Firm Wages in a Frictional Labor Market*

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Abstract

This paper studies a labor market with directed search, where multi-worker firms follow a firm wage policy: They pay equally productive workers the same. The policy reduces wages, due to the influence of firms' existing workers on their wage setting problem, increasing the profitability of hiring. It also introduces a time-inconsistency into the dynamic firm problem, because firms face a less elastic labor supply in the short run. To consider outcomes when firms reoptimize each period, I study Markov perfect equilibria, proposing a tractable solution approach based on standard Euler equations. In two applications, I first show that firm wages dampen wage variation over the business cycle, amplifying that in unemployment, with quantitatively significant effects. Second, I show that firm wage firms may find it profitable to fix wages for a period of time, and that an equilibrium with fixed wages can be good for worker welfare, despite added volatility in the labor market.
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1 Introduction

The work of Truman Bewley (1999), based on interviews of managers in corporate America, sketches a view of labor markets where employee compensation within firms is determined by formal internal pay structures. These structures seek to balance the dual goals of providing incentives on the one hand, and maintaining equity among a firm's employees on the other. The structure is described as a managerial tool emerging in a situation where employee productivity cannot be perfectly measured, but the pay of a large number of employees within the firm must be determined by their respective managers in a mutually consistent way, seeking to avoid favoritism between individuals. Managers believe their employees to be aware of pay differences, even if salaries are not made public, and that inequity antagonizes and embitters employees.[1] To shed light on the implications of such structures for labor market dynamics, this paper develops a macroeconomic theory of multi-worker firms in a frictional labor market that incorporates a notion of firm wages, and studies the consequences.

I study a labor market with search frictions and competitive search, where firms employ a measure of workers and pay equally productive workers the same. I begin by showing, in the context of a static model, that the equal treatment constraint changes the tradeoffs firms face in choosing a wage to offer. In standard competitive search, firms set wages trading off the increased hiring rates associated with higher wages against the increased wage costs involved with these hires. With firm wages, higher wages increase the wage costs involved with the firm's existing workforce as well, however, causing firms to set lower wages. With all firms affected, the equilibrium shifts toward lower wages in a way that hurts workers and benefits firms, encouraging vacancy creation and leading to overhiring in equilibrium.[2]

I then show, in the context of a dynamic infinite horizon model, that the firm's wage setting problem involves a time-inconsistency. The time-inconsistency arises due to the firm wage constraint, which limits the ability of the firm to differentiate among different cohorts of workers in pay, by promising all its workers an identical present value of wages at each point in time. In choosing these wages – assuming the firm has commitment – it effectively faces a less elastic labor supply in the short run, because its initial workforce is predetermined. As a consequence, the firm chooses lower wages in the short run, harvesting its existing workforce,

[1]Blinder and Choi (1990) also surveyed businesses on their wage setting practices, to shed light on the underpinnings of wage rigidities, and found managers to believe workers are concerned with how their wages compare with their peers'.

[2]The competitive search equilibrium with firm wages is inefficient, departing from the typical efficiency result in the absence of constraints discussed, e.g., by Rogerson, Shimer, and Wright (2005). The overhiring occurs for a different reason than in the literature on multi-worker firms with random search and bargaining, however, where it arises due to decreasing returns in technology (see, e.g., Smith (1999)).

but plans on higher wages later on.[3]

The path of wages characterized above requires commitment on the part of the firm, because if the firm were to reoptimize at a later date, it would generally depart from its plan by always choosing lower wages in the reoptimization period than planned. To consider outcomes when firms cannot commit to future wages, I study Markov perfect equilibria, also offering a tractable solution approach to the problem.

Analyzing Markov perfect equilibria in an environment with a time-inconsistency can be challenging because the decision-maker's objective does not coincide with maximizing his/her value function, which means that standard dynamic programming arguments cannot be directly applied. An approach that has been developed for characterizing differentiable Markov perfect equilibria involves deriving a *generalized* Euler equation, which spells out the tradeoffs faced by the decision-maker, as well as serves as a basis for solving the problem numerically. Solving the generalized Euler equation remains challenging, however, due to the dependence of this functional equation on the derivative of choice variables with respect to the state.[4] To avoid this complication, I look for equilibria that are consistent with the size-independence of the firm problem, which implies that the firm's decisions are independent of the relevant endogenous state – firm size – and thus a standard Euler equation approach can be used. In addition to simplifying solving the model, this approach allows incorporating stochastic shocks without difficulty.

I then use the model in two applications, considering the impact of firm wages on the cyclical behavior of wages and unemployment, as well as the profitability and equilibrium implications of infrequent wage adjustment. Intuitively, one would expect firm wages to dampen wage variation over the cycle, because when firms start raising wages to increase hiring in an expansion, the firm wage firm must raise them for existing workers as well as new hires. Moreover, firm wage firms may also find it profitable to fix wages for a period of time, because of the commitment problem.

To study the impact of firm wages on business cycle variation in wages and unemployment, I compare shock propagation in the firm wage model (without commitment) to the standard competitive search model. I show that wages in the firm wage model are less responsive

[3]The time-inconsistency of the dynamic firm problem is reminiscent of that in optimal capital taxation (Chamley 1986, Judd 1985) in that the firm prefers to tax labor (via low wages) more in the short run, where labor supply is less elastic.

[4]Time-inconsistencies appear in multiple contexts, due to either preferences directly or the economic environment, such as in problems of optimal fiscal or monetary policy. See Klein, Krusell, and Rios-Rull (2008) for a discussion on characterizing Markov perfect equilibria in problems with time-inconsistency, in the context of a study of optimal government spending.

to shocks, leading to amplification in the responses of unemployment and vacancy creation. The amplification is significant in magnitude, with a tenfold increase in the response of the vacancy-unemployment ratio to the shock relative to the standard model. Overall, this allows the model mechanism to explain roughly a third of the difference between model and data discussed in the literature (Shimer 2005).[5]

To study the profitability and equilibrium implications of infrequent wage adjustment, I extend the model to allow firms to commit to a simple wage rule of a fixed wage for a probabilistic period of time. In the context of this extended firm wage model, I show that a single firm deviating to a fixed wage when other firms reoptimize each period chooses a higher wage and grows faster, due to being more forward-looking. In particular, firm value increases as a result of the commitment, something that holds also in the presence of shocks, even though the fixed wage limits the firm's ability to respond to them.

Concluding that fixing the wage is profitable for firms, I then consider equilibrium outcomes when all firms fix wages for a probabilistic period of time, in a staggered way. I show that longer wage durations work to undo the equilibrium effects of firm wages. By making firms more forward-looking, they raise the level of wages, shifting the labor market equilibrium to make workers better off, while reducing overhiring, thus improving the efficiency of resource allocation. Moreover, these effects hold also in the presence of aggregate shocks, despite the added volatility in the labor market associated with longer wage durations. Thus, in an environment characterized by firm wage constraints, fixed wages may be welfare-improving, despite the seeming "rigidities" in the labor market.

Finally, the firm wage model also accommodates firm-specific idiosyncratic shocks. In a stationary equilibrium with firm heterogeneity, more productive firms offer higher wages and grow faster than less productive ones, giving rise to cross-sectional dispersion in wages and a large firm wage premium (Brown and Medoff 1989, Mortensen 2003). I show that firm wages dampen the responses of wages to firm shocks also, amplifying those of firm growth, and that the effects of fixed wages carry over to a setting with non-trivial firm risk as well.

Related Literature In terms of the applications, a key motivation for the work of Bewley (1999) was to understand why wages vary so little while unemployment varies so much over the business cycle. This long-standing puzzle re-emerged in the context of search and matching models of the labor market in the work of Hall (2005) and Shimer (2005) and a sizable literature that followed. This literature has sought mechanisms generating amplification in the responses of unemployment and vacancy creation to shocks, typically via rigidity

[5] Taking into account the effect of the convex vacancy costs that are part of the firm wage model.

in wages.[6] In an early contribution in this vein, Menzio (2005) also sought to think about the implications of firm wages for labor market dynamics. His work considers a random search model with on-the-job search where firms have private information about their productivity. The present paper, on the other hand, abstracts from on-the-job search and asymmetric information, focusing on a commitment problem emerging with firm wages and competitive search, and its implications.[7]

Meanwhile, the idea that wages are adjusted only intermittently is discussed by John Taylor (1999, 2016), with evidence going back to his own study of union wage contracts (Taylor 1983). He found that only 15 percent of workers saw contract adjustments each quarter, and only 40 percent each year. Recently, Barattieri, Basu, and Gottschalk (2014) revisited the question with broader data from the Survey of Income and Program Participation, finding a quarterly frequency of wage adjustment ranging from 12 to 27 percent, which implies an average duration of wages of 4 to 8 quarters. For European countries, Lamo and Smets (2009) report that 60 percent of firms changed wages once a year, and 26 percent less frequently, with an average duration of wages of 15 months. Clearly, wage adjustment is less frequent than the monthly, or even weekly, frequencies labor market flows are generally analyzed at.

In terms of related models, a set of papers have introduced multi-worker firms into search and matching models to study the impact on employment dynamics. Such models with random search include Smith (1999), Cooper, Haltiwanger, and Willis (2007), Elsby and Michaels (2013), Acemoglu and Hawkins (2014), and Fujita and Nakajima (2016), and ones with competitive/directed search are Hawkins (2013), Kaas and Kircher (2015), and Schaal (2017). In these papers production technologies feature decreasing returns to scale, with a focus on how such technologies affect labor market dynamics, while the present paper abstracts from decreasing returns to simplify solving the dynamic model with a time-inconsistency.[8]

The papers with random search effectively also feature firm wages, in the sense that all workers within a firm are paid the same wage, which is determined by a bargaining protocol. That literature does not discuss a related time-inconsistency in the dynamic firm problem, however. Among the papers with directed search, the most closely related paper is Kaas and

[6] See, e.g., Rogerson and Shimer (2011) for a discussion.

[7] Snell and Thomas (2010) also consider the implications of equity concerns for the cyclical behavior of wages, in a (non-search) framework where equity concerns combine with the motive of risk-neutral firms to insure risk-averse workers, resulting in wage rigidity.

[8] See also Rudanko (2011) for a model of multi-worker firms with competitive search, where firms post optimal long-term contracts.

Kircher (2015), who extend the competitive search equilibrium of Moen (1997) to a setting with multi-worker firms. Both in their work and here, firms post long-term wage contracts to attract workers. But while their contracts are independent across workers within a firm, here these wages are explicitly linked, imposing an additional restriction that leads to time-inconsistency.[9]

In a contribution also emphasizing equity in pay, Gertler and Trigari (2009) study labor market dynamics in a random search model where multi-worker firms pay all their workers the same and rebargain wages only when a Calvo-draw allows it. The imposed infrequent wage adjustment leads to amplification in the responses of unemployment and vacancy creation to shocks, because firms that have not yet adjusted pay pre-shock wages also to new hires, affecting vacancy creation. In a sense, the present paper offers a basis for such infrequent wage adjustment as an equilibrium outcome, due to the commitment problem, as well as suggesting that it may be welfare-improving in a firm wage environment.[10]

Firm wages are also part of the Burdett and Mortensen (1998) framework of on-the-job search and its dynamic extensions such as Moscarini and Postel-Vinay (2013, 2016). The latter consider equilibria where firms have full commitment to future wages, while Coles (2001) considers the implications of relaxing this assumption, revealing the complexity of the problem.[11]

Stepping back, the notion of firm wages is based on a literature in labor and personnel economics on internal labor markets (Doeringer and Piore 1971) that argues that worker compensation within firms is governed by administrative rules related to the internal hierarchy of positions within the firm, with wages determined more by job characteristics than those of workers. Baker, Gibbs, and Holmstrom (1994) offer an early case study of such a hierarchy, which they argue to be important for explaining compensation. While the hierarchy does not explain wages fully – there remains variation within levels of the hierarchy – it appears to go relatively far.[12] The present paper does not attempt to build a full model of such hierarchies, but rather takes the stand that levels of the hierarchy are subject to equity

[9]Limited commitment plays a role also in the competitive search models of Rudanko (2009) and Menzio and Moen (2010), who study labor market dynamics when firms post optimal long-term contracts. Krusell and Rudanko (2016) emphasize a time-inconsistency and related commitment problem arising when a union sets wages in a frictional labor market.

[10]In support of their assumption of equity in pay, Gertler and Trigari (2009) also provide evidence challenging a conventional view that the wages of job movers are more cyclical than those of stayers (Bils 1985). They argue that these findings are driven by cyclical variation in job quality and vanish with proper controls, an idea they pursue further in Gertler, Huckfeldt, and Trigari (2017).

[11]Other recent related work includes Coles and Mortensen (2016) and Gottfries (2018).

[12]In a similar spirit, Bayer and Kuhn (2018) argue that they explain the bulk of wage differences within firms based on their job-complexity based measures of hierarchy.

constraints or a degree of wage compression, and studies the consequences in a dynamic context.

More recently, the notion of equity among peers finds support also in an experimental literature arguing that relative pay concerns enter worker preferences and affect worker effort and output (Card, Mas, Moretti, and Saez 2012, Bracha, Gneezy, and Loewenstein 2015, Breza, Kaur, and Shamdasani 2018). According to this work, workers appear to prefer equal treatment with peers, unless productivity differences are sufficiently large and evident, with wage differences within the peer group reducing effort and output.

This paper is organized as follows. Section 2 begins with a one-period model to illustrate the static tradeoffs involved with firm wages, while Section 3 turns to a dynamic infinite horizon model to illustrate the time-inconsistency. Section 4 extends the baseline model to allow longer wage commitments/fixed wages. Section 5 considers the implications for business cycles in wages and unemployment, as well as the impact of infrequent wage adjustment, in a quantitative setting. Appendixes A-G contain proofs, a two-period model demonstrating the time-inconsistency in a simpler environment, details on the parametrization and solution methods, an extension to firm-level shocks, as well as additional figures.

2 Static Model

This section begins by considering the impact of firm wages in the context of a static, one-period model, before proceeding to the dynamic model in the next section.

Within a single period, consider a labor market with measure one workers, and a large number I firms. Each firm begins the period with n_i existing workers, for all $i \in I$. The total measure of matched workers in the beginning of the period is thus $N = \sum_{i \in I} n_i$, leaving $1 - N$ unmatched workers looking for jobs. All firms have access to a linear production technology with output z per worker, while workers who do not find jobs have access to a home production technology with output b $(< z)$ per worker.

In addition to their existing workers, firms can hire new workers in a frictional labor market. Firms seeking to hire must post vacancies, where posting v vacancies is subject to a convex cost $\kappa(v, n) = \hat{\kappa}(v/n)n$, where n is the firm's existing workforce and $\hat{\kappa}' > 0, \hat{\kappa}'' > 0$.[13] The search frictions in bringing these vacancies and unmatched workers together

[13]The convexity in the vacancy cost is introduced to help ensure that first order conditions characterize optimizing behavior, while the homothetic form plays an important role in allowing solving the dynamic model in a tractable way. Note that the derivatives $\kappa_v(v, n)$ and $\kappa_n(v, n)$ are functions of the ratio v/n only, and for expositional reasons I hence denote them as $\kappa_v(v/n)$ and $\kappa_n(v/n)$ in what follows.

are formalized with a matching function, with constant returns to scale, and I denote the probability a worker finds a job in a market with tightness θ as $\mu(\theta)$, with $\mu' > 0, \mu'' < 0$, and the probability a vacancy is filled as $q(\theta)$, where $\mu(\theta) = \theta q(\theta)$.

In posting vacancies firms also specify the wage that will apply to those jobs and take into account that the offered wage will affect their ability to fill vacancies. Specifically, they expect the measure of job applicants they attract to be such that job seekers are left indifferent between applying to this firm versus elsewhere, the latter yielding the equilibrium value of search U. Formally, given wage w_i, the tightness θ_i they expect to face is such that

$$U = \mu(\theta_i)w_i + (1 - \mu(\theta_i))b. \tag{1}$$

Here a worker applying to the firm finds a job with probability $\mu(\theta_i)$, attaining the wage w_i, and remains unmatched with probability $1 - \mu(\theta_i)$, attaining b. The firm takes the value of search U as given, because it is small relative to the market, but anticipates that by offering a higher wage it can attract more job applicants, via a lower tightness, which increases the probability its vacancies are filled, $q(\theta_i)$.

Each firm chooses a measure of vacancies and a wage to maximize its profits:

$$\max_{w_i,\theta_i,v_i} (n_i + q(\theta_i)v_i)(z - w_i) - \kappa(v_i, n_i), \tag{2}$$

taking as given n_i and condition (1). The profits reflect the firm's n_i existing workers and $q(\theta_i)v_i$ new hires all producing z units of output at the firm wage w_i, with vacancies subject to the vacancy cost $\kappa(v_i, n_i)$.

Note that this firm problem is independent of the firm's initial size. Defining the firm's rate of vacancy creation as $x_i := v_i/n_i$, one can scale and rewrite the problem as:

$$\max_{w_i,\theta_i,x_i} (1 + q(\theta_i)x_i)(z - w_i) - \hat{\kappa}(x_i), \tag{3}$$

taking as given (1). This means that heterogeneity in initial sizes across firms does not translate into differences in wages or vacancy rates, as well as that firm growth is independent of size (Gibrat's law holds). While larger firms do hire more, they do so only to the extent that initial differences in size are preserved. Assuming firms are equally productive, I thus drop the firm indexes on w_i, θ_i, x_i in what follows.

The firm's first order condition for vacancy creation,

$$\kappa_v(x) = q(\theta)(z - w), \tag{4}$$

states that the firm creates vacancies to a point where the marginal cost of an additional vacancy, on the left, equals the expected profits from the additional workers hired, on the right.

The firm's first order condition for the wage reads

$$1 + q(\theta)x = q'(\theta)x(z - w)g_w(w; U), \tag{5}$$

where I denote by $g(w; U)$ the tightness implied by wage w, as defined by constraint (1), and by g_w the corresponding derivative. The firm raises the wage to a point where the marginal increase in wage costs, on the left, equals the marginal increase in profits from greater vacancy filling rates, on the right. Note that the firm effectively faces a less elastic labor supply due to the influence of its existing workforce on this tradeoff: While the measure of new hires responds to the wage, the measure of existing workers is fixed, at least as long as the wage does not fall below b.[14]

The firm wage policy is embodied in the single wage appearing in problem (2). In the absence of such constraints, the firm problem would instead read:

$$\max_{w_i, \theta_i, v_i} n_i(z - w_i^e) + q(\theta_i)v_i(z - w_i) - \kappa(v_i, n_i), \tag{6}$$

subject to constraint (1) on the hiring wage, and where I denote the average wage of existing workers by w_i^e.[15]

The first order conditions for this unconstrained firm problem include the same condition for optimal vacancy creation as for the firm wage firm (4), together with the condition for the optimal wage-tightness tradeoff:

$$q(\theta)x = q'(\theta)x(z - w)g_w(w; U). \tag{7}$$

Here the firm again raises the wage to a point where the marginal increase in wage costs equals the increase in profits from greater vacancy filling rates. As the wage increase now applies to new hires only, however, the unconstrained firm effectively faces a more elastic labor supply.

Definition 1. *A competitive search equilibrium with firm wages is an allocation* $\{w, \theta, x\}$ *and value of search* U *such that the allocation and value solve the problem* (3) *with each job seeker applying to one firm:* $1 - N = xN/\theta$.

[14] The firm wage firm could in some circumstances also prefer the corner solution of opting out of hiring altogether, while paying its existing workers the minimum to keep them, by setting $v_i = 0, w_i = b$. I focus on interior solutions characterised by first order conditions in what follows, checking in the quantitative exercises that the firm values dominate deviating to such a corner.

[15] If the firm has commitment, the wages of existing workers are predetermined. If it does not, the firm optimally sets them to equal b, retaining the workers at minimum compensation. Either way, hiring is unaffected.

The level effects of firm wages on equilibrium outcomes are summarized in the following proposition:

Proposition 1. *With sufficient curvature in the vacancy cost, the competitive search equilibrium with firm wages satisfying* (1), (4), (5), *and* $1 - N = xN/\theta$ *is unique, with a strictly lower wage* w *and higher tightness* θ*, as well as strictly greater vacancy creation and employment, than without the firm wage policy.*

Intuitively, as firm wage firms face a less elastic labor supply, they tend to choose lower wages. With all firms affected, this downward pressure on wages causes the equilibrium to shift toward lower wages in a way that encourages vacancy creation and hiring, leading to higher employment. Given that the competitive search equilibrium without firm wages is known to be efficient, it follows that the firm wage equilibrium is inefficient, featuring overhiring.[16]

Similar effects emerge in the dynamic model, where the measure of existing matches is endogenous. I turn to this dynamic model next.

3 Dynamic Model

This section extends the firm wage model to a dynamic infinite-horizon setting, revealing a time-inconsistency in the corresponding firm problem. I begin by assuming firms have commitment to future choices, before turning to the case where firms reoptimize each period, considering Markov perfect equilibria. I also relate equilibrium outcomes to those without firm wages, as well as what a benevolent planner would choose.[17]

3.1 Firm Wages

Time is discrete and the horizon infinite. All agents are rational and discount the future at rate β. Each period a large number I firms inherit a measure of existing workers n_{it} from the previous period and hire new ones in a frictional labor market. Employment relationships are long term and end at the end of each period with probability δ. Labor productivity z_t is stochastic, following a Markovian process.

In posting vacancies, firms now specify a full state-contingent wage contract that will apply to those jobs. Given a contract $\{w_{it+k}\}_{k=0}^{\infty}$ offered in period t, the tightness θ_{it} firms

[16] Rogerson, Shimer, and Wright (2005) discuss the efficiency of the competitive search equilibrium.

[17] For a two-period version of the model that illustrates the time-inconsistency in a simpler setting, see Appendix B.

expect to face is such that

$$U_t = \mu(\theta_{it}) E_t \sum_{k=0}^{\infty} \beta^k (1-\delta)^k (w_{it+k} + \beta\delta U_{t+1+k}) + (1-\mu(\theta_{it}))(b + \beta E_t U_{t+1}). \tag{8}$$

Here a worker applying to the firm in period t finds a job with probability $\mu(\theta_{it})$, subsequently receiving the specified wages until a separation returns him to job search, and remains unmatched with probability $1-\mu(\theta_{it})$, receiving b and continuing to search in the following period. Each firm, again, takes the values of search $\{U_{t+k}\}_{k=0}^{\infty}$ as given, but anticipates that by offering a better contract it can attract more job applicants, via a lower tightness, which increases the probability its vacancies are filled.

Note that as far as wages are concerned, what workers care about is the expected present value $E_t \sum_{k=0}^{\infty} \beta^k (1-\delta)^k w_{it+k}$ rather than the details of how these wages are paid out. Given this, it is convenient to rewrite constraint (8) as

$$X_t = \mu(\theta_{it})(W_{it} + Y_t), \tag{9}$$

where W_{it} represents the present value of wages and I define the variables $X_t := U_t - b - \beta E_t U_{t+1}$ and $Y_t := E_t \beta\delta \sum_{k=0}^{\infty} \beta^k (1-\delta)^k U_{t+1+k} - b - \beta E_t U_{t+1}$. By way of interpretation, X_t represents the option value of search and Y_t the value of forgone home production and search during employment.[18] Recall that because firms take the equilibrium values of search $\{U_t\}_{t=0}^{\infty}$ as given, they also take as given $\{X_t, Y_t\}_{t=0}^{\infty}$.

Assuming commitment to future choices, each firm then solves the sequence problem

$$\max_{\{w_{it},\theta_{it},v_{it}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t [(n_{it} + q(\theta_{it})v_{it})(z_t - w_{it}) - \kappa(v_{it}, n_{it})] \tag{10}$$

$$\text{s.t. } n_{it+1} = (1-\delta)(n_{it} + q(\theta_{it})v_{it}), \ \forall t \geq 0, \tag{11}$$

$$X_t = \mu(\theta_{it}) E_t (\sum_{k=0}^{\infty} \beta^k (1-\delta)^k w_{it+k} + Y_t), \ \forall t \geq 0, \tag{12}$$

taking as given n_{i0} and $\{X_t, Y_t\}_{t=0}^{\infty}$. Firms maximize the expected present value of profits, where in each period t the firm's existing and new workers produce z_t units of output at the firm wage w_{it}, with vacancies subject to the vacancy cost $\kappa(v_{it}, n_{it})$. In doing so, they take as given the law of motion for their workforce (11), and the constraint determining the tightnesses corresponding to the contracts offered (12).[19]

[18] To make the interpretations more evident, the equations can be rewritten as: $U_t = b + \beta E_t U_{t+1} + X_t$ and $Y_t = -b - E_t \sum_{k=1}^{\infty} \beta^k (1-\delta)^k (b + X_{t+k})$.

[19] As noted in the context of the static model, I focus on equilibria where firms hire each period and (12) thus always holds. There are circumstances in which firms could prefer to opt out of hiring completely, however, paying their existing workers such low wages as to make them indifferent between remaining employed and quitting to look for a new job (or lower, if possible). I discuss this possibility in Appendix A and provide checks in the quantitative exercises to make sure that such a deviation would not appear profitable for firms.

Because the firm is constrained to pay all its workers the same each period, the contracts offered at different points in time must be consistent with each other. The constraints imply that the expected present value of wages is equated across workers within a firm at each point in time. This does not prevent the firm from offering different present values today and tomorrow, as the hiring period wage today does not enter the present value tomorrow, but implementing a specific plan of present values over time does pin down the timing of wage payments.

For solving this firm problem, it is convenient to note that one can reduce the dimensionality of the problem significantly by substituting wages out with the help of constraint (12). With this, the firm problem becomes:

$$\max_{\{\theta_{it},v_{it}\}_{t=0}^{\infty}} -\frac{n_{i0}X_0}{\mu(\theta_{i0})}+E_0\sum_{t=0}^{\infty}\beta^t[(n_{it}+q(\theta_{it})v_{it})(z_t-b)-\kappa(v_{it},n_{it})-X_t(\frac{v_{it}}{\theta_{it}}+n_{it})] \tag{13}$$
$$\text{s.t. } n_{it+1}=(1-\delta)(n_{it}+q(\theta_{it})v_{it}),\ \forall t\geq 0,$$

taking as given n_{i0} and $\{X_t\}_{t=0}^{\infty}$.

Formally, we have the result:

Proposition 2. *Problem* (10) *is equivalent to problem* (13) *if the firm hires each period.*

This formulation makes it clear that the initial period is different from later periods, with the firm's existing workforce in the initial period playing a role. The difference makes sense if one thinks of the firm as choosing a sequence of present values of wages to offer new hires, where the firm wage constraint imposes the same value to apply to existing workers at each point in time as well. In choosing the initial period value, the firm effectively faces a less elastic labor supply due to the influence of its existing workforce: While the measure of new hires responds to the value, the measure of existing workers is fixed. In making plans for future hiring, on the other hand, the firm does not view its future workforce as exogenous, but instead internalizes it and its influence on subsequent hiring. Hence, the tradeoffs the firm faces in the initial period are different from later periods – indicating time-inconsistency.

The firm's first order conditions reflect the above difference between initial and later periods when it comes to the wage-tightness tradeoff, but the asymmetry does not apply to vacancy creation and hence the first order conditions for vacancy creation are the same for all periods $t\geq 0$:

$$\kappa_v(x_{it})+\frac{X_t}{\theta_{it}}=q(\theta_{it})E_t[z_t-b+\sum_{k=1}^{\infty}\beta^k(1-\delta)^k(z_{t+k}-b-X_{t+k}-\kappa_n(x_{it+k}))]. \tag{14}$$

Here the left hand side represents the marginal costs of additional vacancies, consisting of the vacancy costs and equilibrium value of search X_t the firm must deliver to job applicants, while the right hand side represents the marginal surpluses associated with those vacancies. Vacancies are filled at rate $q(\theta_t)$, with those hires producing at the market productivity instead of remaining at home and continuing to search, as well as reducing the cost of vacancy creation going forward.

The first order condition for the wage-tightness tradeoff in the initial period reads

$$\frac{X_0}{\theta_0^2} + \frac{\mu'(\theta_{i0})X_0}{x_{i0}\mu(\theta_{i0})^2} = -q'(\theta_{i0})E_0[z_0 - b + \sum_{k=1}^{\infty}\beta^k(1-\delta)^k(z_k - b - X_k - \kappa_n(x_{ik}))]. \tag{15}$$

The left hand side now represents the marginal costs of an increase in applicants per vacancy, consisting of the value of search the firm must deliver to those additional applicants, as well as the losses the firm incurs on its existing workers when raising the firm wage to attract more applicants. The right hand side represents the marginal surpluses from an increase in applicants per vacancy, reflecting an increase in the rate at which vacancies are filled.

For future periods $t > 0$, the first order condition reduces to

$$\frac{X_t}{\theta_{it}^2} = -q'(\theta_{it})E_t[z_t - b + \sum_{k=1}^{\infty}\beta^k(1-\delta)^k(z_{t+k} - b - \kappa_n(x_{it+k}) - X_{t+k})]. \tag{16}$$

The costs of an increase in applicants per vacancy, on the left, now amount to only the value of search the firm must deliver to those applicants. While the firm again incurs a loss on its existing workers when raising the firm wage, this does not enter here because the firm does not treat its future workforce as exogenous.

As in the static model, firm wage firms thus face a less elastic labor supply due to the influence of their existing workforce on the wage-tightness tradeoff, resulting in downward pressure on wages. If the firm has commitment, this affects only the initial period, however.

Note that conditions (14)-(16) are again identical across firms, because also the dynamic firm problem is independent of the firm's initial size. I hence drop the firm indexes in what follows.

Proposition 3. *The firm wage firm problem* (10) *is independent of the firm's initial size.*

For understanding the evolution of allocations, as well as in preparation for relaxing the assumption of commitment, it is useful to consider Euler equations written in terms of allocations. For future periods $t > 0$, combining the optimality conditions above yields the inter-temporal Euler equation

$$\frac{\kappa_v(x_t)}{\mu'(\theta_t)} = z_t - b + \beta(1-\delta)E_t[(1-\mu(\theta_{t+1}) + \mu'(\theta_{t+1})\theta_{t+1})\frac{\kappa_v(x_{t+1})}{\mu'(\theta_{t+1})} - \kappa_n(x_{t+1})]. \tag{17}$$

Here the marginal costs of creating new matches today, $\kappa_v(x_t)/\mu'(\theta_t)$, are equated to the flow surpluses from those matches together with the expected present value of the match tomorrow, $\kappa_v(x_{t+1})/\mu'(\theta_{t+1})$, as well as the resulting decrease in vacancy costs. The expectations take into account the probability of a separation, as well as that an increase in hires today reduces hires per vacancy tomorrow, by reducing the measure of unmatched workers.

For the initial period, the influence of the firm's existing workforce introduces a wedge into the corresponding Euler equation, on the left hand side,

$$\frac{\kappa_v(x_0)}{\mu'(\theta_0)}[1 - \frac{1-\mu'(\theta_0)\theta_0/\mu(\theta_0)}{1+q(\theta_0)x_0}] = z_0 - b + \beta(1-\delta)E_0\big[\big(1-\mu(\theta_1)+\mu'(\theta_1)\theta_1\big)\frac{\kappa_v(x_1)}{\mu'(\theta_1)} - \kappa_n(x_1)\big], \tag{18}$$

indicating that the effective cost of creating matches is lower in the initial period than in later periods (consistent with lower equilibrium wages in the initial period).

Similar Euler equations can be derived for firms that reoptimize each period as well, with related wedges appearing. I turn to this problem next.

Reoptimizing firms Due to the time-inconsistency, firms must have commitment to future choices to implement the above plans. But what if they instead expect to reoptimize each period? Ultimately this seems like the more natural specification to consider.

To think about the case without commitment, I consider Markov perfect equilibria, where the firms' choices each period depend on the set of payoff-relevant state variables in the firm problem. In particular, I focus on equilibria that are consistent with the size-independence of the firm problem – an approach that simplifies solving the problem significantly.[20]

Suppose the current aggregate state is denoted as $S := (N, z)$. The firm problem can then be written recursively, based on the sequence problem (13), as:

$$\max_{\theta,v} -\frac{nX(S)}{\mu(\theta)} + (n+q(\theta)v)(z-b) - \kappa(v,n) - X(S)(\frac{v}{\theta}+n) + \beta E_S V(n';S') \tag{19}$$
$$\text{s.t. } n' = (1-\delta)(n+q(\theta)v),$$

with the accounting equation

$$V(n;S) = (n+q(\theta)v)(z-b) - \kappa(v,n) - X(S)(\frac{v}{\theta}+n) + \beta E_S V(n';S'). \tag{20}$$

[20]One could also consider equilibria allowing for a richer dependence on histories. The present paper focuses on Markov perfect ones, however, making use of the size-independence to gain tractability.

Here the firm's existing workforce influences decision making in each period by introducing the term $-nX/\mu(\theta)$ into the objective (19), as it did in the initial period of the sequence problem. In line with the sequence problem, the term does not enter the accounting equation keeping track of continuation values (20), however.

This recursive firm problem also scales by size, leading to a firm problem with no endogenous firm-level state variable. Defining $\hat{V}(S) := V(n; S)/n$ and using the law of motion for the firm's workforce, scaling by n yields the firm problem

$$\max_{\theta,x} -\frac{X(S)}{\mu(\theta)} + (1+q(\theta)x)(z-b+\beta(1-\delta)E_S\hat{V}(S')) - \hat{\kappa}(x) - X(S)(\frac{x}{\theta}+1) \tag{21}$$

with the accounting equation

$$\hat{V}(S) = (1+q(\theta)x)(z-b+\beta(1-\delta)E_S\hat{V}(S')) - \hat{\kappa}(x) - X(S)(\frac{x}{\theta}+1). \tag{22}$$

Note that while the firm's existing workforce continues to influence firm behavior here, the magnitude of the effect does not depend on firm size, as everything scales.

The firm's first order conditions for vacancy creation and the wage-tightness tradeoff read:

$$\kappa_v(x) + \frac{X(S)}{\theta} = q(\theta)(z-b+\beta(1-\delta)E_S\hat{V}(S')), \text{ and} \tag{23}$$

$$\frac{X(S)}{\theta^2}[1+\frac{\mu'(\theta)\theta^2}{x\mu(\theta)^2}] = -q'(\theta)(z-b+\beta(1-\delta)E_S\hat{V}(S')), \tag{24}$$

respectively. While the optimality condition for vacancy creation coincides with that for the commitment firm in general, the condition for the wage-tightness tradeoff now coincides with that for the initial period of the commitment firm problem – in all periods. The reoptimizing firm thus views its labor supply as less elastic in all periods.

The optimality conditions can be combined with the accounting equation to arrive at the inter-temporal Euler equation characterizing allocations for all periods $t \geq 0$[21]

$$\begin{aligned} &\frac{\kappa_v(x_t)}{\mu'(\theta_t)}[1-\frac{1-\frac{\mu'(\theta_t)\theta_t}{\mu(\theta_t)}}{1+q(\theta_t)x_t}] = z_t - b \\ &+\beta(1-\delta)E_t\left\{\frac{\kappa_v(x_{t+1})}{\mu'(\theta_{t+1})}[1-\mu(\theta_{t+1})+\mu'(\theta_{t+1})\theta_{t+1}-(1-\mu(\theta_{t+1}))\frac{1-\frac{\mu'(\theta_{t+1})\theta_{t+1}}{\mu(\theta_{t+1})}}{1+q(\theta_{t+1})x_{t+1}}] - \kappa_n(x_{t+1})\right\}. \end{aligned} \tag{25}$$

Relative to the Euler equations of the commitment firm, the reoptimizing firm's Euler equation features wedges on both the left and right sides of the equation, as the firm's existing

[21]Note that equation (22) can be written as $\hat{V} = z - b - \kappa_n(x) - X + \beta(1-\delta)E\hat{V}'$, using (23) and $\kappa_n(x) = \hat{\kappa}(x) - \kappa_v(x)x$.

workforce influences firm behavior in all periods, with the firm anticipating this to happen going forward.

Importantly, note that equation (25) takes the form of a standard Euler equation, instead of the *generalized* Euler equations that typically appear in problems with time-inconsistencies (see, e.g., Klein, Krusell, and Rios-Rull (2008)). Such generalized Euler equations generally involve derivatives of choice variables with respect to an endogenous state variable – reflecting the decision-maker taking into account the effects of his/her choices on the magnitude of future biases – something that makes the Euler equation a more complicated object to analyze than standard Euler equations.[22] Here the focus on size-independent behavior eliminates such a dependence.

Finally, defining an equilibrium where firm wage firms reoptimize each period, we have:

Definition 2. *A competitive search equilibrium with firm wages is an allocation* $\{w_t, \theta_t, x_t\}_{t=0}^{\infty}$ *and job seeker values* $\{X_t\}_{t=0}^{\infty}$ *such that the allocation and values solve the problem* (21)*, with each job seeker applying to one firm:* $1 - N_t = x_t N_t/\theta_t$, $\forall t$.

Overall, the core of the firm wage equilibrium reduces to a simple three-equation dynamic system in endogenous variables θ_t, x_t, N_t given by the Euler equation (25), law of motion $N_{t+1} = (1-\delta)(N_t + \mu(\theta_t)(1-N_t))$, and adding up constraint $1 - N_t = x_t N_t/\theta_t$, together with an initial value for N_0 and process for shocks.

3.2 Standard Competitive Search and Efficient Allocations

As a benchmark for firm wage firms, this section considers unconstrained firms, as well as what a benevolent planner would choose.

In the absence of firm wage constraints, the problem of a firm deciding on hiring in any period $t \geq 0$ can be written almost independently of hiring at other points in time as:

$$\max_{\{w^t_{it+k}\}_{k=0}^{\infty}, \theta_{it}, v_{it}} E_t[q(\theta_{it})v_{it}\sum_{k=0}^{\infty}\beta^k(1-\delta)^k(z_{t+k} - w^t_{it+k}) - \sum_{k=0}^{\infty}\beta^k\kappa(v_{it+k}, n_{it+k})] \tag{26}$$

$$\text{s.t. } X_t = \mu(\theta_{it})E_t(\sum_{k=0}^{\infty}\beta^k(1-\delta)^k w^t_{it+k} + Y_t), \tag{27}$$

where n_{it} is given. Here the first term in (26) represents the present value of output net of wages associated with workers hired in period t, while the second the costs of creating the

[22]Even solving for a steady state is non-trivial: One cannot simply evaluate the Euler equation in steady state and solve, because the derivative introduces an additional unknown.

vacancies together with the influence of the hiring on vacancy costs in the future, assuming the workforce follows the corresponding law of motion over time.

Substituting out wages and aggregating up across cohorts hired at different points in time, the full firm problem can further be written as[23]

$$\max_{\{\theta_{it},v_{it}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t [(n_{it} + q(\theta_{it})v_{it})(z_t - b) - \kappa(v_{it}, n_{it}) - X_t(\frac{v_{it}}{\theta_{it}} + n_{it})]$$
$$\text{s.t. } n_{it+1} = (1-\delta)(n_{it} + q(\theta_{it})v_{it}), \ \forall t \geq 0.$$

Comparing this problem with the firm wage problem in (13) reveals the two to be identical except for the initial period, with the unconstrained firm's first order conditions coinciding with (14) and (16) for all $t \geq 0$ (absent an initial period wedge). Similarly, the Euler equation characterizing allocations coincides with that in (17) for all $t \geq 0$.

The key difference between the constrained and unconstrained problems is that the firm wage firm faces a less elastic labor supply in the optimization period where its existing workforce is given, due to the firm wage constraint. If the firm has commitment to future choices, it does not treat its future workforce as given, however, meaning that while the tradeoffs the firm faces are affected in the initial period they otherwise coincide with those of the unconstrained firm. If the firm does not have commitment, on the other hand, reoptimization occurs each period with the tradeoffs affected each time.

In the unconstrained case, allocations do not depend on whether the firm has commitment to future wages, because the tradeoffs the firm faces in hiring are effectively independent over time when the firm is free to choose wages independently across cohorts.[24]

Meanwhile, a planner maximizing the expected present value of output would allocate resources according to:

$$\max_{\{\theta_{it},v_{it}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \sum_{i\in I} [(n_{it} + q(\theta_{it})v_{it})z_t - \kappa(v_{it}, n_{it})] + [1 - \sum_{i\in I}(n_{it} + q(\theta_{it})v_{it})]b \right\} \quad (28)$$
$$\text{s.t. } n_{it+1} = (1-\delta)(n_{it} + q(\theta_{it})v_{it}), \ \forall i \in I, t \geq 0,$$
$$\sum_{i\in I} v_{it}/\theta_{it} = 1 - \sum_{i\in I} n_{it}, \ \forall t \geq 0, \quad (29)$$

[23] See Appendix A.

[24] Commitment does matter for the timing of wage payments over the course of an employment relationship, but without affecting the present value of wages the firm can offer new hires, and hence without affecting allocations. If the firm has commitment, it is left indifferent across a variety of alternative ways of paying out any desired present value of wages. But even if it does not, the firm should still be able to implement the same present value with an appropriate wage payment in the hiring period, even if subsequent wages are low enough to make the worker indifferent between remaining employed and quitting to search for a new job. Having commitment within the hiring period is thus enough.

with n_{i0} given for all producers i. The planner maximizes the present value of producer and home output net of the costs of vacancy creation, taking into account the law of motion for the workforce of each producer. In addition, the planner's choices of θ_{it}, v_{it} must be consistent with the measure of unmatched workers each period, as the job seekers allocated to each producer, v_{it}/θ_{it}, must add up to the latter.

Reorganizing terms and including the adding up constraints (29) with corresponding Lagrange multipliers λ_t, the planner objective becomes

$$E_0 \sum_{i \in I} \sum_{t=0}^{\infty} \beta^t [(n_{it} + q(\theta_{it})v_{it})(z_t - b) - \kappa(v_{it}, n_{it}) - \lambda_t(\frac{v_{it}}{\theta_{it}} + n_{it})].$$

Comparing this objective with that of the unconstrained firm reveals the two to be closely related, as the producer-level objective the planner faces is essentially the same as the firm's, with the equilibrium value of search replaced by the shadow value of job seekers. This is consistent with the conclusion of Kaas and Kircher (2015) that the unconstrained competitive search equilibrium allocations are efficient.

Note that it follows that the firm wage equilibrium is inefficient (with or without commitment) due to the influence of the firm's existing workforce on the wage-tightness tradeoff.

4 Infrequent Wage Adjustment

The commitment problem suggests that firms may find it profitable to adopt rules governing their wage setting, rather than optimizing continually. A basic rule firms could consider is simply fixing a wage for a period of time, to be reoptimized only from time to time. In addition to the benefits of the commitment, such a rule would involve costs as well, of course, in preventing firms from responding to shocks.

To explore the idea further, this section extends the model to allow firms to fix wages for a probabilistic period of time. I begin by considering the profitability of such a rule from an individual firm's perspective, before turning to an equilibrium where all firms adopt them.

4.1 Single Firm Deviation to Longer Wage Commitment

Consider a competitive search equilibrium with firm wages, where firms choose wages and vacancy creation each period in the face of aggregate shocks to labor productivity, and where a single firm contemplates a deviation to a fixed wage for a probabilistic period of time.

Recall that the competitive search equilibrium allows individual firms to contemplate deviations from equilibrium behavior by offering an alternative contract, with the ensuing market tightness determined by the job seeker constraint based on the present value of wages. To formally connect a wage and ensuing tightness(es), note that for a firm that deviates to fixed wage w, expecting to revert to equilibrium behavior each period with probability α, the present value of wages is given by

$$\phi(w,S) = \frac{w}{1-\beta(1-\delta)(1-\alpha)} + \Lambda(S), \tag{30}$$

where $\Lambda(S) = E_S \sum_{k=0}^{\infty} \beta^k (1-\delta)^k (1-\alpha)^k \beta(1-\delta)\alpha W(S^{k+1})$ represents the present value associated with reverting to equilibrium present values, denoted $W(S)$. The tightness(es) that prevail during the deviation are then determined by the job seeker constraints $X(S) = \mu(\theta)(\phi(w,S) + Y(S))$ each period.

With this, the problem of the deviating firm is to choose the wage w and vacancy creation v to maximize

$$-\frac{nX(S)}{\mu(\theta)} + (n+q(\theta)v)(z-b) - \kappa(v,n) - X(S)(\frac{v}{\theta}+n) + \beta E_S(\alpha V(n',S') + (1-\alpha)V^f(n',w,S'))$$
$$\text{s.t. } n' = (1-\delta)(n+q(\theta)v),$$
$$X(S) = \mu(\theta)(\phi(w,S)+Y(S)),$$

given n, S. The objective is the same as for firms reoptimizing each period in (19) except for the continuation values, where the firm now attains the equilibrium value $V(n',S')$ only if it reverts immediately after the deviation period and a value $V^f(n',w,S')$ of holding the wage fixed otherwise. The choice of wage has more long-lived effects here, requiring firms to be more forward-looking.

Meanwhile, in subsequent periods while the deviation lasts, the firm only chooses vacancies v to maximize:

$$(n+q(\theta)v)(z-b) - \kappa(v,n) - X(S)(\frac{v}{\theta}+n) + \beta E_S(\alpha V(n',S') + (1-\alpha)V^f(n',w,S'))$$
$$\text{s.t. } n' = (1-\delta)(n+q(\theta)v),$$
$$X(S) = \mu(\theta)(\phi(w,S)+Y(S)),$$

given n, w, S, and where the maximized value determines $V^f(n,w,S)$.

These firm problems, again, scale with size, and I maintain the focus on size-independent behavior in what follows. The first order condition for the deviation wage reads

$$\frac{X(S)}{\theta^2}[1+\frac{\mu'(\theta)\theta^2}{x\mu(\theta)^2}] = -q'(\theta)[z-b+\beta(1-\delta)E_S[\alpha\hat{V}(S') + (1-\alpha)\hat{V}^f(w,S')]] \tag{31}$$
$$-\beta(1-\delta)(1-\alpha)(1+q(\theta)x)/xE_S\hat{V}^f_w(w,S')/\theta_w,$$

where $\hat{V}^f(w,S) := V^f(n,w,S)/n$ and the final term reflects the impact of the deviation wage on future profits,[25] while the first order condition for vacancy creation remains the same throughout the deviation:

$$\kappa_v(x) + \frac{X(S)}{\theta} = q(\theta)(z - b + \beta(1-\delta)E_S(\alpha\hat{V}(S') + (1-\alpha)\hat{V}^f(w,S'))). \tag{32}$$

The differences between the optimality conditions of the deviating firm (with $\alpha < 1$) and equilibrium firms (with $\alpha = 1$) indicate that the deviating firm will choose a different wage and grow at a different rate from the rest. Section 5 demonstrates these differences, and how they depend on the duration of the wage, in the context of a parameterized model. I will show that deviating raises firm value, making it interesting to consider the equilibrium implications of all firms adopting such rules. I turn to this problem next.

4.2 Equilibrium with Infrequent Adjustment

Consider an equilibrium where each firm reoptimizes its wage with probability α each period, and otherwise holds it fixed. I assume reoptimization shocks are independent across firms, so that wage adjustment is staggered. In this context, the equilibrium will generally feature a distribution of wages, which becomes part of the aggregate state S.

The problem of a reoptimizing firm is to choose a wage w and vacancy creation v to maximize

$$-\frac{nX(S)}{\mu(\theta)} + (n + q(\theta)v)(z-b) - \kappa(v,n) - X(S)(\frac{v}{\theta} + n) + \beta E_S(\alpha V^r(n',S') + (1-\alpha)V^f(n',w,S'))$$

s.t. $n' = (1-\delta)(n + q(\theta)v)$,

$X(S) = \mu(\theta)(\phi(w,S) + Y(S))$,

given n, S, and where the implied continuation value satisfies the accounting equation

$$V^r(n,S) = (n + q(\theta)v)(z-b) - \kappa(v,n) - X(S)(\frac{v}{\theta} + n) + \beta E_S(\alpha V^r(n',S') + (1-\alpha)V^f(n',w,S')).$$

[25]From the job seeker constraint, the change in tightness associated with a change in the wage is given by $\theta_w = -\mu(\theta)^2/(\mu'(\theta)X(1-\beta(1-\delta)(1-\alpha)))$, whereas the corresponding change in the continuation value satisfies

$$\begin{aligned}\hat{V}^f_w(w,S) =& xq'(\theta)[z - b + \beta(1-\delta)E_S[\alpha\hat{V}(S') + (1-\alpha)\hat{V}^f(w,S')]]\theta_w \\ &+ \frac{xX(S)}{\theta^2}\theta_w + \beta(1-\delta)(1-\alpha)(1 + q(\theta)x)E_S\hat{V}^f_w(w,S').\end{aligned}$$

Note that the firm objective is the same as for the deviating firm except for the continuation values, where once the wage expires the firm now attains the equilibrium value of choosing a new fixed wage $V^r(n', S')$.[26]

Similarly, in subsequent periods while the wage remains fixed, the firm only chooses vacancies v to maximize

$$
\begin{aligned}
&(n + q(\theta)v)(z - b) - \kappa(v, n) - X(S)(\frac{v}{\theta} + n) + \beta E_S(\alpha V^r(n', S') + (1 - \alpha)V^f(n', w, S')) \\
&\text{s.t. } n' = (1 - \delta)(n + q(\theta)v), \\
&\qquad X(S) = \mu(\theta)(\phi(w, S) + Y(S)),
\end{aligned}
$$

given n, w, S, and where the maximized value determines $V^f(n, w, S)$.

The firm problems again scale with size, as well as yielding first order conditions that coincide with those for the deviating firm (31)-(32), except with continuation values $\hat{V}^r(S) := V^r(n, S)/n$ and $\hat{V}^f(w, S) := V^f(n, w, S)/n$ that now correspond to an equilibrium with fixed wages. As noted above, the fact that the optimality conditions of firms fixing wages differ from those of firms reoptimizing each period suggests differences in firm behavior – something that will likely affect these equilibrium values as well. I turn to illustrating the impact of fixed wages on equilibrium outcomes and welfare in the following section.

5 Quantitative Illustration

This section uses the model to study the implications of firm wages for labor market outcomes: first, how firm wages affect the responses of wages and hiring to aggregate shocks, and then, the profitability and equilibrium impact of infrequent wage adjustment. I begin with an environment where firms face aggregate shocks to labor productivity, but also discuss firm-level shocks toward the end of the section.

5.1 Parameterizing and Solving the Model

I begin with a parametrization and discussion of the solution approach, before turning to results.

Parametrization I adopt a monthly frequency, set the discount rate to $\beta = 1.05^{-1/12}$, and normalize steady-state labor productivity to $z = 1$. To be consistent with an average duration

[26]Similarly, the function ϕ is as defined in (30), but with $\Lambda(S) = E_S \sum_{k=0}^{\infty} \beta^k (1 - \delta)^k (1 - \alpha)^k \beta (1 - \delta)\alpha W^r(S^{k+1})$ where $W^r(S)$ now corresponds to equilibrium values of choosing a new fixed wage.

of employment of 2.5 years, I set the separation rate to $\delta = 0.033$. To then be consistent with an average unemployment rate of 5 percent, when steady-state unemployment in the model is $\mu(\theta)(1-\delta)/(\delta + \mu(\theta)(1-\delta))$, requires a steady-state job-finding rate of $\mu(\theta) = 0.388$. I adopt the matching function $m(v,u) = vu/(v^\ell + u^\ell)^{1/\ell}$ for this discrete time model, as in den Haan, Ramey, and Watson (2000), and target a steady-state level of θ of 0.43, as in Kaas and Kircher (2015). With this, fitting the above job finding probability requires $\ell = 1.85$. Finally, I follow Kaas and Kircher (2015) in adopting the vacancy cost $\kappa(v,n) = \frac{\kappa_0}{1+\gamma}(v/n)^\gamma v$ with $\gamma = 2$. This leaves two remaining free parameters, κ_0 and b.

To then arrive at comparable parametrizations for the firm wage model and the standard competitive search model, I begin with a benchmark parametrization for the latter by following Shimer (2005) in adopting the value $b = 0.4$ and setting κ_0 such that the corresponding Euler equation holds in steady state.

For a parametrization of the firm wage model allowing a fair comparison of shock propagation across models, I seek alternative values of κ_0, b that hold the steady-state profitability of hiring unchanged across models. It turns out that for the two models to yield identical levels for wages and hence the firm profit rate, one must hold the value of κ_0 unchanged across models (see Appendix C for details). I thus adopt the same value of κ_0 for the firm wage model, setting b such that the firm wage model's Euler equation (25) holds. Doing so involves raising the value of b relative to the standard model, to bring wages in the firm wage model to their levels in the standard model, to $b = 0.89$.

Solution Approach The baseline firm wage model where firms reoptimize each period is relatively straightforward to solve, as the equilibrium conditions reduce to a set of nonlinear difference equations that can be solved with standard methods, such as Dynare.[27] The complete system of equations is provided in Appendix D. In solving the model, I also check that the solution characterized by the first order conditions dominates the corner solution of opting out of hiring for a period: zero vacancies and a low wage making existing workers indifferent between remaining employed and quitting to search for a new job (see Appendix A for a discussion). These checks can be found in Appendix G.

The extension to infrequent wage adjustment has two parts: the single deviating firm fixing its wage and the equilibrium with fixed wages. Solving the first involves simply adding the deviating firm's first order conditions to the baseline system and solving as before. The

[27]The tractability is due to the structure of the problem together with the focus on equilibria consistent with the size-independence of the firm problem. Klein, Krusell, and Rios-Rull (2008) discuss solving the more general case.

second requires an adjustment, however, because the distribution of wages becomes a state variable: Individual firms' choices of θ_{it}, x_{it} depend on their wage and in equilibrium these must satisfy the adding up constraint across firms $\sum_i x_{it} n_{it}/\theta_{it} + \sum_i n_{it} = 1$ each period. I solve this extended model using the approach of Gertler and Trigari (2009), by first linearizing the model equations and then aggregating across firms, arriving at a system where the average wage across firms becomes a sufficient statistic for the distribution of wages. The resulting linear system is provided in Appendix E and can again be solved with standard approaches.

In addition to aggregate shocks, I consider an environment where firms face firm-specific idiosyncratic shocks, discussed in more detail in Appendix F. To solve the baseline model with firm shocks, one can either use Dynare with higher-order approximations or solve the non-linear firm problem on a grid for productivity directly, as in this case the only state variable in the firm problem is the firm's current productivity. For the baseline model, the latter approach involves solving a nonlinear system of equations in the equilibrium firm choices of $\{\theta, x\}$ for each possible productivity realization, and simulating the model to find the job seeker value X consistent with the equilibrium adding up constraint. For the equilibrium with fixed wages, the set of unknowns is larger but a similar approach can be used.

Next, I turn to describing the results.

5.2 Firm Wages over the Business Cycle

I now return to the key issue motivating the work of Bewley (1999), the puzzle of why wages vary so little while unemployment varies so much over the business cycle. His work suggests that firm wages play a role in determining worker compensation, but do they help explain the puzzle? How do firm wages affect the dynamics of wages and unemployment?

A side-by-side comparison of the baseline firm wage model and the standard competitive search model reveals that the firm wage model features clearly more rigid wages over the business cycle than the standard model. To illustrate, Figure 1 plots impulse responses to a one percent increase in labor productivity in the two model environments, parameterized to maintain the steady-state levels of unemployment, wages and profits the same across models as described. As the figure shows, the wage increase in the firm wage model is about a quarter of that in the standard model, where the increase is almost identical to that of productivity. This means that the profitability of hiring rises more in the firm wage model, while in the standard model the increase in wages absorbs the bulk of the increase in productivity, leaving limited room for the profitability of hiring to increase. This results

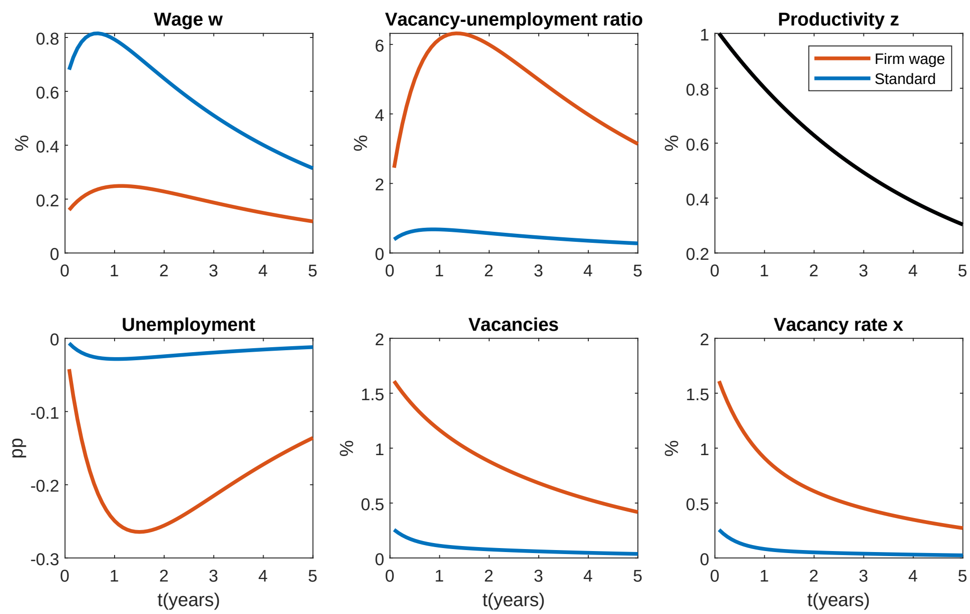

Figure 1: Impulse Responses in Firm Wage vs Standard Model

Notes: The figure plots the percentage responses of model variables to a one percent increase in aggregate labor productivity in the firm wage model and the standard competitive search model without firm wages. Labor productivity follows an $AR(1)$ with autocorrelation $\rho_z = 0.98$ and standard deviation $\sigma_z = 0.02$. The two models compared have the same steady-state levels of wage, tightness, unemployment, as described in the section on parametrization. The plotted vacancy-unemployment ratio is its model counterpart, which differs slightly from θ due to timing.

in an increase in the vacancy-unemployment ratio that is an order of magnitude greater in the firm wage model, with equally significant amplification in the increase in vacancies and drop in unemployment.[28]

A more recent literature has studied the cyclical behavior of unemployment and vacancy creation in the search and matching framework specifically, arguing that the model produces little variation in these variables over the business cycle relative to the data. The impact of firm wages is significant relative to the gap between model and data emphasized in this literature: According to Shimer (2005), the standard model would require a tenfold increase in the volatility of the vacancy-unemployment ratio to be consistent with measured volatility in the same. Taking into account the impact of the convex adjustment costs introduced as part of the firm wage model, firm wages explain roughly a third of the observed volatility:

[28]The amplification in labor market flows indicates the firm wage model generates allocative rigidity in wages, meaning rigidity in the present value of wages. The wages plotted in the figure are per-period wages, pinned down for the standard model by assuming the firm pays all its workers the same wage at each point in time (as in the firm wage model), with the cyclicality of present values thus carrying over to per-period wages in a symmetric way across models.

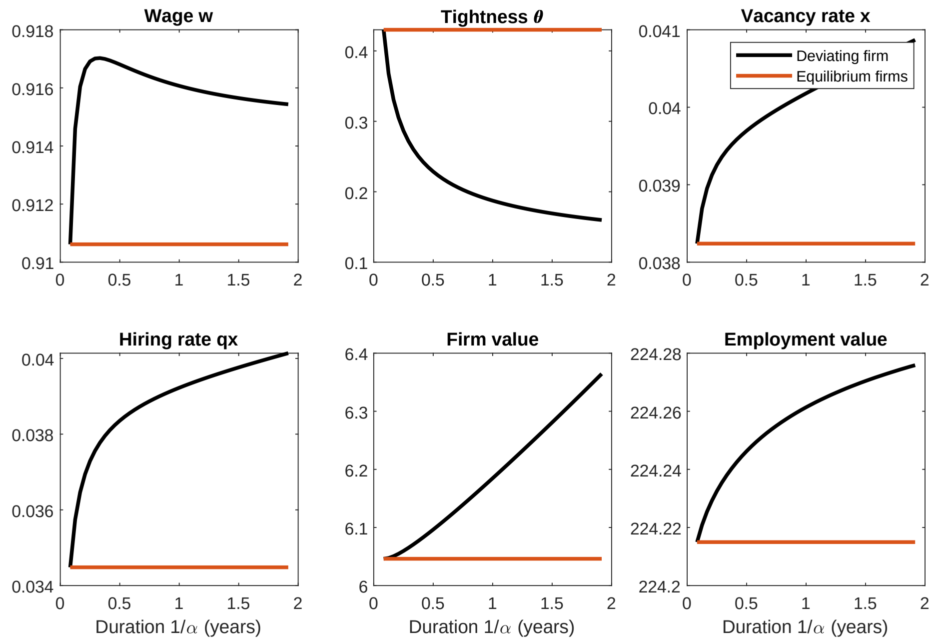

Figure 2: Single Firm Deviating to Longer Wage Commitment

Notes: The figure displays the steady-state values of a number of variables in the stationary equilibrium with firm wages, along with the corresponding values for an individual firm in that equilibrium that is able to set a wage commitment for a probabilistic period of time. The latter are plotted as a function of $1/\alpha$, the expected duration of the wage commitment. The firm value plotted is the scaled firm value per initial size.

In the model the vacancy-unemployment ratio rises by 6.5 percent in response to a one percent shock to labor productivity, while in the data the relative standard deviation of the vacancy-unemployment ratio to the same is $38/2 \approx 19$ (Shimer 2005).[29]

To conclude, firm wages lead to rigidity in wages that amplifies cyclical fluctuations in unemployment and vacancy creation, and the magnitude of the effect is quantitatively significant.

5.3 Infrequent Wage Adjustment

I then turn to an even more extreme form of rigidity in wage setting, considering the effects of firms literally fixing wages and reoptimizing them only from time to time.

[29] It is known from the literature that a higher value of b in itself generally amplifies responses to shocks in the Mortensen-Pissarides model, by reducing steady-state profit margins on hires and thus amplifying the contribution of aggregate shocks to the same. The firm wage model does generate wage rigidity and amplification in the vacancy-unemployment ratio even if the parameters are held unchanged between the two models, but with significant differences in steady-state profit margins working against these effects in percentage terms. See Figure G.1 in Appendix G for level responses.

The Firm Wage Equilibrium: Levels and Welfare Before proceeding to think about the impact of fixed wages, it is useful to begin by describing the level effects of firm wages on equilibrium outcomes, including those on welfare. To that end, consider the competitive search equilibrium with firm wages where firms reoptimize each period, parameterized as described in Section 5.1. I compare the steady state of this model with that of the standard competitive search model/planner allocation below.[30]

In line with the basic mechanism discussed already in the context of the static model, firm wage firms tend to offer lower wages, due to the influence of their existing workers on the wage setting problem, attracting fewer job seekers per vacancy. With all firms affected, this downward pressure on wages shifts the labor market equilibrium to make workers worse off – both the worker value of employment and unemployment fall – despite increased job finding rates. Meanwhile firm value is higher, as is the profitability of vacancy creation, which also increases. Overall, the firm wage equilibrium features overhiring, with higher employment than in the standard model and what is efficient, due to the increased vacancy creation.

Note that perhaps surprisingly, the firm wage constraints thus appear to make firms better off in equilibrium, rather than being purely costly, as one would expect in partial equilibrium. Relatedly, the equity would appear to come at a cost for workers, in addition to distorting allocations from what a benevolent planner would choose.

The Profitability of a Fixed Wage Consider then an individual firm deviating to a fixed wage in this context. Figure 2 shows how the firm compares to equilibrium firms in terms of its choices of wages, market tightness, vacancy rate, hiring rate, as well as firm and worker value. The deviating firm offers a higher present value of wages than equilibrium firms, thus attracting more job seekers and hiring more workers per vacancy – consistent with the deviating firm being more forward-looking in setting wages. It also creates more vacancies than equilibrium firms – consistent with the ability to commit raising the profitability of vacancy creation. And for both reasons, the deviating firm grows faster than equilibrium firms while the deviation lasts. (Note that while the effects are generally monotonic in the duration of the wage, with the present value of wages – the allocative wage variable – rising in duration, this does not require the implied per-period wage to be monotonically increasing, because the probability of reverting back to equilibrium declines in duration.)[31]

[30] These effects can be confirmed in Figure 4, which relates the firm wage equilibrium (at $\alpha = 1$) to the corresponding planner/standard competitive search allocation.

[31] I have also checked that the deviating firm remains small relative to the size of the market in Figure 2. If the deviating firm grows at rate g during the deviation (with $1 + g = (1 + qx)(1 - \delta) > 1$), then expected initial firm size $t > 1$ periods after the deviation started is $[\alpha \sum_{k=0}^{t-2} (1-\alpha)^k (1+g)^k + (1-\alpha)^{t-1}(1+g)^{t-1}] n_1$,

Figure 3: Deviation to Longer Wage Commitment with Aggregate Shocks

Notes: The figure displays simulation means in an equilibrium where wages are reoptimized each period, together with those chosen by a firm deviating each period, as a function of $1/\alpha$, the expected duration of wages. Labor productivity follows an AR(1) with autocorrelation $\rho_z = 0.98$ and standard deviation $\sigma_z = 0.02$. The firm value plotted is the scaled firm value per initial size. For the deviating firm, the variables plotted are the deviation wage, deviation period growth rate, firm value, and worker value. (Note that the standard deviation of this wage is not zero because the prevailing aggregate state affects the choice.)

Importantly, note that the deviation is profitable for the firm, raising firm value relative to equilibrium firms. This is due to the time-inconsistency inherent in the firm problem, which makes commitment valuable. In the standard competitive search model, meanwhile, a similar deviation would have no impact on the firm's choices or value, as Figure G.4 in Appendix G illustrates. Moreover, in addition to the deviating firm itself, also the workers working for the deviating firm are strictly better off as a result of the deviation, because of the higher present value of wages, meaning that the deviation is Pareto-improving.

When firms face shocks, a fixed wage involves costs as well as benefits, however. To compare the deviating firm with equilibrium firms in an environment where firms face aggregate shocks, Figure 3 plots simulation means together with corresponding standard deviation bounds for the deviating firm's choices of wage, hiring rate, as well as resulting firm and worker value, comparing them to those of equilibrium firms. As the standard deviation

where $n_1 = (1+g)n_0$ is initial size after one period of deviation. It follows that firm size remains bounded as t grows if and only if $(1-\alpha)(1+g) < 1$.

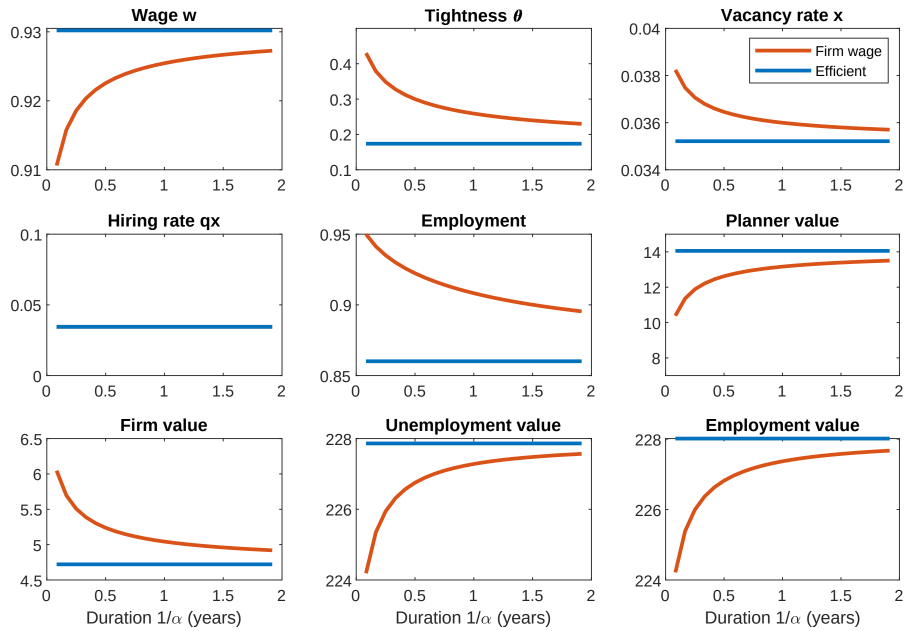

Figure 4: Equilibrium with Longer Wage Commitments

Notes: The figure displays the steady-state values of a number of variables in the stationary equilibrium with firm wages and infrequent adjustment, as a function of $1/\alpha$, the expected duration of wages. The firm value plotted is the scaled firm value per initial size, but also the unscaled firm value declines in wage duration. Correspondingly, the planner value plotted is the scaled value per initial size, but also the unscaled value increases in wage duration. The figure also shows the corresponding values in the efficient allocation.

bounds illustrate, changes in the aggregate state clearly matter for both firms, but ultimately the figure is consistent with the previous one in that the deviating firm chooses higher wages and grows faster than equilibrium firms, with the deviation remaining profitable for the firm.

To illustrate the costs involved with fixed wages, Figure G.5 in Appendix G compares the impulse responses of the deviating firm to those of equilibrium firms. As shown, fixing the wage hampers the firm's ability to respond to an increase in productivity by shutting down one of the two instruments it would normally use in doing so. Instead of raising wages together with other firms, the firm holds its wage fixed, which makes it less attractive to job seekers. The firm still increases vacancy creation in response to the shock, but the profitability of vacancy creation suffers from not being able to offer more attractive terms, dampening the increase. On net, the deviating firm's hiring barely increases with the shock. Overall, the value of the commitment provided by the fixed wage appears to dominate these limitations in responding to shocks, however.

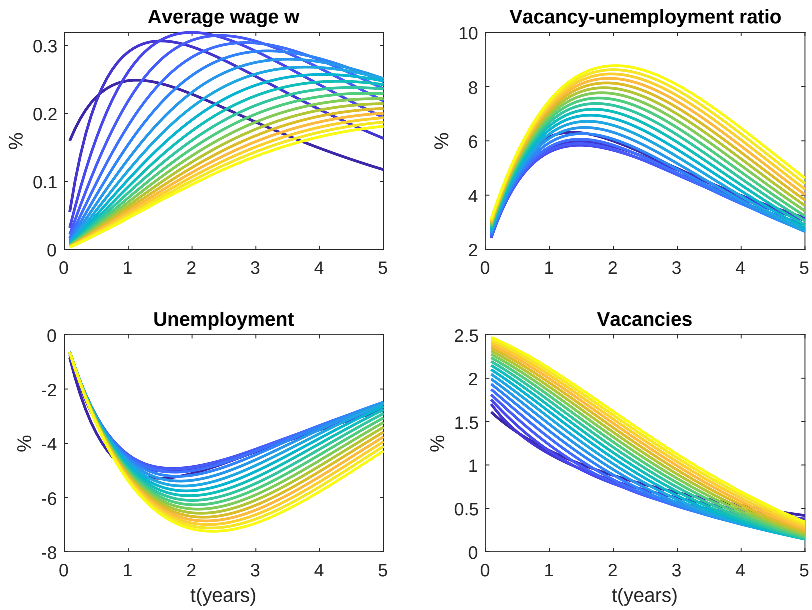

Figure 5: Impact of Duration on Labor Market Volatility

Notes: The figure displays impulse responses to an aggregate shock to productivity, as a function of $1/\alpha$, the expected duration of wages rises from one month to 1.5 years. Labor productivity follows an AR(1) with autocorrelation $\rho_z = 0.98$ and standard deviation $\sigma_z = 0.02$.

Equilibrium with Fixed Wages Given that fixed wages thus appear profitable for firms, I then turn to the question of how they affect equilibrium outcomes.

Beginning again with a version of the model without shocks, Figure 4 shows how the firm wage equilibrium compares with standard competitive search/efficient allocations, and in particular how longer wage durations affect outcomes. As in the case of the deviating firm, longer wage durations raise the level of wages, as firms become more forward-looking, with firms thus attracting more applicants per vacancy. With all firms now affected, however, the upward pressure on wages shifts the labor market equilibrium, undoing the effects of firm wages on equilibrium outcomes. As equilibrium wages rise, both employed and unemployed workers gain, despite reduced job finding rates. Meanwhile firm values fall, as does the profitability of vacancy creation, which also falls. While the hiring rate must remain equal to the separation rate in steady state (by construction), employment falls with the falling job finding rates, reducing overhiring. Longer wage durations thus make workers better off, as well as improving the efficiency of resource allocation.

When firms face aggregate shocks, fixed wages also influence labor market volatility, however. To illustrate, Figure 5 plots impulse responses showing how the responses of the vacancy-unemployment ratio, unemployment and vacancy creation change as the duration

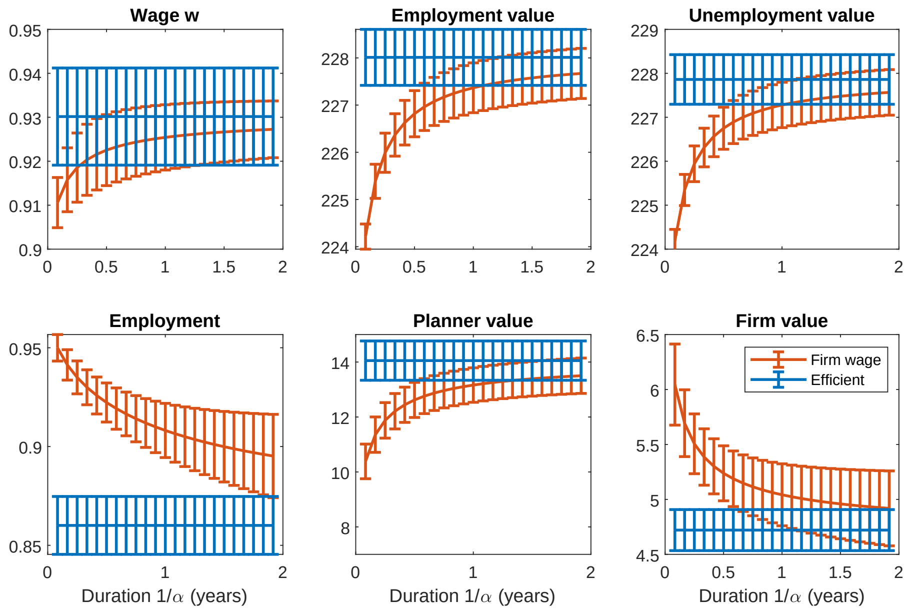

Figure 6: Equilibrium with Longer Wage Commitments with Aggregate Shocks
Notes: The figure displays simulation means in the firm wage equilibrium with infrequent adjustment, as a function of $1/\alpha$, the expected duration of wages. Labor productivity follows an AR(1) with autocorrelation $\rho_z = 0.96$ and standard deviation $\sigma_z = 0.02$. The firm value plotted is the scaled firm value per initial size, but also the unscaled firm value declines in wage duration. Correspondingly, the planner value plotted is the scaled value per initial size, but also the unscaled value increases in wage duration. The figure also shows the corresponding values in the efficient allocation.

of wages increases, with longer wage durations increasing volatility. To revisit the favorable conclusions about the impact of wage durations on equilibrium outcomes with these changes in volatility in mind, Figure 6 again plots simulation means with standard deviation bounds for wages and employment, as well as worker, firm and planner values. From the figure, it is clear that aggregate shocks induce non-trivial variation, with employment volatility increasing in the duration of wages. But at the same time the presence of shocks does not appear to change the main conclusions that longer wage durations raise the level of wages, shifting the equilibrium to make workers better off, as well as reducing overhiring and improving efficiency of allocations.

5.4 Firm-Level Shocks

In practice firms face non-trivial firm-level risk as well. The firm wage model extends to accommodate a stationary equilibrium with idiosyncratic firm-level shocks in a natural way, and this section illustrates the impact of firm wages, as well as fixed wages, on outcomes in that context. I relegate the equations to Appendix F, proceeding directly to discuss the results below.

Firm Wage Equilibrium with Firm Shocks In a stationary equilibrium with idiosyncratic firm-level shocks to productivity, individual firms grow and shrink over time in response to the shocks they face. Figure F.1 illustrates this churn by plotting impulse responses.[32] An increase in firm productivity causes the firm to raise its offered wage, thus attracting more job seekers per vacancy, as well as to increase its vacancy creation. As a result firm growth accelerates, with employment expanding over time relative to other firms. Comparing to the standard competitive search model, firm wages again work to dampen the responses of wages to shocks, amplifying those of hiring and employment as a result.

Note that despite the size-independence of the firm problem, which carries over to the case of firm-level shocks, the above indicates that the model will generate a large firm wage premium in the cross section (Brown and Medoff 1989) if productivity is persistent, as more productive firms both offer higher wages and become larger.

Fixed Wages with Firm Shocks Can fixing wages remain profitable in the face of significant firm-level shocks? To shed light on this, Figure F.2 again considers a firm deviating to a fixed wage when other firms reoptimize each period. Firm productivity is discretized to take on one of three values: low, intermediate or high, with the deviating firm depicted in the intermediate productivity state. The figure shows that, in line with the impulse responses, among the equilibrium firms more productive firms pay higher wages, attract more job seekers per vacancy, and grow faster, as well as having greater firm value. Despite the differences driven by productivity, the effects of the deviation remain very similar, however, with the deviating firm offering a higher wage and creating more vacancies, and thus growing faster than equilibrium firms with similar productivity. In particular, the deviation remains profitable despite productivity being an important determinant of firm value when shocks are large.

Finally, I also revisit the equilibrium effects of fixed wages in this setting with firm shocks. Figure F.3 plots the results, confirming that while prevailing productivity is important for firms, longer wage durations continue to have similar effects as before. As firms become more forward-looking, the equilibrium shifts toward higher wages, to the benefit of workers,

[32] I adjust the model calibration for the case of firm shocks, increasing the size of the adjustment cost in order to curb firm responses to large and persistent firm-level shocks in the context of linear production technologies. Specifically, I lower the target tightness to 0.4, which implies $\ell = 2.67$, $b = 0.81$ and an average cost of vacancies of 3.8 in the firm wage model.

with reduced overhiring as well.

6 Conclusions

Motivated by the work of Truman Bewley (1999), this paper studies a labor market with search frictions and directed search, where firms employ multiple workers and follow a firm-wage policy: They pay equally productive workers the same. I show that the policy results in lower equilibrium wages, due to the influence of existing workers on firms' wage setting, which encourages vacancy creation and leads to overhiring. The policy also introduces a time-inconsistency into the dynamic firm problem, as the firm effectively faces a less elastic labor supply in the short than the long run. To consider outcomes when firms reoptimize each period, I study Markov perfect equilibria, offering a tractable approach to solving based on standard Euler equations. I use the model to show that, as suggested by Bewley, firm wages dampen wage variation over the business cycle and amplify that in unemployment and vacancy creation. Moreover, firm wage firms may also find it profitable to fix wages for a period of time, due to the commitment problem, and such infrequent wage adjustment be welfare-improving for workers, despite added volatility in the labor market.

The model environment could, of course, be enriched in various ways. Instead of imposing constraints on firms, one could seek to incorporate an agency problem between owners, managers and workers that gives rise to them. Instead of imposing complete equity within the firm, one could allow a degree of discretion in wage setting with equity considerations leading to wage compression. And, one could seek to formalize the internal wage structures of firms more fully, including worker transitions from job to job within the firm. At the same time, the stylized approach allows overcoming important modeling challenges involved with time-inconsistency, to highlight that the institutional constraints affecting firms can play an important role for understanding important macroeconomic questions.

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Appendix

A Proofs and Details

Proof of Proposition 1 For concreteness, I consider the vacancy cost $\kappa(v,n) = \kappa_0(v/n)^\gamma n$, where $\gamma > 0$ is assumed large enough.

Eliminating w, x, U from the conditions yields

$$1 + q(\theta)\theta\frac{1-N}{N} = -q'(\theta)\theta\frac{1-N}{N}\frac{\kappa_v(\theta\frac{1-N}{N})}{q(\theta)}\frac{\mu(\theta)}{\mu'(\theta)(z-b-\frac{\kappa_v(\theta\frac{1-N}{N})}{q(\theta)})},$$

or dividing by θ,

$$\frac{1}{\theta} + q(\theta)\frac{1-N}{N} = \frac{1}{1-1/\varepsilon_q(\theta)}\frac{1-N}{N}\frac{\kappa_v(\theta\frac{1-N}{N})}{z-b-\frac{\kappa_v(\theta\frac{1-N}{N})}{q(\theta)}},$$

denoting the matching function elasticity by $\varepsilon_q(\theta) := -q'(\theta)\theta/q(\theta)$.

The left hand side is strictly decreasing in θ, while the second part of the right hand side is strictly increasing, given an increasing and convex vacancy cost. With the functional form used in the paper, $q(\theta) = (1+\theta^{1/\ell})^\ell$, $\frac{1}{1-1/\varepsilon_q(\theta)} = \theta^{1/\ell}$ is also strictly increasing. Hence, the right hand side of the equation is strictly increasing and the equation pins down a unique equilibrium θ. More generally, as long as the curvature in the vacancy cost is sufficient, the right hand side should be increasing and the same hold.

For the standard model one simply leaves out the $1/\theta$ term on the left hand side, which implies that the tightness in the firm wage model is strictly greater, $\theta_{FW} > \theta_{SD}$, and hence employment, $N+\mu(\theta)(1-N)$, is strictly higher in the firm wage model. From $x = \theta(1-N)/N$, the hiring rate in the firm wage model is then strictly greater, $x_{FW} > x_{SD}$, as is total vacancy creation xN. Finally the wage, from $w = b + \kappa_v(x)/q(\theta)$, is strictly lower in the firm wage model, $w_{FW} < w_{SD}$.

Proof of Proposition 2 For convenience, let $y_t := Y_t - \beta(1-\delta)E_t Y_{t+1}$. We have $y_t = E_t[\beta\delta U_{t+1} - b - \beta U_{t+1} + \beta(1-\delta)(b + \beta U_{t+2})] = E_t[-b - \beta(1-\delta)(U_{t+1} - b - \beta U_{t+2})]$, meaning that $y_t = -b - \beta(1-\delta)E_t X_{t+1}$.

First, the firm objective in (10) can be rewritten as

$$E_0[n_0 \sum_{t=0}^{\infty} \beta^t (1-\delta)^t (z - w_t) + \sum_{t=0}^{\infty} \beta^t \sum_{k=0}^{t-1} (1-\delta)^{t-k} q(\theta_k) v_k (z - w_t) - \sum_{t=0}^{\infty} \beta^t \kappa(v_t, n_t)], \quad (33)$$

using that $n_t + q(\theta_t)v_t = (1-\delta)^t n_0 + \sum_{k=0}^{t-1}(1-\delta)^{t-k} q(\theta_k) v_k$.

The first term in (33) can then be rewritten as

$$E_0 n_0 \sum_{t=0}^{\infty} \beta^t (1-\delta)^t (z - w_t) = n_0[Z_0 + Y_0 - \frac{X_0}{\mu(\theta_0)}], \quad (34)$$

using that the job seeker value constraint (12) implies $W_0 = X_0/\mu(\theta_0) - Y_0$.

The second term in (33) can be rewritten as

$$\begin{aligned}
&E_0 \sum_{k=0}^{\infty} \beta^k q(\theta_k) v_k \sum_{t=k}^{\infty} \beta^{t-k} (1-\delta)^{t-k} (z_t - w_t) \\
&= E_0 \sum_{k=0}^{\infty} \beta^k [q(\theta_k) v_k \sum_{t=k}^{\infty} \beta^{t-k} (1-\delta)^{t-k} (z_t + y_t) - \frac{v_k}{\theta_k} \sum_{t=k}^{\infty} \beta^{t-k} (1-\delta)^{t-k} X_t] \\
&= E_0 \sum_{t=0}^{\infty} \beta^t \sum_{k=0}^{t-1} (1-\delta)^{t-k} [q(\theta_k) v_k (z_t + y_t) - \frac{v_k}{\theta_k} X_t], \qquad (35)
\end{aligned}$$

where the first equality follows from rearranging terms, and the second uses the job seeker value constraint to substitute out the present value of wages.

Combining the terms in (34) and (35) and rearranging, the firm objective becomes

$$\begin{aligned}
&E_0 n_0 [\sum_{t=0}^{\infty} \beta^t (1-\delta)^t (z_t + y_t) - \frac{X_0}{\mu(\theta_0)}] \\
&+ E_0 \sum_{t=0}^{\infty} \beta^t \sum_{k=0}^{t-1} (1-\delta)^{t-k} [q(\theta_k) v_k (z_t + y_t) - \frac{v_k}{\theta_k} X_t] - E_0 \sum_{t=0}^{\infty} \beta^t \kappa(v_t, n_t) \\
&= -\frac{n_0 X_0}{\mu(\theta_0)} + E_0 \sum_{t=0}^{\infty} \beta^t [(n_t + q(\theta_t) v_t)(z_t + y_t) - \frac{v_t X_t}{\theta_t} - \kappa(v_t, n_t)]. \qquad (36)
\end{aligned}$$

Using that $y_t = -b - \beta(1-\delta)E_t X_{t+1}$, and rearranging, the firm objective can be written as

$$-\frac{n_{i0}X_0}{\mu(\theta_{i0})} + n_{i0}X_0 + E_t \sum_{t=0}^{\infty} \beta^t [(n_{it} + q(\theta_{it})v_{it})(z_t - b) - \kappa(v_{it}, n_{it}) - X_t(\frac{v_{it}}{\theta_{it}} + n_{it})]. \quad (37)$$

Note that the term $n_{i0}X_0$ is independent of the firm's actions, and has been omitted for brevity in writing the firm problem as in (13) in the text. In calculating the actual firm value, it must be added back.

Opting Out of Hiring Note that because the firm begins with a stock of existing workers, it could potentially find it optimal to, instead of following the interior solution characterized by the first order conditions, not hire at all in the first period and instead set a wage that is so low as to make those existing workers indifferent between remaining with the firm and unemployment. The latter would mean that $W_0 + Y_0 = 0$ and no hiring that $v_0 = 0$. How would this change firm value?

In the derivation above, it would mean that the expression in (34) would reduce to $n_{i0}[Z_0 + Y_0]$, and the expression in (35) would have $v_{i0} = 0$, such that θ_{i0} no longer appears. Firm value, as in (37), would then become

$$n_{i0}X_0 + E_t \sum_{t=0}^{\infty} \beta^t [(n_{it} + q(\theta_{it})v_{it})(z_t - b) - \kappa(v_{it}, n_{it}) - X_t(\frac{v_{it}}{\theta_{it}} + n_{it})]$$

with $v_{i0} = 0$. With commitment, after this initial period the firm problem becomes equivalent to the planner problem, and hence hiring should be consistent with efficient allocations and interior as long as standard conditions are met (z sufficiently above b). In the initial period, one would want to check that this value does not dominate the equilibrium value. Note that due to the size-independence of the firm problem, if one firm prefers to deviate, all firms will.

In the context of no commitment, if a firm in any period were to deviate to this non-hiring option, its value would be

$$\begin{aligned} &nX(S) + n(z-b) - nX(S) + \beta E_S V(n'; S') \\ &\text{s.t. } n' = (1-\delta)n, \end{aligned}$$

where the continuation value $V(n; S)$ follows (20). In solving the model using first order conditions, one would want to make sure this deviation value does not exceed equilibrium values, something that can restrict parameter values. In practice high aggregate levels of existing matches tend to make deviating more attractive, so one would choose parameters such that the desired steady-state measure of matches is sufficiently below this range, keeping the economy below a range where deviating becomes attractive.

Second Order Conditions For the sequence problem, denoting the firm objective as g, second order conditions read, for $t > 0$: $g_{x_t x_t} = -\hat{\kappa}''(x_t) < 0$, $g_{\theta_t\theta_t} = q''(\theta_t)x_t(z_t - b + \beta(1-\delta)E_t\hat{V}_{t+1}) - \frac{2X_t x_t}{\theta_t^3} < 0$, and $det = g_{x_t x_t}g_{\theta_t\theta_t} - g^2_{x_t\theta_t} > 0$, where $g_{x_t\theta_t} = q'(\theta_t)(z_t - b + \beta(1-\delta)E_t\hat{V}_{t+1}) + \frac{X_t}{\theta_t^2} = 0$. For the initial period: $g_{x_0x_0} = -\hat{\kappa}''(x_0)$, $g_{\theta_0\theta_0} = \frac{X_0\mu''(\theta_0)}{\mu(\theta_0)^2} - \frac{2X_0\mu'(\theta_0)^2}{\mu(\theta_0)^3} + q''(\theta_0)x_0(z_0 - b + \beta(1-\delta)E_0\hat{V}_1) - \frac{2X_0x_0}{\theta_0^3}$ and $det = g_{x_0x_0}g_{\theta_0\theta_0} - g^2_{x_0\theta_0} > 0$, where $g_{x_0\theta_0} = q'(\theta_0)(z_0 - b + \beta(1-\delta)E_0\hat{V}_1) + \frac{X_0}{\theta_0^2}$. The periods separate when calculating second order conditions.

For the no commitment case, again denoting the firm objective as g, second order conditions read: $g_{xx} = -\hat{\kappa}''(x) < 0$, $g_{\theta\theta} = \frac{X\mu''(\theta)}{\mu(\theta)^2} - \frac{2X\mu'(\theta)^2}{\mu(\theta)^3} + q''(\theta)x(z-b+\beta(1-\delta)E\hat{V}) - \frac{2Xx}{\theta^3} < 0$, and $det = g_{xx}g_{\theta\theta} - g^2_{x\theta} > 0$, where $g_{x\theta} = q'(\theta)(z - b + \beta(1-\delta)E\hat{V}) + \frac{X}{\theta^2}$.

Proof of Proposition 3 The firm problem (10) is equivalent to the problem

$$\max_{\{w_{it},\theta_{it},x_{it}\}} E_0 \sum_{t=0}^{\infty} \beta^t(1-\delta)^t \prod_{k=0}^{t-1}(1+q(\theta_{ik})x_{ik})[(1+q(\theta_{it})x_{it})(z_t - w_{it}) - \hat{\kappa}(x_{it})]$$
$$\text{s.t. } X_t = \mu(\theta_{it})(E_t \sum_{k=0}^{\infty} \beta^k(1-\delta)^k w_{it+k} + Y_t),\ \forall t \geq 0,$$

which does not depend on n_{i0}. This can be seen by expressing the profits in problem (10) in each period t scaled by size n_{it} and using the law of motion to adjust the discounting for this scaling. Finally, normalizing the firm problem with initial size n_{i0} yields the expression above.

Unconstrained Firm Problem Starting from the firm problem for hiring in period t in (26), one can substitute wages out using (27) to arrive at

$$\max_{\theta_{it},v_{it}} E_t \left[q(\theta_{it})v_{it}[z_t - b + \sum_{k=1}^{\infty} \beta^k(1-\delta)^k(z_{t+k} - b - X_{t+k})] - \frac{X_t v_{it}}{\theta_{it}} - \sum_{k=0}^{\infty} \beta^k \kappa(v_{it+k}, n_{it+k}) \right],$$

using the relationship $Y_t = -b + \beta(1-\delta)E_t(Y_{t+1} - X_{t+1})$, or $Y_t = -b - E_t\sum_{k=1}^{\infty}\beta^k(1-\delta)^k(b + X_{t+k})$.

Adding up over cohorts of workers hired at different points in time (with discounting) yields

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[q(\theta_{it})v_{it}[z_t - b + \sum_{k=1}^{\infty} \beta^k(1-\delta)^k(z_{t+k} - b - X_{t+k})] - \frac{X_t v_{it}}{\theta_{it}} - \kappa(v_{it}, n_{it}) \right]$$
$$= E_0 \sum_{t=0}^{\infty} \beta^t \left[(n^n_{it} + q(\theta_{it})v_{it})(z_t - b) - X_t(n^n_{it} + \frac{v_{it}}{\theta_{it}}) - \kappa(v_{it}, n_{it}) \right] + X_0 n^n_{i0},$$

where n_{it}^n denotes the workforce hired on or after period zero, with $n_{i0}^n = 0$ and $n_{it+1}^n = (1-\delta)(n_{it}^n + q(\theta_{it})v_{it})$ for all $t \geq 0$.

Total firm value in period zero can then be written, adding the present value associated with existing workers at time zero, $n_{i0}[E_0 \sum_{t=0}^{\infty} \beta^t (1-\delta)^t z_t - W_{i0}^e]$, where W_{i0}^e represents the average present value of wages among the initial workforce, as

$$E_0 \sum_{t=0}^{\infty} \beta^t [(n_{it}^n + q(\theta_{it})v_{it})(z_t - b) - X_t(n_{it}^n + \frac{v_{it}}{\theta_{it}}) - \kappa(v_{it}, n_{it})] + X_0 n_{i0}^n + n_{i0}[E_0 \sum_{t=0}^{\infty} \beta^t (1-\delta)^t z_t - W_{i0}^e]$$

$$= E_0 \sum_{t=0}^{\infty} \beta^t [(n_{it} + q(\theta_{it})v_{it})(z_t - b) - X_t(n_{it} + \frac{v_{it}}{\theta_{it}}) - \kappa(v_{it}, n_{it})]$$

$$- n_{i0} E_0 \sum_{t=0}^{\infty} \beta^t (1-\delta)^t (z_t - b - X_t) + X_0 n_{i0}^n + n_{i0}[E_0 \sum_{t=0}^{\infty} \beta^t (1-\delta)^t z_t - W_{i0}^e],$$

where the equality uses that $n_{it} = n_{it}^n + (1-\delta)^t n_{i0}$.

Note that the terms on the last line are either given or independent of the firm's choice variables $\{\theta_{it}, v_{it}\}_{t=0}^{\infty}$ which appear only on the previous line, and thus the firm problem effectively coincides with maximizing the former.

B Two Period Model of Firm Wages

Consider a deterministic, two-period version of the dynamic model in Section 3.

The value of entering period $t = 0, 1$ as an unemployed worker satisfies

$$U_1 = \mu(\theta_{i1})w_{i1} + (1 - \mu(\theta_{i1}))b,$$
$$U_0 = \mu(\theta_{i0})(w_{i0} + \beta(1-\delta)w_{i1} + \beta\delta U_1) + (1 - \mu(\theta_{i0}))(b + \beta U_1).$$

If we define $X_0 := U_0 - b - \beta U_1$, $X_1 := U_1 - b$, $Y_0 := -b - \beta(1-\delta)U_1$, and $Y_1 := -b$, the above can be written as

$$X_t = \mu(\theta_{it})(W_{it} + Y_t), \ t = 0, 1,$$

where $W_{i1} = w_{i1}$ and $W_{i0} = w_{i0} + \beta(1-\delta)w_{i1}$.

Commitment Assuming firms have commitment, the firm problem reads

$$\max_{\{w_{it},\theta_{it},v_{it}\}_{t=0}^{1}} \sum_{t=0}^{1} \beta^t[(n_{it}+q(\theta_{it})v_{it})(z_t-w_{it})-\kappa(v_{it},n_{it})],$$

$$\text{s.t. } n_{i1}=(1-\delta)(n_{i1}+q(\theta_{i1})v_{i1}), \tag{38}$$

$$X_t=\mu(\theta_{it})(\sum_{k=0}^{1}\beta^k(1-\delta)^k w_{it+k}+Y_t), \text{ for } t=0,1, \tag{39}$$

with n_{i0} given for all i. The firm maximizes the present discounted value of profits, taking into account the law of motion for employment relationships, as well as the constraint reflecting job seeker behavior each period, where the firm takes the market-determined values of X_t, Y_t as given.

Using the job seeker constraints (39) to substitute out wages, the law of motion (38), and dividing by initial size, the firm problem can be rewritten as

$$\max -\frac{X_0}{\mu(\theta_{i0})}+X_0+(1+q(\theta_{i0})x_{i0})(z_0-b)-\hat{\kappa}(x_{i0})-X_0(\frac{x_{i0}}{\theta_{i0}}+1)$$
$$+\beta(1-\delta)(1+q(\theta_{i1})x_{i1})[(1+q(\theta_{i1})x_{i1})(z_1-b)-\hat{\kappa}(x_{i1})-X_1(\frac{x_{i1}}{\theta_{i1}}+1)].$$

Note that this problem is independent of firm size, and hence in what follows the firm-level indicators are dropped.

The first order conditions in period one read[33]

$$\kappa_v(x_1)+\frac{X_1}{\theta_1}=q(\theta_1)(z_1-b),$$
$$\frac{X_1}{\theta_1}=-q'(\theta_1)(z_1-b).$$

Taken together, these imply that $\frac{\kappa_v(x_1)}{\mu'(\theta_1)}=z_1-b$, where $X_1=\kappa_v(x_1)\frac{\mu(\theta_1)-\mu'(\theta_1)\theta_1}{\mu'(\theta_1)}$.

Given an allocation θ_1, x_1, period one firm value (normalized by size) is

$$\hat{V}_1^o=-\frac{X_1}{\mu(\theta_1)}+X_1+(1+q(\theta_1)x_1)(z_1-b)-\hat{\kappa}(x_1)-X_1(\frac{x_1}{\theta_1}+1),$$

while the continuation value is

$$\hat{V}_1=(1+q(\theta_1)x_1)(z_1-b)-\hat{\kappa}(x_1)-X_1(\frac{x_1}{\theta_1}+1).$$

[33] Denoting the firm objective as g, second order conditions read: $g_{x_1x_1}=-\hat{\kappa}''(x_1)<0$, $g_{\theta_1\theta_1}=q''(\theta_1)x_1(z_1-b)-\frac{2X_1x_1}{\theta_1^3}<0$, and $det=g_{x_1x_1}g_{\theta_1\theta_1}-g_{x_1\theta_1}^2>0$, where $g_{x_1\theta_1}=q'(\theta_1)(z_1-b)+\frac{X_1}{\theta_1^2}=0$, and $g_{x_0x_0}=-\hat{\kappa}''(x_0)$, $g_{\theta_0\theta_0}=\frac{X_0\mu''(\theta_0)}{\mu(\theta_0)^2}-\frac{2X_0\mu'(\theta_0)^2}{\mu(\theta_0)^3}+q''(\theta_0)x_0(z_0-b+\beta(1-\delta)E_0\hat{V}_1)-\frac{2X_0x_0}{\theta_0^3}$ and $det=g_{x_0x_0}g_{\theta_0\theta_0}-g_{x_0\theta_0}^2>0$, where $g_{x_0\theta_0}=q'(\theta_0)(z_0-b+\beta(1-\delta)E_0\hat{V}_1)+\frac{X_0}{\theta_0^2}<0$. The periods separate when calculating second order conditions.

Using the optimality conditions, these can be written as

$$\hat{V}_1^o = -\frac{X_1}{\mu(\theta_1)} + z_1 - b - \kappa_n(x_1),$$
$$\hat{V}_1 = z_1 - b - \kappa_n(x_1) - X_1.$$

The first order conditions in the initial period read

$$\kappa_v(x_0) + \frac{X_0}{\theta_0} = q(\theta_0)[z_0 - b + \beta(1-\delta)\hat{V}_1],$$
$$\frac{X_0}{\theta_0^2}[1 + \frac{\mu'(\theta_0)\theta_0^2}{x_0\mu(\theta_0)^2}] = -q'(\theta_0)[z_0 - b + \beta(1-\delta)\hat{V}_1],$$

and taken together, imply that

$$\frac{\kappa_v(x_0)}{\mu'(\theta_0)}[1 - \frac{(1-\mu'(\theta_0)\theta_0/\mu(\theta_0))}{1+q(\theta_0)x_0}] = z_0 - b + \beta(1-\delta)\big[\frac{\kappa_v(x_1)}{\mu'(\theta_1)}\big(1-\mu(\theta_1)+\mu'(\theta_1)\theta_1\big) - \kappa_n(x_1)\big],$$

with

$$X_0 = \kappa_v(x_0)\frac{\mu(\theta_0)-\mu'(\theta_0)\theta_0}{\mu'(\theta_0)}\frac{q(\theta_0)x_0}{1+q(\theta_0)x_0}.$$

Given allocations and the continuation value, and using the optimality conditions, the normalized firm value in the initial period can be written as

$$\hat{V}_0^o = -\frac{X_0}{\mu(\theta_0)} + z_0 - b + \beta(1-\delta)\hat{V}_1 - \kappa_n(x_0).$$

Limited Commitment If firms cannot commit, the firm problem is solved backward.

In period one, firms maximize the firm value

$$\max -\frac{X_1}{\mu(\theta_1)} + X_1 + (1+q(\theta_1)x_1)(z_1-b) - \hat{\kappa}(x_1) - X_1(\frac{x_1}{\theta_1}+1).$$

The first order conditions for optimality read

$$\kappa_v(x_1) + \frac{X_1}{\theta_1} = q(\theta_1)(z_1 - b),$$
$$\frac{X_1}{\theta_1^2}[1 + \frac{\mu'(\theta_1)\theta_1^2}{x_1\mu(\theta_1)^2}] = -q'(\theta_1)(z_1 - b),$$

and imply

$$\frac{\kappa_v(x_1)}{\mu'(\theta_1)}[1 - \frac{(1-\mu'(\theta_1)\theta_1/\mu(\theta_1))}{1+q(\theta_1)x_1}] = z_1 - b,$$

with

$$X_1 = \kappa_v(x_1)\frac{\mu(\theta_1) - \mu'(\theta_1)\theta_1}{\mu'(\theta_1)}\frac{q(\theta_1)x_1}{1 + q(\theta_1)x_1}.$$

Given allocations, the normalized firm value then satisfies

$$\hat{V}_1^o = -\frac{X_1}{\mu(\theta_1)} + (1 + q(\theta_1)x_1)(z_1 - b) - \frac{X_1 x_1}{\theta_1} - \hat{\kappa}(x_1) = -\frac{X_1}{\mu(\theta_1)} + z_1 - b - \kappa_n(x_1)$$

and the corresponding continuation value

$$\hat{V}_1 = z_1 - b - \kappa_n(x_1) - X_1.$$

The first order conditions in the initial period read[34]

$$\kappa_v(x_0) + \frac{X_0}{\theta_0} = q(\theta_0)[z_0 - b + \beta(1-\delta)\hat{V}_1],$$
$$\frac{X_0}{\theta_0^2}[1 + \frac{\mu'(\theta_0)\theta_0^2}{x_0\mu(\theta_0)^2}] = -q'(\theta_0)[z_0 - b + \beta(1-\delta)\hat{V}_1],$$

and imply

$$\frac{\kappa_v(x_0)}{\mu'(\theta_0)}[1 - \frac{(1 - \mu'(\theta_0)\theta_0/\mu(\theta_0))}{1 + q(\theta_0)x_0}]$$
$$= z_0 - b + \beta(1-\delta)\big[\frac{\kappa_v(x_1)}{\mu'(\theta_1)}\big(1 - \mu(\theta_1) + \mu'(\theta_1)\theta_1 - (1 - \mu(\theta_1))\frac{(1 - \mu'(\theta_1)\theta_1/\mu(\theta_1))}{1 + q(\theta_1)x_1}\big) - \kappa_n(x_1)\big],$$

with

$$X_0 = \kappa_v(x_0)\frac{\mu(\theta_0) - \mu'(\theta_0)\theta_0}{\mu'(\theta_0)}\frac{q(\theta_0)x_0}{1 + q(\theta_0)x_0}.$$

Given allocations and continuation values, normalized firm value in the initial period equals

$$\hat{V}_0^o = -\frac{X_0}{\mu(\theta_0)} + z_0 - b + \beta(1-\delta)\hat{V}_1 - \kappa_n(x_0).$$

Whether or not firms have commitment, equilibrium requires allocations to be optimal for firms, as well as the total measure of job seekers allocated to firms to be consistent with the measure of job seekers in the market: $x_t = \theta_t(1 - N_t)/N_t$ for $t = 0, 1$.

[34]Denoting the firm objective as g, second order conditions read: $g_{x_1x_1} = -\hat{\kappa}''(x_1) < 0$, $g_{\theta_1\theta_1} = \frac{X_1\mu''(\theta_1)}{\mu(\theta_1)^2} - \frac{2X_1\mu'(\theta_1)^2}{\mu(\theta_1)^3} + q''(\theta_1)x_1(z_1 - b) - \frac{2X_1x_1}{\theta_1^3} < 0$, and $det = g_{x_1x_1}g_{\theta_1\theta_1} - g_{x_1\theta_1}^2 > 0$, where $g_{x_1\theta_1} = q'(\theta_1)(z_1 - b) + \frac{X_1}{\theta_1^2} < 0$, and $g_{x_2x_2} = -\hat{\kappa}''(x_2)$, $g_{\theta_0\theta_0} = \frac{X_0\mu''(\theta_0)}{\mu(\theta_0)^2} - \frac{2X_0\mu'(\theta_0)^2}{\mu(\theta_0)^3} + q''(\theta_0)x_0(z_0 - b + \beta(1-\delta)E_0\hat{V}_1) - \frac{2X_0x_0}{\theta_0^3}$ and $det = g_{x_0x_0}g_{\theta_0\theta_0} - g_{x_0\theta_0}^2 > 0$, where $g_{x_0\theta_0} = q'(\theta_0)(z_0 - b + \beta(1-\delta)E_0\hat{V}_1) + \frac{X_0}{\theta_0^2} < 0$. The periods separate when calculating second order conditions.

Planner Problem The planner problem reads:

$$
\begin{aligned}
\max_{\{\theta_{it},v_{it}\}_{t=0}^{1}} & \sum_{t=0}^{1}\beta^t[\sum_i[(n_{it}+q(\theta_{it})v_{it})z_t-\kappa(v_{it},n_{it})]+(1-\sum_i(n_{it}+q(\theta_{it})v_{it})b)] \\
\text{s.t. } & n_{i1}=(1-\delta)(n_{i0}+q(\theta_{i0})v_{i0}), \\
& \sum_i v_{it}/\theta_{it}=1-\sum_i n_{it}, \text{ for } t=0,1, \qquad (40)
\end{aligned}
$$

with n_{i0} given for all i. The planner maximizes the present discounted value of output produced by employed workers with the market technology and by unemployed workers with the home technology, net of the costs of vacancy creation. The planner takes as given the law of motion for employment relationships, as well as a constraint (40) that imposes that the planner's choices of vacancies and market tightness across markets must be consistent with the total measure of job seekers in each period. In what follows, the latter constraint is associated with a Lagrange multiplier λ_t for $t=0,1$, reflecting the planner's shadow value of job seekers.

The first order conditions for the planner's choice of v_{it},θ_{it} for $t=0,1$, read

$$
\begin{aligned}
\kappa_v(x_{i1})+\frac{\lambda_1}{\theta_{i1}} &= q(\theta_{i1})(z_1-b), \\
\frac{\lambda_1}{\theta_{i1}^2} &= -q'(\theta_{i1})(z_1-b), \\
\kappa_v(x_{i0})+\frac{\lambda_0}{\theta_{i0}} &= q(\theta_{i0})[z_0-b+\beta(1-\delta)(z_1-b-\kappa_n(x_{i1})-\lambda_1)], \\
\frac{\lambda_0}{\theta_{i0}^2} &= -q'(\theta_{i0})[z_0-b+\beta(1-\delta)(z_1-b-\kappa_n(x_{i1})-\lambda_1)].
\end{aligned}
$$

Note that these are independent of producer size, and in what follows I hence drop the producer index i to consider symmetric allocations.

Taken together, the optimality conditions imply that

$$
\begin{aligned}
\frac{\kappa_v(x_1)}{\mu'(\theta_1)} &= z_1-b, \\
\frac{\kappa_v(x_0)}{\mu'(\theta_0)} &= z_0-b+\beta(1-\delta)(\frac{\kappa_v(x_1)}{\mu'(\theta_1)}(1-\mu(\theta_1)+\theta_1\mu'(\theta_1))-\kappa_n(x_1)),
\end{aligned}
$$

with the Lagrange multipliers satisfying

$$
\lambda_t=\kappa_v(x_t)\frac{\mu(\theta_t)-\mu'(\theta_t)\theta_t}{\mu'(\theta_t)}
$$

for $t=0,1$.

In addition, the planner's allocation must also satisfy the constraint (40), $x_t=\theta_t(1-N_t)/N_t$ for $t=0,1$, where the total measure of existing relationships satisfies the law of motion $N_1=(1-\delta)(1+q(\theta_0)x_0)N_0$.

C Calibration Details

The law of motion for matches implies steady-state unemployment:

$$u = 1 - N - \mu(\theta)(1-N) = \frac{\mu(\theta)(1-\delta)}{\mu(\theta)+\delta-\mu(\theta)\delta},$$

and if δ is given, a target for steady-state u determines $\mu(\theta)$.

Given a target for the tightness θ, the matching function parameter γ is then pinned down (uniquely) from $\mu(\theta) = \theta/(1+\theta^{\ell})^{1/\ell}$. This also determines steady-state values of $x = \theta(1-N)/N = \delta\theta/((1-\delta)\mu(\theta))$ and $\mu'(\theta)$.

These labor market flows must also be consistent with the model Euler equation: in the case of the firm wage model, equation (25), and the case of the standard model, equation (17). The Euler equation pins down a unique value of $(z-b)/\kappa_0$ that allows the Euler equation to hold with the flows chosen. This still allows alternative combinations of b, κ_0 consistent with any such value, however.

To consider the implications for wages and profits, note that in either case, the firm's first order conditions for vacancy creation together with the dynamic equation for the continuation value of the firm imply that in steady state:

$$\frac{\kappa_v(x)+\frac{X}{\theta}}{q(\theta)} = z - b + \beta(1-\delta)[\frac{\kappa_v(x)+\frac{X}{\theta}}{q(\theta)} - \kappa_n(x) - X].$$

To connect this to wages, note that the present value of wages satisfies $W = X/\mu(\theta) - Y$, where $Y = -(b+\beta(1-\delta)X)/(1-\beta(1-\delta))$.[35] Using this above, we have

$$\frac{\kappa_v(x)+\frac{X}{\theta}}{q(\theta)} = z + \beta(1-\delta)[\frac{\kappa_v(x)+\frac{X}{\theta}}{q(\theta)} - \kappa_n(x)] + (1-\beta(1-\delta))Y,$$

or

$$\frac{\kappa_v(x)}{q(\theta)} + W = z + \beta(1-\delta)[\frac{\kappa_v(x)}{q(\theta)} + W - \kappa_n(x)],$$

which implies that the steady-state per-period wage $w = W(1-\beta(1-\delta))$ satisfies

$$w = z - \frac{\kappa_v(x)}{q(\theta)} + \beta(1-\delta)[\frac{\kappa_v(x)}{q(\theta)} - \kappa_n(x)].$$

For both models to have the same steady-state wage, conditional on having the same steady-state flows, they must have the same κ_0. If this is the case, it follows that firm profits are also the same across models, as firm profit per worker equals

$$\frac{(n+q(\theta)v)(z-w) - \kappa(v,n)}{n+q(\theta)v} = \frac{(1+q(\theta)x)(z-w)-\kappa(x)}{1+q(\theta)x}.$$

[35] Appendix A shows that $y_t = -b - \beta(1-\delta)X_{t+1}$, and by definition $Y = y/(1-\beta(1-\delta))$.

The calibration approach used first adopts a baseline parametrization for the standard model, involving targets for steady-state flows and a choice of the parameter b, with κ_0 set to satisfy the corresponding Euler equation. For a comparable parametrization of the firm wage model then, the targets for the steady-state flows are held unchanged, as is the value of κ_0, to keep the steady-state wage and profit rate unchanged across models. The value of b is then determined by the Euler equation for that model.

D Solving: Firm Wages with Aggregate Shocks

The full non-linear dynamic system to solve for the firm wage equilibrium with aggregate shocks is given below. The last five equations define some variables of interest based on the solution (employment, unemployment, the vacancy-unemployment ratio, firm value, and firm value if the firm did not hire in the current period at all).

$$
\begin{aligned}
&\hat{\kappa}'(x_t) + \frac{X_t}{\theta_t} = q(\theta_t)(z_t - b + \beta(1-\delta)V_{t+1}) \\
&\frac{X_t}{\theta_t^2}(1 + \frac{\mu'(\theta_t)\theta_t^2}{x_t\mu(\theta_t)^2}) = -q'(\theta_t)(z_t - b + \beta(1-\delta)V_{t+1}) \\
&V_t = z_t - b - X_t + \beta(1-\delta)V_{t+1} - \hat{\kappa}(x) + x\hat{\kappa}'(x) \\
&N_{t+1} = (1-\delta)(N_t + \mu(\theta_t)(1-N_t)) \\
&x_t = v_t/N_t \\
&\theta_t(1-N_t) = v_t \\
&X_t = \mu(\theta_t)(W_t + Y_t) \\
&W_t = w_t + \beta(1-\delta)W_{t+1} \\
&y_t = -b - \beta(1-\delta)X_{t+1} \\
&Y_t = y_t + \beta(1-\delta)Y_{t+1} \\
&z_{t+1} - 1 = \rho_z(z_t - 1) + \epsilon_{zt+1} \\
&e_t = N_t + \mu(\theta_t)(1-N_t) \\
&u_t = 1 - e_t \\
&vuratio_t = v_t/u_t \\
&V_{obj,t} = -X_t/\mu(\theta_t) + z_t - b + \beta(1-\delta)V_{t+1} - \hat{\kappa}(x) + x\hat{\kappa}'(x) \\
&V_{objnh,t} = z_t - b + \beta(1-\delta)V_{t+1}
\end{aligned}
$$

This uses that $\kappa_v(x) = \hat{\kappa}'(x)$ and $\kappa_n(x) = \hat{\kappa}(x) - x\hat{\kappa}'(x)$.

E Solving: Infrequent Adjustment and Aggregate Shocks

This section considers the solution approach adopted for the equilibrium with infrequent adjustment and aggregate shocks. The challenge is that in principle the distribution of wages is a state variable, with individual firm behavior affected by the firm's prevailing wage, and feeding into the equilibrium adding up condition. The model is solved by linearization, following the approach of Gertler and Trigari (2009). Once the equations are linearized, only the average wage appears in the system characterizing equilibrium.

Given a wage w, we have the present value of wages:

$$W(w) = \frac{w}{1-\beta(1-\delta)(1-\alpha)} + \beta(1-\delta)\alpha \sum_{k=0}^{\infty} \beta^k (1-\delta)^k (1-\alpha)^k W_{t+k+1}.$$

For short, let $\Lambda_t = \beta(1-\delta)\alpha \sum_{k=0}^{\infty} \beta^k (1-\delta)^k (1-\alpha)^k W_{t+k+1}$, which satisfies the dynamic equation

$$\frac{\Lambda_t}{\beta(1-\delta)\alpha} = W_{t+1} + \beta(1-\delta)(1-\alpha)\frac{\Lambda_{t+1}}{\beta(1-\delta)\alpha}.$$

First, I solve for a linear approximation to the firm continuation value when the wage is fixed: $V_t^f(w) - \bar{V} = V_t^0 + V_t^1(w - \bar{w})$.

While a firm's wage w is fixed, the present value of wages at the firm follows:

$$W_t(w) - \bar{W} = \frac{w - \bar{w}}{1-\beta(1-\delta)(1-\alpha)} + \Lambda_t - \bar{\Lambda},$$

where the equilibrium contracting wages (not the wage held fixed w) determine Λ_t according to

$$\frac{\Lambda_t - \bar{\Lambda}}{\beta(1-\delta)\alpha} = W_{t+1} - \bar{W} + \beta(1-\delta)(1-\alpha)\frac{\Lambda_{t+1} - \bar{\Lambda}}{\beta(1-\delta)\alpha}.$$

The present value of wages $W_t(w)$ determines the tightness according to:

$$X_t - \bar{X} = \mu'(\bar{\theta})(\bar{W} + \bar{Y})(\theta_t(w) - \bar{\theta}) + \mu(\bar{\theta})(W_t(w) - \bar{W} + Y_t - \bar{Y}),$$

as a linear function $\theta(w, S) - \bar{\theta} = A_t + B(w - \bar{w})$ with

$$B = -\frac{\mu(\bar{\theta})}{\mu'(\bar{\theta})(\bar{W} + \bar{Y})(1-\beta(1-\delta)(1-\alpha))},$$

$$A_t = \frac{1}{\mu'(\bar{\theta})(\bar{W} + \bar{Y})}(X_t - \bar{X} - \mu(\bar{\theta})(\Lambda_t - \bar{\Lambda} + Y_t - \bar{Y})).$$

The firm's choice of x follows:

$$\hat{\kappa}''(\bar{x})(x_t - \bar{x}) + \frac{X_t - \bar{X}}{\bar{\theta}} - \frac{\bar{X}}{\bar{\theta}^2}(\theta_t - \bar{\theta}) = q'(\bar{\theta})(\bar{z} - b + \beta(1-\delta)\bar{V})(\theta_t - \bar{\theta})$$
$$+ q(\bar{\theta})(z_t - \bar{z} + \beta(1-\delta)(\alpha(V_{t+1} - \bar{V}) + (1-\alpha)(V^0_{t+1} + V^1_{t+1}(w - \bar{w}))).$$

Substituting in for $\theta_t(w)$, this gives the hiring rate x as a linear function $x_t(w) - \bar{x} = \hat{A}_t + \hat{B}_t(w - \bar{w})$, where

$$\hat{B}_t = \frac{B\bar{X}}{\hat{\kappa}''(\bar{x})\bar{\theta}^2} + \frac{Bq'(\bar{\theta})(\bar{z} - b + \beta(1-\delta)\bar{V})}{\hat{\kappa}''(\bar{x})} + \frac{q(\bar{\theta})}{\hat{\kappa}''(\bar{x})}\beta(1-\delta)(1-\alpha)V^1_{t+1},$$

$$\hat{A}_t = -\frac{1}{\hat{\kappa}''(\bar{x})\bar{\theta}}(X_t - \bar{X}) + \frac{\bar{X}}{\hat{\kappa}''(\bar{x})\bar{\theta}^2}A_t + \frac{q'(\bar{\theta})}{\hat{\kappa}''(\bar{x})}(\bar{z} - b + \beta(1-\delta)\bar{V})A_t$$
$$+ \frac{q(\bar{\theta})}{\hat{\kappa}''(\bar{x})}(z_t - \bar{z} + \beta(1-\delta)(\alpha(V_{t+1} - \bar{V}) + (1-\alpha)V^0_{t+1})).$$

Finally, the dynamic equation for the value $V^f(w, S)$ implies that for all such w we have:

$$V^0_t + V^1_t(w - \bar{w})$$
$$= z_t - \bar{z} + \beta(1-\delta)(\alpha(V_{t+1} - \bar{V}) + (1-\alpha)(V^0_{t+1} + V^1_{t+1}(w - \bar{w}))) + \bar{x}\hat{\kappa}''(\bar{x})(x_t(w) - \bar{x}) - (X_t - \bar{X}).$$

Using the expression for $x_t(w)$, the expression yields equations for the constant and coefficient on w for this equation to hold.

The coefficient on w thus satisfies:

$$V^1_t = \beta(1-\delta)(1-\alpha)V^1_{t+1} + \frac{\bar{x}\bar{X}B}{\bar{\theta}^2} + \bar{x}q'(\bar{\theta})(\bar{z} - b + \beta(1-\delta)\bar{V})B + \bar{x}q(\bar{\theta})\beta(1-\delta)(1-\alpha)V^1_{t+1}.$$

Note that this is an unstable equation with constant coefficients, implying the coefficient V^1_t is a constant. Further, $\hat{B}_t$ is also then a constant.

The constant satisfies:

$$V^0_t = z_t - \bar{z} + \beta(1-\delta)(\alpha(V_{t+1} - \bar{V}) + (1-\alpha)V^0_{t+1}) - (X_t - \bar{X}) - \frac{\bar{x}}{\bar{\theta}}(X_t - \bar{X}) + \frac{\bar{x}\bar{X}}{\bar{\theta}^2}A_t$$
$$+ \bar{x}q'(\bar{\theta})(\bar{z} - b + \beta(1-\delta)\bar{V})A_t + \bar{x}q(\bar{\theta})(z_t - \bar{z} + \beta(1-\delta)(\alpha(V_{t+1} - \bar{V}) + (1-\alpha)V^0_{t+1})).$$

This is a dynamic equation that is also unstable, but with coefficients that can vary over time. Add this equation into the model system, to determine the coefficients (they enter into the system).

Second, proceed to solve for equilibrium.

Firms that are optimizing this period choose a wage according to:

$$\frac{X_t - \bar{X}}{\bar{\theta}^2} - 2\frac{\bar{X}}{\bar{\theta}^3}(\theta_t - \bar{\theta}) + \frac{\mu'(\bar{\theta})}{\bar{x}\mu(\bar{\theta})^2}(X_t - \bar{X}) - \frac{\mu'(\bar{\theta})\bar{X}}{\bar{x}^2\mu(\bar{\theta})^2}(x_t - \bar{x}) + \frac{\bar{X}}{\bar{x}}\frac{\mu(\bar{\theta})^2\mu''(\bar{\theta}) - 2\mu(\bar{\theta})\mu'(\bar{\theta})^2}{\mu(\bar{\theta})^4}(\theta_t - \bar{\theta})$$
$$= -q''(\bar{\theta})(\bar{z} - b + \beta(1-\delta)\bar{V})(\theta_t - \bar{\theta})$$
$$- q'(\bar{\theta})(z_t - \bar{z} + \beta(1-\delta)(\alpha(V_{t+1} - \bar{V}) + (1-\alpha)(V^0_{t+1} + V^1_{t+1}(w_t - \bar{w})))$$
$$- \beta(1-\delta)(1-\alpha)[\frac{q'(\bar{\theta})\bar{V}^1}{\bar{\theta}_w}(\theta_t - \bar{\theta}) - \frac{\bar{V}^1}{\bar{\theta}_w x^2}(x_t - \bar{x}) + \frac{(1+q(\bar{\theta})\bar{x})}{\bar{\theta}_w\bar{x}}(V^1_{t+1} - \bar{V}^1) - \frac{(1+q(\bar{\theta})\bar{x})\bar{V}^1}{\bar{\theta}_w^2\bar{x}}(\theta_{wt} - \bar{\theta}_w)]$$

with $\theta_t(w) = A_t + B(w - \bar{w})$, $x_t(w) = \hat{A}_t + \hat{B}_t(w - \bar{w})$ from above and

$$\frac{\mu'(\bar{\theta})}{\mu(\bar{\theta})^2}\bar{\theta}_w(X_t - \bar{X}) + \frac{\mu'(\bar{\theta})}{\mu(\bar{\theta})^2}\bar{X}(\theta_{wt} - \bar{\theta}_w) + \frac{\mu(\bar{\theta})^2\mu''(\bar{\theta}) - 2\mu(\bar{\theta})\mu'(\bar{\theta})^2}{\mu(\bar{\theta})^4}\bar{X}\bar{\theta}_w(\theta_t - \bar{\theta}) = 0.$$

The rest of firms apply a previously set wage, and the cross-firm average wage follows: $\hat{w}_t = \alpha w_t + (1-\alpha)\hat{w}_{t-1}$.

The cross-firm average tightness and vacancy rate are: $\hat{\theta}_t = A_t + B(\hat{w}_t - \bar{w})$, $\hat{x}_t = \hat{A}_t + \hat{B}_t(\hat{w}_t - \bar{w})$.

The average firm size follows the law of motion:

$$\hat{n}_{t+1} - \bar{n} = (1-\delta)((1 + q(\bar{\theta})\bar{x})(\hat{n}_t - \bar{n}) + \bar{n}q(\bar{\theta})(\hat{x}_t - \bar{x}) + \bar{n}q'(\bar{\theta})\bar{x}(\hat{\theta}_t - \bar{\theta})).$$

Finally, the equilibrium adding up constraint reads:

$$\frac{\bar{n}}{\bar{\theta}}(\hat{x}_t - \bar{x}) + \frac{\bar{x}}{\bar{\theta}}(\hat{n}_t - \bar{n}) - \frac{\bar{x}\bar{n}}{\bar{\theta}^2}(\hat{\theta}_t - \bar{\theta}) = -(\hat{n}_t - \bar{n}).$$

Steady state: Guess θ. This implies values for $N = \mu(\theta)(1-\delta)/(1-(1-\delta)(1-\mu(\theta)))$ and $x = \theta(1-N)/N$. Firm continuation value satisfies $V = (z - b - \hat{\kappa}(x) + x\hat{\kappa}'(x) - X)/(1 - \beta(1-\delta))$. Plugging this into the optimality condition for vacancies allows solving for X:

$$\hat{\kappa}'(x) + \frac{X}{\theta} = q(\theta)[z - b + \beta(1-\delta)\frac{z - b - \hat{\kappa}(x) + x\hat{\kappa}'(x) - X}{1 - \beta(1-\delta)}],$$

$$X = \frac{-\hat{\kappa}'(x) + q(\theta)[z - b + \beta(1-\delta)\frac{z-b-\hat{\kappa}(x)+x\hat{\kappa}'(x)}{1-\beta(1-\delta)}]}{\frac{1}{\theta} + \frac{q(\theta)\beta(1-\delta)}{1-\beta(1-\delta)}}.$$

One can then solve for θ_w and V_w. Finally, the optimality condition for wage/tightness gives an equation determining θ. Then, $W = X/\mu(\theta) - Y$ and $\Lambda = \beta(1-\delta)\alpha W/(1 - \beta(1-\delta)(1-\alpha))$.

Linearized system:

$$\frac{\Lambda_t}{\beta(1-\delta)\alpha} = W_{t+1} + \beta(1-\delta)(1-\alpha)\frac{\Lambda_{t+1}}{\beta(1-\delta)\alpha}$$

$$W_t - \bar{W} = \frac{w_t - \bar{w}}{1-\beta(1-\delta)(1-\alpha)} + \Lambda_t - \bar{\Lambda}$$

$$\frac{\bar{\mu}'}{\bar{\mu}^2}\bar{\theta}_w(X_t - \bar{X}) + \frac{\bar{\mu}'}{\bar{\mu}^2}\bar{X}(\theta_{wt} - \bar{\theta}_w) + \frac{\bar{\mu}^2\bar{\mu}'' - 2\bar{\mu}(\bar{\mu}')^2}{\bar{\mu}^4}\bar{X}\bar{\theta}_w(\theta_t - \bar{\theta}) = 0$$

$$\frac{X_t - \bar{X}}{\bar{\theta}^2} - 2\frac{\bar{X}}{\bar{\theta}^3}(\theta_t - \bar{\theta}) + \frac{\bar{\mu}'}{\bar{x}\bar{\mu}^2}(X_t - \bar{X}) - \frac{\bar{\mu}'\bar{X}}{\bar{x}^2\bar{\mu}^2}(x_t - \bar{x}) + \frac{\bar{X}}{\bar{x}}\frac{\bar{\mu}^2\bar{\mu}'' - 2\bar{\mu}(\bar{\mu}')^2}{\bar{\mu}^4}(\theta_t - \bar{\theta})$$
$$= -\bar{q}''[\bar{z} - b + \beta(1-\delta)\bar{V}](\theta_t - \bar{\theta})$$
$$- \bar{q}'[z_t - \bar{z} + \beta(1-\delta)(\alpha(V_{t+1} - \bar{V}) + (1-\alpha)(V^0_{t+1} + V^1_{t+1}(w_t - \bar{w})))]$$
$$- \beta(1-\delta)(1-\alpha)[\frac{\bar{q}'\bar{V}^1}{\bar{\theta_w}}(\theta_t - \bar{\theta}) - \frac{\bar{q}\bar{V}^1}{\bar{\theta}_w\bar{x}^2}(x_t - \bar{x}) + \frac{1+\bar{q}\bar{x}}{\bar{\theta}_w\bar{x}}(V^1_{t+1} - \bar{V}^1) - \frac{(1+\bar{q}\bar{x})\bar{V}^1}{\bar{\theta}_w^2\bar{x}}(\theta_{wt} - \bar{\theta}_w)]$$

$$V^0_t = z_t - \bar{z} + \beta(1-\delta)(\alpha(V_{t+1} - \bar{V}) + (1-\alpha)V^0_{t+1}) - (X_t - \bar{X}) - \frac{\bar{x}}{\bar{\theta}}(X_t - \bar{X}) + \frac{\bar{x}\bar{X}}{\bar{\theta}^2}A_t$$
$$+ \bar{x}\bar{q}'[\bar{z} - b + \beta(1-\delta)\bar{V}]A_t + \bar{x}\bar{q}[z_t - \bar{z} + \beta(1-\delta)(\alpha(V_{t+1} - \bar{V}) + (1-\alpha)V^0_{t+1})]$$

$$\hat{A}_t = -\frac{X_t - \bar{X}}{\bar{\kappa}''\bar{\theta}} + \frac{\bar{X}A_t}{\bar{\kappa}''\bar{\theta}^2} + \frac{\bar{q}'}{\bar{\kappa}''}[\bar{z} - b + \beta(1-\delta)\bar{V}]A_t$$
$$+ \frac{\bar{q}}{\bar{\kappa}''}[z_t - \bar{z} + \beta(1-\delta)(\alpha(V_{t+1} - \bar{V}) + (1-\alpha)V^0_{t+1})]$$

$$A_t = \frac{1}{\bar{\mu}'(\bar{W} + \bar{Y})}(X_t - \bar{X} - \bar{\mu}(\Lambda_t - \bar{\Lambda} + Y_t - \bar{Y}))$$

$$\hat{n}_{t+1} - \bar{n} = (1-\delta)((1+\bar{q}\bar{x})(\hat{n}_t - \bar{n}) + \bar{n}\bar{q}(\hat{x}_t - \bar{x}) + \bar{n}\bar{q}'\bar{x}(\hat{\theta}_t - \bar{\theta}))$$

$$\frac{\bar{n}}{\bar{\theta}}(\hat{x}_t - \bar{x}) + \frac{\bar{x}}{\bar{\theta}}(\hat{n}_t - \bar{n}) - \frac{\bar{x}\bar{n}}{\bar{\theta}^2}(\hat{\theta}_t - \bar{\theta}) = -(\hat{n}_t - \bar{n})$$

$$\hat{w}_t = \alpha w_t + (1-\alpha)\hat{w}_{t-1}$$

$$\theta_t - \bar{\theta} = A_t + B(w_t - \bar{w})$$

$$x_t - \bar{x} = \hat{A}_t + \hat{B}(w_t - \bar{w})$$

$$\hat{\theta}_t - \bar{\theta} = A_t + B(\hat{w}_t - \bar{w})$$

$$\hat{x}_t - \bar{x} = \hat{A}_t + \hat{B}(\hat{w}_t - \bar{w})$$

$$V_t - \bar{V} = V^0_t + V^1(w_t - \bar{w})$$

F Model with Firm-Level Shocks

Consider an environment where firms face idiosyncratic shocks to their productivity. In a stationary equilibrium with firm heterogeneity, the aggregate measure of matches N and the value of job seekers X remain constant, while firm shocks lead to reallocation of labor across firms over time.[36]

The firm problem in this context reads:

$$\max_{\theta,v} -\frac{nX}{\mu(\theta)} + (n+q(\theta)v)(z-b) - \kappa(v,n) - X(\frac{v}{\theta}+n) + \beta E_z V(n',z')$$
$$\text{s.t. } n' = (1-\delta)(n+q(\theta)v),$$

where the continuation value satisfies

$$V(n,z) = (n+q(\theta)v)(z-b) - \kappa(v,n) - X(\frac{v}{\theta}+n) + \beta E_z V(n',z').$$

Scaling by size, these equations again yield the size-independent problem:

$$\max_{\theta,x} -\frac{X}{\mu(\theta)} + (1+q(\theta)x)(z-b+\beta(1-\delta)E_z\hat{V}(z')) - \hat{\kappa}(x) - X(\frac{x}{\theta}+1) \tag{41}$$

where

$$\hat{V}(z) = (1+q(\theta)x)(z-b+\beta(1-\delta)E_z\hat{V}(z')) - \hat{\kappa}(x) - X(\frac{x}{\theta}+1). \tag{42}$$

To arrive at an intertemporal Euler equation, one can combine the firm first order conditions to arrive at:

$$\frac{\kappa_v(x_t)+\frac{X}{\theta_t}}{q(\theta_t)} = z_t - b + \beta(1-\delta)[\frac{\kappa_v(x_{t+1})+\frac{X}{\theta_{t+1}}}{q(\theta_{t+1})} - \kappa_n(x_{t+1}) - X],$$

where the value of job seekers is now constant in this stationary setting, satisfying

$$X = \kappa_v(x_t)\frac{\mu(\theta_t)-\mu'(\theta_t)\theta_t}{\mu'(\theta_t)}\frac{q(\theta_t)x_t}{1+q(\theta_t)x_t}.$$

Definition 3. *A stationary competitive search equilibrium with firm wages is an allocation* $\{w_{it},\theta_{it},x_{it}\}_{t=0}^{\infty}$ $\forall i$ *and job seeker value* X *such that the allocation and value solve the problem (41-42), and that each job seeker applies to one firm:* $1-\sum_i n_{it} = \sum_i x_{it}n_{it}/\theta_{it}$, $\forall t$.

Single Firm Deviation to Longer Wage Commitment Consider introducing into the above equilibrium an individual firm, small relative to the size of the market, that today makes a wage commitment for a probabilistic period of time, returning to equilibrium behavior once the commitment expires.

[36] I abstract from firm entry and exit here, but one could incorporate such a margin by adding exit shocks into the firm problem, with new firms replacing exiting ones. The behavior of new and existing firms is identical if new firms are assumed to enter with at least one worker, due to the size-independence.

The deviating firm chooses a wage w, expecting each period going forward to revert to equilibrium behavior with probability α and to maintain the wage with probability $1-\alpha$. To connect the per-period wage to the market tightness, note that the equilibrium firms' market tightnesses imply these firms offer their workers specific present values of wages for each z, due to the job seeker constraint. Taking these equilibrium values as given, one can solve for the present value of wages for the deviating firm as a function of the wage w and productivity z, denoted below as $\phi(w,z)$.[37] Finally, the job seeker constraint gives the implied tightness: $X = \mu(\theta)(\phi(w,z)+Y)$.

In the period the firm deviates to the longer wage commitment, it chooses a wage w and vacancy creation v, to solve the problem

$$\begin{aligned}&\max_{w,v} -\frac{nX}{\mu(\theta)} + (n+q(\theta)v)(z-b) - \kappa(v,n) - X(\frac{v}{\theta}+n) + \beta E_z(\alpha V(n',z') + (1-\alpha)V^f(n',w,z'))\\ &\text{s.t. } n' = (1-\delta)(n+q(\theta)v),\\ &\quad X = \mu(\theta)(\phi(w,z)+Y),\end{aligned}$$

given n, z. Here the firm expects to revert to equilibrium behavior in the following period with probability α, implying the continuation value $V(n',z')$, and to maintain the wage commitment otherwise, implying the continuation value $V^f(n',w,z')$, discussed below.

In periods when the firm maintains the wage commitment, it only chooses vacancies, to solve the problem:

$$\begin{aligned}&\max_{v}(n+q(\theta)v)(z-b) - \kappa(v,n) - X(\frac{v}{\theta}+n) + \beta E_z(\alpha V(n',z') + (1-\alpha)V^f(n',w,z'))\\ &\text{s.t. } n' = (1-\delta)(n+q(\theta)v),\end{aligned}$$

where the tightness θ is determined by the job seeker constraint $X = \mu(\theta)(\phi(w,z)+Y)$. The continuation value $V^f(n',w,z')$ satisfies

$$V^f(n,w,z) = (n+q(\theta)v)(z-b) - \kappa(v,n) - X(\frac{v}{\theta}+n) + \beta E_z(\alpha V(n',z') + (1-\alpha)V^f(n',w,z')).$$

The deviating firm's problems can also be scaled to arrive at size-independent problems. Defining $\hat{V}^f(w,z) := V^f(n,w,z)/n$, the deviating firm chooses w, x to solve

$$\begin{aligned}&\max_{w,x} -\frac{X}{\mu(\theta)} + (1+q(\theta)x)(z-b+\beta(1-\delta)E_z(\alpha\hat{V}(z') + (1-\alpha)\hat{V}^f(w,z'))) - \hat{\kappa}(x) - X(\frac{x}{\theta}+1)\\ &\text{s.t. } X = \mu(\theta)(\phi(w,z)+Y).\end{aligned}$$

[37] Denote the vector of equilibrium present values of wages across z as $\mathbf{W}$ and that of the deviating firm as $\mathbf{W}^f(w)$. We have that $\mathbf{W}^f(w) = w\mathbf{i} + \beta(1-\delta)[\alpha\Pi\mathbf{W} + (1-\alpha)\Pi\mathbf{W}^f(w)]$, where Π is the transition matrix for the productivity process and $\mathbf{i}$ a vector of ones. This gives the deviating firm's present values as $\mathbf{W}^f(w) = (I - \beta(1-\delta)(1-\alpha)\Pi)^{-1}(w\mathbf{i} + \beta(1-\delta)\alpha\Pi\mathbf{W})$. I denote the components of this vector in the text by $\phi(w,z)$. Note that the derivative of the value satisfies $\phi_w(w,z) = (1-\beta(1-\delta)(1-\alpha))^{-1}$.

In periods when the firm maintains the commitment to w, it chooses x to solve

$$\max_x (1+q(\theta)x)(z-b+\beta(1-\delta)E_z(\alpha\hat{V}(z')+(1-\alpha)\hat{V}^f(w,z')))-\hat{\kappa}(x)-X(\frac{x}{\theta}+1),$$

where the tightness θ is determined by the job seeker constraint $X=\mu(\theta)(\phi(w,z)+Y)$. The continuation value satisfies

$$\hat{V}^f(w,z)=(1+q(\theta)x)(z-b+\beta(1-\delta)E_z(\alpha\hat{V}(z')+(1-\alpha)\hat{V}^f(w,z')))-\hat{\kappa}(x)-X(\frac{x}{\theta}+1).$$

The two problems yield the same first order condition for vacancy creation

$$\hat{\kappa}'(x)+\frac{X}{\theta}=q(\theta)(z-b+\beta(1-\delta)E_z(\alpha\hat{V}(z')+(1-\alpha)\hat{V}^f(w,z'))),$$

for the deviation period and periods when the commitment is maintained. Meanwhile, the deviating firm's first order condition characterizing the wage-tightness tradeoff reads

$$\begin{aligned}\frac{X}{\theta^2}[1+\frac{\mu'(\theta)\theta^2}{x\mu(\theta)^2}]=&-q'(\theta)[z-b+\beta(1-\delta)E_z[\alpha\hat{V}(z')+(1-\alpha)\hat{V}^f(w,z')]]\\&-\beta(1-\delta)(1-\alpha)(1+q(\theta)x)/xE_z\hat{V}^f_w(w,z')/\theta_w,\end{aligned}$$

where the derivative of θ with respect to w is $\theta_w=-\mu(\theta)^2/(\mu'(\theta)X(1-\beta(1-\delta)(1-\alpha)))$, while the derivative of the continuation value satisfies

$$\begin{aligned}\hat{V}^f_w(w,z)=&xq'(\theta)[z-b+\beta(1-\delta)E_z[\alpha\hat{V}(z')+(1-\alpha)\hat{V}^f(w,z')]]\theta_w\\&+\frac{xX}{\theta^2}\theta_w+\beta(1-\delta)(1-\alpha)(1+q(\theta)x)E_z\hat{V}^f_w(w,z').\end{aligned}$$

Equilibrium with Infrequent Adjustment If a longer wage commitment is profitable for the deviating firm, it becomes interesting to consider an equilibrium where all firms follow a strategy of infrequent adjustment.

To think about these questions, suppose all firms reoptimize their wage w each period with probability α and maintain their existing wage commitment with probability $1-\alpha$. To connect the per-period wage to the corresponding market tightness, one can again solve for the present value of wages as a function of the wage w and productivity z, denoted $\phi(w,z)$.[38]

[38] Denote the vector of equilibrium present values of wages for a reoptimizing firm across z as $\mathbf{W}^r$ and that of a firm maintaining wage commitment w as $\mathbf{W}^f(w)$. We have that $\mathbf{W}^f(w)=w\mathbf{i}+\beta(1-\delta)[\alpha\Pi\mathbf{W}^r+(1-\alpha)\Pi\mathbf{W}^f(w)]$, where Π is the transition matrix for the productivity process and $\mathbf{i}$ a vector of ones. This gives the deviating firm's present values as $\mathbf{W}^f(w)=(I-\beta(1-\delta)(1-\alpha)\Pi)^{-1}(w\mathbf{i}+\beta(1-\delta)\alpha\Pi\mathbf{W}^r)$. I denote the components of this vector in the text by $\phi(w,z)$.

In this case, firms reoptimizing wages solve

$$\max_{w,v} -\frac{nX}{\mu(\theta)} + (n+q(\theta)v)(z-b) - \kappa(v,n) - X(\frac{v}{\theta}+n) + \beta E_z(\alpha V^r(n',z') + (1-\alpha)V^f(n',w,z'))$$
$$\text{s.t. } n' = (1-\delta)(n+q(\theta)v),$$
$$X = \mu(\theta)(\phi(w,z)+Y),$$

where the implied continuation value satisfies

$$V^r(n,z) = (n+q(\theta)v)(z-b) - \kappa(v,n) - X(\frac{v}{\theta}+n) + \beta E_z(\alpha V^r(n',z') + (1-\alpha)V^f(n',w,z')),$$

and firms holding the wage commitment fixed solve

$$\max_{v}(n+q(\theta)v)(z-b) - \kappa(v,n) - X(\frac{v}{\theta}+n) + \beta E_z(\alpha V^r(n',z') + (1-\alpha)V^f(n',w,z'))$$
$$\text{s.t. } n' = (1-\delta)(n+q(\theta)v),$$

where the tightness θ is determined by the job seeker constraint $X = \mu(\theta)(\phi(w,z)+Y)$ and the continuation value $V^f(n',w,z')$ satisfies

$$V^f(n,w,z) = (n+q(\theta)v)(z-b) - \kappa(v,n) - X(\frac{v}{\theta}+n) + \beta E_z(\alpha V^r(n',z') + (1-\alpha)V^f(n',w,z')).$$

Once again, the problems can be scaled. Thus, firms reoptimizing wages solve

$$\max_{w,x} -\frac{X}{\mu(\theta)} + (1+q(\theta)x)(z-b+\beta E_z(\alpha\hat{V}^r(z') + (1-\alpha)\hat{V}^f(w,z'))) - \hat{\kappa}(x) - X(\frac{x}{\theta}+1)$$

where the tightness θ is determined by the job seeker constraint $X = \mu(\theta)(\phi(w,z)+Y)$, and the implied continuation value satisfies

$$\hat{V}^r(z) = (1+q(\theta)x)(z-b+\beta E_z(\alpha\hat{V}^r(z') + (1-\alpha)\hat{V}^f(w,z'))) - \hat{\kappa}(x) - X(\frac{x}{\theta}+1),$$

and firms holding the wage commitment fixed solve

$$\max_{x}(1+q(\theta)x)(z-b+\beta E_z(\alpha\hat{V}^r(z') + (1-\alpha)\hat{V}^f(w,z'))) - \hat{\kappa}(x) - X(\frac{x}{\theta}+1),$$

where the tightness θ is determined by the job seeker constraint $X = \mu(\theta)(\phi(w,z)+Y)$ and the continuation value $V^f(w,z')$ satisfies

$$\hat{V}^f(w,z) = (1+q(\theta)x)(z-b+\beta E_z(\alpha\hat{V}(z') + (1-\alpha)\hat{V}^f(w,z'))) - \hat{\kappa}(x) - X(\frac{x}{\theta}+1).$$

The first order conditions for the firms' choice of wage and vacancy creation rate coincide with those for the deviating firm, with the continuation values $\hat{V}^r(z)$ and $\hat{V}^f(w,z)$ as characterized above.

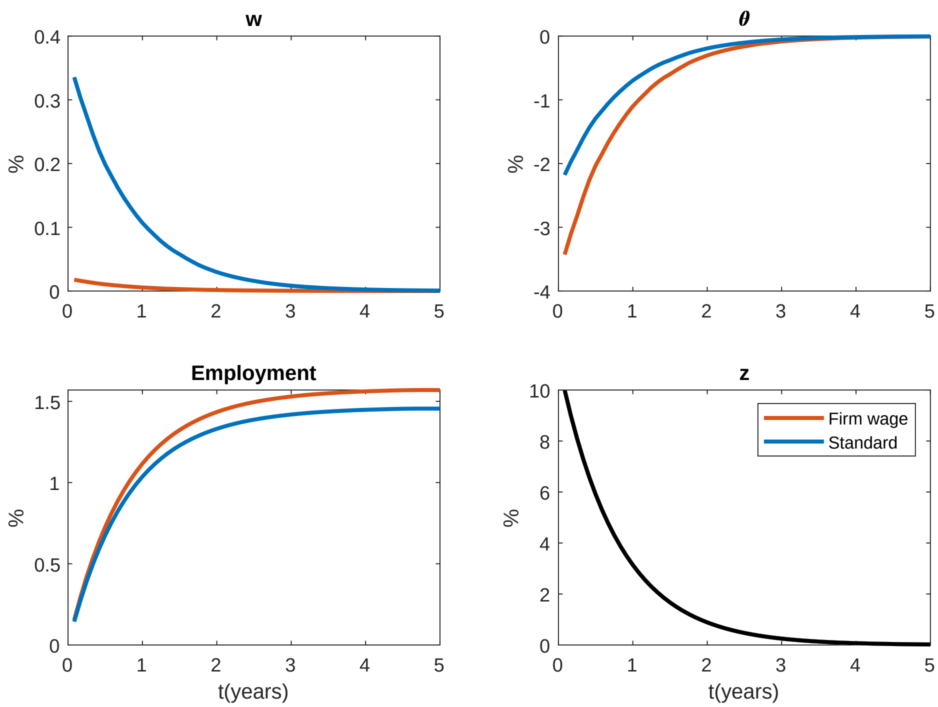

Figure F.1: Impulse Responses in Firm Wage vs Standard Model

Notes: The figure plots the percentage responses of model variables to a ten percent increase in firm-level labor productivity in the firm wage model and the standard competitive search model without firm wages. Labor productivity follows an $AR(1)$ with autocorrelation $\rho_z = 0.9$ and standard deviation $\sigma_z = 0.1$. The response is based on a quadratic approximation, produced with Dynare. The two models compared have the same steady-state levels of wage, tightness, unemployment.

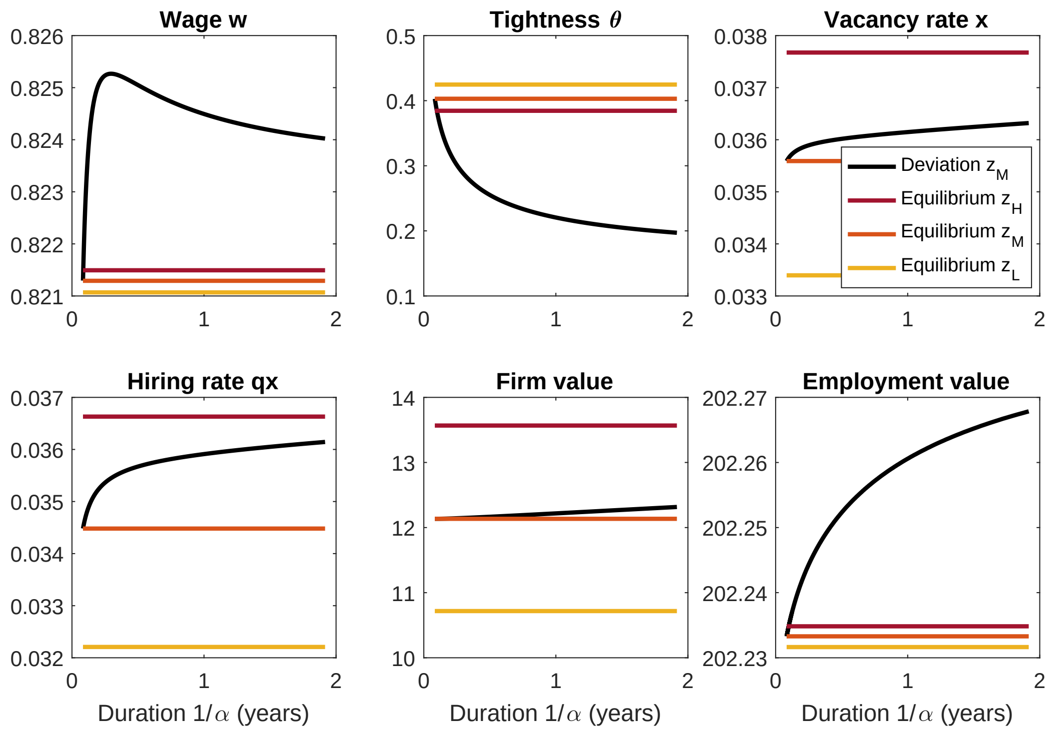

Figure F.2: Single Firm Deviating to Longer Wage Commitment with Firm-Level Shocks

Notes: The figure displays the equilibrium values of a number of variables in the stationary equilibrium with firm wages, along with the corresponding values for an individual firm in that equilibrium that is able to set a wage commitment for a probabilistic period of time. The model is solved on three state grid for productivity, approximating an $AR(1)$ with autocorrelation $\rho_z = 0.9$ and standard deviation $\sigma_z = 0.1$ based on the Rouwenhorst method. The deviating firm is in the intermediate productivity state and its choices are plotted as a function of $1/\alpha$, the expected duration of the wage commitment. The firm value plotted is the scaled value per initial size.

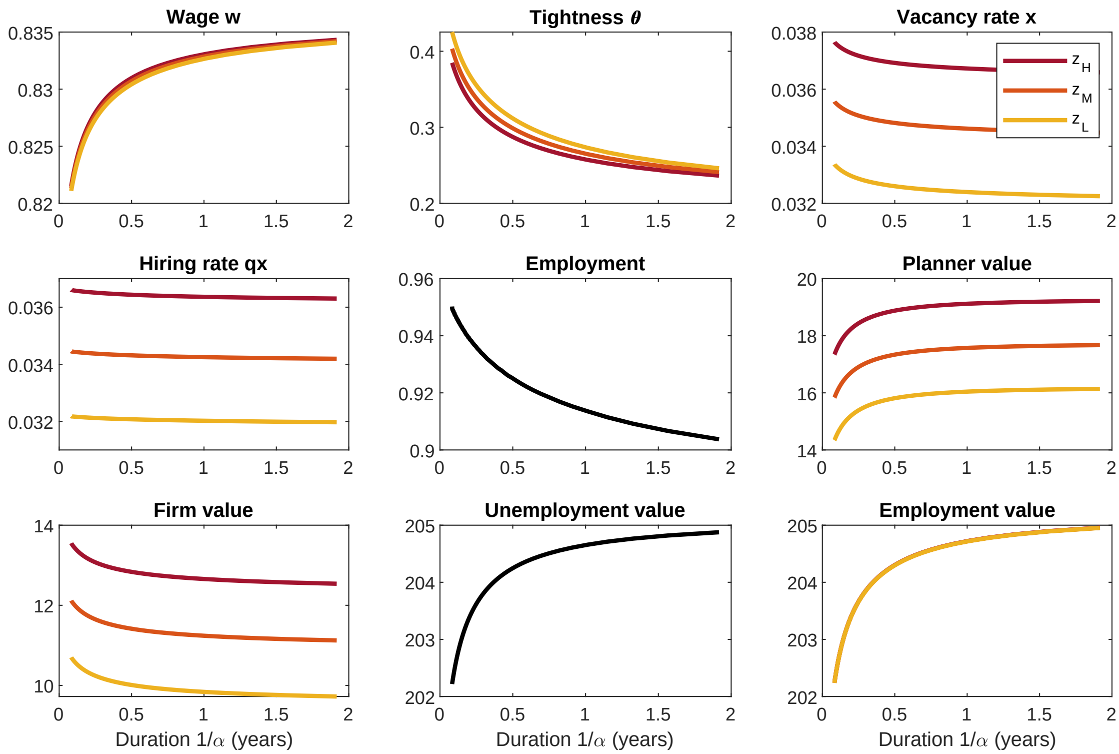

Figure F.3: Equilibrium with Longer Wage Commitments

Notes: The figure displays the values of a number of variables in the stationary equilibrium with firm wages and infrequent adjustment, as a function of $1/\alpha$, the expected duration of wages. The firm value plotted is the scaled value per initial size, but also the unscaled firm value declines in wage duration. Correspondingly, the planner value plotted is the scaled value per initial size. The figure also shows the corresponding values in the efficient allocation.

G Additional Figures

Figure G.1: Impulse Responses with Identical Parameters

Notes: The figure plots the level responses of model variables to a one percent increase in aggregate labor productivity in the firm wage model and the standard competitive search model without firm wages. Productivity follows an $AR(1)$ with autocorrelation $\rho_z = 0.96$ and standard deviation $\sigma_z = 0.02$. The two models compared are parameterized identically. The plotted vacancy-unemployment ratio is its model counterpart, which differs slightly from θ due to timing.

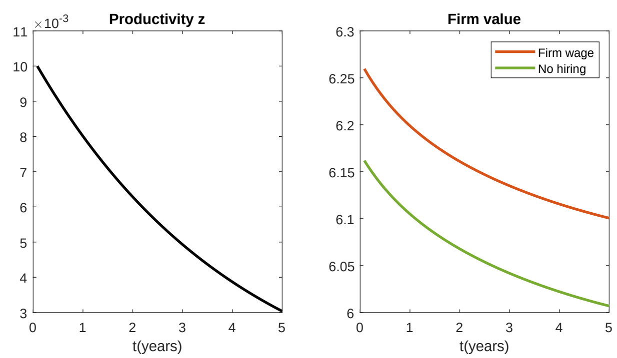

Figure G.2: Impulse Response of Firm Value vs No Hiring Value

Notes: The figure refers to the impulse response in Figure 1. It shows that the firm value attained by following the first order conditions dominates opting out of hiring for a period, throughout the impulse response.

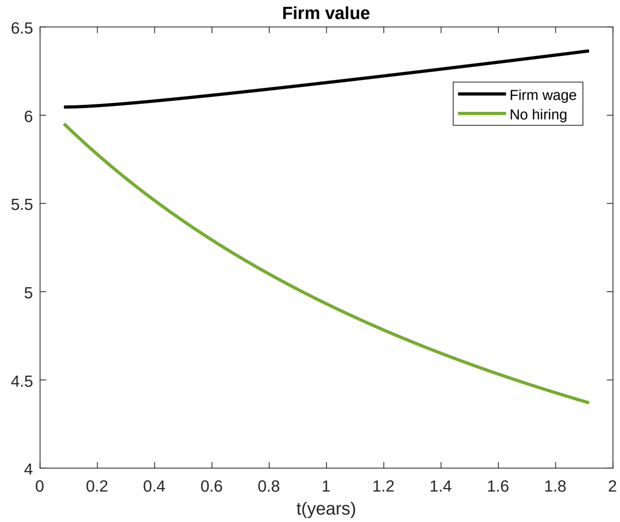

Figure G.3: Single Firm Deviation Value vs No Hiring Value

Notes: The figure refers to the deviating firm in Figure 2. It shows that the firm value attained by following the first order conditions dominates opting out of hiring for the deviation period, across wage durations.

Figure G.4: Single Firm Deviating in the Standard Competitive Search Model

Notes: The figure displays the steady-state values of a number of variables in a stationary equilibrium with competitive search, along with the corresponding values for an individual firm in that equilibrium that is able to set a wage commitment for a probabilistic period of time. The latter are plotted as a function of $1/\alpha$, the expected duration of the wage commitment. The firm value plotted is the scaled firm value per initial size.

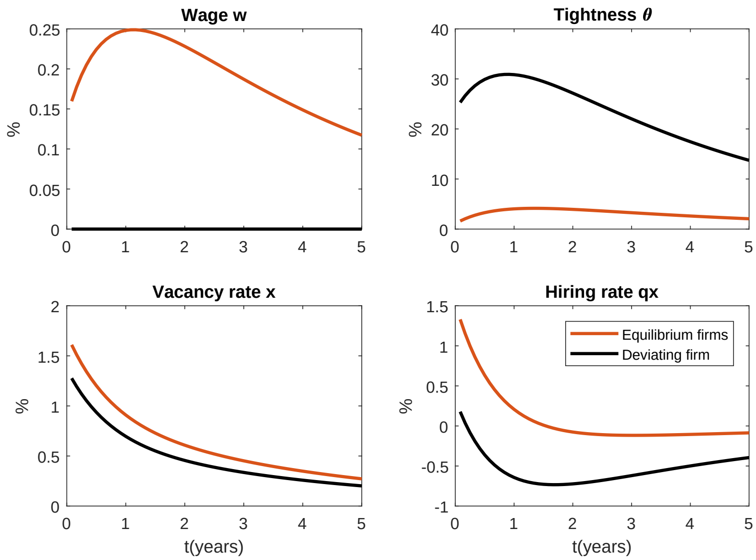

Figure G.5: Impulse Response of Fixed Wage vs Equilibrium Firms

Notes: The figure plots the percentage responses of model variables to a one percent increase in aggregate labor productivity in the firm wage model and for a single firm deviating to a longer wage commitment. Labor productivity follows an $AR(1)$ with autocorrelation $\rho_z = 0.98$ and standard deviation $\sigma_z = 0.02$.

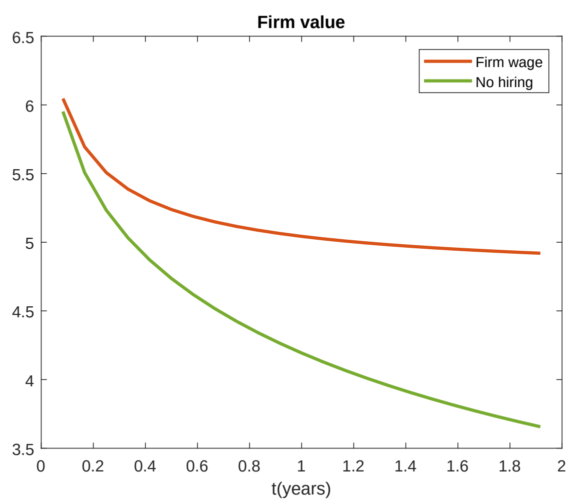

Figure G.6: Equilibrium Firm Value vs No Hiring Value

Notes: The figure refers to the deviating firm in Figure 4. It shows that the firm value attained by following the first order conditions dominates opting out of hiring for the duration of a fixed wage, across wage durations.