Information Acquisition and Liquidity Traps in Over-the-Counter Markets*

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January 12, 2019

(Job Market Paper)

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Abstract

I analyze the interaction between buyers' information acquisition and market liquidity in over-the-counter markets with adverse selection. If a buyer anticipates that future buyers will acquire information about asset quality, she has an incentive to acquire information to avoid buying a lemon that will be hard to sell at a later date. However, when current buyers acquire information, they cream-skim the market, leaving a larger fraction of lemons for sale and giving future buyers an incentive to acquire information. A liquid market can go through a self-fulfilling market freeze when buyers start to acquire information. More importantly, if information acquisition continues for a long enough period of time, the market gets stuck in an information trap with low liquidity: information acquisition worsens the composition of assets remaining on the market, and the bad composition incentivizes information acquisition. This prediction helps explain why the market for non-agency residential mortgage-backed securities experienced a sudden drop in liquidity—as potential buyers realized the need for greater due diligence—but has remained essentially dormant despite a strong recovery in the housing market.

Keyword: Information acquisition; Adverse selection; Market freezes; OTC markets

*I am deeply indebted to my committee members: Itay Goldstein, Guillermo Ordonez, Benjamin Lester, Vincent Glode and Harold Cole for their guidance and support. I also thank Mitchell Berlin, Guillermo Calvo, Alessandro Dovis, Ricardo Lagos, Christian Opp, Victor Rios-Rull, Chaojun Wang, Pierre-Olivier Weill, Shengxing Zhang, as well as seminar participants in the Wharton Finance Seminars, the Macro Seminars at Penn and the Federal Reserve Bank of Philadelphia for useful comments. All errors are my own.

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1 Introduction

During the 2007–2008 financial crisis, many asset markets suffered from periods of illiquidity—sellers found it increasingly hard to sell assets at acceptable prices. Dry-ups in liquidity are especially prominent among classes of assets that are opaque and traded in over-the-counter (OTC) markets, as in the case with mortgage-backed securities (Gorton, 2009) and collateralized debt obligations (Brunnermeier, 2009). A large literature has sought to explain these events of market freezes through the lens of asymmetric information.[1] The standard narrative is that asset owners are better informed of their assets' quality than potential buyers in these markets. Therefore, when the perceived average quality of assets decreases, markets freeze as a result of the exacerbated adverse selection problem.

One decade after the financial crisis, the US economy is on track for the longest expansion ever, and housing prices are on a path of continued growth.[2] However, the impact of the crisis seems rather persistent. The market for non-agency residential mortgage-backed securities (RMBS), which was at the center of the financial crisis, has yet to come back (Ospina and Uhlig, 2018).[3] At the same time, investors have been conducting more due diligence in inspecting and evaluating securitized products since the crisis. Instead of solely relying on external ratings, investors now develop their own models to provide independent assessments of asset quality.[4] These stark differences in market liquidity and the behavior of market participants before and after the crisis, despite similar fundamentals of the market, are hard to reconcile with the standard narrative of adverse selection. Indeed, if the RMBS market freeze was driven by deterioration of the value of the underlying mortgages, the market should have recovered given the current strong economic fundamentals and the bullish housing market.

To explain both the decline in market liquidity and the increase in investors' due diligence, I introduce buyers' information acquisition into a dynamic adverse-selection model with resale considerations. The key result of my model is that an asset market can have multiple steady states, and more importantly, transitions between steady states are asymmetric.

[1] See Tirole (2012), Daley and Green (2012), Camargo and Lester (2014), Guerrieri and Shimer (2014) and Chiu and Koeppl (2016), among many other papers.

[2] See All-Transactions House Price Index for the United States, https://fred.stlouisfed.org/series/USSTHPI.

[3] Non-agency mortgage-backed securities are issued by private entities, and do not carry an explicit or implicit guarantee by the US government. In contrast, agency MBS are issued and backed by government agencies or government-sponsored enterprises, such as Fannie Mae, Freddie Mac and Ginnie Mae.

[4] For instance, see *The Economist* in its January 11, 2014, issue: "Before 2008, . . . , investors piled in with no due diligence to speak of. Aware of the reputational risks of messing up again, they now spend more time dissecting three-letter assets than just about anything else in their portfolio." Also, Kaal (2016) finds that since the financial crisis, private funds have hired more analysts to conduct investors' due diligence using textual analysis of the ADV II filings.

Liquid markets are susceptible to a *self-fulfilling market freeze*, in which buyers suddenly start to acquire information and the market quickly transitions from a liquid state to an illiquid one. As illiquid trading and information acquisition continue for an extended period, the market falls into an *information trap* with low liquidity and information acquisition, in which there is no equilibrium path that leads back to the liquid state. Importantly, while some previous papers have studied sudden market freezes in the framework of multiple equilibria, my findings are different in terms of the sharp prediction of whether the market can recover in a self-fulfilling manner after a market freeze.

Before describing these results in greater detail, it makes sense to first lay out the key ingredients of the model. A continuum of investors trades assets of either high or low quality. Gains from trade arise because asset owners are subject to idiosyncratic liquidity shocks that lower the flow payoff from holding assets. Upon receiving a liquidity shock, an asset owner participates in the market as a seller and trades with potential buyers who arrive sequentially. A seller is privately informed of the quality of her own asset, while the buyer can acquire a noisy signal of the asset's quality by incurring a fixed cost. If the asset is traded, the buyer hold the asset and will return to the market as a seller when receiving a liquidity shock in the future. Otherwise, the seller keeps the asset and waits for the arrival of the next buyer. Although this paper is motived by observations in the non-agency RMBS market, the model can be applied to various OTC markets with asymmetric information.

How does buyers' information acquisition interact with market liquidity? If the current composition of assets for sale is good enough to support pooling trading, buyers' information acquisition reduces current market liquidity. Intuitively, if a buyer acquires information and observes a bad signal, she is unwilling to trade at a pooling price because the posterior belief about the asset's quality becomes worse. In addition to the static relationship between buyers' information acquisition and market liquidity, there is also a dynamic strategic complementarity between buyers' current and future incentives to acquire information, and hence a complementarity between current and future market liquidity. On one hand, current buyers' incentive to acquire information depends on future buyers' information acquisition through the *resale consideration*. If a buyer anticipates that future buyers will acquire information about asset quality, she has an incentive to acquire information so as to avoid buying a low-quality asset that will be hard to sell at a later date. In this sense, expected future market liquidity improves current market liquidity. On the other hand, current buyers' information acquisition changes future buyers' incentives to acquire information through the *cream-skimming effect*. When current buyers acquire information, high-quality assets are traded faster than low-quality assets. As low-quality assets accumulate on the market over time, future buyers have more incentive to acquire information. Therefore, current market illiquidity harms future market liquidity.

The dynamic strategic complementarity in buyers' information acquisition gives rise to the possibility of a self-fulfilling market freeze. Suppose the market is in a liquid state, in which buyers do not acquire information and the composition of assets for sale is good. One day, investors suddenly start to worry that in the future buyers will acquire information, lowering market liquidity . As a result, the resale value of low-quality assets drops abruptly and the current buyers start to acquire information. Because of the cream-skimming effect of information acquisition, the composition of assets for sale deteriorates gradually, giving future buyers more incentive to acquire information. This justifies current investors' belief in future low liquidity. A self-fulfilling market freeze takes place when investors coordinate to follow an equilibrium path with information acquisition.

As the self-fulfilling market freeze continues and the composition of assets for sale declines further, it is impossible for the market to return to liquid trading without outside intervention. This dynamic is apparent if we note that buyers' incentives to acquire information depend on both future market liquidity and the current composition of assets for sale. When the composition is bad enough, even if buyers believe the market will be liquid in the future, it is still optimal for them to acquire information today to avoid buying low-quality assets. Their information acquisition in turn keeps the composition of assets for sale at a low level. The market is therefore "trapped" in an illiquid state with information acquisition and longer trading delays.

The key mechanism that generates the asymmetric transitions between states with different liquidity is the slow-moving property of the composition of assets for sale. Buyers' information acquisition worsens the composition of assets for sale through the cream-skimming effect and has a long-lasting negative impact on future market liquidity. The composition will only improve gradually when buyers stop acquiring information. However, even with the most optimistic belief about future market liquidity, buyers will not stop acquiring information unless the composition of assets is good enough. Buyers' information acquisition and the bad composition of assets for sale reinforce each other, preventing the market from recovering without outside intervention to clean the market.

This paper sheds light on the discussion of regulatory reforms to increase transparency in many asset markets. For example, Dodd-Frank Act Section 942 requires issuers of asset-backed securities (ABS) to provide asset-level information according to specified standards. These measures increase the precision of buyers' idiosyncratic signals when they conduct due diligence. Although these measures can potentially discipline the ABS issuance process, I show that they have the unintended consequence of increasing fragility in the secondary market. When buyers have access to more precise signals, they have a greater incentive to acquire information and provide quotes conditional on the signals. Therefore the cream-skimming effect becomes stronger and the market is more susceptible to an information trap.

This paper also has important implications for the timing of the provision of asset purchase programs aiming to revive the market. During the latest financial crisis, the US Treasury created the Troubled Asset Relief Program (TARP), aimed at restoring a liquid market by purchasing "toxic" assets. I show that the fraction of "toxic" assets on the market is endogenous and depends on investors' information acquisition in the past. As the market gets deeper into a crisis, the asset composition on the market becomes worse and policy makers need to purchase a larger amount of low-quality assets to revive the market.

The paper is organized as follows. I describe the model setup in Section 2. Section 3 focuses on the equilibrium analysis. The stationary equilibria are studied in Section 4. In Section 5 I explore the set of non-stationary equilibria that converge to different steady states. Policy implications are studied in Section 6. Section 7 concludes.

Related Literature

This paper builds on the large literature on adverse selection initiated by the seminal work of Akerlof (1970). Among many other papers, Janssen and Roy (2002); Camargo and Lester (2014); Chari, Shourideh and Zetlin-Jones (2014), and Fuchs and Skrzypacz (2015) analyze dynamic-adverse selection models with centralized or decentralized market structures.[5] These models share the common feature that low-quality assets are sold faster than or at the same speed as high-quality assets. None of these papers feature resale considerations or buyers' acquisition of information about assets' quality.

Taylor (1999), Zhu (2012), Lauermann and Wolinsky (2016), and Kaya and Kim (2018) all considers dynamic adverse-selection models in which each buyer observes a noisy signal about an asset's quality. A new result obtained in this strand of literature is that high-quality assets are traded faster than low-quality assets. This is related to the cream-skimming effect in my model when buyers acquire information. These papers consider a trading environment with a single seller and sequentially arriving buyers, and there is no scope for reselling the asset. In contrast, in my paper, buyers anticipate that they will sell their assets in the same market when they experience liquidity shocks.

In papers that study dynamic adverse-selection models with resale considerations—such as Chiu and Koeppl (2016) and Asriyan, Fuchs and Green (2018)—buyers' valuation of an asset depends on future market liquidity. This gives rise to an intertemporal coordination

[5] See also Hendel and Lizzeri (1999), Blouin (2003), Hörner and Vieille (2009), Moreno and Wooders (2010).

problem which in turn yields multiple steady states with symmetric self-fulfilling transitions. Another closely related study is by Hellwig and Zhang (2012), who analyze a dynamic adverse-selection model with both resale consideration and endogenous information acquisition. While I allow buyers' signals to be noisy, they focus on the situations in which the signals are precise. Therefore, information acquisition has no cream-skimming effect in their model and transitions between steady states are symmetric. In contrast to all of the above papers, mine has the novel feature of generating multiple steady states with unidirectional transitions.

This paper is also related to work by Daley and Green (2012, 2016), who study the role of a publicly observable "news" process in dynamic-adverse selection models. In my paper, buyers make their own decisions on whether to acquire information and the information is not observable to other market participants.

In terms of modeling search frictions, this paper builds on the theoretical papers on OTC markets. Examples are Duffie, Gârleanu and Pedersen (2005, 2007); Vayanos and Weill (2008); and Lagos and Rocheteau (2009). The trading environment is very similar to the investor's life-cycle model in Vayanos and Wang (2007). I contribute to this literature by introducing asymmetric information about asset quality.

There is a large literature that studies information acquisition in financial markets, including Froot, Scharfstein and Stein (1992); Glode, Green and Lowery (2012); Fishman and Parker (2015); as well as Bolton, Santos and Scheinkman (2016).[6] This literature shows that information acquisition can be a strategic complement and excess information acquisition in equilibrium leads to inefficiency. I differ from this line of research by studying information acquisition in a dynamic trading environment. This allows me to characterize transitions between different states of the market, such as episodes of market freezes or recovery.

Lastly, this paper contributes to the literature on the role of transparency and information acquisition in financial crises. Gorton and Ordonez (2014) study how a small shock to the collateral value can be amplified into a large financial crisis when it triggers information acquisition. In my model, a market freeze can arise as a self-fulfilling outcome. Also, I study a topic not addressed in their paper: whether a market can recover after a crisis. In terms of policy implications, this paper is related to the recent discussion of optimal disclosure of information by government and regulators, as in Alvarez and Barlevy (2015); Bouvard, Chaigneau and de Motta (2015); Gorton and Ordonez (2017); and Goldstein and Leitner (2018). A closely related study is that of Pagano and Volpin (2012), who also look at the welfare implications of increasing transparency in the securitization process. My work differs

[6] See also Barlevy and Veronesi (2000), Veldkamp (2006), Hellwig and Veldkamp (2009), Goldstein and Yang (2015).

from the literature in that I argue that information disclosure does not directly reveal the value of an asset; instead, investors need to conduct due diligence to interpret the disclosed information. The noise in the interpretation of disclosed information reflects the complexity of the underlying assets, such as securitized products. Greater transparency reduces noise, but it can also exacerbate adverse selection in the market through the cream-skimming effect.

2 The Model

Time is continuous and infinite. There is a continuum of assets with mass 1. The quality of an asset is either high or low, denoted by $j \in \{H, L\}$. The mass of high-quality and low-quality assets is fixed at $\alpha/(1+\alpha)$ and $1/(1+\alpha)$ respectively, so the ratio of high-quality to low-quality assets is α, which is an exogenous parameter that controls the average quality of the assets. Therefore I will refer to α as the *fundamental* of the market.[7]

The trading environment is populated with a continuum of investors. They are risk-neutral and discount time at rate r. Each of them is restricted to holding either 0 units or 1 unit of an asset. Their preference for holding assets can be either *unshocked* or *shocked*, reflecting the fact that some investors experience liquidity shocks and become financially constrained. Whether an investor is shocked is observable or verifiable. When holding an asset of quality $j \in \{H, L\}$, an unshocked investor enjoys a flow payoff designated as rv_j, while a shocked investor enjoys a flow payoff of rc_j. Throughout this paper, I maintain the assumption that $v_H > c_H > v_L \geq c_L > 0$. Thus, the shocked investors enjoy a lower flow payoff from holding both types of assets. Also, $c_H > v_L$, meaning that the common value component dominates the private value component, which is a necessary condition for the existence of the lemons problem.

Following Vayanos and Wang (2007), I consider a life-cycle model of OTC markets. At any time, there is a flow into the economy of unshocked investors without assets, the buyers in the market. They have a one-time opportunity to trade with the shocked asset owners, who are the sellers in the market. After buying an asset, a buyer becomes an unshocked asset owner. Otherwise, if trade is unsuccessful, the buyer exits the market with zero payoff. Since an investor's liquidity shock is observable, there will be no trade between a buyer and an unshocked asset owner.[8] Therefore, unshocked asset owners only passively hold assets until their preferences change. These investors are labeled as holders. Holders face liquidity shocks that arrive at Poisson rate δ. Upon receiving a liquidity shock, a holder becomes a

[7] I deviate from the conventional notation of using the fraction of high-quality assets to represent the average quality of the assets. The notation adopted here turns out to be convenient for characterizing investors' beliefs and asset distribution.

[8] This is a direct implication of the No-Trade Theorem in Milgrom and Stokey (1982).

seller and offers her asset for sale on the market. For simplicity, I assume that the inflow of buyers at any time equals a constant λ times the mass of sellers in the market. These buyers are matched with sellers randomly. Therefore, from a seller's perspective, buyers arrive at a constant Poisson rate λ. Sellers stay in the market until they sell the assets and exit the economy with zero payoff.

The flow of investors in the economy is summarized in Figure 1. Buyers enter the economy from the pool of outsider investors. When a seller sells an asset, she exits the economy and returns to the pool of outside investors. I use the word *market* to represent the two groups of active traders in the economy, the sellers and the buyers. From a buyer's perspective, the severity of the adverse selection problem is determined by the composition of sellers with high-quality and low-quality assets. Notice that sellers are a subset of asset owners who actively participate in the market. Therefore, the composition of assets among sellers can potentially differ from the fundamental of the market, which is the asset composition among all asset owners. In this sense, the level of adverse selection in my model is endogenous and depends on the asset distribution. Later, I use the word *market composition* to represent the composition of high-quality and low-quality assets among sellers.

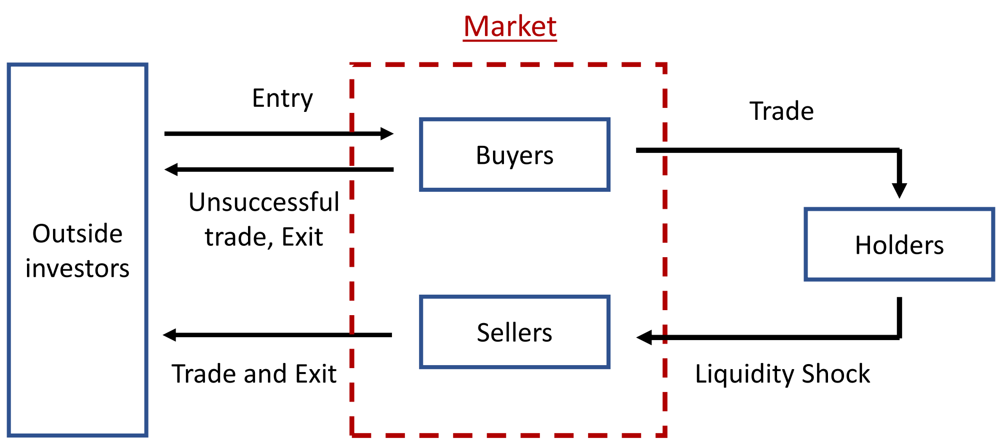

Figure 1: Flow Diagram of the Asset Market

When a buyer meets a seller, the seller is privately informed of the quality of her asset. The buyer does not observe the quality of the seller's asset, nor does she have information regarding the trading history of the seller. Her prior belief is determined by the market composition—i.e., the ratio of high-quality assets and low-quality assets among sellers. In addition, the buyer can pay a fixed cost k to acquire information and obtain a signal $\psi \in \{G, B\}$ of the asset's quality. G represents a good signal and B represents a bad signal. The probability of observing a signal ψ from an asset of quality j is f_j^ψ. Signals obtained by different buyers are jointly independent conditional on the quality of the asset. The assumption that a buyer can only observe a noisy signal of the asset's quality captures the opaque nature of the assets. Different buyers may have different evaluations of the same asset. Without loss of generality, I assume $f_H^G > f_L^G$, so a high-quality asset is more likely to generate a good signal than a low-quality asset. This implies that a good signal improves the buyer's posterior belief about the asset's quality. The trading protocol is deliberately simple. The buyer makes a take-it-or-leave-it offer to the seller. The entire transaction takes place instantly, with the seller and buyer separating immediately afterward.

3 Equilibrium Analysis

In this section I analyze investors' optimal trading strategies and define the equilibrium of the model. Since investors are infinitesimal, they take the continuation value of leaving a match as given. This allows me to separate the equilibrium analysis into three parts. First, I study a static trading game between a seller and a buyer, taking the continuation values as given. Second, I determine the continuation values of different agents. Lastly, I characterize the evolution of the asset distribution.

3.1 The Static Trading Game

The static trading game is played by one seller and one buyer. To define a static trading game, it is sufficient to specify the prior belief of the buyer and the terminal payoffs of both players when they separate. I denote the buyer's prior belief by $\theta(t)$, which equals the probability that the seller carries a high-quality asset divided by the probability that the seller carries a low-quality asset. If θ is small, there is a large fraction of low-quality assets on the market, and the adverse selection problem is severe. In equilibrium, θ must be consistent with the asset distribution among sellers when the buyer meets the seller. If the seller sells the asset or the buyer does not buy the asset, they leave the economy with zero continuation value. If the buyer buys an asset of quality $j \in \{H, L\}$, the continuation value is denoted by $V_j(t)$, which is also the continuation value of a passive holder at time t. If the seller keeps an asset of quality j, the continuation value is denoted by $C_j(t)$. From now on, I omit the time argument of all variables when analyzing the static trading game. A static trading game is therefore defined by the combination of the buyer's prior belief and the continuation values $(\theta; V_H, C_H, V_L, C_L)$. For reasons that will become clear later, we only need to consider the case of $V_H > C_H > V_L, C_L$.

The static game has two stages, the information acquisition stage and the trading stage. We use backward induction to solve the static game. The seller's optimal strategy takes a simple form. A seller with an asset of quality j is going to accept any price higher than the continuation value C_j and reject any offer below C_j. The buyer needs to decide whether to acquire information, and based on her belief about the asset's value after the information acquisition stage, decides upon an optimal offering price. If the buyer acquires information, she will update her belief in a Bayesian way. Her posterior belief about the asset's quality after seeing signal $\psi \in \{G, B\}$ in the form of a high-quality to low-quality ratio is

$$\tilde{\theta}(\theta, \psi) = \frac{f_H^{\psi}}{f_L^{\psi}}\theta. \tag{1}$$

If the buyer doesn't acquire information, the posterior belief $\tilde{\theta}$ equals the prior belief θ. For the consistency of notation, let $\tilde{\theta}(\theta, N) = \theta$ represent the posterior belief if the buyer has chosen not to acquire information.

The following lemma characterized the optimal offering strategy of the buyer conditional on the posterior belief $\tilde{\theta}(\theta, \psi)$.

Lemma 1 *The buyer's strategy is characterized by a threshold belief*

$$\hat{\theta} = \frac{C_H - \min\{C_L, V_L\}}{V_H - C_H}.$$

1. *If $\tilde{\theta}(\theta, \psi) > \hat{\theta}$, the buyer makes a pooling offer C_H,*
2. *If $\tilde{\theta}(\theta, \psi) < \hat{\theta}$ and $V_L > C_L$, the buyer makes a separating offer C_L,*
3. *If $\tilde{\theta}(\theta, \psi) < \hat{\theta}$ and $V_L < C_L$, the buyer makes a no-trade offer $p < C_L$.*

If the buyer's posterior belief $\tilde{\theta}(\theta, \psi)$ is above the threshold $\hat{\theta}$, the buyer should offer a pooling price C_H to trade with both the high-quality and the low-quality seller. However, if the buyer's posterior belief is not good enough, the optimal price to offer depends on the relationship between V_L and C_L or, alternatively, whether there are gains from trade of a low-quality asset. If $V_L > C_L$, the buyer values a low-quality asset more than the seller does, and the buyer can offer a separating price C_L that will only be accepted by a low-type seller. On the other hand, if $V_L < C_L$, the buyer values a low-quality asset less than the seller does, and it is optimal for the buyer to offer a no-trade price, which is lower than a low-type seller's continuation value, to avoid buying the asset. In the knife-edge case of $\tilde{\theta}(\theta, \psi) = \hat{\theta}$, or $V_L = C_L$, the optimal offering strategy of the buyer can be a mixed strategy.

In the information acquisition stage, the buyer will compare the value of information, which is the increase in the expected payoff after the buyer observes the signal, to the cost of information acquisition. She will only acquire information about the asset when the net gain is positive. The signal is potentially valuable to the buyer because it gives the buyer the option of making offers conditional on the signal. Depending on prior belief, the buyer will either improve the offered price when seeing a good signal, or lower the offered price when seeing a bad signal.

Lemma 2 *The value of information is*

$$W(\theta) = \begin{cases} \max\left\{-\frac{\theta}{1+\theta} f_H^B (V_H - C_H) + \frac{1}{1+\theta} f_L^B (C_H - \min\{C_L, V_L\}), 0\right\}, & \text{if } \theta \geq \hat{\theta}, \\ \max\{\frac{\theta}{1+\theta} f_H^G (V_H - C_H) - \frac{1}{1+\theta} f_L^G (C_H - \min\{C_L, V_L\}), 0\}, & \text{if } \theta < \hat{\theta}. \end{cases}$$

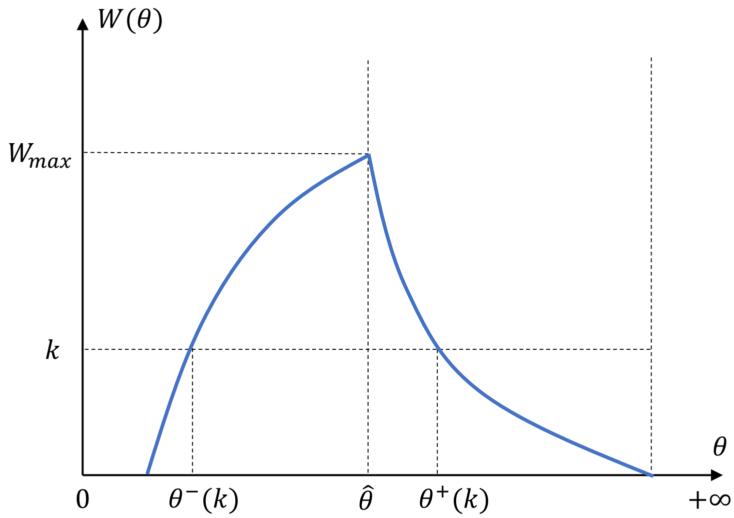

Figure 2: Value of information to the buyer.

Figure 2 depicts the value of information as a function of the prior belief θ. Let W_{max} be the maximum value of information. If the prior belief θ falls at the left or right end of the $[0, 1]$ interval, the value of information is zero. This is because the prior belief is so high (low) that even after observing a bad (good) signal, the posterior is still higher (lower) than the threshold belief. If the prior belief is around the threshold belief $\hat{\theta}$, the value of information first increases from 0, reaches the maximum at $\hat{\theta}$, and then decreases to 0. The buyer will acquire information if and only if the value of information based on the prior belief is greater than the cost of acquiring information. The following lemma summarizes the buyer's optimal strategy in information acquisition.

Lemma 3 *If $k < W_{max}$, the buyer will acquire information if and only if*

$$\theta^-(k, \min\{C_L, V_L\}) \leq \theta \leq \theta^+(k, \min\{C_L, V_L\}),$$

where the two functions are defined as

$$\theta^-(k,\nu) = \frac{f_L^G(C_H-\nu)+k}{f_H^G(V_H-C_H)-k}, \quad \theta^+(k,\nu) = \frac{f_L^B(C_H-\nu)-k}{f_H^B(V_H-C_H)+k}.$$

Both $\theta^-(k,\nu)$ *and* $\theta^+(k,\nu)$ *are decreasing in* ν*.*

When the value of a low-quality asset ($\min\{C_L, V_L\}$) decreases, the loss of buying a low-quality asset at pooling price C_H is higher. Therefore, the buyer is more inclined to avoid low-quality assets on the right boundary of the information-sensitive region and less willing to rely on the noisy signal on the left boundary. The information-sensitive region $[\theta^-(k), \theta^+(k)]$ moves to the right as both C_L and V_L decrease. As we will show later, C_L and V_L are determined by both the flow payoff from holding the asset and the likelihood that a low-quality asset can be sold at the pooling price in the future. The above comparative statics are important because they are related to the resale consideration that links the current buyers' information acquisition decision to future market liquidity. When the current market composition is relatively good (θ on the right boundary of the information-sensitive region), buyers are more willing to acquire information if their belief about future market liquidity deteriorates.

To conclude the analysis of the static trading game, I summarize the trading probability in the equilibrium of the static trading game (for the non-knife-edge cases) when $k < W_{max}$ in Table 1. When θ falls on the boundary of the information region, the equilibrium is not unique. The buyer will use a mixed strategy of information acquisition. Thus, the set of trading probabilities is the convex combination of the set of trading probabilities of the adjacent regions.

	$\theta < \theta^-(k,\nu)$	$\theta^-(k,\nu) < \theta < \theta^+(k,\nu)$	$\theta > \theta^+(k,\nu)$
$V_L < C_L$	$\rho_H = \rho_L = 0$	$\rho_H = f_H^G, \rho_L = f_L^G$	$\rho_H = \rho_L = 1$
$V_L = C_L$	$\rho_H = 0, \rho_L \in [0,1]$	$\rho_H = f_H^G, \rho_L \in [f_L^G, 1]$	$\rho_H = \rho_L = 1$
$V_L > C_L$	$\rho_H = 0, \rho_L = 1$	$\rho_H = f_H^G, \rho_L = 1$	$\rho_H = \rho_L = 1$

Table 1: Trading probability when $k < W_{max}$

3.2 Continuation Values

First I introduce some notations that describe the investors' strategy in the full dynamic game, allowing for both pure strategy and mixed strategy. I use $\mu(p, j, t) \in [0, 1]$ to represent the probability of type j seller accepting offer p at time t. The buyer's strategy is more complicated and can be denoted by a couple of functions $\{i(t), \sigma(p, \psi, t)\}$.[9] $i(t) \in [0,1]$ is the probability that the buyer acquires information at time t. $\sigma(p, \psi, t)$ represents the probability of offering p in a match at time t when seeing signal ψ. If a buyer does not acquire information, $\psi = N$ following the previous notation. Therefore, $\sigma(p, N, t)$ is the buyer's probability of offering p in a match at time t conditional on not acquiring information. In principle, a buyer can draw a price from a mixed distribution. Fortunately, based on the analysis of the static trading game, the buyer will only choose from three relevant offers at any time.[10] Thus it's without loss of generality to assume $\sigma(\cdot, \psi, t)$ is a probability mass function of p.

With the help of the above notations, we can write down $\gamma_j(p, t)$, the probability that a type j seller is offered price p conditional on meeting a buyer at time t.

$$\gamma_j(p,t) = i(t) \sum_{\psi=G,B} f_j^{\psi} \sigma(p, \psi, t) + (1 - i(t))\sigma(p, N, t). \tag{2}$$

$\gamma_j(p, t)$ characterizes the market condition faced by a type j seller at time t. If $\gamma_j(p, t)$ has more weights on high prices of p, the market is more liquid for sellers with assets of quality j because it's easier for them to sell the assets at a high price.

The continuation value of sellers with high-quality assets is at least c_H since the sellers can always hold on to their assets. Also, no buyer will offer a price higher than c_H in equilibrium.[11] Therefore

$$C_H(t) = c_H. \tag{3}$$

The previous analysis of the static trading game shows that only three types of prices will be offered by a buyer at time t: the pooling price $C_H(t) = c_H$, the separating price $C_L(t)$ or the no-trade price $p < C_L(t)$. Getting an offer at the separating price or the no-trade price will not change the continuation value of the seller. Therefore, to compute the continuation value of a low-quality seller, we consider the hypothetical case where the seller always holds on to the asset unless offered c_H. In fact, $\gamma_j(c_H, t)$ can be viewed as a proxy of endogenous market liquidity for owners of an asset of quality j. This is especially important for investors with low-quality assets because it measures the likelihood of extracting information rent in future meetings. Since the arrival rate of a pooling offer c_H for a low-type seller at time

[9] Note that the strategy functions are independent of the identity of any given buyer or seller. This means that we will focus on equilibria with symmetric strategies without loss of generality because for any equilibrium with asymmetric strategies, we can find an equilibrium in symmetric strategies with the same path of asset distributions, trading volume, and average prices.

[10] We can pick any $p < c_L$ to be the no-trade price.

[11] Otherwise the price of high-quality asset will be unbounded when t goes to infinity

τ is $\lambda\gamma_L(c_H, \tau)$, for a low-quality seller remaining in the market at time t, the distribution function of the arrival time of an offer with pooling price c_H is $1 - e^{-\lambda \int_t^\tau \gamma_L(c_H,u)du}$. A low-quality seller's continuation value is characterized by[12]

$$C_L(t) = \int_t^\infty \left[(1 - e^{-r(\tau-t)})c_L + e^{-r(\tau-t)}c_H\right] d(1 - e^{-\lambda \int_t^\tau \gamma_L(c_H,u)du}). \tag{4}$$

The seller enjoys the flow payoff rc_L before a pooling offer arrives, and the value jumps to c_H when the seller accepts the offer. If $\gamma_L(c_H, \tau)$ improves for all future $\tau > t$, the low-type sellers' continuation value $C_L(t)$ increases.

Now let's turn to the continuation value of a holder/buyer. A holder enjoys the flow payoff from an asset and mechanically becomes a seller when hit by a liquidity shock that arrives at Poisson rate δ.[13] The continuation value of a type-j holder at time t is

$$V_j(t) = \int_t^\infty \left[(1 - e^{-r(\tau-t)})v_j + e^{-r(\tau-t)}C_j(\tau)\right] d(1 - e^{-\delta(\tau-t)}). \tag{5}$$

To derive the gains from trade at time t, we need to compare the continuation values of sellers and holders. Notice for the high type, $C_H(t) = c_H$,

$$V_H(t) = \frac{rv_H + \delta c_H}{r + \delta}. \tag{6}$$

As long as $\delta > 0$, $V_H(t) > C_H(t)$ holds at any time. There are always gains from trade for high-quality assets. However, the same result doesn't necessarily hold for low-quality assets although $v_L \geq c_L$. Taking the difference between (5) and (4), we have

$$V_L(t) - C_L(t) = \int_t^\infty \Bigg[\underbrace{(1 - e^{-r(\tau-t)})(v_L - c_L)}_{\text{flow payoff}} \ldots \tag{7}$$

$$- \underbrace{\int_t^\tau e^{-r(u-t)}\lambda\gamma_L(c_H, u)(c_H - C_L(u))\mathrm{d}u}_{\text{information rent}}\Bigg] d(1 - e^{-\delta(\tau-t)}). \tag{8}$$

The first component of the integrand represents the holder's extra benefit from the higher flow payoff. However, the positive gain is offset by the information rent of the low-type seller, represented by the second component of the integrand. Notice $C_L(\tau) \leq \frac{rc_L + \lambda c_H}{r + \lambda} < c_H$.

[12] Equivalently, a low-quality seller's continuation value can be characterized by a differential equation $rC_L(t) = rc_L + \lambda\gamma_L(c_H, t)\,(c_H - C_L(t)) + \frac{\mathrm{d}C_L(t)}{\mathrm{d}t}$.

[13] The continuation value of a type-j holder can be equivalently characterized by a differential equation $rV_j(t) = rv_j + \delta\,(C_j(t) - V_j(t)) + \frac{\mathrm{d}V_j(t)}{\mathrm{d}t}$.

When the low-type seller is likely to be offered a pooling price c_H—i.e., $\gamma_L(c_H, u) > 0$—she can take advantage of the liquid market condition and extract information rent from the buyers. This benefit is not enjoyed by the holder. The buyer/holder has an advantage of holding the asset because of the higher flow payoff. However, she has a disadvantage in reselling the asset because her liquidity shock is observable. The fact that an asset holder seeks to immediately sell her asset on the market reveals that she is holding a low-quality asset. Whether the gain from trade is positive or negative depends on the relative size of the two components. As the market condition becomes uniformly more liquid (higher $\gamma_j(c_H, u)$ for all $u > t$), the gains from trade decrease. Here I state the following assumption regarding the information structure of the signal:

Assumption 1 $f_L^G > \frac{r+\lambda}{\lambda}\frac{v_L - c_L}{c_H - c_L}$.

Given Assumption 1, the gains from trade for low-quality assets could be positive, negative, or zero depending on future market conditions denoted by $\gamma_L(c_H, t)$. A liquid market condition in the future (uniformly higher $\gamma_L(c_H, t)$) increases the low-quality seller's incentive to remain in the market and wait for a pooling offer, therefore lowering the gain from trade. Assumption 1 implies that if future buyers always acquire information, the gains from trade of a low-quality asset are negative. This result is formally stated in Lemma 4.

Lemma 4 *Given Assumption 1, $V_L(t) - C_L(t) < 0$ if $\gamma_L(c_H, \tau) \geq f_L^G$ for any $\tau > t$.*

For Assumption 1 to hold, the value difference between the high-type and low-type assets can not be too small (v_L is relatively close to c_L instead of c_H). Also, buyers' signals must be inaccurate ($f_L^G > 0$) so that when they acquire information, there is a large enough chance that they will offer a pooling price to a low-quality seller.

3.3 The Evolution of Asset Quality

The trading probability of each type of asset at any time can be constructed from the trading strategies. The probability that an asset of quality j is traded in a match at time t is

$$\rho_j(t) = \sum_{\{p:\mu(p,j,t)>0\}} \gamma_j(p, t)\mu(p, j, t). \tag{9}$$

The product $\gamma_a(p, t)\mu(p, a, t)$ represents the probability that a type a asset is sold at price p at time t. The summation of the product over p gives us the trading probability.

Let $m_H^S(t)$ and $m_L^S(t)$ represent the masses of high-quality and low-quality assets held by sellers. Since high-quality and low-quality assets are in fixed supply of $\frac{\alpha}{1+\alpha}$ and $\frac{1}{1+\alpha}$

respectively, mass $\frac{\alpha}{1+\alpha} - m_H^S(t)$ of high-quality assets and mass $\frac{1}{1+\alpha} - m_L^S(t)$ of low-quality assets are held by holders. The evolution of asset distribution is fully characterized by the following differential equations:

$$\dot{m}_H^S(t) = \delta \left(\frac{\alpha}{1+\alpha} - m_H^S(t) \right) - \lambda \rho_H(t) m_H^S(t), \tag{10}$$

$$\dot{m}_L^S(t) = \delta \left(\frac{1}{1+\alpha} - m_L^S(t) \right) - \lambda \rho_L(t) m_L^S(t). \tag{11}$$

In each equation, the right-hand side consists of two terms. The first term represents the inflow of assets brought into the market by holders who just received liquidity shocks. The second term represents the outflow of assets because of trading. Since buyers are assigned to sellers randomly, buyers' prior beliefs about the quality of their counter-parties' assets must be consistent with the market composition of high-quality and low-quality assets. For this reason, we use the same notation $\theta(t)$ to represent both the market composition and the buyers' prior belief

$$\theta(t) = \frac{m_H^S(t)}{m_L^S(t)}. \tag{12}$$

Combining (10) and (11), we can characterize the evolution of the market composition as

$$\frac{\mathrm{d}}{\mathrm{d}t} \ln \theta(t) = \underbrace{\frac{\delta}{m_H^S(t)} \frac{\alpha}{1+\alpha} \left(1 - \theta(t)/\alpha\right)}_{\text{fundamental reversion}} - \underbrace{\lambda(\rho_H(t) - \rho_L(t))}_{\text{trading probability differential}} . \tag{13}$$

The evolution of asset distribution can be equivalently characterized by $m_H^S(t)$ and $\theta(t)$. The change in the quality of assets on the market can be decomposed into two effects. The first effect is the *fundamental reversion*. When $\theta(t) < \alpha$, the composition of assets on the market is worse than the fundamental. Therefore, the inflow of assets because of liquidity shocks improves the quality of assets on the market. On the contrary, the inflow of assets worsens the quality of assets on the market when $\theta(t) > \alpha$. Therefore, the market composition tends to revert to the fundamental. This effect is stronger when the high-quality asset on the market is a smaller fraction of total stock of high-quality asset in the economy. The second term is the *trading-probability differential*. Most previous literature has focused on cases where low-quality assets trade weakly faster than high-quality assets in illiquid markets. In those cases, $\rho_H(t) \leq \rho_L(t)$ so the second effect is always weakly positive. In the analysis of the static trading game, we know that when $\theta(t)$ falls in the information acquisition region and there's negative gain from trade for low-quality assets, $\rho_H(t) > \rho_L(t)$. Therefore, high-quality assets leave the market faster than low-quality assets, so the second effect is negative. The negative trading-probability differential effect generates novel implications for the set of steady states and market transitions in the dynamic equilibrium.

3.4 Equilibrium Definition

The equilibrium of the full dynamic game is defined as follows.[14]

Definition 1 *Given an initial asset distribution* $\left\{\theta(0), m_H^S(0)\right\}$*, an equilibrium consists of paths of asset distribution* $\left\{\theta(t), m_H^S(t)\right\}$*, buyers' strategies* $\{i(t), \sigma(p, \psi, t)\}$ *and continuation value functions* $V_H(t), V_L(t)$*, sellers' strategies* $\mu(p, a, t)$ *and continuation value functions* $C_H(t), C_L(t)$ *such that*

1. *For any time* t*, given the continuation values* $V_L(t)$*,* $V_H(t)$*,* $C_L(t)$*,* $C_H(t)$ *and the prior belief* $\theta(t)$*, a buyer's strategy* $\{i(t), \sigma(p, \psi, t)\}$ *and a seller's strategy* $\mu(p, a, t)$ *form a sequential equilibrium of the static trading game.*

2. *The sellers' continuation values* $C_H(t)$ *and* $C_L(t)$ *are given by (2), (3) and (4). The buyers' continuation values* $V_H(t)$ *and* $V_L(t)$ *are given by (5).*

3. *The asset distribution* $\left\{\theta(t), m_H^S(t)\right\}$ *evolves according to (10) and (13).*

4 Stationary Equilibria

In this section, we characterize the set of stationary equilibria of the dynamic trading game, ignoring the role of the initial asset distribution. A stationary equilibrium is an equilibrium in which the asset distribution and investors' trading strategies remain fixed along the equilibrium path. These stationary equilibria are the steady states of the market in the long run. We mostly focus on the pure-strategy stationary equilibria while leaving most of the analysis of mixed-strategy stationary equilibria in the Appendix. The stationary equilibria can be ranked in terms of the total welfare of the investors.

4.1 Construction of Stationary Equilibria

The set of stationary equilibria can be exhausted by guess-and-verify. We start by assuming a trading strategy for all investors and compute the continuation values $\bar{V}_H, \bar{C}_H, \bar{V}_L, \bar{C}_L$. At the same time, we can compute the stationary asset distribution, especially the market

[14] This definition makes use of some results in the previous analysis. A complete definition of equilibrium is given in the Appendix.

composition $\bar{\theta}$, and check if the assumed trading strategies are consistent with the static trading game $(\bar{\theta}; \bar{V}_H, \bar{C}_H, \bar{V}_L, \bar{C}_L)$.

Let $\bar{\rho}_H$ and $\bar{\rho}_L$ be the trading probability of high-quality and low-quality assets in a match. The stationary market composition is

$$\bar{\theta} = \frac{\delta + \lambda \bar{\rho}_L}{\delta + \lambda \bar{\rho}_H} \alpha. \tag{14}$$

If high-quality assets are traded with higher probability in the stationary equilibrium (i.e., $\bar{\rho}_H > \bar{\rho}_L$), the stationary market composition is worse than the fundamental α. On the contrary, if low-quality assets are traded faster, the stationary market composition is better than the fundamental.

The analysis of the static trading game shows that along any equilibrium path, the continuation values of high-quality assets are fixed at $\bar{C}_H = c_H$ and $\bar{V}_H = \frac{rv_H + \delta c_H}{r + \delta}$, independent of the market conditions. Let $\bar{\gamma}_L(c_H)$ be the constant probability that a low type is offered the pooling price c_H in any given match in a stationary equilibrium. The low-quality sellers' and buyers' continuation values are

$$\bar{C}_L = \frac{rc_L + \lambda \bar{\gamma}_L(c_H) c_H}{r + \lambda \bar{\gamma}_L(c_H)}, \; \bar{V}_L = \frac{rv_L + \delta \bar{C}_L}{r + \delta}. \tag{15}$$

If $\bar{\gamma}_L(c_H)$ is small in a stationary equilibrium, the market features lower liquidity and the value of owning low-quality assets is low.

Depending on the strategy of the buyer, the pure strategy stationary equilibria can be put into three categories. Here we describe the information-insensitive pooling stationary equilibrium and the information-sensitive stationary equilibrium while leaving the analysis of the last case, the information-insensitive separating stationary equilibrium, in the Appendix.

4.1.1 Information-Insensitive Pooling Stationary Equilibrium (S_1)

In the first case, buyers do not acquire information and always offer the pooling price c_H. Therefore, both high-quality and low-quality assets are traded at the same speed, $\bar{\rho}_{H,1} = \bar{\rho}_{L,1} = 1$, and the market composition $\bar{\theta}_1$ is the same as the fundamental α. Since the low-type sellers get a pooling offer in each match, $\gamma_L(c_H) = 1$, the continuation values of the low-type sellers and buyers are

$$\bar{C}_{L,1} = \frac{rc_L + \lambda c_H}{r + \lambda}, \; \bar{V}_{L,1} = \frac{rv_L + \delta \bar{C}_{L,1}}{r + \delta}.$$

Notice Assumption 1 implies that $\bar{V}_{L,1} < \bar{C}_{L,1}$, so there are no gains from trade between a buyer and a low type seller. We can check if offering a pooling price without acquiring information is a buyer's optimal trading strategy given the market composition and the continuation values in the stationary equilibrium. To simplify the notation, we use $\theta_1^-(k)$ and $\theta_1^+(k)$ to represent the upper and lower bound of the information region if the continuation values equal to those in the stationary equilibria S_1.

$$\theta_1^-(k) = \theta^-(k, \bar{V}_{L,1}),\ \theta_1^+(k) = \theta^+(k, \bar{V}_{L,1}).$$

Lemma 5 *An information-insensitive pooling stationary equilibrium S_1 exists when*

$$\alpha \geq \max\left\{\frac{c_H - \bar{V}_{L,1}}{V_H - c_H},\ \theta_1^+(k)\right\}.$$

Lemma 5 gives the sufficient and necessary conditions on the fundamental α for the information-insensitive pooling stationary equilibria to exist. It imposes two lower bounds on the fundamental α. If k is large, buyers have no incentive to acquire information for any market composition. In order for buyers to offer a pooling price, $\bar{\theta}_1$ must exceed the threshold for pooling offers. If k is small, $\bar{\theta}_1$ must fall in the information-insensitive pooling region. Notice the threshold $\theta_1^+(k)$ depends on the low type seller's continuation value in the stationary equilibrium.

S_1 is the stationary equilibrium with highest market liquidity subject to search frictions. Both high-type and low-type assets are transferred to the high valuation investors (buyers) whenever a match is formed. Moreover, buyers do not spend resources on inspecting the assets. This resembles the market condition in many liquid OTC markets before the financial crisis. Investors offer similar prices for assets with the same credit ratings without spending resources to acquire private information regarding the quality of the assets. They do it for two reasons. First, lemons only account for a small fraction of the assets for sale, and the composition of assets for sale is unlikely to deteriorate because the fundamental of the market is strong. Second, the expectation that the market will remain liquid in the future keeps investors from worrying about obtaining a lemon because they know that later they will be able to sell it quickly at a high price.

4.1.2 Information-Sensitive Stationary Equilibrium (S_2)

Now let's consider a pure strategy stationary equilibrium with information acquisition (i.e. $\bar{i} = 1$). From the analysis of the static trading game, we know that the pooling price is offered if and only if a good signal is observed. Therefore, the probability of a low-type seller getting a pooling offer is $\bar{\gamma}_L(c_H) = f_L^G$. The continuation values of the low type sellers and buyers in S_2 are

$$\bar{C}_{L,2} = \frac{rc_L + \lambda f_L^G c_H}{r + \lambda f_L^G},\ \bar{V}_{L,2} = \frac{rv_L + \delta \bar{C}_{L,2}}{r + \delta}. \tag{16}$$

In S_2, low-type sellers expect they will receive the offer c_H with probability f_L^G in a match at any time in the future. Assumption 1 implies that $\bar{C}_{L,2} > \bar{V}_{L,2}$, so there's no gain from trade with low-type sellers. Buyers will offer the pooling price c_H after seeing a good signal and offer a no-trade price $p < \bar{C}_{L,2}$ after seeing a bad signal. The probability that an asset is traded in a match is equal to the probability that a good signal is generated by the asset, so $\bar{\rho}_{H,2} = f_H^G,\ \bar{\rho}_{L,2} = f_L^G$. Since the high-quality assets are traded faster, the stationary market composition is worse than the fundamental.

$$\bar{\theta}_2 = \frac{\delta + \lambda f_L^G}{\delta + \lambda f_H^G} \cdot \alpha < \alpha. \tag{17}$$

To check whether the assumed trading strategies indeed form a stationary equilibrium, we need to verify that the stationary market composition falls in the information-sensitive region given the continuation values. Let

$$\theta_2^-(k) = \theta^-(k, \bar{V}_{L,2}),\ \theta_2^+(k) = \theta^+(k, \bar{V}_{L,2}).$$

be the lower and upper bounds of the information region when the continuation values are equal to those in S_2, Lemma 6 gives the sufficient and necessary conditions for the pure strategy information-sensitive stationary equilibrium to exist.

Lemma 6 *Suppose Assumption 1 is true. An information-sensitive stationary equilibrium S_2 exists if and only if*

$$\frac{\delta + \lambda f_H^G}{\delta + \lambda f_L^G} \cdot \theta_2^-(k) \leq \alpha \leq \frac{\delta + \lambda f_H^G}{\delta + \lambda f_L^G} \cdot \theta_2^+(k).$$

Lemma 6 puts a lower bound and an upper bound on the fundamental. From the expressions for the information region in Lemma 3, we know the information region exists when k is small. Therefore, S_2 doesn't exist when k is above a threshold value. In S_2, the market is less liquid than in the information-insensitive pooling stationary equilibrium S_1. Buyers are cautious about the composition of assets on the market, and they always acquire information. As buyers rely on an inaccurate signal, high-quality sellers sometimes receive bad quotes because their asset is taken to be a lemon. It takes longer for a high-quality seller to find an acceptable price in the market compared with the liquid stationary equilibrium S_1. As for the low-quality sellers, there is still a positive probability that they will receive a pooling offer since the buyers sometimes mistakenly take lemons for good assets. If the signal is noisy enough, as in Assumption 1, the expected information rent received by a low-quality seller is higher than the difference in discounted flow payoff between a seller and a buyer. Therefore, low-quality sellers demand a high price that the buyers are not willing to offer unless a good signal is observed. As a result, low-quality sellers stay in the market longer than high-quality sellers. The rent seeking behavior of low-quality sellers has two negative effects on the allocative efficiency in the market. The first effect is direct: low-quality assets are not traded immediately when a buyer arrives, even if the buyer has a higher flow payoff for holding the asset. The second effect is indirect: as low-quality sellers stay longer in the market, the market composition remains below the fundamental and therefore reduces buyers' incentive to offer pooling prices.

Proposition 1 shows that the information-insensitive pooling stationary equilibrium S_1 and the information-sensitive stationary equilibrium S_2 coexist when the fundamental α is within an intermediate region.

Proposition 1 (Coexistence of S_1 and S_2) *Suppose Assumption 1 is true. Let $A_1(k)$ and $A_2(k)$ be*

$$A_1(k) = \max\left\{\theta_1^+(k),\ \frac{\delta + \lambda f_H^G}{\delta + \lambda f_L^G} \cdot \theta_2^-(k)\right\}, \quad A_2(k) = \frac{\delta + \lambda f_H^G}{\delta + \lambda f_L^G} \cdot \theta_2^+(k).$$

S_1 and S_2 co-exist if and only if $\alpha \in [A_1(k), A_2(k)]$. When k is small, $A_1(k) < A_2(k)$.

When agents hold the belief that the market will be liquid as in S_1 in the future, the value of a low-quality asset is high for both sellers and buyers. Buyers are willing to offer the pooling price without acquiring information for a wide range of the market composition. Also, as buyers acquire assets without any selection, the market composition remains at the fundamental value. However, when agents believe the market will be partially illiquid as in S_2, the value of a low-quality asset becomes lower. The information-insensitive pooling region shrinks. At the same time, as buyers cream-skim the market, the market composition stays below the fundamental. Both the trading effect and the valuation effect justify the buyers' information acquisition behavior.

4.2 Welfare Analysis

The total welfare along an equilibrium path is given by

$$\begin{aligned}\varepsilon =&\frac{\alpha}{1+\alpha}v_H + \frac{1}{1+\alpha}v_L \\ &-\int_0^\infty e^{-rt}\left[rm_H^S(t)(v_H-c_H)+rm_L^S(t)(v_L-c_L)+\lambda(m_H^S(t)+m_L^S(t))i(t)k\right]\mathrm{d}t. \end{aligned} \tag{18}$$

The first line of the right-hand side $\frac{\alpha}{1+\alpha}v_H + \frac{1}{1+\alpha}v_L$ represents the welfare in a frictionless benchmark. In the benchmark, assets can be moved from shocked investors to unshocked investors instantaneously. However, due to search frictions and information frictions, some assets are held by shocked investors in equilibrium. The first and the second term in the integrand of (18) represents the welfare loss because of market illiquidity. The third term represents the welfare loss from the resources devoted to information acquisition.

From (10) and (11) we can solve for the stationary asset distribution characterized by the mass of high-quality and low-quality assets held by sellers,

$$\bar{m}_H^S = \frac{\delta\alpha}{(\delta+\lambda\bar{\rho}_H)(1+\alpha)},\ \bar{m}_L^S = \frac{\delta}{(\delta+\lambda\bar{\rho}_L)(1+\alpha)}. \tag{19}$$

Using the trading probability and (19) for stationary asset distribution, we can write down the welfare loss $\Delta = \alpha v_H + (1-\alpha)v_L - \varepsilon$ in each stationary equilibrium:

$$\begin{aligned}\Delta_1 &= \frac{\delta\alpha}{\delta+\lambda}(v_H-c_H)+\frac{\delta(1-\alpha)}{\delta+\lambda}(v_L-c_L),\\ \Delta_2 &= \frac{\delta\alpha}{\delta+\lambda f_H^G}\left(v_H-c_H+\frac{\lambda k}{r}\right)+\frac{\delta(1-\alpha)}{\delta+\lambda f_L^G}\left(v_L-c_L+\frac{\lambda k}{r}\right).\end{aligned}$$

The welfare loss in S_1 is lower than that in S_2. In S_2, sellers hold a larger mass of both high-quality and low-quality assets, and buyers are paying extra costs of information acquisition compared to S_1. As we previously pointed out, S_1 is the most efficient stationary equilibrium subject to search frictions.

5 Non-Stationary Equilibria

In the previous section we investigated various states of the market in the long run. Now we turn to analyze how investors' trading behavior and market liquidity evolve over time starting from a given initial asset distribution. Particularly, we are interested in the following question. When a liquid steady state and an illiquid steady state co-exist, is it possible for the market to transition from one to the other? In order to answer this question, it is important to study the set of non-stationary equilibria.

To show the existence of a certain equilibrium path from an initial asset distribution to a terminal steady state, we first hypothesize about investors' trading strategies for any $t > 0$. Given the paths of trading probability $\rho_H(t)$ and $\rho_L(t)$ and the initial asset distribution represented by $m_H^S(0)$ and $m_L^S(0)$, the full path of the asset distribution can be analytically solved from (10) and (11) as follows:

$$m_H^S(t) = e^{-\int_0^t \delta+\lambda\rho_H(s)\mathrm{d}s} m_H^S(0) + \frac{\delta\alpha}{1+\alpha}\int_0^t e^{-(\delta+\lambda\rho_H(u)(t-s))\mathrm{d}u}\mathrm{d}s, \tag{20}$$

$$m_L^S(t) = e^{-\int_0^t \delta+\lambda\rho_L(s)\mathrm{d}s} m_L^S(0) + \frac{\delta}{1+\alpha}\int_0^t e^{-(\delta+\lambda\rho_L(u)(t-s))\mathrm{d}u}\mathrm{d}s. \tag{21}$$

Next we can compute the paths of continuation values to verify whether the assumed trading strategies form an equilibrium of the static trading game at any $t > 0$.

In the Appendix, I provide sufficient conditions for the market composition $\theta(t)$ to change monotonically along a non-stationary equilibrium path.

5.1 Self-fulfilling Market Freeze

Suppose the market has an asset distribution as in the liquid state S_1. Is it possible that all investors suddenly change their beliefs and coordinate to follow an equilibrium path that converges to the illiquid state S_2? This question is answered in Proposition 2.

Proposition 2 (Self-fulfilling Market Freeze) *If Assumption 1 holds, for small k there exists*

$$A_3(k) = \theta_2^+(k) \in (A_1(k), A_2(k)),$$

such that, for any $\alpha \in [A_1(k), A_3(k)]$, starting from an initial asset distribution in the neighborhood of S_1, there is an equilibrium path that converges to S_2.

When $\alpha \in [A_1(k), A_3(k)]$, the model has multiple equilibria starting from the asset distribution of S_1. Proposition 2 implies that a liquid market can go through a self-fulfilling market freeze. Starting from the asset distribution in S_1, if all investors believe that future buyers will not acquire information and always offer the pooling price, the current buyers have no incentive to acquire information and they continue to offer the pooling price. The market therefore remains in the liquid steady state of S_1. However, if all investors believe the market liquidity will begin to decline and buyers in the future will begin to acquire information as a way of avoiding low-quality assets, the continuation value of holding low quality assets drops immediately. Thus, for current buyers, the loss incurred by buying a low-quality asset at the pooling price becomes larger, and this gives them more incentive to acquire information. When current buyers acquire information but their independent evaluation of the assets are not accurate enough, high-quality assets are traded faster than low-quality assets, resulting in a cream-skimming effect on the market composition. The market composition deteriorates over time and justifies future buyers' information acquisition. Therefore, the market evolves along a path with information acquisition and converges to the information-sensitive steady state S_2.

Notice that Proposition 2 does not imply that the information-insensitive pooling steady state is unstable. In fact, the liquid steady state is locally stable.

Proposition 3 *If $\alpha, \theta(0) > \theta_1^+(k)$, there exists an equilibrium path with pooling offers and no information acquisition that converges to S_1.*

The results of Propositions 2 and 3 can be illustrated graphically. In Figures 3 and 4 I plot the phase diagram of the evolution of asset distributions according to (10) and (13). The horizontal axis represents the market composition that determines the current investors' trading strategies. The vertical axis represents the mass of sellers with high-quality assets in the market. Although the mass of high-quality sellers does not affect the current investors' trading strategies directly, it shapes the evolution of the asset distribution through the interaction with market composition. Recall that the evolution of the asset distribution depends on the trading probability of different assets, which in turn depends on investors' belief about future market liquidity through resale considerations. Therefore, before we plot a phase diagram, we need to specify investor's continuation values according to their belief about future market liquidity.

Figure 3 shows the phase diagram when all investors believe future buyers will not acquire information but instead will always make pooling offers. Given this belief, the continuation values of owners of low-quality assets are $\bar{V}_{L,1}$ and $\bar{C}_{L,1}$. The corresponding information-sensitive region is given by $[\theta_1^-(k), \theta_1^+(k)]$, represented by the shaded region in the figure. If the fundamental α is above $\theta_1^+(k)$, there exists an information-insensitive pooling steady state, represented by the stationary asset distribution S_1 on the right of the shaded region.

If the investors maintain their belief about a liquid market in the future, the market will stay in S_1. Moreover, as Proposition 3 shows, starting from any asset distribution to the right of the shaded region, there is a path converging to S_1. Along the path, the asset composition is always above $\theta_1^+(k)$, consistent with the investors' belief that there is no need for information acquisition.

What happens when investors' beliefs shifts? Suppose the market starts out with the asset distribution in S_1, but investors suddenly start to believe that investors in the future will acquire information and the market will become illiquid. The phase diagram changes from Figure 3 to 4. The continuation values of owning low-quality assets drop to $\bar{V}_{L,2}$ and $\bar{C}_{L,2}$, the same as in the information-sensitive steady state. Since the continuation values become lower, the information-sensitive region moves to the right, represented by the shaded region in Figure 4. The asset composition is good enough to support pooling trading in S_1 when investors believe in a liquid market in the future. However, after the shift in the investors' beliefs, S_1 is now in the shaded information-sensitive region, reflecting higher incentives to acquire information when investors anticipate lower liquidity in the future. The market will therefore follow the arrows and move to S_2. The whole path lies within the shaded region, meaning that buyers always acquire information along the path, consistent with investors' belief in low liquidity in the future. The transition from S_1 to S_2 is consistent with an event of a self-fulfilling market freeze, in which trading delays suddenly become longer.

5.2 Information Trap

If the market's initial asset distribution is in the illiquid state S_2, is there a non-stationary equilibrium path that converges to liquid trading? The answer depends on the relationship between the market composition in S_2 and the information-sensitive region $[\theta_1^-(k), \theta_1^+(k)]$ in S_1. This can be illustrated in the same set of phase diagrams. In Figure 3 and Figure 4, the information acquisition regions in S_1 and S_2 overlap and the illiquid state S_2 falls in the overlapping region. Starting from the initial asset distribution in S_2, if all investors hold the belief that future buyers will acquire information, S_2 is in the shaded information-sensitive region in Figure 4, consistent with the investors' belief. Now suppose all investors believe that in the future, buyers will not acquire information and will always offer the pooling price. This optimistic belief in future market liquidity improves the continuation values, changing the phase diagram to Figure 3 and shifting the information-sensitive region to $[\theta_1^-(k), \theta_1^+(k)]$. However, since S_2 is also in the shaded information-sensitive region in Figure 3, current buyers will still acquire information and cream-skim the market. Their trading behavior keeps the asset distribution at S_2 and prevents the market from recovering to S_1.

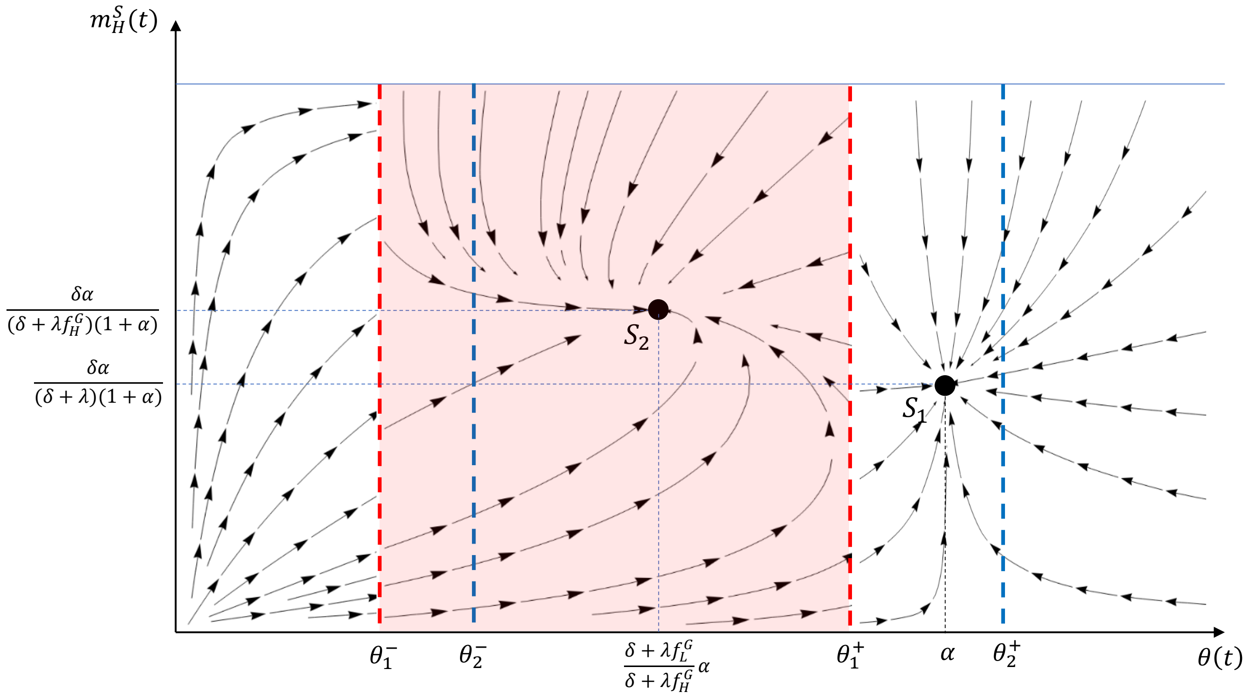

Figure 3: Phase Diagram ($V_L(t) = \bar{V}_{L,1}$, $C_L(t) = \bar{C}_{L,1}$).

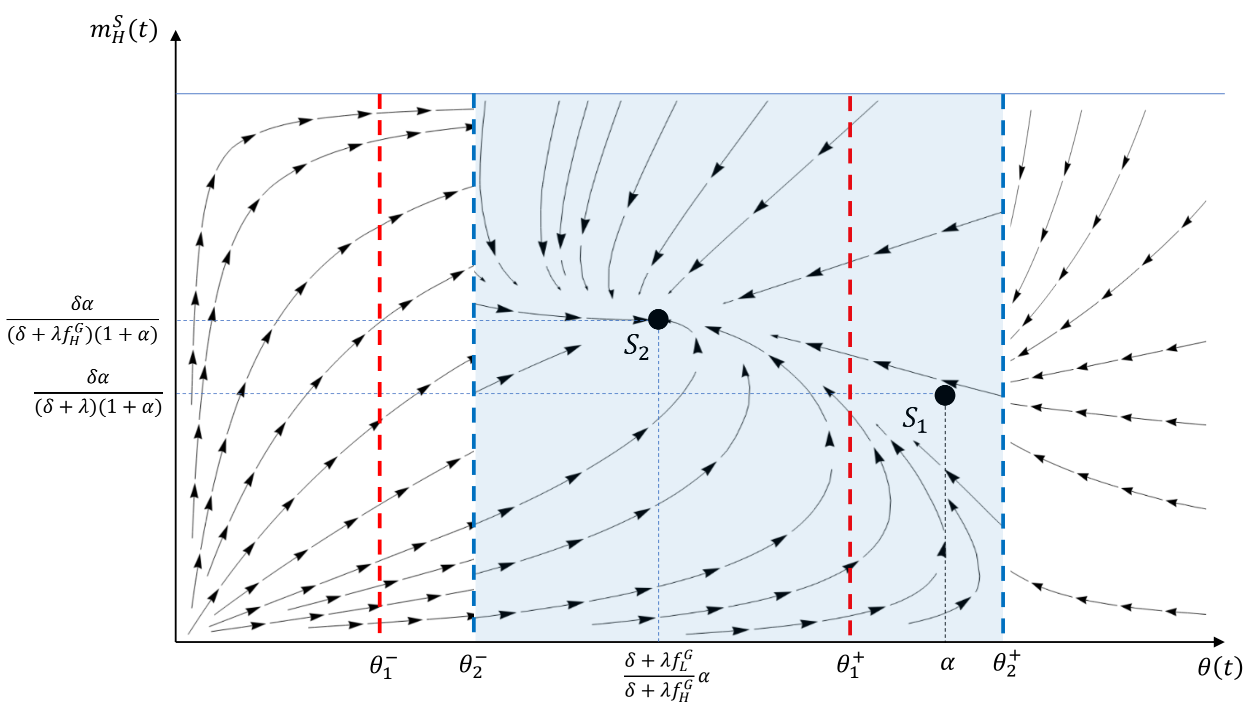

Figure 4: Phase Diagram ($V_L(t) = \bar{V}_{L,2}$, $C_L(t) = \bar{C}_{L,2}$).

To summarize, if the market composition in S_2 satisfies $\bar{\theta}_2 < \theta_1^+(k)$, there is no equilibrium path that converges to the liquid state S_1.

Now let's consider the opposite case if $\bar{\theta}_2 \geq \theta_1^+(k)$. Starting from the initial asset distribution in S_2, when investors believe the market will be liquid in the future, the optimal strategy for a buyer is to stop acquiring information and to instead offer the pooling price. As a result, the market composition will gradually improves and converges to $\bar{\theta}_1$, the market composition in S_1. Along this path, buyers do not acquire information. Therefore, if $\bar{\theta}_2 \geq \theta_1^+(k)$, there exists a non-stationary equilibrium path that transitions from S_2 to S_1.

Assumption 2 $\frac{f_H^G f_L^B}{f_L^G f_H^B} > \frac{c_H - \bar{V}_{L,2}}{c_H - \bar{V}_{L,1}}$.

Assumption 2 is equivalent to the condition $\theta_2^-(0) < \theta_1^+(0)$. If Assumption 2 is true, $\theta_2^-(k) < \theta_1^+(k)$ holds for small k so that the two information acquisition regions overlap. The intuitive interpretation of Assumption 2 is that it requires the signal to be relatively accurate so given any set of continuation values, information acquisition is optimal for a wide range of market composition. Otherwise, if the information available to be buyers is very noisy, information acquisition is irrelevant most of the time.[15]

I call the overlapping part of the two information-sensitive regions $[\theta_2^-(k), \theta_1^+(k)]$ the *information trap* whenever it exists. The information trap is different from the information sensitive regions we just discussed. At any time t, the information sensitive region depends on the continuation values of owning low-quality assets $V_L(t), C_L(t)$. However, by definition, the information trap is time and strategy invariant so it is independent of investors' beliefs and the continuation values. When the market composition is within the information trap, whether or not investors believe that future buyers will acquire information or not, the optimal strategy is to acquire information today, and the cream-skimming effect will be in play. Intuitively speaking, the market composition will be trapped in the region and dragged into the "sink," which is the information-sensitive state S_2.[16]

Proposition 4 formally conveys the condition in which there is no non-stationary equilibrium path that transitions from S_2 to S_1.

[15] Assumption 2 is not in conflict with Assumption 1. Assumption 1 requires that f_L^G is not too small so the buyer can make a mistake in the inspection and take a low-quality asset as a "good" one. However, it does not put any restrictions on the signals observed from a high-quality asset. When f_H^G gets closer to 1, the left-hand side of Assumption 2 goes to ∞.

[16] In Appendix E, I consider whether there exists a non-stationary equilibrium path that converges to the liquid state S_1, starting from an arbitrary initial market composition $\theta(0)$ in the information trap. I provide the sufficient and necessary conditions such that the equilibrium path exists.

Proposition 4 (Information Trap) *If Assumption 1 and 2 hold, for small k there exists*

$$A_4(k) = \frac{\delta + \lambda f_H^G}{\delta + \lambda f_L^G} \cdot \theta_1^+(k) \in (A_1(k), A_2(k)), \tag{22}$$

such that, for any $\alpha \in [A_1(k), A_4(k)]$, if the initial asset distribution is in the neighborhood of S_2, there is no equilibrium path converging to pooling trading.

Propositions 2 and 4 jointly imply that, for $\alpha \in [A_1(k), \min\{A_3(k), A_4(k)\}]$, the liquid steady state S_1 and the illiquid steady state S_2 coexist. More importantly, the transitions between the two steady states are asymmetric. Suppose the market is in the liquid state S_1 where buyers are not paying any attention to the idiosyncratic features of the assets. They simply buy assets at the pooling price from any seller they meet in the market. The market composition remains at a high level. A self-fulfilling market freeze starts from a market-wide panic about a decline in future market liquidity. Investors worry that if they hold low-quality assets in the portfolio, in the future, it will be hard for them to sell these assets at good prices. Because of this concern, buyers start to collect information and carefully evaluate the assets they see on the market. They are only willing to offer a good price for an asset if the aspects of the asset satisfy their own criteria. However, because buyers' evaluations of assets are not perfect, sellers who receive a bad quote will stay in the market with the hope that they will receive a high quote from the next buyer. The trading speeds of both types of assets drop immediately, and the value of low-quality assets to the current owners decline. As the market goes further down the illiquid path, the market composition deteriorates gradually as low-quality assets accumulate in the market. At some point, the market composition becomes bad enough that it falls into the information trap. Even if buyers have optimistic beliefs about future market liquidity, since the current market composition is bad, they keep acquiring information to avoid buying low-quality assets at high prices. The low liquidity and the bad market composition reinforce each other through buyers' information acquisition, and the market can not recover to the liquid state.

6 Policy Implications

In this section we explore two policy implications of the model.

6.1 Issuance Transparency

Transparency in the issuance process of ABS was low before the latest financial crisis. The low issuance transparency has been criticized for generating moral hazard problems in the securitization process and adverse selection problems in the secondary market, which played important roles in the creation and propagation of the financial crisis. After the financial crisis, regulators moved toward a more transparent issuance process. For example, Dodd-Frank Act Section 942 requires issuers of asset-backed securities (ABS) to provide asset-level information according to specified standards.[17] In the context of my model, these regulatory changes could lower the cost of information acquisition and increase the precision of buyers' signals.

Definition 2 *A signal ψ' is (weakly) more precise than a signal ψ if and only if $f_H^{G'} \geq f_H^G$ and $f_L^{G'} \leq f_L^G$.*

We use two simple criteria to evaluate the effect of increasing transparency on the liquidity of the secondary market. First, we look at $\theta_1^+(k)$, since the liquid steady state $S1$ exists if and only if $\alpha > \theta_1^+(k)$. Second, we consider $A_4(k)$. When $\alpha > A_4(k)$, there is no steady state in the information trap.

Proposition 5 *If both ψ' and ψ satisfy Assumption 1 and 2, and ψ' is more precise than ψ, both $\theta_1^+(k)$ and $A_4(k)$ increase when switching from the signal structure ψ to ψ', and when k decreases.*

Proposition 5 implies that increasing transparency in the issuance process can harm market liquidity, judging by our simple criteria. An intuitive explanation of this result is that when issuers provide more information regarding the pool of assets backing the ABS, future investors can better evaluate the assets' quality upon conducting due diligence. This gives buyers more incentive to acquire information, and when they do so, the cream-skimming effect is stronger. It is worth mentioning that I only consider the impact of increasing transparency on the liquidity of the secondary market and ignore the impact on disciplining the issuance process. A complete evaluation of these types of polices should take effects on both the primary and the secondary markets into consideration.

6.2 Asset Purchase Programs

When a market freezes because of the adverse selection problem, a natural solution is to clean the market by removing low-quality assets from the market. Many theoretical papers have studied the design of asset purchase programs in the presence of severe adverse selection, including Philippon and Skreta (2012), Tirole (2012), Camargo and Lester (2014) and Chiu

[17] See https://www.sec.gov/spotlight/dodd-frank-section.shtml#942.

and Koeppl (2016). During the latest financial crisis, the US Treasury created the Public-Private Investment Program (PPIP) to purchase "toxic" assets, aiming at restoring liquidity in the markets for legacy Commercial MBS and non-agency RMBS.

Asset purchase programs can help the target market restore liquid trading through two channels. First, it removes lemons from the market, so the fundamentals of the market improves. Second, if the government purchases assets at a higher price than the market would offer, or selling assets to the government is easier than locating a buyer in the private sector, the asset purchase program effectively increases the value of lemons. As a result, the lemon's problem is mitigated and buyers in the market are more willing to offer pooling prices.

In my model, when the market goes through a self-fulfilling market freeze from S_1 to S_2, the market composition deteriorates gradually and the mass of "toxic" assets on the market increases over time. In the proof of Proposition 2, I show that $\theta(t)$ decreases and $m^s_H(t)$ increases over time along the path of market freeze. There exists a time $\hat{t}$ such that $\theta(\hat{t}) = \theta_1^+(k)$. If the government intervenes before $\hat{t}$, the market composition is above the information trap. There still exists an equilibrium path that converges to liquid trading. Therefore, market liquidity can be boosted by a plan that guarantees a floor-price for all assets. The government does not need to actually purchase assets from the market since the market will immediately return to liquid trading as buyers all stop acquiring information. However, after $\hat{t}$, the market enters the information trap and there is no self-fulfilling equilibrium path that returns to S_1. The government needs to purchase a positive amount of assets to revive the market.

Chiu and Koeppl (2016) study the *announcement effect* of asset purchase programs. Specifically, when the government announces that it will purchase a given amount of lemons at a given price later at a given time, it is possible that the market will restore to liquid trading even before the government actually purchases these assets. Thus the government may be justified in delaying the purchase to lower the intervention cost. However, a direct implication of Proposition 4 is that in an illiquid steady state within the information trap, there is no announcement effect for any asset purchase program with purchasing price $p \in [\bar{C}_{L,2}, \bar{C}_{L,1}]$.

7 Conclusions

In this paper, I present a model for studying the interaction between buyers' information acquisition and market liquidity in over-the-counter markets with adverse-selection problems. Buyers can acquire information to avoid buying low-quality assets, and their incentive for doing so is strong if they expect that the market will be illiquid when they resell their assets. When buyers' signals are inaccurate, information acquisition has a cream-skimming effect on the composition of assets for sale and harms future market liquidity. The interaction of resale consideration and the cream-skimming effect gives rise to multiple steady states and asymmetric transitions between steady states. Specifically, the market can transition from a liquid state without information acquisition to an illiquid state with information acquisition, but it can not transition back. This uni-directional transition between different steady states is a novel feature of my model that, to the best of my knowledge, is not present in the models used in previous papers on dynamic adverse selection. This result helps explain the continued low liquidity in the non-agency residential mortgage-backed-security market in spite of the recovery of the US economy and the housing markets.

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Appendices

A Alternative Definition of Equilibrium

Here I provide a formal but less intuitive equilibrium definition which is equivalent to the definition provided in Section 3.

Definition A.1 *A equilibrium consists of paths of asset distribution* $\{\theta(t), m_H^S(t), m_L^S(t)\}$*, buyers' policy functions* $\{i(t), \sigma(p, \psi, t)\}$ *and value functions* $\{V_H(t), V_L(t)\}$*, seller's policy function* $\mu(p, j, t)$ *and value functions* $\{C_H(t), C_L(t)\}$*, which satisfy the following conditions:*

1. *Seller's optimality condition: For any* $j \in \{H, L\}$*,*

$$\mu(p, j, t) = \begin{cases} 1, & \text{if } p > C_j(t), \\ [0,1], & \text{if } p = C_j(t), \\ 0, & \text{if } p < C_j(t). \end{cases} \tag{A.1}$$

2. *Buyer's optimality conditions:*

   (a) *For* $\psi \in \{G, B\}$*,* $\sigma(p, \psi, t) > 0$ *only if* p *solves*

$$J(\psi, t) = \max_p \frac{\theta(t)}{\theta(t)+1} f_H^{\psi} \mu(p, H, t) \left[V_H(t) - p\right] + \frac{1}{\theta(t)+1} f_L^{\psi} \mu(p, L, t) \left[V_L(t) - p\right];$$

   (b) $\sigma(p, N, t) > 0$ *only if* p *solves*

$$J(N, t) = \max_p \frac{\theta(t)}{\theta(t)+1} \mu(p, H, t) \left[V_H(t) - p\right] + \frac{1}{\theta(t)+1} \mu(p, L, t) \left[V_L(t) - p\right];$$

   (c) *The value of information* $W(t)$ *is*

$$W(t) = \max \left\{J(G, t) + J(B, t) - J(N, t), 0\right\},$$

   and $i(t)$ *satisfies*

$$i(t) = \begin{cases} 1, & \text{if } W(t) > k, \\ [0,1], & \text{if } W(t) = k, \\ 0, & \text{if } W(t) < k. \end{cases} \tag{A.2}$$

3. *The continuation values of sellers* $C_j(t)$ *are given by* (2),(3) *and* (4)*. The continuation values of buyers/holders* $V_j(t)$ *are given by* (5)*.*

4. The asset distribution, characterized by $m_H^S(t)$, $m_L^S(t)$ and $\theta(t)$ evolves according to (11)-(13).

B Other Stationary Equilibria

B.1 Pure-Strategy Stationary Equilibria

B.1.1 Information-Insensitive Separating Stationary Equilibrium (S_3)

When the stationary market composition falls in the information-insensitive region with separating offers, the market is in an information-insensitive separating stationary equilibrium. This is the third and the last type of stationary equilibrium with pure strategies. In S_3, buyers do not acquire information and only offers the separating price. Therefore, the low-quality assets are traded with probability 1 in each match and the high-quality assets are never traded. $\bar{\rho}_{H,3} = 0$, $\bar{\rho}_{L,3} = 1$. The stationary equilibria market composition is better than the fundamental.

$$\bar{\theta}_3 = \frac{\delta + \lambda}{\delta} \cdot \alpha > \alpha. \tag{B.1}$$

Since the pooling price is never offered in equilibrium, the continuation values of low-quality asset owners are

$$\bar{C}_{L,3} = c_L, \ \bar{V}_{L,3} = \frac{rv_L + \delta c_L}{r + \delta}.$$

It's easy to verify that $\bar{V}_{L,3} > \bar{C}_{L,3}$ so there are gains from trade for low-quality assets. Similarly, let

$$\theta_3^-(k) = \theta^-(k, \bar{C}_{L,3}), \ \theta_3^+(k) = \theta^+(k, \bar{C}_{L,3})$$

be the lower and upper bounds of the information-sensitive region when the continuation values are equal to those in S_3

Lemma B.1 *An information-insensitive separating stationary equilibrium S_3 exists if and only if*

$$\alpha \leq \frac{\delta}{\delta + \lambda} \min \left\{ \frac{c_H - c_L}{V_H - c_H}, \theta_3^-(k) \right\}.$$

In S_3, all high-quality assets and a fraction of low-quality assets are on the market. Yet, the fundamental of the market is so bad that the amount of lemons on the market is large enough to prevent any pooling offers or information acquisition from buyers. The continuation values of low-quality asset owners are the lowest in all possible equilibria.

B.2 Mixed-Strategy Equilibria

Here we provide two useful results that restrict the set of possible mixed strategies in equilibrium.

Lemma B.2 *In any equilibrium, if $i(t) > 0$, $\sigma(c_H, G, t) = 1$ and $\sigma(c_H, B, t) = 0$.*

Lemma B.2 applies to all equilibrium path. It implies a buyer will offer the pooling price c_H if and only if a good signal is observed. The proof is intuitive. Based on the analysis of the static trading game, it is clear that given any set of continuation values, buyers only choose between two price. Without loss of generality, assume the buyer offers price p_1 after seeing a good signal and mix between p_1 and p_2 after seeing a bad signal. Since the buyer uses mixed strategy after seeing a bad signal, then the expected payoff from offering the two prices based on the posterior belief of seeing a bad signal must be the same. Therefore, the expected payoff doesn't change if the buyer offer p_1 with probability 1 after seeing a bad signal. This makes the buyer's offer independent of the signal. Thus, the buyer can simply offer p_1 without information acquisition and save the fixed cost. The above reasoning shows the sub-optimality of using mixed strategy after acquiring information. We can us Lemma B.2 to simplify (2), in any equilibrium,

$$\gamma_L(t) = i(t) f_L^G + (1 - \bar{i}(t))\, \sigma(c_H, N, t). \tag{B.2}$$

Do sellers randomize in equilibrium? Obviously, sellers of low-quality assets always accept the pooling price c_H. Also, sellers of high-quality assets always accept the pooling price c_H in any equilibrium. If sellers of high-quality assets accept price c_H with a probability less than 1, a buyer can raise the offer by a tiny amount and increase the surplus by $V_H - C_H$ with a strictly positive probability. Following the same logic, if sellers of low-quality assets randomize when offered a separating price, $\bar{C}_L$ must be equal to $\bar{V}_L$. In stationary equilibria, this implies that $\bar{\gamma}_L = \frac{r}{\lambda}(v_L - c_L)/(c_H - v_L)$. By Assumption 1, $\bar{\gamma}_L < f_L^G$. Using (B.2), we immediately have the following lemma.

Lemma B.3 *If Assumption 1 holds, in any stationary equilibria with sellers of low-quality assets using mixed strategies, we have $\bar{i} < 1$ and $\bar{\sigma}(c_H, N) < f_L^G$.*

If buyers randomize between a separating offer and a no-trade offer, the gains from trade of low-quality assets must be zero, $V_L(t) = C_L(t)$. We say two equilibria are equivalent when sellers and buyers of both high-quality and low-quality assets have the same trading probability and continuation values at any give time. Any equilibrium with buyer mixing between a separating offer and a no-trade offer is equivalent to an equilibrium with buyers only offering the separating price and sellers rejecting the offer with a positive probability. This equivalence allows us to focus on mixed-strategy equilibria in which buyers only choose between the separating offer and the pooling offer.

B.2.1 Mixed-Strategy Stationary Equilibrium without Information Acquisition

Any mixed strategy stationary equilibrium without information acquisition must have buyers using mixed strategies. It is sufficient to consider buyers mixing between the pooling price c_H and the separating price $\bar{C}_L$. Notice in any equilibrium without information acquisition, the probability of buyer offering c_H is equal to γ_L. When buyers do not acquire information, whether they offer the separating price or the no-trade price depends on the relationship between $\bar{V}_L$ and $\bar{C}_L$. Since in a stationary equilibrium, $\bar{V}_L$ is a weighted average of v_L and $\bar{C}_L$, it's equivalent to compare $\bar{C}_L$ and v_L. There are three cases:

1. (S_4) $\bar{C}_L > v_L$. This is the case when buyers offer c_H with probability $\bar{\gamma}_{L,4}$ and the no trade price with probability $1 - \bar{\gamma}_{L,4}$. In each match, either type of asset is traded with probability $\bar{\gamma}_{L,4}$. (15) implies that $\bar{\gamma}_{L,4} > \frac{r}{\lambda}(v_L - c_L)/(c_H - v_L)$. This stationary equilibria exists when the following conditions are satisfied:

$$\frac{c_H - \bar{V}_{L,1}}{V_H - c_H} < \alpha < \frac{c_H - v_L}{V_H - c_H}, \tag{B.3}$$

$$k \geq (f_L^B - f_H^B)(V_H - c_H)\frac{\alpha}{1+\alpha}. \tag{B.4}$$

   The market liquidity $\bar{\gamma}_{L,4}$ is determined by $\alpha = \frac{c_H - \bar{V}_{L,4}}{V_H - c_H}$ and (15).

2. (S_5) $\bar{C}_L < v_L$. In this stationary equilibrium buyers offer c_H with probability $\bar{\gamma}_{L,5}$ and the separating price $\bar{C}_{L,5}$ with probability $1 - \bar{\gamma}_{L,5}$. Low-quality sellers accept the separating offer for sure. In each match, a high-quality asset is traded with probability $\bar{\gamma}_{L,5}$ and a low-quality asset is always traded. If this stationary equilibrium exists, (α, k) must satisfy the following conditions given a market liquidity $\bar{\gamma}_{L,5} \in (0, \frac{r}{\lambda}(v_L -$

$c_L)/(c_H - v_L))$.

$$\bar{C}_{L,5} = \frac{rc_L + \lambda\bar{\gamma}_{L,5}c_H}{r + \lambda\bar{\gamma}_{L,5}},$$
$$\frac{c_H - \bar{C}_{L,5}}{V_H - c_H} = \frac{\delta + \lambda}{\delta + \lambda\bar{\gamma}_{L,5}} \cdot \alpha,$$
$$k \geq (f_L^B - f_H^B)(V_H - c_H) \cdot \frac{c_H - \bar{C}_{L,5}}{V_H - \bar{C}_{L,5}}.$$

3. (S_6) $\bar{C}_L = v_L$. In this stationary equilibria, buyers offer c_H with probability $\bar{\gamma}_{L,6} = \frac{r}{\lambda}(v_L - c_L)/(c_H - v_L)$ and the separating price $\bar{c}_{L,6}$ with probability $1 - \bar{\gamma}_{L,6}$. Low-quality sellers accept the separating offer with probability $\bar{\mu}(v_L, L) \in (0,1)$. For the stationary equilibria to exist, (α, k) must satisfy the following conditions

$$\frac{\delta + \lambda\bar{\gamma}_{L,6}}{\delta + \lambda} \cdot \frac{c_H - v_L}{V_H - c_H} < \alpha < \frac{c_H - v_L}{V_H - c_H},$$
$$k \geq (f_L^b - f_H^b)(V_H - c_H) \cdot \frac{c_H - v_L}{V_H - v_L}.$$

where $\bar{\mu}(v_L, L)$ is the solution to

$$\frac{\delta + \lambda\bar{\gamma}_{L,6}}{\delta + \lambda\left[\bar{\gamma}_{L,6} + \bar{\mu}(v_L, L)(1 - \bar{\gamma}_{L,6})\right]} \cdot \frac{c_H - v_L}{V_H - c_H} = \alpha. \tag{B.5}$$

B.2.2 Mixed-strategy equilibrium with partial information acquisition

Now let's turn to the mixed-strategy stationary equilibria with $\bar{i} \in (0,1)$. In any equilibrium, buyers always offer c_H after observing a good signal.

1. (S_7) First let's consider stationary equilibria with $\bar{\theta}$ located on the right boundary of the information-sensitive region. Since $\bar{\theta} > \hat{\theta}$, when buyers do not acquire information, they offer the pooling price. Therefore $\bar{\gamma}_{L,7} = \bar{i}_7 f_L^G + 1 - \bar{i}_7$. Notice $\bar{\gamma}_{L,7} > f_L^G$. Assumption 1 implies that $\bar{C}_{L,7} > v_L$, so there's no gain from trade for low-quality assets. Low-quality assets will not be traded if a bad signal is observed. High-quality and low-quality assets are traded with probability $\bar{\rho}_{H,7} = \bar{i}_7 f_H^G + 1 - \bar{i}_7$, while low-quality assets are traded with probability $\bar{\rho}_{L,7} = \bar{i}_7 f_L^G + 1 - \bar{i}_7$. The stationary equilibria market composition $\bar{\theta}_7$ is given by (14). S_7 exists if and only if the following conditions are

satisfied:

$$\theta^{+}(k, \bar{V}_{L,7}) \geq \frac{c_H - \bar{V}_{L,7}}{V_H - c_H}, \tag{B.6}$$

$$\alpha = \frac{\delta + \lambda \bar{\rho}_{H,7}}{\delta + \lambda \bar{\rho}_{L,7}} \cdot \theta^{+}(k, \bar{V}_{L,7}) \tag{B.7}$$

2. (S_8) The next group of stationary equilibria we investigate has $\bar{\theta}$ located on the left boundary of the information-sensitive region. Since $\bar{\theta} < \hat{\theta}$, buyers never offer the pooling price without information acquisition. Therefore $\bar{\gamma}_{L,8} = \bar{i}_8 f_L^G$. High-quality assets are traded with probability $\bar{\rho}_{H,8} = \bar{i}_8 f_H^G$. The probability that a low type asset is traded depends on whether there's gain from trade. Given different $\bar{i}_8$, there are three cases:

   - If $\bar{i}_8 > \frac{r}{\lambda f_L^G}(v_L - c_L)/(c_H - v_L)$, the gain from trade of low-quality assets is negative. Low-quality assets are traded with probability $\bar{\rho}_{L,8} = \bar{\gamma}_{L,8}$.
   - If $\bar{i}_8 < \frac{r}{\lambda f_L^G}(v_L - c_L)/(c_H - v_L)$, the gain from trade of low-quality assets is positive. Low-quality assets are traded with probability $\bar{\rho}_{l,8} = 1$.
   - If $\bar{i}_8 = \frac{r}{\lambda f_L^G}(v_L - c_L)/(c_H - v_L)$, the gain from trade of low-quality assets is zero. Sellers of low-quality assets can use mixed strategies when offered the separating price. Low-quality assets are traded with probability $\bar{\rho}_{l,8} \in [\bar{\gamma}_{L,8}, 1]$.

   The continuation values of the owners of low-quality assets are given by (15). The stationary equilibria market composition $\bar{\theta}_8$ is given by (14). Let $\bar{\nu}_8 = \min\left\{\bar{V}_{L,8}, \bar{C}_{L,8}\right\}$. S_8 with a given $\bar{i}_8 \in (0, 1)$ exists if and only the following conditions are satisfied:

$$\theta^{-}(k, \nu_8) \geq \frac{c_H - \bar{\nu}_8}{V_H - c_H}, \tag{B.8}$$

$$\alpha = \frac{\delta + \lambda \bar{\rho}_{H,8}}{\delta + \lambda \bar{\rho}_{L,8}} \cdot \theta^{-}(k, \bar{\nu}_8). \tag{B.9}$$

3. (S_9) The last group of stationary equilibria features buyer's partial information acquisition and mixed offering strategy when information is not acquired. Buyers acquire information with probability $\bar{i}_9$. In case the buyers do not acquire information, they offer the pooling price with probability $\bar{\sigma}(c_H, N)$. Therefore, $\bar{\gamma}_{L,9} = \bar{i}_9 f_L^G + \bar{\sigma}(c_H, N)$. High-quality assets are traded with probability $\bar{\rho}_{h,9} = \bar{i}_9 f_L^G + \bar{\sigma}(c_H, N)$. The probability that low type assets are traded depends on the gain from trade of low-quality assets. There are three cases depending on $\bar{\gamma}_{L,9}$:

- If $\bar{i}_9 > \frac{r}{\lambda}(v_L - c_L)/(c_H - v_L)$, the gain from trade of low-quality assets is negative. Low-quality assets are traded with probability $\bar{\rho}_{L,9} = \bar{\gamma}_{L,9}$.
- If $\bar{i}_9 < \frac{r}{\lambda}(v_L - c_L)/(c_H - v_L)$, the gain from trade of low-quality assets is positive. Low-quality assets are traded with probability $\bar{\rho}_{L,9} = 1$.
- If $\bar{i}_9 = \frac{r}{\lambda}(v_L - c_L)/(c_H - v_L)$, the gain from trade of low-quality assets is zero. Sellers of low-quality assets can use mixed strategies when offered the separating price. Low-quality assets are traded with probability $\bar{\rho}_{l,9} \in [\bar{\gamma}_{L,9}, 1]$.

The continuation values of the owners of low-quality assets are given by (15). The stationary equilibria market composition $\bar{\theta}_9$ is given by (14). Let $\bar{\nu}_9 = \min\left\{\bar{V}_{L,9}, \bar{C}_{L,9}\right\}$. S_9 with given $\bar{i}_9 \in (0,1)$ and $\bar{\sigma}(c_H, N)$ exists if and only if the following conditions are satisfied:

$$k = (f_L^B - f_H^B)(V_H - c_H) \cdot \frac{c_H - \bar{\nu}_9}{V_H - \bar{\nu}_9}, \tag{B.10}$$

$$\alpha = \frac{\delta + \lambda \bar{\rho}_{H,9}}{\delta + \lambda \bar{\rho}_{L,9}} \cdot \frac{c_H - \bar{\nu}_9}{V_H - c_H}. \tag{B.11}$$

C Monotonicity of Paths of Market Composition

Define $\bar{\rho}_{H0}$ and $\bar{\rho}_{L0}$ as

$$\bar{\rho}_{H0} = \frac{\delta}{\lambda}\left(\frac{\alpha}{m_H^S(0)(1+\alpha)} - 1\right), \ \bar{\rho}_{L0} = \frac{\delta}{\lambda}\left(\frac{1}{m_L^S(0)(1+\alpha)} - 1\right). \tag{C.1}$$

Compared with (19), if the initial asset distribution is an stationary distribution, $\bar{\rho}_{H0}$ and $\bar{\rho}_{L0}$ are the corresponding trading probability of high-quality and low-quality assets. A higher $\bar{\rho}_{H0}$ ($\bar{\rho}_{L0}$) is related to a smaller initial mass of high-quality(low-quality) assets in the market. Note that $\bar{\rho}_{H0} > \bar{\rho}_{L0}$ if and only if $\theta(0) < \alpha$, while $\bar{\rho}_{H0} < \bar{\rho}_{L0}$ if and only if $\theta(0) > \alpha$. In the follow lemma, we give two scenarios in which the market composition $\theta(t)$ converges monotonically to a new steady state along an equilibrium path.

Lemma C.1 *Assume $\rho_H(t) = \bar{\rho}_H$ and $\rho_L(t) = \bar{\rho}_L$,*

1. *$\theta(t)$ is decreasing (increasing) in $t \in (0, +\infty)$ if $\bar{\rho}_{L0} \geq \bar{\rho}_{H0} \geq \bar{\rho}_H \geq \bar{\rho}_L$ ($\bar{\rho}_{H0} \geq \bar{\rho}_{L0} \geq \bar{\rho}_L \geq \bar{\rho}_H$);*
2. *if $\bar{\rho}_H = \bar{\rho}_L$, $\theta(t)$ is decreasing (increasing) in $t \in (0, +\infty)$ if and only if $\bar{\rho}_{H0} \leq \bar{\rho}_{L0}$ ($\bar{\rho}_{H0} \geq \bar{\rho}_{L0}$).*

D Alternative Assumptions on Buyers' Entry and Exit

In the model, I make a simplifying assumption with respect to buyers' entry and exit. Namely, the inflow of buyers is proportional to the mass of sellers at any given time, and buyers exit the market immediately if no trade happens within matches. This assumption helps me highlight the effect of buyers' trading strategy on market liquidity without considering the changes in the meeting rate. Here I analyze the robustness of the main results in a model with more conventional assumptions on buyers' entry and exit.

Let's consider a market with a fixed inflow of buyers denoted by ϵ. After unsuccessful trade, buyers do not exit the market. Instead, they stay on the market and are matched randomly with sellers. Denote the mass of buyers at time t by $m^B(t)$. The matching function takes a multiplicative form of $\hat{\lambda} m^B(t)\left[m^S_H(t) + m^S_L(t)\right]$. Therefore, each seller meets a buyer at Poisson rate $\hat{\lambda} m^B(t)$, and each buyer meets a seller at Poisson rate $\hat{\lambda}\left[m^S_H(t) + m^S_L(t)\right]$. Since the matching process is random, the prior belief of a seller—the probability of meeting a high-quality seller to the probability of meeting a low-quality seller—is still $\theta(t)$. Compared to the model described in Section 2, the market liquidity is affected by both the endogenous meeting rate and buyers' trading strategy. In addition, buyers now take into consideration the option value of waiting to buy assets later. Both factors complicate the analysis of the model, especially the analytical characterization of the non-stationary equilibria.

To characterize the equilibrium in the revised model, we need to introduce more notations. Let $\hat{J}(t)$ be the ex ante expected value of a matched buyer and $J(t)$ be the continuation value of an unmatched buyer at time t. They are linked through the following expression.

$$J(t) = \int_t^{+\infty} e^{-r(\tau - t)} \hat{J}(\tau) \mathrm{d}\left(1 - e^{-\int_t^\tau \lambda m^S(t) \mathrm{d}u}\right).$$

The continuation values $C_H(t)$, $V_H(t)$ and $V_L(t)$ still satisfy (3), (5) and (6), while $C_L(t)$ is different because the matching function is different.

$$C_L(t) = \int_t^\infty \left[(1 - e^{-r(\tau - t)})c_L + e^{-r(\tau - t)} c_H\right] d(1 - e^{-\lambda \int_t^\tau m^B(u)\gamma_L(c_H, u) du}).$$

For the static trading game, the previous analyses still apply if we replace the continuation values with $\hat{C}_H(t) = C_H(t)$, $\hat{C}_L(t) = C_L(t)$, $\hat{V}_H(t) = V_H(t) - J(t)$ and $\hat{V}_L(t) = V_L(t) - J(t)$.

Let $\nu(t) = \min\left\{\hat{V}_L(t), \hat{C}_L(t)\right\}$, the expected value of being matching at time t is

$$\hat{J}(t) - J(t) = \begin{cases} \frac{1}{1+\theta(t)}\left(\hat{V}_L(t) - \nu(t)\right), & \theta(t) < \hat{\theta}^-(k, \nu(t)), \\ \frac{1}{1+\theta(t)}\left[\hat{V}_L(t) - f_L^G \hat{C}_H(t) - f_L^B \nu(t)\right] \dots & \\ + \frac{\theta(t)}{1+\theta(t)} f_H^G \left(\hat{V}_H(t) - \hat{C}_H(t)\right) - k, & \theta^-(k, \nu(t)) \leq \theta(t) < \theta^+(k, \nu(t)), \\ \frac{1}{1+\theta(t)}\left(\hat{V}_L(t) - \hat{C}_H(t)\right) + \frac{\theta(t)}{1+\theta(t)}\left(\hat{V}_H(t) - \hat{C}_H(t)\right), & \theta(t) \geq \theta^+(k, \nu(t)). \end{cases}$$

Although the characterization is more complicated, the main result still holds—given certain parametric restrictions, there exists two steady states, a liquid one without information acquisition and an illiquid one with information acquisition. Moreover, given the initial condition in the illiquid steady state, there is no equilibrium that converges to the liquid steady state. Here I provide the intuition without giving the details of the analysis. First, since the static trading game can be represented with a set of modified continuation values, the equilibrium of the static trading game does not change qualitatively. Specifically, the information-sensitive region lies to the left of the information-insensitive pooling region. Second, when buyers acquire information, high-quality assets are still traded faster than low-quality assets. Therefore, the cream-skimming effect of information acquisition is still present in the revised model. Third, although the rate at which sellers meet buyers is higher in an illiquid market, it does not offset the low liquidity caused by buyers' information acquisition. To summarize, the above three components that drive the main results are all present in the revised model.

E Non-Stationary Equilibria from the Information Trap

The following proposition shows that when the current market composition falls in the overlapping part of the two information-sensitive region $[\theta_2^-(k), \theta_1^+(k)]$, it is hard for the market to recover to the liquid state S_1, even if an information-insensitive pooling stationary equilibria exists for the same set of parameters and fundamental α.

Proposition E.1 *If $\theta_2^-(k) \leq \theta(0) < \theta_1^+(k)$, there exists an equilibrium path that converges to pooling trading if and only if the dynamics of the asset distribution characterized by (10) and (11) with $\rho_H(t) \equiv f_H^G$ and $\rho_L(t) \equiv f_L^G$ satisfy $\theta(t) = \theta_1^+(k)$ for some $t \geq 0$.*

F Proofs

Proof of Lemma 1-3 (Solutions to the Static Trading Game).

$\mathbf{V_L < C_L}$**, no gains from trade for low-quality assets.** The buyer has lower continuation value of the low-quality asset than the seller. Therefore, no trade will take place at any price lower than C_H. The buyer will compare the expected payoff from offering the lowest pooling price and withdrawing from trading (or offering a price lower than V_L). The buyer finds it optimal to offer the pooling price C_H if and only if

$$\tilde{\theta}V_H + V_L - (1+\tilde{\theta})C_H \geq 0.$$

It can be written as

$$\tilde{\theta} \geq \hat{\theta} = \frac{C_H - V_L}{V_H - C_H}. \tag{F.1}$$

where $\hat{\theta}$ is the threshold belief.

If the prior belief $\theta \geq \hat{\theta}$, the optimal strategy of a buyer without information is to offer the lowest pooling offer C_H and get the expected revenue $\frac{\theta}{1+\theta}V_H + \frac{1}{1+\theta}V_L - C_H$. However, when observing the signal, the buyer can make offers conditional on the signal. Specifically, if $\theta \geq \hat{\theta}$ and $\tilde{\theta}(\theta, B) \leq \hat{\theta}$, the buyer will offer pooling price C_H when observing G and withdraw from trade if observing B. The expected revenue is $\frac{\theta}{1+\theta}f_H^G(V_H - C_H) + \frac{1}{1+\theta}f_L^G(V_L - C_H)$. If $\tilde{\theta}(\theta, B) > \hat{\theta}$, the buyer is willing to offer the pooling price C_H no matter what the signal is. The expected revenue is $\frac{\theta}{1+\theta}V_H + \frac{1}{1+\theta}V_L - C_H$, the same as if there's no information. Therefore, the value of information for the buyer can be written in the form of an option value

$$W(\theta) = \max\left\{-\frac{\theta}{1+\theta}f_H^B(V_H - C_H) + \frac{1}{1+\theta}f_L^B(C_H - V_L), 0\right\}.$$

The intuition is as following. For prior belief $\theta \geq \hat{\theta}$, the signal allow the buyer to avoid loss $C_H - V_L$ from buying a low-quality asset with probability $\frac{1}{1+\theta}f_L^B$. However the signal can be "false negative" with probability $\frac{\theta}{1+\theta}f_H^B$ and by making conditional offers the buyer loses the trade surplus $V_H - C_H$ from buying a high-quality asset.

On the other hand, if $\theta < \hat{\theta}$, there will be no trade for both types if there's no information. Therefore, using the same reasoning as above, we find the value of information for the buyer is

$$W(\theta) = \max\left\{\frac{\theta}{1+\theta}f_H^G(V_H - C_H) - \frac{1}{1+\theta}f_L^G(C_H - V_L), 0\right\}.$$

After observing the signal, the buyer has the option to make conditional offers. Doing so, the buyer gains the surplus of trading with the high type with probability $\frac{\theta}{1+\theta}f_H^G$, but incurs a loss of trading with the low type with probability $\frac{1}{1+\theta}f_L^G$. The buyer will make conditional offers only if the net gain is positive.

$\mathbf{V_L \geq C_L}$, non-negative gains from trade for low-quality assets. There's a non-negative gain if the buyer offers a low price to only buy low-quality assets. Therefore, the buyer compares the expected gain from offering a pooling price with only buying low-quality assets. The buyer find it optimal to offer pooling price if and only if

$$\frac{\tilde{\theta}}{1+\tilde{\theta}}V_H + \frac{1}{1+\tilde{\theta}}V_L - C_H \geq \frac{1}{1+\tilde{\theta}}(V_L - C_L),$$

which translates into

$$\tilde{\theta} \geq \hat{\theta} = \frac{C_H - C_L}{V_H - C_H}.$$

If $\theta \geq \hat{\theta}$, the buyer will offer pooling price C_H without information. By making conditional offers, the buyer can reduce the price paid for a low-quality asset from C_H to C_L with probability $\frac{\theta}{1+\theta}f_L^B$, but with probability $\frac{\theta}{1+\theta}f_H^B$ she will lose the revenue $V_H - C_H$ from buying a high-quality asset. The value of information to the buyer is

$$W(\theta) = \max\left\{-\frac{\theta}{1+\theta}f_H^B(V_H - C_H) + \frac{1}{1+\theta}f_L^B(C_H - C_L), 0\right\}.$$

If $\theta < \hat{\theta}$, the buyer will only trade with the low type at price C_L without information. By making conditional offers, the buyer can get revenue of $V_H - C_H$ with probability $\frac{\theta}{1+\theta}f_H^G$ from buying a high-quality asset, while loss $C_H - C_L$ with probability $\frac{1}{1+\theta}f_L^G$ buying a low-quality asset at the pooling price. The value of information to the buyer is therefore

$$W(\theta) = \max\left\{\frac{\theta}{1+\theta}f_H^G(V_H - C_H) - \frac{1}{1+\theta}f_L^G(C_H - C_L), 0\right\}.$$

Let $\nu = \min\{V_L, C_L\}$, the value of information can be written in a synthetic form,

$$W(\theta) = \begin{cases} \max\left\{-\frac{\theta}{1+\theta}f_H^B(V_H - C_H) + \frac{1}{1+\theta}f_L^B(C_H - \nu), 0\right\}, & \text{if } \theta \geq \hat{\theta}, \\ \max\{\frac{\theta}{1+\theta}f_H^G(V_H - C_H) - \frac{1}{1+\theta}f_L^G(C_H - \nu), 0\}, & \text{if } \theta < \hat{\theta}. \end{cases}$$

Notice $W(\theta)$ remains at zero for θ close to 0, then increases to its maximum value $W_{max}(\nu) = (f_L^B - f_H^B)(v_H - c_H) \cdot \frac{C_H - \nu}{V_H - \nu}$ at $\theta = \hat{\theta} = \frac{C_H - \nu}{V_H - C_H}$, and decreases to zero at a finite value of θ. For $k < W_{max}(\nu)$, the boundaries of the information-sensitive region can be solved by equating

$W(\theta)$ and k,

$$\theta^-(k,\nu) = \frac{f_L^G(C_H-\nu)+k}{f_H^G(V_H-C_H)-k}, \quad \theta^+(k,\nu) = \frac{f_L^B(C_H-\nu)-k}{f_H^B(V_H-C_H)+k}.$$

■

Proof of Lemma 4. First note that

$$C_L(t) \le \frac{rc_L+\lambda c_H}{r+\lambda}.$$

If $\gamma_L(c_H,\tau) \ge f_L^G$ for any $\tau > t$,

$$\begin{aligned}
&(1-e^{-r(\tau-t)})(v_L-c_L) - \int_t^\tau e^{-r(u-t)}\lambda\gamma_L(c_H,u)(c_H-C_L(u))\mathrm{d}u,\\
\le&(1-e^{-r(\tau-t)})(v_L-c_L) - \int_t^\tau e^{-r(u-t)}\lambda f_L^G\left(c_H-\frac{rc_L+\lambda c_H}{r+\lambda}\right)\mathrm{d}u,\\
=&(1-e^{-r(\tau-t)})(v_L-c_L) - \lambda f_L^G\frac{r(c_H-c_L)}{r+\lambda}\int_t^\tau e^{-r(u-t)}\mathrm{d}u,\\
=&(1-e^{-r(\tau-t)})(v_L-c_L)\left(v_L-c_L-f_L^G\frac{\lambda}{r+\lambda}(c_H-c_L)\right).
\end{aligned}$$

If Assumption 1 holds, the above expression is negative for any $\tau > t$. Therefore $V_L(t) - C_L(t) < 0$. ■

Proof of Prosposition 1. Since $f_L^G < f_H^G$ and $f_L^B > f_H^B$, the interval defined in Lemma 6 has positive measure for small k. Also, when k is small, the condition for the existence of S_1 becomes

$$\alpha \ge \frac{f_L^B(c_H-\bar{V}_{L,1})-k}{f_H^B(V_H-c_H)+k}.$$

Lemma 5 and 6 jointly imply that S_1 and S_2 coexist if and only if $\alpha \in [A_1(k), A_2(k)]$. To show the interval has positive measure for small k, it's sufficient to show that

$$\frac{f_L^B(c_H-\bar{V}_{L,1})}{f_H^B(V_H-c_H)} < \frac{\delta+\lambda f_H^G}{\delta+\lambda f_L^G}\cdot\frac{f_L^B(c_H-\bar{V}_{L,2})}{f_H^B(V_H-c_H)}.$$

In fact, the above inequality always holds since $\bar{V}_{L,1} > \bar{V}_{L,2}$ and $f_L^G > f_L^G$. ■

Proof of Lemma C.1. When $\rho_H(t)$ and $\rho_L(t)$ are constants, they can be further simplified as

$$m_H^S(t) = \frac{\delta\alpha}{\delta + \lambda\rho_H} + \left(m_H^S(0) - \frac{\delta\alpha}{\delta + \lambda\rho_H}\right) e^{-(\delta+\lambda\rho_H)t}, \tag{F.2}$$

$$m_L^S(t) = \frac{\delta(1-\alpha)}{\delta + \lambda\rho_L} + \left(m_L^S(0) - \frac{\delta(1-\alpha)}{\delta + \lambda\rho_L}\right) e^{-(\delta+\lambda\rho_L)t} \tag{F.3}$$

Plugging in (F.2) and (F.3), we can show that the sign of $\frac{\mathrm{d}\theta(t)}{\mathrm{d}t}$ is the same as the sign of

$$\frac{(\delta + \lambda\bar{\rho}_{H0}) - (\delta + \lambda\bar{\rho}_H)}{1 + (\delta + \lambda\bar{\rho}_{H0})\frac{e^{(\delta+\lambda\bar{\rho}_H)t}-1}{\delta+\lambda\bar{\rho}_H}} - \frac{(\delta + \lambda\bar{\rho}_{L0}) - (\delta + \lambda\bar{\rho}_L)}{1 + (\delta + \lambda\bar{\rho}_{L0})\frac{e^{(\delta+\lambda\bar{\rho}_L)t}-1}{\delta+\lambda\bar{\rho}_L}}. \tag{F.4}$$

Note that for any $t > 0$ the function $\frac{x-y}{1+x\frac{e^{yt}-1}{y}}$ is strictly increasing in x and strictly decreasing in y for any $y \le x$. Thus, if $\bar{\rho}_{L0} \ge \bar{\rho}_{H0} \ge \bar{\rho}_H \ge \bar{\rho}_L$ ($\bar{\rho}_{H0} \ge \bar{\rho}_{L0} \ge \bar{\rho}_L \ge \bar{\rho}_H$), (F.4) is non-positive (non-negative), which implies $\theta(t)$ is decreasing (increasing) in t. Similarly, if $\bar{\rho}_H = \bar{\rho}_L$, the sign of (F.4) is the same as the sign of $\bar{\rho}_{H0} - \bar{\rho}_{L0}$. Therefore, $\theta(t)$ is decreasing (increasing) in t if and only if $\bar{\rho}_{H0} \le \bar{\rho}_{L0}$ ($\bar{\rho}_{H0} \ge \bar{\rho}_{L0}$). ■

Proof of Proposition 2. Notice

$$A_2(k) = \frac{\delta + \lambda f_H^G}{\delta + \lambda f_L^G} \cdot \theta^+(k, \bar{V}_{L,2}) = \frac{\delta + \lambda f_H^G}{\delta + \lambda f_L^G} \cdot A_3(k) > A_3(k). \tag{F.5}$$

$A_1(k)$ is the maximum of two values. By Lemma 3 we know $\theta^+(k, \bar{V}_{L,2}) > \theta^+(k, \bar{V}_{L,1})$. To show that $\theta^+(k, \bar{V}_{L,2}) > A_1(k)$ for small enough k, it is sufficient to show that

$$\frac{f_L^B(c_H - \bar{V}_{L,2})}{f_H^B(V_H - c_H)} > \frac{\delta + \lambda f_H^G}{\delta + \lambda f_L^G} \cdot \frac{f_L^G(c_H - \bar{V}_{L,2})}{f_H^G(V_H - c_H)}. \tag{F.6}$$

It follows directly from $f_L^B > f_H^B$ and $f_H^G > f_L^G$.

Given any $\alpha \in (A_1(k), A_3(k))$, the no information pooling stationary equilibria features $\bar{\theta}_1 = \alpha > \theta^+(k, \bar{V}_{L,1})$. Suppose the market starts from an initial asset distribution with $\theta(0)$ in the neighbourhood of α. Let's consider two paths. On the first path buyers always offer the pooling price c_H without acquiring information. Therefore, $\rho_H(t) = \rho_L(t) = 1$. Lemma C.1 implies that $\theta(t)$ converges monotonically to α. Since the continuation values are the same as in the no information pooling stationary equilibria, it is easy to verify that $\theta(t)$ falls in the pooling no information region for any $t > 0$. The first path is indeed an equilibrium path converging to S_1.

For the second path, assume buyers always acquire information and offer the pooling price c_H only if a good signal is observed. Thus, the continuation values are the same as in the information stationary equilibria. Moreover, $\rho_H(t) = f_H^G$ and $\rho_L(t) = f_L^G$ for any $t > 0$. Lemma C.1 implies that starting from the initial distribution close to S_1, $\theta(t)$ decreases monotonically to $\bar{\theta}_2$. Notice by assumption

$$\theta(0) = \alpha < A_3(k) = \theta^+(k, \bar{V}_{L,2}),$$
$$\bar{\theta}(+\infty) = \frac{\delta + \lambda f_L^G}{\delta + \lambda f_H^G} \cdot \alpha \geq \frac{\delta + \lambda f_L^G}{\delta + \lambda f_H^G} \cdot A_1(k) \geq \bar{\theta}^-(k, \bar{V}_{L,2}).$$

The whole path of $\theta(t)$ lies within the information sensitive region. Since $\bar{\theta}_2$ is the only sink in the information region, when starting from an initial distribution close to that of S_1, the path of $\theta(t)$ also stays in the information sensitive region. Therefore, the second path is an equilibrium path converging to S_2. ■

Proof of Proposition 3. Assume buyers do not acquire information and always offer the pooling price c_H for any $t > 0$. Therefore, both high-quality and low-quality assets are traded with probability 1. Also, the continuation values of owners of low-quality assets are fixed at $V_L(t) = \bar{V}_{L,1}$ and $C_L(t) = \bar{C}_{L,1}$. To show the assumed path is indeed an equilibrium path, we only need to verify that the whole path of market composition falls in the pooling information-insensitive region. In fact, Lemma C.1 implies that the market composition $\theta(t)$ increases monotonically from $\theta(0)$ to α. Given that $\alpha, \theta(0) > \theta_1^+(k)$, we know $\theta(t) > \theta_1^+(k)$ for any $t > 0$. The assumed path is an equilibrium path that converges to S_1. ■

Proof of Proposition E.1. First, we prove a lemma that characterizes any equilibrium path that converges to pooling trading.

Lemma F.1 *If $\frac{\delta+\lambda f_L^G}{\delta+\lambda f_H^G}\alpha \leq \theta^+(k, \bar{V}_{L,1})$, along any equilibrium path that converges to pooling trading, $\theta(t)$ must be weakly increasing whenever $\theta(t) < \theta^+(k, \bar{V}_{L,1})$.*

Proof of Lemma F.1. This can be proved by contradiction. Suppose there exist t_1 such that $\dot{\theta}(t_1) < 0$ and $\theta(t_1) < \theta^+(k, \bar{V}_{L,1})$. By continuity of $\theta(t)$, there exists $t_3 > t_2 \geq t_1$ such that $\dot{\theta}(t_2) < 0$, $\theta(t) < \theta^+(k, \bar{V}_{L,1})$ for any $t_2 \leq t < t_3$ and $\theta(t) \geq \theta^+(k, \bar{V}_{L,1})$ for any $t \geq t_3$. Namely, t_3 is the last time that $\theta(t)$ enters the region $\theta \geq \theta^+(k, \bar{V}_{L,1})$ from the left. $\theta(t)$ decreases at t_2 and stays below $\theta^+(k, \bar{V}_{L,1})$ for $t_2 < t < t_3$.

Since $\theta(t) > \theta^+(k, \bar{V}_{L,1})$ for any $t > t_3$, using backward induction, we can show that $C_L(t_3) > V_L(t_3) = \bar{V}_{L,1}$. For t slightly less than t_3, $\theta^-(k, \bar{V}_{L,1}) < \theta(t) < \theta^+(k, \bar{V}_{L,1})$, therefore, buyers acquire information and only offers the pooling price when signal G is observed. So $\rho_H(t) = f_H^G$ and $\rho_L(t) = f_L^G$. Since $\theta(t)$ crosses $\theta^+(k, \bar{V}_{L,1})$ from the left, for t slightly less than t_3, we have

$$\frac{\mathrm{d}}{\mathrm{d}t}\ln\theta(t) = \frac{\delta\alpha}{m_H^S(t)(1+\alpha)}\left(1-\theta(t)/\alpha\right) - \lambda(f_H^G - f_L^G) > 0, \tag{F.7}$$

Taking the limit of t to t_3, it yields

$$\frac{\delta\alpha}{m_H^S(t_3)(1+\alpha)}\left(1-\theta^+(k,\bar{V}_{L,1})/\alpha\right) - \lambda(f_H^G - f_L^G) \geq 0. \tag{F.8}$$

Evaluating the derivative of $\theta(t)$ at $t = t_2$, we have

$$\frac{\delta\alpha}{m_H^S(t_2)(1+\alpha)}\left(1-\theta(t_2)/\alpha\right) - \lambda(\rho_H(t_2) - \rho_L(t_2)) < 0. \tag{F.9}$$

By construction, $\theta(t_2) < \theta^+(k,\bar{V}_{L,1}) < \alpha$. Also notice $\rho_H(t_2) - \rho_L(t_2) < f_H^G - f_L^G$. Comparing (F.8) and (F.9), we have

$$m_H^S(t_2) > m_H^S(t_3). \tag{F.10}$$

On the other hand, since $\theta^+(k,\bar{V}_{L,1}) \geq \frac{\delta+\lambda f_L^G}{\delta+f_H^G}\alpha$, from (F.8) we know

$$m_H^S(t_3) \leq \frac{\delta\alpha}{1+\alpha}\frac{1-\theta^+(k,\bar{V}_{L,1})/\alpha}{\lambda(f_H^G - f_L^G)} \leq \frac{\delta}{\delta+\lambda f_H^G}\frac{\alpha}{1+\alpha}.$$

Rewrite (10),

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(m_H^S(t) - \frac{\delta}{\delta+\lambda f_H^G}\frac{\alpha}{1+\alpha}\right) = -(\delta+\lambda f_H^G)\left(m_H^S(t) - \frac{\delta}{\delta+\lambda f_H^G}\frac{\alpha}{1+\alpha}\right) - \lambda\left(\rho_H(t) - f_H^G\right)m_H^S(t). \tag{F.11}$$

Since $\theta(t) < \theta^+(k,\bar{V}_{L,1})$ for $t_2 \leq t < t_3$, from Table 1 we know $\rho_H(t) \leq f_H^G$. Therefore

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(m_H^S(t) - \frac{\delta}{\delta+\lambda f_H^G}\frac{\alpha}{1+\alpha}\right) \geq -(\delta+\lambda f_H^G)\left(m_H^S(t) - \frac{\delta}{\delta+\lambda f_H^G}\frac{\alpha}{1+\alpha}\right), \tag{F.12}$$

or equivalently

$$\frac{\mathrm{d}}{\mathrm{d}(-t)}\left(\frac{\delta}{\delta+\lambda f_H^G}\frac{\alpha}{1+\alpha} - m_H^S(t)\right) \geq (\delta+\lambda f_H^G)\left(\frac{\delta}{\delta+\lambda f_H^G}\frac{\alpha}{1+\alpha} - m_H^S(t)\right), \tag{F.13}$$

Given $m_H^S(t_3) \leq \frac{\delta}{\delta+\lambda f_H^G}\frac{\alpha}{1+\alpha}$, (F.13) implies that $m_H^S(t_2) \leq m_H^S(t_3)$. This is in contradiction with (F.10). Therefore, $\theta(t)$ must be weakly increasing when $\theta(t) < \theta^+(k,\bar{V}_{L,1})$ along any equilibrium path that converges to pooling trading. ■

Now we can move on to prove the necessity of the given condition. Notice, if $\frac{\delta+\lambda f_L^G}{\delta+\lambda f_H^G}\alpha > \theta(k,\bar{V}_{L,1})$, the path with constant $\rho_H(t) = f_H^G$ and $\rho_L(t) = f_L^G$ converges to $\frac{\delta+\lambda f_L^G}{\delta+\lambda f_H^G}\alpha > \theta(k,\bar{V}_{L,1})$ in the end. On the other hand, if $\frac{\delta+\lambda f_L^G}{\delta+\lambda f_H^G}\alpha \leq \theta(k,\bar{V}_{L,1})$, Lemma F.1 indicates that any path that starts from $\theta(0) < \theta(k,\bar{V}_{L,1})$ and converges to pooling trading only crosses $\theta^+(k,\bar{V}_{L,1})$ once. Again, let t_3 be the earliest time such that $\theta(t_3) = \theta^+(k,\bar{V}_{L,1})$. For any $0 \leq t < t_3$, we must have $\theta^-(k,\bar{V}_{L,2}) \leq \theta(0) \leq \theta(t) < \theta^+(k,\bar{V}_{L,1})$. Using backward induction, it can be easily shown that $\bar{V}_{L,1} < V_L(t) < \bar{V}_{L,2}$ for any $0 \leq t < t_3$. Therefore, from the monotonicity of $\theta^-(k,\cdot)$ and $\theta^+(k,\cdot)$ we know that $\theta^-(k,V_L(t)) < \theta^-(k,V_{L,2}) < \theta(t) < \theta^+(k,\bar{V}_{L,1}) < \theta^+(k,V_L(t))$ for any $0 \leq t < t_3$. Also, Assumption 1 implies that $V_L(t) < C_L(t)$ for any $t \geq 0$. Referring to Table 1, we know buyers acquire information with probability 1, $\rho_H(t) = f_H^G$, $\rho_L(t) = f_L^G$ for any $0 \leq t < t_3$. This shows that if we fix $\rho_H(t) = f_H^G$ and $\rho_L(t) = f_L^G$ for any $t \geq 0$, we must have $\theta(t_3) = \theta^+(k,\bar{V}_{L,1})$.

Now we want to show the given condition is also sufficient. This is done by guess-and-verify. Let t_3 be the first positive value that satisfies $\theta(t) = \theta(k,\bar{V}_{L,1})$ in the hypothetical path with $\rho_H(t) \equiv f_L^G$ and $\rho_L(t) \equiv f_L^G$. Let $i(t) = 1$ and $\sigma(c_H,G,t) = 1$ for any $t < t_3$ and $i = 0$, $\sigma(c_H,N,t) = 1$ for any $t \geq t_3$. It is easy to construct an equilibrium path that's consistent with the above offering strategy. ■

Proof of Proposition 4. Since $\bar{V}_{L,1} > \bar{V}_{L,2}$, by Lemma 3, $\theta^+(k,\bar{V}_{L,1}) < \theta^+(k,\bar{V}_{L,2})$, therefore $A_4(k) < A_2(k)$. Also, Assumption 2 implies that $\theta^-(k,\bar{V}_{L,2}) < \theta^+(k,\bar{V}_{L,1})$. It immediately follows that $A_1(k) < A_4(k)$ for small $k > 0$. By Proposition 1, we know when k is small, for any $\alpha \in (A_1(k), A_4(k))$, S_1 and S_2 coexist. Moreover, the market composition in the information stationary equilibria S_2 satisfies

$$\theta^-(k,\bar{V}_{L,2}) < \bar{\theta}_2 < \theta^+(k,\bar{V}_{L,1}).$$

Therefore, the asset distribution in S_2 falls in the information trap. By Proposition E.1, when the asset distribution is in the neighbourhood of S_2, there's no equilibrium path that converges to S_1. ■

Proof of Proposition 5. It is a direct implication of the monotonicity of $\theta_1^+(k)$ and $A_4(k)$. ■