Borrowing Constraints, Search, and Life-Cycle Inequality[1]

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First Draft: Sept., 2015
This Draft: Apr., 2019
Most Recent Version

Abstract

I develop and estimate a life-cycle directed search model in which risk-averse workers face a repeated portfolio allocation decision between precautionary savings and accumulating human capital, and use the model to study the sources of inequality. In the model, income uncertainty decreases the wage selectivity of borrowing constrained workers and alters the human capital growth of all workers. I show that unemployment risk has a persistent impact on low-wealth workers, causing a precautionary reallocation away from human capital accumulation toward savings to mitigate the cost of an unemployment spell. I find supporting evidence in the data and then estimate the model using indirect inference. While a competitive labor market would imply small effects of wealth, I find that initial wealth causes 3.3% of inequality in earnings and that initial conditions cause 51.5% in total. The precautionary response of low-wealth workers to unemployment risk causes 1.1% of the inequality in human capital, and a decline of 0.8% on average over the life-cycle. Redistributive policies, including student debt relief (1.4% to 3.5% among attendees), the Earned Income Tax Credit (1.4%), and universal basic income (5.6%), are shown to decrease the persistence of inequality.

JEL Classification: E21, E24, J63, J64, D31, I32, J31

Keywords: Directed Search, Borrowing Constraints, Inequality, Human Capital

[1]I thank Peter Rupert, Finn Kydland, Javier Birchenall, Peter Kuhn, and Marek Kapicka for their guidance and support. I am also grateful to Kyle Herkenhoff, Thomas Cooley, Mark Huggett, Marcus Hagedorn, Toni Braun, David DeJong, Pedro Gomis-Porqueras, Dick Startz, Kelly Bedard, Eric Young, Martin Gervais, Tiemen Woutersen, Shu Lin Wee, and David Wiczer for helpful advice and comments, as well as seminar participants at the University of California at Santa Barbara, Washington University St. Louis, University of Leicester, Midwest Macro at Purdue University, Southwest Search and Matching at Simon Fraser University, the Vienna Macro Workshop at NYU Abu Dhabi, the University of Melbourne, Midwest Macro at University of Pittsburgh, The Federal Reserve Bank of Atlanta, and the Vigo Macro Workshop. I would also like to thank the Center for Scientific Computing at UCSB (CNSC) and NSF Grant CNS-0960316 for making this project computationally feasible.

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1 Introduction

It is well-documented that individuals take precautionary measures to mitigate consumption risk in the presence of uncertainty. Employed workers build savings early in their careers, and unemployed workers replace income with unemployment insurance and savings. Once their benefits expire and they begin to exhaust their savings, unemployed workers reduce their selectivity among job offers in order to smooth consumption. These are choices that reduce current consumption and future income, but decrease the variability of a worker's consumption flows over the life-cycle. In this paper, I demonstrate that exposure to income and consumption uncertainty amplify the life-cycle importance of initial wealth inequality, and cause permanently depressed earnings and human capital among workers who are affected by borrowing constraints.

Many workers enter the labor market with negative wealth and a capacity for borrowing that is either limited or entirely restricted. A worker at their first full-time job spends 18% of their income servicing debt, and more than 40% have been denied access to credit (Survey of Consumer Finances, 2013). I explore the role that exposure to consumption risk early in the life-cycle due to borrowing constraints and differences in wealth has on the sources of consumption and earnings inequality. To do this, I construct and estimate a quantitative model that includes initial heterogeneity in wealth as well as human capital and learning ability. While initial wealth is a smaller source of inequality than human capital or learning ability (3.27%, versus 7.19% and 44.34%, respectively), market imperfections amplify its impact on labor market outcomes as well as human capital accumulation, reducing averages (1.59%, 1.11%) and increasing inequality (3.27%, 2.51%). Even absent a separation, the risk of unemployment depresses human capital accumulation over the life-cycle by 0.79%, and increases inequality by 1.15%. Redistributive policies that reduce consumption risk, including student debt relief and universal basic income yield beneifts sizeable benefits by enabling borrowing-constrained workers to find better jobs and accumulate additional human capital.

This differs from previous research that suggests a limited (Huggett et al., 2011) or non-existant (Heathcote et al., 2014) role for initial wealth in determining earnings. What differentiates my work is that I assume workers face search frictions, in the spirit of Mortensen and Pissarides, while the previous literature assumed workers are paid competitively.[3] I in-

[3]A few recent papers have explored the impact of wealth inequality (Yang, 2018) and debt (Ji, 2017), but focus on college attendance as the source of inequality.

troduce a tractable model in which workers face uncertainty over income and consumption due to labor and asset market imperfections as well as unemployment risk.[4] I extend a life-cycle model of on-the-job directed search with wage posting (Menzio et al., 2016) to include risk-averse workers, borrowing constraints, and endogenous human capital accumulation.[5] I allow employed workers to make a portfolio allocation decision each period between spending productive time to accumulate human capital, a risky asset, and building riskless precautionary savings.[6] While incomplete asset markets limit the ability of workers to borrow in order to smooth consumption, they can take precautionary measures by altering their job search or by accumulating savings. Directed search allows workers to choose their exposure to consumption risk by searching for jobs with lower wages, but shorter expected durations of unemployment. Once employed, workers can choose the share of their income to devote between precautionary savings and human capital.

Risk plays an important role in both worker wage selectivity and subsequent human capital accumulation, which interact to further depress human capital accumulation among low-wealth workers. Unemployed workers are able to control their exposure to consumption risk by accepting low-wage jobs that offer a high job-finding rate in equilibrium. While this mitigates the immediate uncertainty associated with an extended unemployment spell, it decreases earnings until the worker can move up the job ladder.[7] Different sources of uncertainty are also important in the accumulation of human capital. The rate of return on human capital is uncertain at the time of investment due to stochastic depreciation. More importantly, human capital does not provide insurance against consumption risk in the event of an unemployment spell. Because low-wealth unemployed workers have no alternative but to accept low-paying employment to smooth consumption, they choose to build precautionary

[4] I achieve tractability by employing a Block Recursive Equilibrium, first described by Shi (2009) and Menzio and Shi (2011).

[5] Other papers that incorporate risk aversion and some form of incomplete asset markets include Burdett and Coles (2003), Lentz and Tranaes (2005), Costain and Reiter (2008), Chaumont and Shi (2017), Krusell et al. (2010),Lise (2013),and Herkenhoff (2014). The latter two are the most closely related, but neither includes human capital accumulation.

[6] To my knowledge, this is the first paper that features a portfolio allocation over Ben-Porath (1967) human capital and precautionary savings in a search environment, but not the first to include Ben-Porath human capital in the search literature(Bowlus and Liu, 2013). Several papers introduce learning-by-doing in search models with risk-neutrality (Bagger et al. (2014) and Yamaguchi (2010)) or complete markets (Carillo-Tudela, 2012), or the analyze the business cycle Herkenhoff et al. (2016), but neither focuses on the importance of risk on inequality. Graber and Lise (2015) includes risk-averse workers, but assumes human capital production is determined by the choice of firms.

[7] Low et al. (2010) use a search model to analyze the impact of wage and employment risk on consumption and welfare. They find that for all education levels, workers are willing to decrease consumption (by 19.2%) instead of experience a 50% increase in wage volatility.

savings while employed instead of accumulating human capital. The effect of unemployment risk on human capital accumulation alone accounts for a 0.79% decline in average human capital, and increases inequality by 1.15%.[8]

To motivate the theory, I use the Survey of Income and Program Participation (SIPP) to show evidence that borrowing constraints alter earnings following an unemployment spell. Similar to Chetty (2008), I exploit differences in replacement rates across states to estimate the differential effect that unemployment insurance replacement rates have on constrained and unconstrained households. I find that workers whose reported wealth places them in the lowest quintile during an unemplyoment spell exhibit a strong response to more generous unemployment insurance. When UI is increased by 10%, these workers find jobs associated with a 4% increase in quarterly earnings, while higher wealth quintiles show no response to more generous UI.

I estimate the model using indirect inference. Indirect inference is a method of moments estimator in which the targets are coefficients from a set of reduced-form specifications that approximate the equilibrium of the structural model. I use findings from the SIPP as well as life-cycle earnings and job transition statistics from the Panel Study of Income Dynamics (PSID) and the National Longitudinal Study of Youth 1979 (NLSY) to discipline key decision rules in my model. I show that the moments from the SIPP yield a piece-wise linear approximation to worker application strategies, while moments from the PSID and NLSY provide inference on the correlations between wealth, human capital growth, and earnings over the life-cycle. I test the fit of the estimated model and find that the model fits the data well.

My findings show that when low-wealth workers are subject to borrowing constraints, they experience persistently worse labor market outcomes than equally capable, but wealthier peers. While initial wealth plays a smaller role (3.27%) than human capital (7.19%) and learning ability (44.34%), it is sizeable given that it plays no direct role in the earnings process. After decomposing inequality, I run a series of policy experiments. Among policies whose stated goal is to decrease poverty and income uncertainty, relieving student debt would increase earnings by between 1.4% and 3.5% and human capital by 1.80% for in-debt college attendees, while the earned income tax credit and universal basic income both provide benefits in excess of the cost of their programs, increasing earnings by 1.37% and

[8]Using an incomplete markets model with a savings-human capital portfolio allocation, Krebs (2003) shows that in allocation, income uncertainty causes human capital growth to slow by 0.13% to 0.83%, nearly identical to my findings.

5.59%, respectively.

The paper is organized as follows: in Section 2 I document liquidity effects on re-employment wages for constrained groups and discuss evidence for the impact of unemployment risk on human capital accumulation. In Section 3, I construct a model that incorporates these findings, and show the equilibrium. In Section 4, I explain the functional form and parameter assumptions, and my construction of targets for indirect inference. In Section 5, I decompose the implications for life-cycle inequality, and compare my findings to the existing literature. In Section 6, I explore the ability of common (UI, EITC), and controversial (UBI, student debt relief) policies to address the sources of inequality in the model. Lastly, in Section 7 I summarize my contributions and discuss routes for future work.

2 Empirical Regularities

Isolating the impact of wealth on earnings and employment outcomes is subject to a clear empirical hurdle: unobserved ability is presumed to be positively correlated with observed wealth, leading (rightfully) to endogeneity concerns should I estimate the impact of wealth on outcomes. Instead, I focus on policies that relax a worker's borrowing constraint or supplement their income, but are likely to be uncorrelated with their individual ability. These policies linearly affect a worker's budget, meaning that the estimated effect can be interpreted as the slope of a line tangent to the worker's constrained optimization. I exploit this intuition in Section 4 when I estimate the model.

I divide the empirical regularities that motivate my study into two areas: evidence for the impact of borrowing constraints on worker selectivity along wages, and evidence for the impact of consumption and unemployment risk on human capital accumulation. Some of these findings are novel to this paper, while some are restated from related work. Briefly, I find constrained individuals who receive more generous unemployment insurance replacement rates match to higher-paying jobs following an unemployment spell.[9] Once employed, I find that uncertainty depresses the growth rate of earnings, and differences in initial wealth are associated with lower earnings and a smaller fraction of employable time spent in activities associated with human capital accumulation. My findings on job search are presented in Section 2.1, and my findings on life-cycle outcomes and human capital are presented in

[9] While there is previous evidence for the effect of borrowing constraints or liquidity effects on unemployment outcomes, (Herkenhoff et al. (2016) on earnings, Chetty (2008) on durations, among others) the effects on re-employment earnings is sparse.

Section 2.2.

2.1 Borrowing Constraints and Worker Selectivity

Although theory makes strong predictions about the effect of borrowing constraints on employment outcomes (Lise (2013), Chatterjee et al. (2007), among others), the empirical literature has found mixed evidence. There are two notable exceptions on which I rely: Herkenhoff et al. (2016) and Nekoei and Weber (2017), both of which provide evidence that increasing access to resources (credit and UI benefits, respectively) improve outcomes following an unemployment spell.[10]

To briefly summarize their approach and findings, Herkenhoff et al. (2016) use matched administrative employer-employee records from the Longitudinal Employer Household Dynamics (LEHD) dataset linked to an administrative dataset on credit access provided by TransUnion. They focus on mass layoffs, and find that an increase in an individual's credit limit equal to 10% of their previous annual earnings translates into a 0.2 to 1.34 percent increase in re-employment earnings, and that affected workers take between 0.33 and 1.24 weeks longer to find a job. They also find that workers with better access to credit match to firms that offer higher average wages and have higher than average productivity. The second, Nekoei and Weber (2017), exploit an age-based discontinuity in the Austrian unemployment insurance system to show that extending the duration of unemployment benefits lead to improvements in re-employment wages. They find that workers respond along both a search effort margin, and a selectivity margin, and that the size of each is heterogeneous across the pool of unemployed. Overall, their findings suggest that benefit extensions of about 33% lead to an increase in average post-unemployment wages of 0.5%, which persists through the duration of the first job after unemployment. They find that workers are both less likely to accept jobs that entail a large (40% or more) decrease in earnings relative to previous employment, and more likely to accept a job that offers an increase between 0 and 10%. I provide additional complementary evidence to show that the largest effects are felt among the least-wealthy unemployed in the following sections.

[10]Related work on student debt is similarly divided. Gervais and Ziebarth (2017) exploits a kink in subsidized stafford loan eligibility to show that an extra $1,000 in student loan debt at graduation decreases earnings by 2.5%. Luo and Mongey (2017) and Rothstein and Rouse (2011) use variation in the ratio of grants to total loans across cohorts, but within institutions. These papers find that debt increases earnings after graduation (1.21% in Luo and Mongey and $978 in Rothstein and Rouse). While the samples differ from those I employ here (all education groups, previous labor market experience), additional exploration would prove valuable to better determine the sources.

2.1.1 Empirical Strategy

To explore the effects of borrowing constraints on labor market outcomes, I estimate the responsiveness of constrained (using liquid wealth as a proxy) individuals to changes in their unemployment insurance replacement rates. I Find that the elasticity of the re-employment wage with respect to unemployment insurance amount is substantial for constrained individuals, but has no effect for unconstrained individuals. I use Survey of Income and Program Participation (SIPP) panels from 1990-2008, as well as data from state unemployment insurance laws provided by the Employment and Training Administration. I restrict my sample to 23 and older males who take up UI within one month of unemployment. More details on the construction of this data is available in Section A.1.

Like the previous literature (Browning and Crossley (2001), Bloemen and Stancanelli (2005), Sullivan (2008), and Chetty (2008), among others), I proxy for the degree to which a worker is constrained using their liquid wealth position.[11] I also follow an approach taken by the previous literature to deal with benefit mismeasurement and use state-month average UI benefits to proxy for individual UI benefits. I also include potential UI duration, defined as the average number of weeks a cohort of unemployed individuals could receive UI, at a state-by-quarter frequency to capture any correlation between replacement rates and duration generosity for a state unemployment insurance system. This means that I am exploiting within-state variation in benefit levels to identify the effect of unemployment insurance. Table C.1 summarizes key employment and demographic characterics by liquidity quintile and UI generosity, and shows that there are no significant differences within a quintile across UI generosity.[12]

My approach to measure the effect of unemployment insurance on re-employment wages is to use a Mincer equation and stratify the sample of unemployed individuals into quintiles of liquid wealth.[13] I include age, race, marital status, education, tenure, industry and occupation (each at 2-digit level), as well as state and year fixed effects. I include interaction

[11] These papers find that unemployment insurance is used as a substitute for income during unemployment spells among illiquid households, which motivates the use of net liquidity as a proxy for borrowing constraints.

[12] Selecting on unemployment insurance recipients may cause bias in my estimates; however, per Table C.1, the rates of UI takeup do not vary across wealth quintiles, which suggest that endogenous takeup is not driving the following results that I find for individuals from the first quintile. Within the first quintile takeup in below median UI states is lower than in states above the median, contrary to what we would expect if the recipients selected along liquidity needs.

[13] In other words, I exploit variation in unemployment insurance over time that is not the result of previous income, UI duration, or choice of location. Similar identification strategies are employed by Engen and Gruber (2001), Chetty (2008), among others.

terms between liquidity quintile and education, to control for the costs of college attendance, and a log-wage spline. My main test uses the following specification:

$$ln(Y_{i,j+1,s,t}) = \alpha_0 + \sum_{q=1}^{5} \delta_0^q \times ln(UI_{s,t}) + \sum_{q=1}^{5} \delta_1^q \times \text{UIDur}_{s,t} \tag{2.1}$$

$$+ \delta_s + \delta_t + X_{i,j,t}\beta + \epsilon_{i,j+1,s,t} \tag{2.2}$$

where j is the previous job and $j+1$, the next job, reported by individual i at time t in net liquidity quintile q. δ_0^q and δ_1^q are the effect of UI replacement rates and potential UI duration for an individual in net liquid wealth quintile q at the start of a spell. A positive δ_0^q indicates that more generous unemployment insurance is associated with better employment outcomes for quintile q. A negative δ_1^q indicates that longer unemployment insurance durations result in worse re-employment outcomes.

2.1.2 Findings

My results show that constrained workers alter their search behavior when presented with additional unemployment insurance. The first column and second column of Table 2.1show the results without stratifying individuals by wealth. Column 1 includes potential UI duration (the average duration until UI expiration for individuals in the state when the individual became unemployed), while column 2 conditions on the realized unemployment duration. Columns 3 and 4 show that UI impacts individuals from the first quintile of the wealth distribution exclusively. Column 3 shows that individuals from the first quintile of liquid wealth find jobs offering 4.2% higher pay the month after unemployment when they receive a 10% increase in UI, while column 4 indicates a similar result. Column 3 includes potential UI duration, while column 4 conditions on realized unemploymen duration. The estimate is significant at the 5-percent level, using Taylor Linearized standard errors, (the suggested variance estimator for the SIPP's complex survey design) but only for the first quintile. I also find that longer potential UI is associated with a decline in wages, though only for the wealthiest population.

Given that employment is highly persistent, while the average unemployment spell in my sample is less than 25 weeks, an elasticity of 0.4 suggests that an additional source of income alters job search behavior. Prior to separation, these individuals had nearly identical labor market characteristics (Table C.1). These results are consistent with those in Herkenhoff et al. (2016) and Nekoei and Weber (2017), and provide additional evidence that the individuals

	(1) Full Sample	(2) Wealth Sample	(3) Wealth Interaction	(4) Wealth Interaction
log UI Benefit	0.117	0.196		
	(0.152)	(0.185)		
Net Liq Q1 X × log UI Benefit			0.420^{*}	0.459^{**}
			(0.227)	(0.232)
Net Liq Q2 X × log UI Benefit			0.239	0.263
			(0.231)	(0.234)
Net Liq Q3 X × log UI Benefit			0.0870	0.0988
			(0.246)	(0.248)
Net Liq Q4 X × log UI Benefit			0.194	0.191
			(0.215)	(0.225)
Net Liq Q5 X × log UI Benefit			0.0688	0.122
			(0.240)	(0.240)
Observations		2129	2129	2135
State FE	X	X	X	X
Year FE	X	X	X	X
Qtile X Wage Spline			X	X
Qtile X Ed			X	X

Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 2.1: Elasticities by net liquidity quintile. Columns 1 and 3 condition on potential UI duration, while columns 2 and 4 condition on observed unemployment duration.

affected are low-wealth.

2.2 Consumption Risk and Human Capital Accumulation

The findings in Section 2.1 suggest that worker placement is affected by borrowing constraints for low-wealth workers. A well-understood characteristic of the labor market is that job placement can have a long-term effect on outcomes (Jacobson et al. (1993), Kahn (2010), among others). Although the SIPP is a short panel, my evidence indicates that wealth effects on worker selectivity cause differences in earnings that persist for at least a quarter. As I summarize in Section 2.2.2, closely related work indicates that these effects are likely to persist beyond the initial placement.

I also document that over the life-cycle, differences in wealth are predictive of both differences in earnings and differences in time allocated for training in Section 2.2.1. This cannot be interpreted causally, but provides additional interpretations of the findings that I discuss in Section 2.1 and Section 2.2.2.

2.2.1 Profiles of Earnings and Training

To examine the correlation between initial wealth and lifetime earnings, I use the Panel Study of Income Dynamics (PSID) and National Longitudinal Survey of Youth 1979 (NLSY), and partition individuals into their wealth quantiles before entering the labor force. I detail the sample selection as well as the construction of these profiles in Appendix A.

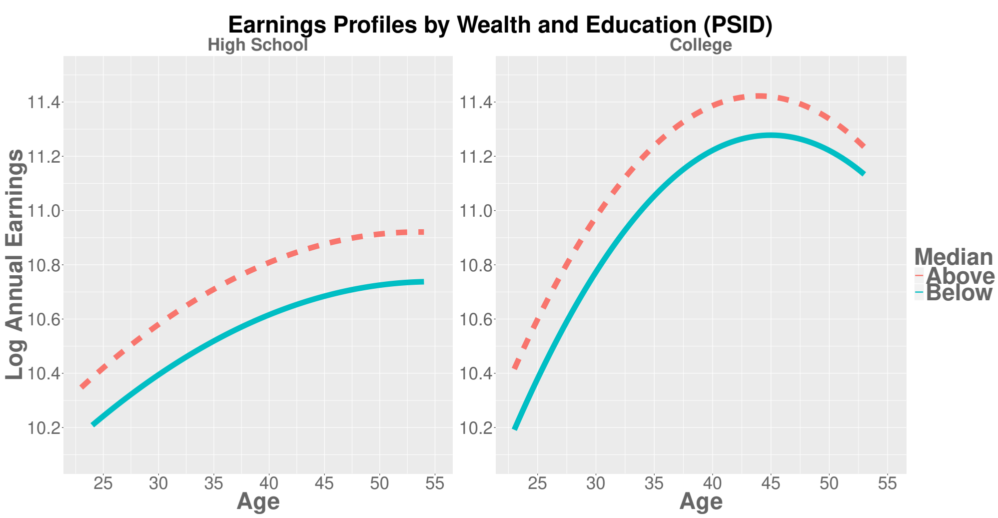


Figure 2.1: Earnings Profiles by Initial Wealth and Education.

There appear to be permanent earnings differences between individuals from different wealth strata. Figure 2.1 shows the average earnings profiles individuals by their liquid wealth prior to entering the labor market. The left panel shows high school educated individuals,

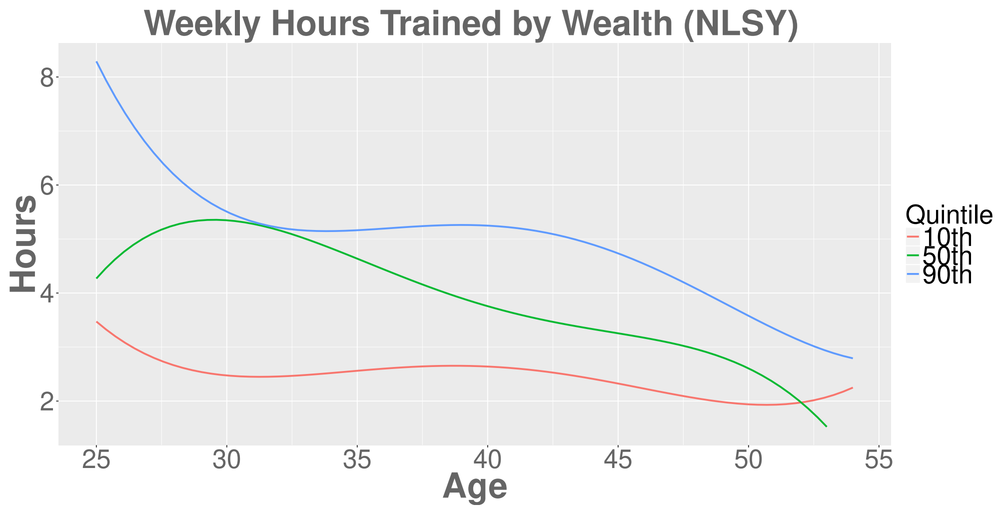


Figure 2.2: Training Hours Per Week by Wealth Quintile.

and the right panel shows college educated individuals. Both show that individuals from the bottom of the wealth distribution experience persistently different earnings profiles from their wealthier peers. Details of the sample selection are available in Section A.2. The profiles show a clear correlation between initial wealth and time training (Figure 2.2). These measures include training outside of work as well as training sponsored by employers. These profiles suggest that there is a correlation between wealth and human capital accumulation while working.

2.2.2 Evidence from Related Literature

A large literature has found that adverse shocks, either displacement of a currently employed worker, or aggregate shocks when first entering the labor market, can have long-lasting effects on earnings. Jacobson et al. (1993) show that the earnings of displaced workers are 19 percent lower one quarter after regaining employment than at their previous job. Perhaps more importantly (as it relates to this study), they find that these earnings losses remain persistent for both workers who separate without a mass layoff (3 to 5 years), and even moreso for those employed at a firm with a mass layoff (25% lower after 6 years).

In a subsequent study, Jacobson et al. (2005) find that enrolling displaced workers in community college or vocational courses decrease the persistence of earnings losses following an unemployment spell (9% to 29%, depending on gender and area of study). They note, that even after attending community college, these displaced workers experience a period of lower wages than non-displaced workers, before realizing the gains to their increased human capital. This suggests that these workers still experience transitory shocks, perhaps due to reduced selectivity, but are not subject to persistent earnings losses associated with productivity losses.

Similarly, the literature that studies the impact of entering the labor market during a recession finds that workers experience large and persistent earnings losses. Kahn (2010) estimates that entering during above average periods of unemployment causes a decline in earnings of between 0.2 and 0.35 log points. Some workers substitute away from work by attending college or graduate school: those who enter during the highest periods of unemployment are 7 percentage points more likely to obtain a further degree. This option would not be available to those most tightly borrowing constrained. The paper further finds that a 1 percentage point increase in the unemployment rate results in a 0.026 log point wage loss 15 years after entering the labor market.[14]

Additional evidence suggests that borrowing constrained workers recognize that human capital investment during a recession or unemployment spell would be beneficial, but are unable due to costs. Barr and Turner (2015) find that increasing the generosity of the unemployment insurance system allows more workers to attend college during sharp downturns, and amplify the benefits of unemployment insurance on labor market outcomes. This suggests an interaction between the selectivity benefits of unemployment insurance and human capital accumulation.

These papers consistently point to the impact of labor market risk on long-term earnings outcomes. Entering the labor market during a recession, a time in which consumption risk is heightened, slows earnings growth even after the economy recovers. Similarly, workers who experience a separation are subject to persistent earnings losses, potentially never returning to the same path of life-cycle earnings.

3 The Model

3.1 Environment

Time is discrete and continues forever, while each agent participates in the labor market deterministically for $T \geq 2$ periods, before retiring. There is a continuum of both firms and workers, each of which discounts future value at the identical rate β. Each worker is born

[14]Wee (2013) shows that recessions have an impact both on human capital and worker job-search behavior. A worker who enters during a recession is less likely to undertake "complex" job-to-job transitions that might be associated with an improved match.

unemployed without benefits, and receives a draw from a correlated trivariate log-normal distribution $\Psi \sim LN(\psi, \Sigma)$ of wealth, human capital, and learning ability (a_0, h_0, ℓ). Over the life-cycle, a worker may be in one of three employment states: employed, unemployed with unemployment insurance, and unemployed without unemployment insurance. Workers in each employment state are allowed to direct their search to contracts posted by firms. Once a worker reaches age $T+1$, they receive exogenous retirement income and exit the model with probability δ_M.

While in the labor market, each worker is endowed with one indivisible unit of labor that they can enjoy as leisure during unemployment or supply inelastically while employed. Leisure utility ν is assumed to be additively separable, $u(c) + (1-e)\nu$, where e denotes employment status. Workers are risk-averse, with utility $u'(c) \geq 0$, $u'(0) = \infty$, and are allowed to smooth consumption over the life-cycle by borrowing and saving at rate r_F. They face a borrowing limit at each working age, $\underline{a}'$, and are not allowed to default on any debt obligations, nor exit the terminal working period T with negative asset holdings because life expectancy post career is uncertain. While employed, workers are allowed to devote productive time τ to accumulating human capital through a Ben-Porath production function, $H(h, \ell, \tau, L)$, which is increasing in its first 3 arguments. L denotes the labor market status E or U. All workers face an *iid* human capital shock between periods, $\epsilon' \sim N(\mu_\epsilon, \sigma_\epsilon)$, that permanently alters human capital. This is modeled as $h' = e^{\epsilon'}(h + H(h, \ell, \tau, L))$.

Workers transition from employment to unemployment in one of two ways: with probability δ, they receive a separation shock and enter unemployment, and with probability $\lambda_E \leq 1$, they are allowed to search while employed for a new job. Employed workers receive $\mu(1-\tau)f(h)$ as income each period, where μ is their piecerate wage, $(1-\tau)$ the time left over after human capital decisions, and $f(h)$ is their productivity given their current human capital. If they receive an unemployment shock, workers receive unemployment benefits $b_{UI} = min\{b\mu(1-\tau)f(h), \bar{b}\}$, where b is the replacement rate, and $\bar{b}$ is the maximum benefit allowed per quarter. Agents stochastically lose benefits with probability γ, and receive $b_L \leq b_{UI}$, which reflects opportunities to earn money outside the labor force. Once a worker's age reaches $t = T+1$, a worker retires with certainty and is entitled to retirement income $b = b_{Ret}$, which is identical for all workers regardless of labor market history. During the retirement period, workers exit the model according to a memoryless process with probability δ_M.[15]

[15]The inclusion of retirement may seem to add unnecessary complication to an already hefty model. The reason is that I use the observed decline in earnings late in the life-cycle to discipline human capital

Firms post vacancies at cost κ. These vacancies are one-firm-one worker contracts that specify the piecerate of output paid as earnings, μ. These contracts are assumed to be renegotiation-proof, and firms are not allowed to respond to outside offers, thus μ is fixed for the duration of the contract. Worker characteristics are assumed to be observable, and thus firms open vacancies into specific submarkets that are indexed by the observables of the worker. Thus, submarkets are identified by the following tuple: $(\mu, a, h, \ell, t) \in \mathbb{R}_+ \times \mathbb{R} \times \mathbb{R}_+ \times \mathbb{R}_+ \times \mathbb{R}_+$. In equilibrium, each submarket has a known probability of employment. Once matched, a firm receives $(1-\mu)(1-\tau)f(h)$ in profits each period. They continue until the match dissolves, either through exogenous separation or on-the-job search.

Following Pissarides (1985), I refer to submarket tightness as $\theta_t(\mu, a, h, \ell) = \frac{v(\mu,a,h,\ell)}{u(\mu,a,h,\ell)}$. The rate at which firms and workers match in each submarket is characterized by a constant returns to scale matching function, $M(u, v)$, where u is the number of unemployed searchers in the submarket and v is the number of firms posting vacancies in the submarket. I define the probability at which firms meet workers as $\frac{M(u,v)}{v} = q(\theta_t(\mu, a, h, \ell))$, and the rate at which workers meet firms as $\frac{M(u,v)}{u} = p(\theta_t(\mu, a, h, \ell))$, both of which I assume to be invertible. I assume that within each submarket the free entry condition holds, meaning that firms compete away any expected profits within a submarket by opening additional vacancies.

The aggregate state of the economy is summarized by the following tuple: $\psi = (z, u, e, \rho)$. The first component is the current level of output in terms of the numeraire for a job in the economy, independent of human capital. The second component is a function that tracks the measure of workers with assets a, human capital h, learning ability ℓ, at age t, $u(a, h, \ell, t)$. The third determines the measure of employment for each of these same types. The last component is the stochastic process that determines newly born workers in each period. By restricting the equilibrium to be block recursive, decision rules do not depend on the distribution of workers or firms. I demonstrate this in Section E.1. Aggregate productivity is assumed to be stationary, $z = \mu_Z$, so I suppress this notation in the model exposition.

depreciation, which requires that workers invest very little in human capital at the end of the life-cycle. Absent a retirement period, workers would have no reason to shift their portfolio from human capital to precautionary savings.

3.2 Worker's Problem

3.2.1 Production, Savings, and Human Capital Accumulation

While in the labor market, each period is divided into two subperiods: job search, and production. During the production subperiod agents choose consumption and savings allocations (c and a'), and the employed workers choose the proportion of time to spend accumulating human capital, τ. All agents are subject to a borrowing constraint $\underline{a}'$, which changes with age. Following these decisions, age advances. Unemployed agents choose consumption and savings and receive benefit and human capital shocks ϵ' once age advances. Their problem is given in Equation 3.1.

$$U_t(b_{UI}, a, h, \ell) = \max_{c, a' \geq \underline{a}'} u(c) + \nu + \beta E[(1-\gamma) R^U_{t+1}(b_{UI}, a', h', \ell) + \gamma R^U_{t+1}(b_L, a', h', \ell)] \quad (3.1)$$

$$\text{s.t. } c + a' \leq (1 + r_F)a + b_{UI} \quad (3.2)$$

$$h' = e^{\epsilon'}(h + H(h, \ell, \tau, U)) \quad (3.3)$$

$$\epsilon' \sim N(\mu_\epsilon, \sigma_\epsilon) \quad (3.4)$$

where b_{UI} is the level of their unemployment benefit. Unemployed agents stochastically lose their benefits with probability γ, and face shocks ϵ' to their human capital both realized at the beginning of the search period. In the calibration, I assume that $H(h, \ell, \tau, U)$ is zero for all unemployed agents. Unemployed agents without unemployment insurance face a similar problem described by Equation 3.5.

$$U_t(b_L, a, h, \ell) = \max_{c, a' \geq \underline{a}'} u(c) + \nu + \beta E[R^U_{t+1}(b_L, a', h', \ell)] \quad (3.5)$$

$$\text{s.t. } c + a' \leq (1 + r_F)a + b_L \quad (3.6)$$

$$h' = e^{\epsilon'}(h + H(h, \ell, \tau, U)) \quad (3.7)$$

$$\epsilon' \sim N(\mu_\epsilon, \sigma_\epsilon) \quad (3.8)$$

with $b_L \leq b_{UI}$. Once an unemployed worker loses unemployment insurance, they must regain employment before resetting their benefits. Unemployed agents in any benefit state retire after T periods with certainty and thus their unemployment utility in period $T + 1$ is given by a Bewley-style dynamic problem with value $U_R(a)$:

$$U_{T+1}(\mathrm{UI}, a, h, \ell) = U_R(a) = \max_{a'} u(c) + \beta(1-\delta_M)U_R(a') \tag{3.9}$$

$$\text{s.t. } c + a' = (1+r)a + b_{Ret} \tag{3.10}$$

where $U_{T+1}(\mathrm{UI}, a, h, \ell) = U_R(a)$ ($\mathrm{UI} \in \{b_{UI}, b_L\}, a, h, \ell$). I do this for computational tractability and ease of exposition.[16] Employed workers solve the problem described in Equation 3.11.

$$W_t(\mu, a, h, \ell) = \max_{c, a' \geq \underline{a}', \tau \in [0,1]} u(c) + \beta E[(1-\delta)R^E_{t+1}(\mu, a', h', \ell) + \delta R^U_{t+1}(b_{UI}, a', h', \ell)] \tag{3.11}$$

$$\text{s.t. } c + a' \leq (1+r_F)a + \mu(1-\tau)f(h) \tag{3.12}$$

$$h' = e^{\epsilon'}(h + H(h, \ell, \tau, E)) \tag{3.13}$$

$$\epsilon' \sim N(\mu_\epsilon, \sigma_\epsilon) \tag{3.14}$$

$$b_{UI} = min\{max\{b(1-\tau)\mu f(h), b_L\}, \bar{b}\} \tag{3.15}$$

The function H determines the accumulation of human capital and is a non-decreasing function of τ, $\frac{\partial H(h,\ell,\tau,E)}{\partial \tau} \geq 0$, $\frac{\partial H^2(h,\ell,\tau,E)}{\partial \tau^2} \leq 0$. Any time allocated to human capital accumulation proportionally decreasing income during the current period as well as unemployment benefits should the worker become unemployed. Employed agents face a probability δ of separating exogenously from their current employer. Newly unemployed agents are assumed to have unemployment benefits for at least one period. Following period $T+1$, employed agents enter retirement with value $W_{T+1}(\mu, a, h, \ell) = U_R(a)$, where $U_R(a)$ is given by Equation 3.9:

Both problems yield insight into the dynamic portfolio allocation problem faced by workers. The decision rules from the employed problem define two difference equations in human capital (Equation 3.16) and precautionary savings (Equation 3.17). In define this and the resulting no-arbitage condition in the following

Proposition 1. *The dynamics of human capital and precautionary savings for employed workers*

[16] If I instead assumed that retirement income were proportional to average lifetime income, this would yield only small changes worker decisions because much of human capital is formed early in the life-cycle when retirement is a small fraction of lifetime utility. This would require an additional state for all dynamic problems and slow down the computational substantially. If I instead assumed that retirement income were proportional to a function of contemporaneous human capital, accumulation decisions nearing retirement would face substantial distortions.

are given by

$$u'(c)\mu h \leq \beta E[(1-\delta)\frac{\partial R^E_{t+1}}{\partial h'}\frac{\partial H}{\partial \tau} + \delta(\frac{\partial R^U_{t+1}}{\partial h'}\frac{\partial H}{\partial \tau} + \frac{\partial R^U_{t+1}}{\partial b_{UI}}\frac{\partial b_{UI}}{\partial \tau})] \tag{3.16}$$

$$u'(c) \geq \beta E[(1-\delta)\frac{\partial R^E_{t+1}}{\partial a'} + \delta\frac{\partial R^U_{t+1}}{\partial a'}] \tag{3.17}$$

if $\tau < 1$. *Together, this system yields a dynamic portfolio allocation decision determined by*

$$E[(1-\delta)\frac{\partial R^E_{t+1}}{\partial h'}\frac{\partial H}{\partial \tau} + \delta(\frac{\partial R^U_{t+1}}{\partial h'}\frac{\partial H}{\partial \tau} + \frac{\partial R^U_{t+1}}{\partial b_{UI}}\frac{\partial b_{UI}}{\partial \tau})] \geq E[(1-\delta)\frac{\partial R^E_{t+1}}{\partial a'} + \delta\frac{\partial R^U_{t+1}}{\partial a'}]\mu h \tag{3.18}$$

with equality if $a' \neq \underline{a}$ *and* $\tau \in (0, 1)$.

The portfolio allocation, or no-arbitrage condition (Equation 3.18) shows that workers may want to substitute future income to the present, but are unable because either the borrowing constraint binds, or they cannot dissave human capital. I discuss this more extensively in Section 3.6.

Human capital changes a workers lifetime utility through two channels: a increase in lifetime income levels due to increased productivity, and an increase in the growth of lifetime income by increasing future marginal productivity of time allocation through H. As the terminal date T approaches, the value of future human capital declines $\frac{\partial H}{\partial h}$, causing workers to substitute toward precautionary savings as they decumulate their portfolios. I describe these trajectories as well as the differences in savings between employed and unemployed workers in Figure 5.4.

3.2.2 Job Search

Age advances and shocks are realized following the production period. Unemployed agents in the job search period solve the problem given by Equation 3.19.

$$\begin{aligned} R^U_t(b_{UI}, a, h, \ell) = \max_{\mu'} & P(\theta_t(\mu', a, h, \ell))W_t(\mu', a, h, \ell) \\ & + (1 - P(\theta_t(\mu', a, h, \ell)))U_t(b_{UI}, a, h, \ell) \end{aligned} \tag{3.19}$$

where b_{UI} denotes their current level of UI and μ' denotes the application strategy $\mu'(w, a, h, \ell, t)$. For agents without unemployment insurance, $b_{UI} = b_L$. Employed workers are allowed to

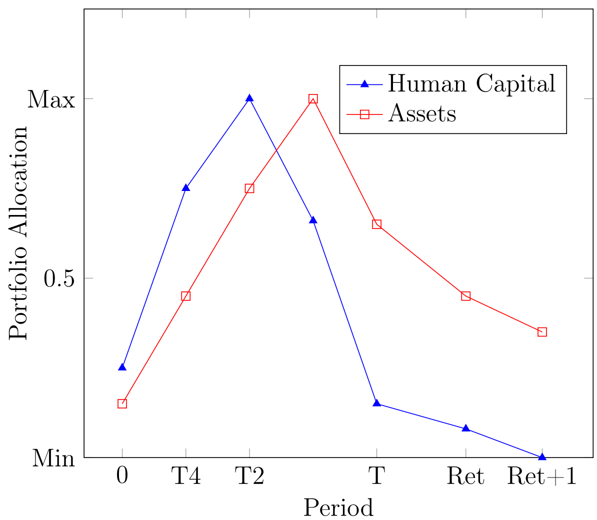


Figure 3.1: Normalized Stocks of Wealth

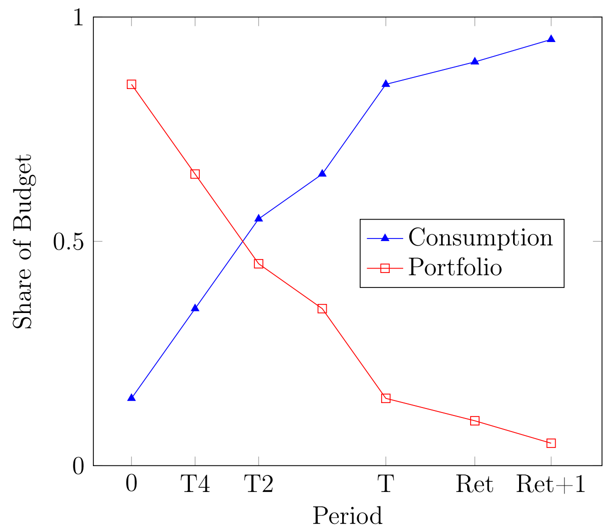


Figure 3.2: Expenditure Shares

search on the job, and solve the problem given by Equation 3.20.

$$\begin{aligned} R_t^E(\mu, a, h, \ell) = \max_{\mu'} \lambda_E P(\theta_t(\mu', a, h, \ell))W_t(\mu', a, h, \ell) \\ + (1 - \lambda_E P(\theta_t(\mu', a, h, \ell)))W_t(\mu, a, h, \ell) \end{aligned} \tag{3.20}$$

Proposition 2. *The application strategy of an unemployed worker is implicitly given by*

$$\mu^{*\prime} \text{ solves } P(\theta_t(\mu', a, h, \ell))\frac{\partial W_t(\mu', a, h, \ell)}{\partial \mu'} = \frac{\partial P(\theta_t(\mu', a, h, \ell))}{\partial \mu'}[W_t(\mu', a, h, \ell) - U_t(b, a, h, \ell)] \tag{3.21}$$

and the application strategy of an employed worker is given by

$$P(\theta_t(\mu', a, h, \ell))\frac{\partial W_t(\mu', a, h, \ell)}{\partial \mu'} = \frac{\partial P(\theta_t(\mu', a, h, \ell))}{\partial \mu'}[W_t(\mu', a, h, \ell) - W_t(\mu, a, h, \ell)] \tag{3.22}$$

Both application strategies are non-decreasing in wealth.

To see the final point of this proposition, consider how the implicit solution for the unemployed changes as wealth changes:

$$\frac{P(\theta_t(\mu', a, h, \ell))}{\frac{\partial P(\theta_t(\mu', a, h, \ell))}{\partial \mu'}} = \frac{\frac{\partial U_t(b, a, h, \ell)}{\partial a} - \frac{\partial W_t(\mu', a, h, \ell)}{\partial a}}{\frac{\partial^2 W_t(\mu', a, h, \ell)}{\partial \mu' \partial a}} \tag{3.23}$$

I assume for simplicity that $\frac{\partial P}{\partial a} = 0$.[17] From this expression, we can see that the right hand side is strictly negative, but increasing in wealth. The left hand side is strictly negative and increasing in μ'. Thus, an increase in wealth requires an increase in the application strategy to maintain equality.

3.3 Firm's Problem

Firms produce using a single worker as an input. New firms post piece-rate wage contracts in submarkets characterized by (μ, a, h, ℓ, t), each of which is assumed to be observable to the firm. Contracts dictate the share of revenue to be received by each side in the match. Wage contracts are assumed to be renegotiation-proof. A firm with a filled vacancy produces using technology $y = (1-\tau)f(h)$, where τ is the time spent accumulating human capital by the worker that cannot be used in production. The firm retains a fraction $(1-\mu)$ of this output as profits and pays the rest out in wages. Matches continue with probability $(1-\delta)(1-\lambda_E P((\theta_{t+1}(\mu', a', h', \ell)))$, the probability that the match does not separate exogenously and the worker does not find a new employer. Firms discount at the same rate as workers, β. The value function of a firm matched with a worker is given in Equation 3.24.

$$J_t(\mu, a, h, \ell) = (1-\mu)(1-\tau)f(h) + \beta E[(1-\delta)(1-\lambda_E P((\theta_{t+1}(\mu', a', h', \ell)))J_{t+1}(\mu, a', h', \ell)] \tag{3.24}$$

$$h' = e^{\epsilon'}(h + H(h, \ell, \tau, E)) \tag{3.25}$$

where $a' = g_a(\mu, a, h, \ell)$ and $\tau = g_\tau(\mu, a, h, \ell)$ are the worker policy decisions over wealth and human capital accumulation. $\mu' = g_\mu(\mu, a', h', \ell)$ is the application strategy of the worker conditional upon his asset and human capital policy rule. Profits from a filled vacancy at age $T+1$ are zero:

$$J_{T+1}(\mu, a, h, \ell) = 0 \tag{3.26}$$

New firms have the option of posting a vacancy at cost κ in any submarket. Each submarket offers a probability of matching with a worker given by $q(\theta_t(\mu, a, h, \ell))$. In expectation, the value of opening a vacancy in submarket (μ, a, h, ℓ) is given by Equation 3.27.

$$V_t(\mu, a, h, \ell) = -\kappa + q(\theta_t(\mu, a, h, \ell))J_t(\mu, a, h, \ell) \tag{3.27}$$

[17]Note that the job-finding rate is decreasing in the job-to-job hazard of employed workers. Therefore, if application strategies are increasing in wealth, the job finding rate is non-decreasing in wealth. Indirect effects through changes in human capital accumulation will be offset by changes in the job-to-job hazard.

I assume that the free entry condition holds for every open submarket. Firms enter until the expected profits of a vacancy, $V_t(\mu, a, h, \ell) = 0$. This means that Equation 3.27 can be rewritten as Equation 3.28.

$$\kappa = q(\theta_t(\mu, a, h, \ell))J_t(\mu, a, h, \ell) \tag{3.28}$$

In equilibrium, this yields the following:

$$q(\theta_t(\mu, a, h, \ell)) = \frac{\kappa}{J_t(\mu, a, h, \ell)} \tag{3.29}$$

$$\theta_t(\mu, a, h, \ell) = q^{-1}(\frac{\kappa}{J_t(\mu, a, h, \ell)}) \tag{3.30}$$

Using the definition of the matching function, $\frac{M(u,v)}{u} = p(\theta_t(\mu, a, h, \ell))$ and $\frac{M(u,v)}{v} = q(\theta_t(\mu, a, h, \ell))$, the equilibrium job-finding rate for workers and firms in a submarket can be expressed as $p(\theta_t(\mu, a, h, \ell)) = \theta q(\theta_t(\mu, a, h, \ell))$.

3.4 Timing

The timing in the model is as follows:

1. Firms open vacancies in submarkets (μ, a, h, ℓ, t).
2. Employed and unemployed workers search for vacancies in submarkets (μ, a, h, ℓ, t).
3. Agents who receive job offers transition employment states. Agents who are not offered a job remain unemployed.
4. All agents make consumption and savings decisions. Employed agents allocate time between production and human capital accumulation.
5. Age advances. Agents receive human capital shocks, benefit duration shocks, and unemployment shocks in that order.

3.5 Equilibrium

A Block Recursive Equilibrium (BRE) in this model economy is a set of policy functions for workers, $\{c, \mu', a', \tau\}$, value functions for workers W_t, U_t, value functions for firms with filled jobs, J_t, and unfilled jobs, V_t, as well as a market tightness function $\theta_t(\mu, a, h, \ell)$.[18] These functions satisfy the following:

1. The policy functions $\{c, \mu', a', \tau\}$ solve the workers problems, W_t, U_t, R_t^E, R_t^U.
2. $\theta_t(\mu, a, h, \ell)$ satisfies the free entry condition for all submarkets (μ, a, h, ℓ, t).
3. The aggregate law of motion is consistent with all policy functions.

3.6 Characterizing Responses to Unemployment and Consumption Risk

Similar to other life-cycle models of inequality, the income process in this model features shocks that may appear either transitory or permanent. However, the persistence of income shocks in this model are determined by worker decisions, and large persistent shocks are concentrated among workers who are unable to insure against present consumption risk.

For workers with large stocks of precautionary savings, unemployment and human capital risk manifest as transitory shocks. Should a wealthy worker lose their high piece-rate job, they are able to self-insure and search for another high piece-rate job with little exposure to consumption risk. Similarly, a negative human capital shock can be easily rectified by a larger allocation of time to accumulation, while anticipating no future consumption risk as a result.

Low-wealth workers are limited in their ability to smooth consumption through borrowing. If they are dealt an unemployment shock, the only margin along which they can mitigate consumption risk is by searching for jobs that are likely to offer employment, those that in equilibrium offer a low piece-rate. That is, for low-wealth workers, unemployment causes a persistent shock to income by returning them to the bottom of the job ladder.[19] In addition, unemployment risk changes the portfolio allocation of employed workers: because an unemployment shock exposes low-wealth workers to substantial consumption risk, these workers allocate less time to human capital accumulation and more time to production.

Returning to 1, I can characterize the portfolio allocation of workers in the presence of unemployment risk across the wealth distribution. Consider the change in the continuation value of search as human capital increases:

[18] A Block Recursive Equilibrium is one in which the first two "blocks" of the equilibrium, i.e. the individual decision rules, can be solved without conditioning upon the aggregate distribution of agents across states, i.e. the third block of the equilibrium. The aggregate state can then be recovered by simulation. For an extended discussion see Section E.2.

[19] A similar point is made by Lise (2013)

$$\frac{\partial R^E_{t+1}}{\partial h'} = \underbrace{P(\theta(\mu'))[\frac{\partial W_{t+1}(\mu')}{\partial h'} - \frac{\partial W_{t+1}(\mu)}{\partial h'}] + \frac{\partial W_{t+1}(\mu)}{\partial h'}}_{Marginal\ Utility\ of\ Human\ Capital} + \underbrace{\frac{\partial P}{\partial \theta}\frac{\partial \theta}{\partial h}(W_{t+1}(\mu') - W_{t+1}(\mu))}_{Increased\ Job-Finding\ Rate} \tag{3.31}$$

Human capital increases lifetime utility by increasing the return to work through productivity gains, and by increasing the job-finding rate when searching for future employment. The job-finding rate is decreasing in future job-to-job mobility, so gains through this channel are depressed due to the moral hazard of on-the-job search to the firm. Increases in human capital result in a direct increase in future income, with per-period gains given by $u'(c)\mu(1-\tau)$. That is, human capital has consumption value if a worker is employed. Should the employed worker experience a separation shock, the change in continuation value is given by

$$\frac{\partial R^U_{t+1}}{\partial h'} = P(\theta(\mu'))[\underbrace{\frac{\partial W_{t+1}(\mu')}{\partial h'}}_{Productivity\ Gains} - \frac{\partial U_{t+1}(b)}{\partial h'}] + \frac{\partial U_{t+1}(\mu)}{\partial h'} + \underbrace{\frac{\partial P}{\partial \theta}\frac{\partial \theta}{\partial h}(W_{t+1}(\mu') - U_{t+1}(b))}_{Increased\ Job-Finding\ Rate} \tag{3.32}$$

Notably, $\frac{\partial W_{t+1}(\mu')}{\partial h'}$ is the only value function in which consumption directly depends on contemporaneous human capital. What this means is that human capital is not capable of mitigating consumption risk for low-wealth workers that experience an unemployment shock. For the arbitrage equation Equation 3.18 to hold, low-wealth workers must accumulate more precautionary savings and less human capital.

Over the life-cycle, this has implications for human capital accumulation. Workers who face consumption risk early in the life-cycle choose to allocate a larger fraction of their portfolio toward precautionary savings. The left panel in Figure 3.6 shows the impact of unemployment risk on these workers. If they experience an unemployment shock, they will be forced to take a low-paying, easily-obtained job, and further depress their future income. The right panel shows their response in terms of human capital accumulation.

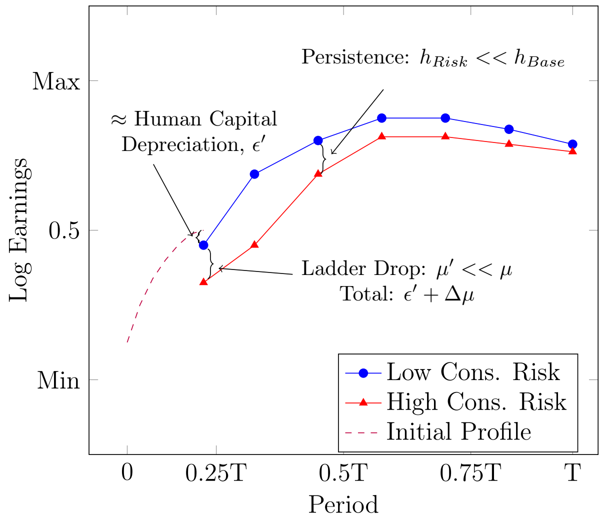


Figure 3.3: Income Profile

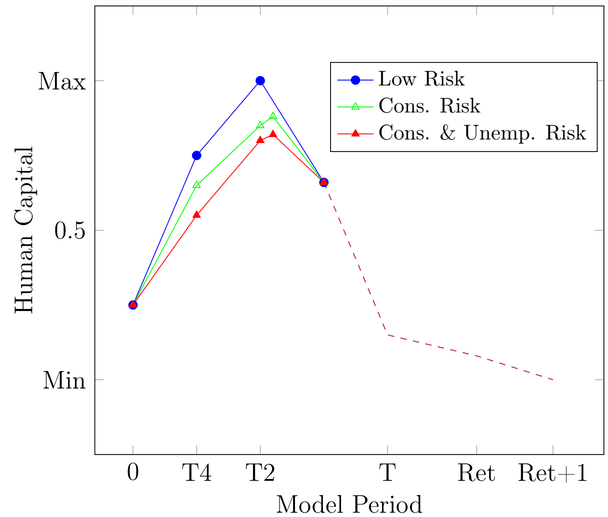


Figure 3.4: Human Capital Profile

4 Estimation

To discipline the model, I use a moment matching technique called indirect inference. Indirect inference is an extension of simulated method of moments in which the targets are coefficients from a collection of conditional moments dubbed the "auxiliary model." Among the advantages of this approach relative to maximum likelihood are that reduced-form specifications are computationally inexpensive to estimate, and data irregularities, like attrition, are easily handled by imposing the same sampling pattern in model-generated data. For these reasons, this technique is popular among papers estimating household response to risk (Guvenen and Smith, 2014), as well as those estimating search behavior over the life-cycle (Lise (2013), Bowlus and Liu (2013)). I discuss this methodology further in Section 4.2.8.

I select empirical specifications for my auxiliary model that are closely related to the impact of borrowing constraints on search behavior, and the heterogeneity in earnings growth over the life-cycle. In Section 4.2, I use a simplified version of my model to show intuition for the identification of the structural parameters.

To implement indirect inference, I preset functional forms and parameters that are ubiquitous throughout the related literature. These choices are detailed in Section 4.1. The remaining parameters are estimated by indirect inference by matching moments from the auxiliary model presented in Section 4.2.

4.1 Empirical Preliminaries

4.1.1 Functional Form and Distributional Assumptions

I set the functional forms to those commonly used in the literatures on search and on inequality. I choose a power utility function of the following form:

$$u(c) = \begin{cases} \frac{c^{1-\sigma}-1}{1-\sigma} & \text{if } \sigma > 0, \sigma \neq 1 \\ \ln(c) & \text{if } \sigma = 1 \end{cases} \tag{4.1}$$

When agents are unemployed, I assume that they receive linear leisure utility, $u(c) + \nu$. I use the matching function from den Haan et al. (2000), which is constant returns to scale and generates well-defined probabilities:

$$M(u, v) = \frac{uv}{(u^\eta + v^\eta)^{\frac{1}{\eta}}} \tag{4.2}$$

I assume the following functional form for production:

$$y = f(h) \tag{4.3}$$

$$f(h) = zh \tag{4.4}$$

where z is a scale factor. Linear production is a common restriction in the search literature when models do not consider physical capital. I assume that all workers face shocks to their human capital each period, $e^{\epsilon'}$ with $\epsilon' \sim N(\mu_\epsilon, \sigma_\epsilon)$, and that unemployed workers are unable to allocate time to human capital production while they are searching for jobs. Workers accumulate human capital using Ben-Porath (1967) technology defined as follows:

$$h' = e^{\epsilon'}(h + H(h, \ell, \tau, E)) \tag{4.5}$$

$$H(h, \ell, \tau, E) = \ell(h\tau)^{\alpha_H} \tag{4.6}$$

$$H(h, \ell, \tau, U) = 0 \tag{4.7}$$

where ℓ is the learning ability of an individual endowed at the beginning of the life-cycle and can be thought of as a fixed effect (it is constant). τ is the fraction of productive time that an employed worker spends accumulating human capital.

Ben-Porath is widely employed among papers on human capital and inequality, which allows for more straightforward comparisons between my findings and the findings of other papers on inequality. However, this assumption is a departure from much of the previous work in the search literature that incorporates human capital. With rare exceptions[20], models of search with human capital assume human capital growth is facilitated exogenously through "learning-by-doing," including many that explore inequality (Bagger et al. (2014) and Carillo-Tudela (2012), among others). The empirical evidence is divided on which approach best fits the data.[21] While this is worthy of concern, I embed learning-by-doing into my model in Section D.2, and show that as long as production and human capital accumulation are partially rival (that is, accumulation is not solely the product of learning-by-doing), my quantitative findings are unaffected. I assume that workers face a natural borrowing constraint that changes with their remaining working-life horizon:

$$\underline{a}'_t = \sum_{j=t}^{T} \frac{b_L}{(1+r_F)^j} \tag{4.8}$$

In each period t, $\underline{a}'_t$ is the amount that any agent could repay if he were in the worst income state (b_L) in every period until the terminal date. Modeling borrowing constraints using the natural borrowing limit is appealing because the Inada Conditions as well as the presence of subsistence income in the model guarantee that this constraint will never bind. In this respect, any estimated wealth effects on worker decisions are likely to be conservative as opposed to a more restrictive approach to asset market imperfections.[22]

Lastly, I assume that initial conditions (a_0, h_0, ℓ) are drawn from a multivariate log-normal distribution, $\Psi \sim LN(\psi, \Sigma)$, with mean ψ and variance-covariance Σ, so that human capital and learning ability are both positive and each marginal distribution can be characterized by a shape and scale parameter. The initial distribution of wealth is shifted by $-\underline{a}'_0$, the borrowing constraint in period 0, while the initial distributions of human capital and learning ability are displaced by $\underline{h}_0$ and $\underline{\ell}$, respectively. These are common assumptions when

[20]Bowlus and Liu (2013) is the only other paper to incorporate Ben-Porath human capital accumulation, of which I am aware. A few papers (Acemoglu and Pischke (1999), Lentz and Roys (2015), among others) have incorporated rival on-the-job training.

[21]Recent evidence from Blandin (2016), who nests learning-by-doing and Ben-Porath within a single model and tests their predictions about life-cycle earnings finds that Ben-Porath fits the data roughly 4 times better than learning-by-doing. It is unclear if those results generalize to a model with labor market frictions.

[22]Some alternatives, like adopting endogenous borrowing limits a la Kehoe and Levine (1993), may better approximate the borrowing environment faced by workers. However, alternate approaches increase the computational expense and are likely to result in larger estimated effects of initial wealth, because the equilibrium outcome is generally tighter borrowing limits for poor applicants.

modeling inequality. I use a Gaussian copula with correlations $\rho_{AH}, \rho_{AL}, \rho_{HL}$ (the pairwise correlations between wealth, human capital, and learning, respectively) to generate correlated draws from this initial distribution. The preset functional forms and initial conditions are summarized in Table 4.1.

Table 4.1: Preset Functional Forms and Distributions

Category	Symbol	Value or Function
Functional Forms		
Utility Function	U(c)	$\frac{c^{1-\sigma}-1}{1-\sigma} + (1-e)\nu$
Production Function	$f(h)$	zh
Human Capital Production	$H(h, \ell, \tau, E)$	$\ell(h\tau)^{\alpha_H}$
Human Capital Evolution	h'	$e^{\epsilon'}(h + H(h, l, \tau, E))$
Matching Function	$M(u, v)$	$\frac{uv}{(u^\eta + v^\eta)^{\frac{1}{\eta}}}$
Borrowing Constraint	$\underline{a}'$	$\sum_{j=t}^{j=T} \frac{b_L}{(1+r_F)^j}$
Distributions		
Depreciation	ϵ'	$\epsilon' \sim N(\mu_\epsilon, \sigma_\epsilon)$
Initial Conditions	Ψ	$\Psi \sim LN(\psi, \Sigma)$
Mean	ψ	$\begin{bmatrix}\mu_A & \mu_H & \mu_L\end{bmatrix}'$
Variance	diag(Σ)	$(\sigma_A, \sigma_H, \sigma_L)$
Correlation		$(\rho_{AH}, \rho_{AL}, \rho_{HL})$
Displacement		(δ_H, δ_L)

4.1.2 Preset Parameter Values

I select a subset of the parameters to be set to common values from the relevant literature. Agents in the model live for $T = 168$ quarters, covering the post-schooling and prime working ages, 25-54. I set the exogenous separation rate to match the average quarterly flows from employment to unemployment (Shimer, 2012), $\delta = 0.030$. An exogenous interest rate is required to for the equilibrium concept used to solve the model, so I set the risk-free rate to a quarterly $r_F = 0.012$, which generates an annual risk-free rate of about 5%. I set $\beta = \frac{1}{1+r_F}$, so that agents smooth consumption in expectation. The elasticity parameter of the matching function, η is set so that the elasticity of the job-finding probability of unemployed workers with respect to submarket tightness is on average 0.5, consistent with the empirical exploration in Shi (2016). The cost of opening a vacancy, κ, is also set at 0.2 using the results from Shi (2016). I use a scale factor, z, equal to the average quarterly income in the PSID at age 25.

I set the unemployment insurance replacement rate to its averagein the data, $b = 0.42$, and cap unemployment insurance at a weekly maximum of \$450, which is the average cap in my data. Both of these considerations are required for identification in my estimation procedure. I assume that unemployment insurance does not fluctuate with human capital depreciation, but can be lost with probability γ. I set $\gamma = 0.54$, which matches the expected max duration of UI in my data (≈ 24.1 weeks).

There are 15 parameters remaining to be estimated (shown in Table 4.3). The preset parameters are summarized in Table 4.2.

Table 4.2: Preset Parameter Values

Category	Symbol	Value or Function	Source
Model Parameters			
Discount Factor	β	0.9882	$\frac{1}{1+r_F}$
Risk Aversion	σ	2	RBC Standard
Quarters	T	168	Working Age 23-60
Elasticity of Matching Function	η	0.5	Shi (2016)
Vacancy Creation Cost	κ	0.2	Shi (2016)
Separation Rate	δ	0.030	Quarterly average 1968-2013
Scale Factor	z	18 165	Mean annual earnings (Age 25, PSID)
Max UI (unscaled)	γ	450	Average UI cap
UI Loss Probability	γ	0.54	Sample max UI duration average
Risk Free Rate	r_F	0.0120	Annual rate of $\approx 5\%$
Distributional Parameters			
UI Replacement Rate	b	0.42	U.S. Average

4.2 Indirect Inference and Auxiliary Model

I estimate the remaining parameters of the model using indirect inference (Gourieroux et al. (1993) and Smith (1993)). Indirect inference is a generalization of simulated method of moments that uses model generated data to match a set of conditional moments that make up an "auxiliary model." The auxiliary model is composed of reduced-form specifications that provide an indirect mapping into the structural parameters of the model. Identification requires that each structural parameter has an independent effect on the auxiliary model.

I select conditional moments can be roughly understood as piece-wise linear approximations of the decision rules in the model, as well as conditional moments that are equilibrium outcomes unrelated to the empirical questions of interest in this paper. I use two reduced-form specifications to approximate the model-implied application strategies and human capital accumulation stratified along the dimensions in which these decision rules are likely to contain the most curvature. In this sense, I require the model to match piece-wise linear approximations of the equivalent observed decision rules.

Beyond the ability to provide clear relationships between the auxiliary model and key components of the theory, indirect inference is computationally inexpensive and provides straightforward approaches to handling flaws in the observed data. When workers attrit, or an empirical specification faces concerns about selection, I mirror the attrition or selection in my model-generated data. It also allows me to estimate the model using multiple data sources by separately sampling my model-generated data.

My model has initial heterogeneity from three sources: differences in wealth, differences in initial human capital, and differences in learning ability, which are jointly distributed at the beginning of the life-cycle. I pick a set of reduced-form moments and estimate an auxiliary model in order to discipline this initial heterogeneity. In each specification, I denote the set of parameters to be matched through indirect inference with β_i, where i indexes the parameter or set of parameters. In my empirical specifications, I use an extensive set of controls that have no analog in my model. I denote these "nuisance" parameters δ.

With the exception of moments characterizing the borrowing constraint and initial wealth and earnings, I estimate my model on agents ages 25 to 54, two years after I start agents in the model (age 23). This is because I observe earnings at first jobs for very few agents, particularly for whom I also observe either wealth or proxies for human capital or learning ability. By matching my model to data on agents who are already employed, I allow for wage growth while still retaining inference on the structural parameters of interest.

4.2.1 A Parsimonious Model

To illustrate the sources of identification as well as their relation to key decision rules, I introduce a two-period version of the model. This "perturbed" model need only approximate the behavior of the structural model presented in Section 3. While this model differs in several important ways, it is qualitative parallel to my theory.

I make several functional form and parametric assumptions that differ from those in Section 4.1 in order to provide analytical characterizations. I assume that the job-finding rate is linear in productivity and application strategy (that is, Cobb-Douglas, $M(u, v) = u^{\eta} v^{1-\eta}$, with elasticity parameter $\eta = \frac{1}{2}$), that unemployed workers lose benefits if they do not match ($\gamma = 1$), and that employed workers face no separation risk ($\delta = 0$). I initially assume that $\lambda_E = 0$ and $\epsilon' = \mu_{\epsilon}$, but depart from these assumptions when describing my identification strategy for each.

In this simplified model, workers start employed or unemployed (with unemployment insurance) in the production subperiod with a random draw from the the initial states (a, h, ℓ) and employment-specific states μ, b. This yields the following problems for an employed worker and unemployed worker, respectively:

$$W_1(\mu, a, h, \ell) = \max_{a', \tau} -((1+r)a + \mu(1-\tau)h - a')^{-1} + \beta[-((1+r)a + \mu e^{\mu_\epsilon}(h + \ell(\tau h)^\alpha))^{-1}] \tag{4.9}$$

$$\begin{aligned} U_1(b, a, h, \ell) = &\max_{a', \mu} -((1+r)a + b - a')^{-1} + \nu \\ &+ \beta[-(\frac{(1-\mu)e^{\mu_\epsilon}h}{\kappa})((1+r)a + \mu e^{\mu_\epsilon}h)^{-1} - (1 - \frac{(1-\mu)e^{\mu_\epsilon}h}{\kappa})(((1+r)a)^{-1} + \nu)] \end{aligned} \tag{4.10}$$

where the optimal choices in the terminal period $a' = 0$ and $\tau = 0$ are imposed. Because wage growth is determined by two worker decisions, application strategies, and time allocations, understanding the relationship between these decision rules in my simplified model and the auxiliary model can yield insight on the sources of identification. In this simplified model, re-employment earnings, the product of piece-rate and undepreciated human capital, are given by

$$\mu e^{\mu_\epsilon}h = ((1+r)a')((\frac{1 + ((1+r)a')^{-1}e^{\mu_\epsilon}h}{1 - \nu(1+r)a'})^{\frac{1}{2}} - 1) \tag{4.11}$$

taking as given the optimal choice of assets, $a' \geq 0$. For a worker who starts period 1 employed, time allocation is optimally given by

$$h\tau = (\frac{\alpha \ell e^{\mu_\epsilon}}{1+r})^{\frac{1}{1-\alpha}} \tag{4.12}$$

4.2.2 Information in Re-Employment Elasticities

A worker's response to unemployment insurance generosity caries information about the tightness of the borrowing constraint as well as leisure utility, ν. I can write the log-earnings upon exiting an unemployment spell using the parsimonious application strategy as

$$ln(\mu e^{\mu_\epsilon} h) = ln((1+r)a') + ln((1 + ((1+r)a')^{-1} e^{\mu_\epsilon} h)^{\frac{1}{2}} - (1 - \nu(1+r)a')^{\frac{1}{2}}) - \frac{1}{2} ln(1 - \nu(1+r)a') \tag{4.13}$$

Note that by assuming there is no income in the no-benefit state, $a' > 0$, with the inequality strict because preferences satisfy the Inada conditions. To help illustrate the identification of borrowing constraints, I make the brief assumption that $\nu = 0$. The derivative of this expression with respect to unemployment benefits, b, yields the following

$$\frac{\partial ln(\mu e^{\mu_\epsilon} h)}{\partial b} = \frac{1}{a'}[1 - \frac{1}{2}\frac{e^{\mu_\epsilon} h}{((1+r)a' + e^{\mu_\epsilon} h)^{\frac{1}{2}}[((1+r)a' + e^{\mu_\epsilon} h)^{\frac{1}{2}} - ((1+r)a')^{\frac{1}{2}}]}]\frac{\partial a'}{\partial b} \tag{4.14}$$

The derivation of this expression is in Section B.1. To see that this expression yields inference on borrowing constraints, consider the derivative with respect to net assets,

$$\frac{\partial ln(\mu e^{\mu_\epsilon} h)}{\partial (a - \underline{a}_1)} = \frac{1}{a'}[1 - \frac{1}{2}\frac{e^{\mu_\epsilon} h}{((1+r)a' + e^{\mu_\epsilon} h)^{\frac{1}{2}}[((1+r)a' + e^{\mu_\epsilon} h)^{\frac{1}{2}} - ((1+r)a')^{\frac{1}{2}}]}]\frac{\partial a'}{\partial (a - \underline{a}_1)} \tag{4.15}$$

Both unemployment benefits and net wealth enter the budget constraint linearly, meaning that $\frac{\partial a'}{\partial b} = \frac{\partial a'}{\partial (a - \underline{a})}$. In isolation, neither expression would be sufficient to determine the tightness of the borrowing constraint for an unemployed worker. A large response to a change in unemployment insurance could indicate that an individual worker is either very poor, or the borrowing constraint is very tight; however, conditional upon an unemployed workers net liquid wealth, these expressions are directly informative of the degree to which an unemployed worker is constrained. They additionally reveal the slope of a worker's application strategy over wealth for a worker of type (b, a, h).

I assume that workers of every wealth level weakly prefer to be employed than be unemployed and receive leisure utility, ν. This implies that $\frac{\frac{\partial U}{\partial a'}}{\nu} \geq 1 \; \forall \; a'$, or in the pasimonious model that $(1+r)\bar{a}^2 \geq \frac{1}{\nu}$. To see how re-employment elasticities yield inference on leisure utility, consider Equation 4.13 as wealth approaches this upper bound, $a' = (\frac{1}{\nu})^{\frac{1}{2}}$:

$$ln(\mu e^{\mu_\epsilon} h) = ln((1+\nu^{\frac{1}{2}} e^{\mu_\epsilon} h)^{\frac{1}{2}} - (1-\nu^{\frac{1}{2}})^{\frac{1}{2}}) - \frac{1}{2}[ln(1-\nu^{\frac{1}{2}}) + ln(\nu)] \tag{4.16}$$

$$\approx ln((1+\frac{1}{2}\nu^{\frac{1}{2}} e^{\mu_\epsilon} h) - (1-\frac{1}{2}\nu^{\frac{1}{2}})) - \frac{1}{2}[ln(\nu) - \nu^{\frac{1}{2}}] \tag{4.17}$$

$$\approx ln(\frac{1}{2}) + ln(1+(1+\mu_\epsilon)h) + \frac{1}{2}\nu^{\frac{1}{2}} \tag{4.18}$$

where I have assume $r = 0$ and introduced linear approximations for quantities that are close to zero. Here, the piece-rate approaches $\frac{1}{2}$ for wealthy households with large stocks of human capital. While the optimal application strategy is rapidly increasing as a worker's wealth moves away from the borrowing constraint, $\frac{\partial \mu}{\partial a'} \approx 0$ for workers unlikely to face consumption risk from extended unemployment spells. This yields two important outcomes: differences in earnings for unconstrained workers are due to differences in human capital, and changes in ν have a level effect on the application strategy for unconstrained workers. Using information on the borrowing constraint from the re-employment elasticities, as well as observed earnings out of unemployment for unconstrained workers give information on ν.

A first-order Taylor approximation of Equation 4.13 yields a specification that can be easily estimated and closely mirrors the specifications from my empirical analysis in Section 2.

$$ln(\mu e^{\mu_\epsilon} h) \approx \underbrace{ln(\tilde{\mu}) + \mu_\epsilon + ln(\tilde{h}) + \frac{\partial ln(\tilde{\mu} e^{\mu_\epsilon} \tilde{h})}{\partial h}(h - \tilde{h})}_{Unobserved} + \underbrace{\frac{\partial ln(\tilde{\mu} e^{\mu_\epsilon} \tilde{h})}{\partial a'}(a' - \tilde{a})}_{Proportional\ to\ UI\ Elasticity} \tag{4.19}$$

Taking the expectation of this expression conditional on net liquid wealth and a set of observables X yields the following:

$$E[ln(\mu e^{\mu_\epsilon} h)|a \in a_q, X] \approx E[ln(\tilde{\mu}) + \mu_\epsilon + ln(\tilde{h})|a \in a_q, X] + \frac{\partial ln(\tilde{\mu} e^{\mu_\epsilon} \tilde{h})}{\partial h} E[(h - \tilde{h})|a \in a_q, X] \tag{4.20}$$

$$+ \frac{\partial ln(\tilde{\mu} e^{\mu_\epsilon} \tilde{h})}{\partial a'} E[(a' - \tilde{a})|a \in a_q, X] \tag{4.21}$$

$$\approx E[ln(\tilde{\mu}) + \mu_\epsilon + ln(\tilde{h})|a \in a_q, X] + \frac{\partial ln(\tilde{\mu} e^{\mu_\epsilon} \tilde{h})}{\partial b}(b - \tilde{b}) \tag{4.22}$$

within a wealth quintile, the log-earnings upon regaining employment are the sum of the average log piece-rate and undepreciated human capital, and the impact that differences in unemployment insurance had on a worker's application strategy through relaxing borrowing constraints. Evaluating and estimating this expression at each quintile of the wealth distribution yields the equivalent of a piece-wise linear interpolation of a worker's application strategy across the wealth distribution, with intercepts differing by quintile-level differences in average human capital as well as the impact of leisure utility. Formally, I discipline borrowing constraints ($\underline{a}_t$) and leisure utility (ν), by matching the coefficients from a regression that is closely related to my specification in Section 2.

$$
\begin{aligned}
ln(Y_{i,j+1}, t) = &\sum_{q=1}^{5}[\beta_0^q 1_q + \beta_1^q 1_q \times ln(UI_{i,j}) + \beta_2^q 1_q ln(Y_{i,j}) + \beta_3^q 1_q ln(Dur_{i,j})] + \beta_4 t \quad (4.23) \\
&+ X\delta + \epsilon_{i,j+1,t} \quad (4.24)
\end{aligned}
$$

Here, t refers to the age of the worker upon exiting unemployment. The vector δ is defined to include "nuissance parameters" that control for heterogeneity present in the data, that is not modeled. These include state and year fixed effects, industry and occupation fixed effects, race, marital status, among others listed in full in Section 2.

Intuition for identifying borrowing constraints and leisure utility is shown graphically in Figure 4.2.2. The left panel depicts the relationship between borrowing constraints faced by unemployed workers for hypothetical borrowing constraints. Holding wealth fixed, a larger level of net liquid wealth in the first quintile (equivalently, a more lax borrowing constraint) would correspond to an estimated elasticity β_1^1 approaching zero, with a resulting application strategy shown by the solid blue line. If there is a minimal response to relaxing the borrowing constraint by increasing unemployment benefits, then the slope of the application strategy evaluated at average wealth in the first quintile would be close to zero. If β_1^1 is large and positive, as shown by the red line, wealth net of the borrowing constraint is smaller (equivalently, there is a tighter borrowing constraint), and the slope of the application strategy is steeper.

The right panel demonstrates the use of unconstrained workers to identify leisure utility (ν). For low-wealth workers, the marginal utility of consumption exceeds the marginal utility of leisure, causing substantial overlap between application strategies in environment with different leisure utilities. For unconstrained workers, a higher marginal utility of leisure causes workers to search for jobs offering a higher piece-rate, to compensate them for the already low marginal utility of consumption. Because the slope of worker application strate-

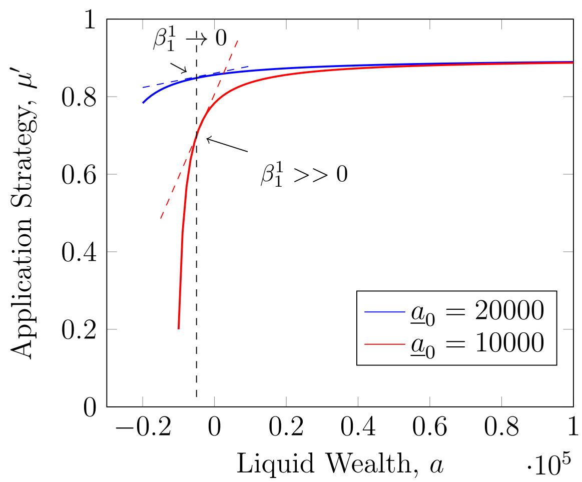


Figure 4.1: Identification of $\underline{a}_t$

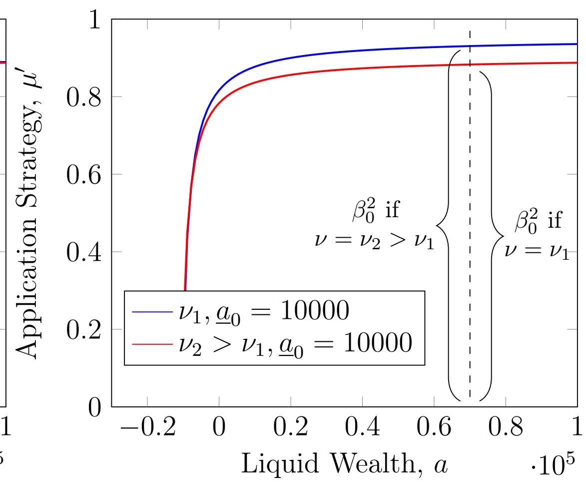


Figure 4.2: Identification of ν

gies approaches zero as workers become unconstrained, changes in ν induce a level-shift in observed earnings.

4.2.3 Information in Life-Cycle Earnings Profiles

A key challenge to identification is separate restrictions for the marginal distribution of learning ability (μ, σ) and the curvature of the human capital production function, α. To do this, I exploit variation over the life-cycle in human capital revealed by observed earnings. Intuitively, the slope of the earnings profile will be closely related to learning ability, over horizons long-enough that the influence of initial human capital diminishes. Deviations from this slope reveal information on contemporaneous and accumulated human capital investment.

In any period t, log-earnings of individual i is given by

$$
\begin{aligned}
ln(Y_{i,t}) &= ln(\mu_{i,t}(1 - \tau_{i,t})h_{i,t}) && (4.25) \\
&= ln(\mu_{i,t}) + ln(1 - \tau_{i,t}) + ln(h_{i,t}) && (4.26)
\end{aligned}
$$

Between period t and $t+1$, the difference equation in human capital yields the change in earnings. Dividing this difference equation by initial human capital yields an expression for the cumulative human capital growth between period-0 and period-$t+1$:

$$h_{t+1} = e^{\mu_\epsilon}(h_t + \ell(h_t\tau_t)^\alpha) \tag{4.27}$$

$$\frac{h_{t+1}}{h_0} = e^{\mu_\epsilon}(\frac{h_t}{h_0} + \frac{\ell}{h_0}(h_t\tau_t)^\alpha) \tag{4.28}$$

$$= e^{\mu_\epsilon}(e^{\mu_\epsilon}(\frac{h_{t-1}}{h_0} + \frac{\ell}{h_0}(h_{t-1}\tau_{t-1})^\alpha) + \frac{\ell}{h_0}(h_t\tau_t)^\alpha) \tag{4.29}$$

$$= \frac{\ell}{h_0}\sum_{j=0}^{t}(e^{\mu_\epsilon})^j(h_j\tau_j)^\alpha) + (e^{\mu_\epsilon})^t \tag{4.30}$$

That is, cumulative human capital growth is the sum of investment between period 0 and $t + 1$, net of undepreciation initial human capital. Over long horizons, the impact of undepreciated initial human capital approaches zero as long as depreciation is strictly negative ($\mu_\epsilon < 0$). Let β_i denote the average growth rate of human capital for individual i, and assume $t + 1 = T$. Then, the human capital diference equation can be written as

$$(1 + \beta_i)^T\frac{h_0}{h_0} = \frac{\ell}{h_0}\sum_{j=0}^{t}(e^{\mu_\epsilon})^j(h_j\tau_j)^\alpha) \tag{4.31}$$

$$Tln(1 + \beta_i) \approx ln(\ell) - ln(h_0) + ln(\sum_{j=0}^{t}(e^{\mu_\epsilon})^j(h_j\tau_j)^\alpha) \tag{4.32}$$

$$\beta_i \approx \frac{1}{T}[ln(\ell) - ln(h_0) + ln(\sum_{j=0}^{t}((e^{\mu_\epsilon})^j(h_j\tau_j)^\alpha))] \tag{4.33}$$

For each individual, human capital growth is approximately equal to the sum of log-learning ability and net human capital investment, averaged over the observed horizon T. Averaging across individuals yields

$$E[\beta_i] \approx \frac{1}{T}E[ln(\ell) - ln(h_0) + ln(\sum_{j=0}^{t}(e^{\mu_\epsilon})^j(h_j\tau_j)^\alpha)] \tag{4.34}$$

$$\beta \approx \frac{1}{T}(\mu_\ell - \mu_h + E[ln(\sum_{j=0}^{t}((e^{\mu_\epsilon})^j(h_j\tau_j)^\alpha))]) \tag{4.35}$$

Using this coefficient allows me to rewrite the expectation of log earnings at age t as

$$E[ln(Y_{i,t})] = \beta_0 + \beta_1 \times t \tag{4.36}$$
$$= \beta_0 + (\beta_1^h + \beta_1^\mu) \times t + \delta \times Hours\ Worked \tag{4.37}$$

where β_1^h is the average growth rate of human capital, and β_1^μ is earnings growth due to increases in worker piece-rate. To separate β_1^h from β_1^μ, I match the average earnings growth of job-stayers and probability of staying over six period of the life-cycle (five year age bins), and include separate slopes in my age-earnings regressions for individuals early in their career (ages 25-40) and later in their career (ages 41-62). While workers are able to allocate time to learning, there is no extensive margin in the model, so I condition on hours to control for both the extensive margin and time allocated to learning. I discuss these approaches further in Section 4.2.4.

To separately identify α from the distribution of learning ability (μ_ℓ, σ_ℓ), I match the earnings profile over the same horizon. For ages in which average earnings exceed those predicted by the age-earnings regression, cumulative human capital investment exceeds its life-cycle average. The decision rules from the parsimonious model help demonstrate the separate identification of the parameters for the human capital production process. Human capital growth varies over the life-cycle due to the present value of future earnings; I therefore prepend an additional period to the parsimonious model to show that investment necessarily varies in ways that identify the structural parameters. I assume that an individual employed in period $t = 0$ faces no unemployment risk and the same parametric assumptions as above. This yields a human capital investment function in period-0

$$h_0\tau_0 = (\frac{\alpha\ell e^{\mu_\epsilon}}{1+r})^{\frac{1}{1-\alpha}}(\frac{1+r+e^{\mu_\epsilon}}{1+r})^{\frac{1}{1-\alpha}} \tag{4.38}$$

and as before, a period-1 investment function

$$h_1\tau_1 = (\frac{\alpha\ell e^{\mu_\epsilon}}{1+r})^{\frac{1}{1-\alpha}} \tag{4.39}$$

Learning ability scales human capital productivity multiplicatively, while α changes the curvature. If productivity allocated to learning, $h\tau$, were fixed with respect to ℓ and α, changes in $h\tau$ over time (equivalently, the growth rate of human capital), would separately identify both parameters. As shown in Equation 4.38 and Equation 4.39, ℓ and α enter both decision rules. However, these decision rules show that the learning allocation varies over time as the future value of human capital falls. Intuitively, the parameters will be separately identified if a change in one cannot be offset by a change in the other parameter while still yielding the same path of human capital. Consider the growth rate of human capital between periods 0 and 1:

$$h_1 = e^{\mu_\epsilon}(h_0 + \ell(h_0\tau_0)^\alpha) \tag{4.40}$$

$$h_1 - e^{\mu_\epsilon}h_0 = e^{\mu_\epsilon}\ell^{\frac{1}{1-\alpha}}(\frac{\alpha e^{\mu_\epsilon}}{1+r})^{\frac{1}{1-\alpha}}(\frac{1+r+e^{\mu_\epsilon}}{1+r})^{\frac{\alpha}{1-\alpha}} \tag{4.41}$$

$$\frac{h_1 - e^{\mu_\epsilon}h_0}{e^{\mu_\epsilon}} = \ell^{\frac{1}{1-\alpha}}(\frac{\alpha e^{\mu_\epsilon}}{1+r})^{\frac{1}{1-\alpha}}(\frac{1+r+e^{\mu_\epsilon}}{1+r})^{\frac{\alpha}{1-\alpha}} \tag{4.42}$$

A change in α can be offset by a change in ℓ and still result in the same observed path of human capital between periods 0 and 1. Because ℓ is fixed over time, the same equation would need to hold between periods 1 and 2, given the compensating change in α. Evaluating the same expression using the period-1 decision rule yields

$$h_2 = e^{\mu_\epsilon}(h_1 + \ell(h_1\tau_1)^\alpha) \tag{4.43}$$

$$h_2 - e^{\mu_\epsilon}h_1 = e^{\mu_\epsilon}\ell(h_1\tau_1)^\alpha) \tag{4.44}$$

$$\frac{h_2 - e^{\mu_\epsilon}h_1}{e^{\mu_\epsilon}} = \ell^{\frac{1}{1-\alpha}}(\frac{\alpha e^{\mu_\epsilon}}{1+r})^{\frac{\alpha}{1-\alpha}} \tag{4.45}$$

$$\tag{4.46}$$

This shows that a simultaneous change in ℓ and α that yields the same growth rate of human capital between periods 0 and 1 cannot keep the growth rate constant between periods 1 and 2 unless the average percent depreciation of human capital, μ_ϵ, is equal to zero. In fact, an increase in ℓ during period-1 requires an increase in α to hold human capital growth constant (workers would like to re-allocate toward savings), but such a change in ℓ requires a decrease in α to keep human capital growth fixed, because human capital investment yields an additional period of returns.

I apply the same intuition in selecting specifications to identify α separately from learning

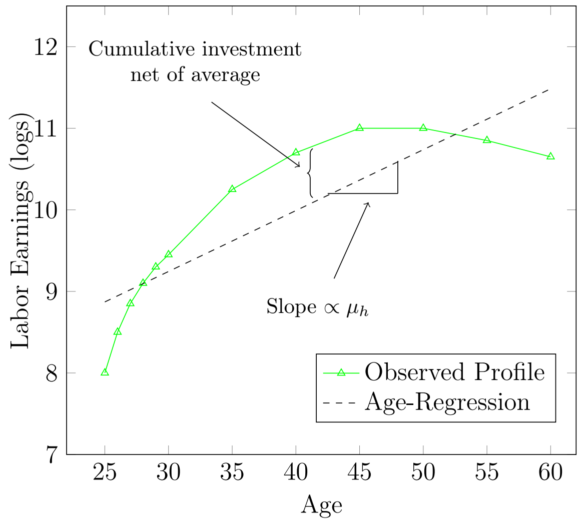


Figure 4.3: Identification of α_H

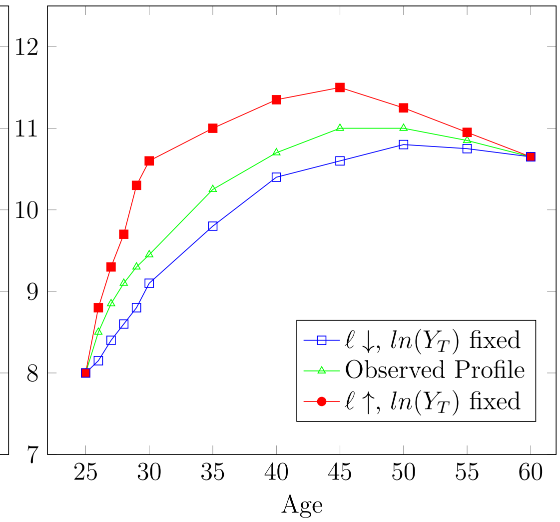


Figure 4.4: Identification of μ_L

ability in my auxiliary model. Any deviations in earnings growth from the life-cycle average cannot be perfectly offset by movements in learning ability and α while still retaining the same earnings profile. Thus, I match the slope and intercept of life-cycle earnings, yielding inference on μ, and the age-earnings profile, yielding inference on α. Figure 4.3 and Figure 4.4 show a graphical depiction of the identification of learning and α.

Both are estimated using data from the Panel Study of Income Dynamics, conditioning on observed hours, and controlling for the observables described in Section A.2. I discuss my use of age-regressions more extensively in Section 4.2.4, where I use them to additionally yield inference on correlations between initial conditions. The age-earnings profile I include in my auxiliary model is given by

$$ln(Y_{i,t}) = \sum_t 1_t \beta_t + X_{i,s,t}\delta + \epsilon_{i,s,t} \tag{4.47}$$

4.2.4 Information from the Age-Earnings Regressions

By including Equation 4.23 in my auxiliary model, a piecewise linear approximation to a worker's application strategy, I can use observed life-cycle earnings to discipline the structural parameters that determine initial worker heterogeneity. There are two sources of wage growth over the life-cycle in my model: workers can move to firms that offer a higher piece-rate, and workers can increase their future human capital by allocating productive time to learning. Within the structure of my model, the first source is jointly identified by a worker's application strategy and observed job-to-job movement. The second is identified using information on earnings growth over the life-cycle.

Observing earnings over three distinct periods of a worker's career will be instrumental in identifying and separating worker heterogeneity. Earnings at first employment by wealth and Armed Forces Qualifying Test (AFQT) scores[23] are informative about the distribution of human capital (μ_H, σ_H) as well as the correlations between initial conditions (ρ_{AH}, ρ_{HL}). I also use the slope of earnings stratified by wealth and AFQT quintile during two subsequent periods: early in a workers career, when a substantial fraction of wage and earnings growth is due to movement up the job ladder, and over the middle of a worker's career, when job mobility has slowed and earnings growth will on average be a result of human capital growth. Both sets of slopes are informative of the joint and marginal distributin of learning ability $(\rho_{AL}, \rho_{HL}, \mu_L, \sigma_L)$.

First, I show how I can exploit initial earnings by wealth and AFQT quintiles to determine the marginal distribution of human capital as well as the correlations with wealth and learning. Consider log-earnings of an individual at a first job after entering the labor market can be written as

$$ln(Y_0) = ln(\mu_0) + ln(1 - \tau_0) + ln(h_0) + \xi_0 \tag{4.48}$$

where ξ is the first appearance of the measurement error described in the calibration. I discuss the identification strategy for the distribution of measurement error in Section 4.2.6. Log earnings controlling for hours worked are given by

$$ln(Y_0) = ln(\mu_0) + ln(h_0) + \xi_0 \tag{4.49}$$

Similar to Equation 4.23, I take a conditional expectation over initial wealth and learning ability to identify the correlations ρ_{AH} and ρ_{HL}, between wealth and human capital, and learning ability and human capital, respectively.[24] Applying this expectation to Equa-

[23]AFQT scores are commonly used to proxy for unobserved ability. While they are a noisy signal, as long as they are unbiased, they can yield inference on correlations.

[24]Generically, the conditional expectation of two jointly normal random variables X, Y with correlation ρ

tion 4.49 over quintiles of initial wealth yields

$$E[ln(y_0)|a_0 \in a_0^q] = E[ln(\mu_0)|a_0 \in a_0^q] + E[ln(h_0)|a_0 \in a_0^q] + E[\xi_0|a_0 \in a_0^q] \tag{4.50}$$
$$= E[ln(\mu_0)|a_0 \in a_0^q] + E[ln(h_0)|a_0 \in a_0^q] \tag{4.51}$$
$$= E[ln(\mu_0)|a_0 \in a_0^q] + \mu_H + \rho_{AH}\frac{\sigma_H}{\sigma_A}(a_0 - \mu_A) \tag{4.52}$$

The parameters characterizing the initial wealth distribution (μ_A, σ_A) are estimated directly from the data, as I describe in Section 4.2.5. The piecewise linear approximation to worker application strategies (Equation 4.23), which conditions on age, helps separately identify $E[ln(\mu_0)|a_0 \in a_0^q]$. As before, matching this expression across the initial wealth distribution yields information on the distribution of human capital (μ_H, σ_H) and the correlation with initial wealth (σ_{AH}).

Because much of earnings growth early in the life-cycle is due to job-to-job movement, I match the slope of the wage profile over two separate periods of the life-cycle. Between ages 25 and 40, wage growth is driven by both job-to-job movement as well as increases in productivity. After age 40, job-to-job movements are roughly fixed in the data, and wage growth slows. I estimate the slope by initial wealth separately for these two periods of the life-cycle. Figure 4.6 shows wage profiles by wealth level for hypothesized values of σ_{AL}.

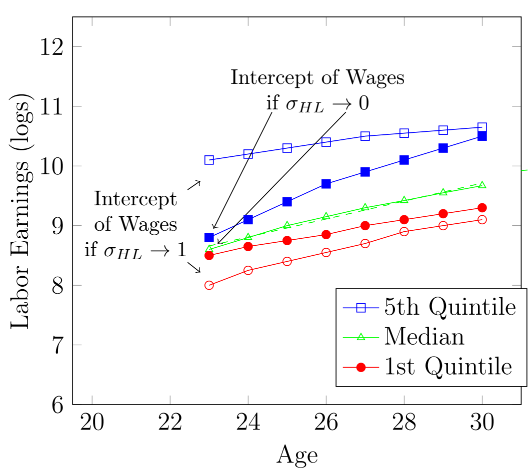


Figure 4.5: Identification of σ_{HL}

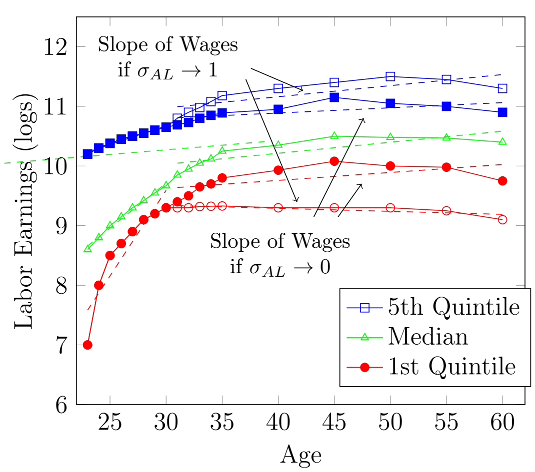


Figure 4.6: Identification of σ_{AL}

is given by $E[X|Y] = E[X] + \rho\frac{\sigma_X}{\sigma_Y}(Y - E[Y])$

If σ_{AL} is approaching 1, wages will fan out along initial wealth, shown by the dashed top and bottom lines. If instead σ_{AL} is approaching 0, the slopes of wages will be approximately the same by wealth. To match both initial earnings and the slope of earnings over the life-cycle, I include the following specification in my auxiliary model:

$$ln(y_t) = \sum_{q=1}^{5} \beta_0^q \mathbb{1}_{a0 \in a_0^q} + \sum_{q=1}^{5} \beta_1^q \mathbb{1}_{a0 \in a_0^q} Age + \sum_{q=1}^{5} \beta_2^q \mathbb{1}_{a0 \in a_0^q, Age \geq 40} Age + \delta' X + \psi_t \qquad (4.53)$$

where q is a worker's initial asset quintile. The vector β_0 helps identify the marginal distribution of human capital as well as the correlation with initial wealth (Equation 4.50). The coefficients in β_1 is the age-slope of wages between ages 25 and 40, where growth is due to both job-to-job movement and human capital growth, and β_2^q is the age-slope of wages between ages 25 and 41. Both β_1 and β_2 yield inference on the correlation between wealth and learning ability. I include additional controls in δ, which are beyond the scope of the model (demographic characteristics, state and year fixed effects, etc.).

I include two periods of earnings growth rather than explicitly condition on job-to-job movement because there are very few individuals in the PSID for whom both an entire history of job-to-job movement as well as inference on initial wealth can be observed. Indirect inference allows me to cope with this by treating my simulated data with the same irregularities–imposing randomly missing data–at the same frequency as the PSID.

Identifying the joint and marginal distribution of learning ability requires a slightly modified argument, but follows the same logic as Equation 4.50. I assume that the Armed Forces Qualifying Test (AFQT) observed for every individual between ages 14 and 22 in the NLSY79 is an imperfect proxy for learning ability. That is, the recorded AFQT scores yield $\ell^{AFQT} = \ell + \omega$ where $\omega \sim N(0, \sigma_\omega)$ is classical measurement error. Initial earnings conditional on AFQT quintile can be expressed as

$$E[ln(y_0)|\ell \in \ell^q] = E[ln(\mu_0)|\ell \in \ell^q] + E[ln(h_0)|\ell \in \ell^q] + E[\omega|\ell \in \ell^q] + E[\xi_0|\ell \in \ell^q] \qquad (4.54)$$

$$= E[ln(\mu_0)|\ell \in \ell^q] + \mu_H + \rho_{HL}\frac{\sigma_L}{\sigma_H}(\ell - \mu_L) + \mu_\omega + \rho_{\omega L}\frac{\sigma_L}{\sigma_\omega}(\ell - \mu_L) \qquad (4.55)$$

$$= E[ln(\mu_0)|\ell \in \ell^q] + \mu_H + \rho_{HL}\frac{\sigma_L}{\sigma_H}(\ell - \mu_L) \qquad (4.56)$$

$$\approx E[ln(\mu_0)] + \mu_H + \rho_{HL}\frac{\sigma_L}{\sigma_H}(\ell - \mu_L) \qquad (4.57)$$

Classical measurement error ω is assumed to be uncorrelated with ℓ, which yields the preceding equation. An underlying assumption is that $E[ln(\mu_0)|\ell \in \ell^q] \approx E[ln(\mu_0)]\forall \ell^q$, and thus differences in initial earnings results from differences in human capital. To see why this assumption is reasonable, consider that firms are unlikely to accrue any rents by offering more generous employment terms to individuals with different learning ability levels. Time allocated to learning will decrease productivity, and workers can freely transport this human capital to future employers. This moral hazard problem depresses any gains in employment probability.

I use the same specification as Equation 4.50, stratified by AFQT quintile observed before entering the labor market. As before, β_0^q is informative of the correlation between initial human capital and learning ability, σ_{HL}. The slope over a short horizon by learning quintile, β_1^q yields information on α. The reason is that α controls the relative importance of initial human capital and learning ability in the human capital production process. If $\alpha = 1$, initial human capital has the same rate of return as initial learning ability. If $\alpha = 0$, initial human capital has no impact on life-cycle human capital accumulation. Over a longer horizon, the model predicts that initial human capital will be less important than learning ability in determining human capital, meaning that β_2^q yields inference on σ_L.

4.2.5 Information from Initial Distributions

To discipline the marginal distributions of wealth and human capital, I ask the estimated model to match the average value of each decile of the reported liquid wealth (prior to entering the labor market) and first job annual earnings, using data from the Panel Study of Income Dynamics.

Matching initial liquid wealth yields inference on μ_A and σ_A, and provides additional inference on the age-23 borrowing constraint, $\underline{a}_0$. While these moments would appear to be a candidate for external calibration, there are several reasons that such an exercise is not straightforward. The model assumes that all wealth can be immediately transformed into consumption, while the liquidity of wealth reported in the PSID (although classified as liquid wealth) may vary. Perhaps more importantly, individuals may face heterogeneity in their ability to borrow that is not observable. As a result, I estimate this distribution jointly within the auxiliary model.

Extracting information on the marginal distribution of human capital using initial earnings rests on the proportionality of earnings to human capital, the ability of the targets in Section 4.2.2 to help identify the curvature of a worker's application strategy near the borrowing constraint, and the approximate linearity of a worker's application strategy far enough away from the borrowing constraint. Then, given the distribution of intial wealth, I can recover the distribution of human capital by using the law of iterated expectations over Equation 4.50.

4.2.6 Information in Wage Growth

To identify the distribution of human capital depreciation $(\mu_\epsilon, \sigma_\epsilon)$, as well as measurement error, I focus on earnings growth as workers near the end of their careers in my model (ages 55-64), ages during which continuation value of savings exceeds that of human capital because of impending retirement, and workers are unlikely to allocate time to human capital investment. Within this group, I focus on individuals who remain with the same employer year to year. With these restrictions, I use the same identification strategy as Huggett et al. (2011): I match the level of wage growth at each age, the variance of wage growth at each age, and the covariance between ages for the subsample between ages 58 and 64, who remain at the same job (i.e., $\mu_{58} = \mu_{59} = ... = \mu_{64}$).

At each age, log earnings is given by

$$ln(Y_t) = ln(\mu_t) + ln(h_t) + \xi_t \tag{4.58}$$

where $\xi_t \sim N(0, \sigma_\xi)$ is mean-zero normally distributed measurement error. Then the change in wages year to year is given by

$$\begin{aligned} ln(y_{t+1}) - ln(y_t) &= ln(\mu_{t+1}) + ln(h_{t+1}) + \xi_{t+1} - ln(\mu_t) - ln(h_t) - \xi_t && (4.59) \\ &= \cancel{(ln(\mu_{t+1}) - ln(\mu_t))} + (ln(h_{t+1}) - ln(h_t)) + (\xi_{t+1} - \xi_t) && (4.60) \end{aligned}$$

$$E[ln(y_{t+n}) - ln(y_t)] = E[ln(h_{t+n}) - ln(h_t)] + \cancel{\mu_\xi - \mu_\xi)} \tag{4.61}$$

$$E[ln(y_{t+n}) - ln(y_t)] = E[\sum_{j=t+1}^{t+n} \epsilon_j + \cancel{ln(h_t) - ln(h_t)}] \tag{4.62}$$

$$E[ln(y_{t+n}) - ln(y_t)] = n\mu_\epsilon \tag{4.63}$$

Thus, the level of wage growth between ages 59 and 64 for workers who remain at the same job year-to-year can identify the average shock to human capital. The variance and covariance of earnings growth helps identify the variance of human capital shocks as well as the variance of measurement error. Measurement error is assumed to be uncorrelated with the human capital shock.

$$Var(ln(y_{t+n}) - ln(y_t)) = Var(ln(h_{t+n}) - ln(h_t)) + Var(\xi_{t+1} - \xi_t) \tag{4.64}$$

$$= Var(ln(h_{t+n})) + Var(ln(h_t)) + Var(\xi_{t+1}) + Var(\xi_t) \tag{4.65}$$

$$= n\sigma_\epsilon^2 + 2\sigma_\xi^2 \tag{4.66}$$

and

$$Cov(ln(y_{t+n}) - ln(y_t), ln(y_{t+m}) - ln(y_t)) = m\sigma_\epsilon^2 + \sigma_\xi^2 \tag{4.67}$$

I assume that measurement error is distributed $\xi \sim N(0, \sigma_\xi)$ in each of the three datasets that I employ. Thus, matching the average growth rate of earnings between ages 59 and 64, as well as the variance and covariance of growth rates over the same period can separately identify $\mu_\epsilon, \sigma_\epsilon$, and σ_ξ

4.2.7 Information in Job Mobility

The final set of targets I employ are used to discipline the job-to-job movement of workers in the model. Job-to-job movement is the product of λ_E, the efficiency of searching while employed, and $P(\theta(\mu'))$, the job-finding rate in a submarket μ'. Common estimates in the literature use average job flows, but are typically estimated for models in which agents are risk-neutral that may have different degrees of heterogeneity in their application strategies.

Here, the majority of job-to-job movement is conducted by low-wealth individuals who move frequently, but experience smaller piece-rate growth than wealthier peers at the same previous employer. Because this is a particularly important effect early in the life-cycle, I match the average job-staying rate for six equal-sized age-bins from 25 to 54 in the NLSY. I choose the job-staying rate rather than the job-to-job transition rate because at an annual frequency (the frequency available in the NLSY), job-mobility is more likely to feature aggregation bias (from multiple moves or unemployment spells), while individuals are directly surveyed on whether they have changed employers within the previous year.

4.2.8 Implementation

Indirect inference can be implemented as either maximum likelihood, by minimizing a Gaussian objective function, or generalized method of moments. Because I use multiple datasets, the generalized method of moments approach is a more natural fit. This makes my estimation analogous to a seemingly unrelated regression (SUR) estimation. Indirect inference proceeds by first specifying an auxiliary model, and minimizing the distance between auxiliary parameters from the data and model simulations. My implementation follows the standard approach, which I restate in Section B.2.

For the model generated data, I average over $S = 100$ simulations for each iteration, and impose identical sample restrictions and attrition rates as in the observed data. I treat simulated data precisely the same as in my empirical analysis: I impose identical sample restrictions (where applicable) in my simulations, and force each sample to contain an identical number of observations as its empirical counterpart. To deal with missing data in the PSID and NLSY, I randomly drop observations at the same frequency as in the data by age. I do this by wealth and AFQT quantiles so that the data generating process from the structural model is as close as possible to that in the data. I simulate separate sets of data for each dataset used in the auxiliary model. I start agents at age 23 with no labor market experience (i.e., unemployed without unemployment insurance) and a random draw from the joint distribution of initial conditions.

4.3 Estimation Results

I use simulated annealing to estimate the model. This allows me to solve for a global minimum distance by sampling from the parameter space and comparing objective function values. With some positive probability, it accepts a new point at which the objective function is higher than previous, and then searches nearby points. This allows the algorithm to test areas of the parameter space that other approaches would have ruled out, giving credibility to the global solution. The parameter estimates are reported in Table 4.3. Notably, the standard errors fit tightly around the estimated values, with the exception of leisure utility. Because of the differences in scale and frequency, some of the parameters are not directly

Table 4.3: Estimation Results

Category	Symbol	Model Value			Comment
Model Parameters					
Subsistence Benefits	b_L	0.0396			
		[,]			
Borrowing Constraint	$\underline{a}_t$				Qtrly Period-0 Value (2011\$): \$12,182
On-the-job Search Efficiency	λ_E	0.6709			Herkenhoff (2014): 0.73
		[,]			
Human Capital Curvature	α_H	0.5167			Browning et al. (1999): [0.5, 0.99]
		[,]			
Leisure Utility	ν	4.0910×10^{-12}			
		[,]			
Initial Conditions					
Initial Wealth	(μ_A, σ_A)	$\mu_A = 1.0949$	$\sigma_A = 1.2775$		Mean (2011\$): $\$16,094$
		[,]	[,]		
Initial Human Capital	(μ_H, σ_H)	$\mu_H = -0.0621$	$\sigma_H = 0.8747$		Mean (2011\$): $\$15,458$
		[,]			
Learning Ability	(μ_L, σ_L)	$\mu_L = -2.7020$	$\sigma_L = 0.5099$	Mean: 0.089	
		[,]	[,]		
Correlations	$\rho_{AH}, \rho_{AL}, \rho_{HL}$	$\rho_{AH} = 0.4250$	$\rho_{AL} = 0.5805$	$\rho_{HL} = 0.3366$	Huggett et al. (2011): $\rho_{HL} = 0.655$
		[,]	[,]	[,]	
Displacement	(δ_H, δ_L)	$\delta_H = 2.2528$	$\delta_L = 0.0124$		
		[,]	[,]		
Other Distributions					
Human Capital Depreciation	$(\mu_\epsilon, \sigma_\epsilon)$	$\mu_\epsilon = -0.0162$	$\sigma_\epsilon = 0.0354$		Huggett et al. (2011): $(-0.029, 0.111)$
		[,]	[,]		
Measurement Error	$(0, \sigma_\xi)$	$\sigma_\xi = 0.0514$			Guvenen (2009): $(0, 0.15)$
		[,]	[,]		

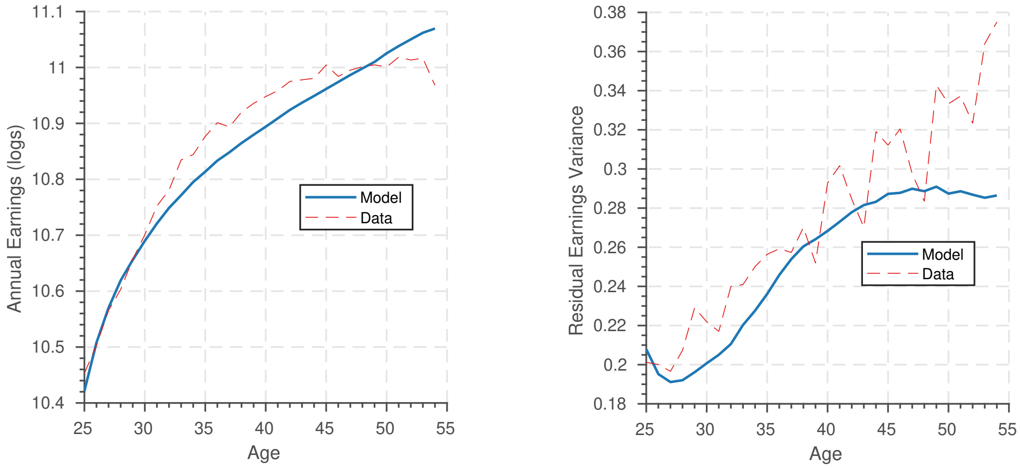
Notes: Mean initial human capital is quoted at a quarterly frequency in terms of total possible productivity. The 95% confidence intervals of the estimates are shown in brackets beneath the structural parameters.

comparable with Huggett et al. (2011). The human capital curvature is higher than estimated in previous search papers, and falls just within the bottom of the estimates from Browning et al. (1999), who put the range at $[0.5, 0.99]$, for models without search frictions.

4.4 Fit

In this section, I highlight moments that are closely associated with the mechanisms in the model, or can be easily displayed graphically. I present the remaining moments in Table C.2 and Table C.3.

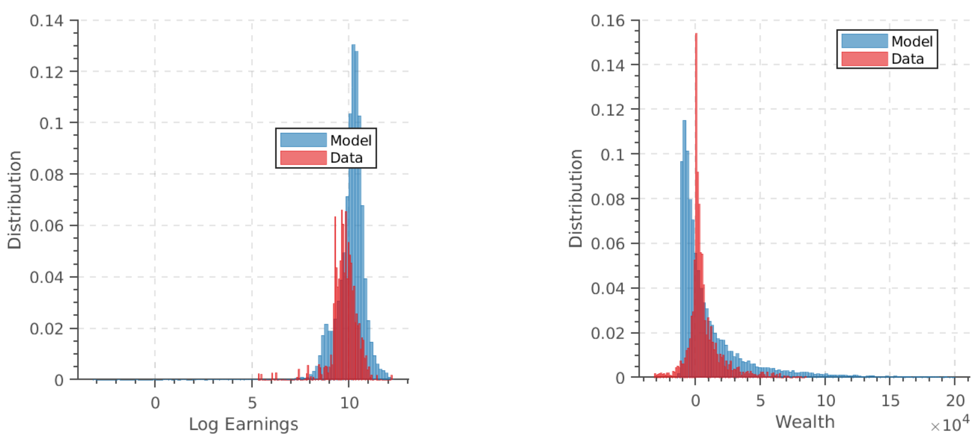
The model does reasonably well at fitting most of the moments in the auxiliary model. Although I am matching 168 auxiliary model coefficients using 18 structural parameters, the model replicates moments that target initial inequality as well as inequality over the life-cycle. In the two panels of Figure 4.7, I show the model's ability to match average earnings profiles over the life-cycle, as well as the estimated variance. In each case, the model-generated data is well within the standard errors of the data observed from the PSID.



(a) Earnings (b) Variance

Figure 4.7: Life-cycle profiles

The model comes reasonably close to replicating the first and second moments of life-cycle earnings, but begins to diverge as the model nears retirement age. As shown by the income growth moments nearing retirement (Table C.3), the model understates the decline in earnings late in the life-cycle, which could explain the estimates. In Figure 4.8, I compare the data on initial distributions with analogues produced by the model.



(a) First Job Earnings (b) Pre-Labor Market Wealth

Figure 4.8: Initial Distributions

The mean initial earnings generated by the model is slightly higher than the mean observed in the data. The model predicts a slightly more right-skewed initial distribution of wealth than is observed in the data. Although initial liquid wealth is observed in the data and would appear to be directly estimable, the translation between reported liquid wealth and wealth that can be used immediately for consumption may not be one-to-one. That is, individuals who report negative wealth below some threshold may have already exceeded their ability to borrow. Another challenge to matching this data is that many individuals report zero liquid wealth, which decreases the skewness of the observed data.

While the model comes reasonably close to replicating initial and life-cycle inequality, there are coefficients that the model has difficulty matching jointly. As I show Table 4.4, under the "Re-Employment Elasticities" heading, the model captures some of the wealth effect associated with relaxing the borrowing constraint, but underpredicts the employment outcome for low-wealth workers in the model. While a closer estimate would clearly be preferable, the estimate indicates that my results are likely to understate the importance of wealth.[25] Similarly, the model underpredicts the slope of the age-earnings regression for low-wealth individuals, while overpredicting their intercept, suggesting that the estimation may understate the degree to which workers are affected by wealth early in the life-cycle. The estimated model overpredicts the intercept of low-AFQT individuals as well, indicating again that there is perhaps too little job ladder movement among low-type individuals early in the life-cycle. For many of the remaining parameters displayed in the table, the model generated data does a reasonable job approximating its empirical analogues.

Given that my auxiliary model contains 168 coefficients, its ability to broadly match the sign and magnitude of most coefficients should not be discounted. Additionally, the areas in which my model falls short are likely to yield more conservative estimates of the impact of wealth on outcomes though the effect on other initial conditions is less clear. I summarize the remaining parameters in Table C.2 and Table C.3.

[25] The difference is due to some selection into unemployment in the data, who are on average less wealthy than the pool of unemployed in my model, as well as my use of a natural borrowing constraint. A tighter borrowing constraint could generate a stronger response.

Table 4.4: Key Auxiliary Parameters

Slopes and Intercepts by Wealth (PSID)				Slopes and Intercepts by AFQT (NLSY)				Re-Employment Elasticities (SIPP)			
Var.	Data	Model	P-Val	Var.	Data	Model	P-Val	Var.	Data	Model	P-Val
Age	0.0476	0.0281	1.0869×10^{-11}	Age	0.0287	0.0160	0.1491	Q1 x Ln(UI)	0.3622	0.2005	0.1680
	(0.1366)	(0.0008)			(0.1308)	(0.0012)			(0.2096)	(0.0099)	
Wealth Q3 x Age	−0.0117	−0.0015	0.0019	AFQT Q3 x Age	−0.0005	0.0063	0.0018	Q2 x Ln(UI)	0.2255	0.0112	0.1642
	(0.0034)	(0.0011)			(0.0018)	(0.0017)			(0.2228)	(0.0129)	
Wealth Q5 x Age	−0.0226	0.0108	3.6659×10^{-13}	AFQT Q5 x Age	0.0213	0.0421	5.6811×10^{-6}	Q3 x Ln(UI)	0.0023	−0.1445	0.0011
	(0.0044)	(0.0016)			(0.0046)	(0.0016)			(1.3289)	(0.0906)	
Cons.	9.5966	9.5150	0.3323	Cons.	9.2944	10.0062	0.0002	Q4 x Ln(UI)	0.0836	0.0171	
	(0.1730)	(0.0255)			(0.2019)	(0.0383)			()	()	
Wealth Q3	0.4219	0.3352	0.2281	AFQT Q3	0.3270	−0.0039	0.0210	Q5 x Ln(UI)	0.0549	0.0891	
	(0.1725)	(0.0354)			(0.0834)	(0.0528)			()	()	
Wealth Q5	0.9423	0.3463	0.0044	AFQT Q5	−0.1206	−0.5559	3.1168×10^{-15}	Q2	0.3580	1.6730	0.3858
	(0.1156)	(0.0517)			(0.0951)	(0.0509)			(1.3631)	(0.2374)	
Wealth Q1 x Age x (Age>=40)	−0.0445	−0.0176	2.7722×10^{-7}	AFQT Q1 x Age x (Age>=40)	−0.0183	−0.0132	0.2532	Q3	1.6347	3.3341	0.0606
	(0.0052)	(0.0014)			(0.0069)	(0.0050)			(1.3697)	(0.3134)	
Wealth Q3 x Age x (Age>=40)	−0.0264	−0.0120	0.0004	AFQT Q3 x Age x (Age>=40)	−0.0200	−0.0044	0.0071	Q4	1.5052	1.3282	0
	(0.0040)	(0.0013)			(0.0057)	(0.0027)			(0)	(0)	
Wealth Q5 x Age x (Age>=40)	−0.0160	−0.0291	0.0283	AFQT Q5 x Age x (Age>=40)	−0.0408	−0.0491	0.1457	Q5	2.3582	0.2583	0
	(0.0063)	(0.0023)			(0.0078)	(0.0023)			(0)	(0)	
Wealth Q1 x (Age>=40)	1.7020	0.6207	1.4962×10^{-6}	AFQT Q1 x (Age>=40)	0.6502	0.4519	0.3346	Cons.	2.5794	6.3206	0.0011
	(0.2251)	(0.0618)			(0.3007)	(0.4271)			(1.3289)	(0.0906)	
Wealth Q3 x (Age>=40)	0.9719	0.4508	0.0024	AFQT Q3 x (Age>=40)	0.7706	0.1455	0.0124				
	(0.1732)	(0.0551)			(0.2511)	(0.1201)					
Wealth Q5 x (Age>=40)	0.5581	1.1078	0.0331	AFQT Q5 x (Age>=40)	1.6345	1.8695	0.2337				
	(0.2775)	(0.0977)			(0.3207)	(0.0993)					

Notes:

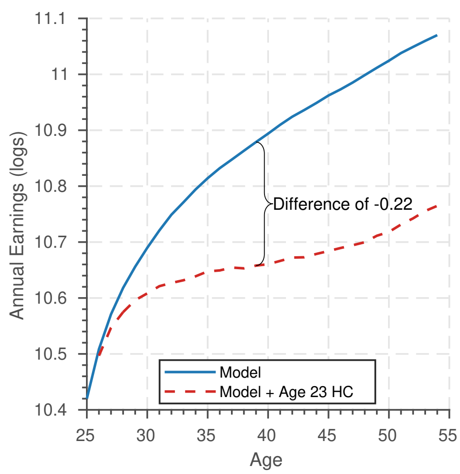
5 The Determinants of Life-Cycle Inequality

Inequality results from differences in initial conditions and shocks throughout the life-cycle. Upon initially entering the labor force, workers differ with respect to wealth, human capital, and learning ability, each of which alters the dynamics of earnings. After entrance, workers are subject to separation shocks, job-finding uncertainty, and stochastic human capital depreciation. I first explore the dynamics of earnings growth in Section 5.1 and then decompose the sources of life-cycle inequality between initial conditions and shocks realized over the life-cycle in Section 5.2.

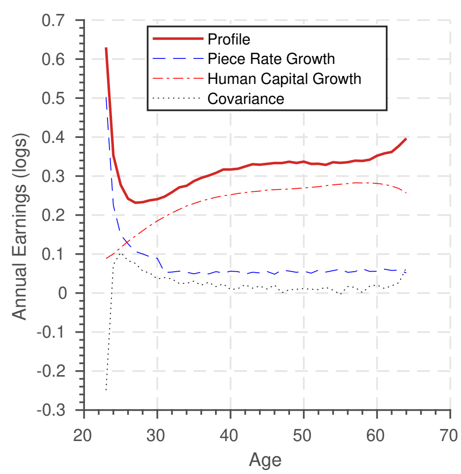
In the concluding section, I explore the contribution of uncertainty and risk to human capital accumulation. Over the life-cycle, workers face uncertainty over their labor market outcomes and the duration of any employment relation, as well as stochastic depreciation of their human capital. Uncertainty interacts with the initial conditions by altering the exposure of borrowing constrained workers to consumption risk throughout the life-cycle, and changing the rate of return on human capital investment for unconstrained individuals. Compared with their unconstrained peers, exposure to consumption risk depresses human capital accumulation and job placement, leading to long-term consequences on earnings of initial inequality in wealth. I use restrictions within the model to decompose the role of unemployment and consumption risk as drivers of earnings growth in Section 5.3.2 and Section 5.3.1, human capital and job search, respectively.

5.1 The Dynamics of Job Search and Human Capital

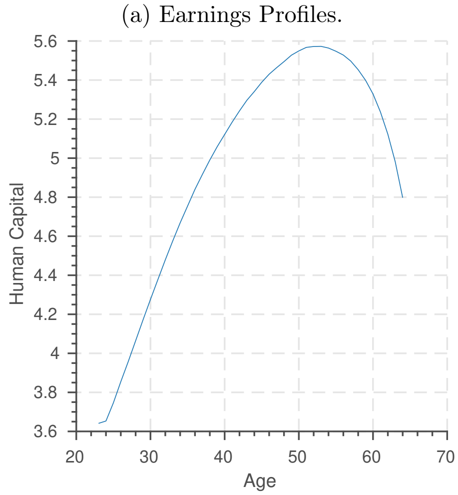
Differences in earnings over the life-cycle are the product of worker selectivity among firms offering different piece-rates and dispersion in productivity once a worker obtains employment. I plot two approaches to understanding the dynamics of each in Figure 5.1 as well as the profiles of average human capital and piece-rate. The top left panel plots a comparison between the model generated earnings profile, and an earnings profile holding human capital fixed at its age-23 value. The top right panel decomposes earnings growth into piece-rate growth, human capital growth, and the residual, which includes both the time allocated to learning and the covariance between these two components.



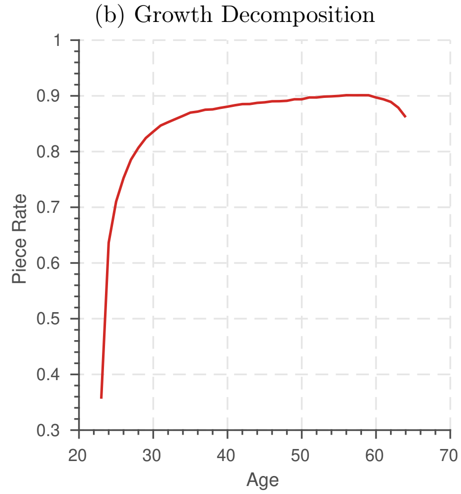
(a) Earnings Profiles.



(b) Growth Decomposition



(c) Average Human Capital (Levels, Unscaled)



(d) Average Piece-Rate (Levels)

Figure 5.1: Life-cycle profiles

Each panel demonstrates that job-to-job mobility is an important driver early in a workers career, but is supplanted in importance by human capital growth as workers become employed in high piece-rate firms. Initially, the majority of inequality in earnings is caused by differences in piece-rates across individuals, but over time these differences disappear as workers move to higher piece-rate employment.

Table 5.1: Contributions of Job Search and Human Capital

Job Search		Human Capital		Covariance
Levels (%)	Inequality (%)	Levels (%)	Inequality (%)	Inequality (%)
38.98	31.38	61.03	73.46	-9.68

Notes: The table presents the fraction (in percentages) of earnings growth and earnings inequality contributed by job search and human capital.

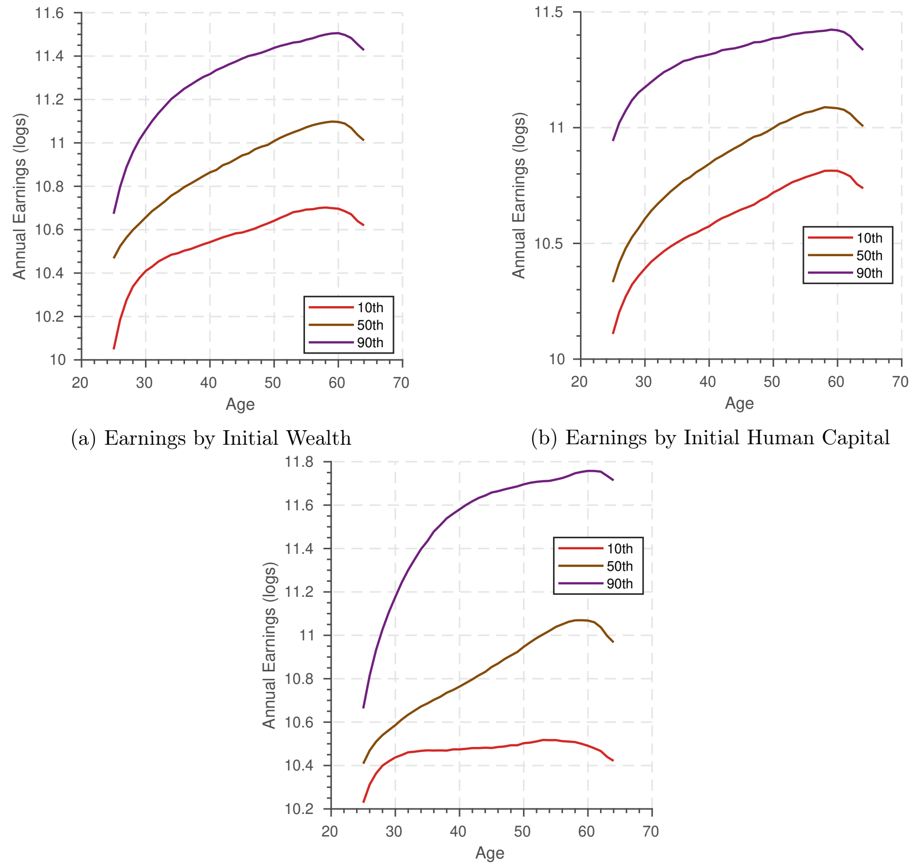
While job search is a larger driver of earnings growth, Table 5.1 demonstrates that human capital is roughly twice as important in determining inequality than job search. The negative covariance suggests that high piece-rate workers are able to take advantage of their higher-income jobs by allocating additional time toward human capital accumulation.

5.2 The Contribution of Initial Conditions to Life-Cycle Inequality

Inequality results from differences in initial conditions and shocks throughout the life-cycle. Upon initially entering the labor force, workers differ with respect to wealth, human capital, and learning ability, each of which alters the dynamics of earnings. Over the life-cycle, workers face uncertainty over their labor market outcomes and the duration of any employment relation, as well as stochastic depreciation of their human capital. Uncertainty interacts with the initial conditions by altering the exposure of borrowing constrained workers to consumption risk throughout the life-cycle, and changing the rate of return on human capital investment for unconstrained individuals. I first explore these dynamics, then decompose inequality among the sources of uncertainty in the model.

Viewed ex-ante, income inequality is determined by realizations of uncertainty in initial conditions and uncertainty from shocks over the life-cycle. This perspective allows me to write total variance as the sum of within-individual variance, determined over the life-cycle, and between-individual variance determined prior to labor market entry.[26] I perform this decomposition for each of the initial conditions as well as uncertainty realized over the life-cycle, and present my findings in Table 5.2.

[26] I use the following definition: $Var(X) = E(Var(X)) + Var(E(X))$



(a) Earnings by Initial Wealth

(b) Earnings by Initial Human Capital

(c) Earnings by Learning Ability

Figure 5.2: Earnings profiles by initial conditions. Each panel shows the life-cycle profile of earnings for individuals classified into the specified percentiles prior to entering the labor market.

Initial conditions account for a substantial fraction of inequality in each of the surveyed variables. Differences in initial conditions generate 51.49% of the overall dispersion in income, and 54.54% of the life-cycle dispersion in consumption. Among the initial conditions, human capital is the primary driver of both income and consumption dispersion, accounting for 7.19% and 9.40% of life-cycle inequality in each. Learning ability plays an important role as well, contributing to an 44.34% and 45.37% reduction in earnings and consumption inequality, respectively. Among the three initial conditions, wealth plays the smallest role, causing 7.19% and 9.40% of life-cycle inequality.

This finding is not surprising: wealth has a second-order effect on earnings, altering income only through effects on application strategies and human capital accumulation, while human capital directly impacts a worker's productivity. Among the three, wealth and learning play the largest role in reducing inequality after entrance. The majority of the reduction in inequality from human capital is due to the immediate change in human capital, with min-

Table 5.2: Sources of Life-Cycle Inequality

Source	Income (%)	Consumption (%)	h (%)	μ (%)
Initial Conditions				
Wealth	3.27	5.23	2.51	10.42
Human Capital	7.19	9.40	13.52	0.81
Learning Ability	44.34	45.37	49.56	2.86
Combined	51.49	54.54	65.39	16.74
Realized over Life-Cycle				
Residual of Combined	48.51	45.46	34.61	83.26

Notes: The table presents the percent of inequality in the surveyed variable that is explained by initial conditions or shocks over the life-cycle. For each of the surveyed variables, income, consumption, human capital (h), and piece-rate (μ), and assets (a), I calculate this statistic using the present discounted value of each variable from the perspective of an age-23 entrant. The contribution of shocks over the life-cycle is calculated as the residual variance left unexplained by initial conditions.

imal subsequent reduction over the life-cycle. Differences in wealth cause income inequality both by altering application strategies (10.42%) and human capital accumulation (2.51%).

This finding starkly contrasts the findings of Huggett et al. (2011), who find that changing initial human capital has a substantially larger impact on lifetime earnings and consumption than either wealth or learning ability.[27]

[27] They do not decompose life-cycle inequality into the contributions of each initial condition, but instead focus on the impact that a standard deviation change in one initial condition, holding the other two fixed at their mean, has on average consumption and income. I repeat similar experiments in Section 6, and find initial wealth to be at least as consequential as human capital or learning ability.

Table 5.3: Percent Change in Averages

Fixed Initial Condition	Income (%)	Consumption (%)	h (%)	μ (%)
Wealth	1.59	1.54	1.11	1.18
Human Capital	2.10	1.82	3.40	0.25
Learning Ability	−4.29	−6.04	0.83	−0.06
Combined	2.50	0.45	8.64	1.58

Notes: The table presents the percent of inequality in the surveyed variable that is explained by initial conditions or shocks over the life-cycle. For each of the surveyed variables, income, consumption, human capital (h), and piece-rate (μ), and assets (a), I calculate this statistic using the present discounted value of each variable from the perspective of an age-23 entrant. The contribution of shocks over the life-cycle is calculated as the residual variance left unexplained by initial conditions.

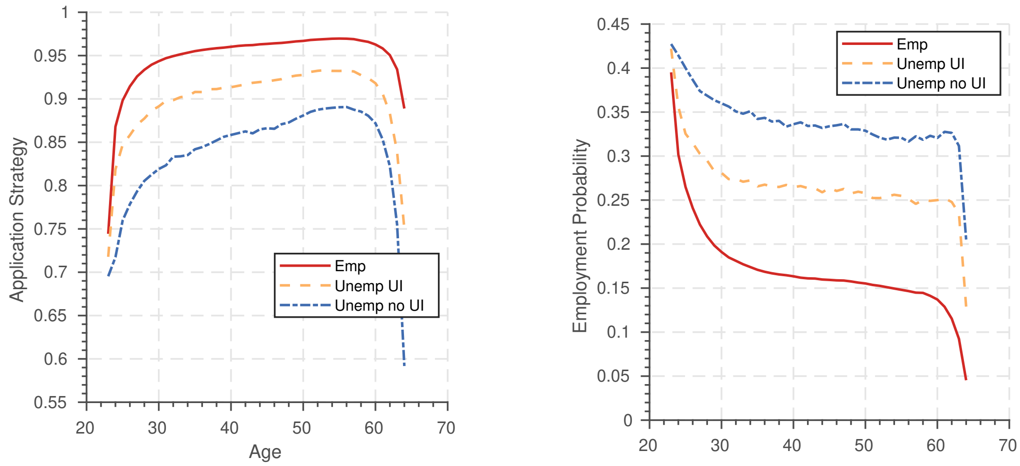
5.3 How Risk Shapes Life-Cycle Earnings

The main idea of this paper is that facing consumption risk early in the life-cycle can create permanent inequality among otherwise identical individuals who must pursue alternatives to borrowing in order to mitigate their exposure to periods of low consumption. There are several sources of uncertainty in the model, but consumption risk primarily manifests itself through the combination of unemployment risk and search frictions. A borrowing constrained worker who begins an unemployment spell has little ability to smooth consumption once their unemployment insurance expires. In response, they alter their application strategies to find employment more rapidly, applying for lower-paying jobs that offer high probabilities of employment in equilibrium.

Employed workers with low-wealth face a decision with permanent consequences as a result of this unemployment risk. They can allocate a larger fraction of their budget to precautionary savings, which they can use to replace lost income if they separate from their current employer. This choice comes at a cost: to allocate a larger fraction of their budget toward savings, they must either decrease current consumption, or decrease their human capital accumulation. The estimated model suggests that workers choose the latter option, forgoing future income as well as future income growth to shield against the immediate consumption risk associated with an unemployment spell.

5.3.1 Consumption Risk and Application Strategies

While workers are directly responsible for their likelihood of employment, they face a trade-off between wages and job-finding rates. Jobs that offer low piece-rates are likely to offer employment in equilibrium, while jobs posting high piece-rates are obtained less frequently. Borrowing constrained workers are unable to smooth consumption over extended unemployment spells, which means that the only option available to mitigate consumption risk is to gain employment. Figure 5.3 shows characteristics of worker application strategies for various states of consumption risk.



(a) Average of application strategies.

(b) Average employment probabilities.

Figure 5.3: Job search characteristics by employment and benefit status.

The left panel shows that unemployed workers with unemployment insurance apply for lower-paying jobs on average than their employed peers throughout the life-cycle. Unemployed workers without unemployment insurance consistently apply for jobs offering yet lower piece-rates. The right panel, which shows the equilibrium job-finding rates by employment status, indicates that such behavior is not driven purely by selection. Workers exposed to larger degrees of consumption risk apply for jobs that offer lower pay, but also higher probabilities of employment.

The impact of consumption risk on application strategies is easily observable in Table 5.2. The fourth column, containing the fraction of inequality in piece-rate explained by different initial conditions, shows that the elimination of initial inequality in wealth explains nearly 10% of the variance in worker piece-rates. The impact is equally as clear when viewing outcomes following an unemployment spell, stratified by wealth (Table 5.4).

While all workers experience a decrease in their earnings, the loss is largest for low-

Table 5.4: Re-Employment Outcomes by Wealth

Wealth Percentile	Δ Earnings (%)	Δ h (%)	Δ μ (%)
Pre-Unemployment			
10th	−7.707	−0.053	−7.655
30th	−0.666	−0.156	−0.510
50th	−1.334	−0.140	−1.194

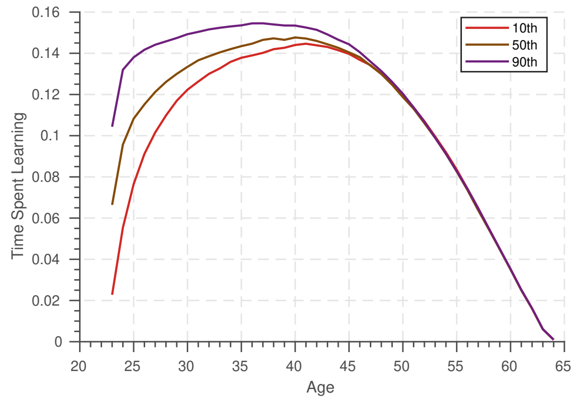
Notes: Wealth percentiles are defined as the contemporaneous percentile at the time of unemployment.

wealth workers. The left-skewness of earnings changes concentrated among low-wealth, low-income individuals has been noted in a number of papers. However, columns 2 and 3 show that earnings losses for low-wealth individuals are primarily the result of movement to jobs offering lower piece-rate than their previous employment, while earnings losses for higher-wealth individuals are the result of human capital depreciation over the duration of the spell. This left-skewness of earnings changes plays a key role in decreasing human capital accumulation, as I discuss in Section 5.3.2.

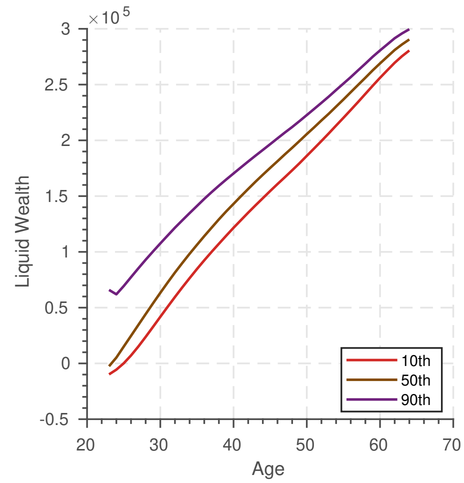
5.3.2 Unemployment Risk and Human Capital Accumulation

Agents in the model face a dynamic portfolio allocation decision: each period they are employed, workers choose the fraction of their time to allocate between human capital accumulation and production, which they may use for precautionary savings. Their solution to this problem depends on their exposure to risk and the expected rate of return on each asset. In Figure 5.4, I show the outcome of this dynamic portfolio allocation for individuals who are ex-ante identical in terms of human capital and learning ability, but differ by initial wealth. The left panel shows profiles of the time allocated to human capital accumulation for employed individuals who entered the labor market with different levels of initial wealth. The right panel shows the savings rate over the life-cycle again grouped by initial wealth percentiles.

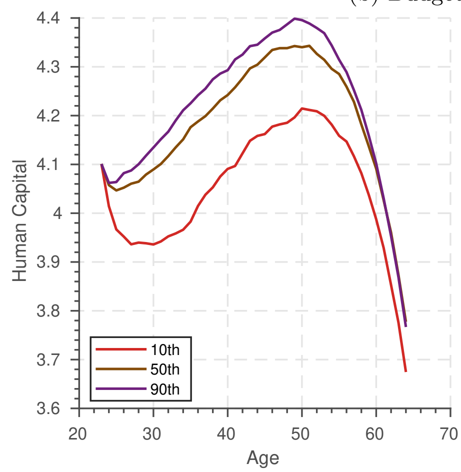
These differences in portfolio allocations are important for long-term outcomes. Heterogeneity in initial wealth is responsible 2.5% of the inequality in human capital over the life-cycle, and reduces overall human capital by 1.1% in levels. While Figure 5.4c demon-



(a) Learning time.



(b) Budget allocated to savings.



(c) Human capital.

Figure 5.4: Life-cycle profiles characterizing the dynamic portfolio allocation decision who are ex-ante identical in human capital and learning ability. The percentiles refer to percentiles within the initial wealth distribution.

strates that the differences in human capital caused by differences in wealth eventually dissipate, they cause a rotation of the human capital profile with the peak occurring later in the life-cycle.

As demonstrated in Table 5.4, unemployment spells cause substantial declines in earnings (due to movement to lower piece-rates) for individuals with limited stocks of precautionary savings. This can cause distortions in the outcome of the portfolio allocation problem when human capital and precautionary savings are rival goods.[28] For unconstrained workers, the solution to this portfolio allocation problem is to equalize the rate of return across both assets, accounting for the risk associated with human capital and differences in present value

[28] In Section D.2, I allow for learning-by-doing, and show that the results are quantitatively similar.

over the life-cycle. For a constrained worker, savings are preferable because the stochastic depreciation of human capital increases the uncertainty over investment and because savings may be used to smooth consumption in the event of separation. While human capital can increase income and the job finding rate, it cannot be used to supplement consumption unless a worker is matched with a firm. This "illiquidity" is particular to low-wealth workers and can cause a large distortion in human capital accumulation compared with equally capable, but wealthier peers. Rather than face extended earnings losses following an unemployment spell, poor workers allocate a larger fraction of their budget to building wealth.

To understand the role that unemployment risk and frictional labor markets play in human capital accumulation, I remove each as sources of uncertainty and then compare simulated outcomes with the baseline model. Specifically, I assume that workers are no longer subject to frictions in the labor market and can immediately find employment offering $\hat{\mu} = E[\mu]$. They continue to face a portfolio allocation decision, choosing a fraction of their productive time to devote to human capital, precautionary savings, and consumption. They are subject to a borrowing constraint $\underline{a}_t$ identical to the constraint faced in the baseline model. Lastly, because workers in the baseline model are unable to invest in human capital during unemployment, I assume that workers in this restricted model face a probability δ_H that they are unable to invest in human capital each period, realized as age advances. I fix this probability to the average unemployment rate in the simulations, $\delta_H = 0.0553$. Succinctly, these restriction results in a Bewley-style model with human capital accumulation. I define the problem fully in Section D.1.

With this restricted model, I perform the same decomposition as in Section 5.2. Any resulting inequality generated by these counterfactuals is due to uncertainty in and differences between the rate of return on human capital investment. I present my findings in Table 5.5.

The frictionless model with the baseline distribution accounts for 3.45% of the inequality in human capital generated by the baseline model, and would result in an increase of 5.15% in human capital over the life-cycle. When the frictionless model is instead simulated with a degenerate wealth distribution set to its initial average, and leaving the other two initial conditions unchanged, I find only a small reduction in human capital inequality. The fixed-wealth frictionless counterfactual explains 4.88% of the overall inequality in human capital, meaning that in a frictionless model, wealth inequality causes only 1.43 percentage points of the dispersion in human capital. A similar calculation shows that eliminating wealth inequality in the absence of unemployment risk increases average human capital by only 0.32%.

Table 5.5: Contributions of Uncertainty to Human Capital Inequality

Impact	Frictionless Labor Market Base Dist.	Frictionless Labor Market Fixed Wealth	Frictionless Labor Market Diff. (pp)	Baseline Fixed Wealth	Overall Diff. (pp)
Human Capital					
%Δ Inequality	3.45	4.88	1.43	2.51	1.08
%Δ Average	5.15	5.47	0.32	1.11	0.79
Earnings					
%Δ Inequality	21.09	21.40	0.32	3.27	2.95
%Δ Average	−0.83	−0.71	0.12	1.59	1.47

Notes: The frictionless labor market baseline counterfactual refers to the restricted model in which the initial conditions are equal to those in the unrestricted model. The fixed wealth counterfactual sets initial wealth to its average value for all workers. The baseline fixed wealth counterfactual reports the same results as in Table 5.2, in which wealth was set to its average in the estimated model.

By contrast, adding unemployment risk (the column headed Baseline/Fixed-Wealth) more than doubles the explained inequality in human capital to 2.51, meaning that the income and consumption uncertainty associated with unemployment causes an increase in inequality of 1.08 percentage points.

Similarly, unemployment risk more than doubles the overall decline in average human capital to 1.11%, an increase of 0.79 percentage points over the economy in which the only uncertainty is due to stochastic returns to human capital.

What this indicates is that workers exhibit a large precautionary response to the consumption and income uncertainty associated with unemployment spells. The persistent earnings losses (explored in Section 5.3.1), as well as the inability to replace income, cause low-wealth workers to re-allocate their portfolio away from human capital and toward precautionary savings. This channel is more than twice as important as the reallocation that results from uncertainty over the rate of return on human capital investment.

6 Policy Experiments

The findings in Section 5 suggest that consumption risk early in the life-cycle plays a quantitatively important role in determining inequality in labor market outcomes. With that in mind, I analyze the effectiveness of labor market and government transfer policies both at alleviating inequality and at improving life-cycle outcomes. I focus on policies that have a clear relation to consumption risk, but also explore additional labor market policies whose direct impact on borrowing constraints is less clear.

6.1 Mean-Preserving Spreads

The first transfer policy I explore is changing the spread of age-23 wealth, while leaving the mean unchanged. Intuitively, a decreased spread could be interpreted as a wealth tax and redistribution implemented by a government costlessly; an increased spread can be interpreted as the equilibrium outcome of leaving the current trend in wealth inequality unchanged. In each case, I change the spread of wealth by 10%: $\sigma_{A,MPS-} = 0.9\sigma_A$ and $\sigma_{A,MPS+} = 1.1\sigma_A$. At the conclusion of this subsection, I briefly explore larger or smaller mean-preserving spreads.

I also explore mean-preserving spreads in initial human capital and learning ability. These could be interpreted as changes in the quality of primary and secondary education, though the relationship is less straightforward.

Table 6.1: Impact of Counterfactual Initial Conditions

Spread Change	Δ Consumption (%)	Δ Earnings (%)	$\Delta\ h$ (%)	$\Delta\ \mu$ (%)	$\Delta\ \tau$ (%)	$\Delta\ \mu'$ (%)
Wealth						
10% Decrease	0.358	0.353	0.238	0.254	0.613	0.139
10% Increase	−0.415	−0.385	−0.257	−0.270	−0.639	−0.147
Human Capital						
10% Decrease	0.304	0.327	0.481	0.041	1.155	0.014
10% Increase	−0.360	−0.373	−0.509	−0.038	−1.057	−0.013
Learning Ability						
10% Decrease	−0.226	−0.069	0.533	0.012	3.527	−0.003
10% Increase	0.110	−0.033	−0.602	−0.014	−3.429	0.003

Notes: The table presents the impact of counterfactual initial conditions on measures of inequality and decision rules that contribute to inequality. Below mean is specific to individuals who experience an increase (decrease) in the corresponding initial condition due to a decrease (increase) in the spread.

Consistent with my findings in Section 5, more equitable distributions of initial wealth result in quantitatively important gains in average earnings and human capital. Decreasing the spread in wealth by 10% increases earnings over the life-cycle by an average of 0.35%, and human capital by 0.24%.[29]. Equal-sized mean-preserving spreads in either human capital or learning ability result in changes that are similar in magnitude to the outcomes from changing wealth.

6.2 Student Debt Relief

Student debt constitutes a substantial fraction of you household's portfolios. Households in the United States whose head is between ages 23 and 30 carried an average of over $16,500 dollars in student debt (Survey of Consumer Finances, 2016). Among those households in which the head had attended college, this number expanded to more than $33,000 per household. This means that college debt makes up 31.9% for college-educated households and 25.8% of debt for all households in this age bracket. My model makes a clear prediction that such high degrees of indebtedness may depress the earnings and productivity of college attendees, as they forgo high-paying jobs and human capital accumulation to repay their debt and build stocks of precautionary savings.

To understand the impact that student debt relief would have on college attendees, I eliminate any debt accumulated by workers who meet a percentile threshold of the initial human capital or learning ability distribution. My model has no college attendance choice, so I vary the thresholds and assess the changes in long-term outcomes. I set these thresholds to be the 25th, 50th, and 75th percentiles of each human capital and learning ability.[30] Any individuals who meet these criteria and enter the labor market with negative wealth are assumed to receive debt relief and have their initial wealth set to zero. I separate the outcomes of these policies by thresholds as well as whether an individual was "treated,"which I define as having received student debt relief. The results are displayed in Table 6.2.

While the size varies, simulated results using each threshold suggest that debt relief could have a non-trivial effect on earnings and human capital among students who attend college. The most expansive policies yield increases in earnings of between 3.5% and 3.5% for thresholds set to the 25th percentiles of the marginal distributions of human capital and learning ability, respectively. When debt relief policies must respect both thresholds, i.e., above the 25th percentile of both human capital and learning ability, the results are similar in magnitude.

[29]For context on the size of this result, an increase in human capital of this size would result in an increase in GDP of $49.71 billion dollars, in 2018Q4, strictly through the increase in worker productivity.

[30]66.7% of high school graduates attended college in 2018. The fraction of individuals ages 16-24 who were not in school and had not attended college was 25%. (BLS, 2018)

Table 6.2: Student Debt Relief

Threshold	Δ Earnings		Δ h		Δ μ	
	Treated	*Overall*	*Treated*	*Overall*	*Treated*	*Overall*
Human Capital						
25th	3.46	1.11	1.61	0.43	1.46	0.45
50th	3.05	0.54	1.29	0.22	1.27	0.22
75th	2.45	0.17	0.89	0.07	1.09	0.07
Learning Ability						
25th	3.52	1.03	1.80	0.49	1.35	0.39
50th	3.24	0.50	1.60	0.27	1.17	0.17
75th	2.15	0.11	0.89	0.06	0.84	0.04

Notes: Treated is defined as any worker who received student debt relief upon first entering the labor market. The thresholds refer to the percentiles of the initial human capital or learning ability marginal distributions above which workers are assumed to have attended college.

6.3 Income Replacement: Unemployment Insurance, EITC, and UBI

Many papers (Acemoglu and Shimer (2000), Chetty (2008), among others) have noted that when workers are risk-averse and face borrowing constraints, unemployment insurance can improve outcomes in the labor market. In my model, unemployment insurance may have additional benefits by altering human capital accumulation. I test the impact of two unemployment insurance policies as well as their interaction: increasing the level of unemployment insurance from 42% to 46%, and increasing the duration of unemployment benefits from the baseline average of 24.1 weeks to 27 weeks. For completeness, I consider reductions in benefit levels and duration as well of the same magnitude (10% reductions).

I lastly use the model to analyze the impact of two additional policies aimed at income replacement: the earned income tax credit (EITC), and universal basic income (UBI). EITC supplements the income of workers who earn less than \$34,001 per year. It increases linearly to a cap of \$2,747 for workers who earn \$8,050 annually, and decreases linearly between \$16,800 and \$34,001. I first explore the impact of introducing EITC in my model on average life-cycle outcomes. I then compare two alternative implementations, one in which workers receive the maximum benefit on the first dollar they earn (i.e., benefits do not slope upward), and a second in which I double the maximum benefit while retaining the original shape of the EITC system.

I consider three possible benefit levels in a UBI scheme. First is \$12,000, the value used

Table 6.3: Impact of Changes in Unemployment Insurance System

Change	Δ Consumption (%)	Δ Earnings (%)	Δ h (%)	Δ μ (%)	Δ Dur (%)	Δ Emp (%)
Replacement Rate						
Increase	0.11	0.04	0.03	−0.01	0.27	−0.02
Decrease	−0.13	−0.12	−0.02	−0.02	−0.57	0.03
Duration						
Increase	0.06	0.06	0.03	0.03	0.34	−0.02
Decrease	−0.05	−0.04	−0.00	−0.03	−0.45	0.03

Notes: Δ Dur is the percent change in unemployment duration, conditional on experiencing an unemployment spell. Δ Emp is the percent change in the employment rate.

Table 6.4: Earned Income Tax Credit

Change	Δ Consumption (%)	Δ Earnings (%)	Δ h (%)	Δ μ (%)	Δ τ (%)	Δ μ' (%)
EITC						
Introduced	1.53	1.37	2.88	−0.14	11.36	−0.12
First Dollar	1.54	1.33	2.88	−0.13	11.39	−0.11
Doubled Cap	3.51	2.61	6.85	−0.44	26.71	−0.22

by Hoynes and Rothstein (2019) in their exploration of UBI. I then analyze the impact when this benefit is doubled, and when this benefit is cut in half.

Table 6.5: Universal Basic Income

Change	Δ Consumption (%)	Δ Earnings (%)	Δ h (%)	Δ μ (%)	Δ τ (%)	Δ μ' (%)
UBI						
\$12,000 (annual)	17.73	5.59	4.86	1.60	15.86	0.80
\$6,000 (50% decrease)	8.63	0.49	2.23	−0.41	7.61	−0.27
\$18,000 (50% increase)	25.59	1.89	5.73	−0.93	18.58	−0.53

6.4 The Role of Labor Market Interventions

While each of these policies vary in benefits and effectiveness, they share one characteristic: re-allocating resources toward workers who are most likely to be constrained provides benefits above the initial injection of wealth. By reducing uncertainty over their path of consumption, these policies allow low-wealth workers to pursue both begin employment at higher rungs of the job ladder and subsequently devote more of their resources to increasing their human capital. No policy is without costs, but policies targeted at low-wealth workers in the simulated model feature large amplification effects by relaxing their borrowing constraints, often in excess of the original cost of the policy.

7 Conclusion

In this paper, I develop a quantitative model that uses incomplete and frictional markets to generate realistic income and consumption risk in order to understand the sources of inequality. The model features risk averse workers who must search for employment and make a portfolio allocation decision between precautionary savings and Ben-Porath human capital accumulation, while subject to consumption risk generated by borrowing constraints. I estimate the model and use it to study the sources of inequality as well as the role that labor market and poverty-reducing policies play in changing long-term outcomes.

Using the SIPP, I show that borrowing constraints affect labor market outcomes following an unemployment spell. Constrained workers in the SIPP match to higher paying jobs when given more generous unemployment insurance replacement rates. I also find evidence that this effect persists. These results help to discipline borrowing constraints when I estimate the model.

I use indirect inference to estimate the model. To do this, I pick reduced-form models that identify key aspects of my structural model in the data. I target re-employment elasticities from the SIPP to gain inference on borrowing constraints, as well as life-cycle moments from the NLSY and PSID to identify the effects of wealth and human capital on growth, as well as their correlations. By matching these moments and treating the data in the same way, the model is asked to match the data generating process of the relevant mechanisms in the data. Despite substantially more moments than estimated parameters, the model fits the reduced-form moments well, indicating that the model can explain the key mechanisms in the data.

With the estimated model, I decompose inequality among its sources. I find that initial conditions cause 51.49% of inequality in earnings over the life-cycle. Differences in learning ability play the largest role, explaining 44.34% of overall inequality. Human capital and wealth both explain similar degrees of inequality over the life-cycle (7.19% and 3.27% for human capital and wealth, respectively). Despite the differences in their explanatory power over inequality, inequality in initial conditions cause similar declines in average earnings and human capital over the life-cycle. Eliminating differences in wealth increase average earnings by 1.59%, while eliminating differences in initial human capital increase earnings by 2.10%. Unlike wealth and human capital, eliminating differences in learning ability cause a decline in earnings of -4.29%.

I assess the ability of redistribution and income replacement policies at changing inequality. I find that decreasing wealth inequality by 10% results in an increase of 0.35% in earnings and 0.24% in human capital, while policies targeted at eliminating student debt yield substantial gains both for attendees (3.46% to 2.15%) and overall (1.11% to 0.11%). Among policies that replace income, I find that both UBI and EITC are effective at increasing earnings (5.59 and 1.37, UBI and EITC, respectively) and human capital (4.86 and 2.88, UBI and EITC, respectively), while increasing the generosity of unemployment insurance yields smaller gains (0.04 and 0.03 for earnings and human capital, respectively).

My findings suggest that borrowing constraints expose low-wealth workers to consumption risk that can cause slower earnings growth than wealthier peers. This occurs because low-wealth workers are less selective among the wages offered by potential employers (1.18%), and because they accumulate less human capital (1.11%). Over the life-cycle, low-wealth workers prefer to accumulate precautionary savings that allow them to insure against the consumption risk and the large negative income shocks that result from unemployment than to invest in human capital. This precautionary channel against unemployment risk causes a decline of 0.79% over the life-cycle.

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A Data Construction

A.1 Survey of Income and Program Participation (SIPP)

I use the SIPP to assess the effect that liquidity has on labor market outcomes. The SIPP is a panel dataset with separate surveys conducted annually from 1984 to 1993, and then during 1996, 2001, 2004, and 2008. Each survey follows a household for 16 to 36 months, with interviews every four months for each "wave" of respondents. Each interview includes detailed information on the employment, income, and unemployment insurance recipiency. Employment variables are coded down to a weekly frequency, which yields an extremely precise picture of a worker's unemployment spells for the duration of the panel. In addition, each wave includes detailed information on special topics in "topical modules." Although information on wealth is not available in the core questionnaire, it is included in some of the topical modules, averaging twice per panel.

My selection criteria is similar to the previous literature on the liquidity effects of unemployment insurance[31]. I first pool SIPP panels from 1990 to 2008. From these panels, I restrict my sample to unemployment spells for males age 23 and older with at least 3 months work experience, who took up UI within one month of job loss, and who are not on a temporary layoff. For each individual, I observe race, marital status, age, years of education, as well as tenure, industry, occupation, and wage at their previous job. Demographic characteristics are shown in Table C.1. This allows me to link 2,311 unemployment spells to a variety of measures of their wealth upon entering an unemployment spell. The selection of individuals who experience unemployment spells but do not report wealth is random, because questions on wealth are only asked during some waves of the panel.

The SIPP employs a stratified sample design whose primary sampling units changed in 1992, 1996, and 2004. I make use of this complex survey structure to obtain accurate estimates of subsample variance, while accounting for design change by specifying the primary sampling units during each design regime (1990-1991,1992-1993,etc.) with a unique identifier. That is, an individual from the first PSU in 1990 would not be assigned to the same variance strata as an individual from the first PSU in 2001. I weight all of my results using person weights for individuals at the start of their unemployment spells.

A.2 Panel Study of Income Dynamics (PSID)

The PSID is a panel that follows a group of households from the United States that ran yearly from 1968 to 1997, and in alternating years through the present. Because the PSID spans nearly 50 years, it has been frequently employed for researchers interested in exploring life-cycle effects within the United States (Storesletten et al. (2004) and Rupert and Zanella (2015), among others), as well as researchers interested in inequality (Huggett et al. (2011), Guvenen (2009), among others). In addition to this, the PSID began recording information on household wealth holdings in their "wealth supplements," in 1984 repeated these questions in 1989, 1994, and 1999, and then in each subsequent interview. In the United States, this is the only publicly available dataset that contains multiple cohorts, long-term observations on

[31] See Chetty (2008) and Meyer (1990) for two examples using the same selection criteria.

earnings, and measures of household wealth at ages close to or before labor market entry[32]. In addition to these variables, the PSID includes rich observations on demographics, labor market experience, as well as family history and behavioral characteristics.

I employ sample restrictions similar to Huggett et al. (2011). First, I require that each individual be head of their household, male, and between the ages of 25 and 54. For constructing the distribution of wealth and earnings at first employment (moments 1 and 4), I require that the individual either be observed before entering employment, or that they report they entered employment during the previous year and the job is their first. I also require that these individuals be no younger than 23 and no older than 27. Over the life-cycle, I require that the individuals in my sample be strongly attached to the labor market: any individual in my sample must work at least 520 hours during the year and earn at least $9,500 in 2011 dollars if they are 31 or older. If they are younger than 30, I lower this requirement to $4,750, and 260 hours, to capture individuals who might choose part-time employment in order to have a steady income stream. I use the same sample restrictions when constructing profiles by initial liquid wealth quantile.

A.2.1 Wealth Quantile Construction

I use net liquid wealth as a measure of liquidity in the PSID. I define this to be any liquid assets, including checking, savings, stocks, bonds, etc. net of any unsecured obligations, including credit cards and student debt. I define earnings to be exclusively labor earnings at an annual frequency, and always in 2011 dollars, identical to the definition that I use in my exploration of the SIPP. Unfortunately, prior to 2011, the PSID did not report the specific composition of the debt held by households other than a few aggregated categories.

To assign individuals to initial quintiles in the wealth distribution, I first exclude observations who do not meet the following characteristics: first, agents must be the head of their household when I observe their assets; second, they must be age 30 or younger during a year in which I observe their assets; third, they must have no labor market experience, having earned no more than $9,750 dollars (2011 dollars) or worked more than 520 hours (one standard deviation less than the sample average) during the previous year[33]. This subsample faces limitations, as few individuals have both observations on their assets at an age younger than 30 and simultaneously have observations on earnings at later ages. I also scale wealth before entering the labor market by the number of individuals in the household. I pool all individuals for whom I observe assets and adjust for growth over time.

Having run this regression, I assign individuals to quantiles within the distribution based on their observed liquid wealth. I assign individuals to the nearest quintile (in terms of their rank) within the distribution. Because the wealth data contains few observations on earnings for individuals, while simultaneously observing their wealth before age 30, I employ a strategy similar to a synthetic control method. I classify individuals into five quintiles as described above, and then using these generated quintiles, I run an ordered logit to classify individuals for whom I do not have observations on wealth, based on their observables. Qualitatively, this technique generates earnings profiles that exhibit the same correlations in earnings for

[32]The NLSY79 contains information on wealth, but for few individuals before labor market entry.

[33]Huggett et al. (2011) use a similar sample selection method.

the ages for which I have wealth observations, but allows me to match my model to earnings at ages greater than 50.

A.3 National Longitudinal Survey of Youth, 1979 (NLSY79)

The National Longitudinal Survey of Youth follows cohorts who were ages 14-22 in 1979 through the present. It was conducted annually from 1979-1994 and bi-annually from 1994 until now, and includes detailed information on labor market status, including current employer, weeks employed, unemployed, and out of the labor force, as well as any training received by the individual since the last interview. Earnings are recorded annually as well as hours worked. In addition, the NLSY recorded a standardized test score, the Armed Forces Qualification Test (AFQT) for every individual in the sample. This allows me to link individuals by their AFQT scores to their outcomes late in the life-cycle. In 1985, the NLSY began recording information on the wealth of individuals. Unfortunately, a large fraction of the sample had already become employed, making its usage challenging in my analysis. I use identical sample restrictions as Section A.2.

B Indirect Inference

B.1 A Parsimonious Model

To illustrate the sources of identification I introduce a 2-period version of the model presented in Section 3, in which workers enter either employed or unemployed, make consumption and savings decisions, and then search for new employment. For simplicity, I set $u(c) = \frac{c^{-1}}{-1}$, $M(u,v) = u^{\frac{1}{2}}v^{\frac{1}{2}}$, i.e., power utility with $\sigma = 2$ and Cobb-Douglas matching with $\eta = \frac{1}{2}$. I also set $\beta = \frac{1}{1+r}$. With these simplifications, an unemployed worker in period-1 solves the following problem:

$$W_1(\mu, a, h, \ell) = \max_{a', \tau} -((1+r)a + \mu(1-\tau)h - a')^{-1} + \beta[-((1+r)a + \mu e^{\mu_\epsilon}(h + \ell(\tau h)^\alpha))^{-1}] \tag{B.1}$$

$$\begin{aligned} U_1(b, a, h, \ell) = \max_{a', \mu} & -((1+r)a + b - a')^{-1} + \nu \\ & + \beta[-(\frac{(1-\mu)e^{\mu_\epsilon}h}{\kappa})((1+r)a + \mu e^{\mu_\epsilon}h)^{-1} - (1 - \frac{(1-\mu)e^{\mu_\epsilon}h}{\kappa})(((1+r)a)^{-1} + \nu)] \end{aligned} \tag{B.2}$$

B.2 Implementation

Indirect inference can be implemented as either maximum likelihood, by minimizing a Gaussian objective function, or generalized method of moments. Because I use multiple datasets, the generalized method of moments approach is a more natural fit. This makes my estimation analogous to a seemingly unrelated regression (SUR) estimation. Indirect inference proceeds by first specifying an auxiliary model, and minimizing the distance between auxiliary parameters from the data and model simulations. Let T denote the number of observations, who need not be observed for every moment included in the auxiliary model. I largely follow the notation from DeJong and Dave (2011) in the following explanation of the procedure. I estimate the following:

$$\beta(Z) = \arg\max_{\delta} \Delta(Z, \delta) \tag{B.3}$$

$$\beta(Y, \theta) = \arg\max_{\delta} \Delta(Y, \delta) \tag{B.4}$$

where $Z = [z_1, ..., z_M]$ and $Y = [y_1, ..., y_M]$ are observed data and model generated data for observations 1,...,M, respectively. Δ are the specifications described in Section 4.2 characterizing the auxiliary model, θ the structural parameters of the model, and β the auxiliary parameters estimated from the auxiliary model.

$$\beta_S(Y, \theta) = \frac{1}{S} \sum_{j=1}^{S} \beta(Y^j, \theta) \tag{B.5}$$

where j is the j^{th} simulation of the model. The goal is to minimize the distance between the model generated auxiliary parameters and their empirical counterparts. I follow DeJong and Dave (2011) and minimize the following objective function:

$$min_{\theta} \Gamma(\theta) = g(Z, \theta)' \times \Omega \times g(Z, \theta) \tag{B.6}$$

$$g(Z, \theta) = \beta(Z) - \beta_S(Y, \delta) \tag{B.7}$$

where Ω is a positive-definite weighting matrix and $g(Z, \theta)$ the moments constructed from the binding functions. For the weighting matrix, I choose the inverse of the variance of the sample moments $var(\beta(Z))^{-1}$. Like Bowlus and Liu (2013), I estimate the variance-covariance matrix using the following:

$$Var(\hat{\theta}) = (1 + \frac{1}{S})[\frac{\partial g}{\partial \theta}' \Omega^{-1} \frac{\partial g}{\partial \theta}]^{-1} \tag{B.8}$$

where the jacobian matrix, $\frac{\partial g}{\partial \theta}$, is approximated using forward differences. For the model generated data, I average over $S = 100$ simulations for each iteration, and impose identical sample restrictions and attrition rates as in the observed data. I treat simulated data precisely the same as in my empirical analysis: I impose identical sample restrictions (where applicable) in my simulations, and force each sample to contain an identical number of observations as its empirical counterpart. To deal with missing data in the PSID and NLSY, I randomly drop observations at the same frequency as in the data by age. I do this by wealth and AFQT quantiles so that the data generating process from the structural model is as close as possible to that in the data. I simulate separate sets of data for each dataset used in the auxiliary model. I start agents at age 23 with no labor market experience (i.e., unemployed without unemployment insurance) and a random draw from the joint distribution of initial conditions.

C Tables and Figures

Table C.1: Summary Statistics by Liquidity Quintile and UI Generosity

	Avg. State UI				Avg. State UI		
	< Med	> Med	P-Val		< Med	> Med	P-Val
White				Duration			
Q1	0.700	0.790	0.0593	Q1	17.27	19.59	0.0939
Q2	0.552	0.684	0.00718	Q2	18.66	20.16	0.215
Q3	0.589	0.683	0.166	Q3	17.52	19.90	0.146
Q4	0.810	0.835	0.553	Q4	18.48	19.78	0.385
Q5	0.896	0.891	0.873	Q5	17.66	19.31	0.285
HS Degree				UI Reported			
Q1	0.353	0.378	0.606	Q1	250.3	329.5	1.62e-90
Q2	0.332	0.452	0.0117	Q2	246.7	324.1	3.20e-95
Q3	0.405	0.415	0.875	Q3	249.6	327.5	4.33e-72
Q4	0.314	0.352	0.465	Q4	253.1	332.2	5.47e-83
Q5	0.332	0.263	0.193	Q5	251.7	336.0	7.81e-95
Coll. Degree				Prev. Ann. Wage			
Q1	0.112	0.0798	0.275	Q1	36391.3	36471.7	0.967
Q2	0.0317	0.0456	0.391	Q2	28051.0	31679.2	0.0357
Q3	0.0536	0.0650	0.664	Q3	31155.4	33229.5	0.323
Q4	0.170	0.127	0.253	Q4	44891.5	46128.4	0.701
Q5	0.154	0.210	0.171	Q5	62213.4	55497.1	0.197
Age				Prev. Tenure (wks)			
Q1	36.62	37.14	0.609	Q1	44.08	43.94	0.969
Q2	37.26	36.81	0.641	Q2	36.82	43.52	0.0338
Q3	37.37	36.13	0.234	Q3	41.80	48.97	0.0955
Q4	40.54	38.89	0.113	Q4	48.73	41.02	0.0329
Q5	43.92	43.93	0.996	Q5	47.67	50.08	0.512
Observations	1210	1144	2354	Observations	1210	1144	2354

Notes: Means are weighted and variance is corrected for the survey design. Number of observations is unweighted.

Table C.2: Estimated Auxiliary Parameters from Elasticity and Age-Regression Moments

Slopes and Intercepts by Wealth (PSID)				Slopes and Intercepts by AFQT (NLSY)				Re-Employment Elasticities (SIPP)			
Var.	Data	Model	P-Val	Var.	Data	Model	P-Val	Var.	Data	Model	P-Val
Age	0.0476	0.0281	1.0869×10^{-11}	Age	0.0287	0.0160	0.1491	Q1 x Ln(UI)	0.3622	0.2005	0.1680
	(0.1366)	(0.0008)			(0.1308)	(0.0012)			(0.2096)	(0.0099)	
Wealth Q2 x Age	−0.0070	−0.0044	0.3332	AFQT Q2 x Age	−0.0006	−0.0010	0.4740	Q2 x Ln(UI)	0.2255	0.0112	0.1642
	(0.0056)	(0.0014)			(0.0026)	(−0.0004)			(0.2228)	(0.0129)	
Wealth Q3 x Age	−0.0117	−0.0015	0.0019	AFQT Q3 x Age	−0.0005	0.0063	0.0018	Q3 x Ln(UI)	0.0023	−0.1445	0.3459
	(0.0034)	(0.0011)			(0.0018)	(0.0017)			(0.2409)	(0.0285)	
Wealth Q4 x Age	−0.0153	0.0037	2.9871×10^{-7}	AFQT Q4 x Age	0.0034	0.0214	4.0378×10^{-7}	Q4 x Ln(UI)	0.0836	0.0171	0.3576
	(0.0037)	(0.0010)			(0.0034)	(0.0016)			(0.1900)	(0.0193)	
Wealth Q5 x Age	−0.0226	0.0108	3.6659×10^{-13}	AFQT Q5 x Age	0.0213	0.0421	5.6811×10^{-6}	Q5 x Ln(UI)	0.0549	0.0891	0.4740
	(0.0044)	(0.0016)			(0.0046)	(0.0016)			(0.2261)	(0.0245)	
Wealth Q2	0.1983	0.3100	6.9669×10^{-9}	AFQT Q2	0.1822	0.1292	0.0619	Q4	0.3580	1.6730	0.2150
	(0.0031)	(0.0463)			(0.0064)	(0.0528)			(1.3373)	(0.1599)	
Wealth Q3	0.4219	0.3352	0.2281	AFQT Q3	0.3270	−0.0039	0.0210	Q3	1.6347	3.3341	0.2759
	(0.1725)	(0.0354)			(0.0834)	(0.0528)			(1.3697)	(0.3134)	
Wealth Q4	0.6762	0.3084	0.1276	AFQT Q4	0.2670	−0.3367	1.5729×10^{-16}	Q4	1.5052	1.3282	0.3858
	(0.1033)	(0.0323)			(0.0653)	(0.0512)			(1.3631)	(0.2374)	
Wealth Q5	0.9423	0.3463	0.0044	AFQT Q5	−0.1206	−0.5559	3.1168×10^{-15}	Q5	2.3582	0.2583	0.0606
	(0.1156)	(0.0517)			(0.0951)	(0.0509)			(1.3697)	(0.3134)	
Cons.	9.5966	9.5150	0.3323	Cons.	9.2944	10.0062	0.0002	Cons.	2.5794	6.3206	0.0011
	(0.1730)	(0.0255)			(0.2019)	(0.0383)			(1.3289)	(0.0906)	

Table C.3: Estimated Auxiliary Parameters from Mobility and Growth Moments

Job-Stay Rate (NLSY)				Growth Variance				Growth Covariance							
Var.	Data	Model	P-Val	Var.	Data	Model	P-Val	Var.	Data	Model	P-Val	Var.	Data	Model	P-Val
Age 25 - 29	0.7790	0.8057	0.0011	$\Delta ln(y_{t+1})\%$	−0.0181	−0.0311	0.4829	$Cov(\Delta_1, \Delta_1)$	0.1886	0.0896	0.4958	$Cov(\Delta_1, \Delta_2)$	0.0993	0.0558	0.4719
	(0.0080)	(0.0037)			(0.0038)	(0.3122)			(0.0318)	(0.3015)			(0.0406)	(0.2372)	
Age 30 - 34	0.8277	0.8458	0.0350	$\Delta ln(y_{t+2})\%$	−0.0694	−0.0608	0.4900	$Cov(\Delta_1, \Delta_3)$	0.0554	0.1098	0.0383	$Cov(\Delta_1, \Delta_4)$	0.0464	0.0494	0.4244
	(0.8277)	(0.0041)			(−0.0694)	(0.3510)			(0.3505)	(0.3312)			(0.0099)	(0.2229)	
Age 35 - 39	0.8524	0.8572	0.2166	$\Delta ln(y_{t+3})\%$	−0.0963	−0.0912	0.4940	$Cov(\Delta_1, \Delta_5)$	0.0480	0.0695	0.3082	$Cov(\Delta_2, \Delta_2)$	0.4668	0.1175	0.1906
	(0.8524)	(0.0053)			(−0.0963)	(0.3686)			(0.0807)	(0.2630)			(0.2544)	(0.3417)	
Age 40 - 44	0.8710	0.8627	0.3117	$\Delta ln(y_{t+4})\%$	−0.1414	−0.1225	0.4805	$Cov(\Delta_2, \Delta_3)$	0.2974	0.0491	0.4501	$Cov(\Delta_2, \Delta_4)$	0.1745	0.0624	0.4503
	(0.8710)	(0.0058)			(−0.1414)	(0.3831)			(0.0168)	(0.2225)			(0.0158)	(0.2505)	
Age 45 - 49	0.8853	0.8627	0.1247	$\Delta ln(y_{t+5})\%$	−0.1413	−0.1594	0.4826	$Cov(\Delta_2, \Delta_5)$	0.1553	0.0771	0.4750	$Cov(\Delta_3, \Delta_3)$	0.5179	0.1256	0.2347
	(0.8853)	(0.0066)			(−0.1413)	(0.3989)			(0.0304)	(0.2778)			(0.1584)	(0.3551)	
Age 50 - 54	0.8971	0.8650	0.0105	$Cov(\Delta_3, \Delta_4)$	0.2732	0.0491	0.3858	$Cov(\Delta_3, \Delta_5)$	0.1890	0.0622	0.4470	$Cov(\Delta_4, \Delta_4)$	0.7008	0.0704	0.4193
	(0.8971)	(0.0098)			(0.2732)	(0.2227)			(0.0182)	(0.2502)			(0.0324)	(0.2655)	
				$Cov(\Delta_4, \Delta_5)$	0.5175	0.0873	0.4720	$Cov(\Delta_5, \Delta_5)$	0.7418	0.1503	0.2528				
					(0.5175)	(0.2959)			(0.2133)	(0.3879)					

D Restricted Models

D.1 Decomposing Unemployment Risk

There are two primary reasons human capital accumulation changes when wealth is altered: Workers intertemporally substitute in the model to smooth consumption (the permanent income effect), and to mitigate the earnings risk from potential unemployment spells in the immediate future (the unemployment risk effect). To disentangle the two, I make the following changes to the model presented in Section 3: agents are paid competitively ($\mu = 1, w = h \; \forall \; t$), and they are continuously employed at every stage of the life-cycle. Because the model only allows employed workers to accumulate human capital, I include a probability δ (same as the calibrated value) that a worker is unable to spend time learning during any period. All parameter values remain the same. The problem is given in Equation D.1.

$$V_t(a,h,\ell,E) = \max_{c,a'\geq \underline{a}',\tau} u(c) + \beta E[(1-\delta)V_{t+1}(a',h',\ell,H) + \delta V_{t+1}(a',h',\ell,D)] \tag{D.1}$$

$$\text{s.t. } c + a' \leq (1+r_F)a + (1-\tau)f(h) \tag{D.2}$$

$$\tau \in \begin{cases} 0 & \text{if } E = D \\ [0,1] & \text{if } E = H \end{cases} \tag{D.3}$$

$$h' = e^{\epsilon'}(h + H(h,\ell,\tau,E)) \tag{D.4}$$

$$\epsilon' \sim N(\mu_\epsilon, \sigma_\epsilon) \tag{D.5}$$

where H means that the worker is able to accumulate human capital and D means the worker is unable to accumulate human capital.

D.2 Embedding Learning-by-Doing

I embed learning-by-doing into my model by changing human capital accumulation according to the following:

$$H(h,\ell,\tau,E) = \ell((h\tau)^\alpha)^{1-\omega}h^\omega \tag{D.6}$$

where ω determines the fraction of human capital produced from learning by doing, and $(1-\omega)$ is the fraction produced by allocating time. Imposing this function in Equation 3.11 changes the portfolio allocation decision to

$$W_t(\mu,a,h,\ell) = \max_{c,a'\geq \underline{a}',\tau\in[0,1]} u(c) + \beta E[(1-\delta)R^E_{t+1}(\mu,a',h',\ell) + \delta R^U_{t+1}(b_{UI},a',h',\ell)] \tag{D.7}$$

$$\text{s.t. } c + a' \leq (1+r_F)a + \mu(1-\tau)f(h) \tag{D.8}$$

$$h' = e^{\epsilon'}(h + \ell((h\tau)^\alpha)^{1-\omega}h^\omega) \tag{D.9}$$

$$\epsilon' \sim N(\mu_\epsilon, \sigma_\epsilon) \tag{D.10}$$

$$b_{UI} = min\{max\{b(1-\tau)\mu f(h), b_L\}, \bar{b}\} \tag{D.11}$$

I set $\omega = 0.5$ and repeat the same decompositions as in Section 5.2, which I present in Table D.1.

While the impact of wealth on human capital inequality is smaller, the impact on overall inequality in earnings and consumption increases due to the additional impact on differences in piece-rate. Overall, the importance of initial conditions decline, as a larger fraction of growth is due to shocks and exogenous human capital growth.

Table D.1: Sources of Inequality with Learning-by-Doing Human Capital

Source	Income (%)	Consumption (%)	h (%)	μ (%)
Initial Conditions				
Wealth	4.12	4.82	1.69	16.81
Human Capital	9.00	8.85	15.58	1.11
Learning Ability	10.74	9.79	19.31	2.81
Combined	22.26	20.67	32.72	21.63
Realized over Life-Cycle				
Residual of Combined	77.74	79.33	67.28	78.37

Notes: The table presents the percent of inequality in the surveyed variable that is explained by initial conditions or shocks over the life-cycle. For each of the surveyed variables, income, consumption, human capital (h), and piece-rate (μ), and assets (a), I calculate this statistic using the present discounted value of each variable from the perspective of an age-23 entrant. The contribution of shocks over the life-cycle is calculated as the residual variance left unexplained by initial conditions.

E Proofs

E.1 Existence of a Block Recursive Equilibrium

The existence proof of a block recursive equilibrium is shown by using backwards induction and at each stage of the life-cycle showing that agents decisions are not conditional on the distribution of workers across states. Throughout, I include aggregate productivity z in the aggregate state, though this is stationary in the model.

Because the value in $T+1$ for all agents is 0, the three worker value functions Equation 3.1, Equation 3.5, and Equation 3.11 respectively, satisfy the following in period T.

$$U_T(b_{UI}, a, h, \ell; \psi) = u((1 + r_F)a + b_{UI}) \tag{E.1}$$

$$U_T(b_L, a, h, \ell; \psi) = u((1 + r_F)a + b_L) \tag{E.2}$$

$$W_T(\mu, a, h, \ell; \psi) = u(\mu f(h) + (1 + r_F)a) \tag{E.3}$$

The optimal policy policy for the terminal period is known: agents will use all accumulated savings to purchase consumption, and spend no time accumulating human capital, because the gains would not be realized until the following period. Because the interest rate is assumed to be the world interest rate and taken as given, each of the value functions do not depend on the distribution of workers across states. Therefore, the distributions, ψ can be dropped from the state space and the value functions rewritten as $U_T(b_{UI}, a, h, \ell; \psi) = U_T(b_{UI}, a, h, \ell; z)$, $U_T(b, a, h, \ell; \psi) = U_T(b_{UI}, a, h, \ell; z)$, and $W_T(\mu, a, h, \ell; \psi) = W_T(\mu, a, h, \ell; z)$.

Since there is no new employment activity for workers of age T, the decision rules of these agents do not depend upon the distribution of agents in the economy. Now, consider the market tightness function for firms posting vacancies for workers who will be age T when they are first employed (i.e., are currently in the search subperiod of age T). Since the continuation value to the firm in period $T+1$ is zero, the period T value of a vacancy is given by

$$J_T(\mu, a, h, \ell; \psi) = (1-\mu) f(h) \tag{E.4}$$

where again, I impose the optimal learning time of age T agents. The vacancy creation conditions can then be solved explicity for every worker state:

$$V(\mu, a, h, \ell; \psi) = -\kappa + q(\theta_T(\mu, a, h, \ell; \psi))(1-\mu) f(h) \tag{E.5}$$

Free entry of firms yields the following:

$$\kappa = q(\theta_T(\mu, a, h, \ell; \psi))(1-\mu) f(h) \tag{E.6}$$

By assumption, q is invertible, and this is imposed in the calibration. Therefore, submarket tightness can be solved for any worker state:

$$\theta_T(\mu, a, h, \ell; \psi) = \begin{cases} q^{-1}\left(\frac{\kappa}{(1-\mu)f(h)}\right) & : \text{if } (1-\mu)f(h) \geq \kappa \\ 0 & : \text{else} \end{cases}$$

This again does not depend upon the distribution of workers; thus, $\theta_T(\mu, a, h, \ell; \psi) = \theta_T(\mu, a, h, \ell; z)$. This means that the vacancy creation condition is known to workers without knowing the distribution of workers across the state space in the rest of the economy. Now, consider the search and matching decision of unemployed workers of age T:

$$\begin{aligned} R_T^U(b_{UI}, a, h, \ell; \psi) = \max_{\mu'} \, & P(\theta_T(\mu', a, h, \ell; \psi)) W_T(\mu', a, h, \ell; \psi) \\ & + (1 - P(\theta_T(\mu', a, h, \ell; \psi))) U_T(b_{UI}, a, h, \ell; \psi) \end{aligned} \tag{E.7}$$

$$\begin{aligned} R_T^U(b_L, a, h, \ell; \psi) = \max_{\mu'} \, & P(\theta_T(\mu', a, h, \ell; \psi)) W_T(\mu', a, h, \ell; \psi) \\ & + (1 - P(\theta_T(\mu', a, h, \ell; \psi)))[\gamma U_T(b_L, a, h, \ell; \psi) \end{aligned} \tag{E.8}$$

Imposing the conditions for θ_T, as well as the value functions in the terminal production and consumption period yields the following

$$\begin{aligned} R_T^U(b_{UI}, a, h, \ell; \psi) = \max_{\mu'} \, & P(\theta_T(\mu', a, h, \ell; z)) W_T(\mu', a, h, \ell; z) \\ & + (1 - P(\theta_T(\mu', a, h, \ell; z))) U_T(b_{UI}, a, h, \ell; z) \end{aligned} \tag{E.9}$$

$$R_T^U(b_L, a, h, \ell; \psi) = \max_{\mu'} P(\theta_T(\mu', a, h, \ell; z))W_T(\mu', a, h, \ell; z) \\ + (1 - P(\theta_T(\mu', a, h, \ell; z)))[\gamma U_T(b_L, a, h, \ell; z) \tag{E.10}$$

Note that neither the probabilities within each submarket, nor the continuation value depend on the distribution of workers across states. Therefore, the job search value functions are independent of the aggregate state and can be written $R_t^U(b_{UI}, a, h, \ell; \psi) = R_t^U(b_{UI}, a, h, \ell; z)$, and $R_t^U(b_L, a, h, \ell; \psi) = R_t^U(b_L, a, h, \ell; z)$, and the optimal application strategy is independent of the aggregate distribution of workers. Performing the same exercise for employed workers similarly yields

$$R_T^E(\mu, a, h, \ell; \psi) = \max_{\mu'} P(\theta_T(\mu', a, h, \ell; \psi))W_T(\mu', a, h, \ell; \psi) \\ + (1 - P(\theta_T(\mu', a, h, \ell; \psi)))W_T(\mu, a, h, \ell; \psi) \tag{E.11}$$

$$R_T^E(\mu, a, h, \ell; \psi) = \max_{\mu'} P(\theta_T(\mu', a, h, \ell; z))W_T(\mu', a, h, \ell; z) \\ + (1 - P(\theta_T(\mu', a, h, \ell; z)))W_T(\mu, a, h, \ell; z) \tag{E.12}$$

which again shows that the employed job searcher's value function does not depend on the aggregate distribution nor does the optimal application strategy, meaning $R_T^E(\mu, a, h, \ell; \psi) = R_T^E(\mu, a, h, \ell; z)$. Now consider the consumption, savings, and human capital decisions of age $T-1$ unemployed workers:

Note that in this economy, the aggregate state is assumed to be $z_t = z \forall t$. To prove that this exhibits a block recursive equilibrium, it must be the case that the value of an employed agent in the same time period is also independent of the distribution of agents across types. Consider the problem of an employed agent at time T - 1:

$$U_{T-1}(UI, a, h, \ell; \psi) = \max_{c,a'} u(c) + \nu + \beta E[(1-\gamma)R_T^U(b_{UI}, a', h', \ell; \psi) + \gamma R_T^U(b_L, a', h', \ell; \psi)] \tag{E.13}$$

$$\text{s.t. } c + a' \leq (1 + r_F)a + b_{UI} \tag{E.14}$$

$$a' \geq \underline{a}' \tag{E.15}$$

$$h' = e^{\epsilon'}(h + H(h, \ell, \tau, U)) \tag{E.16}$$

$$U_{T-1}(b_L, a, h, \ell; \psi) = \max_{c,a'} u(c) + \nu + \beta E[R_T^U(b_L, a', h', \ell; \psi)] \tag{E.17}$$

$$\text{s.t. } c + a' \leq (1 + r_F)a + b_L \tag{E.18}$$

$$a' \geq \underline{a}' \tag{E.19}$$

$$h' = e^{\epsilon'}(h + H(h, \ell, \tau, U)) \tag{E.20}$$

Substituting in the age T value functions yields the following:

$$U_{T-1}(b_{UI}, a, h, \ell; \psi) = \max_{c,a'} u(c) + \nu + \beta E[(1-\gamma) R_T^U(b_{UI}, a', h', \ell; z) + \gamma R_T^U(b_L, a', h', \ell; z)] \tag{E.21}$$

$$\text{s.t. } c + a' \leq (1 + r_F)a + b_{UI} \tag{E.22}$$

$$a' \geq \underline{a}' \tag{E.23}$$

$$h' = e^{\epsilon'}(h + H(h, \ell, \tau, U)) \tag{E.24}$$

$$U_{T-1}(b_L, a, h, \ell; \psi) = \max_{c,a'} u(c) + \nu + \beta E[R_T^U(b_L, a', h', \ell; z)] \tag{E.25}$$

$$\text{s.t. } c + a' \leq (1 + r_F)a + b_L \tag{E.26}$$

$$a' \geq \underline{a}' \tag{E.27}$$

$$h' = e^{\epsilon'}(h + H(h, \ell, \tau, U)) \tag{E.28}$$

Note that the neither the continuation values nor the prices depend on the aggregate distribution of workers, as debt is priced individually (in this case, with one price). This means that the consumption and savings rules of unemployed workers are independent of the distribution of workers, and the value functions can be written $U_{T-1}(\mu, a, h, \ell; \psi) = U_{T-1}(\mu, a, h, \ell; z)$ and $U_{T-1}(b_L, a, h, \ell; \psi) = U_{T-1}(b_L, a, h, \ell; z)$. By essentially the same argument, the value function during the consumptiono and savings period of an employed worker can be written as

$$W_{T-1}(\mu, a, h, \ell; \psi) = \max_{c,a',\tau} u(c) + \beta E[(1-\delta) R_T^E(\mu, a, h', \ell; \psi') + \delta R_T^U(b_{UI}, a', h', \ell; \psi')] \tag{E.29}$$

$$\text{s.t. } c + a' \leq (1 + r_F)a + \mu(1-\tau)f(h) \tag{E.30}$$

$$a' \geq \underline{a} \tag{E.31}$$

$$h' = e^{\epsilon'}(h + H(h, \ell, \tau, E; \psi)) \tag{E.32}$$

$$b_{UI} = b(1-\tau)\mu f(h) \tag{E.33}$$

$$b \sim N(\mu_b, \sigma_b) \tag{E.34}$$

$$\tau \in [0, 1] \tag{E.35}$$

$$
\begin{aligned}
W_{T-1}(\mu,a,h,\ell;\psi) &= \max_{c,a',\tau} u(c) + \beta E[(1-\delta)R^E_T(\mu,a,h',\ell;z) + \delta R^U_T(b_{UI},a',h',\ell;z)] && \text{(E.36)}\\
\text{s.t. } c + a' &\leq (1+r_F)a + \mu(1-\tau)f(h) && \text{(E.37)}\\
a' &\geq \underline{a} && \text{(E.38)}\\
h' &= e^{\epsilon'}(h + H(h,\ell,\tau,E;z)) && \text{(E.39)}\\
b_{UI} &= b(1-\tau)\mu f(h) && \text{(E.40)}\\
b &\sim N(\mu_b,\sigma_b) && \text{(E.41)}\\
\tau &\in [0,1] && \text{(E.42)}
\end{aligned}
$$

Again, neither the consumption, nor savings decisions depend on the distribution of workers across states. Furthermore, because human capital and learning are assumed to be observable, each worker state vector maps to a wage offer by the firm, independent of the distribution of human capital, learning, or wealth and wage. Thus, the human capital accumulation decision is independent of the distribution of workers, and the value function can be written $W_{T-1}(\mu,a,h,\ell;\psi) = W_{T-1}(\mu,a,h,\ell;z)$, and each of the decision rules are independent of the distribution of workers across states.

It's similarly easy to show that the value of a filled vacancy of a worker age $T-1$ does not depend on the distribution of workers across states. The value function of the firm may be written

$$
\begin{aligned}
J_{T-1}(\mu,a,h,\ell;\psi) &= (1-\mu)(1-\tau)f(h)\\
&\quad + \beta E[(1-\delta)(1-P((\theta_T(\mu',a',h',\ell;\psi')))J_T(\mu,a',h',\ell;\psi')] && \text{(E.43)}\\
h' &= e^{\epsilon'}(h + H(h,\ell,\tau,E;\psi)) && \text{(E.44)}\\
\tau &= g_\tau(\mu,a,h,\ell;\psi) && \text{(E.45)}\\
a' &= g_a(\mu,a,h,\ell;\psi) && \text{(E.46)}\\
\mu' &= g_\mu(\mu,a',h',\ell;\psi) && \text{(E.47)}
\end{aligned}
$$

Each of the employed worker decision rules do not depend on the distribution of workers across states. In addition, Θ_T, and J_T do not depend on the distribution as shown earlier. Thus,

$$
\begin{aligned}
J_{T-1}(\mu,a,h,\ell;\psi) &= (1-\mu)(1-\tau)f(h)\\
&\quad + \beta E[(1-\delta)(1-P((\theta_T(\mu',a',h',\ell;z)))J_T(\mu,a',h',\ell;z)] && \text{(E.48)}\\
h' &= e^{\epsilon'}(h + H(h,\ell,\tau,E;z)) && \text{(E.49)}\\
\tau &= g_\tau(\mu,a,h,\ell;z) && \text{(E.50)}\\
a' &= g_a(\mu,a,h,\ell;z) && \text{(E.51)}\\
\mu' &= g_\mu(\mu,a',h',\ell;z) && \text{(E.52)}
\end{aligned}
$$

Therefore, the value function of a filled vacancy for a worker age $T-1$ does not depend on the distribution of workers across states, $J_{T-1}(\mu,a,h,\ell;\psi) = J_{T-1}(\mu,a,h,\ell;z)$. From the free entry condition and the invertibility of $q(\theta)$, this yields

$$\theta_{T-1}(\mu, a, h, \ell; \psi) = \begin{cases} q^{-1}(\frac{\kappa}{J_{T-1}(\mu,a,h,\ell;\psi)} & : \text{if } J_{T-1}(\mu, a, h, \ell; \psi) \geq \kappa \\ 0 & : \text{else} \end{cases}$$

and furthermore, $\theta_{T-1}(\mu, a, h, \ell; \psi) = \theta_{T-1}(\mu, a, h, \ell; z)$.

Finally, it remains to be shown that a worker who is searching during age $T-1$ does not make decisions conditional on the distribution of workers. Similar to before, the value functions of unemployed searchers can be written

$$\begin{aligned} R^U_{T-1}(b_{UI}, a, h, \ell; \psi) = \max_{\mu'} & P(\theta_{T-1}(\mu', a, h, \ell; \psi))W_{T-1}(\mu', a, h, \ell; \psi) \\ & + (1 - P(\theta_{T-1}(\mu', a, h, \ell; \psi)))U_{T-1}(\mu, a, h, \ell; \psi) \end{aligned} \tag{E.53}$$

$$\begin{aligned} R^U_{T-1}(b_L, a, h, \ell; \psi) = \max_{\mu'} & P(\theta_{T-1}(\mu', a, h, \ell; \psi))W_{T-1}(\mu', a, h, \ell; \psi) \\ & + (1 - P(\theta_{T-1}(\mu', a, h, \ell; \psi)))U_{T-1}(b_L, a, h, \ell; \psi) \end{aligned} \tag{E.54}$$

Again, because the continuation values as well as the set of submarket tightnesses do not depend on the distribution, this can be written

$$\begin{aligned} R^U_{T-1}(b_{UI}, a, h, \ell; \psi) = \max_{\mu'} & P(\theta_{T-1}(\mu', a, h, \ell; z))W_{T-1}(\mu', a, h, \ell; z) \\ & + (1 - P(\theta_{T-1}(\mu', a, h, \ell; z)))U_{T-1}(\mu, a, h, \ell; z) \end{aligned} \tag{E.55}$$

$$\begin{aligned} R^U_{T-1}(b_L, a, h, \ell; \psi) = \max_{\mu'} & P(\theta_{T-1}(\mu', a, h, \ell; z))W_{T-1}(\mu', a, h, \ell; z) \\ & + (1 - P(\theta_{T-1}(\mu', a, h, \ell; z)))U_{T-1}(b_L, a, h, \ell; z) \end{aligned} \tag{E.56}$$

where once again, the application strategy is independent of the distribution of workers across states, and therefore $R^U_{T-1}(b_L, a, h, \ell; \psi) = R^U_{T-1}(b_L, a, h, \ell; z)$. Lastly, the same can be shown of employed searchers of age $T-1$:

$$\begin{aligned} R^E_{T-1}(\mu, a, h, \ell; \psi) = \max_{\mu'} & P(\theta_{T-1}(\mu', a, h, \ell; \psi))W_{T-1}(\mu', a, h, \ell; \psi) \\ & + (1 - P(\theta_{T-1}(\mu', a, h, \ell; \psi)))W_{T-1}(\mu, a, h, \ell; \psi) \end{aligned} \tag{E.57}$$

$$\begin{aligned} R^E_{T-1}(\mu, a, h, \ell; \psi) = \max_{\mu'} & P(\theta_{T-1}(\mu', a, h, \ell; z))W_{T-1}(\mu', a, h, \ell; z) \\ & + (1 - P(\theta_{T-1}(\mu', a, h, \ell; z)))W_{T-1}(\mu, a, h, \ell; z) \end{aligned} \tag{E.58}$$

where again, $R^E_{T-1}(\mu, a, h, \ell; \psi) = R^E_{T-1}(\mu, a, h, \ell; z)$; thus, all decision rules for actors in the model in period $T-1$ do not depend on distributions. The proof can be repeated for ages $\{T-2, ..., 1\}$, and by the same logic as above, these value and policy functions will not depend upon the aggregate distribution of agents across states. Thus, the model exhibits a block recursive equilibrium.

E.2 BRE Discussion

A block recursive equilibrium in this economy is possible because of a few assumptions: first, the interest rate cannot depend on the distribution of assets. With this, firms and workers do not have to condition on the distribution of assets in their policy functions. Second, workers must be able to direct their search to submarkets, and in these submarket workers characteristics must either be observable, or be implied by sorting. This assumption allows firms to know the expected profits from opening a vacancy within a submarket, causing policy functions to no longer have to depend upon the distribution of workers across types. Third, the matching function must be constant returns to scale. This implies that the probability of a firm matching with a worker is a function only of the ratio of vacancies to unemployed searchers, which causes policy functions to no longer depend upon the distribution of workers within types. Finally, the probability that firms meet with workers must be invertible, which allows the recovery of the probability a worker meets with a firm in a submarket. With this, workers can select a submarket and know the wage offered and probability of employment.