Information Acquisition and Liquidity Traps in Over-the-Counter Markets

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Abstract

I analyze the interaction between buyers’ information acquisition and market liquidity in over-the-counter markets with adverse selection. If a buyer anticipates that future buyers will acquire information about asset quality, she has an incentive to acquire information to avoid buying a lemon that will be hard to sell at a later date. However, when current buyers acquire information, they cream-skim the market, leaving a larger fraction of lemons for sale and giving future buyers an incentive to acquire information. A liquid market can go through a self-fulfilling market freeze when buyers start to acquire information. More importantly, if information acquisition continues for a long enough period of time, the market gets stuck in an information trap with low liquidity: information acquisition worsens the composition of assets remaining on the market, and the bad composition incentivizes information acquisition. This prediction helps explain why the market for non-agency residential mortgage-backed securities experienced a sudden drop in liquidity—as potential buyers realized the need for greater due diligence—but has remained essentially dormant despite a strong recovery in the housing market.

Keyword: Information acquisition; Adverse selection; Market freezes; OTC markets

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1 Introduction

During the 2007–2008 financial crisis, many asset markets suffered from periods of illiquidity—sellers found it increasingly hard to sell assets at acceptable prices. Dry-ups in liquidity are especially prominent among classes of assets that are opaque and traded in over-the-counter (OTC) markets, such as mortgage-backed securities (Gorton, 2009) and collateralized debt obligations (Brunnermeier, 2009). A large literature has sought to explain these events of market freezes through the lens of asymmetric information. The standard narrative is that asset owners are better informed of their assets’ quality than potential buyers in these markets. Therefore, when the perceived average quality of assets decreases, markets freeze as a result of the exacerbated adverse selection problem.

One decade after the financial crisis, the US economy is on track for the longest expansion ever, and housing prices are on a path of continued growth. However, the impact of the crisis seems rather persistent. The market for non-agency residential mortgage-backed securities (RMBS), which was at the center of the financial crisis, has yet to come back (Ospina and Uhlig, 2018). At the same time, investors are conducting more due diligence in inspecting and evaluating securitized products after the crisis. Instead of solely relying on external ratings, investors now develop their own models to provide independent assessments of asset quality. These stark differences in market liquidity and the behavior of market participants before and after the crisis, despite similar fundamentals of the market, are hard to reconcile with the standard narrative of adverse selection. Indeed, if the RMBS market freeze was driven by deterioration of the value of the underlying mortgages, the market should have recovered given the current strong economic fundamentals and the bullish housing market.

To explain both the decline in market liquidity and the increase in investors’ due diligence, I introduce buyers’ information acquisition into a dynamic adverse-selection model with resale considerations. The key result of my model is that an asset market can have multiple steady states, and more importantly, transitions between steady states are asymmetric. Liquid markets are susceptible to a self-fulfilling market freeze, in which buyers suddenly

2 See All-Transactions House Price Index for the United States, https://fred.stlouisfed.org/series/USSTHPI.
3 Non-agency mortgage-backed securities are issued by private entities, and do not carry an explicit or implicit guarantee by the US government. In contrast, agency MBS are issued and backed by government agencies or government-sponsored enterprises, such as Fannie Mae, Freddie Mac and Ginnie Mae.
4 For instance, see The Economist in its January 11, 2014, issue: “Before 2008, . . . , investors piled in with no due diligence to speak of. Aware of the reputational risks of messing up again, they now spend more time dissecting three-letter assets than just about anything else in their portfolio.” Also, Kaal (2016) finds that since the financial crisis, private funds have hired more analysts to conduct investors’ due diligence using textual analysis of the ADV II filings.
start to acquire information and the market quickly transitions from a liquid state to an illiquid one. As illiquid trading and information acquisition continue for an extended period, the market falls into an information trap with low liquidity and information acquisition, in which there is no equilibrium path that leads back to the liquid state. Importantly, while some previous papers have studied sudden market freezes in the framework of multiple equilibria, my findings are different in terms of the sharp prediction of whether the market can recover in a self-fulfilling manner after a market freeze.

Before describing these results in greater detail, it makes sense to first lay out the key ingredients of the model. A continuum of investors trades assets of either high or low quality. Gains from trade arise because asset owners are subject to idiosyncratic liquidity shocks that lower the flow payoff from holding assets. Upon receiving a liquidity shock, an asset owner participates in the market as a seller and trades with potential buyers who arrive sequentially. A seller is privately informed of the quality of her own asset, while the buyer can acquire a noisy signal of the asset’s quality by incurring a fixed cost. If the asset is traded, the buyer hold the asset and will return to the market as a seller when receiving a liquidity shock in the future. Otherwise, the seller keeps the asset and waits for the arrival of the next buyer. Although this paper is motived by observations in the non-agency RMBS market, the model can be applied to various OTC markets with asymmetric information.

How does buyers’ information acquisition interact with market liquidity? If the current composition of assets for sale is good enough to support pooling trading, buyers’ information acquisition reduces current market liquidity. Intuitively, if a buyer acquires information and observes a bad signal, she is unwilling to trade at a pooling price because the posterior belief about the asset’s quality becomes worse. In addition to the static relationship between buyers’ information acquisition and market liquidity, there is also a dynamic strategic complementarity between buyers’ current and future incentives to acquire information, and hence a complementarity between current and future market liquidity. On one hand, current buyers’ incentive to acquire information depends on future buyers’ information acquisition through the resale consideration. If a buyer anticipates that future buyers will acquire information about asset quality, she has an incentive to acquire information so as to avoid buying a low-quality asset that will be hard to sell at a later date. In this sense, expected future market liquidity improves current market liquidity. On the other hand, current buyers’ information acquisition changes future buyers’ incentives to acquire information through the cream-skimming effect. When current buyers acquire information, high-quality assets are traded faster than low-quality assets. As low-quality assets accumulate on the market over time, future buyers have more incentive to acquire information. Therefore, current market illiquidity harms future market liquidity.
The dynamic strategic complementarity in buyers’ information acquisition gives rise to the possibility of a self-fulfilling market freeze. Suppose the market is in a liquid state, in which buyers do not acquire information and the composition of assets for sale is good. One day, investors suddenly start to worry that in the future buyers will acquire information, lowering market liquidity. As a result, the resale value of low-quality assets drops abruptly and the current buyers start to acquire information. Because of the cream-skimming effect of information acquisition, the composition of assets for sale deteriorates gradually, giving future buyers more incentive to acquire information. This justifies current investors’ belief in future low liquidity. A self-fulfilling market freeze takes place when investors coordinate to follow an equilibrium path with information acquisition.

As the self-fulfilling market freeze continues and the composition of assets for sale declines further, it is impossible for the market to return to liquid trading without outside intervention. Too see this, notice buyers’ incentives to acquire information depend on both future market liquidity and the current composition of assets for sale. When the composition is bad enough, even if buyers believe the market will be liquid in the future, it is still optimal for them to acquire information today to avoid buying low-quality assets. Their information acquisition in turn keeps the composition of assets for sale at a low level. The market is therefore “trapped” in an illiquid state with information acquisition and longer trading delays.

The key mechanism that generates the asymmetric transitions between states with different liquidity is the slow-moving property of the composition of assets for sale. Buyers’ information acquisition worsens the composition of assets for sale through the cream-skimming effect and has a long-lasting negative impact on future market liquidity. The composition will only improve gradually when buyers stop acquiring information. However, even with the most optimistic belief in future market liquidity, buyers will not stop acquiring information unless the composition of assets is good enough. Buyers’ information acquisition and the bad composition of assets for sale reinforce each other, preventing the market from recovering without outside intervention to clean the market.

This paper sheds light on the discussion of regulatory reforms to increase transparency in many asset markets. For example, Dodd-Frank Act Section 942 requires issuers of asset-backed securities (ABS) to provide asset-level information according to specified standards. These measures increase the precision of buyers’ idiosyncratic signals when they conduct due diligence. Although these measures can potentially discipline the ABS issuance process, I show that they have unintended consequence of increasing fragility in the secondary market. When buyers have access to more precise signals, they have a greater incentive to acquire information and provide quotes conditional on the signals. Therefore the cream-skimming
effect becomes stronger and the market is more susceptible to an information trap.

This paper also has important implications for the timing of the provision of asset purchase programs aiming to revive the market. During the latest financial crisis, the US Treasury created the Troubled Asset Relief Program (TARP), aimed at restoring a liquid market by purchasing “toxic” assets. I show that the fraction of “toxic” assets on the market is endogenous and depends on investors’ information acquisition in the past. As the market gets deeper into a crisis, the asset composition on the market becomes worse and policy makers need to purchase a larger amount of low-quality assets to revive the market.

The paper is organized as follows. I describe the model setup in Section 2. Section 3 focuses on the equilibrium analysis. The stationary equilibria are studied in Section 4. In Section 5 I explore the set of non-stationary equilibria that converges to different steady states. Policy implications are studied in Section 6. Section 7 concludes.

Related Literature

This paper builds on the large literature on adverse selection initiated by the seminal work of Akerlof (1970). Among many other papers, Janssen and Roy (2002); Camargo and Lester (2014); Chari, Shourideh and Zetlin-Jones (2014), and Fuchs and Skrzypacz (2015) analyze dynamic-adverse selection models with centralized or decentralized market structures. A common feature is that low-quality assets are sold faster than or at the same speed as high-quality assets. None of these papers feature resale considerations or buyers’ acquisition of information about assets’ quality.

Taylor (1999), Zhu (2012), Lauermann and Wolinsky (2016), and Kaya and Kim (2018) all considers dynamic adverse-selection models in which each buyer observes a noisy signal about an asset’s quality. A new result obtained in this strand of literature is that high-quality assets are traded faster than low-quality assets. This is related to the cream-skimming effect in my model when buyers acquire information. These papers consider a trading environment with a single seller and sequentially arriving buyers, and there is no scope for reselling the asset. In contrast, in my paper, buyers anticipate that they will sell their assets in the same market when they experience liquidity shocks.

In papers that study dynamic adverse-selection models with resale considerations—such as Chiu and Koeppl (2016) and Asriyan, Fuchs and Green (2018)—buyers’ valuation of an asset depends on future market liquidity. This gives rise to an intertemporal coordination problem which in turn yields multiple steady states with symmetric self-fulfilling transitions. Another closely related study is by Hellwig and Zhang (2012), who analyze a dynamic

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5 See also Hendel and Lizzeri (1999), Blouin (2003), Hörner and Vieille (2009), Moreno and Wooders (2010).
adverse-selection model with both resale consideration and endogenous information acquisition. While I allow buyers’ signals to be noisy, they focus on the situations in which the signals are precise. Therefore, information acquisition has no cream-skimming effect in their model and transitions between steady states are symmetric. In contrast to all of the above papers, my paper has the novel feature of generating multiple steady states with unidirectional transitions.

This paper is also related to the work by Daley and Green (2012, 2016), who study the role of a publicly observable “news” process in dynamic-adverse selection models. In my paper, buyers make their own decisions on whether to acquire information and the information is not observable to other market participants.

In terms of modeling search frictions, this paper builds on the theoretical papers on OTC markets. Examples are Duffie, Gărleanu and Pedersen (2005, 2007); Vayanos and Weill (2008); and Lagos and Rocheteau (2009). The trading environment is very similar to the investor’s life-cycle model in Vayanos and Wang (2007). I contribute to this literature by introducing asymmetric information about asset quality.

There is a large literature that studies information acquisition in financial markets, including Froot, Scharfstein and Stein (1992); Glode, Green and Lowery (2012); Fishman and Parker (2015); as well as Bolton, Santos and Scheinkman (2016). This literature shows that information acquisition can be a strategic complement and excess information acquisition in equilibrium leads to inefficiency. I differ from this line of research by studying information acquisition in a dynamic trading environment. This allows me to characterize transitions between different states of the market, such as episodes of market freezes or recovery.

Lastly, this paper contributes to the literature on the role of transparency and information acquisition in financial crises. Gorton and Ordonez (2014) study how a small shock to the collateral value can be amplified into a large financial crisis when it triggers information acquisition. In my model, a market freeze can arise as a self-fulfilling outcome. Also, I study a topic not addressed in their paper: whether a market can recover after a crisis. In terms of policy implications, this paper is related to the recent discussion of optimal disclosure of information by government and regulators, as in Alvarez and Barlevy (2015); Bouvard, Chaigneau and de Motta (2015); Gorton and Ordonez (2017); and Goldstein and Leitner (2018). A closely related study is that of Pagano and Volpin (2012), who also look at the welfare implications of increasing transparency in the securitization process. My work differs from the literature in that I argue that information disclosure does not directly reveal the value of an asset; instead, investors need to conduct due diligence to interpret the disclosed

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information. The noise in the interpretation of disclosed information reflects the complexity of the underlying assets, such as securitized products. Greater transparency reduces noise, but it can also exacerbate adverse selection in the market through the cream-skimming effect.

2 The Model

Time is continuous and infinite. There is a continuum of assets with mass 1. The quality of an asset is either high or low, denoted by \( j \in \{H, L\} \). The mass of high-quality and low-quality assets is fixed at \( \alpha/(1+\alpha) \) and \( 1/(1+\alpha) \) respectively, so the ratio of high-quality to low-quality assets is \( \alpha \). \( \alpha \) is an exogenous parameter that controls the average quality of the assets. Therefore I will refer to \( \alpha \) as the fundamental of the market.\(^7\)

The trading environment is populated with a continuum of investors. They are risk-neutral and discount time at rate \( r \). Each of them is restricted to hold either 0 or 1 unit of an asset. Their preference of holding assets can be either unshocked or shocked, reflecting that some investors experience liquidity shocks and become financially constrained. Whether an investor is shocked is observable or verifiable. When holding an asset of quality \( j \in \{H, L\} \), an unshocked investor enjoy a flow payoff \( rv_j \), while a shocked investor enjoy a flow payoff \( rc_j \). Throughout this paper, I maintain the assumption that \( v_H > c_H > v_L \geq c_L > 0 \). Thus, the shocked investors enjoy a lower flow payoff from holding both type of assets. Also, \( c_H > v_L \), meaning that the common value component dominates the private value component, which is necessary to generate the lemons problem.

Following Vayanos and Wang (2007), I consider a life-cycle model of OTC markets. At any time, there is an flow of unshocked investors without assets into the economy. They are naturally the buyers in the market. They have a one-time opportunity to trade with the shocked asset owners, who are the natural sellers in the market. After buying an asset, a buyer becomes an unshocked asset owner. Otherwise, if trade is unsuccessful, the buyer exits the market with zero payoff. Since an investor’s liquidity shock is observable, there will be no trade between a buyer and an unshocked asset owner.\(^8\) Therefore, unshocked asset owners only passively hold assets until their preferences change. These investors are labeled as holders. Holders face liquidity shocks which arrive at Poisson rate \( \delta \). Upon receiving a liquidity shock, a holder becomes a seller and offer her asset for sale in the market. For simplicity, I assume the inflow of buyers at any time equals to a constant \( \lambda \) times the mass of sellers on the market. These buyers are matched with sellers randomly. Therefore, from a

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\(^7\) I deviate from the conventional notation of using the fraction of high-quality assets to represent the average quality of the assets. The notation adopted here turns out to be convenient in the characterization of investors’ belief and the asset distribution.

\(^8\) This is a direct implication of the No-Trade Theorem in Milgrom and Stokey (1982).
seller’s perspective, buyers arrives with a constant Poisson rate $\lambda$. Sellers stay in the market until they sell the assets and exit the economy with zero payoff.

The flow of investors in the economy is summarized in Figure 1. Buyers enter the economy from the pool of outsider investors. When a seller sells her asset, she exit the economy and returns to the pool of outside investors. I use the word *market* to represent the two groups of active trader in the economy, the sellers and the buyers. From a buyer’s perspective, the severity of the adverse selection problem is determined by the composition of sellers with high-quality and low-quality assets. Notice sellers are a subset of asset owners who actively participate in the market. Therefore, the composition of assets among sellers can potentially differ from the fundamental of the market, which is asset composition among all asset owners. In this sense, the level of adverse selection in my model is endogenous and depends on the asset distribution. Later, I use the word *market composition* to represent the composition of high-quality and low-quality assets among the sellers.

![Figure 1: Flow Diagram of the Asset Market](image)

When a buyer meets a seller, the seller is privately informed of the quality of her asset. The buyer does not observe the quality of the seller’s asset, nor does she have information regarding the trading history of the seller. Her prior belief is determined by the market composition, i.e., the ratio of high-quality assets and low-quality assets among the sellers. In addition, the buyer can pay a fixed cost $k$ to acquire information and obtain a signal $\psi \in \{G, B\}$ of the asset’s quality. $G$ represents a good signal and $B$ represents a bad signal. The probability of observing a signal $\psi$ from an asset of quality $j$ is $f_j^\psi$. Signals obtained by different buyers are jointly independent conditional on the quality of the asset. This is to capture the opaque nature of the assets. Different buyers may have different evaluations of the same asset. Without loss of generality, I assume $f_H^G > f_L^G$, so a high-quality is more likely to generate a good signal than a low-quality asset. This implies that a good signal improves
the buyer’s posterior belief of the asset’s quality. The trading protocol is deliberately simple. The buyer makes a take-it-or-leave-it offer to the seller. All the activities within a match takes no time so the seller and buyer separate in the next instant after they meet.

3 Equilibrium Analysis

In this section I analyze investors’ optimal trading strategies and define the equilibrium. Since investors are infinitesimal, they take the continuation value of leaving the match as given. This allows me to separate the equilibrium analysis into three parts. First, I study a static trading game between a seller and a buyer, taking the continuation values as given. Second, I determine the continuation values of different agents. Lastly, I characterize the evolution of the asset distribution.

3.1 The Static Trading Game

The static trading game is played by one seller and one buyer. To define a static trading game, it is sufficient to specify the prior belief of the buyer and the terminal payoffs of both players when they separate. I denote the buyer’s prior belief by \( \theta(t) \), which equals to the probability that the seller carries a high-quality asset to the probability that the seller carries a low-quality asset. If \( \theta \) is small, there is a large fraction of low-quality assets on the market, and the adverse selection problem is severe. In equilibrium, \( \theta \) must be consistent with the asset distribution among sellers when the buyer meets the seller. If the seller sells the asset or the buyer does not buy the asset, they leaves the economy with zero continuation value. If the buyer buys an asset of quality \( j \), the continuation value is denoted by \( V_j(t) \), which is also the continuation value of a passive holder at time \( t \). If the seller keeps her asset which has quality \( q \), the continuation value is denoted by \( C_j(t) \). From now on, I omit the time argument of all variables when analyzing the static trading game. A static trading game is therefore defined by the combination of the buyer’s prior belief and the continuation values \( (\theta; V_H, C_H, V_L, C_L) \). For reasons that will become clear later, we only need to consider the case of \( V_H > C_H > V_L, C_L \).

The static game has two stages, information acquisition stage and the trading stage. We use backward induction to solve the static game. The seller’s optimal strategy takes a simple form. A seller with an asset of quality \( j \) is going to accept any price higher than the continuation value \( C_j \), and to reject any offer below \( C_j \). The buyer needs to decide whether to acquire information, and based on the belief after the information acquisition stage, what is the optimal price to offer. If the buyer acquires information, she will update her belief in
a Bayesian way. Her posterior belief of the asset’s quality after seeing signal $\psi \in \{G, B\}$ in the form of high-quality to low-quality ratio is

$$
\tilde{\theta}(\theta, \psi) = \frac{f_H^\psi \theta}{f_L^\psi}.
$$

(1)

If the buyer doesn’t acquire information, the posterior belief $\tilde{\theta}$ equals the prior belief $\theta$. For the consistency of the notations, let $\tilde{\theta}(\theta, N) = \theta$ represent the posterior belief if the buyer choose not to acquire information.

The following lemma characterized the optimal offering strategy of the buyer conditional on the posterior belief $\tilde{\theta}(\theta, \psi)$.

**Lemma 1** The buyer’s strategy is characterized by a threshold belief

$$
\hat{\theta} = \frac{C_H - \min \{C_L, V_L\}}{V_H - C_H}.
$$

1. If $\tilde{\theta}(\theta, \psi) > \hat{\theta}$, the buyer makes a pooling offer $C_H$,

2. If $\tilde{\theta}(\theta, \psi) < \hat{\theta}$ and $V_L > C_L$, the buyer makes a separating offer $C_L$,

3. If $\tilde{\theta}(\theta, \psi) < \hat{\theta}$ and $V_L < C_L$, the buyer makes a no-trade offer $p < C_L$.

If the buyer’s posterior belief $\tilde{\theta}(\theta, \psi)$ is above the threshold $\hat{\theta}$, the buyer should offer a pooling price $C_H$ to trade with both the high-quality and the low-quality seller. However, if the buyer’s posterior belief is not good enough, the optimal price to offer depends on the relationship between $V_L$ and $C_L$, or alternatively, whether there are gains from trade for a low-quality asset. If $V_L > C_L$, the buyer values a low-quality asset more than the seller, the buyer can offer a separating price $C_L$ that will only be accepted by a low-type seller. On the other hand, if $V_L < C_L$, the buyer values a low-quality asset less than the seller, it is optimal for the buyer to offer a no-trade price which is lower than a low-type seller’s continuation value to avoid buying the asset. In the knife-edge case of $\tilde{\theta}(\theta, \psi) = \hat{\theta}$, or $V_L = C_L$, the optimal offering strategy of the buyer can be a mixed strategy.

In the information acquisition stage, the buyer will compare the value of information, which is the increase in the expected payoff after the buyer observes the signal, to the cost of information acquisition. She will only acquire information about the asset when the net gain is positive. The signal is potentially valuable to the buyer because it gives the buyer the option value of making offers conditional on the signal. Depending on the prior belief, the buyer will either improve the offered price when seeing a good signal, or lower the offered price when seeing a bad signal.
Lemma 2 The value of information is

\[
W(\theta) = \begin{cases} 
\frac{1}{1+\theta} \max \{-\theta f_B^H(V_H - C_H) + f_B^B(C_H - \min \{C_L, V_L\}), 0\}, & \text{if } \theta \geq \hat{\theta}, \\
\frac{1}{1+\theta} \max \{\theta f_G^H(V_H - C_H) - f_G^G(C_H - \min \{C_L, V_L\}), 0\}, & \text{if } \theta < \hat{\theta}.
\end{cases}
\]

Figure 2: Value of information to the buyer

Figure 2 depicts the value of information as a function of the prior belief \( \theta \). Let \( W_{max} \) be the maximum value of information. If the prior belief \( \theta \) falls in the left or right end of the \([0, 1]\) interval, the value of information is zero. This is because the prior is too high (low) that even after observing a bad (good) signal, the posterior is still higher (lower) than the threshold belief. If the prior belief is around the threshold belief \( \hat{\theta} \), the value of information first increases linearly from 0, reaches the maximum at \( \hat{\theta} \), then decreases linearly to 0. The buyer will acquire information if and only if the value of information based on the prior belief is greater than the cost of acquiring information. The following lemma summarizes the buyer’s optimal strategy in information acquisition.

Lemma 3 If \( k < W_{max} \), the buyer will acquire information if and only if \( \theta^-(k) \leq \theta \leq \theta^+(k) \), where the two bounds are given by

\[
\theta^-(k) = \frac{f_B^G(C_H - \min \{C_L, V_L\}) + k}{f_H^G(V_H - C_H) - k}, \quad \theta^+(k) = \frac{f_B^B(C_H - \min \{C_L, V_L\}) - k}{f_H^B(V_H - C_H) + k}.
\]

Both \( \theta^-(k) \) and \( \theta^+(k) \) are decreasing in \( \min \{C_L, V_L\} \).

When the value of a low-quality asset decreases (\( \min \{C_L, V_L\} \) decreases), the loss of buying a low-quality asset at pooling price \( C_H \) is higher. Therefore, the buyer is more inclined
to avoid low-quality assets on the right boundary of the information sensitive region, and less willing to rely on the noisy signal on the left boundary. Therefore, the information sensitive region \([\theta^-(k), \theta^+(k)]\) moves to the right as both \(C_L\) and \(V_L\) decreases. As we will show later, \(C_L\) and \(V_L\) are determined by both the flow payoff from holding the asset and how likely a low-quality asset can be sold at the pooling price in the future. The above comparative statics are important because they are related to the resale consideration that links the current buyers’ information acquisition decision to future market liquidity. When the current market composition is relatively good (\(\theta\) on the right boundary of the information sensitive region), buyers are more willing to acquire information when their belief of future market liquidity deteriorates.

To conclude the analysis of the static trading game, I summarize the trading probability in the equilibrium the static trading game (for non-knife-edge case) when \(k < W_{max}\) in Table 1. When \(\theta\) falls on the boundary of the information region, the equilibrium is not unique. The buyer will use a mixed strategy of information acquisition. Thus, the set of trading probability is the convex combination of the set of trading probability of the adjacent regions.

<table>
<thead>
<tr>
<th>(V_L &lt; C_L)</th>
<th>(\theta &lt; \theta^-(k, \nu))</th>
<th>(\theta^-(k, \nu) &lt; \theta &lt; \theta^+(k, \nu))</th>
<th>(\theta &gt; \theta^+(k, \nu))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho_H = \rho_L = 0)</td>
<td>(\rho_H = f_H, \rho_L = f_L^C)</td>
<td>(\rho_H = \rho_L = 1)</td>
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<tr>
<td>(V_L = C_L)</td>
<td>(\rho_H = 0, \rho_L \in [0, 1])</td>
<td>(\rho_H = f_H, \rho_L \in [f_L^C, 1])</td>
<td>(\rho_H = \rho_L = 1)</td>
</tr>
<tr>
<td>(V_L &gt; C_L)</td>
<td>(\rho_H = 0, \rho_L = 1)</td>
<td>(\rho_H = f_H, \rho_L = 1)</td>
<td>(\rho_H = \rho_L = 1)</td>
</tr>
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Table 1: Trading probability when \(k < W_{max}\)

### 3.2 Continuation Values

First I introduce some notations that describe the investors’ strategy in the full dynamic game, allowing for both pure strategy and mixed strategy. I use \(\mu(p, j, t) \in [0, 1]\) to represent the probability of type \(j\) seller accepting offer \(p\) at time \(t\). The buyer’s strategy is more complicated and can be denoted by a couple of functions \(\{i(t), \sigma(p, \psi, t)\}\).\(^9\) \(i(t) \in [0, 1]\) is the probability that the buyer acquires information at time \(t\). \(\sigma(p, \psi, t)\) represents the probability of offering \(p\) in a match at time \(t\) when seeing signal \(\psi\). If a buyer does not acquire information, \(\psi = N\) following the previous notation. Therefore, \(\sigma(p, N, t)\) is the buyer’s probability of offering \(p\) in a match at time \(t\) conditional on not acquiring information.

\(^9\) Note that the strategy functions are independent of the identity of a buyer or a seller, this means that we will focus on equilibria with symmetric strategies. This is without loss of generality because for any equilibrium with asymmetric strategies, we can find a equilibrium in symmetric strategies with the same path of asset distributions, trading volume and average prices.
In principle, a buyer can draw a price from a mixed distribution. Fortunately, based on the analysis of the static trading game, the buyer will only choose from three relevant offers at any time.\textsuperscript{10} Thus it’s without loss of generality to assume $\sigma(\cdot, \psi, t)$ is a probability mass functions of $p$.

With the help of the above notations, we can write down $\gamma_j(p, t)$, the probability that a type $j$ seller is offered price $p$ conditional on meeting a buyer at time $t$.

$$
\gamma_j(p, t) = i(t) \sum_{\psi=G,B} f_j^\psi \sigma(p, \psi, t) + (1 - i(t))\sigma(p, N, t).
$$

(2)

$\gamma_j(p, t)$ characterizes the market condition faced by a type $j$ seller at time $t$. If $\gamma_j(p, t)$ has more weights on high prices of $p$, the market is more liquid for sellers with assets of quality $j$ because it’s easier for them to sell the assets at an high price.

The continuation value of sellers with high-quality assets is at least $c_H$ since the sellers can always hold on to their assets. Also, no buyer will offer a price higher than $c_H$ in equilibrium.\textsuperscript{11} Therefore

$$
C_H(t) = c_H.
$$

(3)

The previous analysis of the static trading game shows that only three types of prices will be offered by a buyer at time $t$, the pooling price $C_H(t) = c_H$, the separating price $C_L(t)$ or the no trade price $p < C_L(t)$. Getting an offer at the separating price or the no-trade price will not change the continuation value of the seller. Therefore, to compute the continuation value of a low-quality seller, it’s equivalent to consider the hypothetical case where the seller always hold on to the asset unless offered $c_H$. In fact, $\gamma_j(c_H, t)$ can be viewed as a proxy of endogenous market liquidity for owners of asset of quality $j$. This is especially important for investors with low-quality assets because it measures how likely they will be able to extract information rent in future meetings. Since the arrival rate of a pooling offer $c_H$ for a low type seller at time $\tau$ is $\lambda \gamma_L(c_H, \tau)$, for a low-quality seller remaining in the market at time $t$, the distribution function of the arrival time of an offer with pooling price $c_H$ is $1 - e^{-\lambda \int_t^\tau \gamma_L(c_H, u)du}$. A low-quality seller’s continuation value is characterized by\textsuperscript{12}

$$
C_L(t) = \int_t^\infty [(1 - e^{-r(\tau-t)})c_L + e^{-r(\tau-t)}c_H] d(1 - e^{-\lambda \int_t^\tau \gamma_L(c_H, u)du}).
$$

(4)

\textsuperscript{10} We can pick any $p < c_L$ to be the no trade price.
\textsuperscript{11} Otherwise the price of high-quality asset will be unbounded when $t$ goes to infinity
\textsuperscript{12} Equivalently, a low-quality seller’s continuation value can be characterized by a differential equation $rC_L(t) = rc_L + \lambda \gamma_L(c_H, t) (c_H - C_L(t)) + \frac{dC_L(t)}{d\tau}$. 

13
The seller enjoys flow payoff $rc_L$ before a pooling offer arrives and the value jumps to $c_H$ when the seller accept the offer. If $\gamma_L(c_H, \tau)$ improves for all future $\tau > t$, the low type sellers’ continuation value $C_L(t)$ increases.

Now let’s turn to the continuation value of a holder/buyer. A holder enjoy the flow payoff from the asset and mechanically becomes a seller when hit by a liquidity shock which arrives at Poisson rate $\delta$. The continuation value of a type-$j$ holder at time $t$ is

$$V_j(t) = \int_t^\infty \left[ (1 - e^{-r(\tau - t)})v_j + e^{-r(\tau - t)}C_j(\tau) \right] d(1 - e^{-\delta(\tau - t)}). \tag{5}$$

To derive the gains from trade at time $t$, we need to compare the continuation values of sellers and holders. Notice for the high type, $C_H(t) = c_H$,

$$V_H(t) = \frac{rv_H + \delta c_H}{r + \delta}. \tag{6}$$

As long as $\delta > 0$, $V_H(t) > C_H(t)$ holds at any time. There are always gains from trade for high-quality assets. However, the same result doesn’t necessarily hold for low-quality assets although $v_L \geq c_L$. Taking the difference between (5) and (4), we have

$$V_L(t) - C_L(t) = \int_t^\infty \left[ (1 - e^{-r(\tau - t)})v_L - C_L(\tau) \right] d(1 - e^{-\delta(\tau - t)}). \tag{7}$$

The first component of the integrand represents the holder’s extra benefit from the higher flow payoff. However, the positive gain is offset by the information rent of the low type seller, represented by the second component of the integrand. Notice $C_L(\tau) \leq \frac{rv_L + \lambda c_H}{r + \lambda} < c_H$.

When the low type seller is likely to be offered a pooling price $c_H$, i.e. $\gamma_L(c_H, u) > 0$, she can take advantage of the liquid market condition and extract information rent from the buyers. This benefit is not enjoyed by the holder. The buyer/holder has an advantage of holding the asset because of the higher flow payoff. However, they have a disadvantage in reselling the asset because their liquidity shock is observable. The fact that an asset holder seek to sell her asset on the market immediately reviews that she is holding a low-quality asset. Whether the gain from trade is positive or negative depends on the relative size of the

13 The continuation value of a type-$j$ holder can be equivalently characterized by a differential equation

$$rV_j(t) = rv_j + \delta (C_j(t) - V_j(t)) + \frac{dV_j(t)}{dt}. \tag{8}$$
two components. As the market condition becomes uniformly more liquid (higher $\gamma_j(c_H, u)$ for all $u > t$), the gains from trade decrease. Here I state the following assumption regarding the information structure of the signal.

**Assumption 1** $f^G_L > \frac{r+\lambda}{c_H - c_L}$.

Given Assumption 1, the gains from trade for low-quality assets could be positive, negative or zero depending on future market conditions denoted by $\gamma_L(c_H, t)$. A liquid market condition in the future (uniformly higher $\gamma_L(c_H, t)$) increases the low-quality seller’s incentive to remain in the market and wait for a pooling offer, therefore lower the gain from trade. Assumption 1 implies that if future buyers always acquire information, the gains from trade of a low-quality asset is negative. This result is formally stated in Lemma 4.

**Lemma 4** Given Assumption 1, $V_L(t) - C_L(t) < 0$ if $\gamma_L(c_H, \tau) \geq f^G_L$ for any $\tau > t$.

For Assumption 1 to hold, the value difference between the high type and low type asset can’t be too small ($v_L$ is relatively close to $c_L$ instead of $c_H$). Also, buyers’ signals must be inaccurate ($f^G_L > 0$) so when they acquire information, there is a large enough chance that they will offer a pooling price to a low-quality seller.

### 3.3 The Evolution of Asset Quality

The trading probability of each type of assets at any time can be constructed from the trading strategies. The probability that an asset of quality $j$ is traded in a match at time $t$ is

$$ \rho_j(t) = \sum_{\{p : \mu(p, j, t) > 0\}} \gamma_j(p, t) \mu(p, j, t). \quad (9) $$

The product $\gamma_a(p, t) \mu(p, a, t)$ represents the probability that a type $a$ asset is sold at price $p$ at time $t$. We sum the product over $p$ to get the trading probability.

Let $m^S_H(t)$ and $m^S_L(t)$ represent the masses of high-quality and low-quality assets held by sellers. Since high-quality and low-quality assets are in fixed supply of $\frac{\alpha}{1+\alpha}$ and $\frac{1}{1+\alpha}$ respectively, mass $\frac{\alpha}{1+\alpha} - m^S_H(t)$ of high-quality assets and mass $\frac{1}{1+\alpha} - m^S_L(t)$ of low-quality assets are held by holders. The evolution of asset distribution is fully characterized by the following differential equations,

$$ \dot{m}^S_H(t) = \delta \left( \frac{\alpha}{1+\alpha} - m^S_H(t) \right) - \lambda \rho_H(t) m^S_H(t), \quad (10) $$

$$ \dot{m}^S_L(t) = \delta \left( \frac{1}{1+\alpha} - m^S_L(t) \right) - \lambda \rho_L(t) m^S_L(t). \quad (11) $$
In each equation, the right-hand side consists of two terms. The first term represents the inflow of assets brought into the market by holders who just received liquidity shocks. The second term represents the outflow of assets because of trading. Since buyers are assigned to sellers randomly, buyers’ prior beliefs of the quality of their counter-parties’ assets must be consistent with the market composition of high-quality and low-quality assets. For this reason, we use the same notation \( \theta(t) \) to represent both the market composition and the buyers’ prior belief

\[
\theta(t) = \frac{m^S_H(t)}{m^S_L(t)}. \tag{12}
\]

Combining (10) and (11), we can characterize the evolution of the market composition as

\[
\frac{d}{dt} \ln \theta(t) = \frac{\delta}{m^S_H(t)} \frac{\alpha}{1 + \alpha} (1 - \theta(t)/\alpha) - \frac{\lambda(\rho_H(t) - \rho_L(t))}{\text{fundamental reversion}} - \frac{\lambda(\rho_H(t) - \rho_L(t))}{\text{trading probability differential}}. \tag{13}
\]

The evolution of asset distribution can be equivalently characterized by \( m^S_H(t) \) and \( \theta(t) \). The change in asset quality on the market can be decomposed into two effects. The first effect is the fundamental reversion. When \( \theta(t) < \alpha \), the composition of asset on the market is worse than the fundamental. Therefore, the inflow of assets because of liquidity shocks improves the quality of assets on the market. On the contrary, the inflow of assets worsens the quality of assets on the market when \( \theta(t) > \alpha \). Therefore, the market composition tends to reverse to the fundamental. This effect is stronger when the high-quality asset on the market is a smaller fraction of total stock of high-quality asset in the economy. The second term is the trading-probability differential. Most previous literature has focused the case when low-quality assets trade faster than high-quality asset in illiquid market. In those cases, \( \rho_H(t) \leq \rho_L(t) \) so the second effect is always weakly positive. In the analysis of the static trading game, we know that when \( \theta(t) \) falls in the information acquisition region and there’s negative gain from trade for low-quality assets, \( \rho_H(t) > \rho_L(t) \). Therefore, high-quality assets leave the market faster than low-quality assets, so the second effect is negative. The negative trading-probability differential effect generates novel implications on the set of steady states and market transitions in the dynamic equilibrium.

### 3.4 Equilibrium Definition

The equilibrium of the full dynamic game is defined as follows.\textsuperscript{14}

\textsuperscript{14}This definition makes use of some results in the previous analysis. A complete definition of equilibrium is given in the Appendix.
Definition 1 Given an initial asset distribution \( \{ \theta(0), m^S_H(0) \} \), an equilibrium consists of paths of asset distributions \( \{ \theta(t), m^S_H(t) \} \), buyer’s strategies \( \{ i(t), \sigma(p, \psi, t) \} \) and continuation value functions \( V_H(t), V_L(t) \), seller’s strategies \( \mu(p, a, t) \) and continuation value functions \( C_H(t), C_L(t) \) such that

1. For any time \( t \), given the continuation values \( V_L(t), V_H(t), C_L(t), C_H(t) \) and the prior belief \( \theta(t) \), a buyer’s strategy \( \{ i(t), \sigma(p, \psi, t) \} \) and a seller’s strategy \( \mu(p, a, t) \) form a sequential equilibrium of the static trading game.

2. The sellers’ continuation values \( C_H(t) \) and \( C_L(t) \) are given by (2), (3) and (4). The buyers’ continuation values \( V_H(t) \) and \( V_L(t) \) are given by (5).

3. The asset distributions \( m^S_H(t) \) and \( \theta(t) \) evolves according to (10) and (13).

4 Stationary Equilibria

In this section, we characterize the set of stationary equilibria of the dynamic trading game, ignoring the role of the initial asset distribution. A stationary equilibrium is an equilibrium in which the asset distribution and investors’ trading strategies remain fixed along the equilibrium path. These stationary equilibria are the steady states of the market in the long run. We mostly focus on the pure-strategy stationary equilibria, while leaving most of the analysis of mixed-strategy stationary equilibria in the Appendix. The stationary equilibria can be ranked in terms of the total welfare of the investors.

4.1 Construction of Stationary Equilibria

The set of stationary equilibria can be exhausted by guess-and-verify. We start by assuming a trading strategy for all investors and compute the continuation values \( \bar{V}_H, \bar{C}_H, \bar{V}_L, \bar{C}_L \). At the same time, we can compute the stationary asset distribution, especially the market composition \( \bar{\theta} \) and check if the assumed trading strategies are consistent with the static trading game \( (\bar{\theta}; \bar{V}_H, \bar{C}_H, \bar{V}_L, \bar{C}_L) \).

Let \( \bar{\rho}_H \) and \( \bar{\rho}_L \) be the trading probability of high-quality and low-quality assets in a match. The stationary market composition is

\[
\bar{\theta} = \frac{\delta + \lambda \bar{\rho}_L}{\delta + \lambda \bar{\rho}_H} \alpha.
\]

(14)

If high-quality assets are traded with higher probability in the stationary equilibrium, i.e., \( \bar{\rho}_H > \bar{\rho}_L \), the stationary market composition is worse than the fundamental \( \alpha \). On the
contrary, if low-quality assets are traded faster, the stationary market composition is better than the fundamental.

The analysis of the static trading game shows that along any equilibrium path, the continuation values of high-quality assets is fixed at $\bar{C}_H = c_H$ and $\bar{V}_H = \frac{r v_H + \delta c_H}{r + \delta}$, independent of the market conditions. Let $\bar{\gamma}_L(c_H)$ be the constant probability that a low type is offered the pooling price $c_H$ in any given match in a stationary equilibrium. The low-quality seller and buyer’ continuation values are

$$\bar{C}_L = \frac{r c_L + \lambda \bar{\gamma}_L(c_H)c_H}{r + \lambda \bar{\gamma}_L(c_H)}, \quad \bar{V}_L = \frac{r v_L + \delta \bar{C}_L}{r + \delta}. \quad (15)$$

If $\bar{\gamma}_L(c_H)$ is small in a stationary equilibrium, the market features lower liquid and the value of owning low-quality assets is low.

Depending on the strategy of the buyer, the pure strategy steady states can be put into three categories.

4.1.1 Information-Insensitive Pooling Stationary Equilibrium ($S_1$)

First consider the case that the buyers do not acquire information and always offer the pooling price $c_H$. Therefore, both high-quality and low-quality assets are traded at the same speed, $\bar{\rho}_{H,1} = \bar{\rho}_{L,1} = 1$, and the market composition $\bar{\theta}_1$ is the same as the fundamental $\alpha$. Since the low type sellers get pooling offer in each match, $\gamma_L(c_H) = 1$, the continuation values of the low type sellers and buyers are

$$\bar{C}_{L,1} = \frac{r c_L + \lambda c_H}{r + \lambda}, \quad \bar{V}_{L,1} = \frac{r v_L + \delta \bar{C}_{L,1}}{r + \delta}. \quad (16)$$

Notice Assumption 1 implies that $\bar{V}_{L,1} < \bar{C}_{L,1}$, so there’s no gains from trade between a buyer and a low type seller. We can check if offering pooling price without acquiring information is consistent with the market composition and the continuation values in the stationary equilibria. To simplify notations, we use $\theta_1^-(k)$ and $\theta_1^+(k)$ to represent the upper and lower bound of the information region if the continuation values equal to those in the stationary equilibria $S_1$.

$$\theta_1^-(k) = \theta^-(k, \bar{V}_{L,1}), \quad \theta_1^+(k) = \theta^+(k, \bar{V}_{L,1}).$$
Lemma 5 An information-insensitive pooling stationary equilibrium $S_1$ exists when

$$\alpha \geq \max \left\{ \frac{c_H - \bar{V}_{L,1}}{V_H - c_H}, \theta_1^+(k) \right\}.$$ 

Lemma 5 gives the sufficient and necessary conditions on the fundamental for the information-insensitive pooling stationary equilibria to exist. It imposes two lower bounds on the fundamental $\alpha$, whichever higher binds. If $k$ is large, buyers have no incentive to acquire information for any market composition. For buyers to offer pooling price, $\bar{\theta}_1$ must exceed the threshold for pooling offers. If $k$ is small, $\bar{\theta}_1$ must fall in the upper no information region. Notice the threshold $\theta_1^+(k)$ depends on the low type seller’s continuation value in the stationary equilibria.

$S_1$ is the stationary equilibria with highest market liquidity subject to search frictions. Both high type and low type assets are transferred to the high valuation investors (buyers) whenever a match is formed. Moreover, buyers do not spend resources on inspecting the assets. This resembles the market condition in many liquid OTC markets. Investors offer similar price for assets with the same credit ratings without spending resource to acquire private information regarding the quality of the assets. They do it for two reasons. First, lemons only account for a small fraction of the assets on sale and it’s unlikely to deteriorate because the fundamental of the market is strong. Second, expecting the market will still be liquid in the future, investors do not worry about obtaining a lemon because they know it could be sold immediately at a high price later in the market.

4.1.2 Information-Sensitive Stationary Equilibrium ($S_2$)

Now let’s consider a pure strategy stationary equilibria with information acquisition, i.e. $\hat{i} = 1$. From the analysis of the static trading game, we know that the pooling price is offered if and only if a good signal is observed. Therefore, the probability of a low type seller getting a pooling offer is $\bar{\gamma}_L(c_H) = f^G_L$. The continuation values of the low type sellers and buyers in $S_2$ are

$$\bar{C}_{L,2} = \frac{rc_L + \lambda f^G_L c_H}{r + \lambda f^G_L}, \quad \bar{V}_{L,2} = \frac{rv_L + \delta \bar{C}_{L,2}}{r + \delta}. \quad (16)$$

In $S_2$, low type sellers expect they will receive the offer $c_H$ with probability $f^G_L$ in a match at any time in the future. Assumption 1 implies that $\bar{C}_{L,2} > \bar{V}_{L,2}$, so there’s no gains from trade from low type. Buyers will offer the pooling price $c_H$ after seeing a good signal and
offer a no-trade price \( p < \bar{C}_{L,2} \) after seeing a bad signal. The probability that a asset is traded in a match is equal to the probability that a good signal is generated by the asset, so \( \bar{\rho}_{H,2} = f_{H}^{G}, \bar{\rho}_{L,2} = f_{L}^{G} \). Since the high-quality assets are traded faster, the stationary market composition is worse than the fundamental.

\[
\bar{\theta}_2 = \frac{\delta + \lambda f_{H}^{G}}{\delta + \lambda f_{L}^{G}} \cdot \alpha < \alpha. \tag{17}
\]

To check the assumed trading strategy is indeed an equilibrium, we need to verify if the stationary equilibria market composition falls in the information region given the continuation values. Let

\[
\theta^-_2(k) = \theta^-(k, \bar{V}_{L,2}), \quad \theta^+_2(k) = \theta^+(k, \bar{V}_{L,2})
\]

be the lower and upper bounds of the information region when the continuation values are equal to those in \( S_2 \), Lemma 6 gives the sufficient and necessary conditions for the pure strategy information-sensitive stationary equilibrium to exist.

**Lemma 6** Suppose Assumption 1 is true. An information-sensitive stationary equilibrium \( S_2 \) exists if and only if

\[
\frac{\delta + \lambda f_{H}^{G}}{\delta + \lambda f_{L}^{G}} \cdot \theta^-_2(k) \leq \alpha \leq \frac{\delta + \lambda f_{H}^{G}}{\delta + \lambda f_{L}^{G}} \cdot \theta^+_2(k).
\]

Lemma 6 put a lower bound and an upper bound on the fundamental. From the expressions for the information region in Lemma 3, we know the information region exists when \( k \) is small. Therefore, \( S_2 \) doesn’t exist when \( k \) is above a threshold value. In \( S_2 \), the market is less liquid than in the information-insensitive pooling stationary equilibrium \( S_1 \). Buyers are cautious about the composition of assets on the market and they always acquire information. As buyers rely on the inaccurate signal, high-quality sellers sometimes receive bad quote because their asset is taken as a lemon. In expectation, it takes longer for a high-quality seller to find an acceptable price in the market compared with the liquid stationary equilibrium \( S_1 \). As of the low-quality sellers, there is still a positive probability that they will receive a pooling offer since the buyers sometimes mistakenly think of the lemons as good assets. If the signal is noisy enough as in Assumption 1, the expected information rent received by a low-quality seller is higher than the difference in discounted flow payoff between a seller and a buyer. Therefore, low-quality sellers demand a high price that the sellers are not willing to offer unless a good signal is observed. As a result, low-quality sellers stay in the market
longer than high-quality sellers. The rent seeking behavior of low-quality sellers have two negative effects on the allocation efficiency in the market. The first effect is a direct effect. low-quality assets are not traded immediately when a buyer arrives even when the buyer has higher flow payoff of holding the asset. The second effect is indirect. As low-quality sellers stay longer in the market, the market composition remains below the fundamental, and therefore reduces buyers’ incentive to offer pooling prices.

For an intermediate range of fundamentals, the information-insensitive pooling stationary equilibrium $S_1$ and the information-sensitive stationary equilibrium $S_2$ co-exist.

**Proposition 1 (Coexistence of $S_1$ and $S_2$)** Suppose Assumption 1 is true. Let $A_1(k)$ and $A_2(k)$ be

$$A_1(k) = \max \left\{ \theta^+_1(k), \frac{\delta}{\delta + \lambda f^G_H} \theta^+_2(k) \right\}, \quad A_2(k) = \frac{\delta}{\delta + \lambda f^G_L} \theta^+_2(k).$$

$S_1$ and $S_2$ co-exist if and only if $\alpha \in [A_1(k), A_2(k)]$. When $k$ is small, $A_1(k) < A_2(k)$.

When agents hold the belief that the market will be liquid as in $S_1$ in the future, the value of a low-quality asset is high to both sellers and buyers. Buyers are willing to offer the pooling price without acquiring information for a wide range of market composition. Also, as buyers acquire assets without any selection, the market composition remains at the fundamental value. However, when agents believe the market will be partially liquid as in $S_2$, the value of a low-quality asset becomes lower. The upper no information region is smaller. At the same time, as buyers cherry-pick the market, the market composition stays below the fundamental. Both the trading effect and the valuation effect justify the buyers’ information acquisition behave.

### 4.2 Welfare Analysis

The total welfare along an equilibrium path is given by

$$\varepsilon = \frac{\alpha}{1 + \alpha} v_H + \frac{1}{1 + \alpha} v_L - \int_0^\infty e^{-rt} \left[ rm^S_H(t)(v_H - c_H) + rm^S_L(t)(v_L - c_L) + \lambda(m^S_H(t) + m^S_L(t))i(t)k \right] \, dt. \quad (18)$$

The first line of the right-hand side $\frac{\alpha}{1 + \alpha} v_H + \frac{1}{1 + \alpha} v_L$ represents the welfare in a frictionless benchmark. In the benchmark, assets can be moved from shocked investors to unshocked investors instantaneously. However, due to search frictions and information frictions, some assets are held by shocked investors in equilibrium. The first and the second term in the
The integrand of (18) represents the welfare loss because of market illiquidity. The third term represents the welfare loss from the resources devoted to information acquisition.

From (10) and (11) we can solve for the stationary asset distribution characterized by the mass of high-quality and low-quality assets held by sellers,

\[
\bar{m}_S^H = \frac{\delta \alpha}{(\delta + \lambda \bar{\rho}_H)(1 + \alpha)}, \quad \bar{m}_S^L = \frac{\delta}{(\delta + \lambda \bar{\rho}_L)(1 + \alpha)}.
\]

Using the trading probability and (19) for stationary asset distribution we can write down the welfare loss \( \Delta = \alpha v_H + (1 - \alpha)v_L - \varepsilon \) in each stationary equilibrium,

\[
\Delta_1 = \frac{\delta \alpha}{\delta + \lambda}(v_H - c_H) + \frac{\delta(1 - \alpha)}{\delta + \lambda}(v_L - c_L),
\]
\[
\Delta_2 = \frac{\delta \alpha}{\delta + \lambda f^G_H}(v_H - c_H + \frac{\lambda k}{r}) + \frac{\delta(1 - \alpha)}{\delta + \lambda f^G_L}(v_L - c_L + \frac{\lambda k}{r}).
\]

The welfare loss in \( S_1 \) is the lowest. As we previously pointed out, \( S_1 \) is the most efficient stationary equilibrium subject to the search frictions. The comparison between \( S_2 \) and \( S_3 \) is non-trivial. The mass of high type sellers is higher in \( S_3 \) and the mass of low type sellers is higher in \( S_2 \). Which one is higher depends on the relative size of \( v_H - c_H \) and \( v_L - c_L \). If the flow payoff difference for high type is relatively small compared to the payoff difference for low type and the information acquisition cost, the information-sensitive stationary equilibrium \( S_2 \) may be even less efficient than \( S_3 \), which is usually considered as an “illiquid” state of the market.

5 Non-Stationary Equilibria

In the previous section we investigate various states of the market in the long run. Now we turn to analyze how investors’ trading behavior and market liquidity evolves over time starting from a given initial asset distribution. Particularly, we are interested in the following question. When a liquid steady state and an illiquid steady state co-exist, is it possible for the market to transition from one to the other? In order to answer this question, it is important to study the set of non-stationary equilibria.

To show a certain equilibrium path from an initial asset distribution to a terminal stationary equilibria exists, we first make a guess of the investors’ trading strategies for any \( t > 0 \). Given the paths of trading probability \( \rho_H(t) \) and \( \rho_L(t) \) and the initial asset distribution represented by \( m_H^S(0) \) and \( m_L^S(0) \), the full path of the asset distribution can be
analytically solved from (10) and (11) as follows.

\[
m^S_H(t) = e^{-\int_0^t \delta + \lambda \rho_H(s)ds} m^S_H(0) + \frac{\delta \alpha}{1 + \alpha} \int_0^t e^{-(\delta + \lambda \rho_H(u)(t-s))du} ds,
\]

(20)

\[
m^S_L(t) = e^{-\int_0^t \delta + \lambda \rho_L(s)ds} m^S_L(0) + \frac{\delta}{1 + \alpha} \int_0^t e^{-(\delta + \lambda \rho_L(u)(t-s))du} ds.
\]

(21)

Then we can compute the paths of continuation values verify if the assumed trading strategies form an equilibrium of the static trading game at any \( t > 0 \).

In the Appendix, I provide sufficient conditions for the market composition \( \theta(t) \) to change monotonically along a non-stationary equilibrium path.

### 5.1 Self-fulfilling Market Freeze

Proposition 1 shows that the information-insensitive pooling stationary equilibrium \( S_1 \) and the information-sensitive stationary equilibrium \( S_2 \) coexist when the fundamental \( \alpha \) is within an intermediate region.

Suppose the market has an asset distribution as in the liquid state \( S_1 \), is it possible that all investors suddenly change their beliefs and coordinate to follow an equilibrium path that converges to the illiquid state \( S_2 \)? This question is answered in Proposition 2

**Proposition 2 (Self-fulfilling Market Freeze)** If Assumption 1 holds, for small \( k \), there exists

\[
A_3(k) = \theta_2^+(k) \in (A_1(k), A_2(k)),
\]

such that, for any \( \alpha \in [A_1(k), A_3(k)] \), starting from an initial asset distribution in the neighbourhood of \( S_1 \), there is an equilibrium path that converges to \( S_2 \).

When \( \alpha \in [A_1(k), A_3(k)] \), the model has multiple equilibria starting from the asset distribution of \( S_1 \). Proposition 2 implies that a liquid market can go through a self-fulfilling market freeze. Starting from the asset distribution in \( S_1 \), if all investors believe that future buyers will not acquire information and always offer the pooling price, the current buyers have no incentive to acquire information and they continue to offer the pooling price. The market therefore remains in the liquid steady state of \( S_1 \). However, if all investors believe the market liquidity will decline starting from the moment in a way that future buyers will acquire information, trying to avoid low-quality assets by making offers conditional on their independent evaluation, because of the resale consideration, the continuation value of holding low quality assets drops immediately. Thus, for the current buyers, the loss of buying
a low-quality asset at the pooling price becomes larger, and this gives them more incentive
to acquire information. When current buyers acquire information but their independent
evaluation of the assets are not accurate enough, high-quality assets are traded faster than
low-quality assets, resulting in a cream-skimming effect on the market composition. The
market composition deteriorates over time and justifies future buyers’ information acquisi-
tion. Therefore, the market evolves along a path with information acquisition and converges
to the information-sensitive steady state $S_2$.

Notice that Proposition 2 does not imply the information-insensitive pooling steady state
is unstable. In fact, the liquid steady state is locally stable.

**Proposition 3** If $\alpha, \theta(0) > \theta_1^+(k)$, there exists an equilibrium path with pooling offers and
no information acquisition that converges to $S_1$.

The results in Proposition 2 and 3 can be illustrated graphically. In Figure 3 and 4 I plot
the phase diagram of the evolution of asset distributions according to (10) and (13). The
horizontal axis represents the market composition which determines the current investors’
trading strategies. The vertical axis represents the mass of sellers with high-quality assets in
the market. It does not affect the current investors’ trading strategies directly, but it shapes
the evolution of the asset distribution through the interaction with market composition.
Recall that the evolution of the asset distribution depends on the trading probability of
different assets, which in turn depends on investors’ belief of future market liquidity through
resale considerations. Therefore, before we plot a phase diagram, we need to specify investor’s
continuation values according to their belief of future market liquidity.

Figure 3 shows the phase diagram when all investors believe future buyers do not ac-
quire information and always make pooling offers. Given this belief, the continuation values
are given by $\bar{V}_{L,1}$ and $\bar{C}_{L,1}$. The corresponding information-sensitive region is given by
$[\theta_1^-(k), \theta_1^+(k)]$, represented by the red region in the figure. If the fundamental $\alpha$ is above
$\theta_1^+(k)$, there exists an information-insensitive pooling steady state, represented by the sta-
tionary asset distribution $S_1$ on the right of the red region. If the investors keep their belief
about a liquid market in the future, the market will stay in $S_1$. Moreover, as Proposition
3 shows, starting from any asset distribution to the right of the red region, there is a path
converging to $S_1$. Along the path, the asset composition is always above $\theta_1^+$, consistent with
the investors’ belief of no information acquisition.

What happens when investors’ belief shifts? Suppose the market starts at $S_1$, and in-
vestors suddenly believe that investors in the future will acquire information and the market
will become illiquid. The phase diagram changes from Figure 3 to 4. The continuation values
of owning low-quality assets drop to $\bar{V}_{L,2}$ and $\bar{C}_{L,2}$, the same as in the information-sensitive
steady state. Since the continuation values become lower, the information-sensitive region moves to the right, represented by the blue region in Figure 4. The asset composition was good enough to support pooling trading in S₁ when investors believe in a liquid market in the future. However, after the shift in the investors’ belief, S₁ is now in the blue region of information acquisition, reflecting higher incentives to acquire information after the belief shift. The market will therefore follow the arrow and move to S₂. The whole path is within the blue region, meaning that buyers always acquire information along the path, consistent with investors’ belief of low liquidity in the future. The transition from S₁ to S₂ is consistent with an event of a self-fulfilling market freeze.

### 5.2 Information Trap

If the market starts from the asset distribution in the illiquid state S₂, is there a non-stationary equilibrium path that converges to liquid trading? The answer depends on the relationship between the market composition in S₂ and the information-sensitive region \([θ⁻₁(k), θ⁺₁(k)]\) in S₁. This can be illustrated in the same set of phase diagrams. In Figure 3 and Figure 4, the information acquisition regions in S₁ and S₂ overlap and the illiquid state S₂ falls in the overlapping region. Starting from the initial asset distribution in S₂, if all investors hold the belief that future buyers will acquire information, S₂ is in the highlighted information-sensitive region in Figure 4, consistent with the investors’ belief. Now suppose all investors believe that in the future, buyers will not acquire information and always offer the pooling price. This optimistic belief of future market liquidity improves the continuation values, changes the phase diagram to Figure 3 and shifts the information-sensitive region to \([θ⁻₁(k), θ⁺₁(l)]\). However, since S₂ is also in the highlighted information-sensitive region in Figure 3, current buyers will still acquire information and cream-skim the market. Their trading behavior keeps the asset distribution at S₂ and prevents the market from recovering to S₁. To summarize, if the market composition in S₂ satisfies \(\bar{θ}_2 < θ⁺₁(k)\), there is no equilibrium path that converges to the liquid state S₁.

Now let’s consider the opposite case if \(\bar{θ}_2 ≥ θ⁺₁(k)\). Starting from the initial asset distribution in S₂, when investors believe the market will be liquid in the future, the optimal strategy for a buyer is to stop acquiring information and offer the pooling price. As a result, the market composition will gradually improves and converges to \(θ_1\), the market composition in S₁. Along the path, buyers do not acquire information, which is consistent with investors’ belief. Therefore, if \(\bar{θ}_2 ≥ θ⁺₁(k)\), there exists a non-stationary equilibrium path that transitions from S₂ to S₁.

**Assumption 2** \(\frac{f_H^G f_H^P}{f_L^G f_L^P} > \frac{c_H - V_{L2}}{c_H - V_{L1}}\)
Figure 3: Phase Diagram, Assuming No Information Acquisition in the Future

Figure 4: Phase Diagram, Assuming Information Acquisition in the Future
Assumption 2 is equivalent to the condition $\theta_2^-(0) < \theta_1^+(0)$. If Assumption 2 is true, $\theta_2^-(k) < \theta_1^+(k)$ holds for small $k$ so the two information acquisition regions overlap. The intuitive interpretation of Assumption 2 is that it requires the signal to be relatively accurate so given any set of continuation values, information acquisition is optimal for a wide range of market composition. Otherwise, if the information available to be buyers is very noisy, information acquisition is irrelevant most of the time.\footnote{Assumption 2 is not in conflict with Assumption 1. Assumption 1 requires $f_G^L$ can not be too small so buyer has a large enough chance of thinking of a low-quality asset to be “good”. However, it does not put any restrictions on the probability distribution of the signals sent out observed from a high-quality asset. When $f_H^G$ gets closer to 1, the left-hand side of Assumption 2 goes to $\infty$.}

I call the overlapping region of the two information-sensitive region $[\theta_2^-(k), \theta_1^+(k)]$ the information trap whenever it exists. The information trap is different from the information sensitive regions we discussed previously. At any time $t$, the information sensitive region depends on the continuation values of owning low-quality assets $V_L(t), C_L(t)$. However, by definition, the information trap is time and strategy invariant so it is independent of investors’ belief and the continuation values. When the market composition is within the information trap, no matter whether investors believe the future buyers will acquire information or not, the optimal strategy is to acquire information today and the cream-skimming effect will be at play. Intuitively speaking, the market composition will be trapped in the region and dragged into the “sink”, which is the information-sensitive state $S_2$.\footnote{In Appendix D, I consider whether there exists a non-stationary equilibrium path that converges to the liquid state $S_1$, starting from an arbitrary initial market composition $\theta(0)$ in the information trap. I give the sufficient and necessary condition for such equilibrium path to exist.}

Proposition 4 formally gives the condition such that there is no non-stationary equilibrium path that transitions from $S_2$ to $S_1$.

**Proposition 4 (Information Trap)** If Assumption 1 and 2 hold, for small $k$, there exists

$$A_4(k) = \frac{\delta + \lambda f_H^G}{\delta + \lambda f_L^G} \cdot \theta_1^+(k) \in (A_1(k), A_2(k)), \quad (22)$$

such that, for any $\alpha \in [A_1(k), A_4(k)]$, if the initial asset distribution is in the neighbourhood of $S_2$, there is no equilibrium path converging to pooling trading.

Proposition 2 and 4 jointly implies that, for $\alpha \in [A_1(k), \min \{A_3(k), A_4(k)\}]$, the liquid steady state $S_1$ and the illiquid steady state $S_2$ coexist. More importantly, the transitions between the two steady states are asymmetric. Suppose the market is in the liquid state $S_1$ where buyers are not paying any attention to the idiosyncratic features of the assets. They simply buy assets at the pooling price from any seller they meet in the market. The market composition remains at a high level. A self-fulfilling market freeze starts from a market-wide
panic about a decline in future market liquidity. Investors worry that if they hold low-quality assets in the portfolio, in the future, it will be hard for them to sell these assets at good prices. Because of this concern, buyers start to collect information and carefully evaluate the assets they see on the market. They are only willing to offer a good price for an asset if the aspects of the asset satisfy their own criteria. However, because buyers’ evaluations of assets are not perfect, sellers who receive a bad quote will stay on the market with the hope that they will receive a high quote from the next buyer. The trading speeds of both type of assets drop immediately, and the value of low quality assets to the current owner decline. As the market goes further down the illiquid path, the market composition deteriorates gradually as low-quality assets accumulate in the market. At some point, the market composition becomes bad enough so it falls into the information trap. Even if buyers have optimistic beliefs of future market liquidity, since the current market composition is bad, they keep acquiring information to avoid buying low-quality assets at high prices. The low liquidity and the bad market composition reinforce each other through buyers’ information acquisition and the market can not recover to the liquid state.

6 Policy Implications

In this section we explore two policy implications of the model.

6.1 Issuance Transparency

Transparency in the issuance process of ABS was low before the latest financial crisis. The low issuance transparency has been criticized of generating moral hazard problems in the securitization process and adverse selection problems in the secondary market, which played important roles in the creation and propagation of the financial crisis. After the financial crisis, regulators have moved towards a more transparency issuance process. For example, Dodd-Frank Act Section 942 requires issuers of asset-backed securities (ABS) to provide asset-level information according to specified standards.\footnote{See \url{https://www.sec.gov/spotlight/dodd-frank-section.shtml#942}.} In the context of my model, these regulatory changes could lower the cost of information acquisition and increase the precision of buyers’ signals.

**Definition 2** A signal $\psi'$ is (weakly) more precise than a signal $\psi$ if and only if $f_H^{G'} \geq f_H^G$ and $f_L^{G'} \leq f_L^G$.\footnote{See \url{https://www.sec.gov/spotlight/dodd-frank-section.shtml#942}.}
We use two simple criteria to evaluate the effect of increasing transparency on the liquidity of the secondary market. First, we look at $\theta_1^+(k)$, since the liquid steady state $S_1$ exits if and only if $\alpha > \theta_1^+(k)$. Second, we consider $A_4(k)$. When $\alpha > A_4(k)$, there is no steady states in the information trap.

**Proposition 5** If both $\psi'$ and $\psi$ satisfy Assumption 1 and 2, and $\psi'$ is more precise than $\psi$, both $\theta_1^+(k)$ and $A_4(k)$ increases when switching from the signal structure $\psi$ to $\psi'$, and when $k$ decreases.

Proposition 5 implies that increasing transparency in the issuance process can harm market liquidity, judging by our simple criteria. The intuition is then when issuers provide more information regarding the pool of assets backing the ABS, future investors have better evaluation of the assets’ quality when they conduct due diligence. This gives buyers more incentive to acquire information, and when they do acquire information, the cream-skimming effect is stronger. It is worth mentioning that I only consider the impact of increasing transparency on the liquidity of the secondary market, while ignoring the impact on disciplining the issuance process. A complete evaluation of these type of policies should take both effects on the primary and the secondary market into consideration.

### 6.2 Asset Purchase Programs

When a market freezes because of the adverse selection problem, a natural solution is to clean the market by removing low-quality assets from the market. Many theoretical papers have studied the design of asset purchase programs in the presence of severe adverse selection, including Philippon and Skreta (2012), Tirole (2012), Camargo and Lester (2014) and Chiu and Koeppel (2016). During the latest financial crisis, the US Treasury created the Public-Private Investment Program (PPIP) to purchase “toxic” assets, aiming at restoring liquidity in the markets for legacy Commercial MBS and non-agency RMBS.

Asset purchase programs can help the target market restore liquid trading through two channels. First, it removes lemons from the market, so the fundamental of the market improves. Second, if the government purchase assets at a higher price than the market would offer, or selling assets to the government is easier than locating a buyer in the private sector, the asset purchase program effectively increases the value of lemons. As a result, the lemon’s problem is mitigated and buyers in the market are more willing to offer pooling prices.

In my model, when the market goes through a self-fulfilling market freeze from $S_1$ to $S_2$, the market composition deteriorates gradually and the mass of “toxic” assets increase over
time. In the proof of Proposition 2, I show that $\theta(t)$ decreases and $m^*(t)$ increases over time along the path of market freeze. There exists a time $\hat{t}$ such that $\theta(\hat{t}) = \theta_1^+(k)$. If the government intervene before $\hat{t}$, the market composition is above the information trap. There still exists an equilibrium path that converges to liquid trading. Therefore, market liquidity can be boosted by a plan that guarantees a floor-price for all assets. The government does not need to actually purchase assets from the market since the market will immediately return to liquid trading as buyers all stop to acquire information. However, after $\hat{t}$, the market enters the information trap and there is no self-fulfilling equilibrium path that returns to $S_1$. The government needs to purchase a positive amount of assets to revive the market.

Chiu and Koeppl (2016) studies the announcement effect of asset purchase programs. Specifically, when the government announce that it will purchase a given amount of lemons at a given price later at a given time, it is possible that the market will restore to liquid trading even before the government actually purchase these assets. It provides a justification for delay the purchase to lower the intervention cost when the market is not very important. However, a direct implication of Proposition 4 is that in an illiquid steady state within the information trap, any asset purchase program with purchasing price $p \in [\bar{C}_{L,2}, \bar{C}_{L,1}]$ does not involve an announcement effect.

7 Conclusions

In this paper, I present a model to study the interaction between buyers’ information acquisition and market liquidity in over-the-counter markets with adverse-selection problems. Buyers can acquire information to avoid buying low-quality assets, and their incentive to do so is strong if they expect the market will be illiquid when they resell their assets. When buyers’ signals are inaccurate, information acquisition has a cream-skimming effect on the composition of assets for sale and harms future market liquidity. The interaction of the resale consideration and the cream-skimming effect give rise to multiple steady states and asymmetric transitions between steady states. Specifically, the market can transition from a liquid state without information acquisition to an illiquid state with information acquisition, but can not transition back. This uni-directional transition between different steady states is a novel feature of my model and it is not present in the previous papers of dynamic adverse selection to the best of my knowledge. This result helps explain the continued dormancy of the non-agency residential mortgage-backed-security market in spite of the recovery of the US economy and the housing markets.
References


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Appendices

A Alternative Definition of Equilibrium

Here I provide a formal but less intuitive equilibrium definition which is equivalent to the definition provided in Section 3. I use $\sigma^I(p, \psi, t)$ with $\psi \in \{G, B\}$ to represent the offering strategy of an informed buyer and $\sigma^U(p, t)$ to represent the offering strategy of an uninformed buyer.

**Definition 3** A equilibrium consists of paths of market compositions $\theta(t), m^S_H(t), m^S_L(t)$, buyer’s policy functions $\{i(t), \sigma^I(p, \psi, t), \sigma^U(p, t)\}$, and value functions $V_H(t), V_L(t)$, seller’s policy function $\mu(p, a, t)$ and continuation value functions $C_H(t), C_L(t)$ and low types trading probability $\tau_L(t)$ such that

1. Seller’s optimality condition

$$\mu(p, a, t) = \begin{cases} 1, & \text{if } p > C_a(t), \\ [0, 1], & \text{if } p = C_a(t), \\ 0, & \text{if } p < C_a(t). \end{cases} \quad (A.1)$$

2. Buyer’s optimality condition,

$$i(t) = \begin{cases} 1, & \text{if } W(t) > k, \\ [0, 1], & \text{if } W(t) = k, \\ 0, & \text{if } W(t) < k. \end{cases} \quad (A.2)$$

If $i(t) > 0$, $\sigma^I(p, \psi, t) > 0$ only if $p$ solves

$$J^I(\psi, t) = \max_p \theta(t) f^\psi_H \mu(H, t, p)[V_H(t) - p] + f^\psi_L \mu(L, t, p)[V_L(t) - p],$$

if $i(t) < 1$, $\sigma^U(t, p) > 0$ only if $p$ solves

$$J^U(t) = \max_p \theta(t) \mu(H, t, p)[V_H(t) - p] + \mu(L, t, p)[V_L(t) - p].$$

The value of information $W(t)$ is

$$W(t) = \max \{E_\psi J^I(\psi, t) - J^U(t), 0\}.$$
The continuation values of sellers $C_a(t)$ are given by (2), (4) and (3). The continuation values of buyers(holders) $V_a(t)$ are given by (5).

3. The market composition, characterized by $m^S_H(t)$, $m^S_L(t)$ and $\theta(t)$ evolves according to (13)-(11).

B Other Stationary Equilibria

B.1 Pure-Strategy Equilibria

B.1.1 Information-Insensitive Separating Stationary Equilibrium ($S_3$)

When the stationary market composition falls in the left no-information region, the market is in an information-insensitive separating stationary equilibrium. This is the third and the last type of stationary equilibrium with pure strategies. In $S_3$, buyers do not acquire information and only offers the separating price. Therefore, the low-quality assets are traded with probability 1 in each match and the high-quality assets are never traded. $\bar{\rho}_{H,3} = 0$, $\bar{\rho}_{L,3} = 1$. The stationary equilibria market composition is better than the fundamental.

$$\bar{\theta}_3 = \frac{\delta + \lambda}{\delta} \cdot \alpha > \alpha.$$  \hfill (B.1)

Since the pooling price is never offered in equilibrium, the continuation values of low types are

$$\bar{C}_{L,3} = c_L, \quad \bar{V}_{L,3} = \frac{rv_L + \delta c_L}{r + \delta}.$$  

It’s easy to verify that $\bar{V}_{L,3} > \bar{C}_{L,3}$ so there are gains from trade for low-quality assets. Similarly, let

$$\theta^-_3(k) = \theta^-(k, \bar{C}_{L,3}), \quad \theta^+_3(k) = \theta^+(k, \bar{C}_{L,3})$$

be the lower and upper bounds of the information region when the continuation values are equal to those in $S_3$.

**Lemma 7** An information-insensitive separating stationary equilibrium $S_3$ exists if and only if

$$\alpha \leq \frac{\delta}{\delta + \lambda} \min \left\{ \frac{c_H - c_L}{V_H - c_H}, \theta^-_3(k) \right\}.$$
In $S_3$, all high-quality assets and a fraction of lemons are on the market. Yet, the fundamental of the market is so bad that the amount of lemons on the market is large enough to prevent any pooling offers or information acquisition from buyers. The continuation values of low type sellers and buyers are the lowest in all possible equilibria.

**B.2 Mixed-Strategy Equilibria**

Here we provide two useful results that restrict the set of possible mixed strategies in equilibrium.

**Lemma 8** In any equilibrium, if $i(t) > 0$, $\sigma(c_H, G, t) = 1$ and $\sigma(c_H, B, t) = 0$.

Lemma 8 applies to all equilibrium path. The proof is intuitive. Based on the analysis of the static trading game, it is clear that given any set of continuation values, buyers only choose between two price. Lemma 8 implies the buyer will offer the pooling price $c_H$ if and only if a good signal is observed. Without loss of generality, assume the buyer offers price $p_1$ after seeing signal $G$ and mix between $p_1$ and $p_2$ after seeing signal $B$. Since the buyer uses mixed strategy after seeing signal $B$, then the expected payoff from offering the two prices based on the posterior belief of seeing $B$ must be the same. Therefore, the expected payoff doesn’t change if the buyer offer $p_1$ with probability 1 after seeing $B$. This makes the buyer’s offer independent of the signal. Thus, the buyer can simply offer $p_1$ without information acquisition and save the fixed cost. This shows the sub-optimality of using mixed strategy after acquiring information.

Do sellers randomize in equilibrium? Obviously, low-quality asset sellers always accept the pooling price $c_H$. Also, sellers of high-quality assets always accept the pooling price in any equilibrium. If high-quality asset sellers accept price $c_H$ with a probability less than 1, the buyer can raise the offer by a tiny amount and increase the surplus by $V_H - C_H$ with a positive probability. For the similar reason, we must have $\bar{C}_L = \tilde{V}_L$ if low-quality sellers randomize when offered a separating price. In stationary equilibria, this implies that $\bar{\gamma}_L = \frac{\tilde{\gamma}}{\tilde{x}}(v_L - c_L)/(c_H - v_L)$. By Assumption 1, $\bar{\gamma}_L < f^G_L$. If we apply Lemma 8 to (2), we have

$$\bar{\gamma}_L = \tilde{i}f^G_L + (1 - \tilde{i})\tilde{\sigma}(c_H, N).$$

(B.2)

**Lemma 9** If Assumption 1 is true, in any stationary equilibria with low-quality seller using mixed strategy, we have $\tilde{i} < 1$ and $\tilde{\sigma}(c_H, N) < f^G_L$.
If buyer randomize between a separating offer and a no-trade offer, the gains from trade for low type must be zero, \( V_L = C_L \). We say two equilibria are equivalent when both types have the same trading probability and continuation values at any give time. Any equilibrium with buyer mixing between a separating offer and a no-trade offer can be equivalently expressed as an equilibrium with buyer only offering the separating price and the seller rejecting the offer with positive probability. This allows us to focus on mixed-strategy equilibria in which buyers only choose between the separating price and the pooling price.

**B.2.1 Mixed-Strategy Stationary Equilibrium without Information Acquisition**

Any mixed strategy stationary equilibrium without information acquisition must have buyers using mixed strategies. It is sufficient to consider buyers mixing between the pooling price \( C_H \) and the separating price \( C_L \). Notice in any equilibrium without information acquisition, the probability of buyer offering \( C_H \) is equal to \( \gamma_L \). When buyers do not acquire information, whether they offer the separating price or the no-trade price depends on the relationship between \( \bar{V}_L \) and \( \bar{C}_L \). Since in a stationary equilibrium, \( \bar{V}_L \) is a weighted average of \( v_L \) and \( \bar{C}_L \), it’s equivalent to compare \( \bar{C}_L \) and \( v_L \). There are three cases:

1. \((s_4) \) \( \bar{C}_L > V_L \). This is the case when buyers offer \( c_H \) with probability \( \bar{\gamma}_{L,4} \) and the no trade price with probability \( 1 - \bar{\gamma}_{L,4} \). In each match, either type of asset is traded with probability \( \bar{\gamma}_{L,4} \). This stationary equilibria exists when the following conditions are satisfied.

\[
\frac{c_H - \bar{V}_{L,4}}{V_H - c_H} < \alpha < \frac{c_H - v_L}{V_H - c_H}, \tag{B.3}
\]

\[
k \geq \frac{f_L^B - f_H^B}{V_H - c_H} \alpha. \tag{B.4}
\]

The market liquidity \( \bar{\gamma}_{L,4} \) is determined by \( \alpha = \frac{c_H - \bar{V}_{L,4}}{V_H - c_H} \) and (15).

2. \((s_5) \) \( \bar{C}_L < V_L \). In this stationary equilibrium buyers offer \( c_H \) with probability \( \bar{\gamma}_{L,5} \) and the separating price \( \bar{C}_{L,5} \) with probability \( 1 - \bar{\gamma}_{L,5} \). Low-quality sellers accept the separating offer for sure. In each match, a high-quality asset is traded with probability \( \bar{\gamma}_{L,5} \) and a low-quality asset is always traded. If this stationary equilibrium exists, \( (\alpha, k) \) must satisfy the following conditions given a market liquidity \( \bar{\gamma}_{L,5} \in (0, \frac{c_H}{V_H - c_H}) \):

\[
\frac{c_H - \bar{V}_{L,5}}{V_H - c_H} < \alpha < \frac{c_H - v_L}{V_H - c_H}, \tag{B.3}
\]

\[
k \geq \frac{f_L^B - f_H^B}{V_H - c_H} \alpha. \tag{B.4}
\]
\[ \bar{C}_{L,5} = \frac{rc_L + \lambda \bar{\gamma}_{L,6} c_H}{r + \lambda \bar{\gamma}_{L,5}} , \]
\[ \frac{c_H - \bar{C}_{L,5}}{\bar{V}_H - c_H} = \frac{\delta + \lambda}{\delta + \lambda \bar{\gamma}_{L,5}} \alpha , \]
\[ k \geq \frac{f^B_L - f^B_H}{\bar{V}_H - c_H} \cdot \frac{\delta + \lambda}{\delta + \lambda \bar{\gamma}_{L,5}} \alpha . \]

3. (s6) \( \bar{C}_L = V_L \). In this stationary equilibria, buyers offer \( c_H \) with probability \( \bar{\gamma}_{L,6} = \bar{\gamma}(v_L - c_L)/(c_H - v_L) \) and the separating price \( \bar{c}_{L,6} \) with probability \( 1 - \bar{\gamma}_{L,6} \). Low-quality sellers accept the separating offer with probability \( \bar{\mu}(v_L, l) \). For the stationary equilibria to exist, \((\alpha, k)\) must satisfy the following conditions

\[ \frac{\alpha}{\alpha + \frac{\delta + \lambda \bar{\gamma}_{L,6}}{\delta}(1 - \alpha)} < \frac{c_H - v_L}{v_H - v_L} < \frac{\alpha}{\alpha + \frac{\delta + \lambda \bar{\gamma}_{L,6}}{\delta}(1 - \alpha)} , \]
\[ k \geq \frac{f^B_L - f^B_H}{\bar{V}_H - c_H} \cdot \frac{c_H - v_L}{v_H - v_L} , \]

where \( \bar{\mu}(v_L, l) \) is the solution to

\[ \frac{c_H - v_L}{v_H - v_L} = \frac{\alpha}{\alpha + \frac{\delta + \lambda \bar{\gamma}_{L,6}}{\delta}(1 - \alpha)} . \] (B.5)

**B.2.2 Mixed strategy equilibrium with partial information acquisition**

Now let’s turn to the mixed strategy equilibria with \( \bar{i} \in (0, 1) \). In any equilibrium, buyers always offer \( c_H \) after observing a good signal.

1. (s7) First let’s consider the stationary equilibria equilibrium with \((\bar{\theta}, k)\) located on the right branch of the value of information function. Since \( \bar{\theta} > \hat{\theta} \), when buyers do not acquire information, they offer the pooling price. Therefore \( \bar{\gamma}_{L,7} = \bar{i}_7 f^g_L + 1 - \bar{i}_7 \). Notice \( \bar{\gamma}_{L,7} > f^g_L \). By assumption 1, it immediately implies that \( \bar{c}_{L,7} > v_L \), so there’s no gain from trade for low type. Buyers will offer a no-trade price after observing a bad signal. High-quality assets are traded with probability \( \bar{\rho}_{h,7} = \bar{i}_7 f^g_H + 1 - \bar{i}_7 \), while low-quality assets are traded with probability \( \bar{\rho}_{l,7} = \bar{i}_7 f^g_L + 1 - \bar{i}_7 \). The stationary equilibria market composition \( \bar{\theta}_7 \) is given by (14). If \( s_7 \) exists the following conditions must be satisfied,

\[ \bar{\theta}_7 \geq (c_H - \bar{v}_{L,7})/(v_H - \bar{v}_{L,7}) , \] (B.6)
\[ -\bar{\theta}_7 f^g_H (v_H - c_H) + (1 - \bar{\theta}_7) f^g_L (c_H - \bar{v}_{L,7}) = k . \] (B.7)
2. \((s_8)\) The next stationary equilibrium we investigate has \((\tilde{\theta}, k)\) located on the left branch of the value of information function. Since \(\tilde{\theta} < \hat{\theta}\), when buyers do not acquire information, they do not offer the pooling price without information acquisition. Therefore \(\bar{\gamma}_{L,8} = \tilde{i}_8 f_{L}^g\). High-quality assets are traded with probability \(\bar{\rho}_{h,8} = \tilde{i}_8 f_{H}^g\). The probability that a low type asset is traded depends on whether there’s gain from trade. Given different \(i_8\), there are three cases:

- If \(\tilde{i}_8 > \frac{r}{\lambda} (v_L - c_L) / (c_H - v_L)\), there’s negative gain from trade for low type, while low-quality assets are traded with probability \(\bar{\rho}_{l,8} = \gamma_{L,8}\).
- If \(\tilde{i}_8 < \frac{r}{\lambda} (v_L - c_L) / (c_H - v_L)\), there’s positive gain from trade for low type. High-quality assets are traded with probability \(\bar{\rho}_{h,8} = \tilde{i}_8 f_{H}^g\), while low-quality assets are traded with probability \(\bar{\rho}_{l,8} = 1\).
- If \(\tilde{i}_8 = \frac{r}{\lambda} (v_L - c_L) / (c_H - v_L)\), there’s zero gain from trade for low type. The low type seller can use mixed strategy when offered the separating price. High-quality assets are traded with probability \(\bar{\rho}_{h,8} = \tilde{i}_8 f_{H}^g\), while low-quality assets are traded with probability \(\bar{\rho}_{l,8} \in [\gamma_{L,8}, 1]\).

The continuation values of the low type are given by \((15)\). The stationary equilibria market composition \(\bar{\theta}_8\) is given by \((14)\). Let \(\tilde{\nu}_8 = \min \{\tilde{v}_{L,8}, \bar{c}_{L,8}\}\), the following conditions must be satisfied,

\[
\bar{\theta}_8 \leq \frac{c_H - \tilde{\nu}_8}{v_H - \tilde{\nu}_8}, \quad (B.8)
\]

\[
\tilde{i}_8 f_{H}^g (v_H - c_H) - (1 - \bar{\theta}_8) f_{L}^g (c_H - \tilde{\nu}_8) = k. \quad (B.9)
\]

3. \((s_9)\) The last stationary equilibrium features buyer’s partial information acquisition and mixed offering strategy when information is not acquired. Buyers acquire information with probability \(\tilde{i}_9\). In case the buyers do not acquire information, they offer the pooling price with probability \(\tilde{\sigma}_u(c_H)\). Therefore, the probability that a low-quality asset gets a pooling offer is \(\bar{\gamma}_{L,9} = \tilde{i}_9 f_{L}^g + \tilde{\sigma}_u(c_H)\). High-quality assets are traded with probability \(\bar{\rho}_{h,9} = \tilde{i}_9 f_{H}^g + \tilde{\sigma}_u(c_H)\). The probability that low type assets are traded depends on the gain from trade for low type. There are three cases depending on \(\bar{\gamma}_{L,9}\):

- If \(\bar{\gamma}_{L,9} > \frac{r}{\lambda} (v_L - c_L) / (c_H - v_L)\), there’s negative gain from trade for low type. Low-quality assets are traded with probability \(\bar{\rho}_{l,9} = \bar{\gamma}_{L,9}\).
- If \(\bar{\gamma}_{L,9} < \frac{r}{\lambda} (v_L - c_L) / (c_H - v_L)\), there’s positive gain from trade for low type. Low-quality assets are traded with probability \(\bar{\rho}_{l,9} = 1\).
• If \( \bar{\gamma}_{L,9} = \frac{1}{\lambda} (v_L - c_L) / (c_H - v_L) \), there’s zero gain from trade for low type. The low type seller can use mixed strategy when offered the separating price. Low-quality assets are traded with probability \( \bar{\rho}_{1,9} \in [\bar{\gamma}_{L,9}, 1] \).

The continuation values of the low type are given by (15). Let \( \bar{\nu}_9 = \min \{ \bar{v}_{L,9}, \bar{c}_{L,9} \} \).

The following conditions must be satisfied,

\[
\frac{\alpha}{\alpha + \frac{\delta + \lambda \bar{\rho}_{h,9}}{\delta + \lambda \bar{\rho}_{l,9}} (1 - \alpha)} = \frac{c_H - \bar{v}_9}{v_H - \bar{v}_9}, \quad (B.10)
\]

\[
k = \frac{f_L^h - f_H^h}{v_H - c_H} \cdot \frac{c_H - \bar{v}_9}{v_H - \bar{v}_9}. \quad (B.11)
\]

C Monotonicity of Paths of Market Composition

Define \( \bar{\rho}_{H,0} \) and \( \bar{\rho}_{L,0} \) as

\[
\bar{\rho}_{H0} = \frac{1}{\lambda} \left( \frac{\delta \alpha}{m_H^S(0)(1 + \alpha)} - \delta \right), \quad \bar{\rho}_{L0} = \frac{1}{\lambda} \left( \frac{\delta}{m_L^S(0)(1 + \alpha)} - \delta \right). \quad (C.1)
\]

Compared with (19), if the market is in a stationary equilibria at time 0, \( \bar{\rho}_{H0} \) and \( \bar{\rho}_{L0} \) are the trading probability of high-quality and low-quality assets in the initial stationary equilibria.

Higher \( \bar{\rho}_{H0} \) (\( \bar{\rho}_{L0} \)) corresponds to a lower fraction of high-quality (low-quality) assets in the market. Note that \( \bar{\rho}_{H0} > \bar{\rho}_{L0} \) if and only if \( \theta(0) < \alpha \), while \( \bar{\rho}_{H0} < \bar{\rho}_{L0} \) if and only if \( \theta(0) > \alpha \).

In the follow lemma, we give two scenarios in which the market composition \( \theta(t) \) converges monotonically to the new stationary equilibria level.

**Lemma 10** Assume \( \rho_H(t) = \bar{\rho}_H \) and \( \rho_L(t) = \bar{\rho}_L \),

1. \( \theta(t) \) is decreasing (increasing) in \( t \in (0, +\infty) \) if \( \bar{\rho}_{L0} \geq \bar{\rho}_{H0} \geq \bar{\rho}_L \geq \bar{\rho}_H \);

2. if \( \bar{\rho}_H = \bar{\rho}_L \), \( \theta(t) \) is decreasing (increasing) in \( t \in (0, +\infty) \) if and only if \( \bar{\rho}_{H0} \leq \bar{\rho}_{L0} \) (\( \bar{\rho}_{H0} \geq \bar{\rho}_{L0} \)).

D Non-Stationary Equilibria from the Information Trap

I show in the following proposition that, when the current market composition falls in this information region, it is hard for the market to recover to the liquid state \( S_1 \), even if such stationary equilibria exists.
Proposition 6 If \( \theta_2^-(k) \leq \theta(0) < \theta_1^+(k) \), there exists an equilibrium path that converges to pooling trading if and only if the dynamics of the asset distribution characterized by (10) and (11) with \( \rho_H(t) \equiv f_H^G \) and \( \rho_L(t) \equiv f_L^G \) satisfy \( \theta(t) = \theta_1^+(k) \) for some \( t \geq 0 \).

E Proofs

Proof of Lemma 1-3 (Solutions to the static trading game).

Case 1: No gains from trade for the low type, \( V_L < C_L \). The buyer has lower continuation value of the low-quality asset than the seller. Therefore, no trade will take place at any price lower than \( C_H \). The buyer will compare the payoff of offering the lowest pooling price and withdrawing from trading (or offering a price lower than \( V_L \)). The buyer finds it optimal to offer the pooling price \( C_H \) if and only if

\[
\tilde{\theta} V_H + (1 - \tilde{\theta}) V_L - C_H \geq 0.
\]

It can be written as

\[
\tilde{\theta} \geq \hat{\theta} = \frac{C_H - V_L}{V_H - V_L}.
\]  

(E.1)

where \( \hat{\theta} \) is the threshold belief.

If the prior belief \( \theta \geq \hat{\theta} \), the optimal strategy of a buyer without information is to offer the lowest pooling offer \( C_H \) and get the expected revenue \( \theta V_H + (1 - \theta) V_L - C_H \). However, when observing the signal, the buyer can make offers conditional on the signal. Specifically, if \( \theta \geq \hat{\theta} \) and \( \tilde{\theta}(\theta, B) \leq \hat{\theta} \), the buyer will offer pooling price \( C_H \) when observing \( G \) and withdraw from trade if observing \( B \). The expected revenue is \( \theta f_H^G (V_H - C_H) + (1 - \theta) f_L^G (V_L - C_H) \).

If \( \tilde{\theta}(\theta, B) > \hat{\theta} \), the buyer is willing to offer the pooling price \( C_H \) no matter what the signal is. The expected revenue is \( \theta V_H + (1 - \theta) V_L - C_H \), the same as if there’s no information. Therefore, the value of information for the buyer can be written in the form of an option value

\[
W(\theta) = \max \{-\theta f_H^B (V_H - C_H) + (1 - \theta) f_L^B (C_H - V_L), 0\}.
\]

The intuition is as following. For prior belief \( \theta \geq \hat{\theta} \), the signal allow the buyer to avoid loss \( C_H - V_L \) from trading with the low type with probability \( (1 - \theta) f_L^B \). However the signal can be “false negative” with probability \( \theta f_H^B \) and by making conditional offers the buyer loses the trade surplus \( V_H - C_H \) from trading with the high type.
On the other hand, if $\theta < \hat{\theta}$, there will be no trade for both types if there’s no information. Therefore, using the same reasoning as above, we find the value of information for the buyer is

$$W(\theta) = \max \left\{ \theta f^G_H (V_H - C_H) - (1 - \theta) f^G_L (C_H - V_L), 0 \right\}.$$  

After observing the signal, the buyer has the option to make conditional offers. Doing so, the buyer gains the surplus of trading with the high type with probability $\theta f^G_H$, but incurs a loss of trading with the low type with probability $(1 - \theta) f^G_L$. The buyer will make conditional offers only if the net gain is positive.

**Case 2: Non-negative gains from trade for the low type, $V_L \geq C_L$.** There’s a non-negative gain if the buyer offers a low price to only trade with the low type. Therefore, the buyer compares the expected gain from offering a pooling price with that from only trading with the low type. The buyer find it optimal to offer pooling price if and only if

$$\tilde{\theta} V_H + (1 - \tilde{\theta}) V_L - C_H \geq (1 - \tilde{\theta})(V_L - C_L),$$

which translates into

$$\tilde{\theta} \geq \hat{\theta} = \frac{C_H - C_L}{V_H - C_L}.$$  

If $\theta \geq \hat{\theta}$, the buyer will offer pooling price $C_H$ without information. By making conditional offers, the buyer can reduce the offered price to a low type from $C_H$ to $C_L$ with probability $(1 - \theta) f^B_L$, but with probability $\theta f^B_H$ she will lose the revenue $V_H - C_H$ from trading with a high type. The value of information to the buyer is

$$W(\theta) = \max \left\{ -\theta f^B_H (V_H - C_H) + (1 - \theta) f^B_L (C_H - C_L), 0 \right\}.  

If \theta < \hat{\theta}$, the buyer will only trade with the low type at price $C_L$ without information. By making conditional offers, the buyer can get revenue of $V_H - C_H$ with probability $\theta f^G_H$ from trading with the hight type, while paying extra amount of $C_H - C_L$ with probability $(1 - \theta) f^G_L$ when trading with the low type. The value of information to the buyer is therefore

$$W(\theta) = \max \{ \theta f^G_H (V_H - C_H) - (1 - \theta) f^G_L (C_H - C_L), 0 \}.$$  

**Proof of Proposition 1.** Since $f^G_L < f^G_H$ and $f^B_L > f^B_H$, the interval defined in Lemma
6 has positive measure for small $k$. Also, when $k$ is small, the condition for the existence of $S_1$ becomes

$$\frac{\alpha}{1 - \alpha} \geq \frac{f^B_L(c_H - \bar{V}_{L,1}) - k}{f^B_H(V_H - c_H) + k}.$$ 

Lemma 5 and 6 jointly imply that $S_1$ and $S_2$ coexist if and only if $\frac{\alpha}{1 - \alpha} \in [A_1(k), A_2(k)]$. To show the interval has positive measure for small $k$, it’s sufficient to show that

$$\frac{f^B_L(c_H - \bar{V}_{L,1}) - \delta}{f^B_H(V_H - c_H) + \delta} \geq \frac{f^B_L(c_H - \bar{V}_{L,2}) - \delta + \lambda f^G_L}{f^B_H(V_H - c_H) + \delta + \lambda f^G_L}.$$ 

In fact, the above inequality always holds since $\bar{V}_{L,1} > \bar{V}_{L,2}$ and $f^G_L > f^G_L$. 

**Proof of Lemma 10.** When $\rho_H(t)$ and $\rho_L(t)$ are constants, they can be further simplified as

$$m^S_H(t) = \frac{\delta \alpha}{\delta + \lambda \rho_H} + \left( m^S_H(0) - \frac{\delta \alpha}{\delta + \lambda \rho_H} \right) e^{-(\delta + \lambda \rho_H)t}, \quad (E.2)$$

$$m^S_L(t) = \frac{\delta(1 - \alpha)}{\delta + \lambda \rho_L} + \left( m^S_L(0) - \frac{\delta(1 - \alpha)}{\delta + \lambda \rho_L} \right) e^{-(\delta + \lambda \rho_L)t}. \quad (E.3)$$

Plugging in (E.2) and (E.3), we can show that the sign of $\frac{d\theta(t)}{dt}$ is the same as the sign of

$$\frac{(\delta + \lambda \bar{\rho}_{H0}) - (\delta + \lambda \bar{\rho}_H)}{1 + (\delta + \lambda \bar{\rho}_{H0}) e^{(\delta + \lambda \bar{\rho}_{H0})t - 1}} \times \frac{(\delta + \lambda \bar{\rho}_{L0}) - (\delta + \lambda \bar{\rho}_L)}{1 + (\delta + \lambda \bar{\rho}_{L0}) e^{(\delta + \lambda \bar{\rho}_{L0})t - 1}}. \quad (E.4)$$

Note that for any $t > 0$ the function $\frac{x - y}{1 + x e^{y(t-1)}}$ is strictly increasing in $x$ and strictly decreasing in $y$ for any $y \leq x$. Thus, if $\bar{\rho}_{L0} \geq \bar{\rho}_{H0} \geq \bar{\rho}_H \geq \bar{\rho}_L$ ($\bar{\rho}_{H0} \geq \bar{\rho}_{L0} \geq \bar{\rho}_L \geq \bar{\rho}_H$), (E.4) is non-positive (non-negative), which implies $\theta(t)$ is decreasing (increasing) in $t$. Similarly, if $\bar{\rho}_H = \bar{\rho}_L$, the sign of (E.4) is the same as the sign of $\bar{\rho}_{H0} - \bar{\rho}_{L0}$. Therefore, $\theta(t)$ is decreasing (increasing) in $t$ if and only if $\bar{\rho}_{H0} \leq \bar{\rho}_{L0}$ ($\bar{\rho}_{H0} \geq \bar{\rho}_{L0}$). 

**Proof of Proposition 2.** Notice

$$A_2(k) = \frac{\delta + \lambda f^G_L}{\delta + \lambda f^G_L} \cdot \theta^+(k, \bar{V}_{L,2}) = \frac{\delta + \lambda f^G_L}{\delta + \lambda f^G_L} \cdot A_3(k). \quad (E.5)$$

$A_1(k)$ is the maximum of two values. By Lemma 3 we know $\theta^+(k, \bar{V}_{L,2}) > \theta^+(k, \bar{V}_{L,1})$. To
monotonically to \( \bar{t} > 0 \). Lemma 10 implies that starting from the initial distribution close to \( \theta \) in the neighbourhood of \( \alpha \) offer the pooling price \( c \).

First, we prove a lemma that characterizes any equilibrium path.

**Proof of Proposition 6.** It can be shown easily by examining the functional form of \( \theta^*_t(k) \) and \( A_4(k) \).

**Proof of Proposition 5.** If \( \frac{\delta + \lambda f_H^G}{\delta + \lambda f_H^G} \cdot \frac{1}{1 - \alpha} \) \( A_3(k) = \theta^+(k, \bar{V}_{L,2}) \),

\[
\theta(0) = \frac{\alpha}{1 - \alpha} < A_3(k) = \theta^+(k, \bar{V}_{L,2}),
\]

\[
\theta(+\infty) = \frac{\delta + \lambda f_H^G}{\delta + \lambda f_H^G} \cdot \frac{\alpha}{1 - \alpha} \geq \frac{\delta + \lambda f_H^G}{\delta + \lambda f_H^G} \cdot A_1(k) \geq \bar{\theta}^- (k, \bar{V}_{L,2}).
\]

The whole path of \( \theta(t) \) lies within the information sensitive region. Since \( \bar{\theta} \) is the only sink in the information region, when starting from an initial distribution close to that of \( S_1 \), the path of \( \theta(t) \) also stays in the information sensitive region. Therefore, the second path is an equilibrium path converging to \( S_2 \).

**Proof of Proposition 5.** It can be shown easily by examining the functional form of \( \theta^*_t(k) \) and \( A_4(k) \).

**Proof of Proposition 6.** First, we prove a lemma that characterizes any equilibrium path that converges to pooling trading.

**Lemma 11** If \( \frac{\delta + \lambda f_H^G}{\delta + \lambda f_H^G} \cdot \frac{1}{1 - \alpha} \leq \theta(k, \bar{V}_{L,1}) \), along any equilibrium path that converges to pooling trading, \( \theta(t) \) must be weakly increasing whenever \( \theta(t) < \theta^+(k, \bar{V}_{L,1}) \).

**Proof of Lemma 11.** This can be proved by contradiction. Suppose there exist \( t_1 \) such that \( \hat{\theta}(t_1) < 0 \) and \( \hat{\theta}(t_1) < \theta^+(k, \bar{V}_{L,1}) \). By continuity of \( \theta(t) \), there exists \( t_3 > t_2 \geq t_1 \) such
that $\dot{\theta}(t_2) < 0$, $\theta(t) < \theta^+(k, \bar{V}_{L,1})$ for any $t_2 < t < t_3$ and $\theta(t) > \theta^+(k, \bar{V}_{L,1})$ for any $t > t_3$. Namely, $t_3$ is the last time that $\theta(t)$ enters the region $\theta \geq \theta^+(k, \bar{V}_{L,1})$ from the left. $\theta(t)$ moves to the left at $t_2$ and stays to the left of $\theta^+(k, \bar{V}_{L,1})$ for $t_2 < t < t_3$.

Since $\theta(t) > \theta^+(k, \bar{V}_{L,1})$ for any $t > t_3$, using backward induction, we can show that $C_L(t_3) > V_L(t_3) = \bar{V}_{L,1}$. For $t$ slightly less than $t_3$, $\theta^-(k, \bar{V}_{L,1}) < \theta(t) < \theta^+(k, \bar{V}_{L,1})$, therefore, buyers acquire information and only offers the pooling price when signal $G$ is observed. So $\rho_H(t) = f^G_H$ and $\rho_L(t) = f^G_L$. Since $\theta(t)$ crosses $\theta^+(k, \bar{V}_{L,1})$ from the left, for $t$ slightly less than $t_3$, we have

$$
\frac{d}{dt} \ln \theta(t) = \delta \cdot \frac{\alpha}{m^S_H(t)} \left(1 - \frac{1 - \alpha}{\alpha} \theta(t)\right) - \lambda (f^G_H - f^G_L) > 0, \quad (E.7)
$$

Taking the limit of $t$ to $t_3$, it yields

$$
\delta \cdot \frac{\alpha}{m^S_H(t_3)} \left(1 - \frac{1 - \alpha}{\alpha} \theta^+(k, \bar{V}_{L,1})\right) - \lambda (f^G_H - f^G_L) \geq 0. \quad (E.8)
$$

Evaluating the derivative of $\theta(t)$ at $t = t_2$, we have

$$
\delta \cdot \frac{\alpha}{m^S_H(t_2)} \left(1 - \frac{1 - \alpha}{\alpha} \theta(t_2)\right) - \lambda (\rho_H(t_2) - \rho_L(t_2)) < 0. \quad (E.9)
$$

By construction, $\theta(t_2) < \theta^+(k, \bar{V}_{L,1}) < \frac{\alpha}{1 - \alpha}$. Also notice $\rho_H(t_2) - \rho_L(t_2) < f^G_H - f^G_L$. Comparing (E.8) and (E.9), we have

$$
m^S_H(t_2) > m^S_H(t_3). \quad (E.10)
$$

On the other hand, since $\theta^+(k, \bar{V}_{L,1}) \geq \frac{\delta + \lambda f^G_H}{\delta + \lambda f^G_H} \frac{\alpha}{1 - \alpha}$, from (E.8) we know

$$
m^S_H(t_3) \leq \delta \alpha \frac{1 - \frac{1 - \alpha}{\alpha} \theta^+(k, \bar{V}_{L,1})}{\lambda (f^G_H - f^G_L)} \leq \frac{\delta \alpha}{\delta + \lambda f^G_H}.
$$

Rewrite (10),

$$
\frac{d}{dt} \left(m^S_H(t) - \frac{\delta \alpha}{\delta + \lambda f^G_H}\right) = -(\delta + \lambda f^G_H) \left(m^S_H(t) - \frac{\delta \alpha}{\delta + \lambda f^G_H}\right) - \lambda (\rho_H(t) - f^G_H)m^S_H(t).
$$

(E.11)
Since \( \theta(t) < \theta^+(k, \bar{V}_{L,1}) \) for \( t_2 < t < t_3 \), from Table 1 we know \( \rho_H(t) \leq f^G_H \). Therefore
\[
\frac{d}{dt} \left( m^S_H(t) - \frac{\delta \alpha}{\delta + \lambda f^G_H} \right) \geq -\left( \delta + \lambda f^G_H \right) \left( m^S_H(t) - \frac{\delta \alpha}{\delta + \lambda f^G_H} \right), \tag{E.12}
\]
or equivalently
\[
\frac{d}{d(-t)} \left( \frac{\delta \alpha}{\delta + \lambda f^G_H} - m^S_H(t) \right) \geq \left( \delta + \lambda f^G_H \right) \left( \frac{\delta \alpha}{\delta + \lambda f^G_H} - m^S_H(t) \right), \tag{E.13}
\]
Given \( m^S_H(t_3) \leq \frac{\delta \alpha}{\delta + \lambda f^G_H} \), (E.13) implies that \( m^S_H(t_2) \leq m^S_H(t_3) \). This is in contradiction with (E.10). Therefore, \( \theta(t) \) must be weakly increasing when \( \theta(t) < \theta^+(k, \bar{V}_{L,1}) \) along any equilibrium path that converges to pooling trading. \( \blacksquare \)

Now we can move on to prove the necessity of the given condition. Notice, if \( \frac{\delta + \lambda f^G_H}{\delta + \lambda f^G_H} \frac{\alpha}{1 - \alpha} > \theta(k, \bar{V}_{L,1}) \), the path with constant \( \rho_H(t) = f^G_H \) and \( \rho_L(t) = f^G_L \) converges to \( \frac{\delta + \lambda f^G_H}{\delta + \lambda f^G_H} \frac{\alpha}{1 - \alpha} > \theta(k, \bar{V}_{L,1}) \) in the end. On the other hand, if \( \frac{\delta + \lambda f^G_H}{\delta + \lambda f^G_H} \frac{\alpha}{1 - \alpha} \leq \theta(k, \bar{V}_{L,1}) \), Lemma 11 indicates that any path that starts from \( \theta(0) < \theta(k, \bar{V}_{L,1}) \) and converges to pooling trading only crosses \( \theta^+(k, \bar{V}_{L,1}) \) once. Again, let \( t_3 \) be the earliest time such that \( \theta(t_3) = \theta^+(k, \bar{V}_{L,1}) \). For any \( 0 \leq t < t_3 \), we must have \( \theta^-(k, \bar{V}_{L,2}) \leq \theta(0) \leq \theta(t) < \theta^+(k, \bar{V}_{L,1}) \). Using backward induction, it can be easily shown that \( \bar{V}_{L,1} < V_L(t) < \bar{V}_{L,2} \) for any \( 0 \leq t < t_3 \). Therefore, from the monotonicity of \( \theta^-(k, \cdot) \) and \( \theta^+(k, \cdot) \) we know that \( \theta^-(k, V_L(t)) < \theta^- (k, \bar{V}_{L,2}) < \theta(t) < \theta^+(k, \bar{V}_{L,1}) < \theta^+(k, V_L(t)) \) for any \( 0 \leq t < t_3 \). Also, Assumption 1 implies that \( V_L(t) < C_L(t) \) for any \( t \geq 0 \). Referring to Table 1, we know buyers acquire information with probability 1, \( \rho_H(t) = f^G_H \), \( \rho_L(t) = f^G_L \) for any \( 0 \leq t < t_3 \). This shows that if we fix \( \rho_H(t) = f^G_H \) and \( \rho_L(t) = f^G_L \) for any \( t \geq 0 \), we must have \( \theta(t_3) = \theta^+(k, \bar{V}_{L,1}) \).

Now we want to show the given condition is also sufficient. This is done by guess-and-verify. Let \( t_3 \) be the first positive value that satisfies \( \theta(t) = \theta(k, \bar{V}_{L,1}) \) in the hypothetical path with \( \rho_H(t) = f^G_H \) and \( \rho_L(t) = f^G_L \). Let \( i(t) = 1 \) and \( \sigma^I(c_H, G, t) = 1 \) for any \( t < t_3 \) and \( i = 0, \sigma^{II}(c_H, t) = 1 \) for any \( t \geq t_3 \). It is easy to construct an equilibrium path that’s consistent with the above offering strategy. \( \blacksquare \)

**Proof of Proposition 4.** Since \( \bar{V}_{L,1} > \bar{V}_{L,2} \), by Lemma 3, \( \theta^+(k, \bar{V}_{L,1}) < \theta^+(k, \bar{V}_{L,2}) \), therefore \( A_4(k) < A_2(k) \). Also, Assumption 2 implies that \( \theta^-(k, \bar{V}_{L,2}) < \theta^+(k, \bar{V}_{L,1}) \). It immediately follows that \( A_1(k) < A_4(k) \) for small \( k > 0 \). By Proposition 1, we know when \( k \) is small, for any \( \frac{\alpha}{1 - \alpha} \in (A_1(k), A_4(k)) \), \( S_1 \) and \( S_2 \) coexist. Moreover, the market composition in the information stationary equilibria \( S_2 \) satisfies
\[
\theta^-(k, \bar{V}_{L,2}) < \bar{\theta}_2 < \theta^+(k, \bar{V}_{L,1}).
\]
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Therefore, the asset distribution in $S_2$ falls in the information trap. By Proposition 6, when the asset distribution is in the neighbourhood of $S_2$, there’s no equilibrium path that converges to $S_1$. ■