Firm Heterogeneity & the Transmission of Financial Shocks During the European Debt Crisis[*]

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Abstract

This paper studies the role of firm heterogeneity in the transmission of financial shocks to the real economy. The recent European debt crisis provides a natural experiment with abundant firm-level data on an episode of heterogeneous firm responses to a severe tightening of credit conditions. The data show evidence indicating that smaller firms adjusted more their balance sheets than large firms, and performed better in economies with a more skewed firm size distribution. There is also some evidence suggesting that smaller firms faced larger increases in borrowing costs. This paper proposes a model of heterogeneous firms, in terms of productivity, capital and debt that face financial frictions (defaultable debt and costly equity issuance), a financial intermediation sector, and a sovereign. Financial frictions generate financing structures that depend on firm size, where small firms rely more on equity than debt, which is relatively more costly. Sufficiently large increases in public debt trigger a binding lending constraint for the intermediaries that cause a crowding out of private lending and leads smaller firms to adjust more than large firms. Moreover, firm heterogeneity has aggregate effects that result in larger output declines. Quantitative results show that the model, calibrated to match Spanish firm-level data, is consistent with the empirical facts and explains roughly 50% of Spain's output drop during the crisis.

Keywords: Firm Heterogeneity, Financial Frictions, European Debt Crisis.
JEL Classification: D25, E32, E44, F34.

1 Introduction

This paper studies the role of firm heterogeneity in the transmission of financial shocks and its macroeconomic implications. The analysis focuses on a particular event, namely the severe tightening in credit conditions for firms during the recent European debt crisis. This event was a natural experiment that provided abundant firm-level data on the effects of a sharp, sudden deterioration of credit-market conditions on non-financial firms in the countries most affected by the crisis: Greece, Ireland, Italy, Portugal and Spain (GIIPS). Existing studies have shown that an important channel explaining the deterioration of the financing conditions of firms during this event was the "crowding-out" channel, as documented by Broner et al. (2014) and more recently by Acharya et al. (2018) and Becker and Ivashina (2018). Increased government borrowing and bank holdings of public debt in these countries generated a crowding-out of private lending, reducing loanable funds available to firms and increasing their financing costs.[1]

The first part of this paper uses the Amadeus database to conduct an empirical analysis of the effects of the financial shock described above on non-financial firms in the GIIPS countries. The empirical analysis yields two important new findings regarding the relevance of firm heterogeneity in the responses of firms to the observed surge in sovereign debt: First, smaller firms adjusted their sales, liabilities, assets, and employment more than large firms. Second, smaller firms experienced smaller adjustments in these variables in countries with more skewed firm size distributions. The data also show that financing costs, measured as the ratio of debt service to total debt, increased more for smaller firms.

The second part of the paper proposes a model to explain the above facts and derives its quantitative implications. The model has three main components. First, a heterogeneous firms sector. Firms are heterogeneous in terms of productivity, capital, and debt, and face financial frictions in the form of defaultable debt and costly equity issuance, modeled as imperfect substitutes. These features endogenously generate a time-varying firm distribution. The second component is a financial intermediation sector. A representative financial intermediary lends to firms and

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[1] Another channel through which the government affected the financing conditions of firms during this episode is the pass-through of sovereign risk, as studied by Gennaioli et al. (2014a), Gennaioli et al. (2014b), Bottero et al. (2015), Sosa-Padilla (2015), Bocola (2016). According to this channel, increased sovereign risk negatively affected the balance sheet of intermediaries, who held sovereign bonds, generating a worsening in the financing conditions of firms.

the government, facing an occasionally binding constraint on loanable funds. The last element is the government. For simplicity, aggregate fiscal shocks and a fiscal reaction function drive public debt dynamics. The firms' financial frictions generate different financing structures, with small firms relying relatively more on equity than large firms, where size is defined by a firm's capital stock. The asymmetry in the responses between small and large firms arises from the fact that the former are more financially constrained, so they need to issue equity more often, and thus face a more expensive financing mix.

When public debt is sufficiently high, the bank's constraint binds, so the amount of resources available for firm lending falls (i.e., there is a crowding-out effect), increasing firms' borrowing costs. Since firms have different financing structures depending on their size, due to financial frictions, they respond differently. Because equity issuance is increasingly costly, small firms switching (partially or entirely) from debt to equity financing adjust more aggressively than larger firms that rely relatively more on debt or internal resources. This is because issuing even more equity comes at an increasing cost, pushing small firms closer to the default cutoff, so they have to adjust more their investment and output. The size-dependent responses to the financial shock imply that economies with different firm size distributions should adjust differently. Economies with a higher fraction of small firms (i.e., less skewed firm size distributions) adjust more because small firms adjust more aggressively when financing costs increase.

Interestingly, in this model firm heterogeneity has aggregate implications. The severity of financial frictions and the magnitude of the financial shock affect the evolution of the distribution of firms, and hence the dynamics of aggregate outcomes such as output. In particular, the model predicts a larger fall in aggregate output when the firm size distribution is less skewed. We observe a similar pattern if we compare Portugal v. Spain. During the crisis, Portuguese output declined more and took longer to recover than the Spanish case. Portugal also has a firm size distribution that is significantly less skewed than Spain's.

The model is calibrated to match firm-level features of Spain. For an increase in sovereign debt consistent with the Spanish case, the model predicts that small firms adjust more than large ones, and that smaller firms would reduce less their debt and capital stocks if the economy had

a larger firm-size dispersion. Regarding the aggregate effects of firm heterogeneity, the model predicts an output drop of roughly half the size of the decline observed in Spain during the European debt crisis. Comparing with the results for the same model setup but with a representative firm, output drops by 2 percentage points more (with respect to its long-run average), in the economy with heterogeneous firms. Hence, in the presence of financial frictions, firm heterogeneity amplifies the responses of aggregate variables to a tightening in credit conditions significantly.

The contributions of this paper are threefold. First, it documents the two new facts mentioned earlier using a rich European firm-level dataset, namely that during the European debt crisis small and large firms adjusted differently upon the same financial shock (where the latter adjust less aggressively), and that small firms in countries with more skewed firm size distributions adjusted less during this episode. Empirical findings related to the facts mentioned, such as the ones of Bottero et al. (2015), which uses Italian firm-bank credit relation data, show that small firms adjusted more their investment than large ones. In the same context, Arellano et al. (2017) finds that increases in Italian sovereign spreads decrease more sharply the sales growth of small firms. This paper provides evidence on more balance sheet variables for the set of countries that were most affected during this episode and also studies the role of the skewness of the distribution of firms.

Second, it contributes to the literature on heterogeneous agents in production with aggregate uncertainty. In the model, financial frictions make firms' responses to be non linearly scalable in size, and as a result firm heterogeneity has aggregate implications. This is a key departure from classic approaches, particularly Bernanke et al. (1999), in which the distribution of wealth is irrelevant. The model combines different features of the investment financing literature, such as costly equity issuance (Gomes, 2001, Cooley and Quadrini, 2001) and defaultable debt (Hennessy and Whited, 2007), and extends its traditional framework by including aggregate uncertainty via a government and financial intermediation sector. This paper is closer to the work of Khan and Thomas (2013) and Khan et al. (2014), which studies aggregate fluctuations generated by credit shocks in the context of firm heterogeneity, and it differs in two dimensions. The first one is that this paper explicitly models the financial intermediation sector, where the size of the financial shock is a function of the distribution of firms. The second difference is that the model allows

for (costly) equity issuance. The imperfect substitutability between equity and debt is a key feature of the model and produces size-dependent responses to financial shocks that are consistent with the ones observed during the European debt crisis. In this line, Gilchrist et al. (2014) proposes a setting with heterogeneous firms facing similar financial frictions (but abstracting from financial shocks) to analyze the implications of uncertainty shocks on investment dynamics. In more recent work, Ottonello and Winberry (2018) studies the role of monetary policy shocks in a framework with heterogeneous firms and financial frictions, but not allowing for equity issuance and aggregate uncertainty.

The third contribution is related to the literature on the transmission of sovereign debt crises to the real economy. This paper focuses on studying the crowding-out effect that sovereign debt has on the availability of funds for non-financial firms. It shows that firm heterogeneity along with financial frictions are key to generate an amplification effect, by which a tightening of private financial conditions in response to a sovereign debt crisis has large adverse effects on aggregate output. Empirical work, such as Acharya et al. (2018) and Becker and Ivashina (2018), shows that during the crisis government debt crowded-out private credit and investment. In this regard, Broner et al. (2014) provides a theoretical framework to explain the crowing-out channel, where credit discrimination is the key driver. This paper contributes to this strand of the literature by assessing the role of firm heterogeneity on the crowing-out channel.

This paper is organized as follows. Section 2 provides the empirical analysis of the role of heterogeneity across firms in the effects of the financial shock triggered by the European debt crisis. Section 3 presents the model. Section 4 presents the calibration and assessment of the model's equilibrium outcomes, as well as the quantitative results regarding the transmission of financial shocks and the role of firm heterogeneity. Section 5 concludes.

2 Firm Heterogeneity & the European Debt Crisis

This section presents an empirical analysis of the effects associated on non-financial firms caused by the sharp increase in sovereign borrowing during the European debt crisis. Increased sovereign

borrowing generated a crowding-out effect on firm lending, which affected the financing conditions of non-financial firms in the involved economies. This section focuses on analyzing the effects of this financial shocks at the firm level using the Amadeus database.[2] This database contains roughly 21 million firms across Europe. Firms of all sizes are required to report information about their balance sheets, so that the sample that is possible to access is more representative of the whole economy than other popular datasets, such as Compustat (which only contains publicly listed firms).

In order to study the responses at the firm level to the sovereign debt surge, which is an aggregate shock, a panel dataset is constructed. Firms in Greece, Ireland, Italy, Portugal and Spain are considered, as these countries were the most affected during the debt crisis. The analysis focuses on the time period of 2010 to 2014, in order to avoid the Great Recession.[3] The estimated specification is given by the following equation:

$$x_{i,j,t} = \beta \mathrm{B}_{j,t} + \sum_{k=2}^{5} \delta_k Q_{i,j,t,k} \mathrm{B}_{j,t} + \sum_{k=2}^{5} \phi_k Q_{i,j,t,k} \mathrm{B}_{j,t} \mathrm{Sk}_{j,t} + \kappa \mathrm{B}_{j,t} \mathrm{Sk}_{j,t} + z'_{i,j,t} \gamma + \alpha_{i,j} + \eta_{i,j,t} \qquad (1)$$

where $x_{i,j,t}$ denotes the outcome x for firm i in country j at period t, $\mathrm{B}_{j,t}$ corresponds to the gross public debt-to-output ratio for country j at period t, Q_k is a size dummy that is equal to 1 if the firm belong to the kth quintile of firm size , $\mathrm{Sk}_{j,t}$ represents the skewness of the firm size distribution in country j at period t, z is a vector of controls, α is a firm fixed effect, and η is the error term.[4] The outcome variables studied are the logs of sales, total assets, total liabilities and employment, at the firm level.[5]

This specification allows us to capture the semi-elasticity of the dependent variable (measured in log) with respect to increases in government debt, but allowing for differential effects in

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[2]The database was accessed through Wharton Research Data Services (WRDS). A brief description of how variables were treated and generated is presented in the Appendix.

[3]Unfortunately, the speed at which the Amadeus database is updated does not allow to include 2015 at the moment this paper is written. Years 2015 and 2016 are not well populated in terms of observations, so they are omitted from the analysis.

[4]Control variables include a set of lagged versions of all outcome variables, as well as firm size quintile dummies, and year dummies.

[5]We also performed robustness checks regarding the addition of more controls, in order assess whether it is indeed government debt the variable affecting the firm-level outcomes. In particular, we include 1-year sovereign CDS spreads and also an estimation of the fraction of general government debt held by domestic banks, for each country. For the latter we follow the methodology presented in Arslanalp and Tsuda (2014). The estimation results do not change significantly.

terms of firm size and skewness of the country's firm size distribution. For example, δ_k reflects the differential effect that increases in debt have on firms of different size, while ϕ_k captures a similar effect but considering different levels of skewness of the firm size distribution.[6]

The regression equation is estimated using fixed effects at the firm level.[7] The estimation results are presented in Table 1. The results show that increases in sovereign debt decrease firms' sales, liabilities, assets, and employment. Also, larger firms perform better than small firms when sovereign debt increases, as it can be seen from the coefficient associated with the interaction $B \times Q_k$. However, when considering the triple interaction between debt, firm size and skewness of the firm size distribution, we see that the large firms do not perform better than small ones in economies where skewness is larger. This is the case for all variables except for employment, where the greater the skewness the better the larger firms perform in relation to small ones.

The magnitude of the coefficients is relatively small, but their statistical significance is quite high, as suggested by their p-values. To make the analysis simpler, Table 2 presents the implied semi-elasticities in response to an increase of one percentage point in the ratio of gross public debt-to-output. At the average sample skewness, an increase in sovereign debt decreases the sales of small firms by 0.27%, while for large firms the decrease is smaller, 0.12%.[8] These decreases might seem small, but it is important to note that the average increase in government debt in 2012 was large. For example, in the case of Spain, in 2012 the debt-to-output ratio increased by roughly 16 percentage points. Thus, for this case, the semi-elasticity results predict that sales decreased by 4.4% for small firms, and 1.9% for large ones, a large drop in both cases. We continue to observe an asymmetry in the adjustments of small and large firms, where the former adjust more than the latter, except for the case of liabilities.

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[6]The semi-elasticity of variable y with respect to variable x is given by $\varsigma_{y,x} = \frac{d \ln y}{dx}$.

[7]An alternative version is estimated using random effects and adding controls for industry and country. The results do not change significantly, but the Hausman test strongly rejects the usage of a random effects specification. Another possibility is to use a pooled OLS specification. We present in the Appendix the comparison between the fixed effects, random effects, and pooled OLS specifications.

[8]Coefficients with a p-value above 0.10 are set to 0 when calculating the semi-elasticities.

Table 1: Panel Regression Results

Variable	Log(Sales)	Log(Liabilities)	Log(Assets)	Log(Employment)
B	-0.003	-0.003	-0.001	-0.003
	(0.000)	(0.000)	(0.000)	(0.000)
Q_2	0.327	0.718	0.519	0.161
	(0.000)	(0.000)	(0.000)	(0.000)
Q_3	0.562	1.250	0.909	0.285
	(0.000)	(0.000)	(0.000)	(0.000)
Q_4	0.754	1.712	1.237	0.368
	(0.000)	(0.000)	(0.000)	(0.000)
Q_5	0.864	2.103	1.532	0.424
	(0.000)	(0.000)	(0.000)	(0.000)
B × Q_2	-0.001	-0.001	-0.000	-0.001
	(0.000)	(0.000)	(0.000)	(0.000)
B × Q_3	0	-0.001	0.000	-0.001
	(0.000)	(0.000)	(0.474)	(0.000)
B × Q_4	0	-0.001	0.000	-0.001
	(0.036)	(0.000)	(0.000)	(0.000)
B × Q_5	0.002	0.000	0.000	0.000
	(0.000)	(0.054)	(0.000)	(0.367)
B × Q_2 × Skewness	0.000	-0.000	-0.000	0.000
	(0.058)	(0.003)	(0.000)	(0.000)
B × Q_3 × Skewness	-0.000	-0.000	-0.000	0.000
	(0.727)	(0.000)	(0.000)	(0.000)
B × Q_4 × Skewness	-0.000	-0.000	-0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)
B × Q_5 × Skewness	-0.000	-0.000	-0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.013)
B × Skewness	0.000	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.934)
Year Effects	Yes	Yes	Yes	Yes
Fixed Effects	Yes	Yes	Yes	Yes
Controls	Yes	Yes	Yes	Yes
Observations	3,195,072	3,308,816	3,313,051	3,119,977
R^2	0.58	0.736	0.911	0.585

Notes: B denotes the gross public debt-to-output ratio, Q_k is a dummy variable that is equal to 1 if the firm belongs to the kth quintile of firm size, and Sk is the skewness of the firm size distribution. Countries include Greece, Ireland, Italy, Portugal and Spain. Clustered standard errors at the firm level considered, p-values in parentheses.

The estimated coefficients can be used to compare the responses to increases in the sovereign debt-to-output ratio between small and large firms across countries with different levels of skewness of the firm size distribution. Varying the level of skewness allows us to analyze whether these responses are different between countries. This exercise considers the contrast between the semi-elasticities to increases in public debt-to-output for Portugal (low skewness) and Spain (high skewness). The second and third rows of Table 2 presents the results. We see that (i) small Por-

tuguese and Spanish firms adjust more than large firms in terms of sales, liabilities, assets, and employment, and that (ii) Portuguese firms (small and large) adjust more than Spanish firms for the same set of variables. The first result is consistent with the pattern found for the entire sample, namely that small firms adjust more than large ones. The second one reveals the role of the skewness of the firm size distribution. Firms in countries with a lower skewness adjust more to an increase in the sovereign debt-to-output ratio.

The analysis presented above shows two important new findings regarding the relevance of firm heterogeneity and firm size distribution. First, during the debt crisis, small firms adjusted more than large firms. Second, small firms in economies where the skewness of the firm size distribution adjusted less than their counterparts in economies with lower skewness.

Table 2: Implied Semi-elasticities for Debt Increases (Percentages)

	Sales		Liabilities		Assets		Employment	
	Small	Large	Small	Large	Small	Large	Small	Large
Average Skewness	-0.269	-0.115	-0.273	-0.277	-0.131	0.016	-0.267	-0.263
Portuguese Skewness	-0.319	-0.150	-0.309	-0.294	-0.175	-0.017	-0.267	-0.266
Spanish Skewness	-0.300	-0.136	-0.295	-0.288	-0.158	-0.005	-0.267	-0.265

Notes: Small firms are those that belong to the first quintile of firm size, and large firms correspond to those that belong to the fifth quintile.

The last exercise of this section seeks to assess whether increases in sovereign debt indeed increased the financing cost of firms. Unfortunately, the Amadeus database does not contain information about contractual interest rates paid by firms. It is possible to construct a proxy for the average effective interest rate paid by the firm. In particular, we can compute $r_{i,t} = \frac{\text{Interest Paid}_{i,t}}{\text{Liabilities}_{i,t}}$, which proxies for the average interest rate paid by firms. Unfortunately, the database does not allow us to identify the maturity of the debt.

In order to assess if the financing costs respond to increases in sovereign debt, the following specification is estimated:

$$r_{i,j,t} = \beta B_{j,t} + \sum_{k=2}^{5} \delta_k Q_{i,j,t,k} B_{j,t} + \sum_{k=2}^{5} \phi_k Q_{i,j,t,k} B_{j,t} Sk_{j,t} + \kappa B_{j,t} Sk_{j,t} + z'_{i,j,t}\gamma + \alpha_{i,j} + \eta_{i,j,t} \qquad (2)$$

where the set of independent variables is defined in the same way as in the previous regression

equation. Table 3 presents the estimation results.

Table 3: Panel Regression Results, Financing Costs

Variable	$r_i \times 100$
B	0.000
	(0.000)
Q_2	-0.011
	(0.000)
Q_3	-0.017
	(0.000)
Q_4	-0.022
	(0.000)
Q_5	-0.027
	(0.000)
B $\times Q_2$	-0.006
	(0.222)
B $\times Q_3$	-0.027
	(0.599)
B $\times Q_4$	-0.03
	(0.972)
B $\times Q_5$	-0.043
	(0.637)
B $\times Q_2 \times$ Skewness	0.000
	(0.007)
B $\times Q_3 \times$ Skewness	0.000
	(0.000)
B $\times Q_4 \times$ Skewness	0.000
	(0.000)
B $\times Q_5 \times$ Skewness	0.000
	(0.000)
B $\times$ Skewness	-0.000
	(0.000)
Year Effects	Yes
Fixed Effects	Yes
Controls	Yes
Observations	2,852,913
R^2	0.001

Notes: B denotes the gross public debt-to-output ratio, Q_k is a dummy variable that is equal to 1 if the firm belongs to the kth quintile of firm size, and Sk is the skewness of the firm size distribution. Countries include Greece, Ireland, Italy, Portugal and Spain. Clustered standard errors at the firm level considered, p-values in parentheses.

The magnitudes of the coefficients are quite small, although they are statistically significant. To make the exposition clearer, average responses are also computed. The implied increase in the average interest rate for a small firm in response to a 1-percentage-point increase in the public debt-to-output ratio is of 2 basis points, while for large firms it is 0.9 basis points. For the case of Spain in 2012, the increase (roughly 16 percentage in the debt to output ratio) meant an increase of 32 and 14 basis points for small and large firms, respectively. Although the magnitude

is not large, the effects are statistically significant, which reflects the fact that part of the increase in financing costs of firms is being captured in this analysis.

3 The Model

The model consists of 4 main components. First, is a heterogeneous firm sector, where firms differ in their debt (b) and capital stocks (k), and idiosyncratic productivity (z). Firms face financial frictions (defaultable debt and costly equity issuance) which affects how they finance their investment, leading to non-trivial financing structures which are firm-size dependent (where size is defined by the firm's capital). The time-varying distribution of firms is denoted by Γ. Second, a financial intermediation sector lends to the sovereign and firms. Financial intermediaries live 2 periods and are born with wealth $\bar{A}$. The limited wealth generates an occasionally binding constraint, that is triggered when government debt is sufficiently high. In case the constraint binds, the price at which the government (q^g) and firms (q) can borrow decreases, which is a financing cost shock for firms. Sovereign borrowing crowds-out private borrowing. Third, the sovereign. Sovereign debt (B) dynamics evolve according to a fiscal reaction function and to the realization of aggregate fiscal shocks (ε). The state variables for the sovereign are its outstanding debt and the fiscal shock ($\mathbf{S} = (B, \varepsilon)$), which will also happen to be the aggregate states of the economy. Lastly, a risk-neutral household sector that own firms and receive transfers T from the government.

The timeline of the events of the model between periods t and $t+$ is as follows. At the beginning of the period, the aggregate shock is realized. The sovereign borrows according to its fiscal reaction function, outstanding debt and realization of the shock. Firm productivity for the period is materialized, and the firm decides whether to default on its debt or not. In case of default, the firm exits forever and is replaced by a new one. If it decides to continue operating, it chooses its investment and how to finance it (debt, equity issuance, or internal resources) for the next period. The price at which the sovereign and firms borrow is determined jointly with the problem of the representative financial intermediary.

3.1 Firms

Firms accumulate capital, k, through investment, which can be financed by (i) internal resources of the firm, (ii) debt (b) and (iii) equity issuance. Firms face two kinds of financial frictions, defaultable debt and costly equity issuance. This setting is similar to the one of Hennessy and Whited (2007), which builds on Gomes (2001) and Hennessy and Whited (2005).[9] Firm debt is risky, because can be defaulted on. The price at which firms borrow, q, depends on their observable state variables and on the aggregate states of the economy (which will be specified below). Equity issuance, on the other hand, is costly, reflecting that the firm incurs in costs when trying to raise resources by selling equity. This is a fairly common assumption in the corporate finance literature (Gomes, 2001; Cooley and Quadrini, 2001; Hennessy and Whited, 2007; Jermann and Quadrini, 2012), and allows for the existence of a trade-off between different sources of external financing.

The production technology of a firm is given by $f(z,k) = zk^\alpha$ (with $\alpha < 1$), where z denotes its productivity. Firms are subject to idiosyncratic shocks to their productivity, which follow a finite state Markov process characterized by the CDF $G(z'|z)$. The law of motion for capital is given by $k' = k(1 - \delta) + i$, where i denotes investment and δ is the rate at which capital depreciates. There is an investment adjustment cost that the firm must pay, which is given by $\Psi(k', k) = \frac{\psi}{2} \left(\frac{k'}{k} - 1 \right)^2 k$. Firms are subject to a proportional tax τ on profits, and incur a fixed cost F to operate.

Before production begins each period, firms decide whether to default on their debt or not. If they do, they exit the economy forever and are replaced by a firm with no debt, a level of capital $\underline{k}$, and productivity drawn from the long-run probability distribution of the Markov process specified above ($G^*(z)$).[10] Additionally, in case of default, nothing is recovered by the firm, but creditors recover a fraction θ of the firm's undepreciated capital. Firms obtain a continuation value V in case they decide not to default.

In order to finance investment, firms can rely on internal or external financing. Internal

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[9]These articles provide a partial equilibrium setting featuring heterogeneous firms. Hennessy and Whited (2007) considers firms that differ in their productivity, debt, and capital, that can default on their debt and face equity issuance costs.

[10]This assumption keeps the mass of firms in the economy constant.

resources consist of all the resources the firm has available after producing and paying for the operational costs and outstanding debt. External financing consists of debt issuance and/or equity issuance. The price at which a firm can borrow depends on its productivity z, capital for the next period k', and borrowed amount b'. This is because the price responds to the expected future default probability of the firm, which is a function of how much it borrowed and invested today, and of its expected future productivity.[11] Hence, if borrowing is too high then the likelihood of future default might increase, and this will be reflected in the price of debt. The equilibrium price is determined by the interaction between the government, firms, and financial intermediaries, and is also influenced by aggregate states $\mathbf{S}$ and the distribution of firms Γ. Denote the equilibrium price faced by a firm with the above characteristics by $q(z, k', b'; \mathbf{S}, \Gamma)$. If firms decide to issue equity they incur a cost $\Lambda(e) = \lambda e^2$, where e denotes the amount of equity being issued. The functional form assumed is similar to the one used by Jermann and Quadrini (2012), and Covas and den Hann (2012), and rationalizes the fact that there are increasing marginal costs associated to issuing equity, such as underwriter fees, as shown by Hansen and Torregrosa (1992) and Altinkilic and Hansen (2000).

The problem of a firm that does not default is given by:

$$V(z, k, b; \mathbf{S}, \Gamma) = \max_{\{(k', b') \in \Upsilon(z; \mathbf{S}, \Gamma)\}} \left\{ e + \beta \mathbb{E}_{z', \mathbf{S}'|z, \mathbf{S}} \max\{V(z', k', b'; \mathbf{S}', \Gamma'), 0\} \right\} \tag{3}$$

s.t.

$$e = (1 - \tau)(zk^\alpha - F) - k' + k(1 - \delta) - \Psi(k', k) + q(z, k', b'; \mathbf{S}, \Gamma)b' - b - \Phi\Lambda(e)$$

$$\Gamma' = \Omega(\mathbf{S}, \Gamma)$$

where e denotes the cash flow to shareholders, Φ denotes an indicator variable that is equal to 1 when $e < 0$ (when there is equity issuance) and 0 otherwise, and Ω corresponds to the law of motion for the distribution of firms. Finally, firms choose their capital for next period and debt from sets $\mathcal{K}$ and $\mathcal{B}$, respectively, which are compact sets. As in Gomes (2001), $\mathcal{K}$ is compact because it is a closed and bounded set. It is bounded from below by 0 and bounded above by a level $\tilde{k}$, which is the largest possible capital stock that is profitable for the firm and solves $\alpha \bar{z} \tilde{k}^{\alpha-1} - \delta = 0$. Thus,

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[11]Given that productivity follows a Markov process its realization today gives information about future realizations.

we can define $\mathcal{K} = [0, \tilde{k}]$. Regarding the choice set for debt, given a price schedule $q(z, k', b'; \mathbf{S}, \Gamma)$, debt will be bounded below by 0 (firms cannot save via debt) and above by an endogenous level $\tilde{b}(z, k'; \mathbf{S}, \Gamma)$ that represents the maximum debt a firm can take, which will be a function of productivity and capital choice (and aggregate states) and such that $q(z, k', \tilde{b}(z, k'; \mathbf{S}, \Gamma); \mathbf{S}, \Gamma) = 0$. Hence, we can define $\mathcal{B}$ in a similar way as $\mathcal{K}$. The choice correspondence $\Upsilon(z; \mathbf{S}, \Gamma)$ is defined as $\Upsilon(z; \mathbf{S}, \Gamma) \equiv \{(k', b') : k' \in \mathcal{K}, b' \in [0, \tilde{b}(z, k'; \mathbf{S}, \Gamma)]\}$. Note that this correspondence is convex, compact valued and continuous.

Notice that from the expression for the value function of the firm we can see that a threshold rule for productivity will apply. Let $\mathbf{z}(k, b; \mathbf{S}, \Gamma)$ be defined as:

$$V(\mathbf{z}(k, b; \mathbf{S}, \Gamma), k, b; \mathbf{S}, \Gamma) = 0 \tag{4}$$

$\mathbf{z}(k, b; \mathbf{S}, \Gamma)$ defines the cutoff value of productivity for which the firm will decide to default on its obligations. It follows that we can define $\tilde{d}(z, k, b; \mathbf{S}, \Gamma)$ as a binary variable that takes the value of 1 if the firm defaults today conditional on debt contracted in the previous period, and 0 otherwise:

$$\tilde{d}(z, k, b; \mathbf{S}, \Gamma) = \begin{cases} 1 & \text{if} \quad z < \mathbf{z}(k, b; \mathbf{S}, \Gamma) \\ 0 & \text{if} \quad z > \mathbf{z}(k, b; \mathbf{S}, \Gamma) \end{cases}$$

Using the decision rule $\tilde{d}(z, k, b; \mathbf{S}, \Gamma)$ we can define the probability that a firm defaults in the future. Given The default probability tomorrow for a firm with productivity z that chooses capital k' and borrows b' today is given by:

$$\tilde{p}(z, k', b'; \mathbf{S}, \Gamma) = \mathbb{E}_{z'|z}\left[\tilde{d}(z', k', b'; \mathbf{S}', \Gamma')\right] = \Pr\left(z' < \mathbf{z}(k', b'; \mathbf{S}', \Gamma')|z\right) \tag{5}$$

3.2 Financial Intermediation

Risk-neutral intermediaries that live for 2 periods are born with a level of wealth $\bar{A}$, and have to decide how much to lend to the sovereign and firms.[12] The fact that intermediaries live for 2

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[12]Intermediaries are identical and there is a mass 1 of them.

periods, as in Aguiar et al. (2016) and Coimbra and Rey (2017), simplifies the setting because there is no need to keep track of the evolution of their level of wealth, while maintaining the key idea of the crowding-out between sovereign and firm lending.

If a firm defaults, the intermediaries recover a fraction θ of the undepreciated capital $((1 - \delta)k)$. In case of sovereign default, they recover nothing. Intermediaries face a limited liability constraint (i.e. dividends cannot be negative) in both periods, and loanable funds constraint in the first period, where dividends plus funds lent cannot exceed $\bar{A}$. The problem of the representative financial intermediary is given by:

$$\max_{\{x,x',B',\{b'(z,k,b)\}\}} x + \tilde{\beta}\mathbb{E}_{\mathbf{S}',z'|\mathbf{S},z}[x'] \tag{6}$$

s.t.

$$x + q^g B'(\mathbf{S}) + \int q(z,k'(z,k,b),b'(z,k,b))b'(z,k,b)d\Gamma(z,k,b) \leq \bar{A}$$

$$x' = B'(\mathbf{S})(1 - d^g(B(\mathbf{S}))) + \int [b'(z,k,b)(1 - \tilde{d}_2(z,k'(z,k,b),b'(z,k,b)))$$

$$+ \tilde{d}(z,k'(z,k,b),b'(z,k,b))\theta k'(z,k,b)(1 - \delta)]d\Gamma(z,k,b)$$

$$x, x' \geq 0$$

where x and x' denote the dividends of the first and second period, respectively, $b'(z,k,b)$ denote the amount lent and to a firm with characteristics (z,k,b), $k'(z,k,b)$ is the choice of capital for next period of a firm with the same characteristics, and B' denotes the size of loan going to the sovereign. The discount factor of the representative intermediary is given by $\tilde{\beta}$.

The loanable funds constraint occasionally binds because the sum of sovereign borrowing and aggregate firm borrowing is not necessarily equal the level of wealth $\bar{A}$: if it is less, the constraint does not bind, while if it is equal to the level of wealth the constraint binds. The time-varying nature of public and aggregate private debt generates periods where the constraint may or may not bind.

The first order conditions of the intermediary's problem give the pricing functions for

sovereign and firm debt, as well as a slackness condition:

$$q^g(B'; \mathbf{S}, \Gamma) = \tilde{\beta} \frac{\mathbb{E}[(1 - d^g(B'(\mathbf{S})))]}{1 + \mu(\mathbf{S}, \Gamma)} = \tilde{\beta} \frac{1 - p^g(B'(\mathbf{S}))}{1 + \mu(\mathbf{S}, \Gamma)} \tag{7}$$

$$q(z, k', b'; \mathbf{S}, \Gamma) = \tilde{\beta} \frac{\mathbb{E}[1 - \tilde{d}(z, k', b')(1 - \theta(1 - \delta)k')]}{1 + \mu(\mathbf{S}, \Gamma)} = \tilde{\beta} \frac{1 - \tilde{p}(z, k', b')(1 - \theta(1 - \delta)k')}{1 + \mu(\mathbf{S}, \Gamma)} \tag{8}$$

$$\left(\bar{A} - q^g(B'; \mathbf{S}, \Gamma)B'(\mathbf{S}) - \int q(z, k'(z, k, b), b'(z, k, b); \mathbf{S}, \Gamma)b'(z, k, b)d\Gamma(z, k, b) \right) \mu(\mathbf{S}, \Gamma) = 0 \tag{9}$$

where $\mu(\cdot)$ represents the Lagrange multiplier of the loanable funds constraint of period 1, $d^g(B')$ is a binary variable that is equal to 1 if the sovereign defaults, and 0 otherwise, and $p^g(B')$ is the probability the sovereign defaults in the next period, given a loan B'.[13] μ depends on the aggregate states of the economy and on the distribution of firms because these two elements influence the level of sovereign borrowing and also the aggregate demand for debt of firms. This multiplier reflects the marginal utility of the intermediaries' wealth, so the higher the value it takes the more binding the constraint will be.

Equations (7) and (8) define the price at which the government and a firm with productivity z choosing k' and b' can borrow. Part of the structure of these pricing functions has standard features, such as being decreasing in the expected future probability of default or being increasing in the recovery in case of default. Notice that the Lagrange multiplier for loanable funds is present in these functions. When the loanable funds constraint binds, which is likely to happen when public debt is high enough, the price at which the government and firms can borrow decreases. The latter is equivalent to an increase in the interest rate paid on loans. The value that the multiplier takes is such that the loanable funds constraint is met with equality, so different magnitudes of μ could be observed depending on the dynamics of public debt.

3.3 Sovereign

The sovereign issues one-period non-state contingent bonds B at a price q^g. The primary balance of the sovereign is given by a fiscal reaction function, which depends on an aggregate fiscal shock $\varepsilon \in [\underline{\varepsilon}, \overline{\varepsilon}]$ that follows a Markov process with CDF $G^s(\varepsilon'|\varepsilon)$, and on the level of the outstanding

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[13]The Lagrange multiplier for the limited liability constraint in period 2 is always 0, as the non-negativity constraint will never bind. To see this note that period 2 dividends are composed by the sum of non-negative payouts of assets.

debt in the current period. Thus, at period t, the primary balance is given by $\text{Pb}_t = n(B_t, \varepsilon_t)$. This formulation is in line with the literature on public debt sustainability, initiated by Bohn (1998), and also used in the analysis of debt crises, as in Lorenzoni and Werning (2013). As Bohn (1998) showed, a sufficient condition for the intertemporal budget constraint of the government to hold is that there is a (conditional) positive response of the primary balance to lagged debt. The fiscal reaction function can also exhibit non-linear responses to lagged debt, as it will be specified below, in order to capture the possibility of "fiscal fatigue", as in Ghosh et al. (2013), and still be consistent with fiscal solvency. For low levels of public debt the primary balance slightly increases as debt rises, for higher levels of debt there is a much larger response, but at some point the response of the primary balance weakens.

Finally, the sovereign defaults with probability p^g. This is a function of the amount of borrowing, and which is modeled by the following reduced form structure:

$$p_{t+1}^g = \min\{\max\{\alpha_s + \beta_s B_{t+1}, 0\}, 1\} \tag{10}$$

The likelihood of default will have a direct effect on the price at which the sovereign can borrow, as will be specified in the financial intermediation section. Given that sovereign borrowing can potentially crowd-out private investment, q^g will be a function of the distribution of firms and aggregate states.

The dynamics of debt, then, are obtained from the budget constraint of the sovereign:

$$q_t^g(B_{t+1}; \mathbf{S}_t, \Gamma_t) B_{t+1} = B_t - \text{Pb}_t(\mathbf{S}_t) \tag{11}$$

$$B_{t+1} \leq \arg\max_{\hat{B}_{t+1}} q_t^g(\hat{B}_{t+1}; \mathbf{S}_t, \Gamma_t) \hat{B}_{t+1}$$

where $B_{t+1}(\mathbf{S}_t, \Gamma_t) = \mathcal{B}(\mathbf{S}_t, \Gamma_t)$ is the policy function for debt.

The budget constraint of the sovereign imposes endogenous debt limits. To see this, note that the right-hand-side of equation (11) is given for the current states of the economy $\mathbf{S}_t$. Given $\mathbf{S}_t, \Gamma_t$, the left-hand-side forms the familiar Laffer curve for debt (i.e., for B_{t+1}), but there is no guarantee that even at the level of debt that maximizes the resources raised the equality will hold.

Two cases can occur. In the first one there are two values of B_{t+1} that solve equation (11), which is equivalent to say that these two values produce the same revenue. If this is the case, the lowest amount of debt is selected. In the second case there is no value of B_{t+1} that solves the mentioned equality. Here, debt is set to be equal to $\hat{B}_{t+1} = \arg\max q_t^g(\hat{B}_{t+1}; \mathbf{S}_t, \Gamma_t)\hat{B}_{t+1}$, and the primary balance is adjusted correspondingly.

In order to maintain consistency of the sovereign's problem, the primary balance is redefined by:

$$\mathrm{Pb}_t(\mathbf{S}_t, \Gamma_t) = \begin{cases} n(B_t, \varepsilon_t) & \text{if} \quad B_t - n(B_t, \varepsilon_t) < \hat{B}_{t+1} \\ B_t - \hat{B}_{t+1} & \text{if} \quad B_t - n(B_t, \varepsilon_t) \geq \hat{B}_{t+1} \end{cases} \tag{12}$$

Lastly, government revenues at period t are given by $\hat{\tau}(\mathbf{S}_t, \Gamma_t) = \tau \times \pi_t(\mathbf{S}_t, \Gamma_t)$. In order to ensure that the fiscal reaction function for primary balance is consistent with tax revenues, a transfer T_t to households is considered. This transfer is such that $T_t(\mathbf{S}_t, \Gamma_t) = \hat{\tau}(\mathbf{S}_t, \Gamma_t) - \mathrm{Pb}(\mathbf{S}_t, \Gamma_t)$ for $t \geq 0$.

3.4 Households

Households in this model are assumed to be identical, risk-neutral agents that own firms and consume an endowment $\bar{M}$ plus transfers from the government T_t. The structure assumed for the households simplifies the setting of the model and allows to focus on the firm sector, which is going to be the relevant actor for the transmission of financial shocks towards the real economy.

3.5 Equilibrium

Given an initial distribution Γ_0, a Recursive Markov Equilibrium consists in a value functions for the firm V, policy functions $\hat{b}$, $\hat{k}$, $\hat{d}$, $\mathcal{B}$, pricing functions q, q^g, μ, and a law of motion Ω such that:

(i) Given $\mathcal{B}$, μ, q, Γ and Ω, $V(z, k, b; \mathbf{S}, \Gamma)$ solves the problem of the firm and $\hat{k}(z, k, b; \mathbf{S}, \Gamma)$, $\hat{b}(z, k, b; \mathbf{S}, \Gamma)$ and $\hat{d}(z, k, b; \mathbf{S}, \Gamma)$ are the corresponding policy functions for capital, debt, and

default.

(ii) $q^s(\mathbf{S}, \Gamma)$ and $q(z, k, b; \mathbf{S}, \Gamma)$ solve the pricing equation of financial intermediaries.

(iii) $\mu(\mathbf{S}, \Gamma)$ is consistent with the slackness condition of financial intermediaries.

(iv) Γ evolves according to $\Gamma' = \Omega(\mathbf{S}, \Gamma)$.

(v) Budget constraint of the sovereign holds.

(vi) Aggregate resource constraint holds.

3.6 Understanding the Mechanism of the Model: A 2-Period Case

This section provides a 2-period version of the full model presented above. In order to highlight the economic intuition of the mechanism driving the model.

In the first period firms have an initial capital k_0 and productivity z_0. They decide how much to invest, borrow, produce, and whether to issue equity or not. The production technology is the same as the one specified above. Debt is defaultable, and equity issuance is costly. Firms are subject to an operational fixed cost F in both periods. Firms issue equity whenever dividends in period 1 become negative. In period 2, if firms do not default, they repay their debt and produce.[14]

The problem of the firm is given by:

$$\max_{\{e_0, e_1, k_1, b_1\}} e_0 - \Lambda(e_0)\Phi\{e_0 < 0\} + \beta \mathbb{E}_{z_1|z_0}[e_1] \tag{13}$$

$$\text{s.t.} \tag{14}$$

$$e_0 = z_0 k_0^\alpha - k_1 + q(b_1)b_1 - F$$

$$e_1 = z_1 k_1^\alpha - F - b_1$$

where e_t denote period $t = 0, 1$ dividends, $\Phi = $ is an indicator variable that is equal to 1 in case the firm issues equity, and 0 otherwise, and $\Lambda(e) = e^2$ denotes the equity issuance cost function.

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[14] In period 2 dividends cannot be negative, as the firm would prefer to default. Thus, there is no equity issuance in period 2.

Define the *financing gap* of a firm as the difference between resources needed to operate and invest minus the resources available to do so. Given k_1 and b_1, the financing gap is given by the following expression:

$$\text{Financing Gap} = k_1 + F - z_0 k_0^\alpha - q(b_1)b_1 \tag{15}$$

A firm is *constrained* whenever the financing gap is strictly positive, and *unconstrained* if the opposite occurs. Note that a constrained firm, conditional on k_1 and b_1, does not have enough resources to finance its investment. Hence, it needs to rely also on equity issuance. The amount of equity issued is such that the financing gap is met.

Let k_1^* and b_1^* denote the unconstrained solutions for capital and debt in period 1, respectively. The "marginally constrained firm" has an initial capital given by $k_0^* = \left[\frac{k_1^* + F - q(b_1^*)b_1^*}{z_0}\right]^{\frac{1}{\alpha}}$. Defining size as the amount of capital, we can conclude that a firm with initial size greater or equal than k_0^* will be unconstrained, while if the opposite is true then the firm will be constrained. Hence, whenever $k_0 < k_0^*$ the firm will have a positive financing gap and will issue equity.

The combined first order conditions of the firm's problem (assuming differentiability of the pricing function of debt), for a firm with initial capital k_0 and productivity z_0, yield the following conditions:

$$\beta \mathbb{E}_{z_1|z_0}\left[\alpha z_1 k_1^{\alpha-1}\right] = 1 + \Lambda'(e_0)\Phi\{e_0 < 0\} \tag{16}$$

$$(q(b_1) + q'(b_1)b_1)(1 + \Lambda'(e_0)\Phi\{e_0 < 0\}) = \beta \cdot \Pr(e_1 > 0) \tag{17}$$

These conditions are a central part of the model's mechanism because they imply that constrained and unconstrained firms choose different investment and leverage levels. To see this, consider equation (16) first. For a constrained firm issuing equity, the right-hand-side is strictly greater than 1 (given that $\Lambda'(\cdot) > 0$), which implies that this type of firm has a higher marginal product of capital in period 1 than an unconstrained firm. Given that the production function has decreasing returns to scale, this means that the capital accumulation for period 1 is smaller than for an unconstrained firm. Equation (17) implies that the marginal benefit of debt is higher for a constrained

firm that is issuing equity. The intuition for this is that for every unit borrowed the firm saves the equity issuance cost, which makes debt more attractive. Comparing again with an unconstrained firm, we see that a constrained firm tends to use more debt in relation to its size, and thus has a higher leverage ratio (measured as b_1/k_0). Lastly, note that constrained firms issue equity and use debt at the same time, which implies that they have a more expensive financing mix than unconstrained firms.

The previous paragraphs showed how *small* (constrained) and *large* (unconstrained) behave in terms of their investment and how they finance it. We now explain how these features interact with the rest of the elements in the model. Assume that the sovereign borrows an amount B_1 at a price q^g, and that its initial primary balance and debt are $\text{Pb}_0 = -\varepsilon_0$ and $B_0 > 0$, respectively. Regarding the representative financial intermediary, assume an identical setting as the one specified earlier (where pricing functions are determined in a similar way). Lastly, for simplicity, assume that a fraction m of firms in the economy has initial capital $k_0^s < k_0^*$ (and hence a fraction $1 - m$ has capital $k_0^l > k_0^*$). Given this, the funding constraint that the intermediary faces is $q_1^g(B_1(B_0, \varepsilon))B_1(B_0, \varepsilon) + m \cdot q(b_1^s)b_1^s + (1 - m) \cdot q(b_1^l)b_1^l \leq \overline{A}$, with a corresponding Lagrange multiplier $\mu(B_0, \varepsilon, m)$.

Assume that the realization of ε is such that the lending constraint binds. Then, $q_1^g(B_1)$, $q(b_1^s)$, and $q(b_1^l)$ decrease, as $\mu(B_0, \varepsilon, m) > 0$ because the constraint binds and the demand for funds needs to be consistent with the supply of them. A decrease in the price lowers the marginal benefit of debt, measured as the resources obtained for every unit borrowed, for both types of firms. This makes them substitute debt for equity or internal resources. Initially constrained firms will issue even more equity, which directly affects more their investment in relation to initially unconstrained firms. To see this more clearly, consider again equation (16). Given that $\Lambda(\cdot)'' > 0$, issuing more equity increases even further the marginal product of capital for period 1, which implies that investment is decreasing more for a firm that was constrained initially. This is the key mechanism of the model. The financial frictions that firms face shape their financing structure, and these are size-dependent. Thus, firms of different sizes respond differently to the same financial shock, measured as an increase (decrease) in μ (q).

To conclude this subsection, note that the aggregate effect of the financial shock depends on the fraction of firms that are initially constrained, but there are two channels operating. The first channel is the most evident one, which is determined by the composition of firms in the economy. This is, small firms adjust more their investment than large firms, so an economy that is composed by a large fraction of small firms should have a larger response to the same shock than an economy that has a larger fraction of large firms. The second channel is a general equilibrium channel. Given that the response of the debt of firms also varies across size, economies with different firm composition will have different dynamics for the marginal utility of the intermediaries' wealth. Economies with a lower skewness in firm size distribution (i.e., a larger fraction of small firms) are more sensitive to changes in the value of μ (because small firms adjust more when the lending constraint binds), so a milder increase in μ is required to generate the same adjustment than in a higher skewness economy. Both of these channels are operating at the same time, so a quantitative analysis is required in order to see which type of economy (i.e., less skewed firm size distribution v. more skewed) will adjust more in the presence of a financial shock.

4 Quantitative Analysis

4.1 Model Solution & Calibration

The presence of aggregate risk in the model makes the distribution of firms a relevant state variable that plays a key role in pricing functions of sovereign and firm debt. Firm distribution is a high-dimensional, so keeping track of it is computationally very difficult. In order to deal with this problem, we follow the approach of Krusell and Smith (1998) and assume that agents are boundedly rational in their perception of how the distribution evolves. We assume that the dynamics of the firm size distribution are approximated by a set of moments. In particular, the distribution is summarized by the cross-sectional variance of capital, and firms consider the marginal utility of the intermediaries' wealth (a market clearing price) to be a state variable and use a linear autoregressive linear to forecast future values of these objects. This approach is similar to the one employed in Krusell and Smith (1997), but it differs on the fact that here the price follows an au-

toregressive rule. The algorithm used to solve the model is described in the Appendix.[15] The results from the Krusell-Smith regressions are also presented in the same section of the Appendix.

The calibration of the model proceeds as follows. We start by setting the values of a subset of parameters that can be obtained directly from the data or the literature (external calibration). We then set the rest of the parameters of the model to match specific targets from firm-level data (internal calibration), which are chosen in order to be consistent with observed moments of the cross-section of Spanish firms at a yearly frequency.[16]

External Calibration   The discount factor of firms β is set to 0.96, which is a standard value in the literature. The value for the capital depreciation rate δ is 0.11, which is in line with data from EU KLEMS for depreciation of Spanish capital. The production function parameter, α, is set to be $\alpha = 0.65$, consistent with the estimates of Hennessy and Whited (2007), using cross-sectional data of non-financial, unregulated firms from Compustat. The recovery parameter for the financial intermediaries is set to match the recovery rates of subordinated debt provided by Moody's (22.9%). The recovery rate defined in the model is $\theta k(1 - \delta)/b$. The average leverage ratio of Spanish firms in the Amadeus database for the 2005-2008 period is $b/k = 0.55$, and the depreciation rate of Spanish capital is 0.11. Replacing these values in the recovery rate of the model yields $\theta(1 - 0.11)/0.55 = 0.229$, which implies $\theta = 0.142$. The corporate effective tax rate is set to be $\tau = 0.224$, in line with data from the Spanish Ministry of Finance.

The fiscal reaction function follows the structure of Ghosh et al. (2013). They specify a reaction function of the form: $\mathrm{PB}_t = \nu_0 + \nu_1 B + \nu_2 B^2 + \nu_3 B^3 + \varepsilon$. Hence, this rule requires setting 4 parameter values. For the intercept, ν_0 is set to 6.464 in order to match the average (pre Great Recession) ratio of gross public debt-to-output of Spain (according to the IMF WEO). The rest of the parameters are set to equal those found by Ghosh et al. (2013). Lastly, the parameters of rule for the sovereign's default probability are $\alpha_s = -0.036$ and $\beta_s = 0.094$. We obtain these parameter values from a linear estimation where the independent variable is the gross public debt-to-output ratio, and the dependent variable is the yearly default probability implied by CDS spreads of

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[15]An additional complication that arises in this setting is that the problem of the firm has kinks and non-concavities, which makes difficult to solve this problem using collocation methods or methods that rely on first order conditions. Hence, the problem of the firm is solved via value function iteration.

[16]All of the moments that are targeted in the calibration are those obtained for the 2005-2008 period, in order to avoid the large distortions of the Great Recession and also to check the external validity of the model.

1-year Spanish sovereign bonds.[17] The data sources are IMF WEO and Markit.

Internal Calibration   We choose 6 parameters of the model to match moments of the firm-level data. The problem of the firm is relatively non-linear, and most moments are altered when one parameter value changes. In order to find a reasonable set of initial parameters, we solve a reduced version of the full model. This reduced version is a partial equilibrium model, where firms face an aggregate shock that increases the interest rate at which they borrow. We solve the simplified version 30,000 times. For every time the model is solved there is an initial random draw of a vector of parameters from a uniform distribution. Thus, for every draw of parameters, there is a corresponding set of moments. We then plot the median of different moments for given values of parameters (discretized grids). This provides useful information regarding which moments are more sensitive to each parameter.

We assume that the productivity process z follows a log-AR(1) structure:

$$\log(z') = \rho_z \log(z) + \sigma_z \epsilon \tag{18}$$

where $\epsilon \sim \mathcal{N}(0,1)$, ρ_z is the autoregressive coefficient, and σ_z is the variance of the productivity process. The productivity is discretized following Tauchen (1986). The values of ρ_z and σ_z are internally calibrated, and are set so as to target the autocorrelation of profits-to-capital ratio ($(zk^\alpha - F)/k$), which is equal to 0.858, and the cross-sectional standard deviation of the investment to capital ratio (i/k), equal to 0.219.

The parameter of the adjustment cost of capital, ψ, is set to be $\psi = 0.064$, in order to match the average autocorrelation of i/k observed in the data (0.186). The fixed operational cost, F, targets the default ratio of firms which is roughly 2%, and is set to $F = 0.026$. The equity issuance cost parameter, λ, is equal to 5.428 and is chosen to target the average leverage ratio of firms observed in the data, which is 0.55. Lastly, the endowment of the financial intermediary is set to be $\bar{A} = 0.251$ so the average interest rate faced by firms is roughly 4%, a value that is common in the literature.

Summary of the Calibration & Model Fit   A list of all the parameter values obtained in this

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[17]A 1-year maturity could reflect more accurately sovereign default risk as it is closest to a (potential) default episode.

calibration to Spain are presented in Table 4:

Table 4: Parameter Values

Parameter	Value	Description	Target
ρ_z	0.796	AR Coef. Productivity	Autocorrelation of Profits
σ_z	0.198	Std. Dev Productivity	Std. Dev. of i/k
ψ	0.064	Investment Adjustment Cost	Autocorrelation of i/k
F	0.026	Fixed Cost	Firm Default Rate
λ	5.428	Equity Issuance Cost	Firm Leverage b/k
$\bar{A}$	0.251	Bank Endowment	Average Interest Rate
β	0.960	Firm Discount Factor	Standard
δ	0.110	Capital Depreciation	EU KLEMS
α	0.650	Capital Share	Hennessy and Whited (2007)
θ	0.142	Recovery rate	Moody's
τ	0.224	Tax Rate	Spanish Ministry of Finance
ν_0	6.464	Primary Balance	Spanish Government Debt/GDP
ν_1	-0.225	Primary Balance	Ghosh et al. (2013)
ν_2	0.003	Primary Balance	Ghosh et al. (2013)
ν_3	0.000	Primary Balance	Ghosh et al. (2013)
α_s	-0.036	Sov. Def. Prob. (I)	IMF WEO & Markit
β_s	0.094	Sov. Def. Prob.(S)	IMF WEO & Markit

The process for the aggregate fiscal shock is represented as a 2-state Markov process with high and low values ($\underline{\varepsilon}$, $\overline{\varepsilon}$). In order to calibrate the transition matrix of this process, we use the time series for the Spanish primary fiscal balance, from 1950 to 2010. This information is obtained from the Public Finances in Modern History database of the IMF (Mauro et al., 2013). We define a low-realization period whenever the primary balance is below its long-run average minus one standard deviation, and a high-realization period if the opposite occurs. Using this definition, we construct the transition probabilities by identifying the frequency with which the time series

transitions between the two states. The transition matrix for the process is:

$$\mathcal{P} = \begin{bmatrix} \Pr(\varepsilon' = \overline{\varepsilon}|\varepsilon = \overline{\varepsilon}) & \Pr(\varepsilon' = \underline{\varepsilon}|\varepsilon = \overline{\varepsilon}) \\ \Pr(\varepsilon' = \overline{\varepsilon}|\varepsilon = \underline{\varepsilon}) & \Pr(\varepsilon' = \underline{\varepsilon}|\varepsilon = \underline{\varepsilon}) \end{bmatrix} = \begin{bmatrix} 0.943 & 0.057 \\ 0.250 & 0.750 \end{bmatrix}$$

In order to assess the fit of the model we compute moments from simulations of the model and contrast them with those of the actual data, which we obtain from Spanish firms in the Amadeus database. Table 5 presents the comparison of moments.

Table 5: Targeted and Non-targeted Moments

Targeted	Model	Data
Default rate	0.017	0.020
Leverage (b/k)	0.550	0.547
Cross-sectional Std. Dev. of i/k	0.291	0.219
Autocorrelation of i/k	0.200	0.186
Autocorrelation of π/k	0.772	0.858
Non-targeted		
Investment-capital ratio i/k	0.151	0.128
Equity Issuance Frequency	0.108	0.098
Profits-assets ratio π/k	0.178	0.103
Cross-sectional Std. Dev. π/k	0.126	0.149
Cross-sectional Std. Dev. b/k	0.203	0.262
Autocorrelation of b/k	0.750	0.903

Except for the cross-sectional volatility of the investment to capital ratio, the model does a reasonable job in terms of fitting targeted moments of Spanish firm-level data. Regarding non-targeted moments, the model performs relatively well in terms of approximating non-targeted moments. The fact that the model performs reasonably well in terms of matching these firm-level features suggests that it is a good benchmark for studying the transmission of financial shocks to the real economy.

Policy Functions & Model Mechanism   The 2-period model presented in section 3 suggested

that small firms, which are more financially constrained than large ones, would reduce more their investment in comparison to large firms as a response to the financial shock. Using the solved model we can assess if the policy functions produce results that are consistent with these predictions.

Define the financing gap of the full model by:

$$\text{FG}(z, k, b; \mathbf{S}, \Gamma) = k'(z, k, b; \mathbf{S}, \Gamma) + b + \Psi(k'(z, k, b; \mathbf{S}, \Gamma), k) - (1 - \tau)(zk^\alpha - F) \tag{19}$$
$$- k(1 - \delta) - q(z, k'(z, k, b; \mathbf{S}, \Gamma), b'(z, k, b; \mathbf{S}, \Gamma); \mathbf{S}, \Gamma)b'(z, k, b; \mathbf{S}, \Gamma)$$

where $x'(z, k, b; \mathbf{S}, \Gamma)$ corresponds to the policy function of variable x as a function of the state variables of the firm.

As in section 3, the financing gap reflects the difference between the resources the firm needs to operate, minus the resources the firm can raise without issuing equity. We say that a firm is constrained if the financing gap is strictly positive, and it needs to issue equity in order to operate. We say a firm is unconstrained if the financing gap is negative.

We use the policy functions obtained from the solution of the model to construct the financing gap. The state-space of the model is highly-dimensional, so we focus on one particular set of states: we set the productivity to be equal to the average of the process, and the sovereign debt-to-output ratio is set to the long-run average of Spain. We then proceed to compare the financing gaps and policy functions between the high and low realization of the aggregate shock, which reflects an increase in the financing costs of firms. Figure 1 presents 3 heat maps that show the differences in the financing gap, capital for next period k', and leverage b'/k, between the low and high financing cost scenarios.

Panel (a) of Figure 1 shows the differences in the financing gaps when the financial costs increase. A positive value denotes an increase in the financing gap. The heat map shows that an increase in the financing costs increases the financing gap mostly for highly indebted firms (white part of the map), but also for some medium-sized firms and small firms that were initially highly levered. Panel (b) shows the percentage changes in capital k' when the financing costs

increase. The firms that reduce their capital the most for next period are small and indebted (the same group for which the financing gap increased the most), and also some medium-sized ones. For these groups capital decreases by roughly 20%. Large firms also reduce their capital, but by a much lower amount, around 8%. Panel (c) shows the percentage changes for leverage. We observe a similar pattern than in the case of capital.

Figure 1: Policy Function Heat maps - Increase in Financing Costs
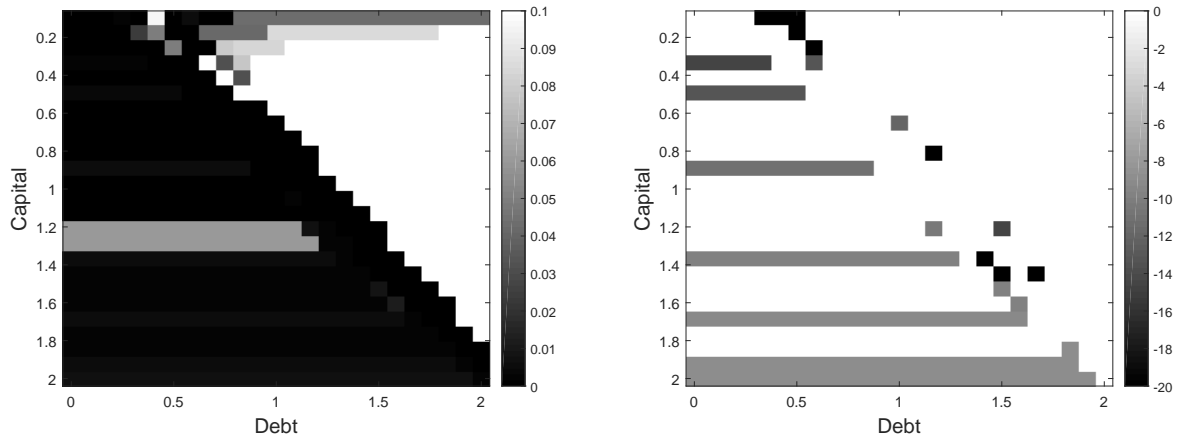
(a) Changes in Financing Gap

(b) Percentage Change in Capital
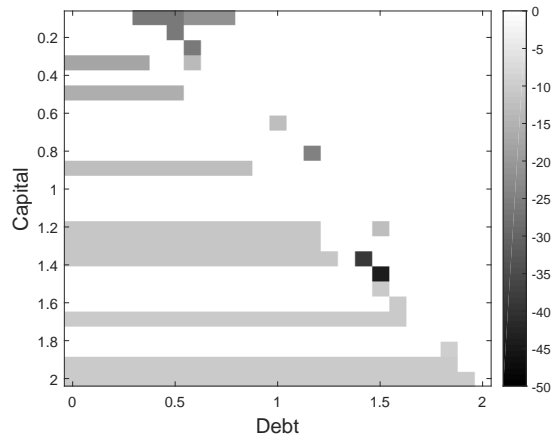
(c) Percentage Change in Leverage

The results presented above are in line with the predictions of the 2-period model. Constrained firms reduce their investment the most upon an increase in financial costs. These firms also tend to be small and using a large amount (relative to their size) of debt. It is important to

note that this analysis is fixing variables of the state space. For a more in-depth understanding, it is necessary to allow for these variable to endogenously adjust. This kind of analysis is presented in the next subsections.

4.2 Aggregate and Firm-Level Dynamics

This subsection focuses on analyzing the responses of the economy upon a sharp increase in sovereign debt. We perform two sets of exercises. The first set focuses on aggregate behavior, assessing to what extent the dynamics of the model are consistent with those of the data. The second one shows the responses conditioning on firm size, to see if the model is able to deliver patterns where small firms adjust more than large ones, as observed in the data.

We examine aggregate and firm-level dynamics by computing impulse response functions to an increase of sovereign debt consistent with the increase in the ratio of gross debt-to-GDP observed in Spain in 2012 (from 69.5% to 85.7%). The procedure to compute the impulse response functions is the one presented in Gilchrist et al. (2014), and is described in the Appendix. Figure 2 presents the impulse response functions (IRFs) for aggregate output, capital, debt, and leverage. We see that output drops by roughly 4%, which is larger than the observed decline in Spanish real output in 2012, which was close to 3%. The economy reduces its leverage, by around 2.5%, which is consistent with the evolution of capital and debt, where the latter adjusts more than the former. These responses show that a sharp increase in sovereign debt can generate important fluctuations in output. However, these responses may not be necessarily the ones observed on the equilibrium path of the model, given that an exogenous sequence was fed to the model in order to generate them.

The observed dynamics in the IRFs are generated by a specific shock at particular initial conditions of the economy. In order to assess whether these observed dynamics are also observed without setting specific initial conditions, we perform an event-study analysis using the simulated dataset. We define the event as a situation in which sovereign debt is one standard deviation above its long-run mean. We show the dynamics of the relevant variables (which are aggregated according to the time-varying distribution) in a 10-period window centered on the event date. These

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dynamics correspond to the average across all the identified events in the simulations. Figure 3 presents the results of the event study.
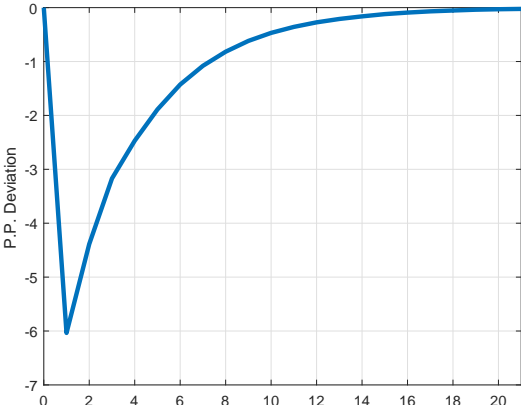
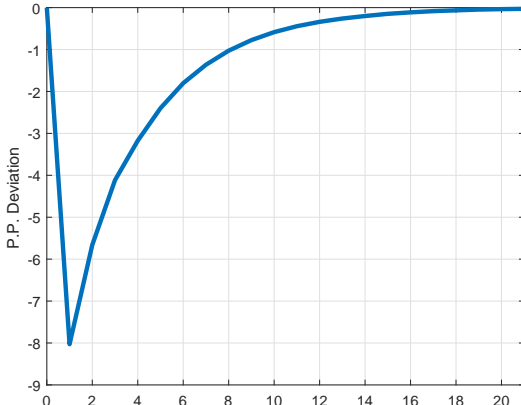**Figure 2:** Impulse Response Functions - Aggregates
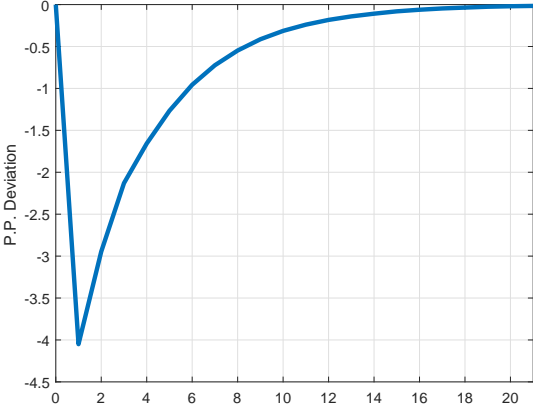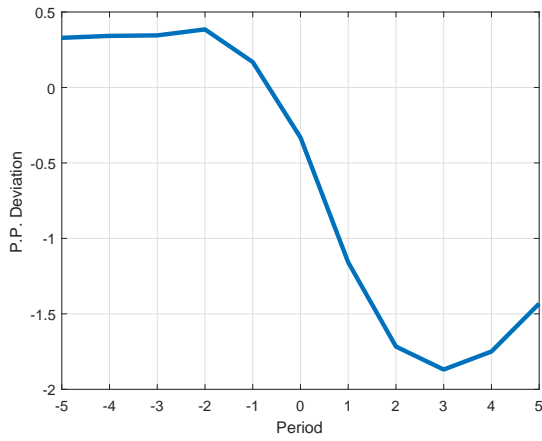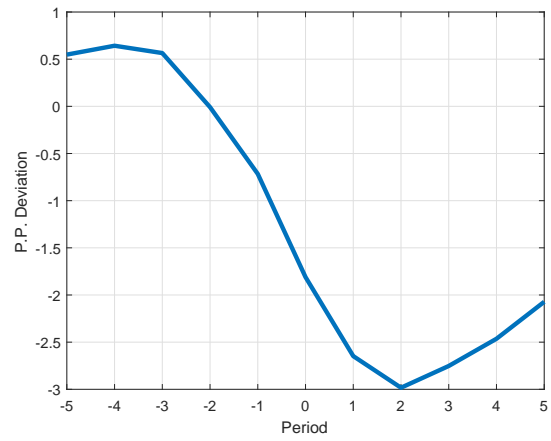

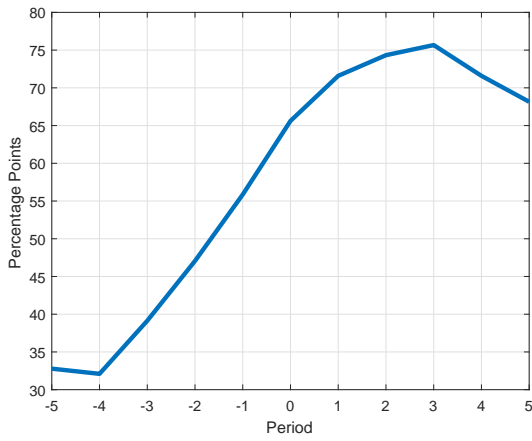(a) Output
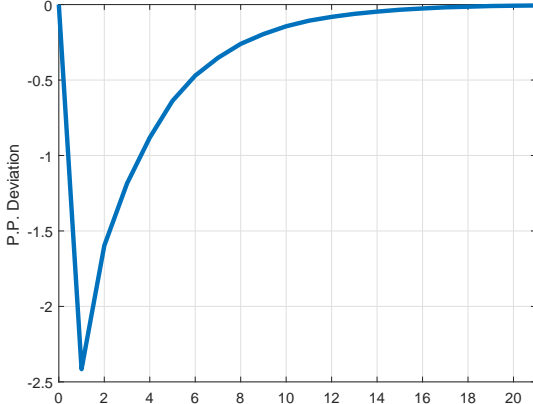
(b) Leverage

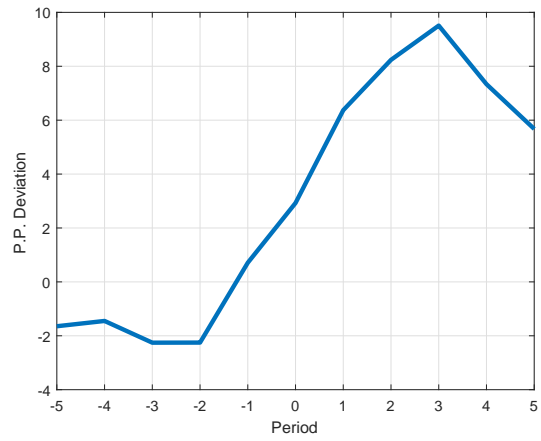(c) Capital

(d) Debt

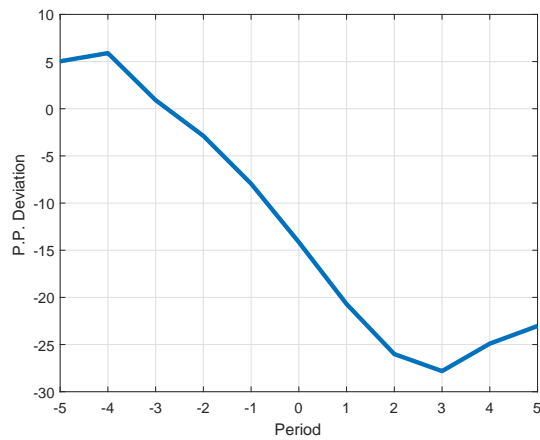**Figure 3:** Event Study

(a) Output

(b) Capital

(c) Sovereign Debt/Output

(d) Marginal Utility of the Intermediaries' Wealth μ

(e) Debt

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The results of the event study show that around the event date the sovereign debt-to-output ratio increases by around 15 percentage points, which is close to the actual increase during 2012 for the Spanish case. Output is about 0.3% below its long-run average at the event and continues dropping to roughly 2% below its mean, 3 years after. This gap of 1.7 percentage points corresponds to the decline in output after the event and is roughly half of the observed decline in real Spanish output. The dynamics of capital also follow a similar pattern to the one of output (given the specification of the production function of the model, where output and capital are proportional at the firm-level). Interestingly, we observe that firm debt (i) adjusts strongly around the event, and (ii) starts reacting even before the large increase in sovereign debt. This forward-looking behavior is consistent with firms internalizing an increasing path in sovereign debt and thus a potentially binding lending constraint for financial intermediaries in the future. In this way firms start adjusting accordingly, especially after the increasing path in sovereign debt is realized. The marginal utility of the intermediaries' wealth takes a bit of time to react, as it starts increasing one period before the event, but it then rises sharply to being roughly 8% above its long-run average.

We study next firm-level responses. The purpose is to assess if the model is able to generate patterns similar to those observed in the firm-level data. Recall from the empirical section that during the European debt crisis small firms reduced more their sales, assets, and liabilities in comparison to large firms. Figure 4 presents the IRFs to the same shock specified for the results in Figure 2. The results show that indeed small firms adjust much more aggressively: their output, leverage, capital, and debt drops more than for large firms (differences of 6.5, 6.25, 8, and 13 percentage points, respectively). These results support the theoretical predictions of the 2-period model of section 3 and the analysis of the policy functions performed in this section: small firms reduce more their investment than large firms when their financing costs increase. This is because small firms rely more on equity. When the cost of debt increases due to the financial shock, these firms are forced to substitute debt for equity, but given that it is increasingly costly and that they were already issuing equity, using this external financing source comes at a large cost. Another factor that makes the responses of small and large firms to differ is that the fixed cost of production pushes small firms closer to the default cutoff. This affects the price at which they can borrow, and
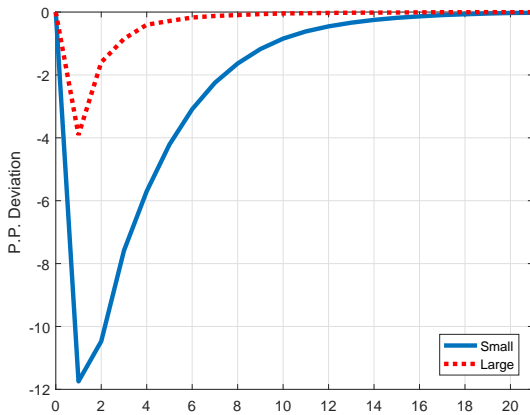
also how far they can go with issuing equity.

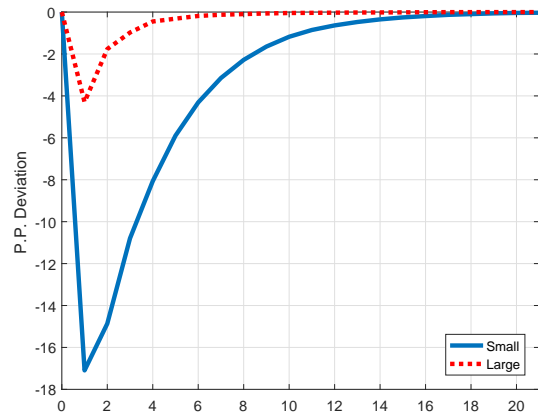**Figure 4:** Impulse Response Functions - Firm-Level
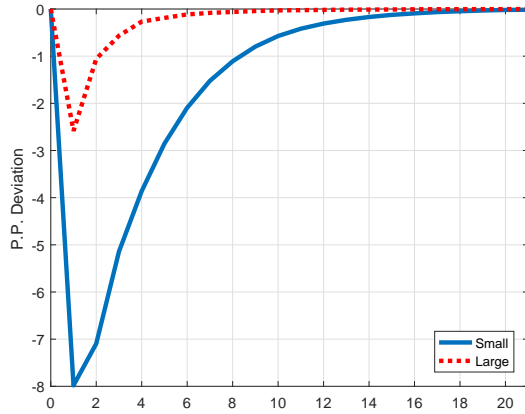
(a) Output
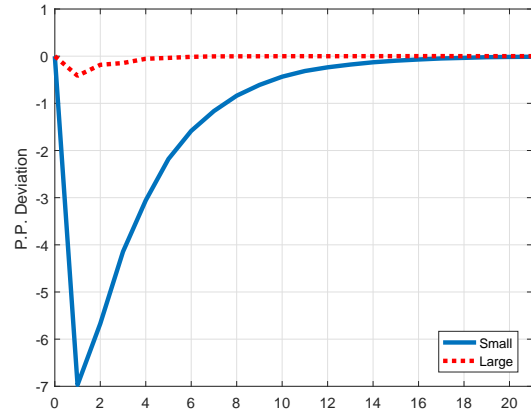
(b) Leverage

(c) Capital

(d) Debt

As shown in section 3, the key elements of the model that allow it to generate a pattern where small firms adjust more than large firms upon a financial shock are the costly equity issuance and the imperfect substitution between external financing alternatives. To illustrate this point, consider what would happen if equity issuance is not allowed. This is a common feature in corporate finance models, as it simplifies the computation. Ottonello and Winberry (2018), for example, find that firms in the US (listed in Compustat) respond to monetary policy shocks, but in their case, less levered firms are more sensitive. In the context of this model, more levered firms

are those who face a larger financing gap and want to avoid excessive equity issuance, so they will be more responsive to financial shocks. Thus, whether there is equity issuance or not in the model can potentially affect the responses that firms will have in terms of their sizes. Figure 5 presents the response of output when equity issuance is not allowed. When there is no equity issuance, the responses are less marked in terms of small, large and the aggregate economy. Also, surprisingly, the type of firm that adjusts most is flipped: when there is no equity issuance, large firms adjust more than small firms, a pattern that is opposite to what is observed during the European debt crisis. The core of this paper is not to explain these differences, but it is an interesting result from the modeling point of view because assuming that firms can not issue equity is not innocuous. When equity issuance is not possible, small firms internalize that by borrowing too much they might be pushed to the default cutoff in the case financing costs sharply increase, as they will not able to rollover their debt. These firms are constrained, and they have no other source of external financing, so they borrow less. Thus, financing cost shocks have less severe effects on them than on large firms, which can rely on internal resources, as they will be unconstrained with a higher likelihood.

Figure 5: Impulse Response Functions - No Equity Issuance
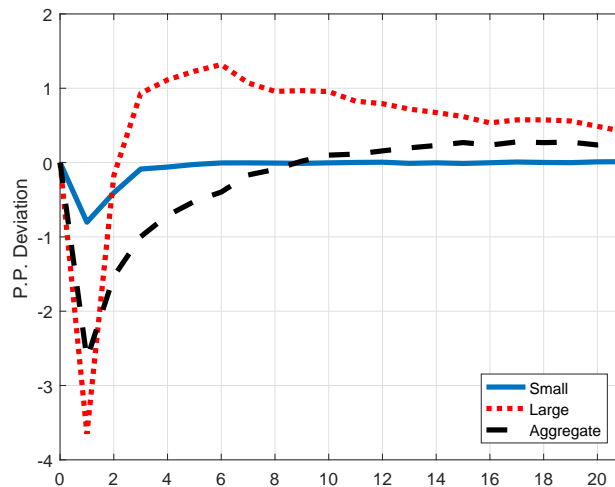
(a) Output

4.3 The Relevance of Firm Heterogeneity

This subsection studies the role of firm heterogeneity on the responses of the economy to an increase in sovereign borrowing. In particular, two economies are contrasted, one with heterogeneous firms and one with a representative firm. Additionally, the responses of two economies composed by heterogeneous firms but with different distributions are computed, to assess the role of firm size distribution on the economy.

As seen in the previous section, small and large firms face different financing conditions and respond differently to the same kind of stimulus. It is important to note that asymmetric responses in terms of firm size do not guarantee that the composition of firms will matter for aggregates. For example, in Krusell and Smith (1998), the policy functions of agents in the model are almost linear and with the same slope. In addition to this, a large fraction of agents is rich enough to be in the linear part of the policy function. Thus, redistribution of wealth virtually has no effect on aggregate consumption. Another example is the case of Bernanke et al. (1999), where firms are linearly scalable in their net worth, rendering the role of the distribution of firms irrelevant. This section studies whether in this setting the heterogeneity of firms and the distribution are relevant.

Two exercises are performed in order to assess the relevance of firm heterogeneity. In the first, firm heterogeneity is nearly shut down by setting the variance of the productivity component equal to 20% of the original, and re-calibrating the rest of the parameters of the model.[18] By proceeding in this way, the "no heterogeneity" framework resembles a model with a representative firm. Then, we compute impulse response functions to a sharp increase in sovereign borrowing. The second exercise consists in computing impulse response functions that are conditional on different levels of skewness of the firm size distribution. This provides an alternative to assess whether firm size distribution is quantitatively important, and also to study if economies with larger skewness adjust more or less than those with a smaller skewness.

Figure 6 presents the IRFs of aggregate responses for the baseline economy ("heterogeneity") and an economy calibrated to have a standard deviation of productivity equal to $\sigma_z^{\text{No Het}} = 0.2 \times \sigma_z$, a substantially lower degree of heterogeneity. The economy with firm heterogeneity re-

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[18]The targeted moments are the same, except for the cross-sectional variance of the investment to capital ratio.

sponds much more to the increase in government debt. Fluctuations in output in the baseline case are nearly twice than those in the no-heterogeneity case, while similar differences are observed for debt and capital. Regarding leverage, the differences are even larger. The baseline economy adjusts nearly 3 times more than in the no-heterogeneity scenario. These differences show that the composition of firms in the economy quantitatively matters, and that firm heterogeneity is a substantial source of amplification in terms of the responses of key macroeconomic aggregates.

Figure 6: Impulse Response Functions - Heterogeneity v. No Heterogeneity

(a) Output

(b) Leverage

(c) Capital

(d) Debt
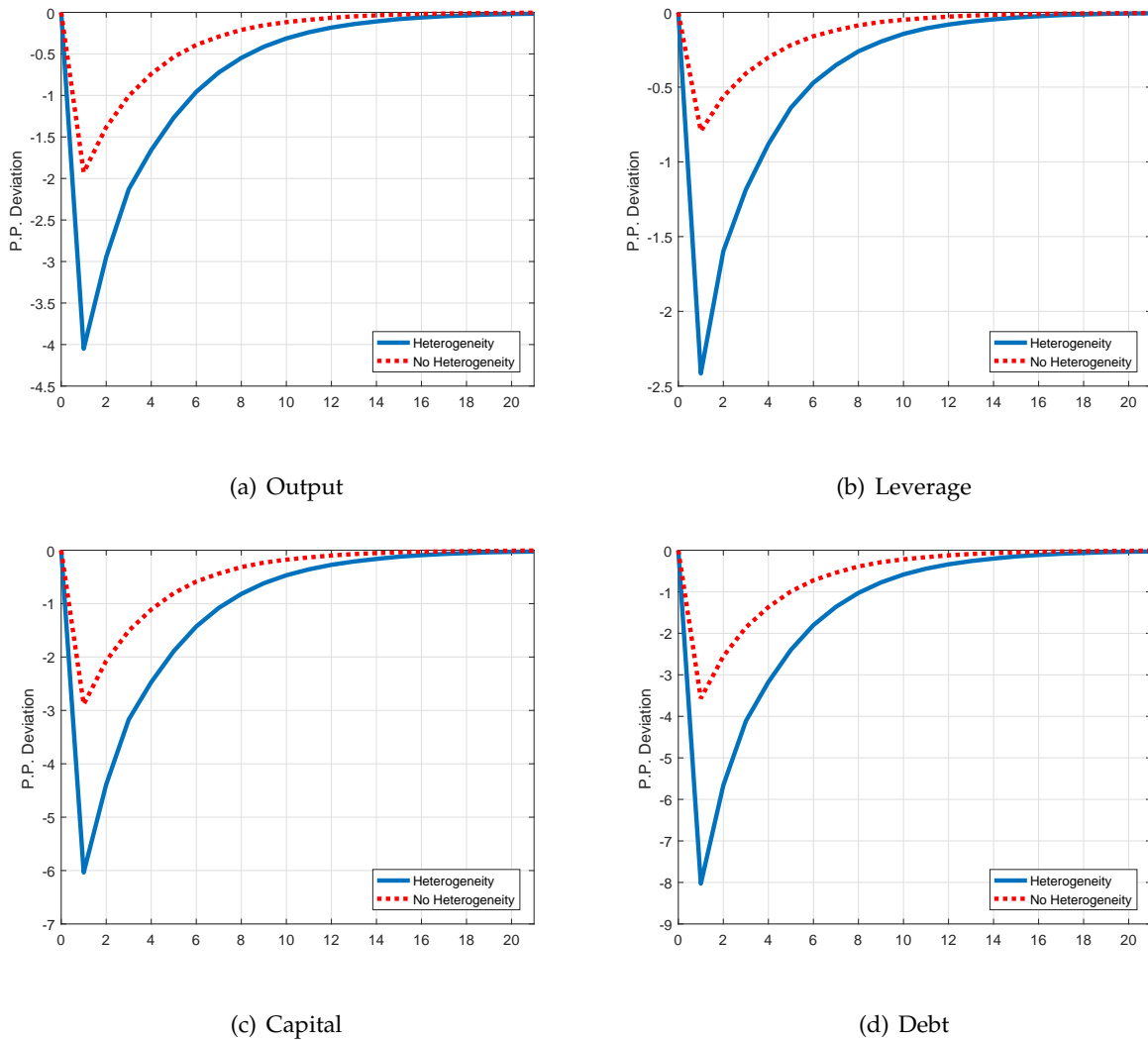
The next exercise compares the responses of economies with high and low skewness of the

firm size distribution.[19] In the first one, there is a bigger share of large firms. Intuitively, given that large firms adjust less when facing a financial shock, we should expect to see that an economy with greater firm size skewness adjusts less in aggregate terms. Figure 7 presents the comparison of the IRFs of these two economies. The IRFs for the "high skewness" case is the benchmark, while the "low skewness" consists on the responses to the same shock as the benchmark economy but conditioning the variance of capital (aggregate moment on the policy function) to be 50% of the baseline scenario.

Figure 7: Impulse Response Functions - Low v. High Skewness

(a) Output

(b) Leverage
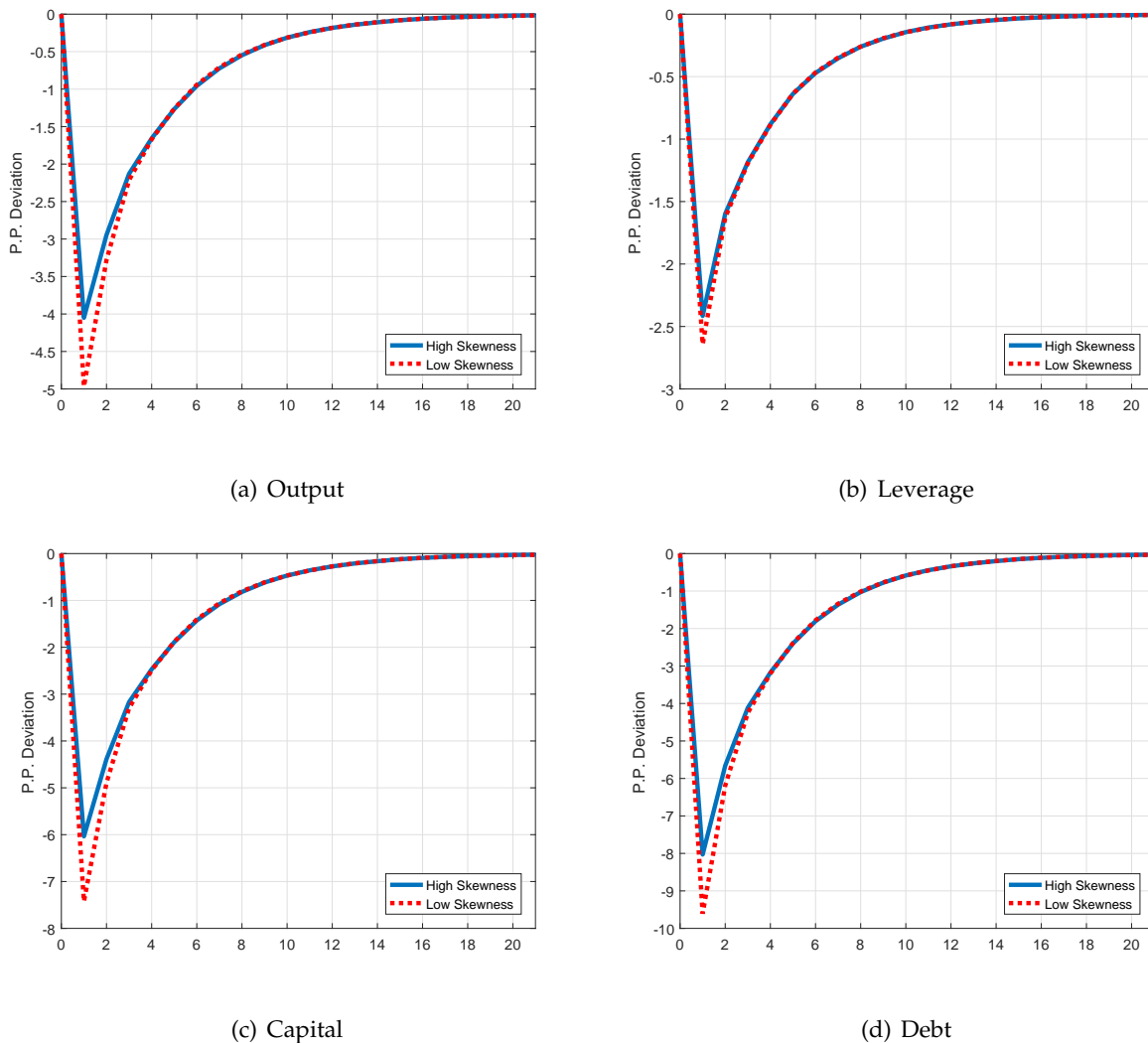
(c) Capital

(d) Debt

Output and capital respond roughly 25% more in the economy with lower skewness. The

[19]The economy with low skewness has a skewness equal to 50% of the high skewness case.

differences are smaller in the case of leverage and debt. These results suggest that the differences come from the right tail of the distribution of firm size, as it is mainly where the two economies differ. A less skewed distribution still includes a similar density of small firms, but it contains a significantly lower density of large firms. Thus, differences in the skewness of distributions can account for observed gaps in the responses of two different economies.

The results of these two exercises show that firm heterogeneity and the distribution of firms are relevant quantitatively. Firm heterogeneity generates responses of output that are twice the size of those of an economy with no firm heterogeneity. Additionally, the skewness of the firm size distribution accounts for up to 25% of the observed differences (in the data across low and high skewness cases) firm in the responses of output.

## 4.4 The Role of Financial Frictions

This subsection studies the role that financial frictions play in the aggregate responses to a financial shock caused by a sharp increase in sovereign borrowing. In particular, it focuses on the role that costly equity issuance has on the responses of firms to the financial shock. Section 3 showed how financial frictions in the model generate different responses for large and small firms when financing costs increase. The imperfect substitution between equity and debt was a key element to generate those asymmetric responses. Figure 8 shows the IRFs for an economy with no equity issuance costs, but with defaultable debt.[20]

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[20]Some degree of financial friction is needed in order to generate responses to the financial shock. In a completely frictionless environment firms would automatically choose their optimal level of capital, and finance investment with any of the two external financing alternatives.

Figure 8: Impulse Response Functions - No Equity Issuance Cost
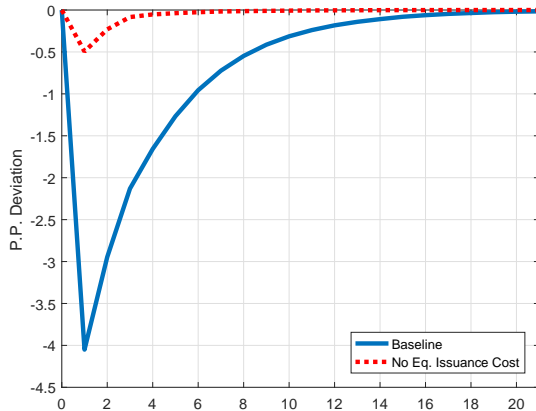
(a) Output
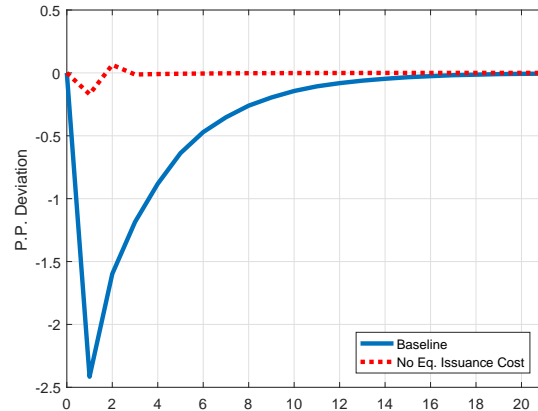
(b) Leverage
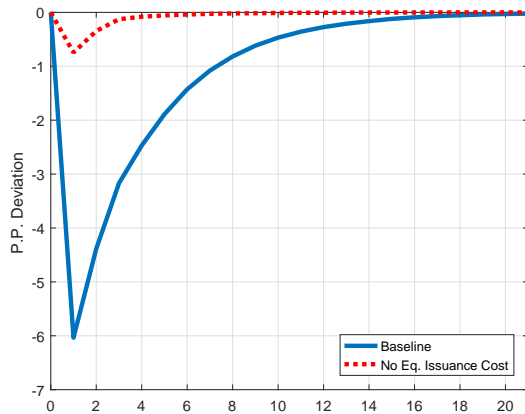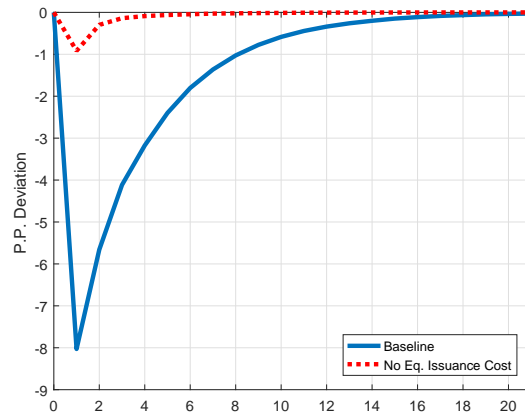
(c) Capital

(d) Debt

Figure 8 presents the responses of the baseline economy and the economy without equity issuance costs. The differences are striking. In the costless equity issuance case, the drop in output, leverage, capital, and debt is barely noticeable when comparing it to the baseline case. For example, we see that output in the baseline case drops by roughly 4%, while in the costless equity issuance scenario it drops by 0.5%. Similar patterns can be drawn from the rest of the variables. These results suggest that financial frictions play a key role in the amplification of the responses to financial shocks. The intuition for this is similar to the one provided in the 2-period model presented in section 3. Debt and equity are imperfect substitutes because debt is defaultable and

equity issuance is costly. Thus, switching debt for equity (which is triggered by the financial shock) comes at a cost, which generates size-dependent adjustments in the investment of firms. If equity issuance is costless, then there is no additional cost for the firm when switching from debt to equity, which implies that the adjustments in investment triggered by financial shocks should be significantly lower. This is corroborated by the results presented in Figure 8.

Lastly, notice that firm heterogeneity loses its amplification in the absence of financial frictions. Recall from Figure 6 that firm heterogeneity amplifies the responses of the variables of interest to the financial shock. In the costless equity issuance scenario, aggregate responses to the financial shock are negligible (in relation to the baseline case), which leaves little space for the amplification that firm heterogeneity generates.

5 Conclusions

This paper examines the role that firm heterogeneity and financial frictions play in terms of the propagation of financial shocks to the real economy. The recent European debt crisis provides a natural experiment to study these issues. During this crisis, a sudden deterioration of the credit conditions faced by firms in the GIIPS countries generated asymmetric responses at the firm level, which differed according to firm size. Empirical evidence suggests that one channel through which this deterioration occurred is a "crowding out" channel, by which increased government borrowing and bank holdings of sovereign debt generated a crowding-out of private lending, reducing funds available to firms and increasing their financing costs.

The empirical section of this paper documented two new facts regarding the response of firms in the GIIPS countries during the debt crisis using a rich panel dataset at the firm-level. It is observed that (i) smaller firms adjusted their sales, liabilities, assets and employment more than large firms, and (ii) smaller firms experienced smaller adjustments in these variables in economies with more skewed firm size distributions. The data also show that financing costs, measured as the ratio of debt service to total debt, increased more for smaller firms.

This paper proposed a model to explain the above two facts, and derived its quantitative

implications. The model has three main components. First, firms that are heterogeneous in terms of productivity, capital and debt, and face financial frictions in the form of defaultable debt and costly equity issuance. Second, financial intermediaries that lend to firms and the government, facing an occasionally binding constraint on loanable funds. Third, a government that causes the occasionally binding constraint to bind when public debt increases sharply. When this occurs, the resources available for firm lending decreases, which generates an increase in their financing costs. Firms have different financing structures conditional on their size because debt and equity are imperfect substitutes. In particular, small firms will tend to rely more on equity than debt and thus face a more expensive financing mix. The financial shock will force firms to substitute debt for internal resources or equity. Small firms issue even more equity (which is increasingly costly), which forces them to adjust their investment more aggressively than large firms.

In this framework firm heterogeneity has aggregate implications. In the presence of financial frictions, the financial shock affects the evolution of the distribution of firms, and hence the dynamics of aggregate outcomes such as output. The model predicts that an economy with a less skewed firm size distribution faces larger drops in output upon a financial shock. The intuition for this result is that economies with a higher fraction of small firms (i.e., less skewed firm size distribution) will tend to adjust more because small firms are more sensitive to the financing cost shock. This pattern resembles the one observed in European debt crisis, where Portugal (an economy with low skewness on firm size distribution) faced larger drops in output than Spain (an economy with larger skewness than Portugal).

The model was calibrated to firm-level and aggregate data for Spain. Quantitative results show that for an increase in sovereign debt consistent with the Spanish case, experienced during the European debt crisis, the model predicts that small firms adjust more than large ones, and that smaller firms would have adjusted less if the economy had a larger firm-size dispersion. Regarding the aggregate effects of firm heterogeneity, the model generates an output drop of roughly half the size of the drop observed in Spain. Comparing with the results for the same model setup but with a representative firm, output drops by 2 percentage points more (with respect to its long-run average), in the economy with heterogeneous firms. Thus, firm heterogeneity along with the presence of financial frictions significantly amplifies the responses of aggregate variables to a

tightening in credit conditions.

Appendices

A Data Appendix

The details of procedure for data cleaning and preparation are presented in this section. I follow steps similar to those of Gopinath et al. (2017). For data cleaning the steps are:

(i) Drop firm-year observations that have missing information on total assets, operational revenues, sales and employment.

(ii) Drop firm-year observations with missing, 0 or negative values for operational revenues and total assets.

(iii) Drop observations with missing industry information.

(iv) Drop firms who have negative total assets in any year, and if employment, sales or tangible fixed assets are negative.

Also, firms from the financial sector and government are dropped from the sample (NACE codes 65, 66, 67, and 75). Variables used in the analysis are constructed in the following way:

(i) Age: Reporting year minus year of incorporation plus 1. Observations with negative values are dropped.

(ii) Liabilities: Total shareholders funds and liabilities minus shareholders funds. Observations with negative values are dropped. Also a consistency check is performed: observations with liabilities different from the sum of current and non current liabilities are dropped.

(iii) Capital Stock: tangible fixed assets plus intangible fixed assets. Observations with negative values of intangible fixed assets are dropped, as well as those with zero tangible fixed assets, with tangible fixed assets that exceed total assets and cases with negative depreciations.

(iv) Equity: shareholders funds. We drop observations whose ratio of shareholders funds and total assets are below percentile 0.1.

(v) Leverage: ratio between liabilities and total assets. We drop observations for which the leverage ratio is below or above the lowest and highest 0.1 percentiles, respectively.

(vi) Size: share of firm's assets over total assets for a given year.

(vii) Average Effective Interest Rate: Observations with negative values for the ratio $\frac{\text{Interest Paid}}{\text{Liabilities}}$ are dropped.

A.1   Additional Specifications

This subsection presents robustness checks regarding the specification of the regression model presented in section 2. We estimate the regression model using a random effects specification and also using (pooled) OLS, and contrast the results with those of the baseline fixed effects specification.

Tables 6 and 7 present the results of the random effects and OLS specifications. Unlike in the case of the fixed effects specification, the random effects and OLS specifications allow for country and industry effects (in addition to the same controls as in the baseline specification).

We see that the effects of government debt on sales, liabilities, and employment and not substantially different between the fixed effects and random effects specifications, but there is a difference for the case of assets of the firm (the effect of government debt loses statistical significance in the random effects specification). When comparing the results from the OLS specification with the baseline, we see that the results are quite different. The latter is because of the nature of the underlying assumptions of each specification. The OLS specification assumes that there is no unobserved heterogeneity (i.e., the firm-specific characteristics are uncorrelated with the regressors), which is unlikely in a setting like the one presented in this paper.[21]

Overall, the results show that the random effects specification, despite not being preferred to the fixed effects specification according to the Hausman test, are not significantly different from those of the fixed effect model.

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[21]The F-statistic of the fixed effects specification provides a way of jointly testing if all individual firm-specific effects are 0. The null hypothesis is that firm-specific effects are 0. The associated p-values are 0.000 for all the estimated regressions, suggesting that the fixed effect specification should be preferred to the pooled OLS one.

Table 6: Panel Regression Results: Additional Specifications - Sales, Liabilities, Assets, and Employment

Variable	Log(Sales)			Log(Liabilities)			Log(Assets)			Log(Employment)		
	FE	RE	OLS	FE	RE	OLS	FE	RE	OLS	FE	RE	OLS
B	-0.003	-0.002	-0.002	-0.003	-0.001	-0.000	-0.001	0.000	0.000	-0.003	-0.000	-0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.294)	(0.207)	(0.000)	(0.000)	(0.020)
Q_2	0.327	0.254	-0.019	0.718	0.708	0.51	0.519	0.596	0.426	0.161	0.091	0.064
	(0.000)	(0.000)	(0.044)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Q_3	0.562	0.448	0.015	1.250	1.14	0.791	0.909	0.987	0.674	0.285	0.171	0.121
	(0.000)	(0.000)	(0.137)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Q_4	0.754	0.674	0.129	1.712	1.547	1.056	1.237	1.370	0.921	0.368	0.271	0.206
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Q_5	0.864	0.934	0.408	2.103	2.171	1.59	1.532	1.914	1.384	0.424	0.417	0.352
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
B $\times$ Q_2	-0.001	0.000	0.002	-0.001	-0.001	-0.000	-0.000	-0.001	0.000	-0.001	-0.001	-0.000
	(0.000)	(0.473)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.706)
B $\times$ Q_3	0.000	0.000	0.002	-0.001	-0.001	-0.000	0.000	-0.001	-0.001	-0.001	-0.001	-0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.474)	(0.000)	(0.000)	(0.000)	(0.000)	(0.763)
B $\times$ Q_4	0.000	0.001	0.001	-0.001	-0.001	-0.000	0.000	-0.001	-0.001	-0.001	-0.001	-0.000
	(0.036)	(0.000)	(0.000)	(0.000)	(0.000)	(0.033)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
B $\times$ Q_5	0.002	0.002	0.002	0.000	0.000	-0.000	0.000	-0.000	-0.001	0.000	-0.000	-0.000
	(0.000)	(0.000)	(0.000)	(0.054)	(0.336)	(0.005)	(0.000)	(0.000)	(0.000)	(0.367)	(0.007)	(0.000)
B $\times$ Q_2 $\times$ Sk	0.000	0.000	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	0.000	0.000	0.000
	(0.058)	(0.000)	(0.000)	(0.003)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.013)	(0.074)
B $\times$ Q_3 $\times$ Sk	-0.000	0.000	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	0.000	0.000	-0.000
	(0.727)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.408)	(0.573)
B $\times$ Q_4 $\times$ Sk	-0.000	0.000	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	0.000	-0.000	-0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.004)	(0.000)
B $\times$ Q_5 $\times$ Sk	-0.000	-0.000	0.000	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000	0.000	-0.000	-0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.013)	(0.000)	(0.000)
B $\times$ Sk	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	(0.000)	(0.012)	(0.012)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.934)	(0.000)	(0.000)
Year Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Fixed Effects	Yes	No	No	Yes	No	No	Yes	No	No	Yes	No	No
Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Country Effects	No	Yes	Yes	No	Yes	Yes	No	Yes	Yes	No	Yes	Yes
Industry Effects	No	Yes	Yes	No	Yes	Yes	No	Yes	Yes	No	Yes	Yes
Observations	3,195,072	3,195,072	3,195,072	3,308,816	3,308,816	3,308,816	3,313,051	3,313,051	3,313,051	3,119,977	3,119,977	3,119,977
R^2	0.58	0.699	0.712	0.736	0.914	0.923	0.911	0.967	0.970	0.585	0.8816	0.885

Notes: B denotes the gross public debt-to-output ratio, Q_i is a dummy variable that is equal to 1 if the firm belongs to the ith quintile of firm size, and Sk is the skewness of the firm size distribution. Countries include Greece, Ireland, Italy, Portugal and Spain. Clustered standard errors at the firm level considered, p-values in parentheses.

Table 7: Panel Regression Results: Additional Specifications - Proxy for Average Effective Interest Rate

Variable	Average Effective Interest Rate $\times$ 100		
	Fixed Effects	Random Effects	Pooled OLS
B	0.000	0.000	0.000
	(0.000)	(0.041)	(0.577)
Q_2	-0.011	-0.015	-0.017
	(0.000)	(0.000)	(0.000)
Q_3	-0.017	-0.022	-0.022
	(0.000)	(0.000)	(0.000)
Q_4	-0.022	-0.026	-0.026
	(0.000)	(0.000)	(0.000)
Q_5	-0.027	-0.032	-0.031
	(0.000)	(0.000)	(0.000)
B $\times$ Q_2	-0.006	0.000	0.000
	(0.222)	(0.000)	(0.000)
B $\times$ Q_3	-0.027	0.000	0.000
	(0.599)	(0.000)	(0.000)
B $\times$ Q_4	-0.03	0.000	0.000
	(0.972)	(0.000)	(0.000)
B $\times$ Q_5	-0.043	0.000	0.000
	(0.637)	(0.000)	(0.000)
B $\times$ Q_2 $\times$ Sk	0.000	0.000	0.000
	(0.007)	(0.000)	(0.000)
B $\times$ Q_3 $\times$ Sk	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)
B $\times$ Q_4 $\times$ Sk	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)
B $\times$ Q_5 $\times$ Sk	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)
B $\times$ Sk	-0.000	-0.000	-0.000
	(0.000)	(0.000)	(0.000)
Year Effects	Yes	Yes	Yes
Fixed Effects	Yes	No	No
Controls	Yes	Yes	Yes
Country Dummies	No	Yes	Yes
Industry Dummies	No	Yes	Yes
Observations	2,852,913	2,852,913	2,852,913
R^2	0.001	0.019	0.020

Notes: B denotes the gross public debt-to-output ratio, Q_i is a dummy variable that is equal to 1 if the firm belongs to the ith quintile of firm size, and Sk is the skewness of the firm size distribution. Countries include Greece, Ireland, Italy, Portugal and Spain. Clustered standard errors at the firm level considered, p-values in parentheses.

B Computational Appendix

B.1 Solution Method

The solution method follows Krusell and Smith (1997) and Krusell and Smith (1998). Unlike these articles, where the distribution of wealth is approximated by the average wealth in the economy, in this paper we assume that the distribution of firms is well approximated by the cross-sectional variance of capital. The individual states for firms are (z, k, b) and the aggregate states are $(B, \sigma^2, \mu, \varepsilon)$.

We conjecture log-linear specifications for variance and the marginal utility of the intermediaries' wealth:

$$\log \sigma^{2\prime} = \alpha_\sigma + \beta_\sigma \log \sigma^2 + \gamma_\sigma \log(1 + B) + \delta_\sigma \log(1 + \mu) + \kappa_\sigma \varepsilon \tag{20}$$

$$\log(1 + \mu') = \alpha_\mu + \beta_\mu \log \sigma^2 + \gamma_\mu \log(1 + B) + \delta_\mu \log(1 + \mu) + \kappa_\mu \varepsilon \tag{21}$$

The steps of the solution algorithm are the following:

1. Start with a guess for the parameters of the forecasting rule $\Theta^{\text{guess}} = (\alpha_\sigma, \beta_\sigma, \gamma_\sigma, \delta_\sigma, \kappa_\sigma, \alpha_\mu, \beta_\mu, \gamma_\mu, \delta_\mu, \kappa_\mu)$

2. Using the forecasting rules solve the problem of sovereign using its budget constraint.

3. Using the forecasting rules and the policy function of the sovereign (which is obtained from solving the budget constraint), solve the problem of the firm. For this:

   (a) Guess a value for the value function $\tilde{V}$.

   (b) Using this guess, construct a default indicator and also the pricing function $\tilde{q}$ for the firm.

   (c) Using $\tilde{V}$ and $\tilde{q}$, solve the problem of the firm using value function iteration. The pricing function is updated after every maximization step (instead of waiting for convergence on V), which increases the speed of the algorithm.[22]

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[22]Solutions of the model under this approach and one where the pricing function is updated after convergence of V are the same.

(d) Using the solution to the firm's problem V, construct an update for the pricing function, denote it q. Notice that a default decision can be directly computed from V, so q can be inferred from it and from aggregate laws of motion. If $||V - \tilde{V}|| < $ tol and $||q - \tilde{q}|| < $ tol, then proceed to the next step. Otherwise go back to step (b) and use V as a guess, construct $\tilde{q}$ using this new guess.

4. Simulate an economy with I firms and the sovereign, for T periods. At every period of the simulation find the value of μ that solves the slackness condition of the intermediaries' problem. For this start by assuming that the constraint does not bind (this is, $\mu = 0$). If the firm aggregate demand plus the sovereign demand for debt do not exceed the level of wealth of the intermediaries', then set $\mu = 0$ and proceed to the next period. Otherwise, perform a bisection algorithm to find the value of μ that makes the slackness condition to hold.

5. Using the completed sequence of simulated variables update forecasting rules and Find Θ^{new}. If $||\Theta^{\text{guess}} - \Theta^{\text{new}}|| < $ tol, stop. Otherwise, go back to step 1 and use $\tilde{\Theta} = \varphi\Theta^{\text{new}} + (1 - \varphi)\Theta^{\text{guess}}$ as the new guess.

The coefficients obtained are presented in Table 8:

Table 8: Regression Results

Coefficient	σ^2	μ
α_i	-0.245	-0.570
	(0.040)	(0.022)
β_i	0.801	0.243
	(0.022)	(0.012)
γ_i	-0.037	0.091
	(0.011)	(0.006)
δ_i	0.311	-5.629
	(0.752)	(0.826)
κ_i	0.000	0.004
	(0.002)	(0.001)
R^2	0.967	0.912

Note: Standard errors in parentheses.

B.2 Construction of Impulse Response Functions

1. Simulate two economies with the same sequence of cross-sectional idiosyncratic productivity for firms (N firms) until a date $\hat{t}$, with government debt fixed at its long run average.

2. In period $\hat{t}$, there is an increase in sovereign debt. Let x_{it}^0 denote the value of variable x for firm i in period t in an economy with no aggregate shock, and x_{it}^1 the equivalent but in the economy with the aggregate shock. Define the impulse response of firm i in period t as $\tilde{x}_{it} = \frac{x_{it}^1 - x_{it}^0}{x_{it}^0} \times 100$. The response of the economy will be given by $\tilde{x}_t = \sum_i^N \tilde{x}_{it}/N$.

3. In order to mitigate potential history dependence, perform this procedure until there are M sequences of responses. The impulse response function is the average of the M sequences.

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