Economics 702, Dirk Krueger

1 The Stochastic Neoclassical Growth Model with Preference Shocks

The social planner in the stochastic neoclassical growth model chooses stochastic consumption, labor and capital allocations $\{c_t, l_t, k_{t+1}\}$ to solve the following maximization problem

$$\begin{aligned} & \max E_0 \sum_{t=0}^{\infty} \beta^t \exp(z_t) U(c_t, l_t) \\ & \text{s.t.} \\ c_t + k_{t+1} &= k_t^{\alpha} l_t^{1-\alpha} \end{aligned}$$

with $\beta \in (0,1)$ and $\alpha \in (0,1)$ being parameters. The initial endowment of capital k_0 and the initial exogenous state z_0 is given. The prefence shock z_t follows a N-state Markov chain. Let $Z = \{z_1, z_2, \ldots z_N\}$ be the state space of the Markov chain, and let $\pi(z_{t+1}|z_t)$ denote the Markov transition matrix of the chain. For questions that involve sequential notation, use z^t to denote a complete history of preference shocks and $\pi_t(z^t)$ to denote its probability. The timing is such that all choices in the current period are taken after the current shock z_t is realized.

1. Let $N = 2$ for the rest of the question, and suppose the Markov transition function is of the form

$$\pi = \begin{pmatrix} 1-\nu & \nu \\ \xi & 1-\xi \end{pmatrix}$$

   For all $\nu, \xi \in [0,1]$ determine the set of stationary distributions.

2. Formulate the problem of the social planner recursively. State clearly what the state variables and control variables are.

3. Use the first order conditions and the envelope conditions to derive the Euler equation and the intratemporal optimality condition (in recursive form).

4. Suppose the utility function is of the form

$$U(c_t, l_t) = \log\left(c_t - \frac{(l_t)^{1+\frac{1}{\chi}}}{1+\frac{1}{\chi}}\right)$$

where $\chi > 0$ is a parameter. Can a *current* negative preference shock (a low z_t) cause a recession (a decline in *current* output)? You have give a formal argument (i.e. use equations) to justify your answer.

5. Now suppose that the utility function is of the form

$$U(c_t, l_t) = \log(c_t) - \frac{(l_t)^{1+\frac{1}{\chi}}}{1+\frac{1}{\chi}}$$

How do your answer to question 4 change? You have to justify your answer with a formal argument. For simplicity, assume that $\nu = \xi = 0.5$.

6. Assume that economy is populated by a unit mass of identical representative households. These households own and trade the physical capital stock as well as a full set of Arrow securities at prices $q_t(z^t, z_{t+1})$ Under the assumptions of the previous question, determine the prices of the Arrow securities in terms of the solution $(c_t(z^t), l_t(z^t))$ of the social planner problem. Is there trade in these Arrow securities in equilibrium? You have to justify your answer.