

Economics 706 Preliminary Examination
August 2018

Do all questions, providing detail and discussion as appropriate. That is, don't just state "answers"; instead, derive and motivate and interpret answers insofar as possible. WRITE CAREFULLY AND CLEARLY. The five questions are equally weighted. Good luck!

Suppose that seasonally-adjusted U.S. quarterly real GDP growth, y_t , follows a covariance stationary $AR(2)$ process with weak white noise innovations.

1. Provide a detailed characterization of y_t via its Wold decomposition. Is it a complete characterization? Are the innovations associated with its Wold representation uncorrelated? Independent? Gaussian?
2. What is the unconditional innovation variance of y_t ? Must it be finite? What is the conditional innovation variance of y_t ? Is it necessarily smaller than the unconditional variance?
3. Provide a complete Bayesian analysis of the model using natural conjugate priors and a Markov Chain Monte Carlo posterior simulator.

Now suppose that y_t follows a covariance stationary $AR(2)$ process with conditionally-Gaussian $GARCH(1, 1)$ innovations.

4. Write down the full conditionally-Gaussian $AR(2) - GARCH(1, 1)$ process for y_t . What must be true of the AR and $GARCH$ parameters to ensure covariance stationarity? How would you modify the process to allow the response of volatility to depend on the signs of innovations? Write down the modified process. Why/when might such a modification be useful?

Now suppose instead that you don't *know* that y_t follows a covariance stationary $AR(2)$ process with conditionally-Gaussian $GARCH(1, 1)$ innovations, but you *think* that it might, so you fit the $AR(2) - GARCH(1, 1)$ model.

5. How would you diagnose the specification adequacy (as regards conditional mean dynamics, conditional variance dynamics, and conditional density) of your fitted $AR(2) - GARCH(1, 1)$ model?