

Monetary Policy that Transmits Through Endogenous Productivity

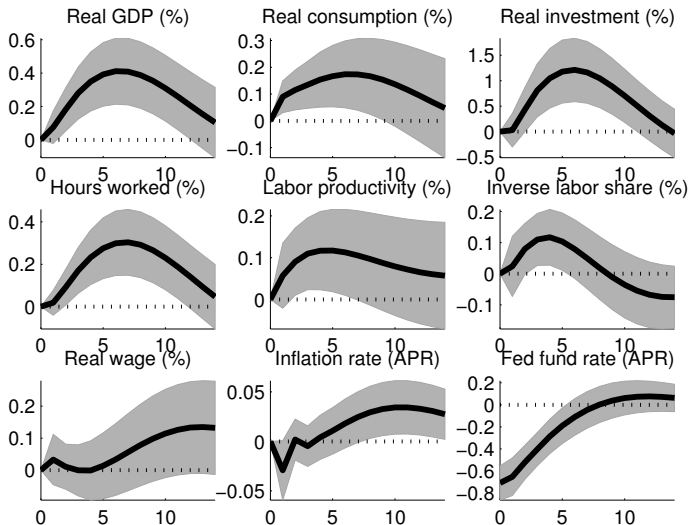
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University of Pennsylvania

- The prevailing NK-DSGEs typically use price and wage rigidity to get the real effects of monetary policy on the supply side:
 1. sticky price distorts firms' production, and
 2. sticky wage distorts households' labor supply.
- Without other channels, those models cannot match the cyclicalities of labor productivity and labor share in the data at the same time.
- We propose a channel through endogenous productivity via competitive search in goods market to fill in this gap.
- Our model is observationally identical to the standard NK-DSGE models in all other aspects, and can be estimated with the same dataset.
- The estimation result will tell us how important our proposed channel is.

- In prevailing NK-DSGEs with price rigidity, monetary shock transmits through $r \downarrow \implies Y \uparrow \implies w \uparrow \implies \textit{markup} \downarrow \implies \pi \uparrow$.
- However, in the data, $r \downarrow \implies \textit{inverse labor share} \uparrow$.
- Empirical defenders focus on why $\textit{markup} \neq \textit{inverse labor share}$, Empirical attackers find $\textit{markup} \uparrow$ after dealing with this issue.
- We propose a model in which $r \downarrow \implies \textit{TFP} \uparrow \ \& \ \textit{desired markup} \uparrow$ by assuming competitive search in goods market.
- Our model is observationally identical to the standard NK-DSGE models in all other aspects, and can be estimated with the same dataset.
- The estimation results shed light on an additional transmission channel of monetary policy (in progress).

Fed Funds Shock VAR(2) (1951q1–2008q4)



Puzzle: Conditional on expansionary monetary shocks, typically

- constant desired markup \Rightarrow procyclical inverse labor share, and
- exogenous TFP \Rightarrow procyclical labor productivity.

Solution: competitive search for varieties in goods market so that

- aggregate demand $\uparrow \Rightarrow$ aggregate tightness $\uparrow \Rightarrow$ TFP \uparrow , which is based on Huo and Ríos-Rull (2013), and
- tightness $\uparrow \Rightarrow$ incentive to cut price $\downarrow \Rightarrow$ desired markup \uparrow , which is new (our theoretical contribution).

Caveat: We cannot exclude the overhead labor story before dealing with this issue in the data for our regressions.

SHOPPING FRICTION IN A OFF-THE-SHELF MODEL

- We start with [shopping friction](#) + Galí (2015).
- The shopping friction is based on Huo and Ríos-Rull (2013), and extended by allowing for [competitive](#) search.
- "Search" allows for [endogenous TFP](#), while "competitive" allows for [non-constant desired markup](#).
- For clearance, we define the model recursively.
- For tractability, we replace Calvo pricing with Rotemberg.
- For simplification, we do not assume wage stickiness. This disables the model to generate countercyclical labor share. We will deal with this issue in the fully quantitative model.

THE ENVIRONMENT

- Shopping friction on top of **Dixit-Stiglitz**.
- Households have to find **varieties** in order to be able to consume them. They exert **search efforts**, but never find all available varieties.
- Firms' **locations** (specified later) have to be matched before producing.
- Households' search efforts and firm's locations look for each other, and meet according to a matching function.
- A competitive search protocol determines the coordination of firms and households via **submarkets** indexed by price and tightness $\{p, q\}$.
- Each household can send shoppers to multiple submarkets while each firm can only go to one of them.
- An **increase in consumption** is implemented via an increase in the **number of varieties** and the **amount consumed** of each variety.
- A **change in price** is implemented via switching to a different submarket that has a different market **tightness** and **demand** for each variety.

THE ENVIRONMENT

- There is a measure one of varieties of goods.
- Each variety is produced by one firm.
- Each firm has a measure one of homogeneous productive locations.
- Each location has to be maintained with preinstalled inputs (labor).
- An active submarket has many firms in it.
- When a firm goes to one submarket, it moves all locations to it.
- Switching to a different submarket incurs adjustment costs.
- Firms also sell goods in a centralized market where only other firms can go (to alleviate adjustment costs, discussed later).
- A central bank pursues its monetary policy as in a standard NK model.

MATCHING FUNCTIONS

- The tightness q of a submarket is its measure of search effort $D(p, q)$ relative to its measure of firms or varieties $J(p, q)$, i.e. $q = \frac{D(p, q)}{J(p, q)}$.
- The number of matches in a submarket depends on a matching function $\psi(D(p, q), J(p, q))$ with constant return to scale.
- For a firm the probability that any of its locations is matched to a shopper is $\psi^f(q) = \psi(q, 1)$.
- For a unit of search effort the probability that it finds a variety is $\psi^h(q) = \psi(1, q^{-1}) = \frac{\psi(q, 1)}{q} = \frac{\psi^f(q)}{q}$.
- Households do not find twice the same variety.

HOUSEHOLDS

- Measure one of ∞ lived hhs that shop in set Φ of available submarkets.
- The representative household chooses
 - $c(p, q)$: goods purchased from each variety it finds in $\{p, q\} \in \Phi$,
 - $d(p, q)$: total shopping effort allocated to $\{p, q\} \in \Phi$,
 - ℓ : labor,
 - b' : one-period zero-coupon bond.
- A household finds $d(p, q)\psi^h(q)$ varieties in submarket $\{p, q\}$.
- Utility function $u(c^A, d^A, \ell)$ where

$$c^A \equiv \left(\int d(p, q)\psi^h(q)c(p, q)^{\frac{\varepsilon-1}{\varepsilon}} dpdq \right)^{\frac{\varepsilon}{\varepsilon-1}}, \text{ with } \varepsilon > 1,$$

$$d^A \equiv \int d(p, q) dpdq.$$

- The discount factor is β .

STATES AND EQUILIBRIUM OBJECTS

- The Aggregate state $S = \{\theta, p_{-}\}$ consists of shocks and the previous period price level.
- The individual state also includes b , nominal bonds of which there are zero in net supply.
- We are looking for a few equilibrium objects
 - $\Phi(S)$: The set of submarkets with economic activity.
 - $\Pi^f(S)$: Profits from all firms .
 - $W(S)$: nominal wage rate,
 - $R(S)$: one-period forward nominal gross interest rate,
 - $H(S, \theta')$: law of motion for aggregate states.
- In particular, we will **guess and verify** that all economic activity takes place in a **unique market** $\Phi(S) = \{p(S), q(S)\}$.

POSING THE HOUSEHOLD'S PROBLEM

- We pose the problem of the household as if many submarkets are available and characterize the properties that have to be satisfied for a household to be willing to go to various submarkets.
- This gives a well defined problem to the firm of which submarket to go to in a similar vein as what is done to solve for the monopolistically competitive price in the standard model.
- Let the value function of a household be $v(S, b)$. We pose its problem conditional on an arbitrary set Φ of available markets.

HOUSEHOLD'S PROBLEM GIVEN AVAILABLE SUBMARKETS Φ

$$\begin{aligned} V(S, b, \Phi) &= \max_{c(p,q), d(p,q), \ell, b'} \{u(c^A, d^A, \ell) + \beta \mathbb{E}[v(S', b')|S]\}, \\ \text{s.t. } e + \frac{b'}{R(S)} &= W(S)\ell + b + \Pi^f(S), \\ S' &= H(S, \theta'), \\ c^A &= \left(\int_{\Phi} d(p, q) \psi^h(q) c(p, q)^{\frac{\epsilon-1}{\epsilon}} dpdq \right)^{\frac{\epsilon}{\epsilon-1}}, \\ d^A &= \int_{\Phi} d(p, q) dpdq, \\ e &= \int_{\Phi} d(p, q) \psi^h(q) p c(p, q) dpdq. \end{aligned}$$

FOC'S GIVEN AVAILABLE SUBMARKETS Φ

The solution is a set of decision rules $\{c(p, q), d(p, q), \ell, c^A, d^A, b'\}$ on (S, b, Φ) satisfying first order conditions (F.O.C.s)

$$0 = \left(\frac{c(p, q)}{c^A} \right)^{-\frac{1}{\varepsilon}} u_{c^A} - \lambda p, \quad \forall \{p, q\} \in \Phi$$

$$0 = \frac{\varepsilon}{\varepsilon - 1} \left(\frac{c(p, q)}{c^A} \right)^{-\frac{1}{\varepsilon}} u_{c^A} + \frac{u_{d^A}}{\psi^h(q)c(p, q)} - \lambda p, \quad \forall \{p, q\} \in \Phi$$

$$0 = u_\ell + \lambda W(S),$$

$$0 = \mathbb{E}[v_b(S', b')|S] - \frac{\lambda}{R(S)}.$$

WHICH SUBMARKET TO GO

The F.O.C.s enable us to derive the following conditions:

- Given $\{\bar{p}, \bar{q}\} \in \Phi$, households would go to submarket $\{p, q\}$ only if

$$\frac{\psi^h(q)}{\psi^h(\bar{q})} = \left(\frac{p}{\bar{p}}\right)^{\varepsilon-1}.$$

The solution for q is denoted as $q^h(S, b, \{\bar{p}, \bar{q}\}, p)$.

- $c(p, q)$ must satisfy

$$\frac{c(p, q)}{c(\bar{p}, \bar{q})} = \left(\frac{p}{\bar{p}}\right)^{-\varepsilon}.$$

The solution for $c(p, q)$ is denoted as $c(S, b, \{\bar{p}, \bar{q}\}, p, q)$.

When $\Phi = \{\bar{p}, \bar{q}\}$, the FOCs collapse to

$$\begin{aligned}0 &= \iota^{\frac{1}{\varepsilon-1}} u_{c^A} - \lambda \bar{p}, \\0 &= \frac{\varepsilon}{\varepsilon-1} \iota^{\frac{1}{\varepsilon-1}} u_{c^A} + \frac{u_{d^A}}{\psi^h(\bar{q})c(\bar{p}, \bar{q}, \bar{p}, \bar{q})} - \lambda \bar{p}, \\0 &= u_\ell + \lambda W(S), \\0 &= \mathbb{E}[v_b(S', b')|S] - \frac{\lambda}{R(S)}.\end{aligned}$$

where $\iota = d^A \psi^h(\bar{q})$ is the measure of varieties that households find and therefore $c^A = \iota^{\frac{\varepsilon}{\varepsilon-1}} c(\bar{p}, \bar{q}, \bar{p}, \bar{q})$. $\varepsilon > 1$ implies households like varieties.

DEGENERATE FUNCTIONS

The degenerate submarket yields decisions as functions of state (S, b) , submarket $\{\bar{p}, \bar{q}\}$, and the possibility of a different $\{p, q\}$ being open

- individual shopping effort aggregator: $d^A(S, b, \{\bar{p}, \bar{q}\})$,
- individual total shopping effort $d(S, b, \{\bar{p}, \bar{q}\}) = d^A(S, b, \{\bar{p}, \bar{q}\})$,
- individual total measure of varieties:
 $\iota(S, b, \{\bar{p}, \bar{q}\}) = d^A(S, b, \{\bar{p}, \bar{q}\}) \psi^h(\bar{q})$,
- individual consumption of each variety it finds: $c(S, b, \{\bar{p}, \bar{q}\})$
- individual consumption aggregator:
 $c^A(S, b, \{\bar{p}, \bar{q}\}) = \iota(S, b, \{\bar{p}, \bar{q}\})^{\frac{\varepsilon}{\varepsilon-1}} c(S, b, \{\bar{p}, \bar{q}\})$,
- the tightness of another submarket $\{p, q\}$ that a household would be willing to go: $q^h(S, b, \{\bar{p}, \bar{q}\}, p)$,
- The consumption of each variety a household finds in a different submarket $\{p, q^h(S, b, \{\bar{p}, \bar{q}\}, p)\}$: $c[S, b, \{\bar{p}, \bar{q}\}, p, q^h(S, b, \{\bar{p}, \bar{q}\}, p)]$.

Imposing Representative Agent ($b = 0$) gives us the aggregate counterparts

- $D(S, \{\bar{p}, \bar{q}\}) = D^A(S, \{\bar{p}, \bar{q}\})$
- $\mathcal{I}(S, \{\bar{p}, \bar{q}\}) = D^A(S, \{\bar{p}, \bar{q}\}) \psi^h(\bar{q})$
- $C(S, \{\bar{p}, \bar{q}\})$
- $C^A(S, \{\bar{p}, \bar{q}\}) = \mathcal{I}(S, \{\bar{p}, \bar{q}\})^{\frac{\varepsilon}{\varepsilon-1}} C(S, \{\bar{p}, \bar{q}\})$
- $Q^h(S, \{\bar{p}, \bar{q}\}, p)$
- $C[S, \{\bar{p}, \bar{q}\}, p, Q^h(S, \{\bar{p}, \bar{q}\}, p)]$

The last two objects are taken as given by the firms when they choose which submarket to go.

FIRMS

- There is a measure one of ∞ lived firms. Each of them has a measure one of homogeneous locations, and produces a differentiated variety j .
- Firm j chooses one and the only submarket $\{p, q\}$ for all its locations and allocates workers $n(p, q)$ to each of its locations before producing.
- Workers are hired in a centralized market at nominal wage rate W , and produce (if matched) $y(p, q)$ in each location using technology

$$y^j(p, q) = \theta^a n^j(p, q),$$

where θ^a is the technology level assumed to be common across all firms.

- Firm j also produces its variety for the intermediate goods market with no search frictions where only other firms can go.
- The intermediate goods are used to alleviate adjustment costs.
- They have to be sold at the **same price** as to the households.
- The aggregate demand for each variety j (specified later) is $X(S, p^j)$.

ADJUSTMENT COSTS

- Firms incur in price adjustment costs in units of a goods aggregate.
- Firms buy all varieties without frictions to pay the adjustment costs.
- Denote by x^j the required adjustment costs for firm j and let x^{ji} be the amount of variety i that firms j purchase to deal with its adjustment costs. The aggregator is

$$x^j = \left(\int_0^1 (x^{ji})^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

- Adjustment costs, to be specified later, are $x^j = \chi(S, p_-^j, p^j)$.

- A firm j that goes to submarket $(p, Q^h(S, \{\bar{p}, \bar{q}\}, p))$ has profits

$$\begin{aligned}\pi^j(S, \{\bar{p}, \bar{q}\}, p) = & p\psi^f [Q^h(S, \{\bar{p}, \bar{q}\}, p)] C [S, \{\bar{p}, \bar{q}\}, p, Q^h(S, \{\bar{p}, \bar{q}\}, p)] \\ & + pX(S, p) - W(S)n - \chi(S, p_-, p),\end{aligned}$$

taking some demand functions $C(S, \{\bar{p}, \bar{q}\}, p, q)$ and $X(S, p)$ and tightness function $Q^h(S, \{\bar{p}, \bar{q}\}, p)$ as given.

- It transfers all profits $\pi^j(S, \{\bar{p}, \bar{q}\}, p)$ to the households.
- Firms have a production requirement:

$$C[S, \{\bar{p}, \bar{q}\}, p, Q^h(S, \{\bar{p}, \bar{q}\}, p)] + X(S, p) = \theta^a n.$$

FIRMS' PROBLEM

- The firm solves

$$\Omega(S, p_-) = \max_p \{ \psi^f [Q^h(S, \{\bar{p}, \bar{q}\}, p)] C[S, \{\bar{p}, \bar{q}\}, p, Q^h(S, \{\bar{p}, \bar{q}\}, p)] + X(S, p) \} \\ - \frac{W(S)}{\theta^a} \{ C[S, \{\bar{p}, \bar{q}\}, p, Q^h(S, \{\bar{p}, \bar{q}\}, p)] + X(S, p) \} - \chi(S, p_-, p) + \mathbb{E}[\Lambda(S, S') \Omega(S' p) | S],$$

$$\text{s.t. } S' = H(S, \theta'),$$

where $\Lambda(S, S')$ is the stochastic discount factor (specified later).

- In shorthand notations,

$$\Omega(S, p_-) = \max_p \{ \psi^f [Q^h(p)] C[p, Q^h(p)] + X(p) \} \\ - \frac{W}{\theta^a} \{ C[p, Q^h(p)] + X(p) \} - \chi(S, p_-, p) + \mathbb{E}[\Lambda' \Omega(S' p) | S].$$

TIGHTNESS AND DEMAND ELASTICITIES

- The households' optimal consumption and shopping decisions imply that

$$\frac{\psi^f[Q^h(p)]}{Q^h(p)} = \left(\frac{p}{\bar{p}}\right)^{\varepsilon-1} \frac{\psi^f[Q^h(\bar{p})]}{Q^h(\bar{p})},$$
$$C[p, Q^h(p)] = \left(\frac{p}{\bar{p}}\right)^{-\varepsilon} C[\bar{p}, Q^h(\bar{p})].$$

- Denote $\mathcal{E}(q) = q\psi_q^f(q)/\psi^f(q)$, these two conditions imply that

$$\frac{pQ_p^h(p)}{Q^h(p)} = -\frac{\varepsilon - 1}{1 - \mathcal{E}[Q^h(p)]},$$
$$\frac{pC_p[p, Q^h(p)]}{C[p, Q^h(p)]} = -\varepsilon,$$
$$C_{Q^h}[p, Q^h(p)] = 0.$$

- We also have

$$\frac{pX_p(p)}{X(p)} = -\varepsilon.$$

- Denote $p(S, p_-, \{\bar{p}, \bar{q}\})$ as the solution of firm's problem
- Consider a fixed point problem

$$\begin{aligned}\bar{p} &= p(S, p_-, \{\bar{p}, \bar{q}\}), \\ \bar{q} &= Q^h(S, \{\bar{p}, \bar{q}\}, \bar{p}).\end{aligned}$$

- The solution is a pair of equilibrium price and market tightness $\{p(S), Q(S)\}$ satisfying

$$\begin{aligned}\frac{\chi_p(S, p_-, p(S))}{C(S) + X(S)} &= \varepsilon \left[\frac{W(S)}{\theta^a p(S)} - \frac{\varepsilon - 1}{\varepsilon} \left(\frac{X(S)}{C(S) + X(S)} \right. \right. \\ &\left. \left. + \frac{\psi^f[Q(S)]}{1 - \varepsilon[Q(S)]} \frac{C(S)}{C(S) + X(S)} \right) \right] - \mathbb{E} \left\{ \Lambda(S, S') \frac{\chi_{p_-}(S', p(S), p(S'))}{C(S) + X(S)} \right\}.\end{aligned}$$

CHOOSING A χ

- Choose a functional form of χ such that

$$\chi(S, p_-, p) = p(S) \cdot \frac{\kappa_p}{2} \left(\frac{p}{p_-} - \bar{\pi} \right)^2 [C(S) + X(S)],$$

- Use the following notations

$$\pi(S) = \frac{p(S)}{p_-},$$

$$\phi^p(S) = \frac{\kappa_p}{2} (\pi(S) - \bar{\pi})^2,$$

$$\mathcal{Y}(S) = C(S) + X(S),$$

- The Phillips Curve can be simplified to

$$\begin{aligned} (\pi(S) - \bar{\pi})\pi(S) = & \frac{\varepsilon}{\kappa_p} \left\{ \frac{W(S)}{\theta^a p(S)} - \frac{\varepsilon - 1}{\varepsilon} \left[\phi^p(S) + \frac{(1 - \phi^p(S))\psi^f[Q(S)]}{1 - \varepsilon[Q(S)]} \right] \right\} \\ & + \mathbb{E} \left\{ \Lambda(S, S') \pi(S') \frac{\mathcal{Y}(S')}{\mathcal{Y}(S)} (\pi(S') - \bar{\pi})\pi(S') \right\}, \end{aligned}$$

The monetary policy is a one-period forward nominal interest rate

$$R = \frac{\bar{\pi}}{\beta} \left(\frac{\pi}{\bar{\pi}} \right)^{r_\pi} \left(\frac{Y}{\bar{Y}} \right)^{r_y} \theta^m,$$

where θ^m is an exogenous process of monetary shocks.

EQUILIBRIUM DEFINITION

The equilibrium is a set of decision rules for the household $\{c(p, q), d(p, q), l, b'\}$ as functions of (S, b, Φ) and value function v as functions of (S, b) , the firms' decision rules and values $\{p, x(p), \Omega\}$ as functions of (S, p_-) , and aggregate functions $\{C, D, L, B', P, \Pi, N, \mathcal{Y}, X, \Pi^f, Q, W, R, H(\cdot, \theta')\}$ on S such that

1. $\{c(p, q), d(p, q), l, b', v\}$ solve the households' problem,
2. $\{p, x(p), \Omega\}$ solve the firms' problem,
3. shopping decisions are consistent with market tightness,
4. individual decisions are consistent with aggregate functions,
5. monetary policy follows the specified rule, and
6. all market clears.

- Endogenous productivity:

$$C(S) = \mathcal{I}(S) \cdot (1 - \phi^p(S))\theta^a L(S).$$

- Non-constant desired markup for $\gamma \neq 0$:

$$\begin{aligned} (\Pi(S) - \Pi_{SS})\Pi(S) = \frac{\varepsilon}{\kappa_p} \left\{ \frac{w(S)}{\theta^a} - \frac{\varepsilon - 1}{\varepsilon} [\phi^p(S) + \mathcal{I}(S)^{1-\gamma}(1 - \phi^p(S))] \right\} \\ + \beta \mathbb{E}[\Lambda(S, S')(\Pi(S') - \Pi_{SS})\Pi(S') | S]. \end{aligned}$$

- The determinant of $\mathcal{I}(S)$ is

$$-(\varepsilon - 1)Q(S)u_{d^A}[S] = C(S)u_c[S],$$

where $\mathcal{I}(S) = (1 + Q(S)^{-\gamma})^{-\frac{1}{\gamma}}$ comes from a CES matching function ψ .

WHY SHOULD WE CARE?

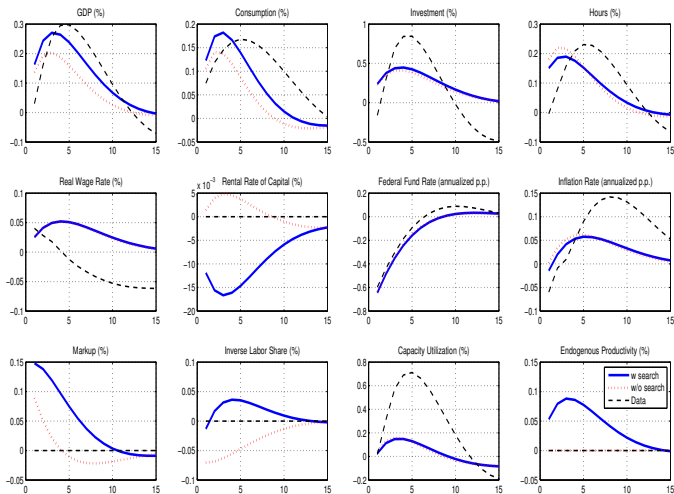
- It is alternative interpretation of **effective demand**.
- It is one way to understand locally **increasing return to scale**.
- It captures **demand pulling inflation** instead of cost pushing inflation, although we have not yet figured out how to identify it.
(An interesting question for another paper)
- It has different **welfare** implications for business cycle fluctuation.

- Follow the baseline model in CET2016, but use Rotemberg sticky wage and drop government purchase for tractability.
- Add shopping friction on top of that (Rotemberg pricing).
- Take the parameters directly from CET2016. Price and wage rigidity parameters are transformed to make the linearized formula identical.
- Choose parameters of the shopping friction to make the IRFs look good.
- Compare these two models to VAR IRFs in CET2016.
- For variables not provided in the VARs of CET2016, we use flat lines.

Caveat: We should carefully deal with the timing of monetary shocks such that it does not have an impact on current variables, as is in VAR, but have not yet done it. This is one of the reasons why our model implied IRFs are not identical to CET2016.

NUMERICAL EXAMPLE

Impulse Responses to a Monetary Policy Shock



- in progress ...
- Our goal is to assess the quantitative importance of the transmission channel we proposed by using variance decomposition.
- Our concern is that the estimation provides no additional insight because we have already known in the data that labor productivity accounts for roughly $1/3$ of the output fluctuation.

- in progress ...

Countercyclical Markup:

- Inverse labor share can be used as a proxy for markup in NKPC and has to be countercyclical (Galí and Gertler, 1999).
- Overhead labor and allocative wage make inverse labor share procyclical and it is a poor measure of markup (Rotemberg and Woodford, 1999).
- Allocative wage is very procyclical so that markup on the margin cannot be procyclical (Basu and House, 2016).

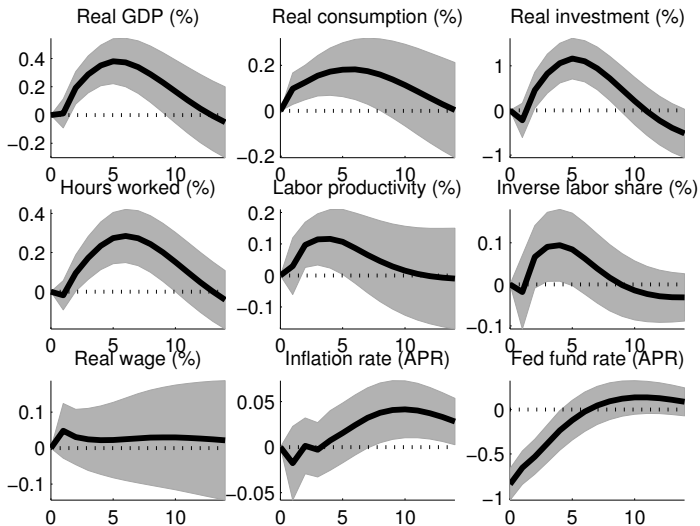
Procyclical Markup:

- labor share based markup is procyclical even after controlling for the role of overhead labor and allocative wage (Nekarda and Ramey, 2013).
- Energy input data also indicates procyclical markup conditional on demand shocks (Kim, 2016).

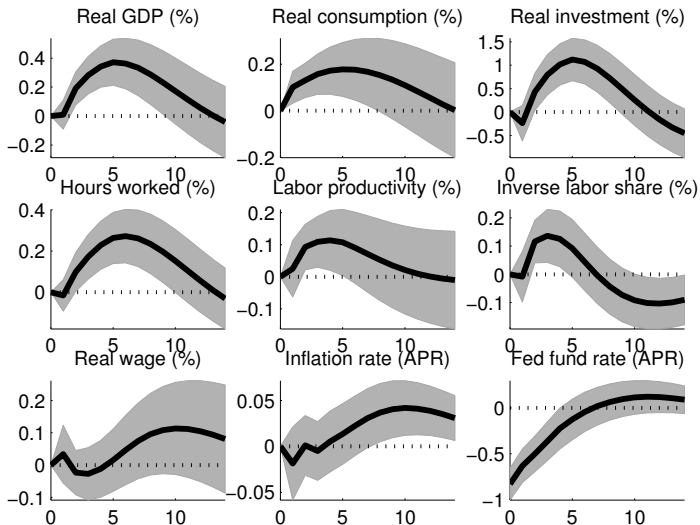
Follow Christiano, Eichenbaum, and Trabandt (2016) (CET2016).

- VAR(2) with [Recursive Identification](#) using quarterly data.
- 8 variables of prices and per capita quantities (a subset of CET2016):
 1. $\Delta \ln(\text{real GDP}_t/\text{hours}_t)$,
 2. $\Delta \ln(\text{GDP deflator}_t)$,
 3. *unemployment rate*_t,
 4. $\ln(\text{hours}_t)$,
 5. $\ln(\text{real GDP}_t/\text{hours}_t) - \ln(\text{real wage}_t)$,
 6. $\ln(\text{nominal consumption}_t/\text{nominal GDP}_t)$,
 7. $\ln(\text{nominal investment}_t/\text{nominal GDP}_t)$,
 8. *Federal Funds Rate*_t.
- $\ln(\text{real GDP}_t/\text{hours}_t) - \ln(\text{real wage}_t)$ is log inverse labor share, so we cannot use both real wage data and labor share data in the same time.
- Sample ranges include
 1. 1965q3-1995q3 (Christiano, Eichenbaum, and Evans, 2005),
 2. 1951q1-2008q4 (Christiano, Eichenbaum, and Trabandt, 2016), and
 3. 1985q1-2008q3 (the great moderation).

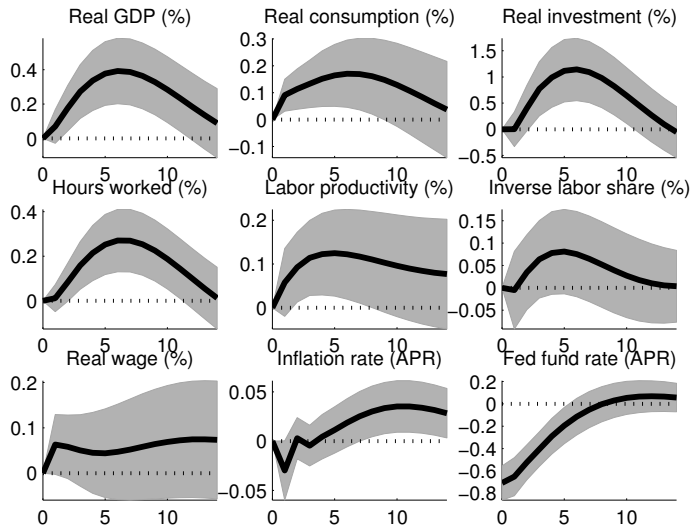
Fed Funds Shock VAR(2) (1965q3-1995q3)



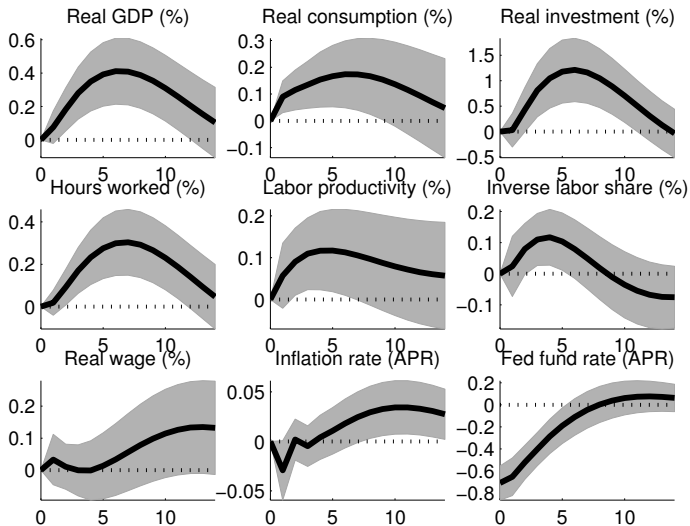
Fed Funds Shock VAR(2) (1965q3-1995q3)



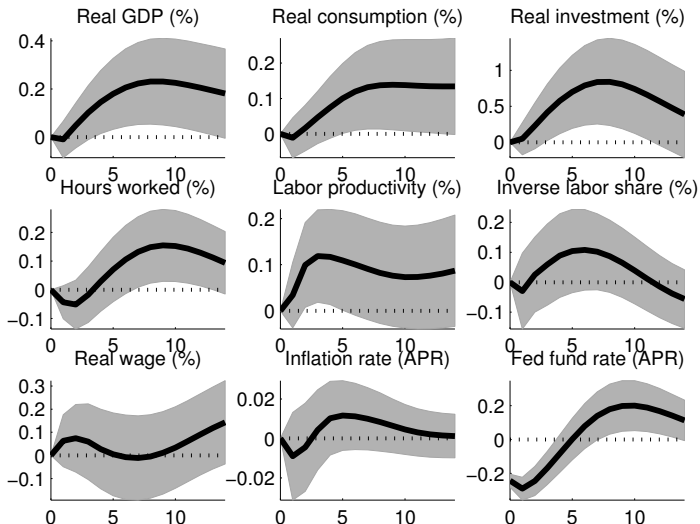
Fed Funds Shock VAR(2) (1951q1–2008q4)



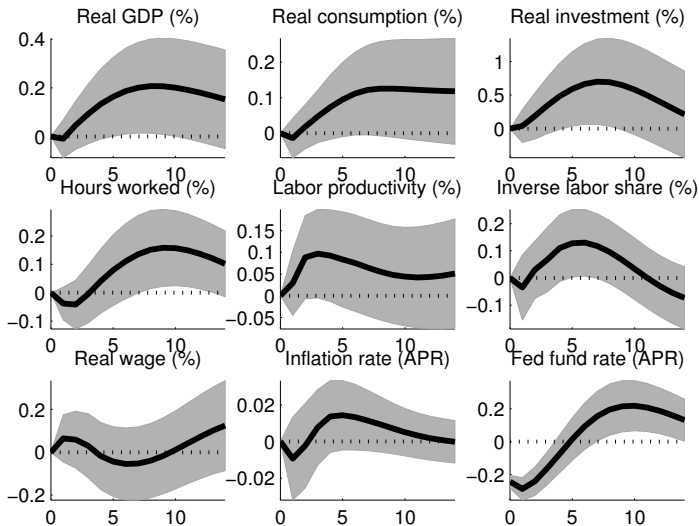
Fed Funds Shock VAR(2) (1951q1-2008q4)



Fed Funds Shock VAR(2) (1985q1–2008q3)



Fed Funds Shock VAR(2) (1985q1–2008q3)



- Cantore et al. (2016) also finds the same labor share pattern robust across different [measures](#) of labor share, different [methods to identify](#) monetary shocks, and data from different [countries](#).
- Our results also include the IRFs of [other variables](#) including labor productivity across different [time ranges](#).

- Given aggregate output Y and individual output y , the firm solves

$$C^y(y, Y) = \min_{\{k, n\}} \{C^k(k, Y) + C^n(n, Y)\},$$

$$\text{s.t. } y \leq A(Y)F(k, n),$$

where $\{C^y, C^k, C^n\}$ are cost functions. The solution $\{k^*, n^*\}$ satisfies

$$C_y^y(y, Y) = \frac{C_k^k(k^*(y, Y), Y)}{A(Y)F_k(k^*(y, Y), n^*(y, Y))} = \frac{C_n^n(n^*(y, Y), Y)}{A(Y)F_n(k^*(y, Y), n^*(y, Y))}.$$

- Use $\mathcal{E}^{F_n}(y, Y)$ and $\mathcal{E}^{C^n}(y, Y)$ to denote the elasticities of production and cost functions w.r.t. labor inputs, then labor share can be derived as

$$ls(y, Y) = \frac{C^n(n^*(y, Y), Y)}{A(Y)F(k^*(y, Y), n^*(y, Y))} = \frac{\mathcal{E}^{F_n}(y, Y)}{\mathcal{E}^{C^n}(y, Y)} C_y^y(y, Y).$$

Denote $lp(y, Y)$ as labor productivity.

1. **Overhead Labor:** $F(k, n - n_0) = k^\alpha(n - n_0)^{1-\alpha}$.

$$\frac{nF_n(k, n)}{F(k, n)} = \frac{(1 - \alpha)n}{n - n_0} \implies \frac{d}{dY} \mathcal{E}^{F_n}(Y, Y) < 0.$$

As good as our story so far.

2. **Allocative wage:** $\mathcal{C}^n(n, Y) = w_0(Y)n_0 + w(Y)(n - n_0)$.

$$\frac{n\mathcal{C}_n^n(n, Y)}{\mathcal{C}^n(n, Y)} = \left[1 - \left(1 - \frac{w_0(Y)}{w(Y)} \right) \frac{n_0}{n} \right]^{-1} \implies \frac{d}{dY} \mathcal{E}^{\mathcal{C}_n^n}(Y, Y) > 0.$$

No effect on lp .

3. **Kimball aggregator:** procyclical desired markup **implies**

$$\frac{d}{dY} \mathcal{C}_y^y(Y, Y) < 0.$$

No effect on lp & $\frac{d}{dY} \{A(Y)\mathcal{C}_y^y(Y, Y)\} < 0$.

- For $F(k, n) = k^\alpha n^{1-\alpha}$, and denote r^k as the rental rate of capital, then

$$C_y^y(Y, Y) = \frac{1}{A(Y)} \left(\frac{r^k(Y)}{\alpha} \right)^\alpha \left(\frac{w(Y)}{1-\alpha} \right)^{1-\alpha}.$$

- Procyclical desired markup **implies**

$$\frac{d}{dY} C_y^y(Y, Y) < 0.$$

- Endogenous TFP allows for

$$\frac{d}{dY} \{A(Y) C_y^y(Y, Y)\} > 0.$$

- Our story and overhead labor can generate both countercyclical labor share and procyclical labor productivity conditional on monetary shocks.
- However, they operate in different ways.
 - In [our story](#), these two patterns are [equilibrium phenomena](#).
 - In [overhead labor](#), they are just [measurement issues](#).
- These two stories should have different implications for inflation, but we have not yet figured out a way to identify it.
- Since our story operates through equilibrium conditions, we cannot use firm-level variation to identify it.

- Knowing that $C_{Q^h} = 0$, the FOC of the firms' problem wrt p is

$$0 = \psi^f C + p\psi^f_{Q^h} Q^h C + p\psi^f C_p + X + pX_p - \frac{W}{\theta^a} (C_p + X_p) - \chi_p + E\{\Lambda' \Omega'_{p-}\}.$$

- Dividing both side by $C + X$ and moving χ_p to the LHS yield

$$\begin{aligned} \frac{\chi_p}{C + X} = & \frac{\psi^f C + X}{C + X} + \frac{pC_p}{C} \frac{\psi^f C}{C + X} + \frac{pX_p}{X} \frac{X}{C + X} + \frac{pQ_p^h}{Q^h} \frac{Q^h \psi^f_{Q^h}}{\psi^f} \frac{\psi^f C}{C + X} \\ & - \frac{W}{\theta^a p} \left(\frac{pC_p}{C} \frac{C}{C + X} + \frac{pX_p}{X} \frac{X}{C + X} \right) + E\left\{ \frac{\Lambda' \Omega'_{p-}}{C + X} \right\}. \end{aligned}$$

- Using the elasticities of $Q^h(p)$, $C[p, Q^h(p)]$ and $X(p)$ that we derived,

$$\frac{\chi_p}{C + X} = \varepsilon \left[\frac{W}{\theta^a p} - \frac{\varepsilon - 1}{\varepsilon} \left(\frac{X}{C + X} + \frac{\psi^f}{1 - \varepsilon} \frac{C}{C + X} \right) \right] + E\left\{ \frac{\Lambda' \Omega'_{p-}}{C + X} \right\}.$$

- More explicitly, the optimal price p satisfies an implicit function

$$\begin{aligned} \frac{\chi_p(S, p_-, p)}{C[p, Q^h(p)] + X(p)} = & \varepsilon \left[\frac{W(S)}{\theta^a p} - \frac{\varepsilon - 1}{\varepsilon} \left(\frac{X(p)}{C[p, Q^h(p)] + X(p)} \right. \right. \\ & \left. \left. + \frac{\psi^f[Q^h(p)]}{1 - \varepsilon[Q^h(p)]} \frac{C[p, Q^h(p)]}{C[p, Q^h(p)] + X(p)} \right) \right] \\ & + E \left\{ \frac{\Lambda(S') \Omega_{p_-}(S', p)}{C[p, Q^h(p)] + X(p)} \right\}. \end{aligned}$$

- The solution is an expression for p that depends on $C^A(S, \{\bar{p}, \bar{q}\})$, on $Q^h(S, \{\bar{p}, \bar{q}\}, p)$, and on p_- via adjustment costs.

- Use notation $\{p(S), Q(S)\}$ for the fixed point $\{\bar{p}, \bar{q}\}$, and $\{C(S), X(S)\}$ as shorthand notations for $\{C[p(S), Q^h(p(S))], X(p(S))\}$.
- The Envelope condition is

$$\Omega_{p-}(S', p(S)) = -\chi_{p-}(S', p(S), p(S')).$$

- Then, we obtain a fixed point condition that $\{p(S), Q(S)\}$ must satisfy

$$\begin{aligned} \frac{\chi_p(S, p-, p(S))}{C(S) + X(S)} = & \varepsilon \left[\frac{W(S)}{\theta^a p(S)} - \frac{\varepsilon - 1}{\varepsilon} \left(\frac{X(S)}{C(S) + X(S)} \right. \right. \\ & \left. \left. + \frac{\psi^f[Q(S)]}{1 - \varepsilon[Q(S)]} \frac{C(S)}{C(S) + X(S)} \right) \right] \\ & - E \left\{ \Lambda(S') \frac{\chi_{p-}(S', p(S), p(S'))}{C(S) + X(S)} \right\}. \end{aligned}$$

$$u_c[S] = \beta R(S) \mathbb{E} \left[\frac{u_c[S']}{\Pi(S')} \middle| S \right],$$

$$-(\varepsilon - 1)Q(S)u_{dA}[S] = C(S)u_c[S],$$

$$-u_\ell[S] = w(S)u_c[S],$$

$$(\Pi(S) - \Pi_{SS})\Pi(S) = \frac{\varepsilon}{\kappa_p} \left\{ \frac{w(S)}{\theta^a} - \frac{\varepsilon - 1}{\varepsilon} \left[\phi^p(S) + \mathcal{I}(S)^{1-\gamma}(1 - \phi^p(S)) \right] \right\}$$

$$+ \beta \mathbb{E} \left[\frac{\mathcal{Y}(S')u_c(S')}{\mathcal{Y}(S)u_c(S)} (\Pi(S') - \Pi_{SS})\Pi(S') \middle| S \right],$$

$$C(S) = (1 - \phi^p(S))\mathcal{I}(S)\mathcal{Y}(S),$$

$$\mathcal{Y}(S) = \theta^a L(S),$$

$$R(S) = \frac{\Pi_{SS}}{\beta} \left(\frac{\Pi(S)}{\Pi_{SS}} \right)^{r_\pi} \left\{ \frac{[\phi^p(S) + (1 - \phi^p(S))\mathcal{I}(S)]\mathcal{Y}(S)}{\mathcal{I}_{SS}\mathcal{Y}_{SS}} \right\}^{r_y} \theta^m,$$

$$\text{where } \phi^p(S) = \frac{\kappa_p}{2} (\Pi(S) - \Pi_{SS})^2,$$

$$\mathcal{I}(S) = (1 + Q(S)^{-\gamma})^{-\frac{1}{\gamma}},$$

$$u_c[S] = \mathcal{I}(S)^{\frac{1}{\varepsilon-1}} \left(\mathcal{I}(S)^{\frac{1}{\varepsilon-1}} C(S) - \zeta \frac{Q(S)^{1+\nu}}{1+\nu} \right)^{-\omega},$$

$$-u_{dA}[S] = \zeta Q(S)^\nu \mathcal{I}(S)^{-\frac{1}{\varepsilon-1}} u_c[S],$$

$$-u_\ell[S] = \eta L(S)^\xi \mathcal{I}(S)^{-\frac{1}{\varepsilon-1}}.$$

Utility function that allows for [habit persistence](#), [wage rigidity](#), and [BGP](#):

$$\begin{aligned} u(c^A, d^A, c_-^A, d_-^A, \{\ell_j\}) \\ = \frac{1}{1-\omega} \left[\left(c^A - \zeta \theta^y \frac{(d^A)^{1+\nu}}{1+\nu} \right) - h \left(c_-^A - \zeta \theta^y \frac{(d_-^A)^{1+\nu}}{1+\nu} \right) \right]^{1-\omega} \\ - \eta (\theta^y)^{1-\omega} \int_0^1 \frac{\ell_j^{1+\xi}}{1+\xi} dj, \end{aligned}$$

where θ^y is the composite technology level for output.

$$\begin{aligned}
& \max_{\{z_{p,q,t}, d_{p,q,t}, w_{j,t}, b_t, u_t^k, k_t, i_t^A\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t^A, d_t^A, c_{t-1}^A, d_{t-1}^A, \{\ell_{j,t}\}_j), \\
& \text{s.t. } z_t^A = \left(\int_{\Phi_t} d_{p,q,t} \psi^h(q) z_{p,q,t}^{\frac{\varepsilon-1}{\varepsilon}} dpdq \right)^{\frac{\varepsilon}{\varepsilon-1}}, \\
& d_t^A = \int_{\Phi_t} d_{p,q,t} dpdq, \\
& e_t^A = \int_{\Phi_t} d_{p,q,t} \psi^h(q) z_{p,q,t} dpdq, \\
& z_t^A = c_t^A + \frac{i_t^A + a(u_t^k)k_{t-1}}{\theta_t^i}, \\
& e_t^A + b_t + p_t \mathcal{Y}_t \int_0^1 \varphi \left(\frac{w_{j,t}}{w_{j,t-1}} \right) dj = p_t \int_0^1 w_{j,t} \ell_{j,t} dj + R_t^k u_t^k k_{t-1} + R_{t-1} b_{t-1} + \Pi_t^f, \\
& \ell_{j,t} = \left(\frac{w_{j,t}}{w_t} \right)^{-\varepsilon_w} \ell_t, \\
& k_t = (1 - \delta)k_{t-1} + \left[1 - \chi \left(\frac{i_t^A}{i_{t-1}^A} \right) \right] i_t^A.
\end{aligned}$$

- Tightness function satisfies

$$\psi^h(Q^h(t, \{\bar{p}, \bar{q}\}, p)) = \left(\frac{p}{\bar{p}}\right)^{\varepsilon-1} \psi^h(Q^h(t, \{\bar{p}, \bar{q}\}, \bar{p})).$$

- Demand function satisfies

$$Z(t, \{\bar{p}, \bar{q}\}, p, Q^h(t, \{\bar{p}, \bar{q}\}, p)) = \left(\frac{p}{\bar{p}}\right)^{-\varepsilon} Z(t, \{\bar{p}, \bar{q}\}, \bar{p}, Q^h(t, \{\bar{p}, \bar{q}\}, \bar{p})).$$

- Use short-hand notation $Q^h(t, p)$ and $Z(t, p)$, and the firms' problem can be derived in the same way as in our off-the-shelf model.

- in progress ...

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