Fertility and Family Leave Policies in Germany: Optimal Policy Design in a Dynamic Framework

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Abstract

I develop and estimate a life-cycle discrete-choice model of fertility and female labor supply to study optimal design of a range of child-related policies. I first evaluate two recent German reforms: A change in parental leave from fixed payments to wage-contingent payments for a shorter duration and an expansion of low-cost public childcare. I find that the parental leave reform increases fertility and lowers employment rates but only among highly-educated women, whereas the childcare reform increases fertility and employment evenly across all women. Second, I solve for an optimal policy portfolio that satisfies the post-reform budget. The objectives I consider are maximizing overall fertility with utility constraints and maximizing welfare with fertility constraints. The first solution increases fixed subsidies and decreases the wage replacement rate to encourage fertility among less educated women. The second increases childcare subsidies to achieve higher employment and consumption. Both solutions cut taxes for single mothers, thereby providing insurance against divorce to married mothers. Compared to post-reform levels of fertility and welfare, substantial improvements are achieved. The first solution increases fertility by 4% and the second results in a welfare gain equivalent to 0.5% of consumption.

JEL Classifications: H21, I21, J13, J24

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1 Introduction

As the number of pensioners per worker grows in most developed countries, policy makers are increasingly interested in developing and implementing programs that increase fertility while at the same time facilitate female employment. Childcare subsidies, paid parental leave, and direct per-child subsidies, often referred to as *family policies*, are commonly-used policy tools. Childcare subsidies defray a portion of childcare costs for working parents, paid parental leave helps stay-at-home parents smooth consumption and per-child subsidies transfer income to parents through fixed payments or income tax deductions. Tax deductions typically depend on household income due to progressive taxation. Moreover additional tax deductions might be granted to single parents.

Although nearly all developed countries make use of some or all of these policies, there is no clear understanding of their combined effects or each program's marginal effect. Eligibility conditions concerning income, employment or marriage can incentivize women to make particular choices to become eligible for subsidies. At times these are conflicting incentives. For instance, childcare subsidies make it more attractive for mothers to return to work, whereas parental leave payments encourage staying home after the birth of a child. With a few exceptions (Geyer et al., 2015; Lalive et al., 2014), previous research focused on effects of single policy reforms in isolation. Understanding how several policies interact is important not only in analyzing policy reforms but also in designing an optimal portfolio of policies that jointly achieve a certain governmental objective.

This paper conducts a comprehensive analysis of a portfolio of policies implemented in Germany, incorporating precise eligibility rules. A subset of these programs – the parental leave program and the child care program – underwent significant reforms in recent years. To analyze the effects of these programs and also of the reforms on women's work and fertility behavior, I develop a dynamic discrete-choice model. In the model, women make annual decisions whether to have a child and whether to work full-time, part-time or not at all. Wage offers depend on education and on labor market experience which evolves endogenously with work choices. Women incur additional disutility from working when they have children as well as monetary costs associated with child consumption and childcare. In addition, exogenous marriage and divorce affect the utility from having children and household income, such that it is less attractive for single women to have children. The risk of divorce is a factor that may discourage married women from becoming mothers and not accumulating labor market experience.

I estimate the model using Simulated Method of Moments (SMM) applied to data from the German Socio-Economic Panel (GSOEP). The moments I choose to fit pertain to fertility and employment for women with different characteristics. I use the estimated model to evaluate two recent policy reforms in Germany. The first reform constituted a switch from a flat to a wage-contingent parental leave payment, along with a decrease in the duration of payment from two years to one year. The second reform introduced an expansion in the availability of low-cost public childcare for children under the age of three around the same time which I model as a decrease in expected cost (following Wrohlich (2011)). I further validate my structural model by comparing simulated employment effects to regression results obtained using an alternative dataset - the Sample of Integrated Labor Market Biographies (SIAB).

Simulations based on the estimated model indicated that the parental leave reform increases completed fertility for college-educated women by about 5%, whereas the fertility effect on women with low levels of education is close to zero. For the former group, the increase in fertility generates negative employment effects, but for the latter the decrease in the length of paid leave increases overall labor supply. For the childcare reform, I find similar effects on women with different levels of education. Fertility increases by around 1.8%, slightly less than for the parental leave reform. Lower childcare costs increases employment, especially among single mothers for whom the savings are substantial. In general, the childcare reform tends to benefit women with lower income while the parental leave reform does the opposite. The effects on welfare are equivalent to a permanent increase of 0.5% and 0.2% in consumption respectively. My results show that compared to the effects of implementing both policies jointly, the added effects of the individual reforms is larger for lifetime earnings and government spending but smaller for utility. This suggest, it may be important to consider changes in different policies jointly.

I also use the estimated model to investigate optimal design of family policies to achieve different government objectives subject to a budget constraint. Specifically, I consider adjustments in policy parameters related to the replacement rate, length, minimum and maximum amount of parental leave as well as childcare subsidies, fixed per-child subsidies and child tax deductions, including deductions specifically for single mothers. I do not solely consider the problem of maximizing welfare, because public statements at the time of the recent reforms indicated fertility as one of the policy goals. As discussed later, this is likely due to positive externalities that individuals may not internalize in their decision-making.

I solve two specifications of the optimal policy problem: First, I maximize welfare with constraints that fertility of women with different levels of education remain at least at their post-reform level. Second, I maximize overall fertility requiring welfare for women of each education level to remain at least as high. The solutions achieve an increase in fertility of 4% and an increase in welfare equivalent to a 0.5% increase in consumption. The solution to the first optimization features cutting the replacement rate in half, increasing monthly fixed subsidies by 30% and raising childcare costs by 73%. I show this relates to the fact that it is less costly to increase births of higher order and among lower educated women. In contrast, the solution to the second optimization that maximizes the utility objective is to cut childcare costs by about 56%. This acts to increase employment among mothers because this is a cost-effective way to raise consumption and welfare. For both objectives, it is beneficial to cut all taxes for single mothers, because in addition to having a high labor supply elasticity, this group tends to have low household income and therefore a high marginal utility of consumption. Given a high risk of divorce this also increases utility for married mothers.

The contributions of this paper are threefold. First, my paper contributes to the literature on optimal policy design. The vast majority of previous work in this area has been centered around optimal taxation as means to increase utility (starting from Mirrlees (1971)). To the best of my knowledge, this is the first paper to investigate optimal policies allowing for endogenous fertility. A few papers consider family policies and all of them either exclude fertility (Haan and Wrohlich, 2010) or restrict attention to exogenous fertility (Ho and Pavoni, 2016; Domeij and Klein, 2013). Given the dynamic and interdependent nature of fertility and female labor supply, responses in fertility are highly relevant when studying the design of family policies. In addition to affecting women's employment decision over the life-cycle, fertility can also be an explicit outcome of concern for the government.

My next two contributions concern the literature on the effect of family policies and models of life-cycle fertility and female labor supply. I add to the former by simultaneously evaluating the fertility, employment and welfare effects of a number of different child-related policies. The majority of papers that study the effects of worldwide reforms use non-structural methods (e.g. Lefebvre and Merrigan (2008), Cohen et al. (2013) for childcare subsidies in Canada and Israel; Schönberg and Ludsteck (2014) and Lalive and Zweimüller (2009) for parental leave reforms in Germany and Austria) whereas only a smaller number of papers make use of structural methods (e.g. Yamaguchi (Forthcoming) for parental leave in Japan and Mukhopadhyay (2012) for the Pregnancy Discrimination Act of 1978 in the United States.)

The structural evaluation approach offers a number of advantages in analyzing these policies. Due to timing and spacing considerations, fertility effects depend considerably on how far in advance women anticipate the change in policy. It can be difficult to capture longer term effects on fertility and labor supply using difference-in-difference regressions or pre-post comparisons. My model can shed light on the differences in short-run and long-run outcomes as well as make long-term predictions. Furthermore, it allows disentangling effects of simultaneous policy changes as in the case of the recent reforms in Germany. Finally, another advantage is the ability to simulate outcomes for hypothetical policies to study optimal policy design.

There is a long-standing tradition of studying female labor supply in life-cycle frameworks (Mincer, 1962; Heckman and Macurdy, 1980; Eckstein and Wolpin, 1989) and endogenous fertility was incorporated into the models starting with Francesconi (2002).[1] Recently, Gayle, Hincapié, and Miller (2018) and Adda et al. (2017) evaluate family policies in their counterfactual analysis. As mentioned there has been little emphasis on examining the joint effects of changing different kinds of policies on women's behavior. The framework developed in this paper, although similar to some existing models in the literature, has rich features needed to study a variety of child-related policies, including changing the replacement rate and length of parental leave pay, and the effects on women with different characteristics. Previous models estimated for Germany with the GSOEP (Adda et al., 2017; Geyer et al., 2015) assumed fixed employment behaviors of mothers with young children and/or did not account for single mothers.

In the following section I give an overview of fertility, female labor supply and family leave policies in developed countries and discuss the specific case of Germany. Section 3 summarizes the data and the structural model is presented in Section 4. This section further discusses estimation and identification. Section 5 shows the model parameter estimates and model fit. Lastly, I evaluate the German policy reforms and discuss optimal policy design in Section 6.

[1] For a more extensive survey of the dynamic female labor supply and fertility literature please refer to Keane et al. (2011).

2 Background

Increasing women's fertility and labor supply are often stated as main goals of child-related policies.[2] The majority of OECD countries have been experiencing low fertility rates well below the replacement rate. On average women have about 1.7 children, but 2.1 children are needed to sustain a constant population size. In Germany, Italy and Japan the number is as low as 1.4-1.5, and although for the U.S. it used to be close to replacement, following the great recession it dropped by about 0.2 children per woman and continues to decline.[3] With progressing economic growth, fertility is also falling for many developing and emerging economies. China, for instance, after pursuing a one-child policy for more than three decades, is now faced with low birth rates of about 1.6 children per woman.[4]

Low fertility rates cause concern because they signify a decline in future working population.[5] With rising life expectancy, governments have increasing difficulty to sustain social infrastructures. The rise in female employment over the past decades, partially mitigates the problem. However, in OECD countries, women's labor force participation is still only about 75% of that of men, in addition, work hours are substantially lower. Both facts are connected through the trade-off that women face between career and fertility. Women incur large career costs when having children because they are primarily responsible for childcare, and taking family leave results in depreciation of human capital and lower wages. This is established by a large body of research that identifies children as the main cause of the gender wage gap.[6]

One immediate implication is that lower educated women tend to have more children due to lower opportunity costs of reducing labor supply.[7] There is global evidence that points to a strong negative correlation between fertility and women's level of education. In the U.S. and Germany for example, women with a college degree have about 0.4 fewer children than their high school educated coun-

[2]Other relevant outcomes include maternal well-being and child-development. I address the former by examining effects on women's utility, but remain agnostic about the latter. Findings regarding the effect of family policies on child outcomes are not conclusive (see e.g. Dustmann and Schönberg (2012) for an analysis of maternity leave in Germany). I did not find significant relationships between the form of childcare and various measures of child development with the GSOEP. This is likely due to the high quality of childcare in Germany.

[3]As reported in e.g. www.nytimes.com/2018/05/17/us/fertility-rate-decline-united-states.html

[4]Birth rates failed to increase substantially even after implementing a two-child policy in 2015. State-owned newspaper ChinaDaily.com.cn reports government officials are considering "birth rewards and subsidies".

[5]In the absence of substantial immigration, which might be infeasible (politically or otherwise).

[6]In OECD countries the gap is around 14%.

[7]There may be broader effects such as investments in education and choice of occupation (high-earning ones typically have a steeper decline in human capital (Yamaguchi, Forthcoming; Adda et al., 2017). I do not address these but my results can give a lower bound on the size of policy effects on employment and wages.

terparts.[8] This gives rise to problems of imbalances in childrearing resources and inter-generational wealth transfers between households, especially in light of assortative marriage market matching.

Given the economic significance of stimulating fertility and female labor supply, many countries have spent millions, if not billions, of dollars on fertility and family-related policies. To reconcile the two objectives, many reforms increased benefits paid to working mothers through parental leave and childcare programs. For instance, the state of New York, Washington and other U.S. states recently implemented paid parental leave.[9] Countries that expanded existing parental leave policies include Germany, Denmark, Portugal, Poland, Japan and South Korea. Cases in which childcare subsidies were increased are, for instance, Quebec's public childcare system from 1997-2000, Sweden's cap on childcare prices in 2002, and the expansion of publicly funded childcare in the UK since 2010.

Findings regarding the effects of the various reforms on fertility and employment vary substantially in size. I discuss these in more detail when describing the results in Section 5.[10] There are a handful of papers studying the fertility and employment effects of the recent German parental leave reform. Raute (2018) and Cygan-Rehm (2016) use difference-in-difference to study fertility effects within five years of the reform date. The former finds a large increase in births for highly educated and high income women, the latter finds a small decline in higher order births. A potential confounding factor for these two studies is the simultaneous increase in childcare availability. Kluve and Tamm (2013) and Geyer et al. (2015) find a negative effect on mothers' employment in the first year after birth and small positive effects in the second year.

Further, a number of structural studies examine the effects of the German childcare reform. Haan and Wrohlich (2011) find an increase in maternal employment and positive fertility effects among childless and highly-educated women but no overall increase. Bick (2016) calibrates a model of paid and unpaid childcare with subsidies financed by labor taxes and also concludes there are positive employment effects but no or even slightly negative fertility effects. While these two studies examine married and cohabiting couples, my findings indicate that single women have the larger positive fertility

[8]These statistics are based on information provided in the webpages of the OECD, US Census Bureau and my own calculations

[9]Paid parental leave does not exist on a federal level in the U.S. but has been constant subject of debate. In the summer of 2018 Marco Rubio introduced the Economic Security Act for New Parents which stipulates that parents can access retirement funds early to cover the leave period.

[10]For an detailed cross-country comparison of family policies and summary of findings please see Olivetti and Petrongolo (2017). Brewer et al. (2009) further give an overview of in-work benefits for lower income families such as the EITC in the U.S. and WFTC in the U.K.

response. Domeij and Klein (2013) calibrate a model with exogenous fertility and find it is optimal to increase childcare subsidies substantially in order to achieve higher employment of mothers. Geyer et al. (2015) also simulate changes in employment and find positive effects for mothers of children under the age of three.

Studying the case of Germany is advantageous for a number of reasons. First, there are many generous fertility and family-related policies (explained further below) with substantial costs amounting to roughly 1.7% of GDP.[11] Second, the traditional “male breadwinner" household model is prevalent: most children are born to married couples with nearly all fathers working full-time and mothers being responsible for childcare. This provides justification to only model the choices of women. Third, in recent public debate it has been emphasized that low fertility rates and maternal labor supply constitute some of the most pressing political challenges. The fact that German mothers predominantly work part-time has been related to a lack of affordable childcare, especially for younger children. Further, it is commonly believed that these career limitations deter women from having children in the first place. The latter is supported by the high rate of childlessness among academics of about 30%, and even among the broader female population the number is around 20%. These factors motivated the two large family policy reforms, which provide an excellent opportunity to validate my structural model and gain insights on the effects of different policies.

Germany introduced a new publicly funded paid parental leave system in 2007 similar to that in Scandinavian countries. Prior to the reform, a fixed benefit of €300 per month was granted for up to two years.[12] The new policy shortened the maximum duration of payment to one year and changed the amount to a two-thirds replacement of net pre-birth earnings.[13] There are lower and upper caps on the payment of €300 and €1800 per month.[14] Hence, for the majority of women, the reform increased the payment in the first year. For example, a woman with monthly net earnings of €2000 (roughly the female average) is paid €1300 under the new policy, €1000 more than before.[15] Even when consid-

[11]Calculated based on information from http://www.bpb.de/politik/innenpolitik/familienpolitik/193715/familienpolitische-geldleistungen?p=all

[12]This benefit was also means-tested. More than half of all parents were not eligible to receive payments in the second year, which I account for in my estimation.

[13]If a mother earns income working part-time while eligible for paid parental leave, she receives two-thirds of the difference between her current and pre-birth labor income. The conditions for part-time leave became more favorable in 2015. Now a month worked part-time on parental leave counts only as half a month of leave.

[14]The minimum cap also applies to parents who were not working before the child was born.

[15]Parents can divide the months of paid leave freely among themselves. In practice, however, the low take-up of fathers likely has little to no effect on the mother's labor supply, so I do not consider paternal leave taking. Until 2007 less than 3% of fathers took any leave. After the reform this number rose to 30%, most likely due to

ering the present value of summed payments over the two years, most women gained with the new policy.

Moreover, since 2005 there has been a gradual expansion of public childcare for small children under the age of three. In Germany, public childcare facilities are heavily subsidized, while private care through nannies and similar arrangements is quite costly. The policy change relaxed a shortage in affordable public childcare for children of this age group and from 2006 to 2013, childcare enrollment rates have nearly doubled from 16% to 30%. The full monthly cost of providing childcare to a child between the ages of three and seven is estimated to be around €700 and for a child under the age of three around €1000.[16] In contrast, parents of children in the older age group only pay an average monthly fee of roughly €130; for younger children the fee is about twice as high. Fees are progressive in parents' income; I estimate that monthly fees increase by about €8 for an increase of €1000 in gross monthly household labor income. The details are shown in Appendix A.2.

In addition to parental leave pay and subsidized childcare, parents further receive either a fixed cash transfer or tax deduction per-child, whichever is more favorable. Fixed subsidies have been at the level of roughly €150 per month for the sample period.[17] Households with incomes of above €66000 per year for married couples or €35000 for a single parent benefit more from tax deductions than fixed subsidies. Tax deductions are set at approximately €7000 per child per year and exempt parents from paying taxes on the deducted amount while also granting them an overall lower tax rate due to progressive taxation.[18] Single parents receive additional tax deductions of around €2400 per year.

3 Data

For the estimation of the structural model I use household data from the German Socio-Economic Panel (GSOEP), a detailed ongoing annual household survey conducted by the German Institute for

a new feature designed to encourage paternal involvement in childcare, so-called 'daddy-months". Parents are granted two more months of paid leave if each parent takes at least two months. The vast majority of fathers who respond to this incentive takes exactly two months, usually simultaneously with the mother. For this reason it is unlikely, that it would affect the mother's labor supply.

[16]This information is based on calculations in government reports: www.bmfsfj.de/blob/94182/763244389dd4e093fa22d4788bbaddeb/kosten-betrieblich-unterstuetzter-kinderbetreuung-data.pdf

[17]In earlier years the payment was increasing in the birth order of the child (so as to encourage higher fertility). Since 2002 the difference has been minimal.

[18]Over my sample period the amount was at about €4500.

Economic Research (DIW). For each adult member of a household, current and retrospective information on education, marital status, employment, wages and children is collected. On the household level there is also information on childcare costs. My sample comprises of West German women born between 1960 and 1985, aged 22-45.[19] I restrict the sample period to the years 1990 to 2006 prior to the childcare and parental leave reforms.[20] I further exclude women who have children prior to the age of 22. This leaves just 5272 women in the GSOEP with about 3500 observations per age on average.

I divide the women into three education groups based on completion of school degrees and apprenticeships: women with a college degree, women with more than 10 years of secondary schooling and a completed apprenticeship, and the remainder. As can be seen from Table 1, the average number of years of education and husband's earnings increase with education, while marriage rates are lower for higher educated women.[21] There are also substantial differences in fertility and employment over the life-cycle across education groups. The more highly educated a women is the more she tends to delay fertility. For the highest education group the number of children almost doubles between the ages of 34 and 44, while for the least educated group the change is only about a fourth of the size. The total number of children over the life cycle is lower for the highly educated, the difference between the highest and lowest educated group is around 0.2 children. Looking at wages and work hours it is not surprisingly that both increase in education. Moreover, the wage gaps between education groups widens with age. At the age of 24, women in the highest education earn less than three Euros more per hour, but by age 44 the difference is more than four Euros. The share of total periods worked full-time is higher for more educated women, the reverse is true for part-time.

I estimate the cost of public childcare as a function of household income using the GSOEP. The Federal Statistical Office provides information on childcare enrollment, which I use to infer the fraction of children in subsidized public childcare. I then calculate the expected cost of childcare as a weighted sum of unsubsidized and subsidized cost, assuming full-time unsubsidized childcare costs €840 per month (similar to Haan and Wrohlich (2011) and Wrohlich (2011)). More details are described in Appendix A.2. When modeling the fertility process, I make use of medical data on women's biological

[19]Large cultural and earnings differences persist between East and West Germany even after reunification. I focus on West Germany because of its larger population and lower fertility as well as maternal employment.

[20]The first law on childcare expansion was passed in 2005, however little change occurred before 2007.

[21]I only model single and married women, cohabitating women are assumed to be single. The literature has shown cohabitation relationships to be less stable and produce fewer children than marriages, hence it is not obvious whether they are more similar to marriage or singledom. To not confound unmarried women in stable relationships too much with those without a partner I predate the year of marriage by one year. Many children are born in this year, in which the women were most likely not single.

probability of successfully conceiving at a given age, detailed in Appendix A.5. This probability declines with age and falls drastically after the age of 35.

As a way to validate my model, I compare the effects of the parental leave policy based on model simulations and regressions using different data set. For this I use a large administrative dataset, the Sample of Integrated labor Market Biographies (SIAB). This data, provided by the Institute for Employment Research (IAB) at the German Federal Employment Agency, is a 2% random sample of all individuals in Germany for whom social security contributions were made or to whom unemployment benefits were paid out.[22] To match the two data sets I exclude women who are self-employed in the GSOEP. The data provides detailed information on employment and wage. Worker characteristics are limited to gender, education and year of birth. In particular, neither marital status nor the number of children are observed. However, by selecting gaps in the data by length, age of the woman and termination reason, using a method developed by Müller and Strauch (2017), it is possible to identify births relatively accurately. I describe this method in more detail and provide data summary statistics in Appendix A.1. Schönberg (2009) proposes a similar method and is able to perform a crosscheck using a subsample that can be linked to administrative birth records. She finds births are identified with nearly 90% accuracy.

4 Model

4.1 Set-up

The model spans ages 22 to 60 of the woman's life.[23] Women can have up to three children, which they can conceive when single or married. For married women, I assume their husband always works. When single, the non-government income consists of only of her income (plus child alimony). Let t denote the woman's age and i the woman and her corresponding household.[24] Objects referring to the woman's husband have superscripts h. I omit the individual superscript whenever possible.

[22]All employers are required to pay social security fees for their employees when certain conditions are met.

[23]The focus of the model is on the fertile period in a woman's life until age 45. Estimation is solely based on moments from this period. To obtain more realistic terminal values, the ages 46-60 are modeled in a slightly simplified way, in particular the fertility choice is eliminated. Furthermore there are no more marital transitions and the non-random component of the husband's income is assumed to stay constant.

[24]I assume children stay with their mother upon divorce and do not consider single fathers

Table 1: Summary Statistics by Education Group

	high	med	low
Yrs of Education	10.3	12.5	16.9
	(1.1)	(1.2)	(1.4)
Age at First Birth	26.7	28.2	30.7
	(3.7)	(3.7)	(3.9)
Share of Childless	.17	.19	.31
	(.22)	(.17)	(.23)
No. of Children:			
Age 24	.28	.14	.03
	(.52)	(.38)	(.18)
Age 34	1.31	1.22	.78
	(.95)	(.95)	(.90)
Age 44	1.48	1.56	1.34
	(.99)	(.93)	(1.08)
Hourly Wage:			
Age 24	7.89	10.02	10.44
	(3.705)	(8.907)	(10.62)
Age 34	12.14	13.03	15.48
	(9.94)	(10.76)	(7.01)
Age 44	11.89	13.46	16.04
	(5.72)	(5.93)	(8.19)
FT Share	.441	.556	.731
	(.427)	(.479)	(.485)
PT Share	.243	.235	.155
	(.429)	(.424)	(.362)
Married Share	.535	.501	.334
	(.499)	(.500)	(.472)
Single Mother Share	.123	.091	.054
	(.329)	(.288)	(.225)
Husband's Earnings	3080.60	3818.69	4168.40
	(1571.11)	(2329.88)	(2717.45)
No. of Obs.	23241	46044	20508

4.1.1 Choices

Each period, the woman, chooses whether to she would like to have a child or not and whether she would like to work full-time, part-time or not at all. I denote the set of choice variables as $\boldsymbol{q_t} = (f_t, \boldsymbol{l_t})$. A woman is fecund, biologically able to conceive a child, in period t with probability p_t^f set using medical data detailed in Appendix A.5. This probability declines with age and is zero by age 45. If the woman is fecund and she decides to have a child f_t takes value one. If the woman is not fecund or does not wish to have a child f_t is equal to zero.

The employment choice is denoted by $\boldsymbol{l_t} = (l_t^F, l_t^P)$, where the first entry is equal to one when working full-time and the second when working part-time work.[25] Part and full-time work are mutually exclusive such that the sum of the two can be at most one. A woman can freely choose her employment level except for when she has not worked in the previous period nor is currently on protected maternity leave.[26] In this case, I assume she has to look for a new employer and only succeeds with probability $p_t^{job} < 1$ due to search friction. The job finding probability assumes the form:

$$\ln\Big(\frac{p_t^{job}}{1-p_t^{job}}\Big) = \Sigma_{e=1}^{3}\pi_{0e}\,ed_e + \pi_1 age_t + \pi_2 age_t^2 + \pi_3 exp_t + \pi_4 exp_t^2$$

Where subscript $e = 1,2,3$ denotes education levels and $\{ed_e\}_{e=1}^3$ the respective education indicators. exp_t is the level of human capital accumulated until period t according to a process detailed further below. As mentioned, if the woman worked in the past period or is under job protection, p_t^{job} is equal to one. Denote $job_t = 1,0$ as an indicator for a woman's job protection status. If the woman does not receive a job offer, the woman cannot work and l_t^F and l_t^P take value zero. Depending on eligibility the woman might receive parental leave payments when she does not work full-time. Furthermore, I assume the woman purchases childcare for the time she works, that is, childcare cost for mothers working part-time is half compared to for those working full-time.

[25] The former corresponds to 40 work hours per week and the latter to 20 hours per week. These are approximately the mean hours worked in the data for each employment level.

[26] Like many other papers I assume workers do not have constraints choosing between part-time and full-time. In principle there is no guarantee that jobs offer a flexible choice of hours. In Germany, this assumption should not be too far from reality. German laws let all full-time workers scale hours down to part-time. It does not guarantee a possibility for all workers to scale back in the future but this flexibility is granted to parents on parental leave. Unpaid parental leave is offered if parents worked prior to giving birth and the youngest child is less than three years old. If the woman gives birth while still under active job protection, the protected period is extended until the newborn turns three years old.

4.1.2 Preference

Period utility of woman i in period t, excluding preference shocks and for n_t children (including newborns), is given by:

$$u_t^i(l_t, c_t; n_t, m_t, a_t) \;=\; \underbrace{\alpha_1 \ln(c_t + \alpha_3)}_{\text{Utility from consumption}}$$

$$+ \underbrace{\Sigma_{k=1}^{3} \alpha_{2k} kid_t^k + \big(1 - m_t\big)\big(\alpha_4 \ln(\Sigma_{k=1}^{3} kid_t^k + 1) + \alpha_5 kid_t^1\big)}_{\text{Utility from children}}$$

$$- \underbrace{\big(l_t^F + \gamma_4 l_t^P\big)\big(\gamma_1 + \gamma_5^i\,(\Sigma_{k=1}^{3} \gamma_{2k} kid_t^k + \Sigma_{a=1}^{5} \gamma_{3a} A_t^a)\big)}_{\text{Disutility from work}}$$

where m_t takes value one if the woman is married and c_t denotes per-capita consumption.[27] Indicators for the kth child $\{kid_t^k\}_{k=1}^3$ are derived from n_t, the number of children. The utility derived from children consists of two terms. The first is the sum of valuations for each kth child and the second a psychic cost for single mothers that depends on the number of children. The term $\alpha_5 kid_t^1$ is non-zero for single women with any children, adding more flexibility to the utility cost.

The disutility from working depends on the employment level of the woman, how many children live in the household and age of the youngest child given by a_t and indicators $\{A_t^a\}_{a=1}^5$ for age groups: 0, 1-2, 3-6, 7-11, 12-18, 18-25.[28] γ_4 captures the differential cost of working full-time versus part-time. Individual preference heterogeneity in the disutility of working with children is captured by γ_5^i which takes one of two possible values. I normalize one value to one and estimate the other value as well as the type distribution by education. The taste type is recorded as state variable *type* $\in \{0, 1\}$ such that formally $\gamma_5^i = type(\gamma_5) + (1 - type)$.

4.1.3 Human Capital, Wages and Husband's Income

The woman's wage shock ξ_t and her husband's income shock ξ_t^h are independently and normally distributed with mean zero and variances σ^2 and σ_h^2. Given a monthly wage w_t, the woman's gross labor

[27] obtained by scaling consumption expenses C_t by the number of adults and children in the household such that $C_t = c_t(1 + 0.5m_t + 0.3n_t)$. This is in line with the modified OECD consumption scale. More information: http://www.oecd.org/eco/growth/OECD-Note-EquivalenceScales.pdf

[28] This is similar as in other recent papers (e.g. (Adda et al., 2017), Yamaguchi (Forthcoming)) and allows the model to fit lower employment rates of mothers with young children.

income is $y_t = w_t(l_t^F + 0.5 l_t^P)$. This assumes that part-time work is paid half the salary of full-time work, which is not unrealistic because German laws explicitly grant part-time workers proportional pay compared to full-time workers for the same job. The wage function is:

$$\ln(w_{e,t}) = \phi_{0e}\, ed_e + \phi_{1e}\, ed_e\, exp_t + \phi_{2e}\, ed_e (exp_t)^2 + \xi_t$$

Work experience exp_t evolves according to

$$exp_{t+1} = \left(exp_t + l_t^F + \Sigma_{e=1}^3 \lambda_{1e}\, ed_e\, l_t^P\right)\left(l_t^F + l_t^P + \delta(1 - l_t^F - l_t^P)\right)$$

In words, experience increases by one if the woman works full-time and decreases at rate $(1-\delta)$ with $\delta < 1$ if the woman does not work. The effect of part-time work on experience depends on the education level, this helps to capture the idea that occupations mainly pursued by women with lower education levels might give different rewards to part-time work than those held by highly educated women.

If a woman is married, her husband's income is modeled as a function of her characteristics to capture assortative matching:

$$y_t^h = \Sigma_{e=1}^3 \psi_{0e}\, ed_e + \psi_1 age_t + \psi_2 age_t^2 + \xi_t^h$$

This assumption avoids adding more variables to the state space as first proposed in Van der Klaauw (1996).

4.1.4 Budget

The budget constraint in period t is given by:

$$\max\Big(G(y_t, y_t^h, n_t) + (1-m_t)\, al_t(n_t)\, ,\, S(m_t, n_t)\Big) + PL_t(\boldsymbol{l_t}, a_t, y_t^b)$$

$$= C_t + \left(1 - l_t^F - 0.5 l_t^P\right) \zeta(n_t, a_t, y_t + y_t^h) + A_t^1 NB$$

The equation states that total income on the right hand side has to be equal to expenses on the left hand side.[29] The term with the maximum operator indicates the government guarantees a minimum income to household through social assistance. G is a function mapping household labor income and number of children to after-tax-income including per-child subsidies. al_t denotes child alimony received by single mothers. I set child alimony to be 200 Euros per child and month and assume it is paid by the father. If the sum of these incomes falls below a threshold S, which varies by marital status and the number of children, the household receives social assistance up to S.

PL_t stands for parental leave pay received by mothers not working full-time, which does not factor into the calculation for social assistance. Before the reform PL_t takes value 300 if the youngest child is less than two years old. After the reform PL_t takes value $\min\big(\max(300, 0.67y_t^b), 1800\big)$ if the youngest child is less than one year old, where y_t^b denotes the mother's net pre-birth wage. Childcare costs are denoted by ζ and depend on number and age of children as well as parents' income. I assume childcare has to be purchased only for children below the age of 7 who are not yet in school. Lastly, households with newborns incur a cost of NB which I set to € 2400 for one year.[30] Details on the German tax code, social assistance and childcare cost functions can be found in Appendix A.3.

4.1.5 Marriage and Divorce

Marital transitions are taken as exogenous and their probabilities differ by age and education. The exact estimates are can be found in Appendix 18. The function is education specific and assumes a logit form. The probability that a single woman marries in a given period is:

$$\ln\Big(\frac{p_{e,t}^{mar}}{1-p_{e,t}^{mar}}\Big) = \theta_{01e} + \theta_{11e}\,age_t + \theta_{21e}\,age_t^2 + \theta_{31e}\,age_t^3$$

The divorce probability $p_{e,t}^{div}$ has the same form.

[29]As Eckstein and Wolpin (1989) pointed out including saving along with labor supply and fertility choices is computationally difficult. Omitting assets is restrictive, as shown for instance in Adda et al. (2017) and Blundell et al. (2016). In particular it might lead to overstating labor supply elasticities, due to the lack of saving as an additional way to smooth consumption. I do indeed find labor supply elasticities that are slightly on the larger side.

[30]This is an estimate from guides for first-time parents e.g. www.t-online.de/leben/familie/baby/id_52172296/erstausstattung-fuers-baby-mit-diesen-kosten-muessen-eltern-rechnen.html

4.1.6 Household Problem

The timing is as follows. At the beginning of each period the woman learns her fertility preference shocks and her fecundity for this period. If fecund, she can choose whether to have a child which would be immediately born. Afterwards, she observes her labor preference shock, wage shock, husband's income shocks as well as whether a job offer has arrived. If she worked in the previous period, enjoyed job protection or received a job offer, she chooses whether to work full-time, part-time or not at all. Accounting for household labor income, social security and income taxes, subsidies from family policies, child alimony and childcare expenses the household consumes the remainder of the budget. At the end of the period a marital transition shock is realized and determines whether married women divorce or single women marry.

The following part formally states the household maximization problem. All preference shocks are assumed to be type I extreme value. This allows expected utility to be partially expressed in analytical form and simplifies the computation. There is one shock for each choice option, two fertility shocks $\boldsymbol{\epsilon_t^f} = \{\epsilon_t^{f1}, \epsilon_t^{f0}\}$ and three labor shocks $\boldsymbol{\epsilon_t^l} = \{\epsilon^{lN}, \epsilon^{lF}, \epsilon^{lP}\}$. Denote the fecundity shock as ν_t, job offer shock as η_t and marital shock as μ_t.

Define the period utility conditional on the fertility choice with the labor preference shocks as:

$$U_t^i(\boldsymbol{l_t}, c_t; n_t, m_t, a_t, \boldsymbol{\epsilon_t^l}) = u_t^i(\boldsymbol{l_t}, c_t; n_t, m_t, a_t) + l_t^F \epsilon_t^{lF} + l_t^P \epsilon_t^{lP} + (1 - l_t^F - l_t^P)\epsilon_t^{lN}$$

Denote state variables at beginning of period t as

$$\Omega_t^* = \{e, type, n_t^*, a_t^*, exp_t, m_t, y_t^b, job_t\}$$

and after the fecundity shock is realized and the fertility choice was made as

$$\Omega_t = \{e, type, n_t, a_t, exp_t, m_t, y_t^b, job_t, \boldsymbol{\epsilon_t^f}, \nu_t\}$$

where $n_t = n_t^* + f_t$ and $a_t = (1 - f_t)\, a_t^* + f_t(0)$. Let $V^*\left(\Omega_t^*\right)$ denote corresponding expected utility at the beginning of the period and $V\left(\Omega_t^0\right)$ and $V\left(\Omega_t^1\right)$ conditional on $f_t = 0, 1$ before the labor preference, wage and income shocks were realized.

Starting from the interim labor decision, given her fertility choice for the period and before the realization of η_t, ξ_t, ξ_t^h and $\boldsymbol{\epsilon}_{\boldsymbol{t}}^{\boldsymbol{l}}$ the women has expected utility:

$$V(\Omega_t^f) = E_{\xi_t,\xi_t^h,\boldsymbol{\epsilon}_{\boldsymbol{t}}^{l}}\Big(p_t^{job}\Big(\max_{\boldsymbol{l_t}} U_t^i(\boldsymbol{l_t}, c_t; n_t, m_t, a_t, \boldsymbol{\epsilon}_{\boldsymbol{t}}^{\boldsymbol{l}}) + \beta E_{\mu^t} V_{t+1}^*(\Omega_{t+1}^* | \, \boldsymbol{l_t}, \Omega_t)\Big)$$

$$+ (1-p_t^{job})(U_t^i(\mathbf{0}, c_t; n_t, m_t, a_t^*, \boldsymbol{\epsilon}_{\boldsymbol{t}}^{\boldsymbol{l}}) + \beta E_{\mu^t} V_{t+1}^*(\Omega_{t+1}^* | \, \mathbf{0}, \Omega_t)\Big)\Big)$$

subject to the budget constraint and for discount factor $\beta < 1$. Ω_{t+1}^* evolves according to the aforementioned human capital, marital transition etc. processes.

At the start of the period the expected utility of the woman is:

$$V^*(\Omega_t^*) = E_{\boldsymbol{\epsilon}_{\boldsymbol{t}}^{\boldsymbol{f}}}\big(p_t^f \max_{f_t} V(\Omega_t^f) + (1-p_t^f) V(\Omega_t^0)\big)$$

That is, conditional on her fertility preference for the period, if she is fecund, she decides optimally whether to have a child or not.

4.2 Identification and Estimation

The functions for husband's income, marital transitions, childcare cost, income tax and social security contributions can be consistently estimated outside of the model in a first step. I present details and results in Appendices A.2, A.3 and A.5. I further set the discount factor β to 0.97, similar to other studies in the literature.[31]

I estimate the remaining model using the Simulated Method of Moments (SMM) (McFadden, 1989). Targeting wage regression coefficients,[32] average wage levels by age and changes in wages for different employment choices allows me to identify the parameters in the wage and human capital accumulation functions. Moments linking the number and age of children to employment and marital status of the woman give information about the parameters of the utility function. Identifying job finding

[31] Blundell et al. (2016) assume a discount factor of 0.98 for women in the UK; Adda et al. (2017) estimate a discount factor of 0.96 using the GSOEP.

[32] I regress log wages on education and experience using the real and simulated data (a form of indirect inference).

parameters is less straight forward, because I do not have job search data. Given the structure of the model, the job finding parameters are adjusted to fit employment outcomes that cannot be rationalized with the assumed functional forms.[33] The variation in the employment decisions of mothers with similar characteristics in each education group determines the parameters governing heterogeneity that match the model closest to the data. A full list of data moments and their simulated counterpart can be found in Appendix A.6.

Define Θ as the vector of the model parameters and $\hat{\Theta}$ as its estimate. For a given set of parameters, the model is solved backwards to obtain expected values at each relevant point in the state space. For continuous parameter exp_t I use interpolation based on eight grid points. The expected values are used to simulate and record choices for a representative sample of 5000 women throughout their life-cycle. For most women the simulation begins at age of 22, for about two-thirds of women in the highest education group I begin simulation at a later age upon completion of university education. I further account for initial conditions regarding marriage and employment prior to the start of the model. More information is given in Appendix A.4.

To estimate the model, I generate moments using the simulated data and compare them to the moments obtained from the real data by evaluating a loss function. The function uses a diagonal weighting matrix consisting of variances of the data moments.[34] Let M_k^d and M_k^m stand for the kth data and model moment respectively. Formally the estimation solves:[35]

$$\min_{\hat{\Theta}} \Sigma_{k=1}^{K}[(M_k^d - M_k^m)^2 / H(Var(M_k^d))]$$

5 Results

In this section I present the estimates and illustrate the model fit. Table 2 shows the utility parameter estimates and the distribution of types among the three education groups. The first section shows that women derive the highest utility from the first child and the lowest from the third. All else equal, the

[33] For instance, if employment is very low for women in their 40s compared to women in their early 20s and this cannot be explained by other possible mechanisms in the model (e.g. children), then the job finding probability is estimated as decreasing in age.

[34] e.g. Adda et al. (2017) and Blundell et al. (2016).

[35] Because the number of observations decreases with age, I adjust moment variances slightly to increase the weight on moments for older women.

additional benefit of having a first child for a non-working woman while holding consumption constant is worth 85% in consumption at the average level of around €1750. However, this calculation does not consider utility costs from working for mothers and the drop in consumption due to lower labor income and monetary expenses for children. Single mothers incur sizeable disutility costs, equivalent to reduction of €200 in monthly per-capita consumption for an average single mother household with a consumption level of €1200.

The second section shows the disutility from working. The cost of working with children does not decrease monotonically in birth order, the largest cost is incurred with the third child. It somewhat declines with the youngest child's age, although the level seems to be similar for children within the age groups 1-6 and 7-18. Part-time work reduces the work disutility by nearly 85%. Furthermore, the preference type with lower cost of working with children only faces 11.5% of the cost of the high-cost type. The prevalence of the low-cost type is below 20% for all education groups and increases in education.

The job finding probability is estimated to less than 50% for most women. It increases in age for women below the age of 28 and then declines with age. The probability increases with more experience, such that it is highest for college-educated women despite this group having the lowest intercept. For an average 45-year-old woman the probability drops to about 32%.[36] Human capital depreciates by 12.8% when the women does not work for one year and decreases by 5.5-16.2% when she works part-time.[37] Due to selection into employment the overall effect of part-time work on *working* women's wages is positive as shown in Table 27 in the Appendix. Baseline wages are highest for women with medium education. This is because many college educated women start their career with accumulated human capital through schooling. The value of human capital increases with education. The average hourly wage increase for an additional unit of human capital is 59 cents, 73 cents and 94 cents (5.0%, 5.2% and 5.4%) for women of low, medium and high education respectively. These numbers are similar to Francesconi (2002), who estimates an increase of 7.5% and 3.8% for women with zero and 20 years

[36]There is no direct data on the job finding probability, however unemployment data from the employment agency shows that the fraction of unemployed women that are unemployed for more than 1 year increases significantly with age. In 2000 around 35% of 15-24-year-olds were searching for one year or more, for 25-49 year old the proportion rises to about 50%. This would imply a minimum bound on the job finding probability that is slightly higher than the one I estimate. However, non-searchers are not included in the data (who might very well have lower job-finding rates due to not actively searching).

[37]The large negative effect of working part-time might seem surprising. Working part-time work leads to a larger decrease in human capital than not working for women in the two lower education groups when the level is below 8 units which is true for most. One explanation is signaling, such that part-time work may reflect very negatively upon desirable traits in workers such as leadership ambition and prioritizing work.

Table 2: Utility Parameter Estimates

Parameter	Estimate
Consumption and Children:	
α_1 Cons. Slope	1.937
α_3 Cons. Shifter	-2.124
α_{21} First Child	1.523
α_{22} Second Child	.517
α_{23} Third Child	.488
α_4 Single Mother Cost	2.350
α_5 Single Mother Cost Shifter	-.555
Work Disutility:	
γ_1 Constant	.012
γ_{21} First Child	3.227
γ_{22} Second Child	1.929
γ_{23} Third Child	4.246
γ_{31} Age of Youngest Child: 0	4.933
γ_{32} Age of Youngest Child: 1-2	1.775
γ_{33} Age of Youngest Child: 3-6	1.773
γ_{34} Age of Youngest Child: 7-11	.627
γ_{35} Age of Youngest Child: 12-18	.641
γ_4 Part-time	.159
γ_5 Type 1	.115
Type 1 Proportions by Education Group	
μ_1 Edu. Group 1	.108
μ_2 Edu. Group 2	.168
μ_3 Edu. Group 3	.176

of experience.

I present Marshallian and Frisch labor supply elasticities as well as fertility elasticities in Table 5 for a 1% increase in net wages.[38] Labor supply is measured as hours of work (not conditional on working). The Marshallian and fertility elasticities are for responses to a permanent wage increase for all periods while the Frisch elasticities for an anticipated wage increase in one given year measured in that particular year. Following Eckstein et al. (Forthcoming) I calculate the Frisch elasticities for responses at three ages: 25, 32 and 40, and Marshallian elasticities for responses in three age groups (22-29, 30-37, 38-45). Frisch elasticities by education and family composition are computed as weighted averages over the three years.

I estimate an overall Marshallian elasticity of around 0.8, which is within the range found in the literature. For instance, Haan and Wrohlich (2011) estimate an elasticity of 0.5, Kaiser, van Essen, and Spahn

[38]I simulate a 5% increase in wages and divide the percentage change by five to smooth out simulation errors.

Table 3: Job Finding and Human Capital Parameter Estimates

Parameter	Estimate
Employment Probability	
π_{11} Intercept Low Ed.	-.699
π_{12} Intercept Medium Ed.	-.659
π_{13} Intercept High Ed.	-.762
π_2 Age	.041
π_3 Age Sq.	-.003
π_4 Experience	.083
π_5 Experience Sq.	.001
Human Capital	
λ_{11} Part-Time Low Ed.	-1.0
λ_{12} Part-Time Medium Ed.	-.995
λ_{13} Part-Time High Ed.	-.340
δ Depreciation	.872

Table 4: Wage Parameter Estimates

Parameter	low	med	high
ϕ_{0e} Intercept	2.576	2.662	2.484
ϕ_{1e} Experience	.054	-.084	.101
ϕ_{2e} Experience Sq.	-.001	-.002	-.002

(1993) of 1.0.[39] Frisch elasticities tend to be lower but move in the same direction with the exception of age. In particular, mothers and women with lower income respond more to wage incentives. Elasticities are highest for low education women and single mothers, a common finding in the literature (see e.g. Blundell et al. (2016)). Marshallian elasticities increase with age and decrease for Frisch elasticities, although the difference in Frisch elasticities is small. Younger women are less likely to be mothers and more likely to work full-time which may explain a lower Marshallian elasticity of 0.5 among 22-29 year olds. When decomposing Marshallian elasticities into intensive and extensive margins, I find that the extensive margin to be more important than the intensive margin (not shown here). This finding agrees with Geyer et al. (2015) but contradicts Haan (2010). The magnitudes are close to findings in Blundell et al. (2016).

Fertility elasticities in the literature are calculated for different types of changes in wages and subsidies and for different subgroups. Although direct comparisons are often not possible, my results approximately fall in the same range as those reported in previous papers. I show fertility elasticities in Table 5 for a permanent 1% net wage change and changes in the number of children born to women in each subcategory. The results confirm the common finding that fertility responds negatively to an increase in wages: higher wages do not only increase myopic work incentives but also the return to human capital. The average elasticity is about -1.2, similar to Francesconi (2002) who finds elasticities in the range of -1.0 to -1.5.[40] Geyer et al. (2015) estimate an elasticity about half the size for a one-period wage change, while Butz and Ward (1979) report an elasticity of about -1.73. Lower educated women and college-educated women react more strongly than women with medium levels of education. Furthermore, elasticities decreases with age. This makes sense, given the stronger motive for younger women to build human capital. Lastly, single women have the largest fertility response, in line with the result on labor supply elasticities.

I report additional elasticities in Table 12 in Section 6.2 for changes in the various types of family policies. For an additional €10 in monthly fixed child subsidies I find average completed fertility increases by about 0.01 or 0.7%.[41] In comparison, for the same amount of money Laroque and Salanié (2014) find an increase of about 1.4% for France, Cohen et al. (2013) an increase of 3.2% for Israel and

[39] Bargain and Peichl (2016) provide an overview of estimated labor supply elasticities across countries.

[40] The author reports elasticities for different subgroups. For example, he estimates that a 10% increase in the full-time wage intercept results in a drop in fertility from 1.97 to 1.76 for women with more than five years of full-time work experience and zero children at the start of marriage.

[41] It is important to bear in mind that this would only affect households on the lower part of the income distribution since higher income households receive tax deductions instead.

Table 5: Labor Supply and Fertility Elasticities

	Frisch	Marshallian	Fertility
Low Ed.	.155	1.047	-1.323
Medium Ed.	.115	.716	-1.055
High Ed.	.108	.543	-1.248
Age 25, 22-29	.139	.520	-1.468
Age 32, 30-37	.124	.903	-1.259
Age 40, 38-45	.108	.918	-1.206
Married Childless Women	.173	.198	-
Married Mothers	.149	.903	-1.226
Single Childless Women	.045	.016	-
Single Mothers	.221	1.040	-3.475

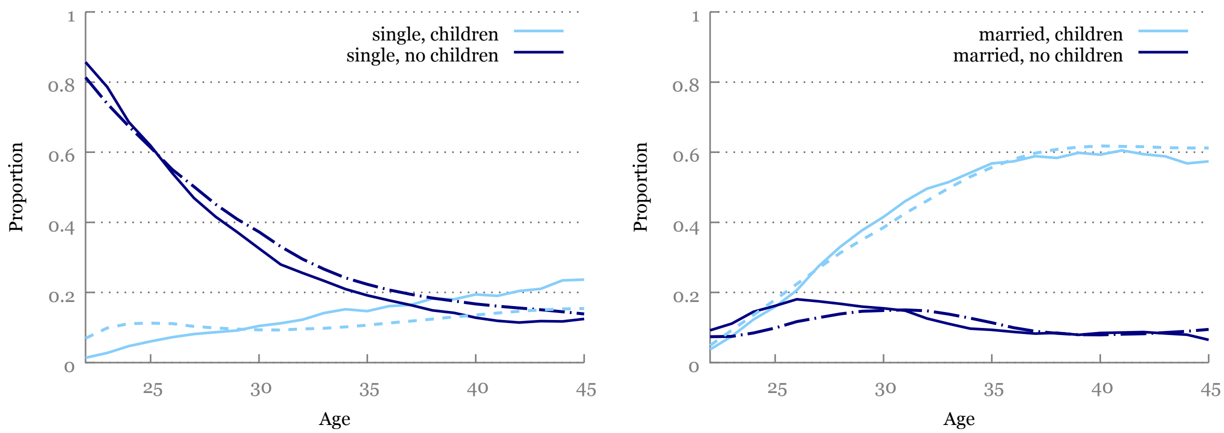
Marshallian (ME) and fertility (FeE) elasticities calculated for permanent 1% net wage increase. Frisch (FrE) elasticities for anticipated one-period 1% net wage increase at given age. (ME) and (FrE) for changes in hours worked unconditional on employment. (FeE) measured for number of births of subgroup; last four rows for married and unmarried women. Age: (ME) and (FeE) as averages within age groups, (FrE) for one age. (FrE) by education and family composition are computed as weighted averages over three ages.

Adda et al. (2017) find close to no long-term effects for Germany.[42] For payments only to households of young children Geyer et al. (2015) and Milligan (2005) find very large effects for Germany and Canada which I cannot confirm.[43] Generally the size of effects from non-structural estimation methods could be larger in magnitude because short-term effects tend to be larger than long-term effects (I demonstrate this in the next section, Adda et al. (2017) show a similar result).

I present the fit for all moments used in the estimation with the respective standard deviations as well as some measures of out-of-sample fit in Appendix A.6. Overall the model fits the data well. Figure 1 shows the distribution of women who are either single or married and have or do not have children. The proportion of single unmarried women is above 80% at age 22 and continuously falls with age. The fraction of single mothers grows from close to zero to 25% of total population at age 45. The size of the group of married women with children increases with age, while the proportion of married childless

[42] Laroque and Salanié (2014) find a 21% increase in birth rates for a simulated €150 increase in monthly subsidies, Cohen et al. (2013) finds a reform that increased subsidies for the third child child by NIS 150 (about €30) increased the yearly birth probability of mothers with two children by 0.99 points from about 10%, Adda et al. (2017) estimate a structural model with savings and simulate effects for a €6000 payment at birth. This roughly corresponds to an increase of €35 in monthly subsidies until the child turns 18. The increase in fertility is less than 0.2%.

[43] Geyer et al. (2015) find an increase of €30 per month for children under the age of three increases fertility by 4.6%, while for Milligan (2005) finds a payment of CAD 1000 (€669) at birth increases birth rates by 16.9%.



Solid lines for data, dashed for model

Figure 1: Proportion of Women by Marital and Family Status Over the Life-Cycle

women first increases with age and then declines because women have children at later ages. The model fits these trends well except for a slightly flatter line for the fraction of single women. At age 45 the model underpredicts by about 8%.

Life-cycle patterns of full and part-time work for women of different levels of education are matched well by the model as can be seen in Figure 2. Full-time employment decreases with age until around age 38. It is uniformly higher for women with higher levels of education. The graph for part-time employment is almost a mirror to full-time employment. As women have more children they work more part-time. Here there is hardly any difference between women in the two lower education groups.

Figure 3 shows averages wages over the life-cycle for women with high, medium and low education by age. Wage data in the GSOEP data is fairly noisy as can be seen from the solid lines. The model simulation produces smooth average wages represented by the dashed lines. Overall the model captures the levels and shapes of the data curves with wage growth being steepest for the highest educated women. Wages increase with higher education for ages over 29 and for all groups wage profiles are increasingly flat with older ages (approximately when most women begin to have children).

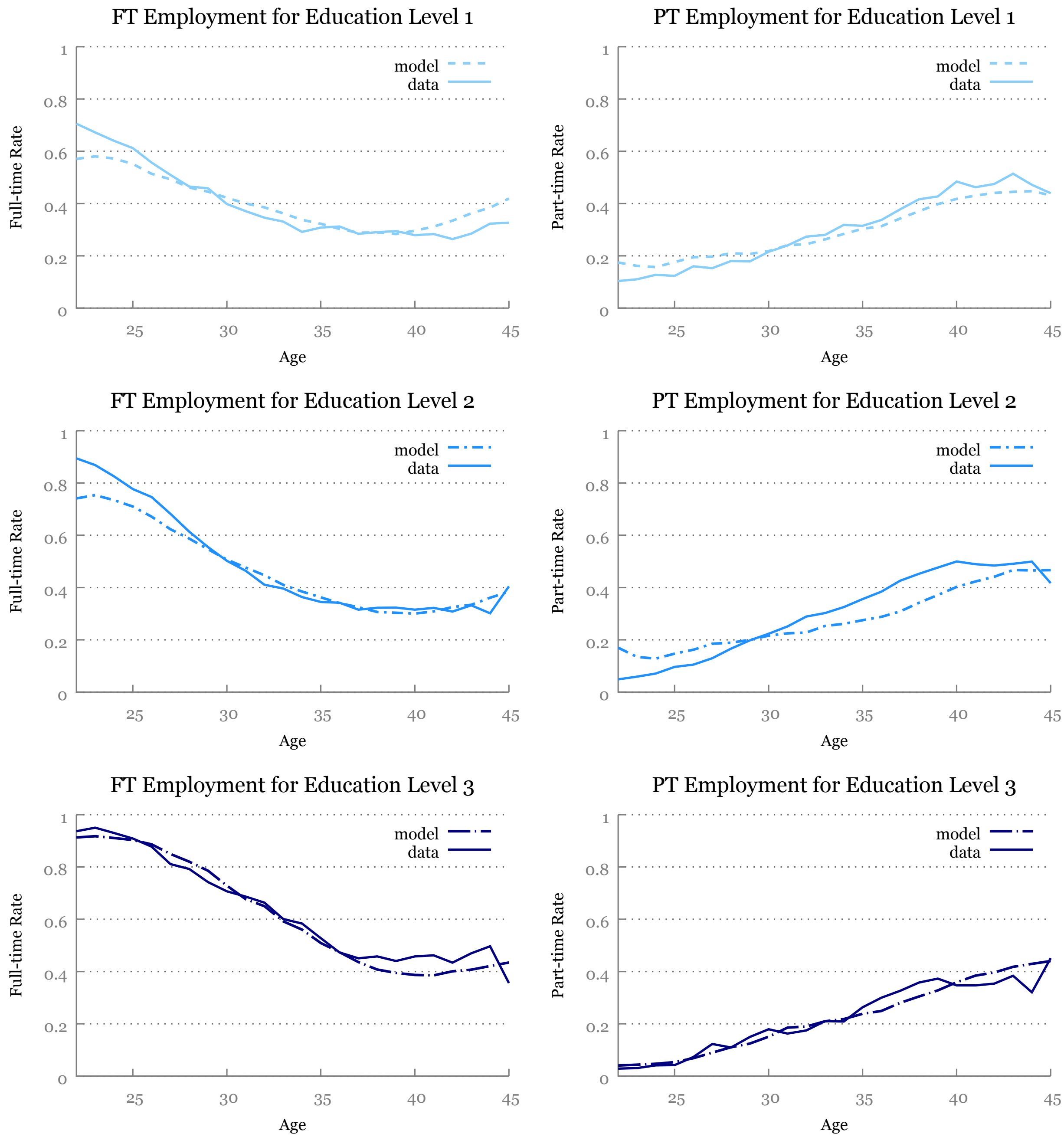


Figure 2: Full-Time and Part-Time Rates Over the Life-Cycle By Education

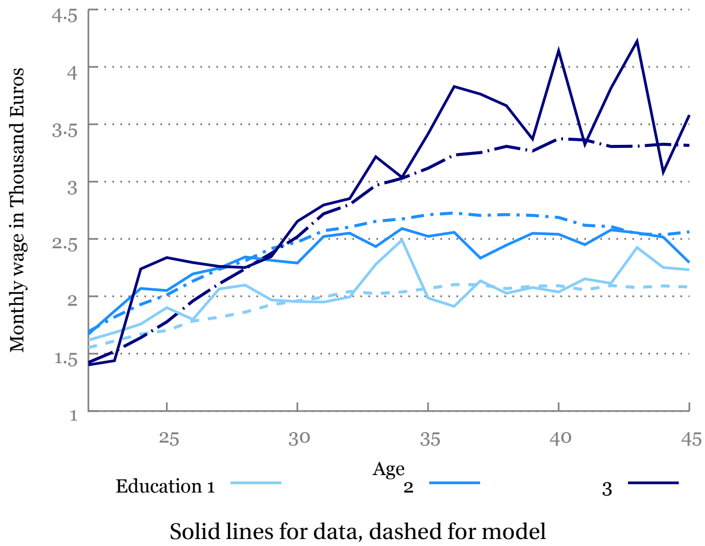


Figure 3: Average Monthly Wage By Education Over the Life-Cycle

6 Policy Simulation

6.1 Model Validation and Effects of the Parental Leave and Childcare Reforms

In this section, I use the estimated model to simulate the individual and joint effects of the parental leave and childcare reforms on a number of fertility and employment outcomes. For the parental leave reform, I reduce the payment period to only cover the year the child was born and set the pay at 66.7% of pre-birth wages, with lower and upper bounds at € 300 and € 1800. For the childcare reform I impose the fraction of government-funded childcare to one as it is stipulated in the childcare law of 2008.[44] To obtain present values of lifetime earnings and government spending, I use a discount rate of 4%, the average German interest rate over the sample period. [45]

Before examining long-term effects of the policy reforms, I verify the model predictions by simulating short-term effects of the parental leave reform and comparing them to non-structural results that I

[44] the goal was set to be achieved in 2013, however it appears there is still a supply shortage of around 25%

[45] approximated using data from the Federal Reserve Bank of St. Louis available at https://fred.stlouisfed.org/series/intgstdem193n.

and other papers obtain using other data sources. A number of papers use the sharp policy break to calculate short (to medium) term effects on fertility and employment. I add to this analysis by calculating effects on employment and earnings using the SIAB data set. For fertility effects, I show that my findings agree qualitatively with the results of a difference-in-difference study, and also give evidence that short-term effects are almost twice as large as long-term effects. Because public childcare availability has been gradually increasing over many years, there are no reduced-form results. Here, I discuss how the long-term results from my model compare with other structural findings in the literature.

First I describe my short-term analysis using the structural model. I assume the reform was unanticipated as argued by Kluve and Tamm (2013).[46] I use the structural model to simulate the behavior of a sample of 60000 women for two cases: (i) no reform is implemented (ii) the reform is implemented at a random age between 22 and 45.[47] To obtain reform effects, I compare outcomes for women, who in the year prior to the reform decide to have children, under (ii) and (i). In practice, to capture the unanticipated nature of the reform, I assume women make their fertility decision and then find out about the reform before making employment choices.

For model validation based on the SIAB data I compare maternal income and employment in the first two years after childbirth of women with children born under the old policy and immediately after the reform. I consider the effects separately by level of education. In the SIAB data, I select a sample of women whose children were born in the first quarter of 2006 and 2007. I regress the outcomes of interest on a reform dummy, effectively comparing means. Selecting the sample to include the same months in both years rules out seasonality effects of births. Further, for children born in the first quarter of 2007, it is unlikely that mothers anticipated the reform.

This comparison follows Geyer et al. (2015) who compare their structural results against findings from the Mikrozensus, a large cross-sectional dataset. Table 6 shows the results from both data sets and methods. The effect in the first year is negative for both methods and all outcomes due to the disincentive to work provided by the new policy. Both methods find a larger decrease in part-time rate

[46]The authors examine Google searches and the timing of public announcements and conclude that the earliest date mothers could have been aware of the impending reform was May 2006. Unanticipated reforms have the benefit of ruling out women selecting into fertility outcomes.

[47]I solve the model twice for the pre and post-reform regime and obtain two sets of expected values at every state space point. For case (ii), in the years prior to the reform, women make decisions based on expected values under the old policy. In the year of the policy change, women update their beliefs that the new policy will be in effect from then on and given the value of their current state variables, make decisions taking into account the expected values under the new policy.

than in full-time rate. In the second year, the effects tend to be positive, because parental leave payments where eliminated. For the most part, regression effects on part-time and earnings in the first year of the reform are significant and all others are not. This is likely because the largest change in payment occurred for the first year, while payment decreased by at most € 300 in the second year. Among the significant regression estimates, the corresponding structural estimates lie within the 95% confidence intervals with the exception of the effect on earnings for women with medium levels of education. Both methods indicate the reform had a larger effect on women with more education in the first year. This is intuitive because more educated women experienced a larger increase in payments. The rest of the estimated effects using both methods have the same sign, except for some for the college-educated women. The sample size for this group is relatively small (around 380 for both years), hence the regression estimates are noisy. Overall Geyer et al. (2015) draw similar conclusions.[48] The authors also note that it is typical for non-structural effects to be larger in size than structural ones. The regression results further agree with Kluve and Tamm (2013) who analyze data from a specifically commissioned survey.

Additional to employment and earnings effects, I compute short-run and long-run effects on fertility. Similar to above, I compare the number of births for women who never experienced the reform with that of women who experienced it for one year and from the beginning of their lives. Table 7 shows the reform causes a surge in the number of births that increases in magnitude for women with more education. Overall, the increase in the short-run is %196 of that in the long-run. This is potentially part of the reason why Raute (2018) finds much larger effects in in the range of 20-30%.[49] The difference in short and long run effects is due to older women having more children when surprised by the reform in the short run, as evident from the increase in average age at birth. In the long-term these women would have anticipated the policy and had children at younger ages. Hence the average age increase is smaller in the long-run. Nevertheless, because parental leave pay is conditional on previous wages, women have incentives to delay birth to achieve higher earnings prior to giving birth. The discrepancy in immediate versus longer term fertility responses illustrates that changes in fertility timing matter and that short-term effects are not always a good proxy for long-term effects. In this case the effect on fertility would have been substantially overestimated.

[48] Instead of grouping results by education, they consider subgoups of high and low income women.

[49] Another might be due to selecting a sample of women who work at the start of the sample period. Repeating the analysis in Table 7, restricting attention to women who worked in the year prior to the reform, raises the overall increase in births to 7.8%, 9.2% for highly educated women.

Table 6: Parental Leave Reform Effect on Maternal Employment and Earnings by Child's Age

	Child's Age	**Simulation**/Regression		
		low	med	high
Full-Time	<1	**-.2** -.2 (-5.9,5.5)	**-.4** -.4 (-5.6,4.8)	**-.8** -.7 (-8.9,7.4)
	1-2	**1.2** 6.9 (.1,13.8)	**1.4** 1.2 (-4.9,7.2)	**3.4** -3.5 (-12.6,5.5)
Part-Time	<1	**-1.3** -5.8 (-11.8,.3)	**-2.7** -5.9 (-11.1,-.7)	**-3.3** -9.1 (-17.4,-.8)
	1-2	**.7** 1.2 (-7.2,9.6)	**1.8** 1.3 (-5.8,8.4)	**-.9** 2.0 (-8.1,12.1)
Daily Earnings	<1	**-.4** -.3 (-4.8,4.2)	**-1.5** -8.2 (-12.4,-3.9)	**-3.4** -11.5 (-21.5,-1.4)
	1-2	**1.4** 4.9 (-.2,10.1)	**1.3** 3.1 (-2.1,8.3)	**1.5** -3.9 (-14.7,7.0)

Upper row: simulated effects from structural model in bold; second row: regression coefficients using SIAB; third row: 95% CI; employment in %, earnings in € /day.

Table 7: Parental Leave Reform – Short-run and Long-run Effect on Births

	Education Group	Baseline	PL SR	Diff	PL LR	Diff
No. of Births	1	885	908	+2.6%	906	+2.4%
	2	1759	1830	+4.0%	1775	+0.9%
	3	501	541	+8.0%	529	+5.6%
Age at Birth	3	33.75	33.93	+.18	33.82	+.07

Yearly births per 60000 women. Baseline: pre-reform; short-run (SR): second year after unanticipated reform; long-run (LR): anticipated new policy from age 22.

Now I turn to the evaluation of the long-term effects of the parental and the childcare policies. For these effects I simulate the behavior of women for whom the same policies were in effect over their entire adult lives. Table 10 depicts the individual and joint effects. Recall the parental leave reform increased the amount paid proportionally to income. Overall the reform causes fertility to increase by 1.9%. For the highest education group the increase is about 4.8% while for women with low levels of education the effect is close to zero at 0.2%. As shown in Table11, the reform is able to reduce the fraction of childless women by about 14%, addressing one of the main public concerns.

On average, women work about 0.1 periods more full-time and 0.1 periods less part-time. Women who do not yet have children increase full-time employment slightly by about 0.8%, anticipating to take advantage of higher parental leave payments in case they have children. Note this is even true for women who remain childless their whole lives, since they cannot rule out the possibility of having children in advance. Moreover, ending payments when the child turns one year old increases employment in the following year and also incentivizes women of all education levels to work more prior to having children. The overall effect on employment and earnings for college educated women is negative because they have more children. Higher educated women earn about € 6 600 Euros less, a sum more than three times larger than for the low education women. Utility increases slightly for the higher two education groups equivalent to a 0.2% permanent increase in consumption. However it drops slightly for the lowest education group, because the increase in payment during the child's first year is not sufficient to compensate for the loss of payment in the second year.

In contrast, childcare subsidies affect women of all education groups relatively evenly. The increase in fertility is around 1.8% for all groups. This is different from Haan and Wrohlich (2011), who find that highly educated women increasing fertility and less educated women decreasing fertility. A priori it is not clear whether the effects increase or decrease with education. While more educated women are more likely to work, the amount of the subsidy is more significant for lower educated women. Further, Haan and Wrohlich (2011) and Bick (2016) find no overall increase in fertility. One possible explanation is that both papers only include married mothers in their models. My results show that the fraction of children born to single mothers increases slightly (Table 14), indicating that the fertility increase for this group is larger. Bick (2016) further considers a simultaneous increase in labor taxes to finance the childcare subsidies and he attributes the lack of a positive response in fertility to this decrease in income.

Table 8: Married and Single Mother's Employment Rate

Employment Rate	Child's Age	Baseline	PL	CC
Married	$a = 0$	11.7	-1.9	+1.3
Single		30.0	-4.2	+9.3
Married	$1 \leq a < 3$	30.9	+1.2	+3.4
Single		37.3	+3.0	+11.7

Parental Leave and childcare reform effects on employment rate (%) by marital status, and for mothers of newborns and children ages 1-2.

Table 9: Average Years as Single and Married Mother

Years	Baseline	PL	CC
As Married Mother	11.0	+.2	+.1
As Single Mother	2.7	-.1	+.1

Parental leave and childcare reform effects on average number of years a woman spends as single and married mother for ages 22-45.

Compared to the parental leave reform childlessness is only slightly reduced with almost no change for higher educated women. However, the drop in wages is much smaller for this group, by about € 5100. This is compensated by a larger decrease for the other two education groups. Wages decrease because even though mothers of young children work more, the overall hours of women decrease because they have more children. The total number of periods worked full-time decreases by approximately the same amount as part-time periods increase. The increase in utility is about three time as large as achieved by the parental leave reform.

Now I describe how both reforms differentially affect single and married mothers. Table 8 contrasts the differences in the effects on the employment rates of mothers who have children below the age of three. Effects for both groups are negative in the child's first year and positive for the second and third year for the parental leave reform. For the childcare reform effects for all years are positive. Both reforms have larger positive effects on single mothers both in absolute and relative terms. This is mostly driven by the fact that single women with children tend to have lower household income, and thus are more affected by the decrease in parental leave payment in the second year and expected childcare costs. The response of single mothers to the childcare reform is about 8 percentage points larger than for married mothers for all years. For the parental leave reform the difference is 1.8 points.

Table 10: Parental Leave and Childcare Reform Effects

	Educ.	Baseline	PL	CC	Joint
Periods Full-Time	1	10.0	.2	-.1	+.1
	2	11.3	.0	-.1	.0
	3	11.5	-.1	-.1	-.1
Periods Part-Time	1	6.9	-.1	+.2	+.1
	2	6.6	-.1	+.1	.0
	3	5.3	-.1	+.1	.0
Cumulative Wages	1	321.5	+4.0	-1.8	+1.7
(in thousands)	2	435.2	-0.9	-1.7	-2.2
	3	487.8	-6.6	-1.5	-6.1
Completed Fertility	1	1.51	.00	+.02	+.03
	2	1.54	+.03	+.03	+.06
	3	1.19	+.06	+.03	+.08
Lifetime Utility	1	159.7	.0	+.2	+.2
	2	166.5	+.1	+.2	+.4
	3	143.0	+.1	+.2	+.3

Individual and joint effects of parental leave and childcare reforms. First two sections: Average years worked full-time and part-time for ages 22-45. Third row: Discounted lifetime earnings for ages 22-60 in thousands of €s. Fifth row: Lifetime discounted utility.

Table 11: Percentage of Women Remaining Childless by Education

Education	Baseline	PL	Diff	CC	Diff
low	20.7	18.5	-2.2	19.9	-.7
med	21.7	19.0	-2.7	21.2	-.5
high	34.8	30.5	-4.2	33.7	-.1

Effects of parental leave and childcare reforms on % of women who never have children.

Note that single mothers are more likely to work when their child is young as a result of low income. In the first year this difference is largest, in the baseline scenario 30% of single mothers work while only 11.7% of married mothers work. This means they do not benefit as much from the increase in payments under the new parental leave regime. Together with the decrease in benefits in the second year, the parental leave reform disadvantages single mothers relative to married mothers, as shown in the decrease in average number of years spent as single mothers in Table 9. For childcare subsidies this pattern is exactly reversed.

The last column of Table 10 shows the joint effects both reforms. Although it is not expected that the effects of the individual reforms are additive when implemented jointly, for some outcomes the difference between joint effects and the sum of individual effects is large. This is most obvious for cumulative wages. For college-educated women both reforms jointly decrease earnings less than only the parental leave reform even though both individual effects are negative. The difference between the sum and the joint effects is around € 2100 per woman for this group. Furthermore, differences vary in sign across education groups, for instance for completed fertility, such that summing individual effects can underestimate effects for one education group while overestimating it for another. I calculate that jointly implementing both reforms is around 2% less costly for the government than the sum of the costs of each individual reform. Nevertheless, joint implementation achieves a slightly higher increase in utility (0.5%).

In conclusion, the parental leave reform achieves a higher increase in fertility at the cost of less employment with the effect increasing in education. Childcare subsidies, on the other hand, increase fertility slightly in a uniform manner without decreasing employment too much, which also leads to a larger increase in women's utility.

6.2 Optimal Family Policies

To consider which policies are optimal, first note that the commonly-used welfare objective, the weighted sum of utility across all individuals, is not suited for this application. It omits the explicit value of increasing fertility. The government statements made at the time of the reforms indicate strongly that part fo the goal of the policies was to increase the number of children. One explanation is that, if the government is altruistic, due to positive externalities of fertility, as mentioned previously, the socially optimal number of children exceeds the privately optimal number in the absence of fertility policies.

Accounting for fertility considerations, I solve for optimal policy for two different problem specifications while maintaining budget balance. The first is maximizing average fertility without harming utility of women of any education level. In this scenario, the government is primarily concerned with increasing the number of children being born while sustaining public approval. The second specification is a counterpart and maximizes average lifetime utility in the population while guaranteeing the current fertility level of for all education groups. This extends the traditional government problem to include constraints to maintain the increase in fertility achieved through the recent reforms. Particularly, the optimal policy cannot decrease fertility among higher educated women, even though sustaining fertility for this group is most expensive for the government. Fertility, utility and budget constraints apply to their respective long-term levels after the implementation of both reforms. For the first two the values are recorded in the last column of Table 10.

The policy instruments I consider are four parameters for parental leave pay and one each for childcare subsidies, fixed child subsidy, general tax-deductions and tax deductions for single mothers. The parameters for parental leave are length, replacement rate, minimum and maximum payment amount. Since the previous reforms raised the childcare availability to one, I now consider setting a new childcare cost level by scaling the post-reform cost. Define the set of policy instruments as x. I require the government's discounted net revenues under the optimal policies to not exceed the post-reform level $\underline{B}$. Discounted net government revenues are discounted revenues minus discounted costs:

$$B(x) = T(x) + CT(x) - PC(x) - CC(x) - PL(x) - S(x)$$

$T(x)$ denotes revenue from income taxes and social security contributions of workers. It does not account for general tax deductions but does for those of single mothers. $CT(x)$ denotes firm contributions to social security.[50] Because per-child subsidies and tax deductions are mutually exclusive I record them jointly in $PC(x)$. When a household receives child tax deductions, the savings consist of the difference in tax rates with and without deductions multiplied by taxable household income (before deductions) and direct tax savings on the amount deducted, which is equal to the new tax rate multiplied by the amount deducted. $CC(x)$ and $PL(x)$ are public expenses on childcare subsidies and

[50] In line with German regulations I set firm contributions equal to worker contributions at 20% of labor income.

parental leave pay respectively. In line with previously mentioned information, the gross cost of providing one childcare spot is set to € 1000 per month for a child below the age of three and € 700 per month for a child aged 3-6. $CC(x)$ is equal to the total gross cost of public childcare minus total private costs born by parents.

Define U as the average expected utility over a woman's life-cycle and F as average completed fertility. These objects are constructed by using population fractions as weights. The education-specific outcomes are denoted with subscript e. Furthermore, I use an underline to denote education-specific post-reform outcomes which serve as constraint values for the maximization. The first problem that maximizes fertility with solution x^F can be formally stated as:

$$\begin{aligned} &\max_x W^F(x) = F(x) \\ &\text{s.t.} \quad B(x) \geq \underline{B} \\ &\qquad U_e(x) \geq \underline{U}_e \quad \text{for } e = 1,2,3 \end{aligned} \tag{1}$$

Analogously, the solution x^U to the second problem maximizing utility solves:

$$\begin{aligned} &\max_x W^U(x) = U(x) \\ &\text{s.t.} \quad B(x) \geq \underline{B} \\ &\qquad F_e(x) \geq \underline{F}_e \quad \text{for } e = 1,2,3 \end{aligned} \tag{2}$$

To gain a sense of the effects and mechanisms of the different policy tools, I show changes in a number of outcomes for increasing the generosity of each policy by 1% of total government budget in Table 12.[51]

The changes are benchmarked to the outcomes under the new parental leave and childcare policies. The third section shows the average gross monthly earnings at age 45 by number of children. This measure contains effects from sorting (different women are induced to have a certain number of children) and changes in employment behavior conditional on number of children. The next two section show completed fertility by education level; the penultimate section shows the percentage of children born to married mothers, the percentage of childless women and the percentage of women with three

[51]In practice, I simulate the model for raising payment through each policy resulting in an approximate decrease of 1% in the budget and rescale to obtain effects for an exact change of 1%

Table 12: Effect of Investing 1% of Government Budget into Individual Policies

Outcome	Ed.	PL % +8%	PL lgt. +40 d.	PL min +€ 80	PL max +€ 600	CC -80%	Fixed +€ 10	TxD +€ 1.1k	STxD +€ 4.8k
Periods FT	-	-3.6	-4.5	-4.9	-1.9	-1.7	-4.1	-3.8	+0.5
Periods PT	-	-0.2	-0.3	-0.6	-1.7	+2.6	-0.8	+1.9	+1.2
Cml. Wages	1	-1.1	-1.7	-2.1	+.1	-.6	-3.0	-.3	-.1
(000s)	2	-1.1	-1.5	-1.3	-.1	-.2	-1.0	-1.2	+.5
	3	-1.6	-1.5	-1.2	-1.9	-.5	-.5	-1.8	-.9
Inc. $n=0$	-	+3	+10	-3	+16	+2	+3	+8	+16
Inc. $n=1$	-	+5	-4	+7	-10	+4	+4	+.4	+35
Inc. $n=2$	-	0	0	0	-6	-8	+7	-15	+26
Inc. $n=3$	-	-9	-7	-10	-2	+12	-7	+7	+12
Compl. Fert.	1	+.011	+.011	+.017	+.008	+.008	+.031	+.000	-.005
	2	+.010	+.008	+.011	+.010	+.007	+.007	+.003	-.004
	3	+.008	+.007	+.008	+.010	+.007	+.001	+.010	-.001
% Born Mar.	-	.0	-.1	-.2	.0	-.2	.0	.0	-.6
% Childless	-	-.3	-.3	-.1	-.4	-.2	-.2	-.3	-.5
% $n=3$	-	+.3	+.4	+.4	+.4	+.2	+.5	-.3	-.3
Lifetime Util.	1	+.05	+.05	+.05	+.03	+.06	+.09	+.05	+.13
	2	+.05	+.05	+.03	+.05	+.07	+.04	+.07	+.18
	3	+.03	+.03	+.02	+.05	+.06	+.02	+.05	+.18
S. mothers	-	+.01	+.08	+.04	+.01	+.11	+.08	+.06	+.38
M. mothers	-	+.01	+.05	+.06	+.01	+.09	+.12	+.09	+.13

Left to right: Parental leave replacemen!t rate (%), length (days), minimum and maximum € /mo., childcare subsidies (% decrease of cost), fixed per-child subsidies € /mo., tax deductions € /yr. and single mother tax deductions € /yr. First section: total number of years women work full or part-time (from ages 22-45). Second section: Discounted lifetime earnings ages 22-65. Third Section: Monthly income at age 45 by number of children. Penultimate section: % of children born to married mothers, % of women remaining childless, % of women having three children in total. Last two rows in last section: Average utility of a single and married mother with one child at age 22 with no work experience.

Table 13: Optimal Policies

	PL %	PL lgt.	PL min	PL max	CC	Fixed	TxD	STxD.
x^F	.283	-.04	0	1882	174%	208	3.6K	53.9K
x^U	.602	.03	212	2003	56%	137	4.1K	53.7K

x^F solution for max. fertility, x^U for max. utility. First four columns parental leave paymt. pars.: replacement rate (%), length (days), min. and max. amt./mo.; next columns: childcare cost (% of post-reform), fixed subsidy/mo., tax deductions/yr., tax deductions for single mothers/yr.

children. The last two numbers concern completed fertility. The last section contains expected utility by education level and for single and married mothers who have their first child at age 22. The last two measures can give insight as to whether policies benefit single or married mothers relatively more.

Generally, policies that pay a fixed amount, such as fixed subsidies and the minimum payment amount of parental leave, tend to increase fertility and utility for lower educated women, whereas policies for which the payment amount depends on income or affect the returns of employment such as tax deductions and the cap on parental leave payments do so for higher educated women. Another way to categorize policies is by whether they have a relatively large effect on utility or fertility. Policies that substantially increase utility tend to have a smaller effect on fertility and vice versa. Single mother tax deductions are an extreme example because they achieve a large increase in utility but have a negative effect on fertility. An opposite example is the minimum parental leave payment. Among the parental leave policy tools, changing the payment floor has the largest effect on fertility but the lowest on utility. Note that, policies which increase welfare more tend to also encourage employment (but not fertility). The reason is the underlying trade-off between fertility and female employment. Although women benefit from having children, a more cost-effective way of increasing welfare is to increase employment and thereby consumption. Increasing the number of children is very costly because it decreases employment for the remaining life-cycle and decreases per-capita consumption. It further increases government expenses for all child subsidies. Increasing employment on the other hand increases current and future earnings as well as tax revenue while decreasing fertility incentives.

Solutions for the optimal policies x^f and x^u are given in Table 13. Depending on the formulation of the problem the results for optimal policy are quite different. One common feature is to cut (almost) all taxes for single mothers, which I discuss in detail below. The solution to maximizing the objective of increasing average fertility, in addition to raising tax deductions for single mothers, stipulates a

Table 14: Outcomes under Optimal Policies

Outcome	Educ.	Current	x^F	x^U
Periods Full-Time	-	11.0	+.1	+.2
Periods Part-Time	-	6.3	-.3	-.1
Cumulative Wages	1	323.3	-7.3	+4.4
(000s)	2	433.1	+5.3	+8.4
	3	481.3	+7.9	+5.5
Earnings No Child	-	2358	+7	+3
Earnings Mothers	-	797	+6	+44
Completed Fertility	1	1.54	+.15	+.01
	2	1.58	+.04	+.00
	3	1.25	-.02	+.01
Pct. Born to Married	-	90.8	-5	-.6
Pct. Childless	-	22.0	+1.1	+.1
Pct. $n=3$	-	16.5	+8.0	+2.5
Life-Time Utility	1	159.90	+.12	+.04
	2	166.89	.00	+.22
	3	143.3	+.10	+.31

x^F solution for max. fertility, x^U for max. utility. Second section: discounted lifetime earnings in thousands of € , third section; average monthly earnings in € of women ages 22-45 with and without children, fifth section: % of children born to married mothers, % of women remaining childless, % of women having three children in total.

decrease the parental leave replacement rate, the elimination of a minimum payment threshold and an increase in fixed child subsidies. Moreover, childcare prices are raised by almost 75% and tax deductions per child are decreased by € 1.2k a year. I present the effect on several outcomes in Table 14. Overall fertility under this policy portfolio is 4% higher, with an increase of about 10% for women in the lowest education group while fertility of college-educated women decreases by 2%. Thus, compared to the pre-reform scenario, this set of policies achieves the twice the increase in fertility as the joint parental leave and childcare reforms. There is further an increase in utility for the highest and lowest education group equivalent to an increase in consumption of about 0.25%.

The policy increases the fraction of childless women across all education levels, while increasing the fraction of women with three children. To increase fertility, it is more cost-effective to target lower educated women and higher order births. The former relationship is immediate, lower educated women have lower income on average and gain more marginal utility from a given amount of subsidies. The second arises because the drop in women's earnings due the first child is larger than for additional children. This is driven by a more substantial decrease in labor supply for the first child. Most childless women work full-time while most women who have at least one child work part-time. Under the post reform regime, monthly earnings of women with no children are around € 1000 higher than for women with one child. Earnings of women with two children are less than € 300 lower than for women with one child.

These mechanisms are important for deriving the optimal set of policies. As Table 12 shows a higher parental leave replacement rate has a larger effect on women becoming first-time mothers, because they receive a greater increase in payments as second or third-time mothers. The solution also has high fixed subsidies, because, in addition to being the most cost-effective policy tool for increasing average fertility, positive effects on higher order fertility are particularly large. This is again due to the fact that mothers with many children tend to work less, hence benefit more from subsidies not conditional on employment. With the same reasoning it is evident that childcare subsidies favor mothers with fewer children over mothers with many children. Hours worked among mothers with one child under three years old are about 38% higher than for mothers with two or three children (and the youngest being less than three years old). Furthermore, for this government objective encouraging mothers to work is not desirable because it disincentivizes having additional children. This is why general child tax-deductions are lowered in the solutions, they decrease the proportion of mothers with three children.

Despite decreasing the payment through many of the employment-inducing policies, maternal earnings increase for the optimal policy. This is entirely driven by single-mothers who increase work hours by more than 2% as a response to large tax-deductions. Overall wages among mothers increases by about 15 cents per hour, because they take into account the possibility of divorce, and there is a small increase in the wages of childless women mostly due to selection of higher educated women into this group. Because they have fewer children, wages of the highest educated group increase. The minimum parental leave pay is decreased to zero and there is almost no change in the length of paid parental leave and the maximum cap. This former is likely due to increases in fixed childcare subsidies somewhat dominating increasing the payment floor for the purpose of this objective. The nature of these two policies is similar, they both benefit lower educated women more and have the largest effect on fertility among this group. The gain in utility is higher for increasing fixed subsidies across education groups, whereas the increase in fertility is more heavily skewed towards lower income women. which is advantageous.

Turning to the second specification of the government's problem, this policy set achieves an increase in overall utility equal to raising consumption by about 0.5%, with the gain being 0.7% for college-educated women and less than 0.1% for women in the lowest education group. Fertility changes little except for a small increase of 0.01 for women in the highest and lowest education groups. Life-time earnings increase for women of all education groups, this is predominantly due to an increase of about 1.2% in hours worked for mothers. This set of policies is similar to the existing policy, except for a larger increase in childcare subsidies (a decrease in cost by 55%) and in tax deductions for single mothers. This is financed through decreasing the parental leave payment floor by € 88, the replacement rate by 6.5%, the general tax deductions by around € 700 per year and fixed subsidies by € 12 per month. The payment cap is raised by about € 2k.

Intuitively, because fertility is constrained to stay at least constant for women of each education level, compared to the previous objective, the focus is not on increasing fertility incentives of women with low levels of education. Instead, the policy changes have to increase utility but leave relative fertility incentives of women with high versus low education constant. As discussed previously, higher utility is best achieved by increasing the generosity of policies that encourage employment in order to increase consumption.

After tax deductions for single mothers, childcare subsidies, as seen from Table 12, are the second most cost-effective policy to increase welfare. In addition to increasing employment, this program provides more subsidies to low income women who have a high marginal utility from consumption. Furthermore, to summarize from the previous section on policy evaluation, mothers of children below the age of three tend to earn lower income than mothers with older children. One reason is that these mothers tend to be younger themselves and have less work experience as well as lower income from their husbands. Another is that they are more likely to work part-time. Furthermore, uptake of childcare subsidies is particularly high among unmarried mothers, who tend to work more. This further amplifies the positive employment effects of the increase in tax deductions for single mothers.

The effects of the changes in the remaining policy parameters can be summarized as increasing employment incentives and benefiting higher educated women. The large amount of tax deductions for single mothers positively affect fertility of lower educated women and higher childcare subsidies are also relatively more significant in magnitude for lower educated women. This is why the other features in the optimal policy decrease payments in policies that have larger positive effects on fertility of lower educated women such as the parental leave replacement rate, the minimum payment and fixed subsidies. The policy increases maximum parental leave payments likely due to the converse reason.

I now summarizing the effects of the increase in tax deductions for single mothers. On average single mothers save around € 145 and € 141 in taxes per month, and monthly consumption increases by about 11% and 9% for the first and second solution respectively.[52] The mean income for single mother households is around € 15 000, even college-educated single mothers working full-time only earn about € 38 500. Hence, under the new policies single mothers pay almost no taxes. As indicated by the numbers in last column of Table 12, tax deductions for single-mothers are very cost-effective for increasing utility of women of all education levels. The last row shows a large utility increase for single mothers of around 0.38, approximately equivalent to permanently raising consumption by 0.7%, for spending an additional 1% of the government budget on single mother tax deductions. However, even for married mothers the increase in utility is substantial, higher than for spending the same amount of public funds on any other policy. This is because around one third of marriages end in divorce. There is large difference between the welfare of single and married mothers, such that insurance for the event of single-motherhood is very valuable for women.

[52]This includes effects from changes in other policy parameters, e.g. an increase in fixed subsidies for the first solution.

It is also interesting to highlight the results in the penultimate section of Table 12. First, tripling single mother tax deductions does not induce many more single women to have children. The fraction of births to single women only increases by about 0.6%. Second, there is a relatively large drop in the fraction of childless women. This is because even though large cuts in taxes make it more attractive for single women to have their first child, they also induces single mothers to work more and have fewer additional children. In fact, overall this policy has little effect on fertility. It decreases and increases fertility slightly for lower and larger amounts of deductions respectively. A potential concern with increasing tax deductions for single mothers substantially could be that couples might hide their relationship or not get married in order for the woman to receive higher tax deductions. This is difficult in practice, because only parents not living in the same household with another adult person (who is not their child) are eligible for this benefit.[53] Another issue could be higher rates of divorce, which the model does not account for. Divorce behavior should not be too responsive for women with children to the substantial psychic cost for single parents and the benefits from tax splitting, although I cannot rule out this possibility.[54]

7 Conclusion

In this paper I developed and estimated a dynamic discrete-choice model of fertility and female labor supply to assess the effects of a variety of family policies on women's behavior in Germany. In particular, I examined paid parental leave, childcare fixed per-child subsidies and per-child tax deductions including those exclusively for single-parents. The model allows for heterogeneous characteristics of women along several dimensions, namely education, marital transitions and unobserved preference heterogeneity in the cost of working as a mother. Depending on the characteristics, women have varying incentives and react differently to each of the policies. For instance, because higher educated women tend to work more and earn higher incomes, they respond more strongly to subsidies that increase in relation to earnings, such as tax deductions.

[53] It should not be hard for tax authorities to enforce this in practice.

[54] The psychic cost for single mothers with one child is around 0.2 utils per period. Additionally through marriage tax splitting couples can save up to € 6000 yearly if the woman does not work. On average per-capita period consumption of single mothers is 30% lower than that of married mothers. A tentative argument in favor of inelastic divorce decision is that when marital tax splitting was introduced in 1958 there appeared to be be no change in divorce rates. This development resulted in financial benefits for staying married of approximately the same magnitude as the proposed tax deductions would for divorcing. Another example of no divorce responses is when alimony for ex-spouses was lowered substantially in 2008.

I tested the validity of my model in a number of ways: I cross-checked labor supply and fertility elasticities and predicted policy effects with findings in the literature. I further compare simulated employment effects to regression results obtained using a large administrative data set. Then I used the model to evaluate the long-term welfare and fertility effects of two major recent changes in Germany family policies, a parental leave and a childcare reform. I found that the parental leave reform only had a positive effect on fertility of highly educated women and increases employment for lower educated women. Increasing childcare subsidies in contrast, affected women of different education levels similarly and increased fertility and maternal employment. The welfare effects were larger for the childcare reform and more concentrated among low-income groups such as single mothers and women with less education. Compared with the sum of the individual effects, implementing the two policies jointly can have larger or smaller effects for different outcomes and education groups.

In the last section, I proposed two sets of optimal policies that achieve different objectives, increasing fertility with a utility constraint and increasing utility with a fertility constraint. The optimal policies can achieve significant improvements compared to the post-reform scenario while maintaining a constant level of government spending. The first set of optimal policies increases fertility by 4% and the second set of policies increases welfare by a consumption equivalent of 0.5%. For the first objective it is more cost-efficient to increase fertility among lower educated women and for additional children. The solution therefore has a high fixed subsidy and low replacement rate. For the second objective, the solution increases employment and consumption by offering higher childcare subsidies as a cost-effective way of increasing welfare. Both solutions increase tax deductions for single women.

This paper provided an analysis of the complex incentives given in a system of family policies for women of varying characteristics and in different stages of their lives. The results can inform about the optimal design of such policies for different government objectives, taking into account dynamic interactions. As a next step, it would be interesting to conduct a complementary analysis to study whether firms respond to policy. Employers' beliefs about a woman's labor market attachment can potentially affect hiring or promotion decisions. Although Germany offers employees protection against discrimination, this might not hold in all cases or in other countries. It is left to investigate whether this channel could contribute to family policy effects.

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A Appendix

A.1 Identifying Births in the IAB Sample

Here I describe the procedure developed by Müller and Strauch (2017) to identify certain gap in the IAB data as births. Despite its drawbacks, this method offers the possibility of verifying the employment behavior of mothers of young children. Women going into maternity leave while receiving unemployment benefits are recorded directly as such. Women who leave employment when becoming mothers are recorded with a code indicating reason of termination of employment that also encompasses sickness. In order to select the cases of birth a series of restrictions is imposed. First, the woman has to appear in the data again some time after she left (hence a "gap"), otherwise it is possible she became self-employment or passed away. For a gap to be classified as a potential birth it can be no shorter than three months. This is because mothers have a mandatory resting time of two months ("Mutterschutz") that for most women begins 6-8 weeks prior to the predicted day of birth. Furthermore, the woman has to be below the age of 40 and if two gaps are spaced more closely than 224 days the second gap is deleted, because it is unlikely births are spaced too closely apart.

It should be noted that this method is not suitable to determine the number of children a woman has since many women might not be recorded in between births when out of the labor force, and hence large gaps in which two children were born would be classified as one birth. Also a range of births will not be captured. As mentioned above if a woman remained out of the labor force, moved out of the country or became self-employed following birth, the birth will not be identified. Lastly, some extended illnesses will be falsely labeled as births.

Once I identify all the mothers, I save a list of their IDs and the birth dates. Then I transform the dataset, which originally is structured according to spells into a panel using the code provided by (Eberle, Schmucker, and Seth, 2013). Then I merge the list of mothers with the panel and select for the women who had births in the first quarter of 2006 and 2007.

A.2 Childcare Costs

I model expected CC cost for children under three weighted sum of cost of public ι and private $1-\iota$ care. Furthermore the cost is decreasing in the birth order:

$$\zeta(n_t, a_t < 3, y_t^f + y_t^h) = \Big(\iota\, \zeta^{pu}(a_t < 3, y_t^f + y_t^h) + (1-\iota)\, \zeta^{pr}\Big)\Big(kid_t^1 + 0.66 kid_t^2 + 0.5 kid_t^3\Big)$$

In line with information from government reports, childcare for three to six-year-olds is not rationed. The equation for older children is therefore:[55]

$$\zeta(n_t, a_t \geq 3, y_t^f + y_t^h) = \zeta^{pu}\big(a_t \geq 3, y_t^f + y_t^h\big)\big(kid_t^1 + 0.66 kid_t^2 + 0.5 kid_t^3\big)$$

As mentioned in the main part of the paper private monthly cost ζ^{pr} is set to € 840. I estimate the public cost based on data from three years in the GSOEP for children in which detailed childcare questions were asked (1996, 2002, 2005). Because data is very limited for very young children, I estimate costs for children aged 3-6 and assume costs for children under three years old are twice of that. This seems to be a reasonable approximation (see e.g. Wrohlich (2011) and Hank and Kreyenfeld (2003)). The estimated equation for the monthly fee is:

$$\zeta^{pu}(a_t \geq 3, y_t^f + y_t^h) = 100 + 0.008\,(y_t^f + y_t^h)$$

Where $y_t^f + y_t^h$ is the monthly household labor income. I set the probability of obtaining a public childcare spot for young children ι to the fraction of children of working mothers that are in public childcare. As can be seen from Figure 4 below, only around 6-8% of children below the age of three were in public childcare during my sample period, while around 25% of mothers were employed. This sets a range of $.25 - .30$ for ι. Wrohlich (2011) estimates a value of about 0.63, taking into account informal childcare arrangements. Therefore I use the upper limit and set $\iota = \frac{1}{3}$.[56]

Figure 4 is created using data from the Federal Agency for Civic Education (2006-2012) and Hank and Kreyenfeld (2003)(1990-1999). It shows prior to 2006 the percentage of young children enrolled in public childcare was flat at 6-8% and then increased dramatically to 28% in 2012.

[55] Here the enrollment rate is above 80%

[56] In government statistics it is often not distinguished between full-time and part-time childcare spots, I therefor use overall enrolment and employment.

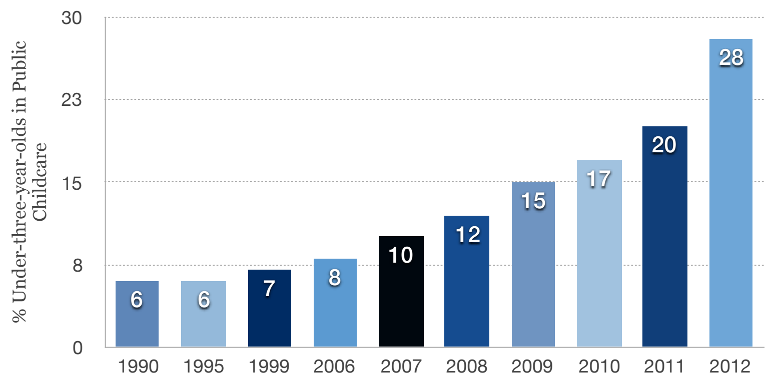

Figure 4: Public Childcare enrollment for Children Below the Age of Three

A.3 Budget Constraint Components

German Tax Code. Employees pay income tax and social security contributions on gross labor income. The share of social security contributions is 20% of wages for most, however there is a cap for very high earners and a workers with monthly income of less than 400 Euros are exempt.

I use the following piece-wise linear function of monthly individual labor income y to approximate the share:

$$ss(y) = \begin{cases} 0 & y \leq 400 \\ .00025y & 400 < y \leq 800 \\ .2 & 800 < y \leq 4300 \\ \frac{860}{y} & 4300 < y \end{cases}$$

The income tax rate is determined by the amount of taxable income which I take as income after social security contributions and child tax deductions. For a woman without children it is $y_s = y(1 - ss(y))$. The tax schedule is progressive and further depends on marital status. Married couples are taxed jointly as if each earned half of the joint income ("Splitting"), this leads to tax benefits especially when the difference between the spouses' income is large, for instance if one spouse does not work. I represent the tax rate on monthly taxable labor income of single individuals y_s, equal to gross income after social security contributions and tax deductions, as follows:

$$\tau(y_s) = \begin{cases} 0 & y_s \leq 700 \\ -.09 + .00015 y_s & 700 < y_s \leq 1700 \\ .075 + .000045 y_s & 1700 < y_s \leq 5000 \\ .4 + .00002 y_s & 5000 < y_s \leq 15000 \\ .5 & 15000 < y_s \end{cases}$$

The tax rate for married couples is $\tau(\frac{y_s + y_s^h}{2})$.

Social Assistance. German households are guaranteed a subsistence level depending on how many adults and children there are. Similar to Haan and Prowse (2017) I model this as an consumption floor using the following function:

$$S(m,n) = 1 + 0.75m + \big(0.65 + 0.2(1-m)\big)n$$

Note that a single parent receives higher benefits per child than a married household. Social assistance acts as a floor for disposal income if the sum of after-tax labor income, per-child benefits and alimony payments falls below. One exception is parental leave pay, which is not included in the calculation.

A.4 Simulation Starting Values

There are differences in employment and marital status at age 21 across education groups which I mimic in the simulation. Table 15 shows the distribution. Furthermore, almost half of all women of the highest education level have not completed schooling at age 25. Because almost no women have children when still in school I begin simulation when schooling is complete (this does not affect any women of lower education levels). Table 16 shows the initial age distribution of women in the highest education group when they first enter the simulation. I assume that during schooling women accumulate human capital as if working full-time. For example, a women who finishes her education when turning 28 has 6 units of human capital.

Table 15: Fraction of Employed and Married Women at Age 21 by Education Group

	low	med	high
Married	.182	.095	.033
Employed	734	.952	.951

Table 16: Age Distribution at Start of Simulation of Women in Highest Education Group

Age	22	23	24	25	26	27	28	29	30
Fraction	.33	.02	.04	.10	.09	.14	.08	.10	.10

A.5 Exogenous Data and Non-Structurally Estimated Parameters

Fecundity by Age. Probability of conception in one year p_t^f, conditional on trying, declines with the woman's age. The following data is taken from Khatamee and Rosenthal (2002).

Table 17: Yearly Probability of Conception by Woman's Age

Age	22	23	24	25	26	27	28	29	30	31	32	33
Prob.	.85	.83	.82	.80	.78	.76	.74	.72	.69	.65	.60	.58

Age	34	35	36	37	38	39	40	41	42	43	44	45
Prob.	.56	.54	.52	.5	.47	.44	.39	.35	.28	.21	.13	.05

First-Step Estimates of Marital Transitions and Husband's Income. Table 18 shows the logit coefficients for marital transition rates for women aged 45 or below. For older women I do not allow for further marital transitions. Similarly the non-random part of a husband's income is assumed to be fixed after the age of 45. The estimates are given in Table 19.

Table 18: Marriage and Divorce Rate by Education Group

	Marriage			Divorce		
	low	med	high	low	med	high
$\theta_{0m,e}$ Intercept	-2.822	-3.508	-4.851	-3.673	-3.810	-3.639
	(.218)	(.145)	(.341)	(.318)	(.34)	(.644)
$\theta_{1m,e}$ Age	.254	.574	.580	-2.822	.017	-.038
	(.091)	(.058)	(.115)	(.064)	(.074)	(.095)
$\theta_{2m,e}$ Age Sq.	-.025	-.058	-.035	-.014	-.001	.001
	(.010)	(.006)	(.011)	(.003)	(.003)	(.004)
$\theta_{3m,e}$ Age Cb.	.001	.002	.001	-	-	-
	(.0003)	(.0002)	(.0003)			

Table 19: Husband's Income by Education Group

	low	med	high
ϕ_{0e} Intercept	22.764	22.713	20.994
	(.218)	(.145)	(4.071)
ϕ_{1e} Age	.254	.574	4.829
	(.091)	(.058)	(.648)
ϕ_{2e} Age Sq.	-.025	-.058	-.035
	(.010)	(.006)	(.011)
ϕ_{3e} Age Cb.	.001	.002	.001
	(.0003)	(.0002)	(.0003)

A.6 Model Fit

Moments Here I list the complete set of data moments, their simulated counterparts, the data standard deviation and the difference normalized by the standard deviations in Tables 20-34.

Table 20: Avg. Fertility By Education and Age

Education Level	Ages	Model	Data	Std. Dev	Norm. Diff.
1	22−29	0.505	0.4676	0.5937	0.063
	30−36	1.1336	1.2018	0.5347	0.128
	37−45	1.5021	1.4817	0.5076	0.040
2	22−29	0.3667	0.3106	0.5130	0.109
	30−36	1.0366	1.0838	0.5130	0.092
	37−45	1.508	1.4329	0.5041	0.149
3	22−29	0.0999	0.0976	0.5275	0.004
	30−36	0.5585	0.6342	0.5094	0.149
	37−45	1.1085	1.1693	0.5034	0.121

Table 21: Fertility Transition Rates by Education, Marital Status and Age

Ed. Level	Number of Children	Age	Model	Data	Std. Dev	Norm. Diff.
1	None to One Married	22−29	0.080	0.028	0.156	0.328
		30−36	0.222	0.135	0.345	0.254
		37−45	0.318	0.259	0.440	0.134
	One to Two Married	22−29	0.050	0.026	0.127	0.191
		30−36	0.164	0.138	0.334	0.079
		37−45	0.242	0.179	0.377	0.168
	Two to Three Married	22−29	0.023	0.015	0.108	0.075
		30−36	0.038	0.035	0.184	0.012
		37−45	0.024	0.063	0.235	0.165
	None to One Single	22−29	0.000	0.004	0.089	0.051
		30−36	0.001	0.039	0.192	0.199
		37−45	0.031	0.036	0.193	0.026
	One to Two Single	22−29	0.000	0.002	0.105	0.017
		30−36	0.005	0.032	0.198	0.138
		37−45	0.034	0.048	0.238	0.057
2	None to One Married	22−29	0.112	0.052	0.195	0.309
		30−36	0.216	0.218	0.392	0.006
		37−45	0.269	0.228	0.427	0.094
	One to Two Married	22−29	0.066	0.035	0.174	0.180
		30−36	0.196	0.169	0.351	0.077
		37−45	0.234	0.193	0.366	0.113
	Two to Three Married	22−29	0.030	0.011	0.085	0.224
		30−36	0.045	0.038	0.185	0.040
		37−45	0.022	0.083	0.232	0.266
	None to One Single	22−29	0.001	0.010	0.129	0.070
		30−36	0.003	0.024	0.168	0.127
		37−45	0.025	0.018	0.169	0.046
	One to Two Single	22−29	0.000	0.011	0.093	0.111
		30−36	0.002	0.049	0.212	0.223
		37−45	0.015	0.052	0.214	0.173
3	None to One Married	22−29	0.118	0.078	0.300	0.131
		30−36	0.249	0.199	0.419	0.119
		37−45	0.191	0.159	0.396	0.081
	One to Two Married	22−29	0.053	0.104	0.283	0.179
		30−36	0.197	0.199	0.399	0.005
		37−45	0.207	0.156	0.389	0.133
	Two to Three Married	22−29	0.033	0.032	0.144	0.002
		30−36	0.079	0.056	0.243	0.093
		37−45	0.040	0.048	0.274	0.030
	None to One Single	22−29	0.002	0.015	0.136	0.092
		30−36	0.012	0.024	0.169	0.071
		37−45	0.013	0.006	0.108	0.065
	One to Two Single	22−29	0.000	0.012	0.149	0.078
		30−36	0.001	0.058	0.227	0.253
		37−45	0.002	0.083	0.247	0.327

Table 22: Fraction of Population By Marital Status, Family Status, Age and Education

Marital/Family Status	Ed. Level	Age	Model	Data	Std. Dev	Norm. Diff.
Married with No Child	1	22−29	0.072	0.086	0.262	0.051
		30−36	0.631	0.593	0.499	0.076
		37−45	0.164	0.204	0.370	0.108
	2	22−29	0.101	0.103	0.291	0.009
		30−36	0.553	0.532	0.499	0.043
		37−45	0.117	0.166	0.405	0.122
	3	22−29	0.111	0.147	0.355	0.102
		30−36	0.272	0.252	0.434	0.047
		37−45	0.097	0.084	0.305	0.043
Married with Children	1	22−29	0.063	0.066	0.257	0.011
		30−36	0.643	0.607	0.498	0.073
		37−45	0.143	0.182	0.434	0.089
	2	22−29	0.133	0.117	0.296	0.054
		30−36	0.538	0.523	0.500	0.029
		37−45	0.094	0.130	0.389	0.092
	3	22−29	0.117	0.150	0.346	0.096
		30−36	0.197	0.186	0.397	0.030
		37−45	0.084	0.051	0.277	0.118
Single with Children	1	22−29	0.104	0.149	0.277	0.163
		30−36	0.560	0.500	0.500	0.121
		37−45	0.101	0.154	0.410	0.131
	2	22−29	0.145	0.168	0.335	0.069
		30−36	0.313	0.327	0.485	0.031
		37−45	0.084	0.081	0.334	0.009
	3	22−29	0.049	0.094	0.297	0.151
		30−36	0.040	0.059	0.286	0.068
		37−45	0.048	0.021	0.181	0.147

Table 23: Full-time Rate By Education and Age

Education Level	Age	Model	Data	Std. Dev	Norm. Diff.
1	22−29	0.547	0.574	0.500	0.054
	30−36	0.368	0.341	0.458	0.060
	37−45	0.324	0.287	0.208	0.175
2	22−29	0.684	0.739	0.460	0.121
	30−36	0.418	0.411	0.496	0.013
	37−45	0.32	0.321	0.407	0.002
3	22−29	0.877	0.868	0.363	0.025
	30−36	0.598	0.620	0.493	0.044
	37−45	0.396	0.453	0.405	0.140

Table 24: Part-time Rate By Education and Age

Education Level	Age	Model	Data	Std. Dev	Norm. Diff.
1	22−29	0.174	0.143	0.369	0.084
	30−36	0.261	0.277	0.464	0.036
	37−45	0.419	0.442	0.208	0.107
2	22−29	0.153	0.111	0.338	0.125
	30−36	0.244	0.297	0.463	0.115
	37−45	0.414	0.465	0.407	0.127
3	22−29	0.068	0.074	0.282	0.022
	30−36	0.200	0.206	0.429	0.014
	37−45	0.371	0.351	0.403	0.051

Table 25: Avg. Monthly Wage By Education and Age

Education Level	Age	Model	Data	Std. Dev	Norm. Diff.
1	22−29	1,739.51	1,815.070	1,109.000	0.068
	30−36	2,036.76	2,052.400	1,865.540	0.008
	37−45	2,092.93	2,087.510	1,274.980	0.004
2	22−29	2,059.33	2,146.700	1,341.490	0.065
	30−36	2,631.19	2,520.390	1,612.700	0.069
	37−45	2,631.43	2,510.420	1,088.190	0.111
3	22−29	1,999.50	2,070.100	1,474.600	0.048
	30−36	2,901.38	2,982.420	2,318.510	0.035
	37−45	3,320.19	3,569.720	2,067.650	0.121

Table 26: Log Wage Regression Coefficients

Regressor	Model	Data	Std. Dev	Norm. Diff.
Constant	2.636	2.583	0.079	0.664
Ed. Level 2 Constant	0.144	0.170	0.070	0.373
Ed. Level 3 Constant	−0.035	−0.073	0.120	0.314
Full-time Exp.	0.043	0.061	0.015	1.169
Full-Time Exp. Sq.	0.000	−0.002	0.001	1.556
Part-time Exp.	−0.043	−0.029	0.020	0.669
Part-Time Exp. Sq.	0.002	0.003	0.001	1.141
Ed. Level 3 FT	0.019	0.032	0.011	1.236
Ed. Level 3 PT	−0.003	0.026	0.018	1.616

Table 27: Log Wage Increase By Education and Past Employment

Education Level	Emp. Level	Model	Data	Std. Dev	Norm. Diff.
1	FT	0.031	0.023	0.491	0.016
	PT	0.011	0.014	0.667	0.005
2	FT	0.047	0.042	0.619	0.009
	PT	−0.001	0.012	0.658	0.019
3	FT	0.053	0.063	0.488	0.019
	PT	0.011	0.048	0.638	0.058

Table 28: Work Transition Rates By Education, Fertility Status and Current Work Status

Transition Type	Ed. Level	Model	Data	Std. Dev	Norm. Diff.
No Child	1	0.311	0.197	0.412	0.278
Work-to-Work	2	0.323	0.322	0.473	0.002
	3	0.428	0.365	0.493	0.128
No Child	1	0.933	0.951	0.216	0.085
No Work-to-Work	2	0.935	0.965	0.189	0.159
	3	0.958	0.975	0.154	0.114
Child	1	0.249	0.130	0.348	0.341
Work-to-Work	2	0.264	0.164	0.414	0.242
	3	0.311	0.198	0.419	0.271
No Child	1	0.852	0.896	0.290	0.152
No Work-to-Work	2	0.860	0.902	0.273	0.155
	3	0.862	0.893	0.267	0.117

Table 29: Employment By Age of Youngest Child - Education Level 1

Child's Age	Emp. Level	Model	Data	Std. Dev	Norm. Diff.
No Child	FT	0.833	0.790	0.427	0.102
	PT	0.127	0.104	0.329	0.072
0	FT	0.010	0.037	0.295	0.093
	PT	0.092	0.081	0.442	0.025
$1-2$	FT	0.056	0.057	0.295	0.002
	PT	0.273	0.226	0.442	0.107
$3-6$	FT	0.095	0.115	0.354	0.055
	PT	0.355	0.380	0.481	0.053
$7-11$	FT	0.178	0.164	0.371	0.037
	PT	0.477	0.457	0.499	0.041
11+	FT	0.304	0.254	0.443	0.114
	PT	0.568	0.500	0.499	0.136

Table 30: Employment By Age of Youngest Child - Education Level 1

Child's Age	Emp. Level	Model	Data	Std. Dev	Norm. Diff.
No Child	FT	0.848	0.900	0.348	0.148
	PT	0.141	0.066	0.292	0.255
0	FT	0.020	0.037	0.295	0.058
	PT	0.110	0.099	0.442	0.026
1−2	FT	0.100	0.069	0.360	0.085
	PT	0.278	0.307	0.468	0.061
3−6	FT	0.162	0.095	0.434	0.154
	PT	0.379	0.440	0.497	0.123
7−11	FT	0.247	0.200	0.449	0.106
	PT	0.452	0.589	0.500	0.275
11+	FT	0.339	0.365	0.496	0.051
	PT	0.553	0.465	0.492	0.178

Table 31: Employment By Age of Youngest Child - Education Level 3

Child's Age	Emp. Level	Model	Data	Std. Dev	Norm. Diff.
No Child	FT	0.871	0.886	0.318	0.046
	PT	0.115	0.085	0.290	0.105
0	FT	0.037	0.084	0.295	0.161
	PT	0.146	0.152	0.442	0.015
1−2	FT	0.143	0.117	0.394	0.065
	PT	0.329	0.317	0.486	0.026
3−6	FT	0.190	0.122	0.443	0.153
	PT	0.438	0.464	0.500	0.052
7−11	FT	0.324	0.378	0.466	0.117
	PT	0.440	0.420	0.500	0.039
11+	FT	0.446	0.420	0.500	0.052
	PT	0.470	0.457	0.499	0.026

Table 32: Employment by Number of Children

No. of Children	Emp. Level	Model	Data	Std. Dev	Norm. Diff.
0	FT	0.849	0.860	0.283	0.038
	PT	0.133	0.081	0.283	0.183
1	FT	0.281	0.280	0.480	0.001
	PT	0.389	0.350	0.480	0.081
2	FT	0.135	0.130	0.339	0.013
	PT	0.416	0.425	0.499	0.017
3	FT	0.081	0.060	0.235	0.091
	PT	0.307	0.330	0.495	0.046

Table 33: Employment Rate of Single Women By Education and Age of Youngest Child

Education Level	Child's Age	Model	Data	Std. Dev	Norm. Diff.
1	No Child	0.958	0.917	0.304	0.134
	≤ 3	0.426	0.493	0.492	0.136
	> 3	0.708	0.694	0.465	0.030
2	No Child	0.991	0.969	0.200	0.115
	≤ 3	0.616	0.584	0.494	0.064
	> 3	0.784	0.878	0.370	0.255
3	No Child	0.987	0.978	0.154	0.061
	≥ 3	0.782	0.656	0.430	0.293
	> 3	0.892	0.872	0.287	0.068

Table 34: Employment Rate of Married Women By Education and Age of Youngest Child

Education Level	Child's Age	Model	Data	Std. Dev	Norm. Diff.
1	No Child	0.969	0.838	0.355	0.369
	≤ 3	0.377	0.353	0.474	0.050
	> 3	0.655	0.598	0.489	0.118
2	No Child	0.982	0.958	0.242	0.099
	≤ 3	0.414	0.445	0.503	0.061
	> 3	0.672	0.692	0.422	0.048
3	No Child	0.980	0.937	0.216	0.199
	≥ 3	0.487	0.541	0.498	0.108
	> 3	0.677	0.751	0.423	0.174

Out-of-Sample Fit Here I verify the predictions of the model for a selection of outcomes not directly used in the estimation. Because many employment-related outcomes are included as moments I present monthly earnings as a composite measure of employment and wages. I further show total periods worked full and part-time and measures for birth timing and spacing. The model captures the overall level of employment until age 45 well. It can also replicate the delay in first births with the level of education and the increase in spacing from the first to the second child and the second to the third, although it slightly overestimates the gap.

I examine earnings, employment and age of youngest child for older women because behavior past the age of 45 matters the analysis of optimal policy. The number of older women is limited in my sample, so I do compute mean outcomes for women aged 50. The fit for the different measures across education group is good, the only larger deviation is for the earnings of college-educated women. One concern to bear in mind is that there might be considerable cohort effects for the data means.

Table 35: Timing and Spacing of Births

	Ed. Level	Model	Data
Age at First Birth	1	27.4	26.8
	2	28.7	28.6
	3	32.1	31.5
Spacing 1st-2nd Child	-	4.1	3.7
Spacing 2nd-3rd Child	-	4.5	4.2

Table 36: Monthly Earnings of Women by Family and Marital Status

	Model	Data
No Children	2367	1980
Married with Children	665	692
Single with Children	1217	1038

Table 37: Periods Worked until Age 45 and at Birth

	Ed. Level	Model	Data
Full-Time	1	9.7	9.5
until 45	2	11.4	11.6
	3	14.9	15.0
Part-Time	1	6.2	5.8
until 45	2	5.2	4.6
	3	32.1	31.5
Full-Time	1	4.3	4.4
at Birth	2	5.9	5.6
	3	8.9	8.0

Table 38: Outcomes at Age 50

	Ed. Level	Model	Data
Monthly Earnings	1	1692	1621
	2	1947	1925
	3	2609	2216
Employment Rate	1	0.93	0.88
	2	0.93	0.93
	3	0.96	0.91
Age of Youngest Child	1	15.3	16.2
	2	17.6	19.1
	3	19.5	20.8