Competitive pricing despite search costs when lower price signals quality

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Abstract

The Diamond paradox demonstrates that when learning prices is costly for consumers, each firm has market power. However, making firms privately informed about their quality and cost restores competitive pricing if quality and cost are negatively correlated. Such correlation arises from e.g. regulation, differing equipment or skill, or economies of scale. If good quality firms have lower costs, then they can signal quality by cutting prices, in which case bad quality firms must cut prices to retain customers. This price-cutting race to the bottom ends in an equilibrium in which all firms price nearly competitively and cheap talk reveals quality.

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The Diamond paradox illustrates starkly the possibility that when consumers find it costly to learn prices, each firm has market power. In the unique equilibrium of

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the Diamond paradox, all firms set the same price. Due to equal prices, no consumer engages in costly learning. Since no consumers learn, the price that the firms choose is the monopoly price.

Consumer search is an important part of many product markets. Yet, if consumers do not expect any variability in price or quality, then there is no incentive to search. Even if price and quality differ, consumers may be indifferent between good quality at a high price and bad quality at a cheap price. In this situation, which arises when a high quality firm has a higher cost and consumers are homogeneous, there is still no search, as shown in Appendix C.

Moreover, heterogeneity of consumers (who prefer one price-quality combination enough to search for it) need not put downward pressure on prices, as the Online Appendix proves. A high quality, high cost firm signals its privately known quality by increasing its price, which drives some consumers to switch to a low quality seller. The latter can then profitably raise its price above the monopoly level to hold up the switchers whose search cost is sunk.

This paper identifies a setting in where consumer search does discipline firm pricing. If quality and cost are negatively related and private, then higher quality is signalled through a lower price. While this might seem counterintuitive, there are markets where it has been empirically verified that higher quality is indeed correlated with lower prices. Examples are mutual funds, private-label foods and some categories of electronics. Section 4 provides additional empirical evidence and explains how the negative association of cost and quality can be caused by for example regulation, differing equipment or skill, economies of scale. By combining higher quality with lower costs, not only do high quality firms price below their monopoly level, but all firms set a price that is close to perfectly competitive. Private information thus neutralises the market power coming from costly learning.

In the markets this paper studies, there are at least two firms, each of which draws an independent type, either *good* or *bad*. The good type has lower marginal cost and higher quality than the bad. Each firm knows its own type, but other players only have a common prior over a firm's type. First the firms simultaneously set prices. Second, each consumer observes the price of one firm and chooses either to buy from this firm, leave the market or pay a small cost to learn the price of another firm. Finally, each consumer who learned chooses either to buy from one of the firms whose price he knows or leave the market. The consumers have a distribution of valuations. A higher-valuation consumer values high quality relatively more. Consumers update their beliefs about the type of a firm whose price they see, using Bayes' rule whenever possible. The equilibrium concept is perfect Bayesian equilibrium (PBE). A unique equilibrium remains after refining with the Intuitive Criterion.

In equilibrium, prices are close to competitive due to a *race to the bottom* consisting of two forces. One is downward price signalling: the good quality firm reduces price to distinguish itself from the bad quality firm and attract greater demand. The Intuitive Criterion determines consumers' beliefs off the equilibrium path, ruling out belief threats that would prevent a good type from signalling quality via a lower price. The second force is that a bad quality firm cuts price to retain its customers and attract those at the other firm if the other also has bad quality. The bad quality firms are in Bertrand competition over the consumers who learn more than one price, which all consumers do when faced with a price indicative of a bad quality firm.

After the bad quality firms undercut each other's price, the good types must cut price further to separate themselves. Then the bad quality firms again undercut each other, etc. The race to the bottom ends when both types price at the marginal cost of the bad quality firm, same as under complete-information Bertrand competition between two bad quality firms. Bertrand competition between two known good quality firms leads to a lower price than under incomplete information, but between a known good quality and bad quality firm to a higher price. Averaging across the Bertrand prices for different type combinations according to the prior used for the incomplete information environment, the expected price may be higher or lower than under incomplete information. The *ex ante* expected price is higher under complete information iff the quality difference between the types is large enough relative to the cost difference. The *ex ante* variance of prices is always larger under public types than under private. If the cost and quality differences between the types go to zero, then the prices in the Bertrand and the incomplete-information environments converge to the same level.

The pricing pattern in this paper differs dramatically from the result of Diamond (1971) that without uncertainty about the costs and qualities of the firms, the unique equilibrium features the monopoly price and no consumer learning. In the current paper, consumers learn if they initially find themselves at a bad quality firm. Consumer learning makes the bad types undercut each other's price down to their competitive level. As long as prices are above the marginal cost of the bad quality type, good quality firms signal their type by a price strictly below that of bad quality firms. At price equal to the bad type's cost, cheap talk can distinguish the qualities, because the bad type has no incentive to raise demand by claiming to be good.[1]

The equilibrium prices of this paper contrast with a privately informed monopolist, and with competition when quality is learned together with the price. The good type of a monopolist still signals its quality by reducing its price to a level that the bad type prefers not to mimic. However, the bad type has no incentive to cut price below its monopoly level.

In competition, when paying a learning cost leads to observing both price and quality,[2] bad quality firms are still in a Bertrand-like situation and compete to a low price. However, a good quality firm has no incentive to cut price to signal, because customers see its quality and stay with it even at a high price.

The Bertrand pricing found in the present work implies that profit is lower for firms with private information, and total and consumer surplus are higher. The competitive outcome does not depend on whether the firms observe each other's cost or quality, but relies on consumers not observing these. Thus firms are better off and consumers worse off when consumers have more information.

The equilibrium in this paper still exists at zero learning cost, so is upper hemi-continuous, unlike Diamond (1971). Of course, other equilibria appear when price observation becomes free for consumers.

The next section sets up the model. Section 2 constructs an equilibrium with near-competitive pricing and shows that this equilibrium is the unique one that survives the Intuitive Criterion of Cho and Kreps (1987). The robustness of the results to relaxing various assumptions is discussed in Section 3. After that, Section 4 discusses the theoretical reasons and empirical evidence of a negative correlation between cost and quality, and Section 5 compares the present work to the previous theoretical literature.

[1] If prices are chosen from a discrete grid, then the good quality type prices strictly below the bad in equilibrium and cheap talk can be dispensed with.

[2] Observing quality as well as price can be interpreted as perfectly enforced mandatory disclosure.

1 Price competition under costly learning of prices

There are two firms, indexed by $i \in \{X, Y\}$, each with a type $\theta \in \{G, B\}$, interpreted respectively as good and bad. Types are i.i.d. with $\Pr(G) = \mu_0 \in (0, 1)$. There is a continuum of consumers of mass 1 with types $v \in [0, \overline{v}]$ distributed according to the strictly positive continuous pdf f_v, with cdf F_v, independently of firm types. Firms and consumers know their own type, but not the types of other players. There is a common prior belief over the types of all players.

The timeline of the game is as follows.

1. Nature draws independent types for firms and consumers, and assigns half the consumers to one firm, half to the other, independently of types. Each player observes his own type, but not the types of the others.
2. Firms simultaneously set prices and choose a cheap talk message[3] about their type.
3. Each consumer observes the price and message of his assigned firm and chooses either to buy from this firm, learn the price and message of the other firm, or leave the market.
4. Each consumer who chose to learn observes both firms' prices and cheap talk and chooses either to buy from his assigned firm, buy from the other firm, or leave the market.

A type G firm has marginal cost c_G, normalised to 0, and type B has $c_B > 0$. The quality of a type G firm is better, in the sense that a type v consumer values firm type B's product at v and G's product at $h(v) \geq v$, with $h' > 1$ and $h(\overline{v}) < \infty$. Relaxing the negative correlation of cost and quality (and many other assumptions) is discussed in Section 3. To ensure that demand for B's good is positive, but not all consumers buy at price equal to the bad type's cost, assume $\overline{v} > c_B \geq h(0)$. Consumers and firms are risk-neutral. Each consumer has unit demand.

After the firms' cost and quality are determined, the firms simultaneously set prices $P_X, P_Y \in \mathbb{R}_+$ and choose cheap talk messages $t_X, t_Y \in \{G, B\}$. A behavioural strategy

[3] The cheap talk is needed for types to separate when they both price at the marginal cost of the bad type. Otherwise equilibrium existence becomes a problem, as explained in Section 3. Cheap talk can be removed when prices are restricted to a discrete grid, as in the previous version of this paper available on `https://sanderheinsalu.com/`.

of firm i maps its type to $\Delta\mathbb{R}_+$.[4] The probability that type θ of firm i puts on message t and prices below P is denoted $\sigma_i^\theta(P,t)$, so $\sigma_i^\theta(\cdot,t)$ is the cdf of price. The corresponding pdf is $\frac{d\sigma_i^\theta(P,t)}{dP}$ if it exists.

A consumer sees the price and cheap talk of his assigned firm and can learn those of the other firm at cost $c_\ell > 0$. Assume that $c_\ell \leq \mu_0(h(c_B) - c_B)$, i.e. the learning cost is small relative to the prior probability of the good type firm and the quality difference between the types. The cost difference $c_B - 0$ between the types, as well as the quality difference $h(0) - 0$ may be small, provided the learning cost is even smaller.

After seeing the price of his assigned firm, a consumer decides whether to buy from this firm (denoted b), learn the other firm's price (ℓ) or not buy at all (n). Upon learning the price of the other firm, the consumer decides whether to buy from firm X (denoted b_X), firm Y (b_Y) or not at all (n_ℓ). A consumer's behavioural strategy maps his valuation, the price(s) and cheap talk message(s) to a decision via the functions $\sigma_1 : [0,\overline{v}] \times \mathbb{R}_+ \times \{G,B\} \to \Delta\{b,n,\ell\}$ and $\sigma_2 : [0,\overline{v}] \times \mathbb{R}_+^2 \times \{G,B\}^2 \to \Delta\{b_X, b_Y, n_\ell\}$. For example, $\sigma_2(v, P_i, P_j, t_i, t_j)(b_j)$ is the probability that a consumer type v initially at firm i buys from $j \neq i$ after learning P_j, t_j.

A type θ firm's *ex post* payoff if mass D of consumers buy from it at price P is $(P - c_\theta)D$. Assume that the full-information monopoly profit function $P[1 - F_v(h^{-1}(P))]$ of firm type G strictly increases in P on $[0, c_B + \epsilon]$ for some $\epsilon > 0$, so that the full-information monopoly price P_G^m of G is strictly above c_B.

A consumer's posterior belief about firm i after observing its price P and message t and expecting the firm to choose strategy σ_i^* is

$$\mu_i(P,t) := \frac{\mu_0 \frac{d}{dP}\sigma_i^{G*}(P,t)}{\mu_0 \frac{d}{dP}\sigma_i^{G*}(P,t) + (1-\mu_0)\frac{d}{dP}\sigma_i^{B*}(P,t)} \tag{1}$$

whenever the denominator is positive. A discontinuity of height h_θ in the cdf $\sigma_i^{\theta*}$ is interpreted in the pdf as a Dirac δ function times h_θ. Therefore an atom in $\sigma_i^{G*}(\cdot,t)$, but not $\sigma_i^{B*}(\cdot,t)$ at P results in $\mu_i(P,t) = 1$, and an atom in $\sigma_i^{B*}(\cdot,t)$, but not $\sigma_i^{G*}(\cdot,t)$ yields $\mu_i(P,t) = 0$. If $\sigma_i^{\theta*}$ has an atom of size h_θ at P for both $\theta \in \{G,B\}$, then $\mu_i(P,t) = \frac{\mu_0 h_G}{\mu_0 h_G + (1-\mu_0)h_B}$. Finally, if the denominator of (1) is zero, then belief is arbitrary.

The solution concept used is perfect Bayesian equilibrium (PBE), hereafter simply called equilibrium. The formal definition is notationally cumbersome and relegated to

[4]For a set S, denote the set of probability distributions on S by ΔS.

Appendix A, but the idea is standard: each firm maximises profit given the strategies of the consumers and the other firm, the consumers maximise their profits given their beliefs, and the beliefs are derived from Bayes' rule when possible. Later, a unique equilibrium is selected using the Intuitive Criterion.

The demand that firm i expects at price P and message t given the expected strategies of firm j and the consumers is

$$D_i(P,t) := \frac{1}{2}\int_0^{\overline{v}} \sigma_1^*(v,P,t)(b) + \int_0^{\infty} \sum_{t_j\in\{G,B\}} \{\sigma_1^*(v,P,t)(\ell)\sigma_2^*(v,P,P_j,t,t_j)(b_i) \tag{2}$$

$$+\,\sigma_1^*(v,P_j,t)(\ell)\sigma_2^*(v,P_j,P,t_j,t)(b_i)\}[\mu_0 d\sigma_j^{G*}(P_j,t_j) + (1-\mu_0)d\sigma_j^{B*}(P_j,t_j)]dF_v(v).$$

The first term under the integral in (2) is the consumers initially at i who buy immediately. The second term is consumers who buy from i after learning both prices and messages, which consists of (the first term in the curly braces) consumers at i who learn and then buy from i and (the second term in the braces) the consumers initially at j who learn and then buy from i. The probability that a consumer who learns buys from firm i depends on the price and message of j. The probability of message t_j and price below P_j under the prior μ_0 and strategy $\sigma_j^{\theta*}$ is $\mu_0\sigma_j^{G*}(P_j,t_j)+(1-\mu_0)\sigma_j^{B*}(P_j,t_j)$. The integral reflects the expectation over consumer valuations. The $\frac{1}{2}$ describes the mass of consumers initially at each firm.

The equilibrium profit of type θ of firm i is denoted $\pi_{i\theta}^*$; it equals $(P-c_\theta)D_i(P,t)$ for any P,t in the support of $\sigma_i^{\theta*}$. The next section constructively proves equilibrium existence by guessing and verifying.

2 Equilibrium

This section constructs an equilibrium in which consumers put probability one on a firm being the good type if the price is strictly below the bad type's cost, probability one on the bad type if the price is strictly above the bad type's cost, and ignore the cheap talk in these cases. At price equal to the bad type's cost, consumers interpret the cheap talk as the truth (are certain that the firm's type equals its message). Both firms set price equal to the bad type's cost and claim their type in cheap talk. A consumer who believes that his initial firm is the good type either buys (when his valuation for the good type is above the price) or leaves the market. A consumer believing himself to face the bad type learns when his expected valuation for the other firm is above c_ℓ, otherwise leaves the market. After learning, all consumers buy from the lower-priced firm or leave the market, breaking ties in favour of the firm claiming to be the good type and in favour of buying, with the remaining ties broken uniformly randomly. The gain from trade that consumer type v expects from buying from firm i at price P is denoted $w(v,i,P,t) := \mu_i(P,t)h(v) + (1-\mu_i(P,t))v - P$. The formal definition of the **conjectured equilibrium** is the following:

1. Beliefs: $P < c_B \Rightarrow \mu_i(P,t) = 1$ and $P > c_B \Rightarrow \mu_i(P,t) = 0$ and $P = c_B \Rightarrow \mu_i(P,t) = \mathbf{1}\{t = G\}$ for $i \in \{X, Y\}$.[5]

2. Each firm i and type θ sets price c_B and sends message $t_i = \theta$.

3. If $\mu_i(P,t) = 1$, then $\sigma_1^*(v,P,t)(b) = \mathbf{1}\{h(v) \geq P\}$.

4. If $\mu_i(P,t) = 0$, then $\sigma_1^*(v,P,t)(\ell) = \mathbf{1}\{\mu_0(h(v) - c_B) + (1-\mu_0)(v - c_B) \geq c_\ell\}$.

5. If $w(v,i,P_i,t_i) \geq \max\{0,\ w(v,j,P_j,t_j)\}$, then $\sigma_2^*(v,P_i,P_j,t_i,t_j)(b_i) = 1$, and if in addition $w(v,i,P_i,t_i) > w(v,j,P_j,t_j)$, then $\sigma_2^*(v,P_j,P_i,t_j,t_i)(b_i) = 1$. However, if $\max\{w(v,i,P_i,t_i),\ w(v,j,P_j,t_j)\} < 0$, then $\sigma_2^*(v,P_i,P_j,t_i,t_j)(n_\ell) = 1$.

Appendix A proves that no player can profitably deviate from the conjectured equilibrium. The idea of the proof is as follows. Consumers are clearly best responding to their belief, which is consistent with firm strategies. The bad type does not price below c_B, because it guarantees nonpositive profit. If all consumers at a bad type learn and find the other firm to be a good type, then all consumers leave the bad type. Conditional on the other firm being a bad type, the two bad types are in Bertrand competition over the consumers who learn. So the bad types undercut each other's price until $P_i = c_B$. A good type does not increase price above c_B, because the resulting fall in belief reduces demand and expected profit to zero, regardless of the type of the other firm. At prices less than c_B, the Diamond paradox reasoning applies to the good types: each can raise its price above that of the rival by less than c_ℓ without losing demand. A price slightly greater than that expected from the other firm does not motivate consumers to learn, unless their belief also decreases after the price increase.

[5] The indicator function $\mathbf{1}\{X\}$ equals 1 if condition X holds, and 0 otherwise.

The conjectured equilibrium already partly resolves the Diamond paradox, because the price is below the monopoly level and search occurs. Prices in the conjectured equilibrium are close to competitive. Both types price the same as under Bertrand competition between the B types with zero search cost and complete information. The price in the conjectured equilibrium is higher than when two known G types Bertrand compete and $c_\ell = 0$, but lower than when a known G type competes with a known B type. When the quality and cost difference between the types is small, all three Bertrand prices are close to that in the conjectured equilibrium.

For a stronger resolution of the Diamond paradox, subsequent results will show that the conjectured equilibrium introduced above is the unique one that survives the Intuitive Criterion. Without refinement, belief threats support other equilibria. For example, for high enough μ_0, both firms pool on $c_B + \epsilon$ for some $\epsilon > 0$, justified by the belief $\mu_i(c_B + \epsilon, t) = \mu_0$ and if $P \neq c_B + \epsilon$, then $\mu_i(P, t) = 0$.

As a first step towards proving uniqueness of equilibrium, the following lemma shows that the good type's price is lower and demand higher than the bad type's in any equilibrium. Given the ranking of the costs and qualities of the types, the results are intuitive—the lower-cost type G sets a lower price and the higher quality type G receives higher demand. Based on Lemma 1, there cannot be two prices on which both types put positive probability and at one of which, demand is positive.

Lemma 1. *In any equilibrium, for any* P_θ, t_θ *in the support of* $\sigma_i^{\theta *}$*,* $D_i(P_G, t_G) \geq D_i(P_B, t_B)$*, and if in addition* $0 < D_i(P_B, t_B) \leq D_i(P_G, t_G)$*, then* $P_G \leq P_B$*.*

The proofs of this and subsequent results are in Appendix B.

The next lemma shows that pooling fails the Intuitive Criterion and proves the natural result that the good type makes positive profit.

Lemma 2. *Any equilibrium satisfying the Intuitive Criterion has disjoint supports of* σ_i^{G*} *and* σ_i^{B*}*, and has* $\pi_{iG}^* > 0$ *for* $i \in \{X, Y\}$*.*

The intuition for the proof of Lemma 2 is that for any candidate pooling equilibrium price, there is a cutoff price below which a bad type firm makes less profit than in the candidate equilibrium even under the most favourable consumer belief (probability 1 of the good type). At prices close to this cutoff, under the most favourable belief, the good type firm makes strictly more profit than at the candidate pooling price, because the good type has strictly lower cost than the bad type who is indifferent at the cutoff. If the good type, but not the bad, deviates to a price, then the Intuitive Criterion sets consumers' belief to certainty of the good type after such a deviation. Probability one of good quality at the deviation price in turn motivates the good type to set that price.

Lemma 2 provides the first component of the race to the bottom, namely the good types separating from the bad by setting a lower price. The Intuitive Criterion drives the separation, because it eliminates belief threats at low prices, which would otherwise deter the good types from price-cutting.

The next lemma establishes a lower bound on the equilibrium price by showing that the good types price weakly above the cost of the bad type.

Lemma 3. *For any $i \in \{X, Y\}$, $P_i < c_B$ and $t \in \{G, B\}$, in any equilibrium satisfying the Intuitive Criterion, $\sigma_i^{G*}(P_i, t) = 0$.*

The intuition for Lemma 3 is that the firms' good types are in a *race to the top* at prices in $[0, c_B)$.[6] Neither firm's good type loses customers to the other firm when raising price slightly, because the small price difference does not motivate customers to pay the learning cost. The reason that a good type does not increase price strictly above c_B is that belief and demand drop discretely.

In the unique[7] equilibrium surviving the Intuitive Criterion, each type sets price c_B and the types separate using cheap talk, as shown in the following Theorem. The proof provides the second component of the race to the bottom: a bad type reduces price to deter its customers from learning, and to undercut the other firm's bad type. The motive for the customers to learn comes from the good types separating (the first component of the race to the bottom, Lemma 2), which makes the other firm's price or message informative, enabling the customer to choose the better quality firm.

Theorem 4. *In the unique equilibrium satisfying the Intuitive Criterion, both types of both firms set price c_B and send different messages with probability* 1.

Theorem 4 shows that the unique equilibrium that satisfies the Intuitive Criterion is the conjectured equilibrium from above. Prices are close to competitive. The equilibrium is robust to changing the prior, the learning cost, the distribution of consumer

[6] A similar race occurs in Diamond (1971) at all prices below the monopoly level.

[7] Uniqueness is up to permutation of the cheap talk messages. Formally, there are two equilibria: in one, each type θ sends message $t_\theta = \theta$; in the other, each θ sends $t_\theta \neq \theta$.

valuations and the good type's cost in a range of parameters[8] (Section 3 discusses cases outside this range and shows that in general the equilibrium remains the same or is continuous in the parameters).

The equilibrium in Theorem 4 is distinct from signalling by a monopoly, because a bad type monopolist does not have an incentive to cut price when the good type's price is low enough. This is because there is no competing firm for the customers to learn about and leave to. Thus the bad type sets its monopoly price. Under the Intuitive Criterion, Lemmas 1–3 still apply, so the good type monopolist sets a price between c_B and P_G^m. Separation from the bad type usually requires the good type's price to be strictly below P_G^m, so unobservable type has some of the same pro-competitive effect with one firm as with two. However, more than one firm is needed for both types' prices to be close to competitive.

Section 3.1 below contrasts Theorem 4 with competition when the type is learned together with the price. The comparisons of the conjectured equilibrium to monopoly and observable type show that the combination of signalling and multiple firms is necessary as well as sufficient to overcome the effect of the positive learning cost.

Bertrand competition under zero learning cost between two known bad or two known good types leads to equal profits (zero) for the firms and no price dispersion, unlike in the equilibrium in Theorem 4. Bertrand competition between a good and a bad firm yields zero demand for the bad firm, but positive demand and profit for the good firm, which sets a strictly higher price than the bad. This differs from the outcome in Theorem 4 where both types set the same price and obtain positive demand and profit.

The next section relaxes some of the assumptions made above. The equilibrium remains qualitatively similar, in particular the Diamond paradox is still resolved.

3 Robustness

Low monopoly price. Relaxing the assumption that the full-information monopoly price P_G^m of the good type is above the cost of the bad type, the equilibrium price of the good type is either c_B as above, or $P_G^m < c_B$. In the latter case, the only modification of the equilibrium in Section 2 is that G sets price $P_G^m \in (0, c_B)$.

[8]The range is the nonempty open set of parameters defined by $h(v) \geq v$, $h' > 1$, $h(\overline{v}) < \infty$, $c_\ell \leq \mu_0(h(c_B) - c_B)$, $c_B \geq h(0) > 0$ and $\frac{d}{dP}P[1 - F_v(h^{-1}(P))] > 0$ for $P \in [0, c_B + \epsilon]$.

Learning cost. If the learning cost is larger than $\mu_0[h(c_B) - c_B]$, then some customers initially at a bad type setting price c_B buy immediately instead of learning the other firm's price. These customers are called *captive*. Then the bad types mix over prices $P > c_B$, getting positive profit from the captive customers. The price distribution is atomless, because atoms motivate undercutting. As the learning cost increases, the support of the bad type's price distribution shifts up and eventually even the good type starts putting positive probability on $P > c_B$. The good types then also mix, because consumers switching from type B are captive for G. As long as $c_\ell < \mu_0[h(\overline{v}) - \overline{v}]$, the qualitative features of the model are preserved: prices are below the monopoly level, lower than under complete information, and some consumers learn.

If there is a distribution of learning costs with support between some $\epsilon > 0$ and $\mu_0[h(c_B) - c_B]$, then the equilibrium is unchanged. Learning costs above $\mu_0[h(c_B) - c_B]$ for some consumers make them captive and motivate the firms to mix over prices.

Nonpositive learning costs for some consumers eliminate the Diamond paradox even without incomplete information, as the previous literature showed. In the current model, consumers with a nonpositive learning cost create an equilibrium in which the good types mix over prices below c_B. The positive probability of the other firm having a bad type ensures that the good types never price at their marginal cost, because the customers initially at a bad type are captive for the other firm's good type.

Even if all consumers have zero learning cost, the conjectured equilibrium in Section 2 survives, but the strategy of the consumers becomes weakly dominated: those initially at a good type choose[9] not to learn. Given the unchanged consumer strategy, the firms' best responses remain the same. Then consumers at a good type get no benefit from learning.

High valuations. If all consumers buy at the prior belief μ_0 and price $c_B + \epsilon$ for some $\epsilon > 0$ (formally, $\mu_0 h(0) > c_B$), then there is no reason for a good type to reduce price below $c_B + \epsilon$ to increase belief. Both firms pooling on $c_B + \epsilon$ then survives the Intuitive Criterion. The conjectured equilibrium from above is no longer unique, but still exists if not all consumers buy at belief zero and price c_B.

Homogeneous consumers. If consumers all have valuation type v_B s.t. $h(v_B) \geq$

[9] The distinction between costless sequential search and Bertrand competition becomes important here. Bertrand competition means that consumers automatically see all prices. If consumers cannot choose not to learn, then the conjectured equilibrium disappears.

c_B, then the conjectured equilibrium still survives.[10] If $v_B < c_B$, then the bad types get zero demand. Other equilibria appear, e.g. pooling on any price between c_B and $\mu_0 h(v_B) + (1-\mu_0)v_B$. At the pooling price, all consumers already buy, so a higher belief does not increase demand, so firms have no incentive to cut price. The nondegenerate demand curve in Section 1 is thus not necessary for the conjectured equilibrium, but guarantees uniqueness.

Nonexistence without cheap talk. If the firms cannot send cheap talk messages, then an equilibrium satisfying the Intuitive Criterion does not exist. The proofs of Lemmas 1–3 still work, so the good types raise price from any $P_G < c_B$, and in Theorem 4, the bad types Bertrand compete down to price c_B. Then belief at c_B is strictly lower than 1, the belief at any $P < c_B$. This makes the payoff of a good type drop discontinuously at c_B, so a best response of a good type does not exist. Instead of cheap talk, restricting prices to a discrete grid also guarantees existence, as shown in an earlier version of this paper, available at `https://sanderheinsalu.com/`.

More than two firms or types. Having more than two firms only strengthens competition. Because a bad type does not price below c_B and the consumers initially at a good type do not learn, pricing cannot get more competitive than with two firms. The outcome is the same as in Section 2.

More than two types (with higher quality implying lower cost) are conceptually similar to two. To simplify notation in this case, suppose firms have a continuum of types $\theta \in [\underline{\theta}, \overline{\theta}] \subset \mathbb{R}$ with pdf $f_\theta > 0$. Higher types are better: if $\theta_a < \theta_b$, then $c_{\theta_b} < c_{\theta_a}$ and consumer valuations are $h(v, \theta_a) < h(v, \theta_b)$, and if $v_1 < v_2$, then $h(v_1, \theta_b) - h(v_1, \theta_a) < h(v_2, \theta_b) - h(v_2, \theta_a)$. Then in any equilibrium, higher types set lower prices, which can be shown by combining ICs as in Lemma 1.

If the difference between the worst and the average quality motivates all consumers initially at the worst type $\underline{\theta}$ to learn or leave (formally $\int_{\underline{\theta}}^{\overline{\theta}} h(v, x) f_\theta(x) dx - c_\ell \geq h(v, \underline{\theta})$ for all v s.t. $h(v, \underline{\theta}) \geq c_{\underline{\theta}}$), then the worst type only gets positive demand if the other firm also has the worst type. The atomless f_θ then implies that the equilibrium profit of the worst type is zero. Every non-worst type $\theta > \underline{\theta}$ gets positive demand at any $P < P(\underline{\theta})$ and some message t_θ, because it faces a worse type $\hat{\theta} \in [\underline{\theta}, \theta)$ charging a higher price with positive probability. If a non-worst type was pricing above $c_{\underline{\theta}}$, then $\underline{\theta}$

[10] The case $h(v_B) < c_B$ is covered under 'Low monopoly price' above. The good types price at $h(v_B)$, otherwise the equilibrium is unchanged.

could imitate and get positive profit. Thus any $\theta > \underline{\theta}$ sets $P(\theta) \leq c_{\underline{\theta}} \leq P(\underline{\theta})$, similarly to the two-type model. Because demand is positive at any $P < c_{\underline{\theta}}$, every non-worst type gets positive profit and prices strictly above its marginal cost, which also parallels the two-type model.

Three or more types result in positive price dispersion, as does combining a discrete price grid with at least two types.

Multidimensional types. Two-dimensional types with combinations of cost and quality $(c_G, \hat{q}_G)$, $(c_G, \hat{q}_B)$, $(c_B, \hat{q}_G)$ and $(c_B, \hat{q}_B)$ are similar to the two-type case when cost and quality are negatively correlated. Type $(c_\theta, \hat{q}_G)$ cannot separate from $(c_\theta, \hat{q}_B)$ in any equilibrium, because if the consumers expect (partial) separation, then $(c_\theta, \hat{q}_B)$ can follow the strategy of $(c_\theta, \hat{q}_G)$ at the same cost as $(c_\theta, \hat{q}_G)$ and strictly increase belief and demand. The model with multidimensional types and negative correlation of cost and quality thus reduces to the two-type model in Section 1, with $q_\theta = \hat{q}_G \Pr(\hat{q}_G|c_\theta) + \hat{q}_B \Pr(\hat{q}_B|c_\theta)$ for $\theta \in \{G, B\}$.

If the correlation of cost and quality is positive, then the four-type model reduces to two types, with higher cost implying higher quality. This case is covered in Appendix C and the Online Appendix. Price signalling is then directed upward (type G sets a price greater than B). The race to the bottom does not occur. The bad type sets a price weakly higher than its monopoly price.

If the correlation of cost and quality is zero, then signalling is impossible in either direction. Consumers expect the average quality after each price set in equilibrium, and each type of firm sets its monopoly price given the prior expected quality.

Other ways to signal. Suppose that the firms signal using advertisements as well as price. If ads reveal prices to some consumers, then competition increases and the good types mix over prices below c_B, but bounded away from zero. The bad types still set price c_B. Unsurprisingly, free price observations are similar to zero learning cost for some consumers.

If ads do not reveal prices, but are just wasteful signalling which for some reason is cheaper for the good type, then the results depend on the noisiness, timing and cost of the ads. If consumers cannot see the advertising expenditure, but must infer it from noisily observed ad quality and quantity, then ads seen before the prices only change the prior. The results are unaffected by the prior μ_0 in the range $\mu_0 > \frac{c_\ell}{h(c_B) - c_B}$. Ads seen after the prices have no effect, because the prices already reveal the types. Even if ads are free for the good type, the good type still signals by price, because ads are noisy, so revealing the type via price discretely increases demand.

Suppose that ads are perfect signals of the money spent on them. Then the relative cost to the types per unit of ads vs per unit of price decrease determines which signalling channel the good type uses. If revealing the type via ads is relatively cheaper, then the good type sets its full-information monopoly price and signals using ads. If the ad costs for the types are similar relative to the difference between the profits lost by cutting price, then ads are not used and the outcome is the equilibrium found above. A similar reasoning applies to any other signals, e.g. warranties, quality certificates, etc.

3.1 Comparison to observable types

In this section, the only difference from Section 1 is that the type is not inferred from the price, but seen directly together with the price. The consumers initially at firm i see the price, message and type of firm i, but have to pay c_ℓ to learn the price, message and type of firm j. In such a market, prices are not competitive and the good type may set its monopoly price P_G^m, as shown below. The equilibrium definition omits part (g) of Definition 1 and replaces $\mu_i(P_i)$ with 1 if firm i is of type G and 0 if B. The following Proposition puts a lower bound on the price of type G.

Proposition 5. *In any equilibrium with observable types, $\pi_{iG}^* > 0$ and any price in the support of σ_i^{G*} is above $\min\{P_G^m,\ h(c_B)\}$ for $i \in \{X, Y\}$.*

The idea for Proposition 5 is that the race to the top between the good types now continues at prices above c_B, as long as the profit increases in the price and consumers initially at a good type do not learn. If the consumers learn, then with positive probability they switch to the other firm (otherwise there would be no reason to pay the learning cost) and the good type loses demand. The prices of the good types stay close to each other throughout the race to the top, so the motive for a consumer to learn is to find a bad type of the other firm at a price low enough to compensate for the quality difference and the learning cost. So the good types can price above c_B by at least the quality difference plus the learning cost.

The race to the top may end at the good type's monopoly price or below it, and if the race ends below it, then consumers initially at a good type learn and switch with positive probability. The bad type then gets positive demand, even when pricing above the other firm's bad type. The captive customers of the bad type then motivate it to raise price above c_B, which loosens the good type' constraint on price increases. Higher prices of the good types in turn allow the bad types to raise their price, etc. In summary, if quality is seen together with the price, then either the good type sets its monopoly price or both types price strictly higher than with unobservable types.

Total surplus is strictly smaller when firm types are observed (e.g. under mandatory disclosure) than when unobserved, because the prices are higher with observed types, so some consumers leave the market. Their gains from trade ($v - c_B$ or $h(v) - 0$) are thus lost. The only potential increase of surplus comes from consumers initially at a bad type who learn with unobserved types, but buy immediately with observed types, thus saving c_ℓ per consumer. However, by revealed preference, the consumer with the lowest valuation who learns with unobserved types has a gain from trade with the prior expected type that exceeds the gain with the bad type by c_ℓ. If this consumer does not learn, then his increase in the gain from trade is lost, which exactly cancels the saved learning cost. Consumers with a greater valuation v have even larger relative gains $\mu_0(h(v) - v)$ from trading with the average type compared to the bad type, so their loss when firm types are observed strictly outweighs the saved learning cost. Consumer surplus is also strictly smaller with observed types, because the total surplus is smaller and the prices higher, thus firms get a greater share of the surplus.

4 Negatively correlated quality and cost

This section discusses the theoretical causes of a negative correlation of quality and marginal cost, and presents empirical evidence on both cost and price decreasing in quality, which matches both the assumptions and the predictions of the main model.

A more skilled tradesperson, or a firm with better equipment, can provide higher quality service with less time and effort, thus at a lower cost. Examples are tire change and rotation using a specialised machine versus 'by hand', ironing a shirt using a dummy extruding hot air (e.g. Siemens Dressman), measuring distance with a laser rangefinder instead of tape, or measuring temperature with an infrared detector instead of a mercury thermometer relying on physical contact.

Economies of scale imply a lower marginal cost for larger producers, and learning by doing improves their quality, e.g. in aircraft or car manufacturing. A larger insurer is less risky (better for policyholders) and has lower overhead costs per policy.[11] Amazon has greater variety, faster delivery and a lower cost per package delivered than smaller online sellers. Airlines with larger fleets can negotiate lower airport fees and fuel prices (lower cost per customer), and have more replacement aircraft available, so cancel a smaller fraction of flights (better quality). In the data of Sheen (2014), larger firms have both higher quality and lower price.

Regulation can cause a bad quality firm to have a higher cost. For instance, a bad quality firm is more likely to be fined or sued for faulty products, which increases its unit cost. Regulation can also turn a low cost into an incentive to improve quality. Low cost firms optimally price lower than their high cost counterparts, so if there is no quality difference, then demand is greater for a low cost firm. If a regulator checks firms with a bigger market share more, and punishes bad quality, then larger producers (with a lower cost) have a greater incentive to improve quality. Higher quality further increases demand for a low cost firm, leading to a feedback loop between demand and quality.

Innovation may both increase quality and decrease cost. Nelson et al. (2009) find that among 2128 medical innovations, 72% have higher cost and quality than the best alternative, 9% have higher cost but lower quality, 16% have lower cost and higher quality, and 1.6% have both lower cost and quality.[12] The conditional probability of the innovation having higher quality than the best alternative is thus $\frac{72}{72+9} = \frac{8}{9}$ if the innovation has higher cost, but $\frac{16}{16+1.6} = \frac{10}{11} > \frac{8}{9}$ if it has lower cost.

The optimal allocation of managerial talent (or some other resource) between cost reduction and quality improvement also links lower cost to higher quality. Improving quality is subject to moral hazard, because consumers pay based on the quality they expect, not the quality that the firm chooses. Cost reduction benefits the firm directly, so it is optimal to reduce cost maximally before improving quality. If managerial talent differs between firms, then those with high talent reduce their cost to a minimum and then might as well improve quality. The low talent firms stay above the minimal cost and do not improve quality. Empirically, Bloom et al. (2013) find that better

[11] Buffett (2014) p. 11 describes the higher underwriting profit, lower cost and smaller risk of default of Berkshire Hathaway insurance businesses relative to competitors.

[12] The percentages do not sum to 100% because some innovations provide insufficient information about cost and quality to classify them.

management practices cause higher quality, profit and TFP, thus lower unit cost.

Evidence for the conjectured equilibrium can be found in the car industry, where good quality models have lower prices and production costs. According to Vasilash (1997), the assembly cost of more reliable car models is smaller, controlling for vehicle category, e.g. subcompact, compact, etc. The rank correlation between the price[13] and the number of repair incidents[14] is positive but statistically insignificant in the sample of 40 cars that belong to both the top 288 new cars by number sold in the US in 2015[15] and the 100 most reliable in 2015 according to CarMD. The average cost per repair is also greater for more expensive cars, according to CarMD, but this is less surprising, because parts for more expensive vehicles cost more. The cheapest cars to maintain are those of East Asian manufacturers, with Toyota leading (Martin, 2016). These are also the cheapest to buy, according to US News & World Report. Unsurprisingly, the profit per car is higher for Toyota than for Detroit's Big 3 automakers (Wayland, 2015). Competition has resulted in approximately zero profit for the high cost manufacturers—the US government had to bail out Detroit's Big 3 carmakers during the 2008 financial crisis. Price close to marginal cost (profit close to zero) for bad quality producers is consistent with the model. The almost zero correlation of price and quality also corresponds to the equilibrium above.

Similar pricing is also prevalent among airlines, which are oligopolists on a typical route, and often make a loss. For example, in 2017, PenAir, Dynamic International Airways and Island Air filed for Chapter 11 bankruptcy protection in the US. Empirically, low-cost airlines like Ryanair are more punctual and set lower prices (Vahter, 2010). The low-cost carriers are also safer according to the Jet Airliner Crash Data Evaluation Centre.

Since Riesz (1979) found that the price of frozen foods is negatively related to quality, a long empirical literature has documented the negative correlation of quality with price: Olbrich and Jansen (2014) find this among private-label foods, Caves and Greene (1996) for a third of all product categories, Pan et al. (2002) for eight types of electronics, Bartelink (2016) for public tender offers, Reuter and Caulkins (2004) for

[13] The price data was retrieved from US News & World Report. The price is the average of the average low and high price paid. Author's calculations.

[14] The index of repair incidents retrieved from CarMD, where a higher index means more frequent repairs. Author's calculations of the correlation.

[15] According to Cain (2016).

street heroin. Among mutual funds, Gil-Bazo and Ruiz-Verdú (2009) show not just a correlation, but that higher fees even predict lower future before-fee returns. The nontrivial fraction of product categories with negatively associated price and quality should not be surprising given the many reasons for cost and quality to be negatively related.

Theoretically, firms with a high cost and low quality should go out of business. However, reaching this long-run outcome may take significant time, and the pool of bad firms may be replenished by entry. The data of Bloom and Van Reenen (2007) shows that high-cost, low quality firms stay in business because competition is low (e.g. for government-owned firms) or the owner ignores the opportunity costs of capital and time (e.g. family- and government-owned firms).

5 Literature

The foremost article on costly learning of prices is Diamond (1971), in which competing firms set the monopoly price. The monopoly price or above is also found in Diamond (1987); Axell (1977); Reinganum (1979); Klemperer (1987) and Garcia et al. (2017).

A number of solutions to the Diamond paradox have been proposed. When a positive fraction of consumers can learn at zero cost, as in Butters (1977); Stahl (1996); Klemperer (1987) and Benabou (1993), firms put a positive probability on the competitive price. However, with positive learning costs for some consumers, firms mix over prices that are weakly above the competitive level. Both mixing and above-competitive pricing differ from the current work. A similar idea to zero learning cost is that consumers observe multiple prices with positive probability, for example because firms send them price advertisements (Salop and Stiglitz, 1977; Burdett and Judd, 1983; Robert and Stahl, 1993). If consumers have private taste shocks, then that generates search and below-monopoly pricing (Wolinsky, 1986; Anderson and Renault, 1999; Zhou, 2014).

Learning, or the motive to learn, is exogenous for at least some consumers in the above papers.[16] In the current work, the learning motive is always endogenous. Consumers pay to observe a firm's price and cheap talk in order learn the firm's quality.

[16]For other consumers, the learning motive may be endogenous. For example, if consumers with a zero search cost or a large taste shock learn multiple prices, then firms mix over prices. The resulting price dispersion may endogenously incentivise the rest of the consumers to learn.

Learning gives consumers the option to buy from a better quality firm. Price and cheap talk are only informative about quality because the firms play a separating equilibrium. The firms in turn separate because the consumers learn. If all firms pooled, then no consumer would have any incentive to pay the learning cost.

With consumer taste shocks (horizontal differentiation of firms), some consumers initially at each firm learn another firm's price and leave. This differs from the current work, which models vertical differentiation and shows that consumers initially at a good firm do not learn or leave.

Prices below the monopoly level also occur with repeat purchases, as in Salop and Stiglitz (1982); Bagwell and Ramey (1992) and Poeschel (2018), where in some equilibria, raising the price is punished in subsequent stage games. However, the markets described by repeated games with high discount factors differ from the markets studied here, which involve infrequent buying (repair services, insurance, durable goods such as cars) and are thus closer to one-shot interactions. The present article does not rely on repeat purchases, a zero learning cost, multiple free price observations or taste shocks.

To the author's knowledge, this work is the first to combine consumer learning costs and signalling in the sense of Spence (1973).[17] Signalling relies on private information about vertical differentiation, and to the author's knowledge, the present paper is the first to combine privately known quality differences with sequential search, either costly or costless. Public quality differences are combined with non-sequential consumer search in Wildenbeest (2011). The benchmark of Bertrand competition considered in this paper is similar to costless simultaneous (non-sequential) search. Bertrand competition has been combined with price signalling in e.g. Daughety and Reinganum (2007); Janssen and Roy (2015); Sengupta (2015).

Downward[18] price signalling by a single firm has been studied in Shieh (1993). A similar idea is found in Simester (1995), where multi-product firms (whose prices for all products are positively correlated) signal by a low price on one product. Kihlstrom and Riordan (1984) also allow quality and cost to be negatively correlated in a signalling context. Pricing is not competitive in these articles, because the signaller is a

[17]Among the thousands of Google Scholar results citing Diamond (1971) or Spence (1973), only 79 cite both. These are either review articles or very distantly related.

[18]As opposed to the upward price signalling (higher quality firm sets a higher price) studied by Milgrom and Roberts (1986) and the subsequent literature.

monopolist.

In the present article, the receivers of the price signal are the consumers and signalling increases competition. This differs from limit pricing (Milgrom and Roberts (1982) and the literature following), where the receivers are potential entrants whom the incumbent keeps out of the market.

If firm types only differ in their private marginal cost, but not quality, and consumers observe both firms' prices for free, then the high cost type prices at its marginal cost. The low cost type mixes over a range of prices strictly above its marginal cost and weakly below the price of the high-cost type (Spulber, 1995). If instead the consumers have a positive learning cost, then the low-cost firm prices at its monopoly price and the high-cost c_ℓ above that (just enough to deter consumers from learning) or at its monopoly price, whichever is lower. This outcome resembles the Diamond paradox. There is no quality difference to signal, so no reason for the low-cost firm to cut price. Thus the race to the bottom does not start.

6 Conclusion

The famous paradox of Diamond (1971) is that a market with multiple firms is not competitive if consumers have to pay a cost to learn prices. However, as shown in the current article, negatively correlated private production cost and quality surprisingly restore competitive pricing. This result is robust to a wide range of quality and cost differences, prior distributions and learning costs. There are several mechanisms that make cost and quality negatively correlated across firms, for example economies of scale, regulation, or differing managerial talent. These mechanisms operate in many markets, including oligopolistic ones in which price is close to the marginal cost of at least some firms, e.g. among car manufacturers and airlines. Private information about cost and quality, as well as prices close to the competitive level are empirically reasonable in skilled services, construction and insurance, among others.

The previous literature resolves the Diamond paradox assuming either (a) zero learning cost for a positive fraction of consumers, (b) that consumers observe multiple prices at once, (c) large private taste shocks, or (d) repeat purchases. The current work models markets in which a given consumer purchases rarely, e.g. cars, insurance, repair services, and in which the vertical quality difference is more important than the horizontal taste shock. The predictions of the current article differ from zero search costs and observing multiple prices at once, because the firms set deterministic prices instead of mixing, and the mark-up and profit are larger for a lower-price firm. The present article assumes no repeat buying of the same good (insurance policies and car models change by the time the consumer purchases a replacement), which distinguishes the model from the literature on repeat purchases. With taste shocks, prices decrease in the number of firms and the degree of product differentiation. In this article, prices stay constant when the number of firms rises above two or when the quality difference changes within some bounds.

If lower cost implies higher quality, then a low price is a credible signal of quality, because it is differentially costly to the firm types. In some markets, other costly signals are available, e.g. warranties or advertising. In other applications like insurance, warranties are uncommon, so price signalling is more likely. Even if feasible, signalling by ads or warranties may not be optimal, for example when price signals are cheaper or more precise.

Signalling by a low price resembles limit pricing, in which an incumbent tries to keep an entrant out of the market. The incumbent sets a low price to convince the entrant that the incumbent has a low cost and is likely to start a price war. The low price in limit pricing is anti-competitive. In the current work, the low price results from competition, thus has different policy implications.

Total and consumer surplus are strictly greater when the price is competitive and consumers pay the learning cost than when there is monopoly pricing and no learning. A regulator maximising total or consumer surplus should encourage the race to the bottom in prices, for example by punishing low quality or checking the quality of a firm with a larger market share more frequently. The regulator should not facilitate verifiable disclosure of quality, because this would allow both types of firms to increase prices, possibly to monopoly levels. This ability to raise prices after disclosing quality explains the large sums firms spend on certification and ratings.

Similarly, industry policy should focus on improving the quality of the low-cost firms, rather than reducing the cost of the high-cost enterprises. The optimal policy is more nuanced than supporting national champions (the lowest-cost, largest producers), because the assistance in improving quality should be targeted only to firms whose quality and cost are uncertain, e.g. start-ups, firms launching a novel product.

An implication of this article for competition policy is that a merger to duopoly need not increase price above the competitive level if there is uncertainty about the (negatively correlated) costs and qualities of the duopolists.

A Equilibrium definition and existence

Denote $\mu_0 d\sigma_j^{G*}(P_j,t_j)+(1-\mu_0)d\sigma_j^{B*}(P_j,t_j)$ by $d\sigma_j^{\mu_0 *}(P_j,t_j)$.

Definition 1. An equilibrium consists of $\sigma_X^*,\sigma_Y^*,\sigma_1^*,\sigma_2^*$ and μ_X,μ_Y satisfying the following for $\theta\in\{G,B\}$, $v\in[0,\overline{v}]$, $i,j\in\{X,Y\}$, $i\neq j$:

(a) if $w(v,i,P_i,t_i)\geq\max\{0,\ w(v,j,P_j,t_j)\}$, then $\sigma_2^*(v,P_i,P_j,t_i,t_j)(b_i)=1$, and if in addition $w(v,i,P_i,t_i)>w(v,j,P_j,t_j)$, then $\sigma_2^*(v,P_j,P_i,t_j,t_i)(b_i)=1$,

(b) if $\max\{w(v,i,P_i,t_i),\ w(v,j,P_j,t_j)\}<0$, then $\sigma_2^*(v,P_i,P_j,t_i,t_j)(n_\ell)=1$,

(c) if $w(v,i,P_i,t_i)>\max\{0,\ \int_0^\infty\sum_{t_j\in\{G,B\}}\max\{w(v,i,P_i,t_i),\ w(v,j,P_j,t_j)\}d\sigma_j^{\mu_0*}(P_j,t_j)-c_\ell\}$, then $\sigma_1^*(v,P_i,t_i)(b)=1$,

(d) if $w(v,i,P_i,t_i)\leq\int_0^\infty\sum_{t_j\in\{G,B\}}\max\{0,\ w(v,i,P_i,t_i),\ w(v,j,P_j,t_j)\}d\sigma_j^{\mu_0*}(P_j,t_j)-c_\ell\geq 0$, then $\sigma_1^*(v,P_i,t_i)(\ell)=1$,

(e) if $\max\{w(v,i,P_i,t_i),\ \int_0^\infty\sum_{t_j\in\{G,B\}}\max\{0,\ w(v,j,P_j,t_j)\}d\sigma_j^{\mu_0*}(P_j,t_j)-c_\ell\}<0$, then $\sigma_1^*(v,P_i,t_i)(n)=1$,

(f) if (P_i,t_i) is in the support of $\sigma_i^{\theta*}$, then $(P_i,t_i)\in\arg\max_{P,t}(P-c_\theta)D_i(P,t)$, where $D_i(P,t)$ is given by (2),

(g) if (P,t) is in the support of σ_i^{G*} or σ_i^{B*}, then $\mu_i(P,t)$ is derived from (1).

Consumers are clearly best responding to their beliefs in parts 3–5 of the conjectured equilibrium. Beliefs in part 1 are consistent with part 2. It remains to check whether firms are best responding in part 2. First, deviations of type G to lower prices are ruled out. The preparative Lemma 6 derives the profit function of G from setting $P<c_B$.

Lemma 6. *In the conjectured equilibrium, the profit of type G from $P<c_B$ and t is*

$$\frac{1}{2}P\left[1-F_v(h^{-1}(P))+(1-\mu_0)\int_{h^{-1}(P)}^{\overline{v}}\sigma_1^*(v,c_B,B)(\ell)dF_v(v)\right].\qquad(3)$$

Proof. The profit (3) is derived from (2) by substituting in the consumers' strategies in the conjectured equilibrium: $\sigma_1^*(v, P_i, t)(b) = 1$ and $\sigma_1^*(v, P_i, t)(\ell) = 0$ for consumers initially at i, because $P_i < c_B$ and $\mu_i(P_i, t) = 1$. Consumers with $v \geq h^{-1}(P_i)$ buy from i, and they form a fraction $1 - F_v(h^{-1}(P_i))$ of the mass of consumers initially at i.

If firm j is type G, then $P_j = c_B$, $t_j = G$ and $\sigma_1^*(v, P_j, t_j)(\ell) = 0$. With probability $1 - \mu_0$, firm j is type B, in which case consumer v at firm j learns P_i with probability $\sigma_1^*(v, c_B, B)(\ell)$ and then buys if $v \geq h^{-1}(P_i)$. $\square$

Next, the technical Lemma 7 simplifies (3) by showing that if σ_i^{B*} puts probability 1 on $(P, t) = (c_B, B)$ and σ_i^{G*} puts probability 1 on (c_B, G) for $i \in \{X, Y\}$, then $\sigma_1^*(v, c_B, B)(\ell)$ is a step function increasing in v.

Lemma 7. *For customers initially at a type B firm, there exists $v_{01} \in [h^{-1}(c_B), \overline{v}]$ s.t. $\sigma_1^*(v, c_B, B)(\ell) = 0$ for $v < v_{01}$ and 1 for $v > v_{01}$.*

Proof. Suppose firm i has type B. Due to $P_j \leq c_B$, in Definition 1(d), $v - c_B$ may be dropped under the max w.l.o.g. If $\int_0^\infty \sum_{t_j \in \{G,B\}} \max\{0,\ w(v, j, P_j, t_j)\}[\mu_0 d\sigma_j^{G*}(P_j, t_j) + (1 - \mu_0) d\sigma_j^{B*}(P_j, t_j)] - c_\ell \geq 0$, for consumer v, then for all $\hat{v} > v$, the inequality is strict.

If $w(v, j, P_j, t_j) \leq 0$, then $v - c_B < 0$, so the first inequality in Definition 1(d) holds. If $w(v, j, P_j, t_j) > 0$, then 0 may be dropped under the max w.l.o.g. Then from $h' > 1$ and $\int_0^\infty \sum_{t_j \in \{G,B\}} [\mu_0 d\sigma_j^{G*}(P_j, t_j) + (1 - \mu_0) d\sigma_j^{B*}(P_j, t_j)] = 1$, the first inequality in Definition 1(d) is strict for all $\hat{v} > v_1$. So if $\sigma_1^*(v, c_B, B)(\ell) > 0$, then for all $\hat{v} > v$, $\sigma_1^*(\hat{v}, c_B, B)(\ell) = 1$. Taking $v_{01} := \inf\{v : \sigma_1^*(v, c_B, B)(\ell) > 0\}$ ensures that $\sigma_1^*(\hat{v}, c_B, B)(\ell) = 0$ for $\hat{v} < v_{01}$ and 1 for $\hat{v} > v_{01}$.

To prove $v_{01} \geq h^{-1}(c_B)$, note that $h^{-1}(x) < x\ \forall x$, so $h^{-1}(c_B) - c_B < 0$. If $P_j \geq c_B$, then $w(h^{-1}(c_B), j, P_j, t_j) \leq 0$ for any t_j. The $-c_\ell$ term in Definition 1(d) then ensures $\sigma_1^*(h^{-1}(c_B), c_B, B)(\ell) = 0$. $\square$

Downward price deviations by a type G firm are ruled out in the following Lemma. After that, the incentives of firm type B are discussed, and then the deviations of G to $P_G > c_B$ are ruled out.

Lemma 8. *A type G firm's best response to the strategies of other players in the conjectured equilibrium satisfies $P \geq c_B$.*

Proof. Based on Lemma 7, $\sigma_1^*(v, c_B, B)(\ell) = 0$ for all $v \leq h^{-1}(c_B) \geq h^{-1}(P)$, where $P \leq c_B$. Therefore (3) reduces to $\frac{1}{2}P[1 - F_v(h^{-1}(P)) + (1 - \mu_0)[1 - F_v(v_{01})]]$, with v_{01}

independent of P. The assumption $P_G^m := \arg\max_P P[1 - F_v(h^{-1}(P))] \geq c_B + \epsilon$ for some $\epsilon > 0$ then implies $\arg\max_P \frac{1}{2}P[1 - F_v(h^{-1}(P)) + (1 - \mu_0)[1 - F_v(v_{01})]] > c_B$, because if $P_G^m D(P_G^m) \geq PD(P)$ for all $P \leq P_G^m$, then for any $\bar{D} > 0$ and $P \leq P_G^m$, we have $P_G^m D(P_G^m) + P_G^m \bar{D} \geq PD(P) + P\bar{D}$. So type G optimally sets a price $P \geq c_B$. $\square$

Lemma 9. *In the conjectured equilibrium, a type B firm's best response to the strategies of other players is $P = c_B$ and $t = B$.*

Proof. A type B firm clearly does not deviate to $P < c_B$ with any message. Consider B's deviations to $P > c_B$ and some $t \in \{G, B\}$. Parts 1 and 4 of the conjectured equilibrium ensure that each customer initially at firm i charging $P > c_B$ either leaves the market or learns the price and message of j. By part 5 of the conjectured equilibrium, a customer who learns at firm i will choose firm j, which has both a lower price $P_j = c_B < P$ and a higher belief $\mu_j(c_B, t_j) \geq 0 = \mu_i(P, t)$ for any t_j, t.

At $P = c_B$, type B is indifferent between demand levels and thus between messages $t \in \{G, B\}$. Therefore $t = B$ is part of a best response. $\square$

Having ruled out deviations by B, the final step (Lemma 10) is to eliminate deviations by a type G firm.

Lemma 10. *A type G firm's best response to the strategies of other players in the conjectured equilibrium is $P = c_B$ and $t = G$.*

Proof. Lemma 8 established $P \geq c_B$. If firm i's type G sets $P > c_B$, with any $t \in \{G, B\}$, then it gets zero demand in the conjectured equilibrium, because $\mu_i(P, t) = 0$ and the other firm j is expected to set price $P_j \leq c_B < P$. At $P = c_B$, message $t = B$ leads to belief $\mu_i(c_B, B) = 0$, but message $t = G$ to $\mu_i(c_B, G) = 1$, thus greater demand. Therefore (c_B, G) is the unique best response for type G. $\square$

B Proofs omitted from the main text

Proof of Lemma 1. In any equilibrium, the incentive constraints (ICs) $P_G D_i(P_G, t_G) \geq PD_i(P, t)$ and $(P_B - c_B)D_i(P_B, t_B) \geq (P - c_B)D_i(P, t)$ hold for any P, P_θ, t, t_θ with (P_θ, t_θ) in the support of $\sigma_i^{\theta *}$. Demand and price are nonnegative and finite by definition. From $(P_B - c_B)D_i(P_B, t_B) \geq (P_G - c_B)D_i(P_G, t_G)$ and $P_G D_i(P_G, t_G) \geq P_B D_i(P_B, t_B)$, we get $(P_B - c_B)D_i(P_B, t_B) \geq P_G D_i(P_G, t_G) - c_B D_i(P_G, t_G) \geq P_B D_i(P_B, t_B) - c_B D_i(P_G, t_G)$, so $D_i(P_B, t_B) \leq D_i(P_G, t_G)$.

If $0 < D_i(P_B, t_B) \leq D_i(P_G, t_G)$ and $(P_B - c_B)D_i(P_B, t_B) \geq (P_G - c_B)D_i(P_G, t_G)$, then $P_B - c_B \geq P_G - c_B$, so $P_B \geq P_G$. □

Proof of Lemma 2. Suppose $\pi^*_{iG} = 0$ and use the Intuitive Criterion to derive a contradiction. Fix some $P_i \in (0, \min\{c_\ell, c_B\})$ and $t \in \{G, B\}$. Set belief to $\mu_i(P_i, t) = 1$. No consumer learns at P_i, t and belief $\mu_i(P_i, t) = 1$, because firm j is expected to have weakly lower quality and a price $P_j \geq 0$ lower by at most c_ℓ. The greatest possible price decrease $|P_j - P_i| < c_\ell$ from switching to j does not justify paying the learning cost c_ℓ. By assumption, $h(P_i) > P_i > 0$, so consumers with valuations $v \leq P_i$ buy at $P_i, t, \mu_i(P_i, t)$ and yield positive demand and profit to type G. Type B can ensure nonnegative profit by setting $P \geq c_B$, thus must get nonnegative equilibrium profit $\pi^*_{iB} \geq 0$. Choosing P_i, t gives B positive demand, so strictly negative profit. Thus belief $\mu_i(P_i, t) = 1$ is justified and any supposed equilibrium with $\pi^*_{iG} = 0$ is eliminated.

Next, the Intuitive Criterion is used to eliminate pooling and semi-pooling on any $P_{i0} > c_B$, t_{i0}. By $\pi^*_{iG} > 0$, demand is positive at P_{i0}, t_{i0}, so $\pi^*_{iB} > 0$. Lemma 1 implies that any price in the support of σ_i^{B*} is above P_{i0} and any price in the support of σ_i^{G*} is below P_{i0}.

Denote demand at the fixed belief μ by $D_i^\mu(P)$; it does not depend on t due to the fixed μ. Demand $D_i^\mu(P)$ increases in μ and decreases in P, so the profit $(P - c_\theta)D_i^\mu(P)$ as a function of P does not have upward jumps. At $P = c_B$, the profit of B is $(c_B - c_B)D_i^\mu(c_B) = 0$ for any μ, but at $P_{i0} > c_B$, the equilibrium profit is $\pi^*_{iB} > 0$. Pooling implies $\mu_i(P_{i0}, t_{i0}) < 1$, so $D_i(P_{i0}, t_{i0}) < D_i^1(P_{i0})$ and therefore $(P_{i0} - c_B)D_i^1(P_{i0}) > \pi^*_{iB} > 0$. Thus there exists $P_{d*} \in (c_B, P_{i0})$ s.t. for any $P < P_{d*}$, $(P - c_B)D_i^1(P) < \pi^*_{iB}$. Focus on the maximal such P_{d*}. The lack of upward jumps in $(P - c_B)D_i^1(P)$ implies that for any $\epsilon > 0$ there exists $\delta > 0$ s.t. $(P_{d*} - \delta - c_B)D_i^1(P_{d*} - \delta) \geq \pi^*_{iB} - \epsilon$. If δ is small enough s.t. $\epsilon < D_i^1(P_{d*} - \delta) - D_i(P_{i0}, t_{i0})$, then type G strictly prefers to deviate to $P_{d*} - \delta$ and t at $\mu_i(P_{d*} - \delta, t) = 1$, because $(P_{d*} - \delta - 0)D_i^1(P_{d*} - \delta) \geq \pi^*_{iB} + (c_B - 0)D_i^1(P_{d*} - \delta) - \epsilon > \pi^*_{iB} + (c_B - 0)D_i(P_{i0}, t_{i0}) = \pi^*_{iG}$. By the definition of P_{d*}, type B strictly prefers the equilibrium to $P_{d*} - \delta$, which justifies $\mu_i(P_{d*} - \delta, t) = 1$ and eliminates pooling on any $P_{i0} > c_B$ and t_{i0}.

Pooling and semi-pooling on $P_{i0} = c_B$ and some t_{i0} is eliminated by the Intuitive Criterion as follows. For $\epsilon > 0$ small and some t_i, set $\mu_i(c_B - \epsilon, t_i) = 1$. Due to pooling,

$\mu_i(c_B, t_{i0}) < 1$, so $D_i^1(c_B) > D_i^{\mu_i(c_B,t_{i0})}(c_B) = D_i(c_B, t_{i0})$. At $c_B - \epsilon$ and t_i, demand is $D_i^1(c_B - \epsilon) > D_i^{\mu_i(c_B,t_{i0})}(c_B) = D_i(c_B, t_{i0})$. Thus for ϵ small enough, G strictly prefers $c_B - \epsilon, t_i$ to c_B, t_{i0}. By $\pi_{iG}^* > 0$, demand is positive at c_B, t_{i0}, so B strictly prefers c_B, t_{i0} to $c_B - \epsilon, t_i$. These strict preferences justify $\mu_i(c_B - \epsilon, t_i) = 1$ and eliminate pooling on c_B, t_{i0}.

Pooling and semi-pooling on $P_{i0} < c_B$ and some t_{i0} cannot occur, because $\pi_{iG}^* > 0$ implies $D_i(P_{i0}, t_{i0}) > 0$, which would yield $\pi_{iB}^* < 0$. $\square$

Proof of Lemma 3. By Lemma 2, $\pi_{iG}^* > 0$ for both $i \in \{X, Y\}$, and type B strictly prefers its equilibrium price to any $P_i < c_B$ for any t_i. Thus if G strictly prefers P_i, t_i to its equilibrium action when $\mu_i(P_i, t_i) = 1$, then the equilibrium fails the Intuitive Criterion.

Denote by $\underline{P}_{i\theta} := \inf\left\{P : \sigma_i^{\theta*}(P, t) = 0 \; \forall t\right\}$ the lower bound of the prices that firm i's type θ sets. Assume w.l.o.g. that $\underline{P}_{iG} \leq \underline{P}_{jG}$. If firm i raises price to $\underline{P}_{iG} + \epsilon$ for $\epsilon \in (0, \min\{c_\ell, c_B - \underline{P}_{iG}\})$ and belief is $\mu_i(\underline{P}_{iG} + \epsilon, t) = 1$ for some t, then consumers initially at firm i still choose $\sigma_1(v, \underline{P}_{iG} + \epsilon, t)(\ell) = 0$, because a price difference less than c_ℓ does not justify the learning cost. The customers at j who chose ℓ anticipating σ_i^{G*} do not know about G's deviation, so still choose ℓ. Upon learning $\underline{P}_{iG} + \epsilon, t$, a customer initially at j's type B has a choice between B at $P_B \geq c_B$ and G at $\underline{P}_{iG} + \epsilon < c_B$, so still buys from i's type G. If a customer initially at j's type G learns both firms' prices and messages and believes $\mu_i(\underline{P}_{iG} + \epsilon, t) = 1$, then he still buys from i if $P_{jG} \geq \underline{P}_{iG} + \epsilon$. If $P_{jG} \leq \underline{P}_{iG} + \epsilon$, then no customer facing P_{jG} learns, because firm i has weakly lower quality and a price lower by at most $\epsilon < c_\ell$. At prices $P \leq c_B$, the profit of G is then given by (3). By Lemmas 7–8, G strictly prefers to increase price. $\square$

Proof of Theorem 4. The assumptions $\overline{v} > c_B$ and $f_v > 0$ ensure that there exists $\epsilon > 0$ s.t. total demand is positive at $P_i = c_B + \epsilon$ for any P_j, t_j, t_i. Suppose by way of contradiction that σ_j^{B*} puts positive probability on P_j, t_j at which $D_j(P_j, t_j) = 0$. Then the expected demand for firm i is positive at $P_i = c_B + \epsilon$, implying that both types of firm i make positive profit, thus $\underline{P}_{iB} > c_B$. By Lemma 2, the supports of σ_i^{B*} and σ_i^{G*} are disjoint, so $\mu_i(P, t) = 0$ for any (P, t) in the support of σ_i^{B*}. Any P_{jd}, t_{jd} with $P_{jd} \in (c_B, \underline{P}_{iB})$ then attracts positive demand in expectation, because $\mu_j(P_{jd}, t_{jd}) \geq \mu_i(P, t) = 0$. Firm j's type B deviates from any zero-demand price to P_{jd}, t_{jd} and makes positive profit. This contradicts $D_j(P_j, t_j) = 0$ for any P_j, t_j in the support of σ_j^{B*}. Therefore demand is positive in expectation for both types of both firms in any equilibrium satisfying the Intuitive Criterion.

By Lemma 1, positive demand implies $P_G \leq P_B \geq c_B$ for any P_θ in the support of $\sigma_i^{\theta*}$. As shown next, all consumers initially at type B of at least one firm learn before buying. Assume $\overline{P}_{iG} \geq \overline{P}_{jG}$ w.l.o.g. Then $\underline{P}_{iB} \geq \overline{P}_{jG}$, so a consumer with valuation v facing $P_{iB} \geq \underline{P}_{iB}$ and any t_i gets payoff $v - P_{iB}$ from buying immediately. On the other hand, learning yields consumer v a payoff of at least $h(v) - \overline{P}_{jG}$ with probability μ_0, and $v - P_{iB}$ with probability $1 - \mu_0$. If $\mu_0[h(v) - v] \geq c_\ell$, then consumer v prefers learning to buying. Consumers $v < P_{iB}$ do not buy at P_{iB} and $\mu_i(P_{iB}, t) = 0$, thus either leave the market or learn. For $v \geq P_{iB} \geq c_B$, the assumption $\mu_0[h(c_B) - c_B] \geq c_\ell$ implies learning instead of immediate buying.

Having $\overline{P}_{iB} > \overline{P}_{jB}$ when all consumers at i learn or leave contradicts positive demand for i. The previous paragraph proves that all customers at j learn or leave when $\overline{P}_{iB} \leq \overline{P}_{jB}$. Given that all consumers who end up buying from type B have learned both firms' prices and messages, the B types are in Bertrand competition. The following undercutting argument then shows that B prices at c_B with certainty. Having $\overline{P}_{iB} \neq \overline{P}_{jB}$ contradicts positive demand for one firm. If σ_j^{B*} has an atom at $\overline{P}_{jB} = \overline{P}_{iB} > c_B$, then for small enough $\epsilon > 0$, firm i's type B strictly prefers $\overline{P}_{iB} - \epsilon$ to $\overline{P}_{iB}$. Supposing σ_j^{B*} has no atom at $\overline{P}_{jB} > c_B$ implies probability 1 of $\overline{P}_{iB} > P_{jB}$, which contradicts $D_i(\overline{P}_{iB}, t) > 0$.

Lemmas 3 and 1 with $\overline{P}_{iB} = c_B$ imply that both types price at c_B with certainty. Lemma 2 proves disjoint supports of σ_i^{B*} and σ_i^{G*}, so $t_{iB} \neq t_{iG}$ with certainty. $\square$

Proof of Proposition 5. Type is observed, so cheap talk messages are ignored. Drop the message from the notation w.l.o.g. Price $\epsilon \in (0, c_\ell)$ is available to type G, with $D_i(\epsilon) > 0$ regardless of σ_j. Therefore $\pi_{iG}^* > 0$ for $i \in \{X, Y\}$.

Assume w.l.o.g. $\underline{P}_{iG} \leq \underline{P}_{jG} < h(\overline{v})$. A customer type $v \geq h^{-1}(\underline{P}_{iG} + \epsilon)$ initially at firm i who sees that the firm is type G and charges $\underline{P}_{iG} + \epsilon$ chooses $\sigma_1^*(v, P_{iG} + \epsilon)(\ell) = 0$ if $h(v) - \underline{P}_{iG} - \epsilon > V := \mu_0 \int_0^\infty \max\{h(v) - P_j,\ h(v) - \underline{P}_{iG} - \epsilon\} d\sigma_j^{G*}(P_j) + (1 - \mu_0) \int_0^\infty \max\{v - P_j,\ h(v) - P_{iG} - \epsilon\} d\sigma_j^{B*}(P_j) - c_\ell$. Type B sets $P \geq c_B$ (which weakly dominates $P < c_B$), firm j's type G sets $P_{jG} \geq \underline{P}_{iG}$ by assumption, and $\int_0^\infty d\sigma_j^{\theta*}(P_j) = 1$, so $V \leq \mu_0[h(v) - \underline{P}_{iG}] + (1 - \mu_0) \max\{v - c_B,\ h(v) - \underline{P}_{iG} - \epsilon\} - c_\ell$. Sufficient for customer type $v \geq h^{-1}(\underline{P}_{iG} + \epsilon)$ facing $\underline{P}_{iG} + \epsilon$ not to learn is $(1 - \mu_0)[h(v) - \underline{P}_{iG} - \epsilon] > \mu_0 \epsilon - c_\ell + (1 - \mu_0) \max\{v - c_B,\ h(v) - \underline{P}_{iG} - \epsilon\}$, which holds if $\underline{P}_{iG} + \epsilon < h\left(c_B + \frac{c_\ell - \mu_0 \epsilon}{1 - \mu_0}\right)$

So type G of firm i increases price to at least $\min\{P_G^m,\ h(c_B)-\epsilon\}$.

Firm i was arbitrary, so the same reasoning applies to firm j. $\square$

C Positively correlated cost and quality

Consumers are assumed homogeneous and cheap talk absent, both for simplicity and for better comparability to the literature. Adding cheap talk does not change the prices or consumers' strategies. The Online Appendix studies the heterogeneous consumer case. In this section, all consumers have valuation $v_B > c_B$ for type B and $v_G := h(v_B) > v_B$ for G. The marginal cost of G is $c_G > c_B$.

There are multiple separating equilibria with the same outcome: $P_{i\theta} = v_\theta$ for $i \in \{X, Y\}$, $\theta \in \{B, G\}$, no consumers learn, all buy at $P \leq v_B$, fraction $\frac{v_B - c_B}{v_G - c_B}$ buy at $P > v_B$. Beliefs that support these strategies are $\mu_i(P) = \frac{P - v_B}{v_G - v_B}$ for $P \in [v_B, v_G]$, and arbitrary beliefs for $P > v_G$ and $P < v_B$. Other equilibria with the same outcome have $\mu_i(P) \leq \frac{P - v_B}{v_G - v_B}$ for $P \in [v_B, v_G)$, $\mu_i(v_G) = 1$ and fraction less than $\frac{v_B - c_B}{P - c_B}$ of consumers buying at $P \in (v_B, v_G)$. The fraction is 0 if $\mu_i(P) < \frac{P - v_B}{v_G - v_B}$. The beliefs in all these separating equilibria pass the Intuitive Criterion, because if G wants to deviate to price $P_d \in (v_B, v_G)$ with belief $\mu_i(P_d) = 1$, then B strictly prefers P_d, which yields the same demand as $P_{iB} = v_B$, but strictly greater margin. The equilibrium outcome is the natural analogue of Diamond (1971). The uniqueness of this outcome is shown next.

An equilibrium with $P_{iB} < v_B$ and $P_{iB} \leq P_{jB}$ cannot exist, because if consumers who see P_{iB} do not learn, then firm i's type B can raise its price by $\epsilon \in (0, c_\ell)$. For consumers who see P_{iB} to learn, they must expect $\mu_0(v_G - P_{jG}) + (1-\mu_0)(v_B - P_{jB}) - c_\ell \geq v_B - P_{iB}$, i.e. $\mu_0(v_G - P_{jG} - v_B + P_{jB}) \geq P_{jB} - P_{iB} + c_\ell > 0$. However, if $v_G - P_{jG} > v_B - P_{jB}$, then demand is weakly greater at P_{jG} than at P_{jB}, thus type B of firm j strictly prefers to deviate to P_G.

An equilibrium where firm i does not pool and $P_{iG} < v_G$ cannot exist, because all consumers would buy at P_{iG}. In this case, demand is weakly greater at P_{iG} than at P_{iB}, so type B of firm i strictly prefers to deviate to P_{iG}. Thus the only non-pooling equilibria feature $P_{i\theta} = v_\theta$.

Pooling on any $P_{i0} \leq \mu_0 v_G + (1-\mu_0)v_B$ fails the Intuitive Criterion, because at the deviation price $P_d = v_G$ and the most favourable belief $\mu_i(v_G) = 1$, a (mixed) best response of the consumers exists for which G prefers to deviate from P_{i0} and B prefers not to.

The unique separating outcome (no learning, $P_{i\theta} = v_\theta$, some consumers do not buy at $P_{iG} = v_G$) stands in contrast to Janssen and Roy (2015), regardless of which equilibrium characterisation in their Proof of Proposition 2 is used. The first paragraph of Janssen and Roy (2015) Proof of Proposition 2 describes the unique symmetric D1 equilibrium as follows.

(a) If $\frac{v_B - c_B}{v_G - c_B} > \frac{1}{2}$, then $P_{iG} = v_G$. If both firms are type G, then some consumers do not buy, otherwise all buy.

(b) If $\frac{v_B - c_B}{v_G - c_B} \leq \frac{1}{2}$, then $P_{iG} = \max\{c_G, c_B + 2(v_G - v_B)\}$ and all consumers buy.

From the second paragraph on, Janssen and Roy (2015) Proof of Proposition 2 claims:

(a) If $\frac{v_B - c_B}{v_G - c_B} \geq \frac{1}{2}$, then $P_{iG} = c_B + 2(v_G - v_B)$, type B mixes over $P_{iB} \in [c_B + \mu_0(v_G - v_B),\ c_B + v_G - v_B]$, all consumers buy at the lowest price, breaking ties uniformly randomly.

(b) If $\frac{v_B - c_B}{v_G - c_B} < \frac{1}{2}$, then $P_{iG} = v_G$, type B mixes over $P_{iB} \in [c_B + \mu_0(v_B - c_B),\ v_B]$. If both firms charge v_G, then a consumer buys from each with probability $\frac{v_B - c_B}{v_G - c_B}$ and leaves the market with probability $\frac{v_G - c_B - 2v_B + 2c_B}{v_G - c_B}$. If at least one firm charges $P \leq v_B$, then the consumer buys at the lowest price with certainty.

Unlike in the incomplete information Bertrand model of Janssen and Roy (2015), the equilibrium under costly learning in this section features $P_{iB} = v_B$ (instead of B mixing on lower prices), zero consumer surplus, consumers never switching (as opposed to always switching when the types of the firms differ), and not all consumers buying when the types of the firms differ. Depending on the parameters in Janssen and Roy (2015), the equilibria also differ in P_{iG} and the probability of consumers purchasing when both firms are type G.

Several of the differences are the expected ones between Bertrand and Diamond environments—no search, monopoly pricing and the corresponding surplus extraction. The contrast between freely observed prices and costly learning when quality and cost are positively correlated makes it the more surprising that under costly learning and negatively associated marginal cost and quality, price is close to competitive. This paper's low price is similar to the Bertrand model in the appendix of Janssen and Roy (2015), but in their Bertrand environment, a competitive outcome is unsurprising.

With heterogeneous consumers and positively related cost and quality, the Online Appendix shows that the results are analogous to the current section and Diamond (1971), thus contrasting Section 2. In particular, type B still prices above its complete-information monopoly level, G prices above B by at least the quality difference between the types, and consumers do not learn at B. The heterogeneous consumer case differs from homogeneous in that G may set a price different from its complete-information monopoly level, some consumers learn at G and switch to B given the chance, and at both types of firms, low-valuation consumers leave the market.

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