discussed earlier, which implies that the debt tax has effects akin to those of a capital income tax.

**Conditional frontiers.** To shed more light on the output-financial stability tradeoff we study next the *conditional* frontier. The conditional frontier shows the trade-off for a given current state with MPP active and for any arbitrary government policies in the future. Unlike the unconditional frontier, which describes the relationship between unconditional averages of the economy across government policies, the conditional frontier isolates the effects of fixing a given initial state and a future government policy. We can consider different policies at time 0 and trace the effects over the next period by considering the continuation competitive equilibrium associated with the chosen future government policies.

We examine a scenario in which there is a debt tax at date zero, denoted by $\tau_0$, and from $t = 1$ onward there are MPP instruments in place, and the equilibrium is therefore the unregulated one. Critically, however, $\tau_0$ affects current allocations and prices, and hence it also affects continuation values for the future through the two endogenous state variables, capital and bonds. This experiment is also equivalent to calculating the effects of an unanticipated debt tax introduced only for one period.

Figure 7 shows the conditional frontiers for allocations and prices (in the vertical axes) for date-0 debt taxes in the 0 to 3 percent range (in the horizontal axes) assuming that the economy starts with bonds (capital) about 10 percent below (above) its long-run average and low TFP shock. For these initial states, the collateral constraint is not binding in the first period, but it binds in some states (i.e. with positive probability) in the next period in the unregulated equilibrium. For $\tau_0 = 0$, the allocations and prices coincide with the unregulated equilibrium and these cases are illustrated with solid dots.

As Figure 7 shows, a higher debt tax does its job in terms of reducing the debt agents choose (panel b) but at the expense of reducing the capital they decide to accumulate (panel a). Moreover, panel (d) shows that both tradable and nontradable inputs fall (recall they are gross complements) while panel (e) shows that tradables consumption falls but nontradables consumption rises (recall they are gross substitutes). This occurs because the tax on debt makes tradable goods more expensive, reducing both investment and consumption. Moreover, the stock of non-tradable goods is given (since initial $k$ and the initial TFP shock are not changing), and so the relative price adjustment together with the difference in elasticities (higher for investment than for consumption) that makes sectoral inputs gross complements but sectoral consumptions...
gross complements explains why $c^N$ increases and $x^N$ decreases.

The dynamic benefits of the tax on debt can be seen in panel (f), which shows the level of next-period investment in case of a low TFP realization in the following period. A higher debt tax induces higher investment in the future because of the lower investment today, but it is also important to note that the lower accumulation of debt leaves more resources to invest in the future, especially when the constraints binds. Moreover, there is also an amplification effect: A lower aggregate debt level implies that demand for consumption and investment is relatively higher tomorrow, which in turn leads to higher market price for assets tomorrow, and mitigates the Fisherian deflation that arises when a negative shock triggers a binding collateral constraint.

Panel (c) shows an important result: The welfare-maximizing debt tax is strictly positive (date-0 welfare peaks with a tax of about 1 percent). By taxing debt, the government induces less borrowing, which in turn mitigates the general equilibrium effects that tighten the credit constraint via the pecuniary externality. The tax on debt weakens this externality and prevents over-consumption (Bianchi, 2011, Bianchi and Mendoza, 2018). Hence, the economy is better off increasing the debt tax if there is no debt tax, but after a 1 percent debt tax welfare declines with the tax, first slowly as some states with significant financial instability remain, and then rapidly as eventually the efficiency losses of the higher debt tax captured in the fall in investment dominate the financial stability benefits.

![Figure 7: Conditional Frontiers - Debt Tax](image_url)
Over-borrowing and also over-investment. The previous experiment shows how a tax on debt reduces investment, borrowing, and consumption and can also improve welfare. An interesting question that emerges, however, is whether there is an efficient level of investment for a given level of consumption. We show next that there is not. In fact the model economy features over-investment.

Consider an initial state \((b, k, z)\) and recall the resource constraint for tradables

\[ c^T + x^T + qb' = ak^\alpha + b \]  

(39)

To evaluate the efficiency of investment decisions, we compute what would happen if a social planner were to induce different levels of debt and investment today for a given level of \(c^T\) today and a given continuation competitive equilibrium. In particular, we want to uncover how different combinations of \((b', k')\) translate into different output and crisis exposure tomorrow. To this end, we solve for all possible combinations of \(b', k'\) consistent with the resource constraint (39), a certain level of consumption (e.g. the optimal one \(c^T(b, k, z)\)), and competitive market clearing for the shares of tradable and non-tradable consumption and investment. Notice that for a given \(c^T\) and the portfolio \(b', k'\), we can infer \(x^N\) from using households’ and firms’ first-order conditions, together with market clearing as follows:

\[
1 - \frac{\omega}{\omega} \left( \frac{ak^\alpha + b - x^T - qb'}{(1 - a)k^\alpha - x^N} \right)^{\theta+1} = \frac{1 - \pi}{\pi} \left( \frac{x^T}{x^N} \right)^{\theta+1}
\]

In turn, once we obtain \(x^N\), we can compute \(k'\) using the value of \(x^T\) obtained from (39), as well as \(c^N\) from market clearing.

Figure 8 presents the results of this experiment. In all panels, we show next-period debt (i.e. \(-b'\)) in the x-axis, and in the y-axis we report the outcomes for the choice of capital, current utility, and the choice of investment in the following period. The solid dots identify the competitive equilibrium without regulation. An important point seen in panel (b) of this Figure, which echoes the one made in Figure 7, is that the amount of debt that maximizes current welfare is lower than the unregulated equilibrium amount. There is over-investment too because, as illustrated in panel (a), at the amount of debt that maximizes date-0 welfare we have a smaller value of \(k'\) than in the unregulated equilibrium. Echoing also the results above, the benefit of reducing debt and investment chosen at date 0 is that there is higher investment in the following period, as shown in panel (c). Hence, this experiment illustrates that, conditional on a given level of consumption, it is socially desirable to have lower borrowing and lower investment.\(^{17}\)

\(^{17}\)It is possible to recover the tax on debt and capital that would implement this level of debt and capital, by computing

\[ 1 + \tau_0 = \frac{u_T(0)}{\beta R E_u u_T(1)} \]  

(40)
5.4 Welfare & Welfare-Maximizing Policies

We now study the welfare gains and costs resulting from the use of MPP instruments. For every initial state \((b, k, s)\), we compute the compensating variation in the argument of the utility function that would make households indifferent between remaining in the unregulated economy and switching to the regulated economy. Then the welfare gain or cost of the policy corresponds to the average of these state-by-state gains computed using the stationary distribution of the unregulated economy.

Panel (a) of Figure 9 shows the welfare effects of varying a constant debt tax between 2 and 2.5 percent, and Panel (b) shows the welfare effects obtained for \(b\) values in the -0.56 to -0.38 range at low, average and high values of \(k\) and with the debt tax set at the value that maximizes welfare in Panel (a) (denoted the “best fixed tax”). Panel (a) shows that welfare is maximized with a positive debt tax (of about 1.6 percent), but it also shows that the welfare gains are small and fall sharply and turn into bigger losses for taxes higher than 1.8 percent. To shed light on this, Panel (b) shows that when debt is low enough, the tax on debt delivers large welfare losses. As explained in Bianchi and Mendoza (2018), the fact that taxes on debt have a contractionary effect on asset prices, can have perverse effects in states in which the collateral constraint is binding, which occurs at high levels of debt. Moreover, this occurs even if current taxes are zero. Expectations of future taxes are sufficient to drive down current asset prices and induce these damaging effects.

\[
s^k_0 = \frac{q_0 u_{c,t}(0)}{\beta E_t \{ u_{c,t}(1) [(1 - \delta) q_{t+1} + \alpha z_{t+1} k_{t+1}^{\alpha - 1}((1 - a) p_{t+1}^N + a)] \}} - 1 \tag{41}
\]
5.5 Macroprudential Policy Rules

We evaluate here two forms of macroprudential policies: taxes on debt and LTV (pending). The goal is to study simple implementable policy rules following Bianchi-Mendoza and similar in spirit to those studied and implemented in monetary policy.

We start by studying the effects of constant debt taxes. Column 2 of Table 2 shows the key statistics for the economy regulated with the welfare-maximizing debt tax, which roughly 1.6 percent (and recall also the tax is set to zero if the collateral constraint binds), which roughly 1.6 percent (and recall also the tax is set to zero if the collateral constraint binds). This policy reduces the probability of Sudden Stops to 1.1 percent and also makes the average effects of Sudden Stops slightly less severe. The policy yields a small welfare gain of only 0.6 percent. Figure 10 shows this debt tax dampens slightly the fluctuations observed around Sudden Stop episodes.
Figure 10: Sudden Stops Events - Comparison

Note: All variables except those measured as output ratios are plotted as percent deviations of their corresponding long-run averages. Variables measured as output ratios are shown as differences relative to the long-run average of the corresponding ratio and expressed in percent.
6 Conclusions

Our review of the quantitative literature on Fisherian models of Sudden Stops summarizes the positive and normative contributions of this research program to the analysis of financial crises. From a positive perspective, the literature has shown that Fisherian deflation is a powerful financial amplification mechanism capable of producing model-generated Sudden Stops with realistic features, particularly their low long-run frequency and their large current account reversals and collapses of real exchange rates and asset prices. From a normative perspective, the literature has found that the large corrections in collateral prices induce large pecuniary externalities that distort borrowing choices and reduce social welfare. Optimal macroprudential policy is very effective at reducing the magnitude and frequency of financial crises, but requires complex state-contingent policies. Simpler policy rules closer to the ones used in practice (e.g. the countercyclical capital buffer and regulatory LTV and DTI ratios with limited flexibility) are much less effective and need to be evaluated carefully because they can easily produce welfare-reducing outcomes. To date, finding an effective but simple MPP rule remains elusive.

In this paper we also aimed to add to the literature by conducting an analysis of MPP tradeoffs using a new model that combines features of the DTI and LTV classes of Fisherian models and allows for capital accumulation and sectoral allocation of inputs. We considered the use of macroprudential debt taxes and regulatory LTVs and showed that both distort capital accumulation with effects analogous to those of capital income taxation. For this reason, we also explored the possibility of using subsidies to offset the distortionary effects of macroprudential policies on investment. Despite the fact that they both distort investment in the same direction, regulation akin to debt taxes and LTV regulation are not equivalent. Regulatory LTVs have a self-adjusting mechanism that neutralizes the policy when debt is sufficiently low for it not to warrant the use of macroprudential debt taxes, and at equilibrium they would yield higher asset prices at a given date if they were to be used to match the consumption and debt choices under a given debt tax. Since they would yield different asset prices, they would also yield different allocations prior to that date, due to the forward-looking nature of asset prices.

In the absence of policy intervention, the model generates Sudden Stop dynamics in line with previous findings. The mechanism for production of investment goods using nontradables and tradables generates sectoral reallocation that delays the collapse of nontradables consumption when a Sudden Stop occurs. Simple MPP rules in the form of constant policy instruments, albeit with the flexibility to be turned off when credit is already constrained, are relatively ineffective when set to values that maximize welfare. Small variations in setting the values of these instruments produce sizable investment and output tradeoffs and non-trivial welfare losses.
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Appendix

Debt taxes & LTV regulation are not equivalent

This part of the Appendix shows that debt taxes and LTV regulation are not equivalent, even though both reduce credit and distort investment. Consider for simplicity a one-sector variant of the model with an asset in unitary fixed supply. The budget and collateral constraints with debt taxes and LTV regulation are:

\[ \frac{b_{t+1}}{R(1 + \tau_t)} + z_t F(k_t) \geq c_t - b_t + q_t(k_t - k_{t+1}) + T_t \]

\[ \frac{b_{t+1}}{R} \leq \kappa(1 - \chi_t)q_t k_{t+1} \]

and the first-order conditions of the household’s problem (evaluated at \( k_{t+1} = 1 \)) are:

\[ u'(c_t) = \beta R(1 + \tau_t) E u'(c_{t+1}) + \mu_t \]

\[ [u'(c_t) - \mu_t \kappa(1 - \chi_t)] q_t = \beta E u'(c_{t+1})[q_{t+1} + z_{t+1}] \]

where we simplified by assuming \( F'(1) = 1 \) without loss of generality.

**Proposition** Suppose there is a set of equilibrium allocations \( \{b^*_{t+1}, c^*_t\} \) and prices \( \{q^*_t\} \) associated with a debt-tax policy \( \{\tau_t\} \). Then, \( \{b^*_{t+1}, c^*_t, q^*_t\} \) cannot be implemented with a regulatory LTV ratio \( \{\chi_t\} \), and in particular implementing \( \{b^*_{t+1}, c^*_t\} \) yields higher asset prices under LTV regulation (\( q^*_t > q^*_t \)).

Proof: We show that implementing \( \{b^*_{t+1}, c^*_t\} \) with regulatory LTVs given by \( \{\chi_t\} \) yields higher equilibrium asset prices (i.e. \( q^*_t > q^*_t \)), and hence the two regimes yield different equilibria. Consider first the equilibrium in which only taxes are used (i.e. \( \chi_t = 0 \)) and a state of nature in which the constraint is not binding at date \( t \) (which is when a positive debt tax would be used). The Euler equations of bonds and capital are:

\[ u'(c_t) = \beta R(1 + \tau_t) E u'(c_{t+1}) \]

\[ u'(c_t)q_t = \beta E u'(c_{t+1})[q_{t+1} + z_{t+1}] \] (42)

To implement the same level of debt with LTV regulation requires that the credit constraint binds (\( \mu_t > 0 \)) at the amount of debt of the debt-tax regime:

\[ (1 - \chi_t) = \frac{b^*_{t+1}}{R\kappa q^*_t} \] (43)
In order to sustain also the same equilibrium asset prices, the Euler equations of the LTV regime must hold with the same allocations and prices using regulatory LTVs instead of taxes (i.e. $\tau_t = 0$). Combining the two Euler equations under the LTV regime yields the following:

\[
\left\{ u'(c_t^\tau) - \kappa (1 - \chi_t) \left[ u'(c_t^\tau) - \beta R E u'(c_{t+1}^\tau) \right] \right\} q_t^\chi = \beta E u'(c_{t+1}^\tau)(q_{t+1}^\tau + z_{t+1}) \\
\left\{ u'(c_t^\tau) - \kappa \left[ u'(c_t^\tau) - \beta R E u'(c_{t+1}^\tau) \right] + \kappa \chi_t \left[ u'(c_t^\tau) - \beta R E u'(c_{t+1}^\tau) \right] \right\} q_t^\chi = \beta E u'(c_{t+1}^\tau)(q_{t+1}^\tau + z_{t+1})
\]

(44)

Subtracting (44) from (42) and simplifying we obtain:

\[
u'(c_t^\tau) (q_t^\chi - q_t^\tau) = -\kappa (1 - \chi_t) \mu_t^\chi q_t^\chi
\]

(45)

Since in order to tighten credit without taxes the regulatory LTV must bind ($\mu_t^\chi > 0$) and since the LTV regulation requires $0 < \chi_t < 1$, the right-hand-side of this expression is negative and hence satisfying this condition requires $q_t^\chi > q_t^\tau$, which implies that asset prices would be higher under the LTV regime than with the debt tax. This also implies that allocations would be different in previous periods, because of the forward looking nature of asset prices. Moreover, this result suggests that it may be the case that LTVs dominate other macroprudential policy instruments, particularly debt-tax-like instruments (e.g. capital controls), because of their ability to boost asset prices.

**Sudden Stops Event Analysis**

We constructed the Sudden Stops event analysis using the same methodology as in Korinek and Mendoza (2014), extending the dataset to cover the 1979-2016 period. Their methodology is in turn based on the one developed by Calvo et al. (2006), including Sudden Stops with both large and mild output collapses. Sudden Stops are identified by applying two filters: the capital flow reversal filter and the systemic filter. The capital flow filter flags years with a large fall in capital flows, measured as an increase in the current account-GDP ratio larger than two standard deviations. The systemic filter identifies years in which there were either aggregate EMBI spread spikes for emerging economies or aggregate VIX spikes for advanced economies. See the Appendix of Korinek and Mendoza (2014) for full details.

A Sudden Stop event is identified for a particular country in a particular year, when the two filter conditions are satisfied. Then we construct five-year event windows using macro data based on medians across all Sudden Stop events for emerging and advanced economies. For Sudden Stop events before 2004, we use the list of events identified by Calvo et al. (2006), except for the events they identified for Morocco in 1981 and 1995, which were too close to Sudden Stops.
they also identified for 1983 and 1997. We consider the 1981 and 1983 (1995 and 1997) events as part of a single event dated in 1983 (1997), which is the year with the largest current account reversal.

We used for the most part the same data sources as in Korinek and Mendoza (2014), but using additional data for the real exchange rate and real equity prices. For countries and/or periods where the Korinek-Mendoza data sources do not show data, we construct bilateral real exchange rates using domestic CPI, US CPI, and the nominal exchange rate of the domestic currency v. the US dollar for December of each year. For stock prices, missing country series were completed (when possible) using stock price index data from Bloomberg (December averages). Lastly, when possible, the stock price data were extended by extrapolating using the annual percentage change from S&P Global Equity Indices, obtained from the World Development Indicators.