

# Employer Credit Checks: Poverty Traps versus Matching Efficiency

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## Abstract

We build a model that rationalizes the increasing use of credit checks in hiring due to adverse selection in credit and labor markets. Workers differ in their patience, with more patient workers repaying debts more frequently and accumulating more human capital. In equilibrium, a better credit history correlates with higher productivity. A poverty trap may arise: an unemployed agent with a low credit score has a low job finding rate, but cannot improve her credit score without a job. A policy that bans employer credit checks must balance their benefits (labor market matching efficiency and improved credit repayment incentives) against their costs (idiosyncratic poverty trap risk).

**Preliminary, comments welcome.**

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“We want people who have bad credit to get good jobs. Then they are able to pay their bills, and get the bad credit report removed from their records. Unfortunately, the overuse of credit reports takes you down when you are down.” Michael Barrett (State Senator, D-Lexington, MA).

## 1 Introduction

The three primary consumer credit agencies (Equifax Persona, Experian Employment Insight, and TransUnion PEER) market credit reports to employers, which include not only personal information (such as addresses and social security numbers) and previous employment history but also any public record (such as bankruptcy, liens and judgments) as well as credit history. According to a Survey by the Society for Human Resource Management (2010), 60% of human resource representatives who were interviewed in 2009 indicated that the companies they worked for ran credit checks on potential employees. Furthermore, a report by the policy think tank DEMOS found that 1 in 7 job applicants with bad credit had been denied employment because of their credit history (Traub [32]).

Until recently, pre-employment credit screening (PECS) was largely unregulated and remains so at the federal level – the FTC writes “As an employer, you may use consumer reports when you hire new employees and when you evaluate employees for promotion, reassignment, and retention as long as you comply with the Fair Credit Reporting Act (FCRA).”<sup>1</sup> However, since 2005, numerous state and federal laws with the goal of limiting or banning employer credit checks have been introduced and, as of 2017, eleven states have enacted such laws.<sup>2</sup> Legislators often express concern of a “poverty trap” arising due to employer credit checks; a worker loses her job, cannot pay her debts, which negatively impacts her credit report and thereby makes her unable to find a job. We assess the welfare consequences of policies to ban PECS in a simple general equilibrium model of unsecured credit and labor market search with adverse selection.

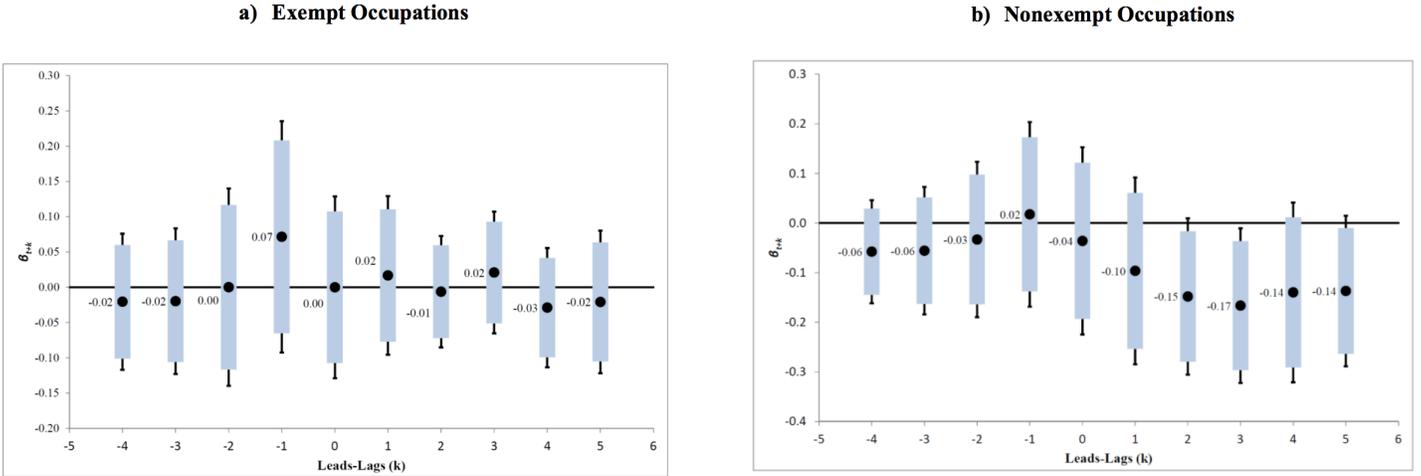
In response to these bans, an empirical literature seeks to estimate their effect on labor market outcomes. Most directly related to our theory is Cortes, et. al. [9], who document a fall in job creation following the implementation of employer credit check bans, but not in occupations that are exempted (typically finance jobs).<sup>3</sup> We reproduce

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<sup>1</sup><http://www.ftc.gov/bcp/edu/pubs/business/credit/bus08.shtm>

<sup>2</sup>The states with bans are CA, CO, CT, DE, HI, IL, MD, NV, OR, VT, WA.

<sup>3</sup>Bartik and Nelson [2] use a statistical discrimination model to study the impact of PECS bans on different racial groups. They find that the bans significantly reduce job-finding rates for blacks but that the results for Hispanics and whites are less conclusive. Their findings are consistent with PECS bans



(a) (b)  
Lead-lags are in quarters, with 5 representing more than one year post ban. Blue boxes are 90% confidence intervals.

Figure 1: Effect of ECCB on Log-Vacancies

their plots in Figure (1), showing that affected occupations see a 10 to 15 percent decline in vacancies following the ban, which persists even after a year, whereas exempt occupations are unaffected. Furthermore, they estimate an increase in delinquencies by subprime borrowers living in counties affected by employer check bans. Their labor market estimates are directly related to the demand effect of our theory and their delinquency estimates confirm the possibility of a feedback from labor to credit markets.

A related literature uses changes in the credit score to instrument for credit access in order to estimate labor supply effects. In a series of papers, Herkenhoff, et al ([17], [18]) show that increased credit access leads workers to become more selective in their job search (longer unemployment duration, higher post-employment earnings) and more likely to start their own business. We do not model the search decision of unemployed workers, but note that in our model an unemployed worker with bad credit would have a strong incentive to find a job in order to begin rebuilding her credit history.

Our model features heterogeneously patient households and has four main components: unobservable time preference so there is an adverse selection problem, an initial human capital investment, labor search frictions, and unsecured credit with endogenous reducing the match quality of newly hired black job applicants (more high match-quality applicants are rejected and more low match-quality applicants are hired after the ban).

default. Employers value the PECS process because credit scores are an externally verifiable and inexpensive signal about an unobservable component of labor productivity. In equilibrium, high productivity workers are also less likely to default, *ceteris paribus*, which means that workers with a high credit score are more valuable as employees. The correlation between productivity and default likelihood is generated by unobserved type differences in agents' discount factors; an agent with a high discount factor is more likely to invest in productive human capital and care about the punishment associated with default than those with a low discount factor. The labor market is modeled with search frictions, which generates both wage and employment differences across credit scores, with high score workers enjoying both higher job finding rates and higher wages conditional upon finding a job.

We then use this model as a laboratory to assess the effect of a policy bans PECS (i.e. forces employers to ignore credit scores in the hiring decision). This has both direct and indirect effects on the equilibrium. First, as expected by policy makers, there is a redistribution of wages from high to low credit score workers, which in equilibrium also translates into a redistribution of wages from high to low productivity workers. However, there is also an indirect effect on incentives that lowers welfare for everyone. When credit scores are not used in the labor market, workers lose some of their incentives to repay debts. This leads to higher interest rates and less borrowing. In our calibrated economy, the negative effect on credit markets dominates across the board and everyone loses in welfare terms from the policy. This cost of the policy has not been considered, even by those who advocate on behalf of lower income households with bad credit. Note that when there is a policy change in the labor market (e.g. banning credit checks), this can potentially affect credit market outcomes and updating functions so it is important to be explicit about the labor and credit market frictions to avoid the Lucas critique.

Our paper is related to the literature on asymmetric information in unsecured consumer credit markets with default. Some closely related papers are ours are Athreya, et. al. [1], Chatterjee, et. al. [4], Chatterjee, et. al. [5], Livshits, et. al. [23], and Narajabad [26] so we briefly describe how our approach differs from theirs.<sup>4</sup> First, we include labor market search frictions as in Mortensen and Pissarides [25]. Second, in the credit market we employ a different equilibrium concept proposed by Netzer and Scheuer [27]. They study the robust sub-game perfect equilibrium of a sequential game of pri-

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<sup>4</sup>The paper is also related to the reputation based model of Cole and Kehoe [6], who demonstrate how an exogenous utility loss in the labor market can incentivize sovereigns not to default in the credit market.

vate information between firms competing for one period loans. The salient assumption that we share with Netzer and Scheuer is that competitive lenders endogenously choose *both* the level of debt *and* the price at which it is offered as opposed to offering a risk adjusted competitive (break even) price for each *given* level of debt as in, for instance, Chatterjee, et. al. [5]. The equilibrium allocation of this game solves a constrained optimization problem with incentive compatibility constraints and generates separating equilibria with (potential) cross-subsidization.<sup>5</sup>

We are also related to the literature on the effect of asset markets on labor markets. These papers focus on how financial status (i.e. ability to borrow or dis-save to fund current consumption) affect job-finding rates. Lentz and Tranaes [21] study the effect of precautionary savings on workers' search intensity and job-finding rates in partial equilibrium. Krusell, et al [19] extend the Diamond-Mortensen-Pissarides general equilibrium model with random search and ex-post bargaining to include risk aversion and precautionary savings. Lise [22] studies the effect of precautionary savings on wage-dispersion in a model with on-the-job search (and exogenous wage distributions) and Chaumont and Shi [10] endogenize the equilibrium wage distribution in a model of precautionary savings and on-the-job directed search. While workers do not accumulate wealth in our model, credit access has a similar effect because it affects the worker's ability to smooth consumption and therefore their valuation of a job, which in turn affects finding rates and wages.

We proceed as follows. In Section 2 we describe the economic environment and in Section 3 we define and characterize equilibrium for our adverse selection environment as well as compare it to a full information version. In Section 4 we calibrate the economy and describe properties of the adverse selection equilibrium such as a poverty trap and quantify labor market inefficiencies. In Section 5 we study the welfare consequences of a ban on using credit checks in the labor market.

## 2 Environment

Time is discrete and infinite. Each period is split into two subperiods (e.g. a beginning and end of the month). The economy is composed of a large number of workers, firms, lenders, and credit scorers.

A newborn starts life unemployed and draws a discount factor  $\beta_i$ , which determines

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<sup>5</sup>We discuss the relationship between our allocations and the fully separating equilibria in Guerrieri, et. al. [14] in Section 3.3 where we present the programming problem.

her type  $i \in \{H, L\}$ . The probability the agent draws  $\beta_H > \beta_L$  is given by  $\pi_H$ . We call a worker “patient” if her discount factor is  $\beta_H$ . A worker keeps her discount factor throughout her probabilistic life; a worker dies with probability  $\delta$ . A newborn worker of type  $i$  makes a one-time choice of her human capital  $h_i \in \{\underline{h}, \bar{h}\}$  at cost  $\phi \times h_i$  where  $\underline{h} < \bar{h}$ .<sup>6</sup> The human capital choice is observed only by the agent and her eventual employer after the PECS hiring decision, but not by the eventual employer during the PECS hiring decision nor by lenders or credit scorers. Since the cost of the human capital choice is born today and payoffs come in the future, patient workers will tend to accumulate more human capital in the equilibrium we consider.

In any period  $t$ , workers have one unit of time in the first subperiod and zero in the second subperiod. They can either be unemployed ( $n_t = 0$ ) or employed ( $n_t = 1$ , which means they work for a firm). Worker preferences are represented by the function  $\mathcal{U}(c_{1,t}, c_{2,t}, n_t) = c_{1,t} + z(1 - n_t) + \psi c_{2,t}$  with the unemployed getting  $\mathcal{U}(0, 0, 1)$  and the employed getting  $\mathcal{U}(c_{1,t}, c_{2,t}, 0)$  (i.e. the employed derive disutility from work). We assume that  $\psi < 1$  so that workers prefer consumption in the first subperiod to the second. Since an unemployed worker receives no income that period, she cannot borrow against it and hence her flow utility is simply  $z$ .

Once employed, a worker’s human capital is observable to the firm. Production takes place in two stages: the worker puts in effort ( $n_t = 1$ ) in the first subperiod which generates output  $y_t = h_i n_t$  in the second subperiod. The worker and firm Nash bargain over her wage  $w_t$  in the first subperiod to be paid when her effort yields output in the second subperiod. The worker’s bargaining weight is  $\lambda$  and her outside option is to walk away, receive  $z$  utility from leisure in this period, and to search for another match tomorrow. The outside option for the firm is to produce nothing this period and post another vacancy at cost  $\kappa$  (in equilibrium the firm’s outside option will be zero due to free entry). The firm sells its second subperiod output, yielding period  $t$  profits of the firm given by  $h_i - w_t$ , which are valued as  $\psi(h_i - w_t)$  in the first subperiod of  $t$ .<sup>7</sup> After production, the worker and firm may exogenously separate with probability  $\sigma$ .

Since an employed worker is paid at the end of the period, if she wants to consume

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<sup>6</sup>Under our parametric assumptions, a patient household will choose  $\bar{h}$  and an impatient will choose  $\underline{h}$  in equilibrium. An alternative modeling choice would be to introduce moral hazard in the form of theft on the job. Since punishment occurs in the future (being fired if caught) and the benefit in the present (the amount stolen), the patient households would have less propensity to steal and therefore a higher marginal-revenue product than the impatient (net of theft).

<sup>7</sup>The firm’s profits are affected by the assessment of a worker’s type even after the worker’s type is observed because it will affect the worker’s threat point.

at the beginning of the period and has no savings, she can borrow  $Q_t$  from a lender.<sup>8</sup> When an employed worker borrows in the first subperiod, she is expected to repay the unsecured debt  $b_t$  once she is paid in the second subperiod provided she does not default. In the second subperiod, however, an employed worker receives an expenditure shock,  $\tau$ , drawn from a distribution with CDF  $F(\tau)$ .<sup>9</sup> The expenditure shock is unobservable to anyone but herself. Her choice of whether to repay in the second subperiod  $d_t \in \{0, 1\}$  is recorded by a credit scorer. If the worker does not repay (i.e.  $d_t = 1$ ) we say she is delinquent at time  $t$  and defaults at  $t + 1$ . Default bears a bankruptcy cost  $\epsilon$  in the second subperiod at  $t + 1$ .

A credit scorer records the history of repayments by a worker, which is summarized by a score  $s_t$ . This score is the probability that a given worker is type  $H$  with discount factor  $\beta_H$  at the beginning of any period  $t$ . Given the prior  $s_t$  and the repayment decision  $d_t$ , the credit scorer updates the assessment of a worker's type  $s_{t+1}$  via Bayes Rule.<sup>10</sup> Since a patient worker cares about their future ability to borrow more than an impatient worker, repayment is a signal to a scorer that the worker is more likely to be a high type. Our type score  $s_t$  is therefore not directly comparable to empirical credit scores such as FICO, which orders repayment likelihood on an index from 300 to 850. However, we can rank people by their expected repayment rate within the model, which allows us to group them into credit ratings (subprime, prime, and super prime) based on their ordering in the population, as in the data.

Since a worker's type influences their human capital and default decisions, a worker's score (which is simply one of their state variables) may be used in hiring and lending decisions. We assume that matches between job seekers with score  $s_t$ , denoted  $u(s_t)$ , and firms posting vacancies for such workers, denoted  $v(s_t)$ , are governed by a constant returns to scale matching function,  $M(u(s_t), v(s_t))$ . Therefore, an unemployed worker with score  $s_t$  matches with a firm with probability  $f(\theta_t(s_t)) = \frac{M(u(s_t), v(s_t))}{u(s_t)} = M\left(1, \frac{v(s_t)}{u(s_t)}\right)$ . We will assume that a tighter labor market (higher  $\theta_t(s_t)$ ) increases the job finding rate for workers (i.e.  $f'(\theta_t(s_t)) > 0$ ). The cost to a firm of posting a vacancy for workers with score  $s_t$  is denoted  $\kappa$  and the job filling rate is denoted  $q(\theta_t(s_t))$ , which is decreasing in

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<sup>8</sup>We will develop the model without intertemporal savings, but will assume that  $\beta_H \leq R^{-1}$  which, along with the linearity of preferences, ensures that households do not want to save.

<sup>9</sup>For simplicity, we assume unemployed workers do not receive expenditure shocks since they cannot repay them because they have no income in the second subperiod.

<sup>10</sup>Since all unemployed workers receive no income in the second subperiod, if they received an i.i.d. expenditure shock, there is no new relevant information from default and their score would remain the same.

tightness (i.e.  $q'(\theta_t(s_t)) < 0$ ). Future profits of the firm are discounted at rate  $R^{-1}$ .

There are a large number of competitive lenders who have access to consumption goods in the first subperiod, for which they must pay an exogenously given worldwide interest rate of  $R$  in the second subperiod and bear intermediation costs  $\omega$ . Lenders observe each potential borrower's type score  $s_t$  and post a menu of contracts  $\mathcal{C}_t(s_t) = \{(Q_{jt}(s_t), b_{jt}(s_t))\}_{j=1}^J$  which specifies an amount to be lent in the first subperiod (i.e. at the beginning of the month),  $Q_{jt}$ , and a promised repayment in the second subperiod (i.e. at the end of the month),  $b_{jt}$ . Lenders realize that households may default on their debt and the probability may differ by worker type, which affects their expected profits for a given contract. As in Netzer and Scheuer [27], after posting these menus the lenders observe all other menus posted and then may withdraw from the market at a cost  $k$ .<sup>11</sup>

Specifically, a large number of lenders play a game against one another by posting menus of contracts (including  $(0, 0)$  so that a worker need not borrow) for each observable credit score  $\mathcal{C}_t(s_t)$ . The game has three stages:

Stage 1: Lenders simultaneously post menus of contracts.

Stage 2: Each lender observes all other menus from stage 1. Lenders simultaneously decide whether to withdraw from the market or remain. Withdrawal entails removing the lender's entire menu of contracts with a payoff of  $-k$  (i.e. it is costly to withdraw).

Stage 3: Workers simultaneously choose the contract they most prefer.

To summarize the information structure, workers observe everything  $(i, h, s_t, \tau_t)$ . Before hiring a worker, a firm only observes the worker's score  $s_t$ . After hiring a worker, a firm observes their type  $i$  and human capital  $h$ . Lenders only observe the worker's score  $s_t$ . Credit scorers observe a worker's current score  $s_t$  and default decision  $d_t$ . Credit and labor markets are segmented in the sense that lenders and scorers cannot communicate with firms who know the worker's type after the hiring decision.

Having described the environment for workers, firms, lenders, and credit scorers, we now describe the timing of actions. Under the assumption that workers do not start the period with assets (which we will show is an optimal decision when assets holdings are

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<sup>11</sup>The ability to withdraw contracts after observing all others posted is key to ensuring that an equilibrium exists, counter to purely competitive models with adverse selection. That the withdrawal of contracts is costly ensures that the equilibrium is unique.

unobservable to everyone but the worker), a worker of type  $i$  with credit score  $s_t$  and human capital  $h$  begins the period either unemployed or employed.

For an unemployed worker:<sup>12</sup>

1. Enjoy utility  $z_t$  from leisure  $n_t = 0$ .
2. Die with probability  $\delta$ .
3. Surviving workers with score  $s_t$  are matched with a firm in labor sub-market  $s_t$  with probability  $f(\theta_t(s_t))$

For an employed worker:

1. First Subperiod:
  - 1.1 Determine earnings  $w_t$  via Nash Bargaining and work  $n_t = 1$ .
  - 1.2 Choose debt contract  $(Q_{jt}(s_t), b_{jt}(s_t))$  and consume  $Q_{jt}$ .
2. Second Subperiod:
  - 2.1 First subperiod work yields output  $y_t = h_i \cdot n_t$  from which earnings  $w_t$  are paid.
  - 2.2 Draw expenditure shock  $\tau_t$  from CDF  $F(\tau_t)$
  - 2.3 Choose whether to default  $d_t \in \{0, 1\}$  and pay  $(1 - d_t)(b_{jt} + \tau_t)$ .
  - 2.4 Type score updated  $s_{t+1}(s_t, d_t)$ .
  - 2.5 Separate from employer exogenously with probability  $\sigma$  and die with probability  $\delta$ .

### 3 Equilibrium

We now provide the decision problems for all agents in recursive form. To that end, we let variable  $x_t$  be denoted  $x$  and  $x_{t+1}$  be denoted  $x'$ . Further, to save on notation we denote  $s_{t+1}(s_t, d_t)$  as  $s'_d$  and will use  $x_{i^*}$  in place of  $x_{i, h_i^*}$  whenever we are evaluating an equilibrium variable at the optimal human capital choice of an  $i$  type worker.

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<sup>12</sup>Since unemployed workers do not receive income in the second subperiod, they do not borrow. This means that the unemployed do not experience a change in their credit score.

### 3.1 Worker Decisions

The value function for an unemployed worker of type  $i$  with human capital  $h$  and score  $s$  is given by

$$U_{i,h}(s) = z + (1 - \delta)\beta_i \left[ f(\theta(s))W_{i,h}^*(s) + \left(1 - f(\theta(s))\right)U_{i,h}(s) \right] \quad (1)$$

where  $W_{i,h}^*(s)$  is the value function for an employed agent, evaluated at equilibrium credit contracts and wages, as described below. The unemployed worker receives current flow utility  $z$  and if they survive till next period with probability  $1 - \delta$  they will transit to employment next period with probability  $f(\theta(s))$  and remain unemployed with probability  $1 - f(\theta(s))$ . Note that with no credit market activity, the unemployed worker's score remains constant.

The value function for an employed worker of type  $i$  with human capital  $h$  and score  $s$  who has chosen contract  $(b, Q)$  and wage  $w$  is given by

$$\begin{aligned} W_{i,h}(b, Q, w, s) &= Q + \psi w \\ &+ \psi \int_0^\infty \max_d \left[ \beta_i(1 - \delta) \left( V_{i,h}(s'_d) - d\psi\epsilon \right) - (1 - d)(b + \tau) \right] dF(\tau) \end{aligned} \quad (2)$$

where we have used an intermediate value function:

$$V_{i,h}(s'_d) = \left[ (1 - \sigma)W_{i,h}^*(s'_d) + \sigma U_{i,h}(s'_d) \right]. \quad (3)$$

The first line in (3) reflects borrowing  $Q(s)$  to pay for first subperiod consumption and the second subperiod wage  $w$  payment. The second line in (3) reflects the strategic decision of whether to go delinquent to avoid paying off  $b + \tau$  in the second subperiod followed by default which bears bankruptcy cost  $\epsilon$  the following period. Note that the scorer updates  $s'_d$  his assessment of the agent's type given the worker's default decision  $d$ .

We start by characterizing the worker's default choice, taking all other objects (in particular their contract choice) as given consistent with the timing assumptions. The worker defaults if and only if:

$$\tau > \tau_{i,h}^*(s, b) \equiv \beta_i(1 - \delta) \left[ \psi\epsilon + V_{i,h}(s'_0) - V_{i,h}(s'_1) \right] - b \quad (4)$$

Thus, higher debt and higher expenditure shocks make default more likely. Furthermore, a lower discount factor and value from a good reputation (i.e.  $V_{i,h}(s'_0) - V_{i,h}(s'_1)$ ) make default more likely. Using  $\tau^*$ , after integrating by parts and some cancelation, this allows us to evaluate the integral in  $W_{i,h}$  for given values of  $(b, Q, w)$ :

$$W_{i,h}(b, Q, w, s) = Q + \psi w + \psi \beta_i (1 - \delta) \left[ V_{i,h}(s'_1) - \psi \epsilon + \int_0^{\tau_{i,h}^*(s,b)} F(\tau) d\tau \right] \quad (5)$$

We can then write the worker's surplus (i.e. utility when employed versus unemployed) evaluated at the equilibrium contracts  $(Q_{i^*}^*(s), b_{i^*}^*(s))$  as the difference:

$$S_{i,h}^w(w, s) = W_{i,h}(b_{i^*}^*(s), Q_{i^*}^*(s), w, s) - U_{i,h}(s). \quad (6)$$

Finally, since a newborn starts unemployed and there are only two values for human capital, their human capital choice must satisfy:

$$h_i^* = \underset{h \in \{\underline{h}, \bar{h}\}}{\operatorname{argmax}} \left[ U_{i,h}(\pi_H) - \phi h \right] \quad (7)$$

We will provide sufficient conditions on parameters and distributions such that high types  $H$  choose a high level of human capital  $\bar{h}$  while low types  $L$  choose a low level of human capital  $\underline{h}$ .

### 3.2 Firm's Problem and Wage Determination

Recall that after a firm and worker are matched, the worker's type and human capital choice is observable to the firm. The value function for a firm matched with a worker of type  $i$  with human capital  $h$  and current type score  $s$  for a given wage  $w$  is:

$$J_{i,h}(w, s) = \psi \int_0^\infty \left[ h - w + R^{-1}(1 - \sigma)(1 - \delta) J_{i,h}(w_{i,h}^*(s'_d), s'_d) \right] dF(\tau). \quad (8)$$

While  $s$  does not add information for the firm's inference about worker type, it influences the worker's bargaining position since it determines their credit contract and hence the worker's flow surplus from being employed. Since Nash Bargaining ensures that the firm receives a constant fraction of the match surplus as in (12) below, the firm's surplus will also depend on  $s$ .

Evaluating (8), we can then write the firm's surplus (i.e. expected discounted profits in a match  $J_{i,h}(w, s)$  versus value from posting a vacancy (which given free entry is just

zero)) as:

$$\begin{aligned}
S_{i,h}^f(w, s) &= J_{i,h}(w, s) - 0 = \psi \left[ h - w \right. \\
&\quad + R^{-1}(1 - \delta)(1 - \sigma)F(\tau_{i,h}^*(s, b))J_{i,h}(w_{i,h}^*(s'_0), s'_0) \\
&\quad \left. + R^{-1}(1 - \delta) \left( 1 - F(\tau_{i,h}^*(s, b)) \right) J_{i,h}(w_{i,h}^*(s'_1), s'_1) \right]. \tag{9}
\end{aligned}$$

The wage is then determined by generalized Nash Bargaining in which the worker's bargaining weight is  $\lambda$ . The wage solves:

$$w_{i,h}^*(s) = \operatorname{argmax}_w S_{i,h}^w(w, s)^\lambda S_{i,h}^f(w, s)^{1-\lambda} \tag{10}$$

Given that worker utility and firm profits are linear in earnings, (10) amounts to a simple splitting rule for the total surplus:

$$S_{i,h}^w(w_{i,h}^*(s), s) = \lambda \left( S_{i,h}^w(w_{i,h}^*(s), s) + S_{i,h}^f(w_{i,h}^*(s), s) \right). \tag{11}$$

Therefore (9) can be written:

$$S_{i,h}^f(w_{i,h}^*(s), s) = (1 - \lambda) \left( S_{i,h}^w(w_{i,h}^*(s), s) + S_{i,h}^f(w_{i,h}^*(s), s) \right). \tag{12}$$

Note that the current wage does not directly affect the repayment decision or optimal debt choice of a household due to the linearity of preferences. If these choices were to depend on the wage, then the wage would affect both the size of the worker's surplus and the split of the total surplus, creating a nonconvexity.

Firms post vacancies in labor "sub-markets" indexed by an unemployed worker's score  $s$  so that labor "sub-market" tightness is given by  $\theta(s)$ .<sup>13</sup> The expected profits from posting a vacancy must be equal to the cost of the vacancy in equilibrium:

$$\kappa = R^{-1}q(\theta(s)) \left[ sJ_{H^*}(w_{H^*}^*(s), s) + (1 - s)J_{L^*}(w_{L^*}^*(s), s) \right] \tag{13}$$

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<sup>13</sup>Our sub-markets are indexed by score rather than contract terms as in the models of directed search. A form of block recursivity as in Menzio and Shi [24] exists when firms can screen using scores because the score corresponds to the fraction of good types with that score and hence firms do not need to know the entire distribution of workers over scores to evaluate the expected value of posting a vacancy in that sub-market.

where  $h_i^*$  is determined in (7).

### 3.3 Lender's Problem and Credit Contract Determination

Invoking Proposition 2 from Netzer and Scheuer [27], for sufficiently small  $k > 0$ , the unique equilibrium to the above game for credit sub-markets with score  $s$  is the two-contract menu  $\{(Q_H(s), b_H(s)), (Q_L(s), b_L(s))\}$  that solves the following constrained optimization problem:

$$\max_{\{Q_H, b_H, Q_L, b_L\}} Q_H + \psi \int_0^{\tau_{H^*}^*(s, b_H)} F(\tau) d\tau \quad (14)$$

s.t.

$$s \left[ -Q_H + R^{-1} F(\tau_{H^*}^*(s, b_H)) b_H \right] + \quad (15)$$

$$(1-s) \left[ -Q_L + R^{-1} F(\tau_{L^*}^*(s, b_L)) b_L \right] \geq 0$$

$$Q_L + \psi \int_0^{\tau_{L^*}^*(s, b_L)} F(\tau) d\tau \geq Q_H + \psi \int_0^{\tau_{L^*}^*(s, b_H)} F(\tau) d\tau \quad (16)$$

$$Q_H + \psi \int_0^{\tau_{H^*}^*(s, b_H)} F(\tau) d\tau \geq Q_L + \psi \int_0^{\tau_{H^*}^*(s, b_L)} F(\tau) d\tau \quad (17)$$

$$Q_L + \psi \int_0^{\tau_{L^*}^*(s, b_L)} F(\tau) d\tau \geq \quad (18)$$

$$\max_b R^{-1} F(\tau_{L^*}^*(s, b)) b + \psi \int_0^{\tau_{L^*}^*(s, b)} F(\tau) d\tau$$

This problem says that the credit contract for a worker whose score is  $s$  is designed to maximize the utility of the type  $H$  (low-risk) borrower subject to profitability, incentive compatibility, and participation constraints. The first constraint (15) says that the lender must make non-negative profits on the contract for each score. The first term is the profit (or loss) per type  $H$  borrowers' contract times the number of patient borrowers with score  $s$ . The second term is profit (or loss) for type  $L$  borrowers' contract times the number of impatient borrowers with score  $s$ . Note that (15) does not rule out cross-subsidization. The second and third inequalities ((16) and (17)) are incentive compatibility constraints. For instance, (16) says that impatient borrowers must choose the contract designed for them rather than the one designed for patient borrowers. The

final constraint (18) says that an impatient borrower must get at least the utility from a credit contract that breaks even and maximizes her utility. That is, the equilibrium contract must give the impatient borrower at least her utility from her least cost separating contract.

The robust sub-game perfect equilibrium is found by taking the limit of this game as  $k \rightarrow 0$ . The equilibrium allocation solves a constrained planner’s problem; the credit market contracts are designed to be separating and cross-subsidize the impatient household for some values of  $s$  while always constraining the borrowing of patient households by imposing a binding credit limit on them.

We note some special properties of this game and its solution. First, we need a well defined solution for all credit scores, which would not be the case in the competitive model of Rothschild and Stiglitz [28]. In that model there would be no equilibrium for a score close enough to one, whereas in this model an equilibrium always.<sup>14</sup> Furthermore, the Netzer and Scheur equilibrium contract can be one of three types: least cost separating (denoted LCS), cross-subsidized separating (denoted CSS), and pooling contracts (denoted PC). Unlike Rothschild and Stiglitz, cross-subsidization can occur in a Netzer and Scheuer equilibrium because lenders can withdraw their contracts. If another lender posted a contract that cream-skimmed (ie, attracted only patient borrowers) then the lender posting the cross-subsidizing contract would make losses and withdraw for sufficiently low  $k$ . Impatient households would then choose the cream-skimming contract, which would then cease to make profits. Second, we want a model where workers care about their future scores *because their score improves credit contract terms (lower rates or looser constraints)* and the fact that credit contracts are cross-subsidizing for high scores ensures this. This would not be the case in a model where the credit contracts were always least-cost separating, such as the competitive search model of Guerrieri, et. al. [14].<sup>15</sup> In that case, absent the employer credit checks, an individual’s credit score would have no affect on their credit contract in equilibrium, which is inconsistent with data.<sup>16</sup> Furthermore, the Netzer and Scheuer equilibrium concept ensures that credit

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<sup>14</sup>Non-existence follows from the standard argument of Rothschild and Stiglitz: the competitive equilibrium cannot include a pooling contract, since lenders could “cream skim” the patient borrowers by posting a contract with a slightly tighter borrowing constraint but lower interest rate. On the other hand, if there were very few impatient borrowers and all other lenders were offering separating contracts with borrowing limits then a lender could post a pooling contract and attract the entire market at a profit. Hence, there would be no competitive equilibrium.

<sup>15</sup>Their equilibrium concept also has search in the credit market and hence an extra endogenous variable. Their framework is directly comparable with the least-cost separating contracts in our work if the cost of posting credit contracts approached zero.

<sup>16</sup>For instance, in states with employer credit check bans, interest rates on debt would be independent

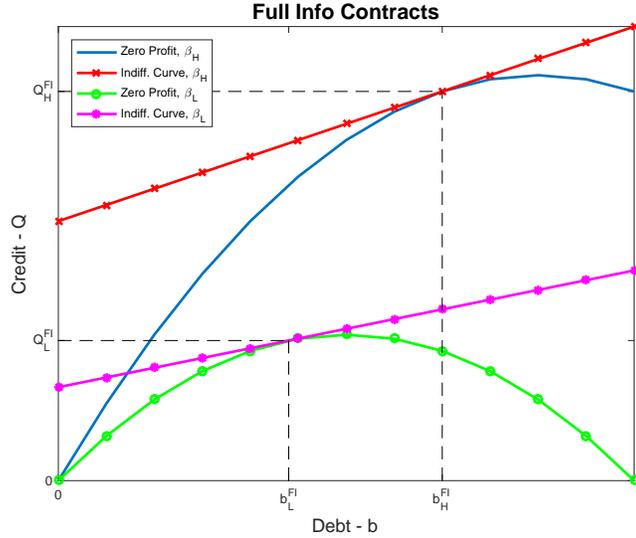


Figure 2: Full Information Example

market allocations are always statically constrained efficient. In our calibration, most workers are patient and have scores in the region where the least-cost separating allocation is dominated by the cross-subsidizing contract, so the welfare gains from using the Netzer and Scheuer equilibrium are substantial.

In order to understand how type score  $s$  affects the credit contract, we first consider the full-information allocation and then demonstrate the general form of optimal constrained allocations that arise for different scores. The full-information allocation is shown in Figure (2).<sup>17</sup> The patient worker chooses more debt and receives a lower interest rate on this debt since she is less likely to default. But then, if type was private information, an impatient worker would choose the patient worker's contract, violating incentive compatibility in (16).

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of credit scores.

<sup>17</sup>The full information contract maximizes an employed borrower type  $i$ 's utility subject to zero expected profits on the type  $i$  contract. This corresponds to maximizing  $Q_i + \psi \int_0^{\tau_{i^*}^*(s, b_i)} F(\tau) d\tau$  (as in (14)) for each type  $i$ , subject to  $Q_i \leq R^{-1} F(\tau_{i^*}^*(s, b_i)) b_i$  (as in (15)). Graphically, this gives us indifference curves with slopes  $\frac{dQ_i}{db_i} = \psi F(\tau_{i^*}^*(s, b_i)) \geq 0$  and isoprofit curves with slopes  $\frac{dQ_i}{db_i} = R^{-1} [F(\tau_{i^*}^*(s, b_i)) - F'(\tau_{i^*}^*(s, b_i)) b_i]$ . Since for a given  $(s, b)$ ,  $\tau_{L^*}^*(s, b) < \tau_{H^*}^*(s, b)$ , the slope of the type  $H$  indifference curve is greater than the slope of the type  $L$ . Furthermore, since the interest rate on these contracts is given by  $\frac{b_i}{Q_i}$ , the interest rate can be seen as the inverse of the slope of a ray from the origin to the contract point. This is analogous to the continuous asset version of Chatterjee, et. al. [3].

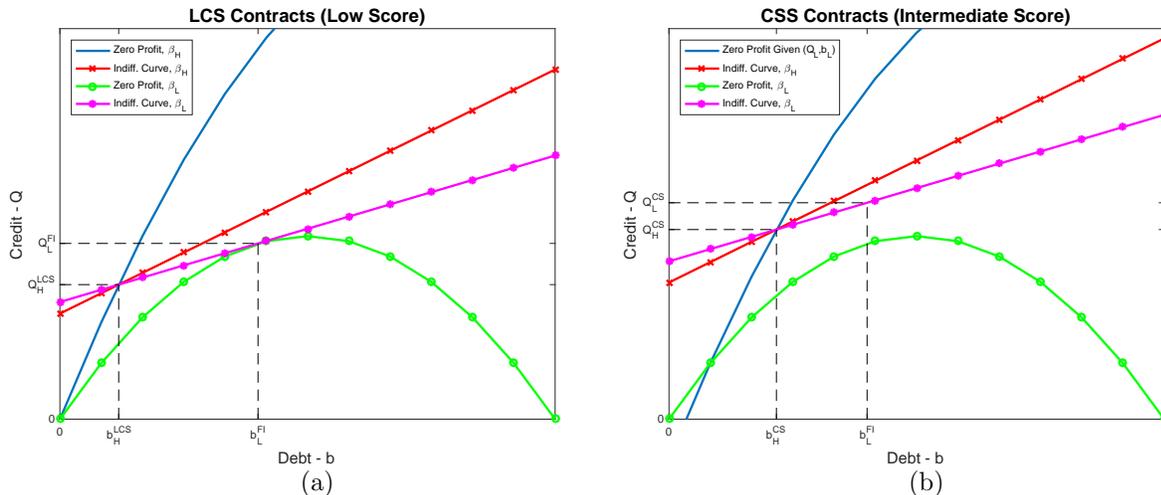


Figure 3: Least Cost vs. Cross-Subsidized Separating Contracts

Figure (3) compares two different types of allocations under private information. As discussed above, in this case the impatient worker's incentive compatibility constraint (16) is binding (as well as their participation constraint (18)). The least cost separating (LCS) contracts are shown in the left box Figure (3a). These types of contracts arise for low scores (in our calibrated model, they arise for  $s < 0.28$ ).

The impatient borrower receives the same amount of debt as under full information and pays the risk-adjusted break-even interest rate. On the other hand, the patient borrower's contract is distorted because of the binding incentive compatibility constraint of the impatient worker. In particular, the patient borrower receives less debt than the impatient borrower, although her interest rate is still equal to the risk-adjusted break even rate on her loan. This puts the patient worker on a lower indifference curve than in Figure (2).

As a worker's score rises the optimal contract switches from LCS to CSS. For CSS contracts, the impatient worker's participation constraint (18) is slack, because she still receives the full-information level of debt but pays a lower interest rate (illustrated by  $Q_L$  being above the impatient zero profit curve in Figure (3b)). This moves the impatient borrower to a higher indifference curve, while shifting the effective zero-profit curve for patient borrowers downward by the total subsidies to impatient borrowers. The patient borrower's contract is given by the intersection of the impatient borrower's

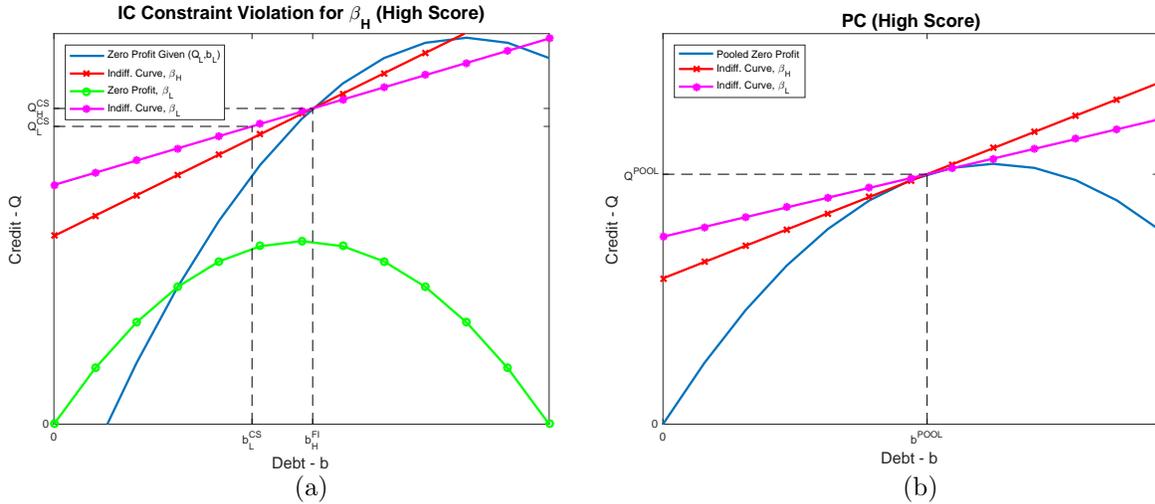


Figure 4: Violation of Incentive Compatibility in Cross-Subsidized Separating Vs. Pooling Contracts

new indifference curve and the patient borrower’s effective zero-profit curve. The CSS contract delivers more debt to the patient borrower than the LCS contract for the same score, but carries a higher interest rate than the LCS contract. The CSS contract dominates the LCS for intermediate scores ( $0.28 \leq s < 0.42$  in our calibration) because the extra interest paid *per patient worker* to subsidize impatient workers is more than offset by the patient worker’s utility gains from receiving more debt (e.g. loosening her credit limit).

The third contract type is pooling (PC), which can arise as  $s$  increases further (above 0.42 in our calibrated model) as the interest rate cross-subsidy to impatient workers becomes extremely generous. In this case, unlike the previous two, the patient household’s incentive constraint (17) binds.<sup>18</sup> That this constraint binds can be seen in Figure (4a), where the interest rate paid by an impatient borrower in the CSS is so low that a patient borrower would prefer the impatient contract to the one prescribed to her. With so few impatient borrowers with a high score, the subsidy *per* impatient contract is too generous and the patient borrower would rather have the impatient borrower’s subsidized

<sup>18</sup>In some settings, such as the constant risk model in Netzer and Scheuer, the high-type incentive compatibility constraint never binds. This is not the case in our model because of the way in which default rates (and therefore the indifference curves and zero-profit curves) depend on debt for each borrower.

rate, even though this gives her less credit. Therefore both incentive compatibility constraints bind, which means that the contract must be pooling (i.e. each type receives the same debt and interest rate). We find this contract by maximizing the utility of the patient borrower subject to the pooled zero-profit condition. Graphically, this is given by the tangency between the patient worker's indifference curve and the pooled zero-profit curve, as in Figure (4b).<sup>19</sup>

### 3.4 Type Scoring

Given the prior probability  $s$  that a worker is type  $H$ , the credit scorer forms a Bayesian posterior  $s'$  the worker is type  $H$  conditional on seeing whether she repays  $d$ :

$$s'_d(s) = \frac{F_d\left(\tau_{H^*}^*(s, b_{H^*}^*(s))\right)s}{F_d\left(\tau_{H^*}^*(s, b_{H^*}^*(s))\right)s + F_d\left(\tau_{L^*}^*(s, b_{L^*}^*(s))\right)(1-s)} \quad (19)$$

where the probability of receiving a shock lower than  $\tau$  is given by  $F_0(\tau) \equiv F(\tau)$  and the probability of receiving a shock larger than  $\tau$  is given by  $F_1(\tau) \equiv 1 - F(\tau)$ .

Typically a credit score is a measure of how likely the borrower is to repay. In the context of our model,  $s$  is a “type” score. In equilibrium we can map  $s$  to a credit score (i.e. the probability of repayment given  $s$ ) as follows:

$$\Pr(d = 0|s) = F_0\left(\tau_{H^*}^*(s, b_{H^*}^*(s))\right)s + F_0\left(\tau_{L^*}^*(s, b_{L^*}^*(s))\right)(1-s) \quad (20)$$

### 3.5 Distributions

We denote the measure of workers of type  $i$  over employment status  $n \in \{0, 1\}$  (where 1 denotes employed and 0 denotes unemployed) and score  $s$  in period  $t$  as  $\mu_{i,n}(s)$ . Given  $\mu_{i,n}(s)$ , we can compute  $t+1$  measures (denoted  $\mu'_{i,n}(S)$ ) for some set of scores  $S$  using decision rules and the updating function (recalling that  $h_i^*$  is constant over time). For

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<sup>19</sup>The formula for the patient borrower's indifference curve is the same as before. The slope of the pooled zero-profit curve is given by  $\frac{dQ}{db} = \frac{d}{db} \left\{ R^{-1} \left[ sF(\tau_{H^*}^*(s, b)) + (1-s)F(\tau_{L^*}^*(s, b)) \right] b \right\}$ .

the employed we have:

$$\begin{aligned} \mu'_{i,1}(s') &= (1 - \delta) \int_0^{s'} f(\theta(s)) d\mu_{i,0}(s) \\ &+ (1 - \delta)(1 - \sigma) \int_0^1 \left\{ \mathbb{I}_{\{s'_0(s) \leq s'\}} F_0(\tau_{i^*}^*(s, b_{i^*}^*(s))) + \mathbb{I}_{\{s'_1(s) \leq s'\}} F_1(\tau_{i^*}^*(s, b_{i^*}^*(s))) \right\} d\mu_{i,1}(s) \end{aligned} \quad (21)$$

where  $\mathbb{I}_{\{s'_d(s) \leq s'\}}$  is an indicator function which takes the value one if  $s'_d(s) \leq s'$  and zero otherwise.

For the unemployed we have two regions. For scores lower than the population share of patient workers (i.e., for  $s < \pi_H$ ):

$$\begin{aligned} \mu'_{i,0}(s') &= (1 - \delta) \int_0^{s'} [1 - f(\theta(s))] d\mu_{i,0}(s) \\ &+ (1 - \delta)\sigma \int_0^1 \left\{ \mathbb{I}_{\{s'_0(s) \leq s'\}} F_0(\tau_{i^*}^*(s, b_{i^*}^*(s))) + \mathbb{I}_{\{s'_1(s) \leq s'\}} F_1(\tau_{i^*}^*(s, b_{i^*}^*(s))) \right\} d\mu_{i,1}(s) \end{aligned} \quad (22)$$

For scores above  $\pi_H$  we must add the newborns who start unemployed with  $s = \pi_H$ . That is, for  $s \geq \pi_H$ :

$$\begin{aligned} \mu'_{i,0}(s') &= \delta + (1 - \delta) \int_0^{s'} [1 - f(\theta(s))] d\mu_{i,0}(s) \\ &+ (1 - \delta)\sigma \int_0^1 \left\{ \mathbb{I}_{\{s'_0(s) \leq s'\}} F_0(\tau_{i^*}^*(s, b_{i^*}^*(s))) + \mathbb{I}_{\{s'_1(s) \leq s'\}} F_1(\tau_{i^*}^*(s, b_{i^*}^*(s))) \right\} d\mu_{i,1}(s) \end{aligned} \quad (23)$$

### 3.6 Definition of Equilibrium

A steady-state Markov equilibrium consists of the following functions:

1. Worker value functions,  $U_{i,h}(s), W_{i,h}(s)$ , satisfy (1) and (3).
2. Default threshold functions,  $\tau_{i,h}^*(s, b)$ , satisfies (4).
3. Human capital investment,  $h_i^*$ , satisfies (7).
4. Firm value functions,  $J_{i,h}(s)$ , satisfies (8).
5. Wage functions,  $w_{i,h}^*(s)$ , satisfies (10).
6. Market tightness functions,  $\theta^*(s)$ , satisfies the free entry condition (7).
7. Credit market contracts,  $\{(Q_{i,h}^*(s), b_{i,h}^*(s))\}_{i \in \{H,L\}}$ , satisfy (14)-(18).

8. The updating function,  $s'_d$ , satisfies (19).
9. Stationary measures of each worker type over human capital levels and scores,  $\mu_{i,1}^*(s), \mu_{i,0}^*(s)$  that satisfy Equations (21) through (23) with  $\mu'_{i,n}(s) = \mu_{i,n}(s) = \mu_{i,n}^*(s)$  for  $n \in \{0, 1\}$  and  $i \in \{L, H\}$ .

### 3.7 Full Information Equilibrium Characterization

We will define a poverty trap relative to the equilibrium outcomes of a full information model, so we provide a characterization.

We now provide parametric assumptions to guarantee that workers borrow within a period and do not save across periods (A.1), that the match surplus of both workers is positive (A.2), that credit contracts are unique (A.3), and that patient workers choose a high level of human capital while impatient workers choose a low level (A.4). We also ensure that all workers would repay some positive level of debt (A.5) and that all workers default with positive probability (A.6).

**Assumption 1 .**

A.1  $\psi < (\omega R)^{-1}, \beta_L < \beta_H \leq R^{-1}$

A.2  $z < \underline{h}$

A.3  $F''(\tau) \leq 0$

A.4

$$\begin{aligned} & \frac{(1 - \beta_H(1 - \delta))^{-1} \beta_H(1 - \delta)}{1 - (1 - \delta)R^{-1}(1 - \sigma)(1 - \lambda) + \beta_H(1 - \delta)\sigma\lambda} \geq \phi \\ & > \frac{(1 - \beta_L(1 - \delta))^{-1} \beta_L(1 - \delta)}{1 - (1 - \delta)R^{-1}(1 - \sigma)(1 - \lambda) + \beta_L(1 - \delta)(\sigma - 1)\lambda} \end{aligned}$$

A.5  $F(\beta_L(1 - \delta)\psi\epsilon) > 0$

A.6 *The support of  $\tau$  is unbounded above.*

**Theorem 1** *Under Assumption 1, there exists a full information steady-state Markov equilibrium where  $i$  and  $h$  are publicly observable that is characterized by the following*

equations:

$$h_H^* = \bar{h}, h_L^* = \underline{h} \quad (24)$$

$$\theta_H^* > \theta_L^* \rightarrow f(\theta_H^*) > f(\theta_L^*) \quad (25)$$

$$w_H > w_L \quad (26)$$

$$F_0(\tau_H^*(b_H^*)) > F_0(\tau_L^*(b_L^*)) \quad (27)$$

The proof is in the appendix. Importantly, with full information under the parametric restrictions in Assumption 1, patient workers choose higher human capital than impatient workers, have higher job finding rates (in (25)), have higher wages (in (26)), and have lower default rates ((27)) implies higher repayment rates for patient workers).

### 3.8 Existence of Private Information Equilibrium

We build an equilibrium in which patient households choose high human capital (i.e.  $\bar{h}$ ), impatient households choose low human capital (i.e.  $\underline{h}$ ), and repayment leads to a higher future score via Bayesian updating than default (i.e. updating function  $s'_0(s) \geq s'_1(s)$  (with equality only when  $s = 0$  or  $s = 1$ )). Existence is complicated by the scoring functions, which are not contractions. We must therefore make additional technical assumptions to guarantee existence.

**Theorem 2** *Under the restrictions in Assumption 1 and additional conditions on  $F(\tau)$ ,  $\psi$ ,  $\omega$ ,  $R$ ,  $\beta_L$ ,  $\beta_H$ ,  $f(\theta)$ , and  $q(\theta)$ , there exists an equilibrium as defined in Section 3.6 with  $h_L^* = \underline{h}$  and  $h_H^* = \bar{h}$ .*

The proof and additional conditions are in the appendix. The idea is to define a mapping using the equilibrium functions defined in Section (3.6). In the appendix in Section (7), we define this operator, show how to find a Lipschitz space of functions for which the operator is a continuous self mapping, and then apply Schauder's fixed point theorem.

Economically speaking, existence requires that the marginal effect of default or repayment is sufficiently small so that the updating functions do not change rapidly across scores. This in turn requires that the odds-ratios for default and repayment are sufficiently independent of changes in score and continuation utilities, which in turn requires the same for the optimal contracts of each household type. We accomplish this by assuming that expenditure shocks are sufficiently volatile (i.e.  $\sup_{\tau \geq 0} F'(\tau)$  is small) and that the slope of each  $Q_i$  and  $b_i$  with respect to  $s$  and  $V_{i^*}(s'_0(s)) - V_{i^*}(s'_1(s))$  is sufficiently small.

## 4 Quantitative Exercise

To demonstrate how a poverty trap may arise and how markets respond to a policy banning employer credit checks, we compute an equilibrium of the economy and then change the determination of market tightness so that it is independent of type score (consistent with a ban).<sup>20</sup>

### 4.1 Calibration

A model period is taken to be a month. We use a Cobb-Douglas matching technology so that the job-finding and filling rates are given by  $f(\theta) = \theta^\alpha$  and  $q(\theta) = \theta^{\alpha-1}$ . We assume that expenditure shocks have an exponential CDF:  $F(\tau) = 1 - e^{-\gamma\tau}$ .<sup>21</sup> Once these functional forms are set, we must choose parameter values. Some values we take externally, while the remainder we choose to match data and model moments. The parameter values are listed in Table (1).

Table 1: Parameter Values

Externally Calibrated Parameters		
Parameter	Value	Source or Informative Moment
$\beta_H$	0.997	No inter-temporal savings condition
$R - 1$	0.33%	Risk free rate 4%
$\delta$	0.21%	45 Years in Market
$\alpha$	0.50	Matching Elasticity <sup>22</sup>
$\lambda$	0.50	Hosios Condition
$\sigma$	2.6%	Separation Rate, Shimer (2005)
$h_H$	1	Normalization
$z$	0.4	Shimer (2005)
Internally Calibrated Parameters		
$\pi_H$	55.0%	Super sub prime - super prime rates, CFPB (2015)
$\epsilon$	0.670	Super sub prime - prime rates, CFPB (2015)
$\beta_L$	0.672	Super sub prime - sub-prime rates, CFPB (2015)
$\psi$	0.982	Debt to Labor Income, CFPB (2015)
$h_L$	0.572	Residual Earnings 50 – 10, Lemieux (2006)
$\kappa$	1.45	Job-finding rate, Shimer (2005)
$\gamma$	13	Delinq. debt share, CFPB (2015)

<sup>20</sup>The algorithm for computing an equilibrium is available upon request.

<sup>21</sup>In order to guarantee model convergence, we include a small fixed probability of a shock that is too large to pay for any borrower. See the computational appendix for details.

Table 2: Model Fit

Moment	Data Value	Model Value
Super Prime CC Rate, top 49%	0.87%	0.88%
Prime CC Rate, 34 – 50%	1.17%	1.20%
Sub-Prime CC Rate, 0 – 33%	1.60%	1.61%
Debt to Labor Income	21.24%	21.34%
Delinq. Rate	0.95%	0.92%
Residual Earnings 50 – 10	0.57	0.57
Monthly Job Finding Rate	45.0%	45.1%

Note: Appendix 2 has definitions of model moments.

Many of our parameters are taken from previous papers or otherwise calibrated externally. We choose the bargaining weight for workers so that the Hosios condition ( $\lambda = \alpha$ ) holds. In a full information environment, the Hosios condition implies that total vacancies created is efficient. We will use that fact when comparing our results to a full information model of the labor market. While we cannot guarantee that the data represents a constrained efficient allocation, this ensures that our welfare results are not amplified due to beginning with an inefficient labor market equilibrium.

We have chosen moments on credit card debt from various sources, some of which are new to the quantitative household credit literature (to our knowledge). The average credit card rate and share of borrowers in each credit bracket are from the Consumer Financial Protection Bureau’s “Consumer Credit Card Market” report [7]. The interest rates are “total costs of credit” for each credit bracket in 2015, less 2% for inflation, and reported as monthly rates. These are the most comparable numbers to the model interest rates, since some people pay all balances monthly in the data (and therefore do not pay interest) whereas everyone pays interest in the model.

We also use the CFPB’s data to compute credit card debt to income and the share of debt that is defaulted upon. Total credit card debt was \$779 Billion in 2015, which we divide by labor’s share of average monthly GDP, which was  $0.60 \times \$6.108$  Trillion. Finally, we use the CFPB’s reported share of debt that is more than three months past due to total debt (Figure 27 in the report).

Our moments on labor market outcomes are taken from economy wide reports since we do not have merged data with credit scores and earnings or job-finding rates. For the residual earnings 50 – 10 ratio, we use the log of median earnings minus the log of the

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<sup>22</sup>Hall [15] uses a value of  $\alpha = 0.24$ . Shimer [29] uses  $\alpha = 0.72$ . Other authors have used values in between, with many settling on 0.5. See Gertler and Trigari [13].

earnings of the tenth percentile, which is reported by Lemieux [20]. For the job finding rate we use the monthly rate implied by Shimer [29].

## 4.2 Properties of Stationary Equilibrium

The equilibrium stationary distribution of workers over “type” scores and employment status is determined by the relative solvency and default rates as well as job-finding rates. Since type scores are not directly observable, we construct a data comparable distribution by sorting borrowers by their default probability and then assigning credit ratings consistent with the empirical shares of households within each rating. This means that as in the data, the bottom third are labeled “sub prime”, the next 15% are “prime” and the top 50% are “super prime”. Figure (5a) plots the histogram of workers over credit ratings constructed in this way.

While the population shares over credit ratings are defined to match the data, the share of workers of each type within each credit rating is endogenous – it depends on the relative default rates of each worker type in equilibrium. We plot these distributions in Figure (5b), where it is clear that the most impatient workers have sub prime credit, while less than 1% of patient workers have such poor credit since they only default due to extremely large expenditure shocks. Likewise, nearly 90% of patient workers have scores in the super prime range.

The composition of types over ratings determines the gradient of interest rates, default rates, and debt-to-income ratios with respect to credit rating. This can be understood by considering the average and type-specific default rates by credit rating, which we report in red text in Figures (5a) and (5b). The average default rate is falling with credit rating, from 1.34% to 0.64%, but this is because the composition of borrowers in each group is changing, not because an individual always defaults less when her score is higher. For example, the average super-prime patient borrower actually defaults four times more than the average subprime patient borrower. This is because she receives much less credit when subprime and because she has a strong incentive to repay. In fact, a patient borrower in the prime category has the strongest incentive to repay and therefore the lowest average default rate because default generates the largest drop in score in the updating function in Figure (6b).

The stationary distribution is derived from the law of motion for a worker’s employment status and score, which depends on the job-finding rate for unemployed and the average change in score for employed workers. Figure (6a) plots the job-finding rate

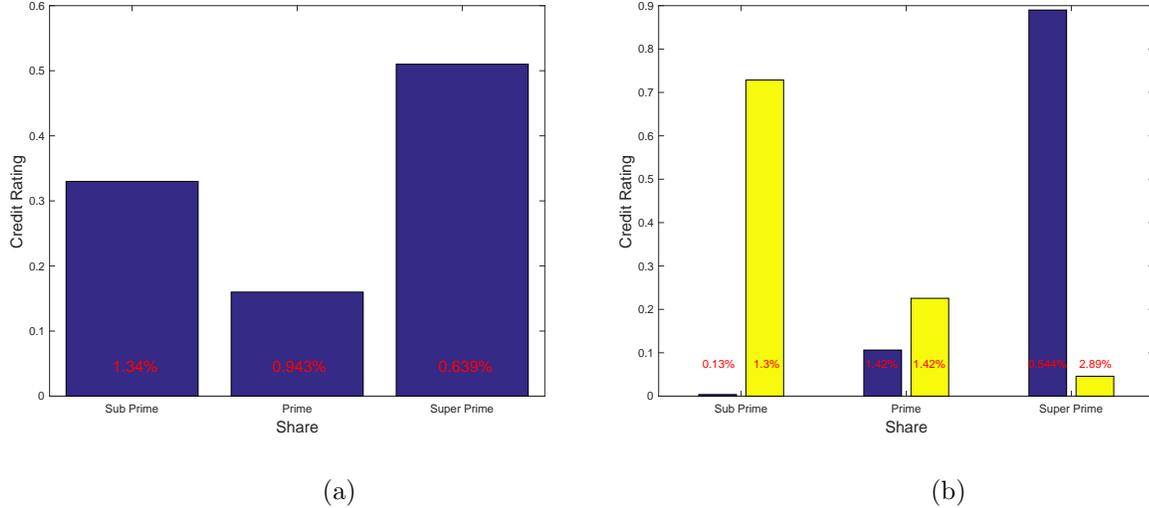


Figure 5: Histograms over Credit Ratings

$f(\theta(s))$ , which is bounded below by the impatient worker's full information rate and above by the patient worker's (both of which are efficient under the Hosios condition). The finding rate rises monotonically for scores between zero and one, reflecting the rising surplus associated with patient (and more productive) workers. Since most unemployed patient workers have scores above 0.80 while most impatient are below 0.014, patient workers find jobs at a substantially higher rate than impatient on average. Of course, some unlucky patient workers have substantially lower scores than average and therefore experience lower job-finding rates due to being pooled with the impatient. The median unemployed worker, marked by  $p_U^{50}$  on the graph, has a score of 0.55 and therefore a job finding rate of nearly 47%.<sup>23</sup>

The score updating functions are plotted in Figure (6b), the shape of which can be understood by the relative solvency and default rates of the two worker types. Because both worker types repay with a high probability at all scores, there is very little information revealed by repayment.<sup>24</sup> The score therefore updates very slowly in the positive

<sup>23</sup>Throughout, we use  $p^x$  to denote the  $x^{th}$  percentile of scores. If we condition on type or status then we use a subscript, so that the notation  $p_U^x$  is the score held by  $x^{th}$  percentile of the unemployed and  $p_H^x$  is the score held by the  $x^{th}$  percentile of high (patient) types. Likewise,  $p_{HU}^x$  is the score held by the  $x^{th}$  percentile of the patient unemployed.

<sup>24</sup>These rates are implied by the interest rate targets, which are relatively low relative to the risk-free rate.

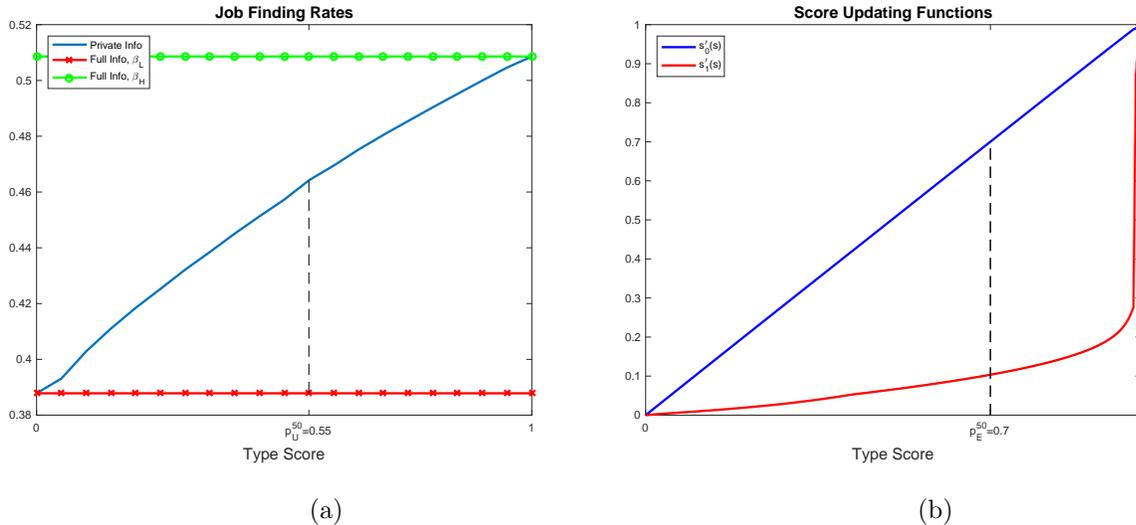


Figure 6: Job Finding Rates and Score Updates

direction, with  $s'_0(s)$  just slightly above the forty-five degree line. However, the relative default rate of impatient workers is quite high - the average default rate for impatient workers is ten times that of the patient. This implies that observing default leads to a dramatic downward update and  $s'_1(s)$  is much lower than  $s$  for most scores. The median employed borrower has a score of 0.68, implying that a default would reduce her score to 0.11 (the bottom third of scores in the stationary distribution).

### 4.3 Fit of Untargeted Moments

Our model is consistent with some untargeted moments (i.e dimensions of the data that were not used to fit the model). Figure (7a) reproduces the fit of the model's interest rates with data, while Figure (7b) shows the shares of debt held by borrowers with each credit rating, both in the data and our model.<sup>25</sup> The fact that credit shares are increasing with rating is a success of the Netzer and Scheuer equilibrium concept and would not be generated by models in which credit contracts were least cost separating for all scores (since patient households would always have less debt than impatient households in such a model) to maintain incentive compatibility as is clear in Figure (3a).

Furthermore, the policy experiment in Section 5 shows that our model closely matches

<sup>25</sup>The data is from the Consumer Financial Protection Bureau's 2017 credit card report [8].

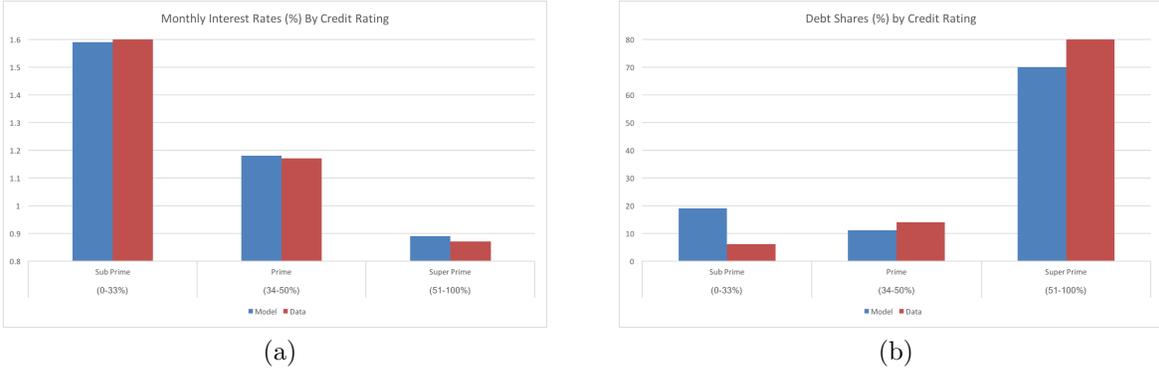


Figure 7: Average Interest Rates and Credit Usage by Rating

the effect of employer credit check bans on the job finding rate of subprime workers. Friedberg, Hynes, and Pattison [12] estimate that workers in the bottom quintile of financial health enjoy a 25% decline in expected unemployment duration when employer credit check bans are enacted at the state level, while our calibrated model predicts that the bottom quintile of borrowers would enjoy a 27% reduction in unemployment duration. While the bottom quintile in our model is not precisely the same as the bottom quintile of Friedberg, Hynes, and Pattison [12], we are encouraged that our calibration generates similar labor market effects for financially distressed workers.

#### 4.4 Poverty Traps

The definition of a poverty trap is not universally agreed upon, so we discuss two possible definitions. The first is a situation in which a worker's experience is made worse due to her credit score relative to an otherwise identical worker. In our case, this happens for the patient households. A patient worker who becomes unemployed with a bad score has a harder time finding a job than one who becomes unemployed with a good score. This leads to further divergence between the two, since the worker with good credit will find a job sooner *and therefore* have an even better credit score in the future. This is because employed patient workers experience an increase in their credit score on average while the unemployed do not. We say that the patient household is subject to a poverty trap because, on average, she experiences a decrease in her score (relative to being employed) and the decrease in score makes it harder to find a job in the next period.

We use two figures to understand how such a poverty trap may arise. Figure (8a) uses the job-finding rates (as in Figure (6a)) to compute the expected unemployment

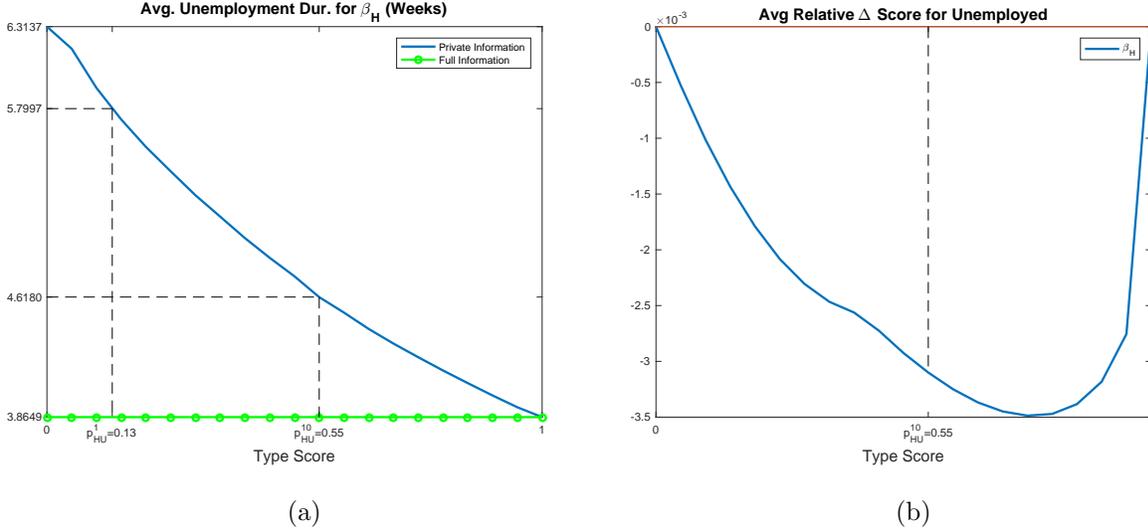


Figure 8: Poverty Trap for Patient Workers

duration of an unemployed patient household as a function of her score  $s$ . It is falling with score, reflecting the fact that patient workers are more productive in equilibrium and tend to have higher scores. Note that there are some patient workers who end up with low scores, illustrated by the vertical bar at the tenth percentile. This is the first part of the poverty trap: an unlucky patient worker with a bad credit history has a hard time finding a job and therefore expects longer unemployment spells than if her score was higher.

We next look at the average change in a worker's score when unemployed relative to when she is employed.<sup>26</sup> Figure (8b) plots this function for patient workers. On average, an employed patient worker experiences a rising score, while her score remains constant during an unemployment spell. It is evident from the figure that an unlucky patient worker with a low score therefore experiences a deterioration in her score relative to if she was employed, which reinforces the longer unemployment duration.

Another way of defining the poverty trap is relative to the full information equi-

<sup>26</sup>The average relative change in score is defined as:

$$\Delta(s) = s - F_0\left(\tau_{H^*}^*(s, b_{H^*}^*(s))\right)s'_0(s) - F_1\left(\tau_{H^*}^*(s, b_{H^*}^*(s))\right)s'_1(s)$$

The change while unemployed is 0 while the average change while employed is the negative of the above expression. Thus, the relative average change is  $\Delta(s)$ .

librium. The idea is that the job-finding rate for a worker with a low score may be strictly lower than if her human capital was observable. Again, consider Figure (6a) and compare the finding rates between the private and full information economies. The patient worker experiences a lower job-finding rate for all  $s < 1$  while the opposite is true for the impatient worker. For example, the bottom quintile of unemployed patient workers have scores below 0.68 and a job-finding rate below 47.9%, which is 3% below the full information rate of patient workers. Private information has the opposite effect for the impatient workers, 10% of whom have scores above 0.55 and therefore finding rates above 46.4%, which is 7.6% above their full information rate.

The extent of the poverty trap relative to full information depends on the patient worker’s score. Using the score percentiles in Figure (8a) we can say that the poverty trap adds just over two days to the median patient worker’s unemployment duration, five days for the 25<sup>th</sup> percentile, and just under a week for the lowest decile of patient job seekers.

A useful summary of the labor market impact of default can be computed as the present value of wages conditional on repayment minus the same value conditional on default. We compute these measures for each worker type and employment status, as well as the unconditional average, amortize them over 10 years, and report this measure relative to the average wage in Table (3). Our model predicts substantial expected wage losses from default through two mechanisms. First, the job-finding rate falls due to a lower score. Second, the worker’s bargaining position becomes weaker and therefore their wages fall even conditional on being employed. The average across all worker types, scores, and employment statuses amounts to 2.34% of earnings in each month for ten years.

Table 3: Wage Losses From Default

	Employed	Unemployed	Overall
Patient ( $\beta_H$ )	3.19%	3.07%	3.18%
Impatient ( $\beta_L$ )	1.36%	0.66%	1.32%
Overall	1.93%	1.52%	2.34%

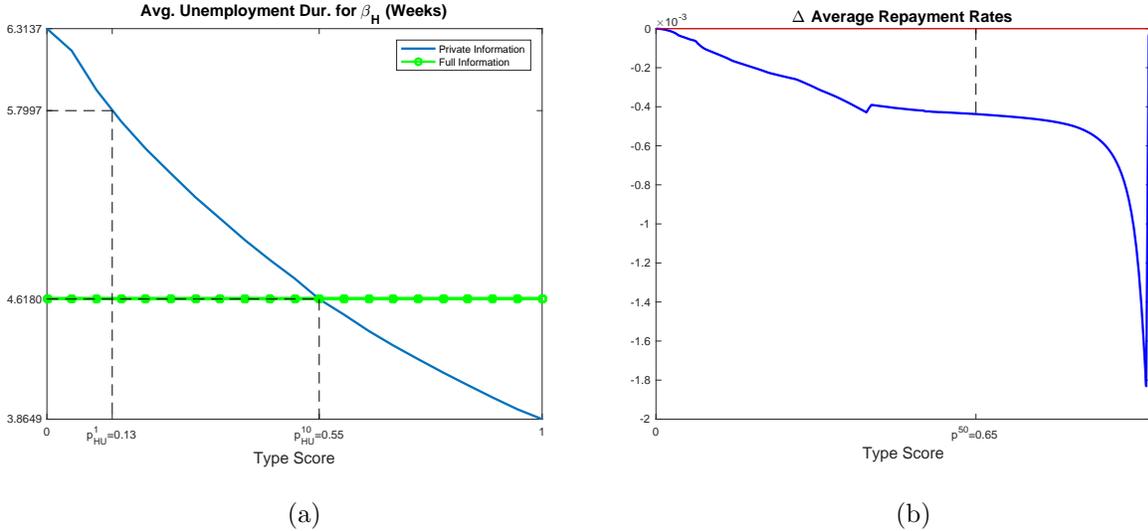


Figure 9: Equilibrium Effects of Ban

## 4.5 Labor Market Efficiency

Since we have assumed that the Hosios condition holds, we know that the full-information finding rates are efficient. We can therefore define a measure of labor market efficiency by considering the average difference between each worker type's average finding rate in the economy with private information relative to the full information economy. For the patient households in the calibrated economy, the monthly job-finding rate averages 49.7%, which is 1.3 percentage points lower than the efficient 50.9%. On the other hand, impatient households have an inefficiently high job-finding rate. In the calibrated economy their monthly job-finding rate is 40.5%, which is 1.7 percentage points higher than the efficient rate.

## 5 Policy Experiment: Banning Credit Checks

We now solve the economy with the same parameters, except that vacancies cannot be conditioned on a worker's score which implies market tightness  $\theta$  is independent of  $s$ . That is, we substitute  $q(\theta)$  for  $q(\theta(s))$  in the free entry condition in (13). While market tightness and the job-finding rate are therefore independent of  $s$  (and independent of  $\beta_i$  as before), match surplus and therefore bargained wages still depend on  $s$  since the worker's

score affects her bargaining position post match. Credit markets operate as before the ban, except that the workers’ incentives to repay endogenously fall: since default (and a lower credit score) do not affect your job finding rate, there is less punishment associated with default.

The ban’s effect on aggregate variables can be seen in Table (4). The median finding rate falls slightly, from 46.42% to 46.40%, although we emphasize that the changes across the credit score distribution are often quite large and in the opposite direction of what would be dictated by labor market efficiency (i.e. unproductive workers have higher finding rates on average, while productive workers have lower finding rates).<sup>27</sup> In fact, we find that the job finding rate for workers with very low scores rises substantially, which causes the average duration of unemployment for the bottom quintile of workers to decline 27%, which is close to the empirical estimates of Friedberg, et al [12]. There is a decline in labor productivity overall since the composition of the labor force shifts towards lower-productivity impatient workers: average labor productivity falls from 80.8% to 80.7%.

The ban affects on the credit market through the repayment decisions of borrowers, again seen in Table (4). The average interest rate rises from 1.17% to 1.18% as the average default rate rises from 0.92% per year to 0.93%. Furthermore, credit access is reduced as the the average debt-to-income ratio falls from 21.34% to 20.50%. Again, the small aggregate changes mask larger changes at the micro level, as seen in Figure (9b).<sup>28</sup>

Table 4: Effect of Employer Credit Ban

Moment	Baseline	After Ban
Median Job Finding Rate	46.42%	46.40%
Avg. Labor Prod. $\times 100$	80.78%	80.65%
Default Rate	0.92%	0.93%
Average CC Rate	1.17%	1.18%
Average Debt to Income	21.34%	20.50%

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<sup>27</sup>We plot changes expected unemployment duration in Figure (9a) since it is in more easily interpreted units (weeks). The relationship with the job finding rate is monotone - a higher finding rate implies a lower duration.

<sup>28</sup>Note that the default rate is very steep near  $s = 1$  since there is no endogenous incentive to repay if lenders are certain of the worker’s type.

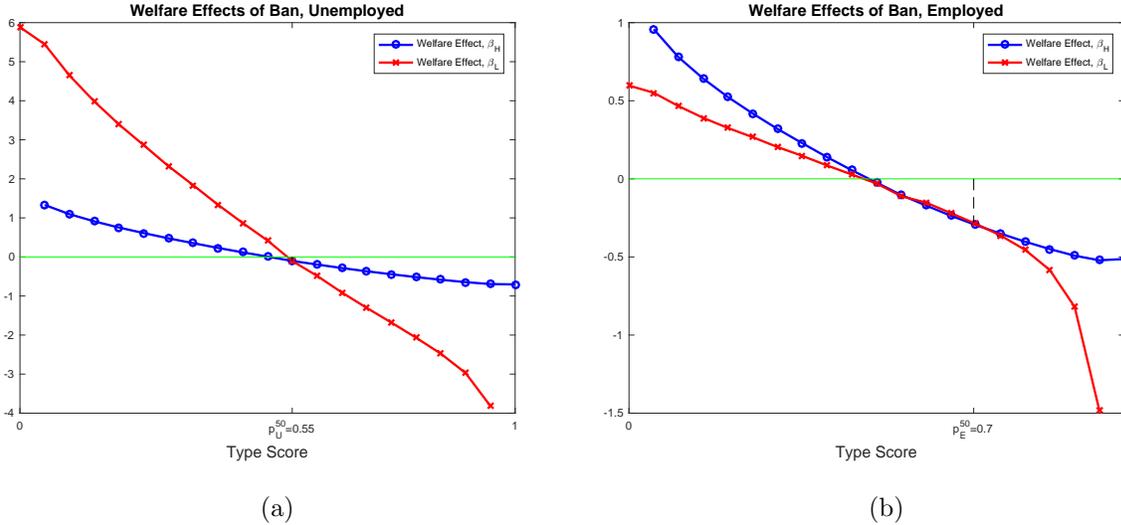


Figure 10: Welfare Effects of Ban

Even though workers with a low credit rating experience a substantial rise in their job-finding rate when employer credit checks are banned, this represents a reduction in matching efficiency. This is because the higher finding rate of these workers is further above their full-information finding rate. This rise, from 42.4% to 46.4%, means that the impatient worker's finding rate is 7.6% higher than the efficient level after the ban. On the other hand, patient workers are now pooled with more low-patience workers and therefore experience a more inefficiently low finding rate than in the economy with employer credit checks. Their finding rates fall from 49.7% to 46.4%, which is 3.25% lower than the efficient level. If we average over these absolute changes, then the ban reduces job-finding efficiency by 5.42%.

This implies that the poverty trap has been eliminated, since workers with bad credit are no longer subject to lower finding rates, yet most of the people affected by the poverty trap experience worse job-finding rates. Furthermore, the net effect is a decrease in labor market efficiency in spite of the increase in average job-finding rates. This can be seen by comparing the finding rate after the ban to the full-information finding rates in Figure (6a). The full-information rates are efficient since our bargaining weight satisfies the Hosios condition. The economy with employer credit checks experience partial separation through type scores, so on average each type has a job finding rate closer to their full information value than under the pooled finding rate that arises after

the ban goes into effect.

Banning pre-employment credit screening also affects the size and split of rents after a match has occurred by affecting a worker's bargaining position. We demonstrate this in Figures (11a)-(11d). Prior to the ban, there is a clear positive effect of credit rating on wages for both worker types and, likewise, a downward effect on profits. Wages are rising for two reasons: first, a higher credit score increases the match surplus and second, by increasing the worker's threat point since the job finding rate is rising with score. This means that a worker captures a larger share of the surplus as her credit score rises. Of course, the unconditional wage rises even faster with credit rating since patient workers have higher wages at all scores. The opposite profile appears in profits - conditional on worker type, profits are highest when for workers with bad credit ratings. On the other hand, the level of profits is strictly higher for patient workers than for impatient, due to their higher labor productivities, which generates the positive profile of vacancies with respect to score.

Once the ban goes into place, job finding rates are no longer score specific, which means that a worker's outside option is less affected by her score. This leads to a near complete flattening of the wage profiles in Figures (11a) and (11b) and profit profiles in Figures (11a) and (11b).<sup>29</sup> Relative to the baseline, this causes a decline in wages for workers with high scores but a rise in wages for sub prime and prime, while profits move in the opposite direction.

We next plot the net effect of the ban on welfare<sup>30</sup> for the unemployed in Figure (10a), since the direct change on market tightness and finding rates affects these workers. Workers with low type scores experience a gain in welfare, since they experience a higher job finding rate than when firms can discriminate based on score.<sup>31</sup> Furthermore, the patient workers gain more since they put a higher weight on finding a job due to their higher  $\beta$ . The welfare gains are falling for both worker types as scores rise, eventually becoming negative for those with high scores. Likewise the welfare effect is positive but falling for employed workers, as seen in Figure (10b). On average, impatient workers gain from the ban and patient workers lose, with the effects are magnified for unemployed

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<sup>29</sup>Quantitatively, our wage profiles are flat to three decimal places and therefore appear as such in the plots, but do still vary in theory. Likewise, the discounted profit lines are quite flat, though less so than wages.

<sup>30</sup>See the appendix for the definition of these welfare measures.

<sup>31</sup>We can evaluate the welfare effects for workers at each score, even if the theoretical measure of them is zero. For example, we calculate the value function of patient workers at  $s = 0$  and impatient at  $s = 1$  when we solve the model. However, we omit these points from our plots because they create non-monotonicities.

Table 5: Welfare Effects of Banning PECS

	Employed	Unemployed	Overall
Patient ( $\beta_H$ )	-0.44%	-0.54%	-0.43%
Impatient ( $\beta_L$ )	0.46%	4.78%	0.72%
Overall	-0.04%	2.11 %	0.09%

workers since any change in job finding rates affects them immediately.

We summarize these conditional averages in Table (5).<sup>32</sup> On the other hand, the average worker in the initial stationary distribution gains slightly, by 0.09% of monthly consumption. This is driven by large gain for a small share of the population, however, because only 43% of workers gain from the policy, starting from the stationary distribution in which employer credit checks are legal. From a political perspective, it is hard to justify employer credit check bans.<sup>33</sup>

## 6 Conclusion

We have presented a theoretical foundation for why employers may use credit histories in the hiring process and how this practice can create a poverty trap. Our theory extends the workhorse Diamond-Mortensen-Pissarides model to include ex-ante private information about worker productivity, while also building a novel framework for including credit scores when borrowers have private information about their repayment rates. Combining these two microeconomic models highlights the connection across markets in the presence of private information. It also allows us to avoid the Lucas Critique as we conduct the PECS ban since a policy change in the labor market spills over to the credit market (which due to general equilibrium effects spills back over to the labor market).

We have used our model to complement the empirical literature on the effect of banning employer credit checks. Specifically, we address the effect on unmeasurable outcomes – labor market efficiency and welfare. While efficiency is unequivocally reduced,

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<sup>32</sup>If private information persisted after hiring, then we would expect reduced expected profits due to overpaying the low-productivity type. This would make scores more valuable than in our baseline model. So, getting rid of PECS would have bigger negative effects on matching and welfare losses would be larger than what we are estimating.

<sup>33</sup>We can also calculate the expected lifetime utility of a newborn agent (i.e. who has a  $\pi_H$  probability of being patient and begins unemployed). This welfare measure gives a welfare loss of 1.3% of monthly consumption for a worker born into the economy without employer credit checks, relative to being born into an economy that allows them.

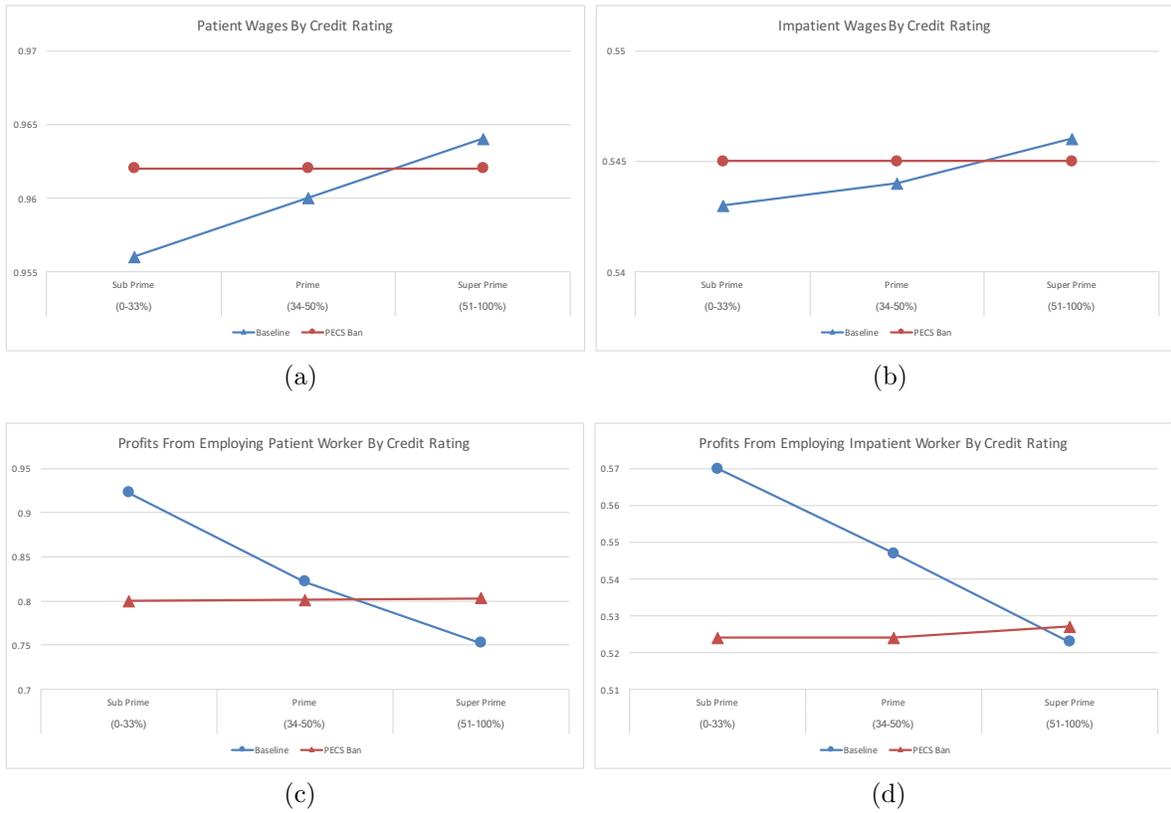


Figure 11: Effect of PECS Ban on Bargaining

the welfare effects are more nuanced. Impatient workers, who tend to have relatively bad credit, gain substantially while patient workers lose. Policy makers must consider the trade-off between equity and efficiency when considering employer credit check bans.

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## 7 Appendix: Proofs

We first fix notation. The domain of all functions is fixed as  $[0, 1]$  throughout, so will be omitted. The family of bounded Lipschitz continuous functions on this domain with slopes bounded by  $M$  is given by

$$\mathcal{L}_M = \{h : [0, 1] \rightarrow \mathbb{R} : \|h\| \text{ is bounded and } \forall (s_x, s_y), |h(s_x) - h(s_y)| \leq M|s_x - s_y|\},$$

where the norm of a function  $f \in \mathcal{L}_M$  is given by  $\|h\| = \max_{0 \leq s \leq 1} |h(s)|$ . We will work with a vector operator  $T : \mathcal{L} \Rightarrow \mathbb{C}$ , where:

$$\mathcal{L} \equiv \prod_{\ell=1}^{17} \mathcal{L}_{M_\ell}$$

and  $\mathbb{C} \subset \mathcal{L}$  is the space of continuously bounded functions. The vector of norms for  $v \in \mathbb{L}$  will be denoted by  $\|v\|$  and the norm on this space is the Euclidian norm on

this vector, denoted  $\|v\|_2 = \sqrt{\|v\| \cdot \|v\|}$ . We will verify that  $T$  is continuous and that  $T : \mathcal{L} \rightrightarrows \mathcal{L}$ . Denoting a vector of functions in  $\mathcal{L}$  by

$$v = \left\langle W_H, W_L, U_H, U_L, \psi w_H, \psi w_L, J_H, J_L, \theta, s'_0, s'_1, \Gamma_0, \Gamma_1, b_H, b_L, Q_H, Q_L \right\rangle,$$

we define  $T$  element-by-element. We fix  $h_H = \bar{h}$  and  $h_L = \underline{h}$  throughout, so suppress these arguments, then verify that these choices are consistent under Assumption 1. Note that, for our proofs, we assume that there is a probability  $p > 0$  that borrowers receive an expenditure shock  $\tau = 0$  and a probability  $(1-p)q > 0$  that the shock is sufficiently large that the worker cannot pay. First, the operators for the value functions are given by:

$$T_H^W(s) = T_1[v](s) \equiv \psi w_H(s) + Q_H(s) + p \left[ \beta_H(1-\delta)V_H(s'_0(s)) - \psi b_H(s) \right] + (28)$$

$$(1-p) \left[ \beta_H(1-\delta)V_H(s'_1(s)) - \psi \beta_H(1-\delta)\epsilon \right] + (1-p)(1-q)\psi \int_0^{\lambda_H(s;v)-b_H(s)} F(\tau) d\tau,$$

$$T_L^W(s) = T_2[v](s) \equiv \psi w_L(s) + Q_L(s) + p \left[ \beta_L(1-\delta)V_L(s'_0(s)) - \psi b_L(s) \right] + (29)$$

$$(1-p) \left[ \beta_L(1-\delta)V_L(s'_1(s)) - \psi \beta_L(1-\delta)\epsilon \right] + (1-p)(1-q)\psi \int_0^{\lambda_L(s;v)-b_L(s)} F(\tau) d\tau,$$

where the functions  $V_i$  and  $\lambda_i$  are defined by:

$$V_i(s) \equiv \sigma U_i(s) + (1-\sigma)W_i(s), \quad (30)$$

$$\lambda_i(s;v) \equiv \beta_i(1-\delta) \left[ \psi \epsilon + V_i(s'_0(s)) - V_i(s'_1(s)) \right]. \quad (31)$$

The next two operator elements map into unemployment values and are then given by:

$$T_H^U(s) = T_3[v](s) \equiv z + \beta_H(1-\delta) \left[ f(\theta(s))W_H(s) + (1-f(\theta(s)))U_H(s) \right], \quad (32)$$

$$T_L^U(s) = T_4[v](s) \equiv z + \beta_L(1-\delta) \left[ f(\theta(s))W_L(s) + (1-f(\theta(s)))U_L(s) \right]. \quad (33)$$

The next two operator elements update wages from the definition of firm values:

$$T_H^w(s) = T_5[v](s) \equiv \psi \bar{h} - J_H(s) + R^{-1}(1 - \sigma)(1 - \delta) \sum_{d \in \{0,1\}} G_d(\lambda_H(s; v) - b_H(s)) J_H(s'_d(s)), \quad (34)$$

$$T_L^w(s) = T_6[v](s) \equiv \psi \underline{h} - J_L(s) + R^{-1}(1 - \sigma)(1 - \delta) \sum_{d \in \{0,1\}} G_d(\lambda_L(s; v) - b_L(s)) J_L(s'_d(s)). \quad (35)$$

The next two operator elements map into firm values using Nash Bargaining:

$$T_H^J(s) = T_7[v](s) \equiv (1 - \lambda) \left[ W_H(s) - U_H(s) + J_H(s) \right], \quad (36)$$

$$T_L^J(s) = T_8[v](s) \equiv (1 - \lambda) \left[ W_L(s) - U_L(s) + J_L(s) \right]. \quad (37)$$

The next operator element updates the market tightness:

$$T^\theta(s) = T_9[v](s) \equiv q^{-1} \left( \kappa R \left[ s J_H(s) + (1 - s) J_L(s) \right]^{-1} \right) \quad (38)$$

The next two operator elements update the scoring functions:

$$T_0^s(s) = T_{10}[v](s) \equiv \frac{s}{s + (1 - s) \Gamma_0(s)}, \quad (39)$$

$$T_1^s(s) = T_{11}[v](s) \equiv \frac{s}{s + (1 - s) \Gamma_1(s)}. \quad (40)$$

The next two operator elements are the definitions of each odds ratio used to update the scoring functions:

$$T_0^\Gamma(s) = T_{12}[v](s) \equiv \frac{p + (1 - p)(1 - q) F(\lambda_L(s; v) - b_L(s))}{p + (1 - p)(1 - q) F(\lambda_H(s; v) - b_H(s))},$$

$$T_1^\Gamma(s) = T_{13}[v](s) \equiv \frac{(1 - p) \left( q + (1 - q) \left( 1 - F(\lambda_L(s; v) - b_L(s)) \right) \right)}{(1 - p) \left( q + (1 - q) \left( 1 - F(\lambda_H(s; v) - b_H(s)) \right) \right)}.$$

The final four operator elements are the credit contracts that solve the optimization

problem below, taking  $v$  as given.

$$\left\langle T_{14}[v](s), T_{15}[v](s), T_{16}[v](s), T_{17}[v](s) \right\rangle =$$

$$\operatorname{argmax}_{\{b_H, b_L, Q_H, Q_L\}} Q_H + \psi \left[ (1-p)(1-q) \int_0^{\lambda_H(s;v)-b_H} F(\tau) d\tau - pb_H \right]$$

s.t.

$$s \left[ -Q_H + (\omega R)^{-1} G_0(\lambda_H(s;v) - b_H) b_H \right] +$$

$$(1-s) \left[ -Q_L + (\omega R)^{-1} G_0(\lambda_L(s;v) - b_L) b_L \right] \geq 0$$

$$Q_L + \psi \left[ (1-p)(1-q) \int_0^{\lambda_L(s;v)-b_L} F(\tau) d\tau - pb_L \right] \geq$$

$$Q_H + \psi \left[ (1-p)(1-q) \int_0^{\lambda_L(s;v)-b_H} F(\tau) d\tau - pb_H \right]$$

$$Q_L + \psi \left[ (1-p)(1-q) \int_0^{\lambda_L(s;v)-b_L} F(\tau) d\tau - pb_L \right] \geq$$

$$\max_b \left\{ (\omega R)^{-1} G_0(\lambda_L(s;v) - b) b + \psi \left[ (1-p)(1-q) \int_0^{\lambda_L(s;v)-b} F(\tau) d\tau - pb \right] \right\}$$

We now study the slopes and bounds of each element of the operator and seek conditions to ensure that  $T : \mathcal{L} \rightrightarrows \mathcal{L}$ .

**Lemma 1** *If the finding rate and its derivative have bounds, so that  $\sup_{\theta \geq 0} f(\theta) \leq B_f$  and  $\sup_{\theta \geq 0} f'(\theta) \leq B_{f'}$ ,<sup>34</sup> then for  $i = 1, 2, 3, 4$  the functions  $T_i[v](s)$  are Lipschitz*

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<sup>34</sup>Conceptually, we have  $B_f = 1$ , although not all matching functions used in practice satisfy this for  $\theta \geq 0$ . Furthermore, in practice it is enough that  $f'(\theta)$  be bounded on a compact interval of  $\theta$  which excludes zero.

continuous with bounding constant  $\tilde{M}_i$ :

$$\begin{aligned}
\tilde{M}_1 &= M_H^w + M_H^Q + p \left[ \psi M_H^b + \beta_H(1-\delta)(\sigma M_H^U + (1-\sigma)M_H^W)M_0^s \right] \\
&+ (1-p)\beta_H(1-\delta) \left( \sigma M_H^U + (1-\sigma)M_H^W \right) M_1^s \\
&+ (1-p)(1-q)\psi \left[ \beta_H(1-\delta)(\sigma M_H^U + (1-\sigma)M_H^W)(M_0^s + M_1^s) + M_H^b \right], \\
\tilde{M}_2 &= M_L^w + M_L^Q + p \left[ \psi M_L^b + \beta_L(1-\delta)(\sigma M_L^U + (1-\sigma)M_L^W)M_0^s \right] \\
&+ (1-p)\beta_L(1-\delta) \left( \sigma M_L^U + (1-\sigma)M_L^W \right) M_1^s \\
&+ (1-p)(1-q)\psi \left[ \beta_L(1-\delta)(\sigma M_L^U + (1-\sigma)M_L^W)(M_0^s + M_1^s) + M_L^b \right], \\
\tilde{M}_3 &= \beta_H(1-\delta) \left[ B_f(M_H^W + M_H^U) + M^\theta \frac{B_f \bar{h}}{1-\beta_H} + M_H^U \right], \\
\tilde{M}_4 &= \beta_L(1-\delta) \left[ B_f(M_L^W + M_L^U) + M^\theta \frac{B_f \underline{h}}{1-\beta_L} + M_L^U \right].
\end{aligned}$$

If there exists some  $m_f$  such that  $\sup_{\tau \geq 0} |F'(\tau)| \leq m_f$  then the functions  $T_5[v]$  and  $T_6[v]$  are Lipschitz with constants:

$$\begin{aligned}
\tilde{M}_5 &= M_H^J + R^{-1}(1-\delta)(1-\sigma) \left\{ (p + (1-p)(1-q))M_H^J M_0^s \right. \\
&+ (1-p)(1-q)m_f \frac{\bar{h}}{1-R^{-1}} \left( M_H^b + \beta_H(1-\delta)(\sigma M_H^U + (1-\sigma)M_H^W)(M_0^s + M_1^s) \right) \\
&+ (1-p)M_H^J M_1^s + (1-p)(1-q)m_f \frac{\bar{h}}{1-R^{-1}} \left( M_H^b + \beta_H(1-\delta)(\sigma M_H^U + (1-\sigma)M_H^W)(M_0^s + M_1^s) \right) \Big\} \\
\tilde{M}_6 &= M_L^J + R^{-1}(1-\delta)(1-\sigma) \left\{ (p + (1-p)(1-q))M_L^J M_0^s \right. \\
&+ (1-p)(1-q)m_f \frac{\underline{h}}{1-R^{-1}} \left( M_L^b + \beta_L(1-\delta)(\sigma M_L^U + (1-\sigma)M_L^W)(M_0^s + M_1^s) \right) \\
&+ (1-p)M_L^J M_1^s + (1-p)(1-q)m_f \frac{\underline{h}}{1-R^{-1}} \left( M_L^b + \beta_L(1-\delta)(\sigma M_L^U + (1-\sigma)M_L^W)(M_0^s + M_1^s) \right) \Big\}
\end{aligned}$$

The operators  $T_7$  and  $T_8$  generate functions with Lipschitz constants:

$$\begin{aligned}
\tilde{M}_7 &= (1-\lambda)(M_H^W + M_H^U + M_H^J), \\
\tilde{M}_8 &= (1-\lambda)(M_L^W + M_L^U + M_L^J).
\end{aligned}$$

If there exists some  $m_q$  such that  $\sup_{\theta \geq 0} \left| \frac{dq^{-1}(\theta)}{d\theta} \right| \leq m_q$ ,<sup>35</sup> then the function  $T_9[v]$  is Lipschitz with constant:

$$\tilde{M}_9 = m_q \kappa R \max_{0 \leq s \leq 1} \left| \left( \frac{1}{sJ_H(s) + (1-s)J_L(s)} \right)^2 \left[ 2M_L^J + M_H^J + \max_{0 \leq s \leq 1} |J_H(s) - J_L(s)| \right] \right|.$$

The functions implied by the operators  $T_{10}$  and  $T_{11}$  are Lipschitz with constants:

$$\begin{aligned} \tilde{M}_{10} &= \frac{M_0^\Gamma + \max_{0 \leq s \leq 1} \Gamma_0(s)}{\min_{0 \leq s \leq 1} |(s + (1-s)\Gamma_0(s))^2|}, \\ \tilde{M}_{11} &= \frac{M_1^\Gamma + \max_{0 \leq s \leq 1} \Gamma_1(s)}{\min_{0 \leq s \leq 1} |(s + (1-s)\Gamma_1(s))^2|}. \end{aligned}$$

If  $\sup_\tau F'(\tau) \leq m_f$ , then the functions  $T_{16}[v]$  and  $T_{17}[v]$  are Lipschitz with constants:

$$\begin{aligned} \tilde{M}_{12} &= \frac{1}{p}(1-p)(1-q)m \left[ M_L^b + \beta_L(1-\delta)(\sigma M_L^U + (1-\sigma)M_L^W)(M_0^s + M_1^s) \right] \\ &\quad + \left( \frac{p + (1-p)(1-q)}{p^2} \right) (1-p)(1-q)m \left[ M_H^b + \beta_H(1-\delta)(\sigma M_H^U + (1-\sigma)M_H^W)(M_0^s + M_1^s) \right], \\ \tilde{M}_{13} &= \frac{1}{(1-p)q}(1-p)(1-q)m \left[ M_L^b + \beta_L(1-\delta)(\sigma M_L^U + (1-\sigma)M_L^W)(M_0^s + M_1^s) \right] \\ &\quad + \left( \frac{1-p}{((1-p)q)^2} \right) (1-p)(1-q)m \left[ M_H^b + \beta_H(1-\delta)(\sigma M_H^U + (1-\sigma)M_H^W)(M_0^s + M_1^s) \right]. \end{aligned}$$

Finally, for the general problem, the distances for  $T_{14}, T_{15}, T_{16}$ , and  $T_{17}$  depend on the form and parameters of  $F$  and the parameters  $p, q, \kappa, R, \beta_L$ , and  $\beta_H$ . We will therefore assume that there exist some values of  $M_H^b, M_L^b, M_H^Q, M_L^Q$  such that  $\tilde{M}_{14} \leq M_H^b, \tilde{M}_{15} \leq M_L^b, \tilde{M}_{16} \leq M_H^Q$ , and  $\tilde{M}_{17} \leq M_L^Q$ .

Each of these is derived from the definition of a Lipschitz constant, the triangle inequality, and the mean value theorem. We can use these expressions to prove the following lemma:

**Lemma 2** Assume that there exists  $m_f$  such that  $\sup_{\tau \geq 0} |F'(\tau)| \leq m_f$ ,  $m_q$  such that  $\sup_{\theta \geq 0} \left| \frac{dq^{-1}(\theta)}{d\theta} \right| \leq m_q$ ,  $B_f$  and  $B_{f'}$  such that  $\sup_{\theta \geq 0} f(\theta) \leq B_f$  and  $\sup_{\theta \geq 0} f'(\theta) \leq B_{f'}$ . If  $m_f, \beta_H, \beta_L, \psi, 1-\lambda$ , and  $\kappa$  are sufficiently small, then there exists some vector of bounds,  $M$ , such that  $T : \mathcal{L} \Rightarrow \mathcal{L}$ .

Note that we must only find a vector  $M$  for which the operator is self mapping. We do

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<sup>35</sup>As with  $f'(\theta)$ , we only require that  $\frac{dq^{-1}(\theta)}{d\theta}$  be bounded on a compact interval of  $\theta$  which excludes zero in practice.

this by construction for each operator element. For the first, we set  $M_i^w, M_i^W$ , and  $M_i^Q$  such that  $M_i^w + M_i^Q < M_i^W$ . Then we can set  $\psi, \beta_L$ , and  $\beta_H$  sufficiently low to ensure that  $\tilde{M}_1 \leq M_H^W$  and  $\tilde{M}_2 \leq M_L^W$ . Ensuring that  $\tilde{M}_3 \leq M_H^U$  and  $\tilde{M}_4 \leq M_L^U$  requires again setting  $\beta_L$  and  $\beta_H$  sufficiently low. If we set  $M_H^J < M_H^w$  and  $M_L^J < M_L^w$  then  $\tilde{M}_5 \leq M_H^w$  and  $\tilde{M}_6 \leq M_L^w$  for  $R^{-1}$  sufficiently small. If we set  $M_i^J \geq \frac{1-\lambda}{\lambda}(M_i^W + M_i^U)$ , which can be guaranteed for sufficiently small  $\lambda$ , then  $\tilde{M}_7 \leq M_H^U$  and  $\tilde{M}_8 \leq M_L^U$ . We can then set  $M^\theta$  to the expression defining  $\tilde{M}_9, M_0^s$  to  $\tilde{M}_{10}$ , and  $M_1^s$  to  $\tilde{M}_{11}$ . Finally, for sufficiently small  $m$  we can guarantee that  $\tilde{M}_{12} \leq M_0^\Gamma$  and  $\tilde{M}_{13} \leq M_1^\Gamma$ . By assumption we can find values of  $M_H^b, M_L^b, M_H^Q, M_L^Q$  such that  $\tilde{M}_{14} \leq M_H^b, \tilde{M}_{15} \leq M_L^b, \tilde{M}_{16} \leq M_H^Q$ , and  $\tilde{M}_{17} \leq M_L^Q$ .

QED

In order to apply Schauder's fixed point theorem, we must also ensure that  $T$  is a continuous mapping. It is sufficient to check that, for each  $s \in [0, 1]$ , and vector  $v_1, v_2$  with each  $v_i \in \mathcal{L}$ , there exists some matrix  $Z$  such that  $|T[v_1](s) - T[v_2](s)| \leq Z \cdot |v_1(s) - v_2(s)|$ . For the first 13 elements of  $T$  this follows from the definition and applications of the triangle inequality and mean value theorem. For a general contracting problem we again assume that this is true. We now write the expressions for  $\|T_i[v_x] - T_i[v_y]\|$  for  $i = 1, 2, \dots, 13$ .

**Lemma 3** *Take  $v_x$  and  $v_y$  in  $\mathcal{L}_M$ . If  $\sup_{\theta \geq 0} f(\theta) \leq B_f$  and  $\sup_{\theta \geq 0} f'(\theta) \leq B_{f'}$  for some*

$B_f, B_{f'}$ , then for  $i = 1, 2, 3, 4$ , the distances  $\|T_i[v_x] - T_i[v_y]\|$  are bounded by

$$\begin{aligned} \|T_1[v_x] - T_1[v_y]\| &\leq \|\psi w_{H,x} - \psi w_{H,y}\| + \|Q_{H,x} - Q_{H,y}\| \\ &+ p\beta_H(1-\delta)(\sigma M_H^U + (1-\sigma)M_H^W) \left( \|s'_{0,x} - s'_{0,y}\| + \psi \|b_{H,x} - b_{H,y}\| \right) \\ &+ (1-p)\beta_H(1-\delta)(\sigma M_H^U + (1-\sigma)M_H^W) \|s'_{1,x} - s'_{1,y}\| \\ &+ \psi(1-p)(1-q) \left( \|b_{H,x} - b_{H,y}\| + \sigma \|U_{H,x} - U_{H,y}\| + (1-\sigma) \|W_{H,x} - W_{H,y}\| \right), \end{aligned} \quad (41)$$

$$\begin{aligned} \|T_2[v_x] - T_2[v_y]\| &\leq \|\psi w_{L,x} - \psi w_{L,y}\| + \|Q_{L,x} - Q_{L,y}\| \\ &+ p\beta_L(1-\delta)(\sigma M_L^U + (1-\sigma)M_L^W) \left( \|s'_{0,x} - s'_{0,y}\| + \psi \|b_{L,x} - b_{L,y}\| \right) \\ &+ (1-p)\beta_L(1-\delta)(\sigma M_L^U + (1-\sigma)M_L^W) \|s'_{1,x} - s'_{1,y}\| \\ &+ \psi(1-p)(1-q) \left( \|b_{L,x} - b_{L,y}\| + \sigma \|U_{L,x} - U_{L,y}\| + (1-\sigma) \|W_{L,x} - W_{L,y}\| \right), \end{aligned} \quad (42)$$

$$\begin{aligned} \|T_3[v_x] - T_3[v_y]\| &\leq \beta_H(1-\delta) \left[ (1+B_f) \|U_{H,x} - U_{H,y}\| \right. \\ &\left. + B_f \|W_{H,x} - W_{H,y}\| + \frac{1}{1-\beta_H} B_{f'} \|\theta_x - \theta_y\| \right], \end{aligned} \quad (43)$$

$$\begin{aligned} \|T_4[v_x] - T_4[v_y]\| &\leq \beta_L(1-\delta) \left[ (1+B_f) \|U_{L,x} - U_{L,y}\| \right. \\ &\left. + B_f \|W_{L,x} - W_{L,y}\| + \frac{1}{1-\beta_L} B_{f'} \|\theta_x - \theta_y\| \right]. \end{aligned} \quad (44)$$

If there is some  $m_f > 0$  such that  $\sup_{\tau \geq 0} F'(\tau) \leq m_f$  then, for  $i = 5, 6$ , the distances  $\|T_i[v_x] - T_i[v_y]\|$  are bounded by

$$\begin{aligned} \|T_5[v_x] - T_5[v_y]\| &\leq \|J_{H,x} - J_{H,y}\| + R^{-1}(1-\delta)(1-\sigma) \left[ M_H^J \left( \|s'_{0,x} - s'_{0,y}\| + \|s'_{1,x} - s'_{1,y}\| \right) \right. \\ &\left. + \frac{\bar{h}}{1-R^{-1}} m_f \left( (\sigma M_H^U + (1-\sigma)M_H^W) (\|s'_{0,x} - s'_{0,y}\| + \|s'_{1,x} - s'_{1,y}\|) + 2\|b_{H,x} - b_{H,y}\| \right) \right] \\ \|T_6[v_x] - T_6[v_y]\| &\leq \|J_{L,x} - J_{L,y}\| + R^{-1}(1-\delta)(1-\sigma) \left[ M_L^J \left( \|s'_{0,x} - s'_{0,y}\| + \|s'_{1,x} - s'_{1,y}\| \right) \right. \\ &\left. + \frac{\underline{h}}{1-R^{-1}} m_f \left( (\sigma M_L^U + (1-\sigma)M_L^W) (\|s'_{0,x} - s'_{0,y}\| + \|s'_{1,x} - s'_{1,y}\|) + 2\|b_{L,x} - b_{L,y}\| \right) \right] \end{aligned} \quad (45)$$

For  $i = 7, 8$ , the distances  $\|T_i[v_x] - T_i[v_y]\|$  are given by

$$\|T_7[v_x] - T_7[v_y]\| = (1 - \lambda) \left[ \|W_{H,x} - W_{H,y}\| + \|U_{H,x} - U_{H,y}\| + \|J_{H,x} - J_{H,y}\| \right], \quad (47)$$

$$\|T_8[v_x] - T_8[v_y]\| = (1 - \lambda) \left[ \|W_{L,x} - W_{L,y}\| + \|U_{L,x} - U_{L,y}\| + \|J_{L,x} - J_{L,y}\| \right]. \quad (48)$$

If there exists some  $m_q$  such that  $\sup_{\theta \geq 0} \left| \frac{dq^{-1}(\theta)}{d\theta} \right| \leq m_q$  and  $\underline{J} > 0$  such that  $J_i(s) \geq \underline{J}$ ,<sup>36</sup> then the distance  $\|T_9[v_x] - T_9[v_y]\|$  is bounded by

$$\|T_9[v_x] - T_9[v_y]\| \leq \kappa R m_q \underline{J}^{-2} \left[ \|J_{H,x} - J_{H,y}\| + \|J_{L,x} - J_{L,y}\| \right]. \quad (49)$$

For  $i = 10, 11$ , the distances  $\|T_i[v_x] - T_i[v_y]\|$  are bounded by

$$\|T_{10}[v_x] - T_{10}[v_y]\| \leq 0.25 \left( \frac{p + (1-p)(1-q)}{p} \right)^2 \|\Gamma_{0,x} - \Gamma_{0,y}\|, \quad (50)$$

$$\|T_{11}[v_x] - T_{11}[v_y]\| \leq 0.25 q^{-2} \|\Gamma_{1,x} - \Gamma_{1,y}\|. \quad (51)$$

If there is some  $m_f > 0$  such that  $\sup_{\tau \geq 0} F'(\tau) \leq m_f$  then, for  $i = 12, 13$ , the distances  $\|T_i[v_x] - T_i[v_y]\|$  are bounded by

$$\|T_{12}[v_x] - T_{12}[v_y]\| \leq \frac{p + (1-p)(1-q)}{p^2} (1-p)(1-q) m_f \left[ \|b_{H,x} - b_{H,y}\| \right] \quad (52)$$

$$+ \beta_H (1 - \delta) (\sigma M_H^U + (1 - \sigma) M_H^W) \left( \|s'_{0,x} - s'_{0,y}\| + \|s'_{1,x} - s'_{1,y}\| \right)$$

$$+ p^{-1} (1-p)(1-q) m_f \left[ \|b_{L,x} - b_{L,y}\| \right]$$

$$+ \beta_L (1 - \delta) (\sigma M_L^U + (1 - \sigma) M_H^U) \left( \|s'_{0,x} - s'_{0,y}\| + \|s'_{1,x} - s'_{1,y}\| \right),$$

$$\|T_{13}[v_x] - T_{13}[v_y]\| \leq \frac{1-p}{((1-p)q)^2} (1-p)(1-q) m_f \left[ \|b_{H,x} - b_{H,y}\| \right] \quad (53)$$

$$+ \beta_H (1 - \delta) (\sigma M_H^U + (1 - \sigma) M_H^W) \left( \|s'_{0,x} - s'_{0,y}\| + \|s'_{1,x} - s'_{1,y}\| \right)$$

$$+ \frac{1}{(1-p)q} (1-p)(1-q) m_f \left[ \|b_{L,x} - b_{L,y}\| \right]$$

$$+ \beta_L (1 - \delta) (\sigma M_L^U + (1 - \sigma) M_H^U) \left( \|s'_{0,x} - s'_{0,y}\| + \|s'_{1,x} - s'_{1,y}\| \right).$$

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<sup>36</sup>Our assumption that  $\underline{h} > z$  and  $\lambda < 1$  ensures that such a lower bound on  $J_i(s)$  can be imposed.

These are again derived algebraically by evaluating the difference  $\|T_i[v_x] - T_i[v_y]\|$ . These bounds immediately imply the following lemma:

**Lemma 4** *Suppose that there exists some  $4 \times 17$  matrix  $z$  such that, for any  $v_x$  and  $v_y$  in  $\mathcal{L}_M$ , for  $i = 14, 15, 16$ , and  $17$ ,*

$$\|T_i[v_x] - T_i[v_y]\| \leq z\|v_x - v_y\|. \quad (54)$$

*Then, if the conditions stated for each bound to hold in Lemma (3) are satisfied, the operator  $T : \mathcal{L}_M \Rightarrow \mathbb{C}$  is continuous.*

Let the assumptions of Lemma (4) hold and denote the matrix implied by the inequalities in Lemma (1) (after appending  $z$  for rows 14 – 17) by  $Z$ . This means that for any  $v_x, v_y$  in  $\mathcal{L}$  we have  $\|T[v_x] - T[v_y]\| \leq Z\|v_x - v_y\|$ . Denote  $z^*$  as the element of  $Z$  with the largest value. For any  $\epsilon > 0$ , the distance  $\|T[v_x] - T[v_y]\|_2$  is smaller than  $\epsilon$  for any pair  $v_x, v_y$  for which  $\|v_x - v_y\|_2 < z^*\epsilon \equiv \delta$ .

QED

We can now invoke Schauder's Fixed Point Theorem to ensure existence of an equilibrium under the conditions from these lemmas.

**Theorem 3** *Under the conditions in Lemmas (1) and (3), there exists some  $v^* \in \mathcal{L}$  such that  $\forall s \in [0, 1] : T[v^*](s) = v^*(s)$ .*

We apply the statement of Schauder's Fixed Point Theorem in Stokey, et al [31]. The space  $\mathcal{L}$  is closed, bounded, and convex.  $T : \mathcal{L} \Rightarrow \mathcal{L}$  is continuous by Lemma (3) and the image is an equicontinuous family by virtue of  $\mathcal{L}$  being Lipschitz and  $T$  being self-mapping on  $\mathcal{L}$ .

QED

## 7.1 Application of Existence Theorem to Calibrated Economy

Our existence proof requires that the contracting problem that defines  $T_{14} - T_{17}$  be well behaved. In general, the solution to this problem satisfies a set of first-order conditions and the Lipschitz continuity of  $v$  can be used to get slopes for  $T_{14} - T_{17}$  as well as to calculate  $T_i[v_x] - T_i[v_y]$ . However, these calculations require further restrictions on differences and sums of partial derivatives of  $F$  in conjunction with the parameters  $p, q, \psi, \omega, R, \beta_L$ , and  $\beta_H$ . To illustrate, we consider the parameterization for our calibrated economy.

We use  $F(\tau) = \tau_0^{-1}\tau$  in our calibrated economy to facilitate accurate computation - for a uniform distribution on expenditure shocks, the contracting problem's first-order conditions are polynomials and can be solved precisely in each iteration. The uniform distribution also ensures relatively simple expressions for  $T_{14}$  through  $T_{17}$  as simple functions of  $s$  and  $\lambda_i(s; v)$ . The condition defining  $T_{17}[v](s)$ , or  $b_L(s)$ , is

$$(\omega R)^{-1} \left[ p + (1-p)(1-q)\tau_0^{-1} \left( \lambda_L(s; v) - T_{17}[v](s) \right) - (1-p)(1-q)\tau_0^{-1} T_{17}[v](s) \right] = (55)$$

$$\psi \left[ p + (1-p)(1-q)\tau_0^{-1} \left( \lambda_L(s; v) - T_{17}[v](s) \right) \right].$$

This means that  $T_{17}[v](s)$  is linear in  $\lambda_L(s; v) = \beta_L(1-\delta) \left[ \psi\epsilon + V_L(s'_0(s)) - V_L(s'_1(s)) \right]$  and *otherwise* independent of  $s$ . The other elements,  $T_{15} - T_{17}$ , are defined piecewise in  $s$  because they depend on whether the impatient borrower's participation constraint binds (whether the contract is least-cost separating or cross subsidizing). They are polynomials in  $s$ ,  $\lambda_L(s; v)$ , and  $\lambda_H(s; v)$  for values of  $s$  where the optimal contract is cross-subsidizing. For values of  $s$  where the contract is least-cost separating,  $T_{15}[v]$  is a polynomial, while  $T_{14}[v]$  is the root of a quadratic equality in  $s$ ,  $\lambda_L(s; v)$ , and  $\lambda_H(s; v)$  (which means that  $T_{16}[v]$  is as well, since it is equal to  $(\omega R)^{-1}G_0(\lambda_H(s; v) - T_{14}[v](s))T_{14}[v](s)$  in this case).

## 8 Appendix: Definitions of Moments

For model moments we use the stationary distribution. For the average value of an endogenous variable  $x_{i\ell}(s)$  where  $i$  is worker type,  $\ell$  is worker employment status, and  $s$  is score we compute:

$$\bar{x} = \int_0^1 \sum_{i \in \{L, H\}} \sum_{\ell \in \{U, E\}} x_{i\ell}(s) d\mu_{i\ell}^*(s)$$

$$\bar{x}_i = \frac{\int_0^1 \sum_{\ell \in \{U, E\}} x_{i\ell}(s) d\mu_{i\ell}^*(s)}{\sum_{\ell} \mu_{i\ell}^*(1)}$$

$$\bar{x}_{\ell} = \frac{\int_0^1 \sum_{i \in \{U, E\}} x_{i\ell}(s) d\mu_{i\ell}^*(s)}{\sum_i \mu_{i\ell}^*(1)}$$

So, for example, the quarterly repayment rate is conditional on employment and is therefore defined as:

$$\frac{\int_0^1 \sum_{i \in \{L, H\}} G_0(\tau_i^*(s, b_i^*(s))) d\mu_{iE}^*(s)}{\sum_i \mu_{iE}^*(1)}$$

In order to compute percentiles of the score distribution, we first define the cumulative distribution for the level of aggregation of interest. For unconditional percentiles, we use CDF:

$$\mu^*(s) \equiv \sum_{i \in \{L, H\}} \sum_{\ell \in \{U, E\}} \mu_{i\ell}^*(s) \quad (56)$$

Unconditional percentiles are then found by first solving for the type score of that percentile. For percentile  $x \in [0, 1]$  we solve:

$$x = \mu^*(p^x) \quad (57)$$

Likewise we define conditional percentiles using the conditional cumulative distributions. So, for example, the  $x^{th}$  percentile of unemployed uses the CDF of unemployed households defined by:

$$\mu_U^*(s) \equiv \frac{\sum_{i \in \{L, H\}} \mu_{iU}^*(s)}{\sum_{i \in \{L, H\}} \mu_{iU}^*(1)} \quad (58)$$

which is then used to solve for  $p_U^x$ :

$$x = \mu_U^*(p_U^x) \quad (59)$$

These percentiles are used to report conditional means. We also use the stationary distribution to create distributions over other endogenous variables. For example, to compute a percentile of earnings we create a grid  $\mathcal{W} \equiv \{w_i^*(s) | s \in \{s_0, s_1, \dots, s_N\}, i \in \{L, H\}\}$  and create the approximate probability distribution:

$$PDF^w(w_i^*(s_j)) \equiv \frac{\mu_{iE}^*(s_{j+1}) - \mu_{iE}^*(s_j)}{\sum_{i \in \{L, H\}} \mu_{iE}^*(1)} \quad (60)$$

We then arrange  $\mathcal{W}$  in ascending order and for any  $w \in \mathcal{W}$  create:

$$CDF^w(w) = \sum_{m \in \mathcal{W}, m \leq w} PDF^w(m) \quad (61)$$

And finally we use these approximate cumulative densities to compute percentiles of the earnings distribution.

For our welfare measures, we use the consumption equivalent concept. Since our preferences are linear, this corresponds to the percentage change in welfare. We ask “what fraction of total consumption in each period of the economy with employer credit checks would the worker exchange in order to switch to the economy without employer credit checks?” When this number is negative the household gains from the ban and when it is positive the household loses. We scale consumption in each sub-period in each date by a number  $1 + \gamma_{ij}(s)$ , where  $i$  is worker type and  $j$  is employment status. Denoting  $W^{nc}$  and  $U^{nc}$  as the value functions without employer credit checks, we define  $\gamma_{ij}(s)$  by:

$$W_{ih_i^*}(s)[1 + \gamma_{iE}(s)] = W_{ih_i^*}^{nc}(s) \tag{62}$$

$$U_{ih_i^*}(s)[1 + \gamma_{iU}(s)] = U_{ih_i^*}^{nc}(s) \tag{63}$$