

Currency Attacks and Government Communication *

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Abstract

This paper studies a model of currency attacks in which the government can choose some credible signal about the fundamentals of the economy. The government initially pegs the exchange rate and speculators decide whether to attack the currency or not. Speculators observe, in addition to the public signal, a private noisy signal. Public signals create partial common knowledge that can lead to multiple equilibria. It is possible to find disclosure policies that dominate an uninformative public signal, regardless of the equilibrium strategy played by speculators. Commitment is key to this result. The optimal policy with commitment is characterized when, if there is multiplicity, the government only cares about its lowest equilibrium payoff. In this case, the public signal is informative and leads to a unique equilibrium, which is preferred to a full disclosure policy. Our results indicate that the government has incentives for being vague in its communication.

1 Introduction

Informed governmental agencies are often criticized for the poor quality of the information they release. Referring to the early years of Alan Greenspan as head of the Fed, Blinder and Reis (2005) write:

*We thank Harold Cole, Dirk Krueger and Guillermo Ordonez for helpful comments.

Soon Greenspan, who is far from plainspoken in any case, became known for such memorable phrases as 'mumbling with great incoherence' - which he used (with a hint of humor) to characterize his own version of FedSpeak.

In this paper, we argue that it is optimal for the government to be vague in its communication. This happens because government's preferences do not coincide with preferences of other economic agents. When the government has access to payoff relevant information, it needs to be vague in order to induce agents to take the government's most preferred action.

We analyze the environment where a government can release a public signal about the fundamentals of the economy. In our model, the government would like to maintain a currency peg. The peg can be attacked by a continuum of speculators, who wish to profit from a currency devaluation. Payoffs depend on the state of fundamentals of the economy, the action taken by speculators, and the government's choice between defending or abandoning the peg. If fundamentals are weak (low states), speculators can have large profits from attacking the currency and the government has to pay a high cost to maintain the peg; if fundamentals are strong (high states), speculators can have at most small profits from attacking the currency and the cost of defending the peg is low. The cost of maintain the peg is increasing in the number of speculators that attack the currency.

Following Morris and Shin (1998), we assume that speculators receive noisy private signals about the fundamentals. Thus, if public signals are imprecise, one could expect them to have small effects on speculators beliefs about the state of fundamentals. This raises the question: why are vague announcements effective?

When information is dispersed across speculators, an imprecise signal about the fundamentals can have large effects because it changes the beliefs of a speculator about what other speculators believe. If the government can delegate to an informed and independent agency (such as the Fed) the mission to send a public signal (such as the FOMC statements) about the state of fundamentals, this public signal generates partial common knowledge about the unknown state. Thus, government communication induces coordination among speculators, even if the public signal has a low precision.

In our model, the government chooses an arbitrary partition of the space of fundamentals. The public signal reveals in which element of the partition the true fundamentals lie. Only truthful signals are allowed. Given the common prior and the private and public signals, speculators use Bayesian updating and then decide whether to attack the currency or not.

In a model where the state of fundamentals is common knowledge, multiple equilibria arise because of the coordination problem faced by speculators. However, Morris and Shin (1998) show that the introduction of noisy private signals about the fundamentals leads to a unique equilibrium,

where speculators use cutoff strategies based on their private signals. Our introduction of public signals breaks the uniqueness result in Morris and Shin (1998).¹ To characterize the optimal disclosure policy we thus assume that, in the case of multiplicity, the government only cares about the worst equilibrium outcome.² Under this assumption, we show that the optimal disclosure policy is, without loss of generality, a policy with two signals (a two interval partition). We interpret this result as a deliberate decision from the government to be vague - indeed, if the government could reveal the exact state of the economy, it would choose not to do so.

We then move to a characterization of the optimal disclosure policy. Two signals are sent when the optimal policy is implemented: a low one, corresponding to *bad* states of fundamentals (a coordinated attack in this region is always profitable), and a high one, for the *not too bad* states (a coordinated attack is not always profitable). We find that the government “hides” some intermediate states with strong ones in the *not too bad* region. Intuitively, this is the optimal signal because, after observing a high signal, speculators assign a sufficiently high probability to states where it is not profitable to attack, which allows the government to prevent attacks in intermediate states. In order to do this, the government commits to acknowledging the really bad states of fundamentals.

We find that the subgame that follows the optimal disclosure policy has a unique equilibrium. After observing the low signal, speculators coordinate on attacking and the government abandons the currency peg. When a high signal is observed, speculators refrain from attacking and the peg is maintained. If the government had included too many states in the *not too bad* region, this would have led to an equilibrium with currency attacks after the high signal, which the government wishes to avoid. In other words, the government wants to minimize the revelation of bad states by reducing the *bad* region up to the limit where not attacking after observing the high signal is still the unique possible action to be taken in equilibrium.

The final result of this paper is that commitment is essential for the government to benefit from disclosing information. When the government cannot commit to a disclosure policy, there exist equilibria in which the government is made worse off by sending a public signal. Without commitment, the government wishes to fully reveal the good states, which allows the speculators to coordinate on attacking in bad and intermediate states.

¹ In a different setting, Angeletos et al. (2006) study a model where policy interventions generate endogenous information, leading to multiple equilibria. We, however, assume that the government can commit ex-ante to a disclosure policy. If we remove this assumption, our results change significantly. See Section 5.

² There are two ways to justify this assumption. First, this selection mechanism maximizes speculators’ payoffs. Second, we take Morris and Shin (1998) as a benchmark to ask whether the government is better off by sending an informative signal. The optimal signal derived from our equilibrium selection provides a strictly positive lower bound for the government’s benefits from sending a public signal.

Related literature.

This paper is related to the literature on self-fulfilling currency crises when payoffs are not common knowledge among speculators. The idea that *small* deviations from common knowledge can have a large impact on equilibrium outcomes dates back at least to Rubinstein's mail game (Rubinstein (1989)), and has gained great attention since Carlsson and van Damme (1993) and Morris and Shin (1998).

We build on the model of Morris and Shin (1998) to introduce a public signal that generates partial common knowledge. In different settings, the interaction between public and private signals in coordination games has been studied in Morris and Shin (2002), Morris and Shin (2003), Hellwig (2002).³

In our model, public signals induce coordination among speculators, as in Angeletos et al. (2006), Angeletos and Pavan (2007, 2009) and Angeletos and Pavan (2013). Breaking the uniqueness result in Morris and Shin (1998), Angeletos et al. (2006) point out that policy interventions that convey some information about the fundamentals may lead to multiple equilibria. We focus on optimal government communication, thus policy in our model is the revelation of information itself, and, as opposed to the literature, does not change payoff relevant parameters. The government has incentives to release information about the fundamentals in order to influence the final outcome of the game. This is true even if, by restoring partial common knowledge, the game that follows the government's decision admits multiple equilibria.

The paper also relates to the literature on coordination motives in information acquisition (e.g., Hellwig and Veldkamp (2009), Myatt and Wallace (2012)). Hellwig and Veldkamp (2009) show that, when there are complementarities in the actions, agents "*want to know what others know*". In line with their findings, our equilibrium displays speculators that coordinate on the public signal and take the same action regardless of their private information.

Finally, the paper relates to the literature on Bayesian persuasion (e.g., Kamenica and Gentzkow (2011)), which studies the optimal signal structure from the perspective of a sender who wants to influence a rational Bayesian receiver to take the sender's preferred action. This is done by affecting the receiver's beliefs. In addition to this effect, our model also takes into account the interaction of speculators who have private information, where a public signal can play an important role on coordination. The optimal policy is designed to maximize the probability that speculators coordinate on not attacking the currency peg.

Structure of the paper. The remainder of this paper is divided as follows. In Section 2, a motivating

³ In those papers, the public signal cannot generate common knowledge about dominance regions. In Morris and Shin (1998), this lack of common knowledge is important for equilibrium uniqueness. In our model, we allow for the public signal to make such revelations.

example is presented. Section 3 presents the model and some of its equilibrium properties. The main results are described in Section 4. Section 5 analyzes the model without the commitment assumption and Section 6 concludes the paper. A numerical example is provided in Appendix A, and the proofs that are omitted in the main text are presented in Appendix B.

2 A simple example

Before the full model is introduced, we present an example that conveys the main ideas in this paper. It illustrates the effects of adding a public signal to a model of currency attacks in which speculators observe a private signal about the fundamentals.

Consider an economy where the state of the fundamentals is given by $\theta \in \Theta = \{0, 0.5, 1\}$, and the common prior assigns the same probability for each state. There is a continuum of speculators with unit mass. The government initially pegs the exchange rate at 1, and the equilibrium rate without intervention is θ . Speculators decide simultaneously whether to attack the currency peg or not. Given θ and the size of the attack, the government decides whether to abandon the peg or defend the currency.

Each speculator pays a cost of $t = 0.4$ to attack the currency peg, and the gross payoff is $1 - \theta$ if the peg is abandoned (a successful attack). The speculator's net payoff from a successful attack is thus given by

$$u(\theta) = \begin{cases} 0.6, & \text{if } \theta = 0, \\ 0.1, & \text{if } \theta = 0.5, \\ -0.4, & \text{if } \theta = 1, \end{cases}$$

and the payoff from an unsuccessful attack is -0.4 . If the speculator refrains from attacking, his payoff is 0.

The government derives a value $v = 1$ from the currency peg, and the cost of defending it is given by $c(\theta, \alpha) = 1.3 - \theta + \alpha$, where α is the mass of speculators who attack the peg. If the government abandons the peg, its payoff is 0. The critical mass of attackers necessary for the government to abandon the peg is given by

$$a(\theta) = \begin{cases} 0, & \text{if } \theta = 0, \\ 0.2, & \text{if } \theta = 0.5, \\ 0.7, & \text{if } \theta = 1, \end{cases}$$

that is, if θ is the state and α is the fraction of speculators who attack the currency, then the government abandons the peg if $\alpha > a(\theta)$, and defends the peg if $\alpha < a(\theta)$.

2.1 Common knowledge

If the state of the fundamentals is common knowledge, the game admits two equilibria. In one equilibrium there is a coordinated attack on the currency peg after $\theta = 0.5$ is observed, which forces the government to abandon the peg. The other equilibrium features no attack after $\theta = 0.5$ is observed, and the government chooses to maintain the peg. In both equilibria there is no attack when $\theta = 1$, since the speculators know that it is not profitable to attack; and every speculator attacks the currency when $\theta = 0$, since they know that the the government will abandon the peg regardless of the size of the attack.

2.2 Private signal

Suppose that the true state is unknown, but the speculators observe a private signal $x \in \{0, 0.5, 1\}$, with conditional probability $\mathbb{P}(x|\theta)$ as follows:

	$x = 0$	$x = 0.5$	$x = 1$
$\mathbb{P}(x \theta = 0)$.50	.25	.25
$\mathbb{P}(x \theta = .5)$.25	.50	.25
$\mathbb{P}(x \theta = 1)$.25	.25	.50

As in Morris and Shin (1998), this game admits a unique equilibrium. In equilibrium, speculators attack the currency if the private signal is $x \leq 0.5$, and the government abandons the peg if $\theta \leq 0.5$.⁴

⁴ The proof of is constructed in 6 steps:

1. If $\theta = 0$, the government abandons the peg in equilibrium regardless of the size of the attack.
2. If $x = 0$, since the peg is abandoned when $\theta = 0$, the equilibrium payoff from attacking is at least $\mathbb{P}(\theta = 0|x = 0) - 0.4 = 0.1$; thus, speculators attack when $x = 0$.
3. If $\theta = 0.5$, since speculators attack when $x = 0$, the size of the attack is at least $\mathbb{P}(x = 0|\theta = 0.5) = 0.25 > a(0.5) = 0.2$: thus, the government abandons the peg when $\theta = 0.5$.
4. If $x = 0.5$, since the peg is abandoned when $\theta \leq 0.5$, the equilibrium payoff from attacking is at least $\mathbb{P}(\theta = 0|x = 0.5) + 0.5\mathbb{P}(\theta = 0.5|x = 0.5) - 0.4 = 0.25 + 0.5^2 - 0.4 = 0.1$: speculators attack.
5. If $x = 1$, since the payoff from attacking when $\theta = 1$ is -0.4 (whether it is successful or not), the equilibrium payoff from attacking is $\mathbb{P}(\theta = 0|x = 1) + 0.5\mathbb{P}(\theta = 0.5|x = 1) - 0.4 = -0.025$; thus, speculators refrain from attacking when $x = 1$.
6. Finally, if $\theta = 1$, the size of the attack is $\mathbb{P}(x < 1|\theta = 1) = 0.5 < 0.7 = a(1)$, and the government defends the peg.

2.3 Private and public signals

Now suppose that the government can commit to a disclosure policy as follows. First, the government partitions the state space Θ , and then it sends a public signal y that reveals in which element of the partition the true fundamentals lie. The introduction of a public signal can lead to multiple equilibria. For example, if the government chooses to fully disclose the fundamentals by choosing the partition $\{\{0\}, \{0.5\}, \{1\}\}$, we are back to the common knowledge case and there are two equilibria.

Suppose that, in case of multiplicity, the government only cares about the worst equilibrium outcome. We claim that, in this case, the optimal partition is $\{\{0\}, \{0.5, 1\}\}$. The government thus sends two signals:

$$y = \begin{cases} y_l & , \quad \text{if } \theta = 0, \\ y_h & , \quad \text{if } \theta \in \{0.5, 1\}. \end{cases}$$

If the public signal is $y = y_l$, it becomes common knowledge that the true state is $\theta = 0$, which means that the currency peg will be abandoned and it is profitable to attack, regardless of the other speculators' behavior and the private signal. Hence, every speculator must attack in equilibrium after observing $y = y_l$. If $y = y_h$, it becomes common knowledge that it is never profitable to attack the currency peg when no one else attacks. This leads to an equilibrium in which speculators coordinate on not attacking if $y = y_h$.

It turns out the equilibrium is unique. Speculators follow the public signal and attack if and only if $y = y_l$, and the government abandons the peg if only if $\theta = 0$. The government is strictly better off by sending a public signal, since it eliminates attacks when $\theta = 1$. Without a public signal, when $\theta = 1$ half the speculators observe a private signal $x \leq 0.5$ and attack the peg, so the government has to pay a cost to defend the currency. Furthermore, in the worst equilibrium with full disclosure, the peg is abandoned if $\theta \in \{0, 0.5\}$. Thus, the optimal policy strictly dominates full disclosure (common knowledge) or no disclosure (uninformative public signal) for the government.

The remainder of paper shows that the results in this section still hold in a more general framework. In the full model, when the government only cares about the worst equilibrium outcome, it is without loss of generality to consider only two-signal structures. Public policy thus divides the fundamentals into two intervals: a lower interval, where the peg is abandoned and speculators coordinate on attacking; an upper interval, where the peg is maintained and no speculator attacks. The optimal policy involves maximizing the size of the upper interval, while keeping the equilibrium unique.

3 Model

3.1 Actions and payoffs

The model is similar to the one in Morris and Shin (1998), with the addition of a public signal. There is a currency peg in the economy and speculators have to decide whether to attack it or not. There is a continuum of speculators of measure one, who are indexed by i and uniformly distributed on $[0, 1]$. The state of fundamentals in the economy is given by θ , which is uniformly distributed on $\Theta = [0, 1]$. In the absence of government intervention, the exchange rate is a function $f(\cdot)$ of the state θ , where $f(\cdot)$ is continuous and strictly increasing. The exchange rate is initially pegged by the government at e^* , with $e^* \geq f(\theta)$ for all θ .

A speculator attacks the currency by selling short one unit of currency at a cost $t > 0$. If the speculator attacks and the peg is abandoned, his payoff is $e^* - f(\theta) - t$, whereas the payoff from attacking when the currency is defended is $-t$. If the speculator does not attack the currency, his payoff is zero.

The government derives a value $v > 0$ from maintaining the currency peg. There is a cost $c(\alpha, \theta)$ to defend the peg, where α is a mass of speculators who attack the currency. The cost c is continuous, strictly increasing in α and strictly decreasing in θ . Hence, the payoff from defending the peg is $v - c(\alpha, \theta)$, and the payoff from abandoning the peg is zero. The following assumptions are made:

- $c(0, 0) > v$: the government abandons the peg if fundamentals are sufficiently weak, even if no one attacks;
- $c(1, 1) > v$: the government abandons the peg if everyone attacks, even if fundamentals are good;
- $e^* - f(1) - t < 0$: it is not profitable for speculators to attack the currency if fundamentals are good enough.

Denote by $\underline{\theta}$ the value of θ that solves $v = c(0, \theta)$. If $\theta \leq \underline{\theta}$, the government finds it optimal to abandon the peg regardless of the size of the attack. Denote by $\bar{\theta}$ the value of θ such that $e^* - f(\theta) - t = 0$. If $\theta > \bar{\theta}$, attacking is not profitable even if the peg is abandoned.

We assume that $\underline{\theta} < \bar{\theta}$.⁵ When the state is common knowledge, we can divide Θ in three intervals, as it has been pointed out in the literature.⁶ Following the terminology in Morris and Shin (1998):

⁵ This condition holds for a large v and a small t .

⁶ See, for example, Obstfeld (1996) and Morris and Shin (1998).

- if $\theta \in [0, \underline{\theta}]$, the currency is *unstable*: the government always abandons the peg;
- if $\theta \in (\underline{\theta}, \bar{\theta})$, the currency is *ripe for attack*: a coordinated attack is profitable and, if there is coordination on not attacking, attacking is not profitable;
- if $\theta \in [\bar{\theta}, 1]$, the currency is *stable*: it is never profitable to attack the peg.

3.2 Timing and information

The game has three stages. In the first stage, before observing θ , the government commits to a disclosure policy, which is announced to the speculators. In the second stage, once θ is realized, a public signal y is sent according to the disclosure policy.⁷ Speculators do not observe θ , just the public signal y and a private signal x . Given x and y , speculators simultaneously decide whether to attack the currency or not. In the last stage, the government observes θ and the size of the attack, and decides whether to defend the currency or abandon the peg. The structure of the game is assumed to be common knowledge.

We denote a partition of the interval $[0, 1]$ by $P = \{m_n\}_{n=0}^N$, where $0 = m_0 < m_1 < \dots < m_N = 1$, and $N \in \mathbb{N}$.⁸ The n -th interval of the partition P is denoted by y_n , with

$$y_1 = [0, m_1], y_2 = (m_1, m_2], \dots, y_n = (m_{n-1}, m_n], \dots, y_N = (m_{N-1}, 1].$$

When the public signal $y = y_n$ is sent, it becomes common knowledge that $\theta \in y_n$. When $N = 1$, the public signal is uninformative.

It is important to stress that, since the government commits to a choice of P before learning the true state θ , there is no *strategic learning*, i.e., the choice of P does not change the speculators' beliefs about what the government knows.⁹ In Section ??, we show that commitment is essential for our results.

In addition to the public signal, speculator i observes a private signal x_i , where

$$x_i = \theta + \sigma \varepsilon_i,$$

with $\sigma > 0$. The idiosyncratic noise ε_i is drawn from a distribution with probability density

⁷ There are two interpretations for the disclosure policy. One is that the government commits to a disclosure rule, observes θ , and then sends the prescribed signal $y(\theta)$. Another interpretation is that the government commits to an information acquisition procedure and, if the state is θ , the government observes $y(\theta)$ and announces it. The latter is in line with the Bayesian persuasion literature (see, for example, Kamenica and Gentzkow (2011)). In this case, the government is not more informed than the speculators when the public signal is sent.

⁸ In this presentation, we restrict the analysis to partitions with a finite number of intervals. The results still hold if the partitions can have a countable number of intervals.

⁹ This is in contrast to Angeletos et al. (2006).

function (pdf) $g(\cdot)$, and cumulative distribution function (cdf) $G(\cdot)$. Each ε_i is independently and identically distributed across agents and independent of θ . We assume that $\text{supp}(g) = [-\bar{\varepsilon}, \bar{\varepsilon}]$, $\bar{\varepsilon} > 0$, and that $g(\cdot)$ is differentiable on $(-\bar{\varepsilon}, \bar{\varepsilon})$. Define $\varepsilon = \sigma\bar{\varepsilon}$, and let $2\varepsilon < \min\{\underline{\theta}, 1 - \bar{\theta}\}$.

The derivative of $g(\cdot)$, $g'(\cdot)$, is assumed to be bounded and such that

$$\text{if } g'(\bar{\varepsilon}) < 0, \text{ then } g'(\hat{\varepsilon}) \leq 0 \quad \forall \hat{\varepsilon} \in (\bar{\varepsilon}, \bar{\varepsilon}). \quad (1)$$

Since the common prior on θ is uniform on $[0, 1]$, the posterior distribution of θ given private signal x and public signal y has probability density function $\phi_y(\theta|x)$, where

$$\phi_{y_n}(\theta|x) = \begin{cases} \frac{\frac{1}{\sigma} g(\frac{x-\theta}{\sigma})}{G(\frac{x-m_{n-1}}{\sigma}) - G(\frac{x-m_n}{\sigma})}, & \text{if } \theta \in y_n \\ 0, & \text{otherwise} \end{cases}. \quad (2)$$

The derivation of $\phi_{y_n}(\theta|x)$ is presented in Appendix B.1.¹⁰

3.3 Equilibrium

We solve this game by backward induction. In the last stage, given an attack of size α and a state θ , the government optimally chooses to abandon the peg if and only if $c(\alpha, \theta) \geq v$. In the second stage, given a partition P , speculators observe the public signal and their own private signal. Anticipating the government's decision in the next stage, they simultaneously decide whether to attack the currency or not. In the first stage, the government chooses a partition P . The multiplicity in the second stage of the game poses a selection problem that we solve by assuming that the government only cares about the worst equilibrium outcome. Alternatively, we could assume that speculators play the equilibrium strategy that maximizes their own payoff (or, equivalently, the one that minimizes the government's payoff).

More formally, suppose the government chooses a partition $P = \{m_n\}_{n=0}^N$. Let $p_n = \mathbb{P}(\theta \in y_n)$ be the probability that θ lies in the interval y_n of the partition.¹¹ In addition, consider the subgame that follows the disclosure of $y = y_n$. Denote V_n the infimum of all government's equilibrium payoffs when $y = y_n$.¹² We let $V(P) = \sum_{n=1}^N p_n V_n$. The government's problem is to choose P to maximize $V(P)$.

¹⁰ There is a finite number of pairs (x, y) that fully reveal θ : when $y = y_n$ and $x = m_n + \varepsilon$, we have $\mathbb{P}(\theta = m_n | y = y_n, x = m_n + \varepsilon) = 1$; likewise, when $y = y_1$ and $x = -\varepsilon$, then $\mathbb{P}(\theta = 0 | y = y_1, x = -\varepsilon) = 1$. For all other pairs (x, y) , the conditional density of θ is given by (2).

¹¹ Since we assume that θ is uniformly distributed on $[0, 1]$, we have $p_n = m_n - m_{n-1}$.

¹² Such infimum always exists as the government always has the option to abandon the peg, so the equilibrium payoff is bounded below by 0.

The problem of the government in the last stage is simple. For each θ , let $a(\theta)$ be the solution to $v = c(a, \theta)$. This function represents the critical mass of speculators that have to attack the currency in order to induce the government to abandon the peg. Note that, given our assumptions on $c(\cdot, \cdot)$, we have that $a(\cdot)$ is continuous, $a(\theta) = 0$ for $\theta \leq \underline{\theta}$, and $a(\cdot)$ is strictly increasing for $\theta > \underline{\theta}$.

For a given profile of strategies for the speculators, the measure of speculators who attack the currency given a pair of signals (x, y) is denoted by $\pi(x, y)$. If the state is θ , the proportion of speculators who attack the currency is given by

$$s(\theta, \pi) = \int_{\theta-\varepsilon}^{\theta+\varepsilon} \pi(x, y_{n(\theta)}) \frac{1}{\sigma} g\left(\frac{x-\theta}{\sigma}\right) dx. \quad (3)$$

where $y_{n(\theta)}$ is the public signal sent according to P when the state is θ . The government maintains the peg when

$$s(\theta, \pi) < a(\theta). \quad (4)$$

Thus, the event where there is a regime change is given by

$$A(\pi) = \{\theta : s(\theta, \pi) \geq a(\theta)\}. \quad (5)$$

The payoff to a speculator from attacking the currency at state θ , when the aggregate strategy is π , is given by

$$h(\theta, \pi) = \begin{cases} e^* - f(\theta) - t, & \text{if } \theta \in A(\pi) \\ -t, & \text{if } \theta \notin A(\pi) \end{cases}. \quad (6)$$

The expected payoff from attacking the currency given a pair of signals (x, y_n) is given by¹³

$$\begin{aligned} u_{y_n}(x, \pi) &= \int_{[x-\varepsilon, x+\varepsilon] \cap y_n} h(\theta, \pi) \phi_{y_n}(\theta|x) d\theta \\ &= \int_{[x-\varepsilon, x+\varepsilon] \cap y_n \cap A(\pi)} [e^* - f(\theta)] \phi_{y_n}(\theta|x) d\theta - t. \end{aligned} \quad (8)$$

¹³ Equation (7) holds for all but a finite number of pairs (x, y) , as described in footnote 10. If $y = y_n$ and $x = m_n + \varepsilon$,

$$u_{y_n}(m_n + \varepsilon, \pi) = [e^* - f(m_n)]I(m_n) - t, \quad (7)$$

where $I(\theta)$ is an indicator function that equals 1 if the peg is abandoned at state θ . Similarly,

$$u_{y_1}(-\varepsilon, \pi) = e^* - f(0) - t,$$

since the peg is always abandoned when $\theta = 0$.

In equilibrium, $\pi(x, y) = 1$ if $u_y(x, \pi) > 0$, and $\pi(x, y) = 0$ if $u_y(x, \pi) < 0$.

3.4 Equilibrium properties

We now present some auxiliary results, which are similar to the ones in Morris and Shin (1998). The first result shows that, if other speculators are more likely to attack the currency peg for every private signal x , then the expected payoff from attacking increases.

Lemma 1. *For a given public signal y , if $\pi(x, y) \geq \pi'(x, y)$ for all x , then $u_y(x, \pi) \geq u_y(x, \pi')$ for all x .*

Proof: See Appendix B.2. □

For $k \in [-\varepsilon, 1 + \varepsilon]$, let the indicator function I_k be defined as

$$I_k(x) = \begin{cases} 1, & \text{if } x < k \\ 0, & \text{if } x \geq k \end{cases}. \quad (9)$$

When aggregate short sales are given by I_k (in particular, short sales will not depend on the public signal y), the proportion of speculators who attack the currency at state θ is given by

$$s(\theta, I_k) = G\left(\frac{k - \theta}{\sigma}\right). \quad (10)$$

Note that $s(\theta, I_k)$ is strictly decreasing in θ for $\theta \in (k - \varepsilon, k + \varepsilon)$, and constant otherwise.

We denote by θ_k the largest value of θ at which the government finds it optimal to abandon the currency peg when the speculators' aggregate short sales are given by I_k . As in Morris and Shin (1998), let $\psi(k) = \min\{\theta_k - k, \varepsilon\}$. Appendix B.3 provides a derivation of θ_k and ψ . The threshold θ_k is increasing in k , and the government finds it optimal to abandon the peg for all $\theta \leq \theta_k$. The function $\psi(\cdot)$ is continuous, $\psi(k) = \varepsilon$ for $k \leq \underline{\theta} - \varepsilon$, $\psi(1 + \varepsilon) = -\varepsilon$, and $\psi(\cdot)$ is strictly decreasing for $k \in (\underline{\theta} - \varepsilon, 1 + \varepsilon]$.

Let X_y denote the set of private signals that can be received by the speculators when the public signal is y . Then $X_{y_1} = [-\varepsilon, m_1 + \varepsilon]$ and, for $n > 1$, $X_{y_n} = (m_{n-1} - \varepsilon, m_n + \varepsilon]$.

Since the currency peg is abandoned if and only if $\theta \in [0, k + \psi(k)]$, the payoff function $u_{y_n}(k, I_k)$ is given by

$$u_{y_n}(k, I_k) = \int_{[k-\varepsilon, k+\psi(k)] \cap y_n} [e^* - f(\theta)] \phi_{y_n}(\theta|k) d\theta - t, \quad (11)$$

for all $k \in X_{y_n}$.¹⁴

¹⁴ As in footnote 13, equation (11) holds for all but a finite number of (x, y) that fully reveal θ . The reader can check

Lemma 2. For a given public signal y , $u_y(k, I_k)$ is continuous in k , for all possible private signals $k \in X_y$.

Proof: See Appendix B.4. □

Let $u(k, I_k)$ be the payoff function when there is no public signal. Then

$$u(k, I_k) = \int_a^b [e^* - f(\theta)] \frac{\frac{1}{\sigma} g\left(\frac{x-\theta}{\sigma}\right)}{G\left(\frac{x}{\sigma}\right) - G\left(\frac{x-1}{\sigma}\right)} d\theta - t, \quad (12)$$

where $a = \max\{k - \varepsilon, 0\}$, and $b = k + \psi(k)$. Note that the payoff function is continuous in k . The following lemma shows that it is also strictly decreasing in k .

Lemma 3. For $k \in (\varepsilon, 1 - \varepsilon)$, the payoff function $u(k, I_k)$ is strictly decreasing in k .

Proof: See Appendix B.5. □

4 Optimal signal structure

This section presents the results of the model with commitment when, in case of multiplicity after a partition choice, the government only cares about the worst equilibrium outcome.¹⁵ First, we show that there is no loss of generality in considering partitions with at most two intervals. Then, we prove that it is not optimal for the government to chose a one-interval partition and send the uninformative signal. Finally, the optimal partition is characterized.

4.1 No loss of generality in two-interval partitions

Let $\Phi_y(\theta|x)$ denote the cumulative distribution function of θ conditional on private signal x and public signal y . To find the optimal partition, the following assumption is made.

that

$$\begin{aligned} \lim_{k \rightarrow m_n + \varepsilon} u_{y_n}(k, I_k) &= u_{y_n}(m_n + \varepsilon, I_{m_n + \varepsilon}), \forall n, \\ \lim_{k \rightarrow -\varepsilon} u_{y_1}(k, I_k) &= u_{y_1}(-\varepsilon, I_{-\varepsilon}), \end{aligned}$$

and that for a fixed k ,

$$\begin{aligned} \lim_{x \rightarrow m_n + \varepsilon} u_{y_n}(x, I_k) &= u_{y_n}(m_n + \varepsilon, I_k), \forall n, \\ \lim_{x \rightarrow -\varepsilon} u_{y_1}(x, I_k) &= u_{y_1}(-\varepsilon, I_k). \end{aligned}$$

For the sake of brevity, in the remainder of the paper we omit these finite number of cases. The limits above guarantee that our results still hold.

¹⁵ When there is no ambiguity, we say equilibrium when we mean the equilibrium of the subgame that follows the choice of P .

Assumption 1. Let the public signal be y . For any pair of private signals x_1 and x_2 , with $x_1 < x_2$, $\Phi_y(\theta|x_2) \leq \Phi_y(\theta|x_1)$ for all θ .

This assumption means that the distribution of θ conditional on y and x_2 first-order stochastically dominates the distribution of θ conditional on y and x_1 . In Appendix B.6, it is shown that Assumption 1 is satisfied, for example, if the idiosyncratic noise on $[-\bar{\varepsilon}, \bar{\varepsilon}]$ follows a concave or a truncated normal distribution. Assumption 1 leads to the following lemma.

Lemma 4. Suppose that Assumption 1 is satisfied. When the aggregate strategy is given by I_k , the payoff from attacking the currency, $u_y(x, I_k)$, is decreasing in the private signal x .

Proof: Suppose that the aggregate strategy is given by I_k . Let $I(\theta)$ be an indicator function that equals 1 if the currency peg is abandoned when the state is θ . Since, by assumption, speculators follow a cutoff rule, $I(\theta)$ is weakly decreasing in θ .¹⁶ Define

$$U(\theta) = [f(\theta) - e^*]I(\theta),$$

which is negative and increasing. Consider a public signal y and a pair of private signals x_1 and x_2 , with $x_1 < x_2$. Then

$$\int_0^1 U(\theta)d\Phi_y(\theta|x_2) \geq \int_0^1 U(\theta)d\Phi_y(\theta|x_1),$$

where the inequality comes from Assumption 1 and the fact that U is increasing. Hence

$$\begin{aligned} u_y(x_1, I_k) &= - \int_0^1 U(\theta)d\Phi_y(\theta|x_1) - t \\ &\geq - \int_0^1 U(\theta)d\Phi_y(\theta|x_2) - t \\ &= u_y(x_2, I_k), \end{aligned}$$

which completes the proof. □

The following two lemmas are needed for the main results. The first one shows a sufficient condition for a cutoff strategy for the speculators to exist in equilibrium, while the second lemma characterizes the speculators' equilibrium strategy that minimizes the government's payoff for a given public signal y .

Lemma 5. Let the public signal be y , and suppose that Assumption 1 is satisfied. If k solves $u_y(k, I_k) = 0$, then there is an equilibrium where the aggregate short sales after y is observed are given by I_k .

¹⁶ $I(\theta) = 1$, if $\theta \leq \theta_k$; and $I(\theta) = 0$, if $\theta > \theta_k$.

Proof: Fix y . Suppose that k solves $u_y(k, I_k) = 0$, and that the aggregate short sales are given by I_k . If a speculator receives a signal $x < k$, his payoff from attacking the currency is given by

$$u_y(x, I_k) \geq u_y(k, I_k) = 0,$$

where the inequality comes from Lemma 4. Hence the payoff from attacking is (weakly) larger than the payoff from not attacking. Similarly, if $x \geq k$, the payoff from attacking the currency is

$$u_y(x, I_k) \leq u_y(k, I_k) = 0,$$

therefore not attacking yields a (weakly) larger payoff than attacking. Both statements imply that following a cutoff rule I_k is optimal for the speculator, given that all other speculators are using the same rule. This means that there exists an equilibrium in which I_k is the aggregate selling strategy. \square

Lemma 6. *Suppose that Assumption 1 is satisfied. For a given public signal y ,*

- i. if $u_y(k, I_k) < 0$ for all $k \in X_y$, then, in any equilibrium, $\pi(x, y) = 0$ for all $x \in X_y$.*
- ii. if $u_y(k', I_{k'}) \geq 0$ for some $k' \in X_y$, then, in the worst equilibrium for the government, speculators use the cutoff rule I_k after observing y , where $k = \sup\{k' \in X_{y_n} : u_{y_n}(k', I_{k'}) \geq 0\}$.*

Proof:

- i. Suppose that $u_y(k, I_k) < 0$ for all $k \in X_y$. Let π be any equilibrium strategy, and suppose by way of contradiction that there is $x' \in X_y$ such that $\pi(x', y) > 0$. If this is true, then the set $\{x \in X_y : \pi(x, y) > 0\}$ is non-empty and we can define \bar{x}_y as*

$$\bar{x}_y = \sup\{x \in X_y : \pi(x, y) > 0\}.$$

Note that $\bar{x}_y \in X_y$ because X_y is right-closed. Also note that, if π is an equilibrium strategy, then for any $\tilde{x} \in \{x \in X_y : \pi(x, y) > 0\}$, it has to be true that $u_y(\tilde{x}, \pi) \geq 0$. By the continuity of u_y in the private signal, $u_y(\bar{x}_y, \pi) \geq 0$. From Lemma 1,

$$u_y(\bar{x}_y, I_{\bar{x}_y}) \geq u_y(\bar{x}_y, \pi) \geq 0$$

Thus, $u_y(\bar{x}_y, I_{\bar{x}_y}) \geq 0$, which contradicts the assumption that $u_y(k, I_k) < 0$ for all $k \in X_y$.

- ii. If $u(k, I_k) > 0$, by continuity (Lemma 2), it has to be true that k is the right bound of the interval X_y and, by the decreasing property of u_y in x (Lemma 4), I_k is an equilibrium strategy. If*

$u(k, I_k) = 0$, then we know that I_k is an equilibrium strategy (Lemma 5). Using the same arguments from part *i.*, assume by way of contradiction that there is an equilibrium with $\pi(x', y) > 0$ for some $x' > k$. Let $\bar{x}_y = \sup\{x \in X_y : \pi(x, y) > 0\} \in X_y$. By Lemma 1, $u_y(\bar{x}_y, I_{\bar{x}_y}) \geq u_y(\bar{x}_y, \pi) \geq 0$, which contradicts the assumption that k is the supremum of the set $\{k' \in X_y : u_y(k', I_{k'}) \geq 0\}$.

□

Recall that $\bar{\theta}$ defines the threshold θ at which investors' payoff from a successful attack is 0. The following lemma characterizes the worst equilibrium outcome for the government given a choice of P .

Lemma 7. *Suppose that Assumption 1 is satisfied and consider an arbitrary partition $P = \{m_n\}_{n=0}^N$. Given P , the equilibrium that minimizes the government's payoff involves the following:*

- i. for all n such that $m_n \leq \bar{\theta}$, speculators always attack the currency if $y = y_n$;*
- ii. for all n such that $m_{n-1} \geq \bar{\theta}$, speculators never attack the currency if $y = y_n$;*
- iii. for all n such that $m_{n-1} < \bar{\theta}$ and $m_n > \bar{\theta}$,¹⁷ speculators never attack if $u_{y_n}(k, I_k) < 0$ for all $k \in X_{y_n}$. Otherwise, speculators follow I_{k_n} after observing y_n , where $k_n = \sup\{k \in X_{y_n} : u_{y_n}(k, I_k) \geq 0\}$.*

Proof: *i.* Let n be such that $m_n \leq \bar{\theta}$. If speculators always attack the currency after observing y_n , then the government abandons the peg (because $c(1, 1) > v$) and the speculators have a positive payoff (by the definition of $\bar{\theta}$). Hence always attacking after observing y_n is an equilibrium strategy for the speculators, and no other strategy can yield a lower payoff for the government when $\theta \in y_n$.

ii. Let n be such that $m_{n-1} \geq \bar{\theta}$. If a speculator attacks after observing y_n , his expected payoff is strictly negative. Hence there is no equilibrium where speculators attack when $\theta \in y_n$.

iii. Follows immediately from Lemma 6. □

Lemma 7 provides the intuition as to why there is no loss of generality in considering only two-interval partitions. If there are several n such that $m_n \leq \bar{\theta}$, then the government could group all these y_n . Likewise, if there are several n such that $m_{n-1} \geq \bar{\theta}$, the government can group these y_n . This rules out any partition P with four or more intervals.

Now consider a partition P with three intervals, that is, $P = \{0, m_1, m_2, 1\}$, where $m_1 < \bar{\theta} < m_2$. If the government maintains the peg for all $\theta \in (m_1, m_2]$, then the government could have chosen the partition $P' = \{0, m_1, 1\}$. If the government abandons the peg for all $\theta \in (m_1, m_2]$, then the government could have chosen the partition $P' = \{0, m_2, 1\}$. If the government abandons the peg for some but not all $\theta \in (m_1, m_2]$, then, by Lemma 7, speculators use a cutoff rule when $y = (m_1, m_2]$.

¹⁷ There is at most one such n .

This cutoff strategy generates a threshold $\theta' \in y_2$, such that the government abandons the peg if $\theta \in (m_1, \theta']$ and maintains the peg if $\theta \in (\theta', m_2]$. But if this is the case, then the government could have chosen the partition $P' = \{0, \theta', 1\}$. We use Lemma 9 (in the appendix B.7) to formalize this result, which is presented in Theorem 1.

Theorem 1. *Suppose that Assumption 1 is satisfied. Then, for any partition $P = \{m_n\}_{n=0}^N$ with $N > 2$, there exists $P' = \{m'_n\}_{n=0}^N$ with $N' = 2$, such that $V(P') \geq V(P)$.*

Proof: Given Lemma 7, the only non trivial result left to show is that, for any $P = \{0, m_1, m_2, 1\}$, with $m_1 < \bar{\theta} < m_2$, there is a $P' = \{0, m', 1\}$ such that $V(P') \geq V(P)$.

- *Case 1: the government maintains the peg for all θ in y_2 .* Consider the alternative partition $P' = \{0, m_1, 1\}$. The government cannot be worse off if $\theta \leq m_1$.

We know from Lemma 7 that $u_{(m_1, m_2]}(k, I_k) < 0$ for any $k \in X_{(m_1, m_2]}$. Since $m_2 > \bar{\theta}$, we also know that $u_{(m_2, 1]}(k, I_k) < 0$ for any $k \in X_{(m_2, 1]}$. From Lemma 9,

$$u_{(m_1, 1]}(k, I_k) \leq u_{(m_1, m_2]}(k, I_k) < 0, \quad \text{for any } k \in (m_1 - \varepsilon, m_2 + \varepsilon],$$

and

$$u_{(m_1, 1]}(k, I_k) = u_{(m_2, 1]}(k, I_k) < 0, \quad \text{for any } k \in (m_2 + \varepsilon, 1 + \varepsilon].$$

The inequalities imply that $u_{(m_1, 1]}(k, I_k) < 0$ for $k \in X_{(m_1, 1]}$. From Lemma 6, no one attacks if $\theta > m_1$. Thus, $V(P') \geq V(P)$.

- *Case 2: the government abandons the peg for all θ in y_2 .* Consider the partition $P' = \{0, m_2, 1\}$. The government is not worse off if $\theta \leq m_2$. If $\theta > m_2$, speculators observe the public signal $(m_2, 1]$, and since $m_2 > \bar{\theta}$, no one attacks. Thus, $V(P') \geq V(P)$.
- *Case 3: the government abandons the peg at some but not all θ in y_2 .* From Lemma 6, speculators use a cutoff rule I_{k_2} .

From Lemma 7, speculators follow a cutoff rule I_{k_2} after observing y_2 , where $k_2 = \sup\{k \in X_{y_2} : u_{y_2}(k, I_k) = 0\}$. Given the speculators' strategy, there exists $\theta_{k_2} \in (m_1, m_2]$ such that the peg is abandoned if and only if $\theta \leq \theta_{k_2}$. From Lemma 9, increasing m_1 would never increase the cutoff k , and it would never increase the threshold θ_k .

This implies that, with the partition $\tilde{P}' = \{0, \theta_{k_2}, m_2, 1\}$, no one attacks if $\theta \in (\theta_{k_2}, m_2]$. From Case 2, when the partition is $P' = \{0, \theta_{k_2}, 1\}$, there is no attack if $\theta \in (\theta_{k_2}, m_2]$. By changing the partition from P to P' , the government no longer has to pay a cost to defend the currency on $(\theta_{k_2}, \theta_{k_2} + \varepsilon)$, therefore $V(P') > V(P)$.

□

For the remainder of the paper, we denote the two element partition $P = \{0, m, 1\}$ by P^m .

4.2 No disclosure is not optimal

This subsection shows that it is not optimal for the government to send the uninformative public signal, i.e., to set $N = 1$. The result is obtained by proving that there exist partition choices with $N = 2$ that strictly dominate the uninformative partition with $N = 1$. When $N = 2$, the government's problem is equivalent to a choice of $m \in [0, 1]$ such that speculators will learn whether $\theta \leq m$ or $\theta > m$. Given the choice of m , they observe the public signal $y \in \{y_l, y_h\}$, drawn as follows:¹⁸

$$y = \begin{cases} y_l, & \text{if } \theta \in [0, m] \\ y_h, & \text{if } \theta \in (m, 1] \end{cases} . \quad (13)$$

In the model without a public signal, which has the same outcome as the case $m = 1$, it is known from Morris and Shin (1998) that the equilibrium is unique. In that equilibrium, speculators follow a cutoff rule and attack the currency if and only if their private signal is below x^* , where x^* solves $u(x^*, I_{x^*}) = 0$. The currency peg is thus abandoned if and only if $\theta \leq \theta^*$, where θ^* makes the government indifferent between defending the peg or not.¹⁹

The next lemma shows that, for any choice of $m < 1$, there is always an equilibrium where government and speculators coordinate on the public signal for at least one realization of y .

Lemma 8. *Consider the subgame that follows the choice of $m < 1$ by the government. If $m \leq \bar{\theta}$, there exists an equilibrium where the government abandons the peg when $\theta \in [0, m]$, and the speculators attack the currency after observing $y = y_l$. If $m \geq \underline{\theta}$, there exists an equilibrium where the government defends the peg if $\theta \in (m, 1]$, and there is no attack following the signal y_h .*

Proof: Let $m \leq \bar{\theta}$ and suppose that all speculators attack the currency after observing $y = y_l$. Given the speculators' aggregate strategy, the government abandons the currency peg if $\theta \in [0, m]$, and it is indeed optimal for each speculator to attack if $y = y_l$. Now let $m \geq \underline{\theta}$ and suppose that no speculator attacks the currency after observing $y = y_h$. Given the speculators' strategy, the government defends the peg if $\theta \in (m, 1]$, and therefore it is indeed optimal for each speculator not to attack if $y = y_h$. □

For any choice of $m \in [\underline{\theta}, \bar{\theta}]$, there exist an equilibrium where speculators follow the public signal: they coordinate on attacking if $y = y_l$, and they refrain from attacking if $y = y_h$. In this

¹⁸ To distinguish the case where the government is restricted to $N \leq 2$ from the general case, we change the notation: we use m instead of m_1 ; y_l and y_h instead of y_1 and y_2 .

¹⁹ That is, θ^* solves $s(\theta, I_{x^*}) = a(\theta)$.

equilibrium, the currency peg is abandoned if $\theta \leq m$. This result is presented in Corollary 1 below.

Corollary 1. *For all $m \in [\underline{\theta}, \bar{\theta}]$, there exists an equilibrium where the currency peg is abandoned if and only if $\theta \in [0, m]$, and speculators attack the currency if and only if $\theta \in [0, m]$.*

The following theorem compares the equilibrium outcomes for a given choice of m with the unique equilibrium outcome in the absence of a public signal.

Theorem 2. *Fix m . If $m = \theta^*$, there is a unique equilibrium, in which speculators follow the public signal. If $m \neq \theta^*$, the equilibrium may not be unique. There are bounds $\underline{x}^* \geq x^*$ and $\bar{x}^* \leq x^*$ such that, in any equilibrium, $\pi(x, y_l) \geq I_{\underline{x}^*}(x)$ and $\pi(x, y_h) \leq I_{\bar{x}^*}(x)$ for all x . The equilibria are as follows:*

- i. if $m < \theta^*$: speculators always attack the currency and the peg is abandoned if $y = y_l$; moreover, if $m \in (x^* - \varepsilon, \theta^*)$, then $\bar{x}^* < x^*$;*
- ii. if $m > \theta^*$: the currency is not attacked and the peg is defended if $y = y_h$; moreover, if $m \in (\theta^*, x^* + \varepsilon)$, then $\underline{x}^* > x^*$.*

Proof: See Appendix B.8. □

Part *i.* of Theorem 2 states that, when the government chooses $m < \theta^*$ and the public signal is $y = y_h$, the set of private signals that induce attack is contained in the set of private signals that would induce attack in the absence of a public signal. Thus, for any $\theta \in y_h$, the size of the attack does not increase. Moreover, if Assumption 1 holds, we use Lemma 7 to conclude that, in the worst equilibrium for the government, the cutoff used when $y = y_h$ is below the cutoff when there is no public signal. Conversely, part *ii.* and Assumption 1 imply that, when $m > \theta^*$, speculators will use a higher cutoff when $y = y_l$. This can be seen in Figure 1, which is constructed from the numerical example in Appendix A.

If the government chooses $m = \theta^*$, the equilibrium is unique and the currency peg is abandoned if and only if $\theta \leq \theta^*$, as in the equilibrium of the game without a public signal. However, no speculator attacks the currency when $\theta > \theta^*$, whereas without the public signal some speculators would still attack the currency for some $\theta > \theta^*$, increasing the cost of maintaining the peg. Thus, the government is strictly better off by the introduction of the public signal. Note that the speculators are also strictly better off now that all of them attack when the currency peg is abandoned, and no one attacks when peg is maintained.

Table 1 summarizes the results in Theorem 2.²⁰

²⁰ If $m > \theta^*$, the currency is not attacked when $y = y_h$. For m close enough to θ^* , the government's payoff can be higher with the public signal if the increase in the probability of devaluation when $y = y_l$ is offsetted by a lower cost of defending the currency when $y = y_h$.

		Probability of Devaluation	Equilibrium payoff
	$m \leq x^* - \varepsilon$	weakly lower	weakly higher
$x^* - \varepsilon <$	$m \leq \theta^*$	strictly lower	strictly higher
$\theta^* <$	$m < x^* + \varepsilon$	strictly higher	effect is ambiguous
	$m \geq x^* + \varepsilon$	weakly higher	weakly lower

Table 1: Given m , the second column compares the equilibrium probability of currency devaluation with the case without a public signal. The last column compares the government's possible equilibrium payoffs with the unique equilibrium payoff in the game without a public signal.

Note that any choice $m > \theta^*$ is strictly dominated by $m = \theta^*$. Compared to the unique equilibrium with $m = \theta^*$, any equilibrium with $m > \theta^*$ features a strictly higher probability of devaluation and, for all θ , there is a weakly larger mass of speculators attacking the currency. This leads to the following corollary.

Corollary 2. *The choice of any $m > \theta^*$ is strictly dominated by $m = \theta^*$.*

Corollary 2 implies that the choice of $m = 1$ is strictly dominated by choosing $N = 2$ and $m = \theta^*$. Thus sending an uninformative public signal is not optimal for the government.

4.3 Characterization of the optimal signal structure

We are now ready to find the optimal partition for the government. Define M as

$$M = \{m : \text{there is no attack in any equilibrium after } y_h\}.$$

Note that $M \neq \emptyset$ because $\theta^* \in M$. Define \underline{m} as

$$\underline{m} = \inf M.$$

Figure 2 from Appendix A gives the intuition about how the optimal partition should be. Any choice of $m > \theta^*$ is strictly dominated by $m = \theta^*$, which leads to a unique equilibrium, with no attacks when $\theta \in (m, 1]$. Starting from $m = \theta^*$, as m decreases, the government is strictly better off as long as the equilibrium is still unique. Decreasing m will increase the range of fundamentals that lead to no currency attacks. However, there is a discontinuity in the government's payoff at \underline{m} . Decreasing m even further to the region where it leads to multiple equilibria makes the government strictly worse off. Thus, the government wants decrease m up to the limit where the equilibrium is still unique, \underline{m} . This result is formalized in the following theorem.

Theorem 3. *Suppose that Assumption 1 is satisfied. For every partition P , $V(P) \leq \bar{V}$, where*

$$\bar{V} = \lim_{m \downarrow \underline{m}} V(P^m).$$

Then

- i. *if $\underline{m} \in M$, the government's equilibrium payoff is \bar{V} . In equilibrium, when $\theta > \underline{m}$, there are no attacks and the peg is maintained; and when $\theta \leq \underline{m}$, every speculator attacks the currency and the peg is abandoned. The government can achieve the payoff \bar{V} with the two-interval partition $P^{\underline{m}} = \{0, \underline{m}, 1\}$.*
- ii. *if $\underline{m} \notin M$, no equilibrium exists. However, the government can achieve a payoff arbitrarily close to \bar{V} .*

Proof: See Appendix B.10. □

The optimal policy involves setting m as low as possible, up to the limit where not attacking is the unique equilibrium action for speculators on $(m, 1]$. Note that for any m close enough to \underline{m} ²¹ any disclosure policy with $y_N = (m, 1]$ yields the same payoff for the government, regardless of the signal structure when $\theta \in [0, m]$. It is still true that when $\theta > m$, there is no attack and the peg is maintained; and when $\theta \leq m$, every speculator attacks the currency and the peg is abandoned. Thus the government could be arbitrarily precise when the state of fundamentals is very bad, but when the state is “not too bad” the government needs to be vague. This vagueness is used by the government to make the speculators uncertain about whether the state is intermediate (where a coordinated attack is profitable) or good (when attacking is never profitable), preventing them from attacking.

4.4 Vagueness

We conclude this section by showing that, even if the government could fully disclose the state, it would not be optimal to do so.

In Lemma 12 (Appendix B.9), we show that that $\underline{m} < \bar{\theta}$. Since $\underline{m} < \bar{\theta}$, there exists $m \in M \cap (\underline{m}, \bar{\theta})$ such that the partition $P^m = \{0, m, 1\}$ is strictly preferred to full disclosure. If the state is fully revealed, in the worst equilibrium for the government, speculators coordinate on attacking whenever $\theta < \bar{\theta}$. With the partition P^m , the government eliminates currency attacks between $(m, \bar{\theta})$. This leads to Proposition 1.

Proposition 1. *Full disclosure of the state is not an optimal policy for the government.*

²¹ That is, $m \in (\underline{m}, \bar{\theta})$.

5 No commitment

In this section, we drop the assumption that the government can commit to a disclosure policy. Here the government chooses the public signal after observing the true state θ . For simplicity, the government's strategy in the last stage of the game is taken as given.

The game between government and speculators becomes a signaling game, where θ can be interpreted as the government's type. A strategy for the government is a function $y : \Theta \rightarrow \Theta^2$ such that when the state is θ , the public signal is $y(\theta) = [\underline{y}(\theta), \bar{y}(\theta)]$ and speculators learn that $\theta \in [\underline{y}, \bar{y}]$.²² As before, we require that the government must make truthful announcements, that is, $\underline{y}(\theta) \leq \theta \leq \bar{y}(\theta)$ for all θ . Note that, if the government is not restricted to truthful announcements, there exists an equilibrium in which the speculators ignore the public signal. In this case, the equilibrium outcome is the same as the one in the game without a public signal. (And, possibly, there are even worse equilibria.)

A strategy for speculators is a function that gives, for every private signal x and every public signal, the corresponding action to be taken (attack or not). As before, let $\pi(x, y)$ be the aggregate selling strategy. The equilibrium concept in this section is the Perfect Bayesian Equilibrium (PBE) with symmetric strategies for the speculators.

Definition. The strategy profile (y, π) is a PBE if

1. for all $\theta \in [0, 1]$, $y(\theta)$ maximizes the government's payoff given π ;
2. for each possible signal y , there exist beliefs $\mu_{x,y}$ about θ such that $\pi(x, y)$ maximizes the speculator's expected payoff given $\mu_{x,y}$, the aggregate strategy π , and signals x and y ;
3. for each signal y such that $\int_{\{\tilde{\theta}: y(\tilde{\theta})=y\}} 1d\tilde{\theta} > 0$,

$$\mu_{x,y}(\theta) = \begin{cases} \frac{\frac{1}{\sigma} g(\frac{x-\theta}{\sigma})}{\int_{\{\tilde{\theta}: y(\tilde{\theta})=y\}} 1d\tilde{\theta}}, & \text{if } y(\theta) = y \\ 0, & \text{otherwise} \end{cases} . \quad (14)$$

4. for each signal y such that $\int_{\{\tilde{\theta}: y(\tilde{\theta})=y\}} 1d\tilde{\theta} = 0$

$$\text{support}(\mu_{x,y}(\theta)) \subset [x - \varepsilon, x + \varepsilon] \cap y \quad (15)$$

²² The restriction to closed intervals is made only for simplicity.

Consider the following profile of (y, π, μ) :

$$\begin{aligned}
y(\theta) &= \{\theta\}, \quad \forall \theta, \\
\mu_{x,y}(\theta) &= \begin{cases} 1, & \text{if } \theta = \max\{x - \varepsilon, \underline{y}\} \\ 0, & \text{otherwise} \end{cases} \\
\pi(x, y) &= \begin{cases} 1, & \text{if } \max\{x - \varepsilon, \underline{y}\} \leq \bar{\theta} \\ 0, & \text{otherwise} \end{cases}.
\end{aligned}$$

In equilibrium, the public signal reveals the true state of the fundamentals, and speculators attack if and only if $\theta < \bar{\theta}$. Since $\theta^* < \bar{\theta}$, the government is ex ante strictly worse off compared to the unique equilibrium of the game without a public signal. Furthermore, for all types θ the equilibrium payoff is weakly smaller.

To see that (y, π) is in fact an equilibrium with beliefs μ , first consider speculator i 's problem. If $\underline{y} = \bar{y} = \theta$, given that speculators follow π , it is only profitable for i to attack if $\underline{y} < \bar{\theta}$, which means that π is optimal on path. Now consider off path signals where $\underline{y} < \bar{y}$. When $\underline{y} \geq \bar{\theta}$, the speculators know that $\theta \geq \bar{\theta}$ and attacking is indeed not profitable. If $\underline{y} < \bar{\theta}$ and speculator i receives a private signal $x_i \leq \bar{\theta} + \varepsilon$, he believes that $\theta = \max\{x - \varepsilon, \underline{y}\} < \bar{\theta}$. The speculator also believes that everyone else received a private signal below $\bar{\theta} + \varepsilon$, and if aggregate sales are given by π , the speculator believes that other speculators will attack. Hence, attacking is profitable given μ , π , x_i , and y . Finally, if $\underline{y} < \bar{\theta}$ and $x_i > \bar{\theta} + \varepsilon$, the speculator knows that $\theta \geq \bar{\theta}$, and it is not profitable to attack. Thus, following π is optimal when $\underline{y} < \bar{\theta}$ and $x_i > \bar{\theta} + \varepsilon$.

Now we show that strategy y is optimal for the government given π and μ . Suppose that the government has a profitable deviation from y for some type $\theta' \in [0, 1]$.²³ Since there is no attack for $\theta \geq \bar{\theta}$, there can only be a profitable deviation if $\theta' < \bar{\theta}$. According to μ , speculators believe that $\theta < \bar{\theta}$ and attack. Thus, there is no profitable deviation from y .

The PBE above passes the intuitive criterion of Cho and Kreps (1987). Note that only types in $[0, \bar{\theta})$ could benefit from a deviation. However, if the speculators know that $\theta < \bar{\theta}$, they can coordinate on attacking the currency peg, thus a deviation would not be profitable.

This example shows that commitment is essential for the government to benefit from a public signal. When only truthful announcements are allowed, the speculators can exploit the fact that the government wants to reveal its type for $\theta \geq \bar{\theta}$. In this case, speculators are able to coordinate on attacking the currency peg whenever $\theta < \bar{\theta}$. If the government is allowed to lie, the speculators can simply ignore the public signal. The results are summarized in the following proposition.

Proposition 2. *Suppose that the government only cares about its lowest equilibrium payoff. If the govern-*

²³A deviation here is a signal $y = [\underline{y}, \bar{y}]$, with $\underline{y} < \bar{y}$ and $\theta' \in [\underline{y}, \bar{y}]$.

ment cannot commit to a disclosure policy, then it does not benefit from sending a public signal. Furthermore, when only truthful announcements are allowed, the government is strictly worse off with the introduction of a public signal.

6 Conclusion

This paper analyzes a model of currency attacks in which the government sends a credible public signal about the fundamentals of the economy. The government can partition the space of fundamentals and reveal in which interval the unknown state lies. The introduction of a public signal generates partial common knowledge about the fundamentals and it can lead to multiple equilibria. We find informative policies that strictly dominate no disclosure even if multiplicity arises. We also derive the optimal policy by assuming that the government only cares about the worst equilibrium outcome of each policy.

We find that sending very precise public signals can be harmful to the government. In fact, for any signal structure, there exists a two-signal policy that is preferred by the government. The optimal disclosure policy thus partitions the space of fundamentals into two intervals. In the lower interval, speculators coordinate on attacking the currency and the peg is abandoned; in the higher interval, no speculator attacks and the peg is maintained. The government is deliberately vague in order to induce speculators not to attack the currency in the higher interval. After the public signal reveals that the state is in the higher interval, speculators are not sure whether a coordinated attack is profitable or not, thus they refrain from attacking. If the government had chosen a finer partition, with more precise signals, speculators could have been able to coordinate on attacking for a wider range of fundamentals, making a devaluation more likely.

When the government cannot commit to a disclosure policy, we find equilibria in which the government is made worse off by sending a public signal. Thus commitment is key for the government to benefit from disclosing information.

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Appendices

A A numerical example

This section provides a numerical example for the baseline model with commitment. The state of fundamentals is uniformly distributed in $[0, 1]$:

$$\theta \sim U(0, 1).$$

The exchange rate in the absence of government intervention is

$$f(\theta) = \theta.$$

If the government maintains the peg, the exchange rate is

$$e^* = 1.$$

The cost of short selling is

$$t = 0.25.$$

The government's value of defending the currency is

$$v = 0.75.$$

The cost of defending the currency is

$$c(\alpha, \theta) = 1 - \theta + \alpha,$$

where α is the measure of speculators that attack the currency.²⁴

For a given θ , the government decides to abandon the peg if the fraction α of speculators attacking the currency is at least $a(\theta)$, where

$$a(\theta) = \begin{cases} 0, & \text{if } \theta \in [0, 0.25] \\ \theta - 0.25, & \text{if } \theta \in (0.25, 1] \end{cases}.$$

²⁴ Note that $c(0, 0) = 1 > v$ and $c(1, 1) = 1 > v$, but $c(0.1, 0.8) = 1 + 0.1 - 0.8 = 0.3 < v$, so there are regions where the government decides to maintain the peg.

Define $\underline{\theta}$ as the solution to $c(0, \theta) = v$, and define $\bar{\theta}$ as the solution to $e^* - f(\theta) = t$. We have

$$\underline{\theta} = 0.25, \bar{\theta} = 0.75.$$

Speculators receive a signal $x = \theta + \tilde{\varepsilon}$, where

$$\tilde{\varepsilon} \sim U(-\varepsilon, \varepsilon).$$

The precision of the signal is affected by the parameter ε :

$$\varepsilon = 0.1.$$

If a speculator receives the signal $x \in [-\varepsilon, 1 + \varepsilon]$, he will believe that θ is uniformly distributed in $[x - \varepsilon, x + \varepsilon] \cap [0, 1]$.

Let $\psi(k)$ solve

$$\frac{1}{2} - \frac{\psi(k)}{2\varepsilon} = k + \psi(k) - 0.25$$

or

$$\psi(k) \left(1 + \frac{1}{2\varepsilon}\right) = \frac{3}{4} - k$$

$$\psi(k) = \left(\frac{3}{4} - k\right) \left(1 + \frac{1}{.2}\right)^{-1} = \left(\frac{3}{4} - k\right) (6)^{-1} = \frac{1}{8} - \frac{k}{6}.$$

The speculators payoff from following I_k when there is no public signal is given by²⁵

$$\begin{aligned} u(k, I_k) &= \frac{1}{2\varepsilon} \int_{k-\varepsilon}^{k+\psi(k)} e^* - f(\theta) d\theta - t = 5 \int_{k-\varepsilon}^{k+\psi(k)} 1 - \theta d\theta - 0.25. \\ &= 5 \left[\theta - \frac{\theta^2}{2} \right] \Big|_{k-\varepsilon}^{k+\psi(k)} - 0.25 \\ &= 0.764k^2 - 1.854k + 0.861. \end{aligned}$$

Proceeding numerically,

$$u(x^*, I_{x^*}) = 0 \iff x^* \cong 0.626,$$

which implies

$$\theta^* \cong 0.64.$$

A speculator will attack the currency $x < x^*$, and will not attack if $x > x^*$. Given this rule, the

²⁵ For $k \in (0.1, 0.9)$.

government will abandon the peg when $\theta < \theta^*$, and maintain the peg if $\theta > \theta^*$.

For a given θ , the fraction of speculators attacking the currency is

$$\mathbb{P}(\theta + \tilde{\varepsilon} < x^*) = \mathbb{P}(\tilde{\varepsilon} < x^* - \theta) = \begin{cases} 0, & \text{if } \theta \geq x^* + \varepsilon \\ \frac{\varepsilon + x^* - \theta}{2\varepsilon}, & \text{if } \theta \in (x^* - \varepsilon, x^* + \varepsilon) \\ 1, & \text{if } \theta \leq x^* - \varepsilon \end{cases}$$

The expected payoff to the government is

$$\mathbb{P}(\theta < \theta^*) \cdot 0 + \mathbb{P}(\theta > \theta^*) \int_{\theta^*}^1 v - (1 - \theta + \alpha(\theta)) d\theta.$$

A.1 Public signal

Suppose that the government can commit to a public signal. The government chooses $m \in [0, 1]$ and emits a signal

$$y(\theta) = \begin{cases} y_l, & \text{if } \theta \leq m \\ y_h, & \text{if } \theta > m \end{cases}.$$

Given the parameters above, $\underline{m} = x^* - \varepsilon = 0.526$. Figure 1 depicts the mass of speculators who attack the currency for different choices of m , compared to the case where there is no public signal. It also shows the critical mass of attackers needed for the government to abandon the peg, $a(\theta)$.

Suppose that, in the case of multiplicity given a choice of m , the government only cares about the lowest equilibrium payoff. In this case, the government chooses a partition P to maximize $V(P)$. From Theorem 3, the government wants to set m as close as possible to \underline{m} . Since $\underline{m} = x^* - \varepsilon$, from Theorem 2, for $m = \underline{m}$, there exists an equilibrium where speculators follow I_{x^*} when $y = y_h$. Hence, the choice $m = \underline{m}$ leads to multiple equilibria, and it is dominated by any $m \in (\underline{m}, \theta^*]$, which lead to a unique equilibrium. Theorem 3 thus implies that there is no equilibrium for this game. However, the government can still achieve a payoff arbitrarily close to $\bar{V} = \sup_P V(P) = \lim_{m \downarrow \underline{m}} V(P^m)$. Figure 2 depicts the government's payoff $V(P^m)$ for all possible choices of $m \in [0, 1]$.

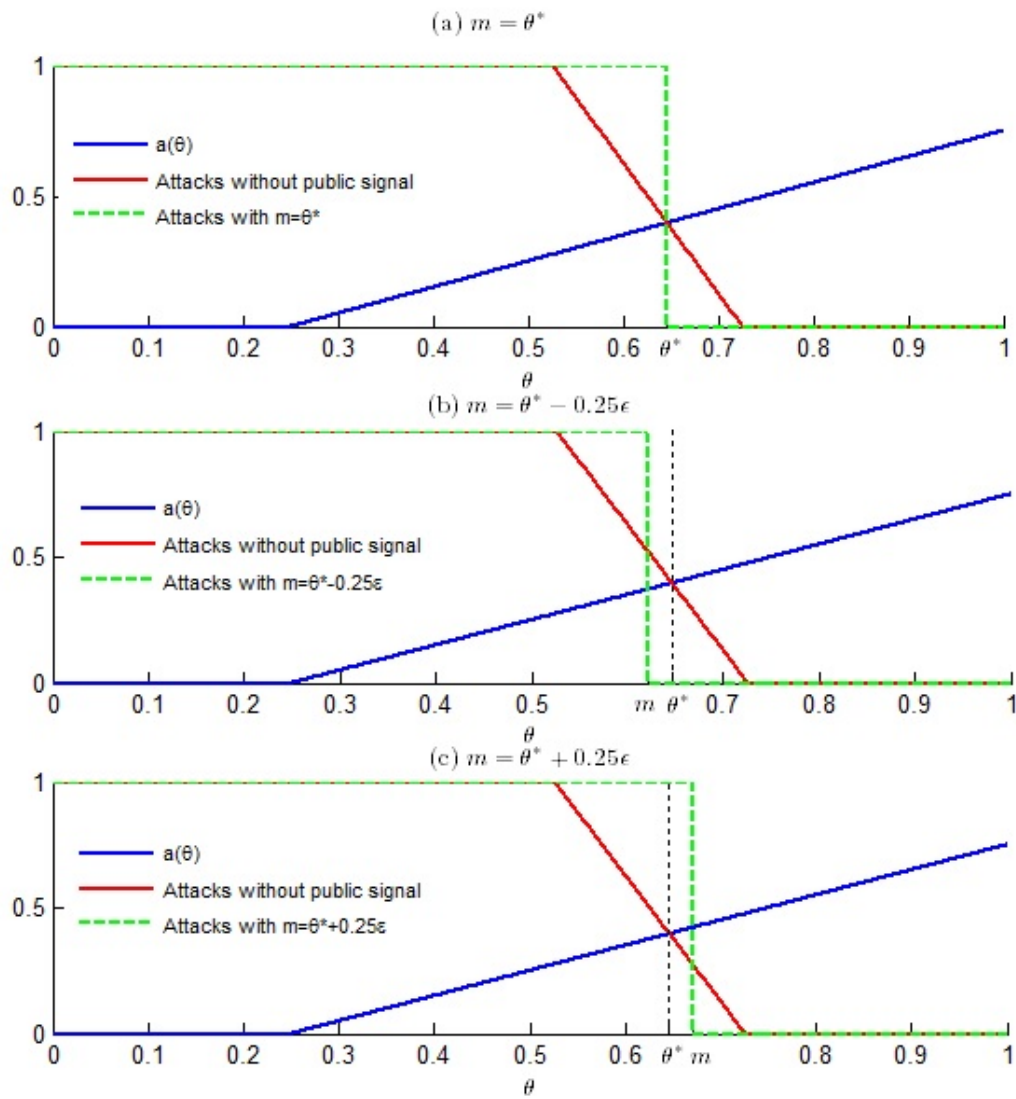


Figure 1: Mass of speculators attacking the currency for different choices of m , compared to the case with no public signal.

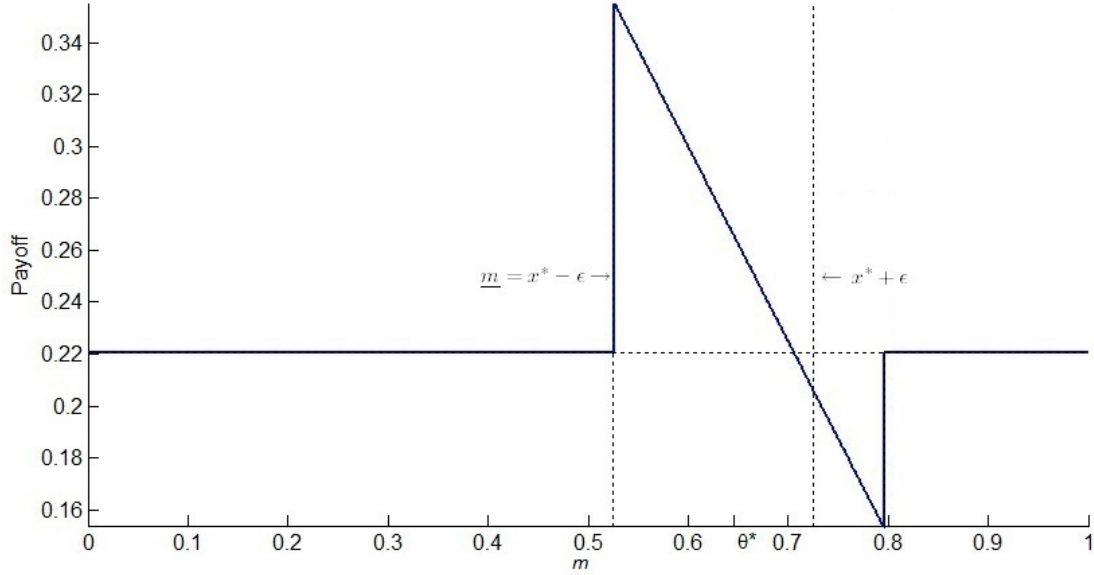


Figure 2: Government's payoff $V(P^m)$ given the choice of m .

B Proofs

B.1 Posteriors

For any pair of continuous random variables A and B , let g_{AB} denote their joint pdf. Let g_A and g_B denote the marginal pdfs, and let $g_{A|B}$ denote the pdf of A conditional on B . Finally, denote the cdfs by G_A and G_B . Following the main text, we denote the pdf of the idiosyncratic noise by g , and its cdf by G , omitting the subscripts.

B.1.1 No public signal

For $x \in (-\varepsilon, 1 + \varepsilon)$:

$$\begin{aligned} g_{\theta|x}(\theta|x) &= \frac{g_{\theta x}(\theta, x)}{g_x(x)} = \frac{\frac{1}{\sigma} g\left(\frac{x-\theta}{\sigma}\right) g_{\theta}(\theta)}{\int_{-\infty}^{+\infty} \frac{1}{\sigma} g\left(\frac{x-\tilde{\theta}}{\sigma}\right) g_{\theta}(\tilde{\theta}) d\tilde{\theta}} \\ &= \frac{\frac{1}{\sigma} g\left(\frac{x-\theta}{\sigma}\right)}{G\left(\frac{x}{\sigma}\right) - G\left(\frac{x-1}{\sigma}\right)}, \text{ if } \theta \text{ is uniform on } [0, 1]. \end{aligned}$$

For $x \in \{-\varepsilon, 1 + \varepsilon\}$: $\mathbb{P}(\theta = 0|x = -\varepsilon) = 1$; $\mathbb{P}(\theta = 1|x = 1 + \varepsilon) = 1$.

B.1.2 Public signal

Posterior of $\theta \in y_n$ conditional on the public signal $y = y_n$:

$$g_{\theta}(\theta|y_n) = \frac{g_{\theta}(\theta)}{P(m_{n-1} \leq \theta \leq m_n)} = \frac{g_{\theta}(\theta)}{G_{\theta}(m_n) - G_{\theta}(m_{n-1})}.$$

Distribution of x_i conditional on $y = y_n$:

$$\begin{aligned} P(x_i \leq x|y_n) &= \frac{\int_{-\infty}^x \int_{m_{n-1}}^{m_n} g_{\theta x}(\theta, \tilde{x}) d\theta d\tilde{x}}{G_{\theta}(m_n) - G_{\theta}(m_{n-1})} = \frac{\int_{-\infty}^x \int_{m_{n-1}}^{m_n} \frac{1}{\sigma} g\left(\frac{\tilde{x}-\theta}{\sigma}\right) g_{\theta}(\theta) d\theta d\tilde{x}}{G_{\theta}(m_n) - G_{\theta}(m_{n-1})} \\ \Rightarrow g_x(x|y_n) &= \frac{\int_{m_{n-1}}^{m_n} \frac{1}{\sigma} g\left(\frac{x-\theta}{\sigma}\right) g_{\theta}(\theta) d\theta}{G_{\theta}(m_n) - G_{\theta}(m_{n-1})}. \end{aligned}$$

Hence, the posterior of θ conditional on (x, y_n) is

$$\begin{aligned} g_{\theta|x}(\theta|x; y_n) &= \frac{g_{\theta x}(\theta, x|y_n)}{g_x(x|y_n)} = \frac{g_{x|\theta}(x|\theta; y_n) g_{\theta}(\theta|y_n)}{g_x(x|y_n)} \\ &= \frac{\frac{1}{\sigma} g\left(\frac{x-\theta}{\sigma}\right) [g_{\theta}(\theta) / (G_{\theta}(m_n) - G_{\theta}(m_{n-1}))]}{\left[\int_{m_{n-1}}^{m_n} \frac{1}{\sigma} g\left(\frac{x-\tilde{\theta}}{\sigma}\right) g_{\theta}(\tilde{\theta}) d\tilde{\theta} / (G_{\theta}(m_n) - G_{\theta}(m_{n-1})) \right]} \\ &= \frac{\frac{1}{\sigma} g\left(\frac{x-\theta}{\sigma}\right) g_{\theta}(\theta)}{\int_{m_{n-1}}^{m_n} \frac{1}{\sigma} g\left(\frac{x-\tilde{\theta}}{\sigma}\right) g_{\theta}(\tilde{\theta}) d\tilde{\theta}} \\ &= \frac{\frac{1}{\sigma} g\left(\frac{x-\theta}{\sigma}\right)}{G\left(\frac{x-m_{n-1}}{\sigma}\right) - G\left(\frac{x-m_n}{\sigma}\right)}, \text{ if } \theta \text{ is uniform on } [0, 1]. \end{aligned}$$

B.1.3 Comparison

In the case of a two-element partition, when $y = y_l$ ($\theta \leq m$):

$$g_{\theta|x}(\theta|x; y_l) = g_{\theta|x}(\theta|x) \gamma_{y_l}(x),$$

where $\gamma_{y_l}(x) = \left[\int_{-\infty}^{+\infty} \frac{1}{\sigma} g\left(\frac{x-\tilde{\theta}}{\sigma}\right) g_{\theta}(\tilde{\theta}) d\tilde{\theta} / \int_{-\infty}^m \frac{1}{\sigma} g\left(\frac{x-\tilde{\theta}}{\sigma}\right) g_{\theta}(\tilde{\theta}) d\tilde{\theta} \right] \geq 1$. And if $y = y_h$ ($\theta > m$):

$$g_{\theta|x}(\theta|x; y_h) = g_{\theta|x}(\theta|x) \gamma_{y_h}(x),$$

where $\gamma_{y_h}(x) = \left[\int_{-\infty}^{+\infty} \frac{1}{\sigma} g\left(\frac{x-\tilde{\theta}}{\sigma}\right) g_{\theta}(\tilde{\theta}) d\tilde{\theta} / \int_m^{+\infty} \frac{1}{\sigma} g\left(\frac{x-\tilde{\theta}}{\sigma}\right) g_{\theta}(\tilde{\theta}) d\tilde{\theta} \right] \geq 1$.

B.2 Proof of Lemma 1

Lemma 1. For a given public signal y , if $\pi(x, y) \geq \pi'(x, y)$ for all x , then $u_y(x, \pi) \geq u_y(x, \pi')$ for all x .

Proof:

$$\pi(x, y) \geq \pi'(y, x) \forall x \Rightarrow s(\theta, \pi) \geq s(\theta, \pi') \Rightarrow A(\pi) \cap y \supseteq A(\pi') \cap y \Rightarrow u_y(x, \pi) \geq u_y(x, \pi').$$

□

B.3 Derivation of ψ

For $k \in [-\varepsilon, 1 + \varepsilon]$, define θ_k as

$$\theta_k = \sup\{\theta : s(\theta, I_k) \geq a(\theta)\}.$$

θ_k is the largest value of θ such that the government finds it optimal to abandon the peg when speculators' aggregate short sales are given by I_k . Since $s(\cdot, I_k)$ is decreasing and $a(\cdot)$ is increasing, the government abandons the peg if and only if $\theta \leq \theta_k$.

If $k \leq \underline{\theta} - \varepsilon$,

$$s(k + \varepsilon, I_k) = G\left(\frac{-\varepsilon}{\sigma}\right) = 0,$$

which implies that $\theta_k = \underline{\theta}$. If $k \in (\underline{\theta} - \varepsilon, 1 + \varepsilon]$,

$$s(k - \varepsilon, I_k) = G\left(\frac{\varepsilon}{\sigma}\right) = 1 > a(k - \varepsilon),$$

therefore θ_k is well-defined.²⁶ Note that θ_k is continuous in k .

Define \bar{k} as the unique value of k that solves

$$G\left(\frac{k - 1}{\sigma}\right) = a(1).$$

Hence $\bar{k} = 1 + \sigma G^{-1}(a(1))$. Since $a(1) \in (0, 1)$, $\bar{k} \in (1 - \varepsilon, 1 + \varepsilon)$.

For $k \in (\underline{\theta} - \varepsilon, \bar{k}]$, θ_k is then the unique value of θ that solves

$$G\left(\frac{k - \theta}{\sigma}\right) = a(\theta). \tag{16}$$

If $\theta < \underline{\theta}$, the LHS of (16) is strictly positive, while the RHS equals 0, thus $\theta_k > \underline{\theta}$. Note that the LHS of (16) is strictly decreasing in θ , for $\theta \in (k - \varepsilon, k + \varepsilon)$, and constant otherwise. For $\theta > \underline{\theta}$,

²⁶ $c(1, 1) > v$ implies that $a(1) < 1$, thus $a(\theta) < 1$ for all θ .

$a(\theta) \in (0, 1)$, and it is strictly increasing. This implies that $\theta_k \in (k - \varepsilon, k + \varepsilon)$, and that θ_k is strictly increasing in k . Finally, if $k > \bar{k}$, $\theta_k = 1$.

Define the function ψ as $\psi(k) = \min\{\theta_k - k, \varepsilon\}$, for $k \in [-\varepsilon, 1 + \varepsilon]$. Thus

$$\psi(k) = \begin{cases} \varepsilon, & \text{if } k \leq \underline{\theta} - \varepsilon \\ -\sigma G^{-1}(a(\theta_k)) \in (-\varepsilon, \varepsilon), & \text{if } k \in (\underline{\theta} - \varepsilon, \bar{k}] \\ 1 - k \in [-\varepsilon, \varepsilon), & \text{if } k > \bar{k} \end{cases} .$$

From the continuity of θ_k , it follows that $\psi(k)$ is continuous. Since θ_k is strictly increasing for $k \in (\underline{\theta} - \varepsilon, \bar{k}]$, then $\psi(k)$ is strictly decreasing for $k > \underline{\theta} - \varepsilon$.

B.4 Proof of Lemma 2

Lemma 2. For a given public signal y , $u_y(k, I_k)$ is continuous in k , for all possible private signals $k \in X_y$.

Proof: Using 11, the payoff function when $y = y_n$ is given by

$$u_{y_n}(k, I_k) = \int_{a_{y_n}}^{b_{y_n}} [e^* - f(\theta)] \phi_{y_n}(\theta|k) d\theta - t, \quad (17)$$

where $a_{y_n} = \max\{k - \varepsilon, m_{n-1}\}$, and $b_{y_n} = \max\{\min\{k + \psi(k), m_n\}, m_{n-1}\}$. Since $\phi_{y_n}(\cdot|k)$ and the limits of integration are continuous in k (because $\psi(\cdot)$ is continuous), $u_{y_n}(k, I_k)$ is continuous in k . \square

B.5 Proof of Lemma 3

Lemma 3. For $k \in (\varepsilon, 1 - \varepsilon)$, the payoff function $u(k, I_k)$ is strictly decreasing in k .

Proof: For $k \in (\varepsilon, 1 - \varepsilon)$

$$u(k, I_k) = \int_{k-\varepsilon}^{k+\psi(k)} [e^* - f(\theta)] \frac{\frac{1}{\sigma} g\left(\frac{k-\theta}{\sigma}\right)}{G\left(\frac{k}{\sigma}\right) - G\left(\frac{k-1}{\sigma}\right)} d\theta - t.$$

Differentiating $u(k, I_k)$ with respect to k and using the fact that $G\left(\frac{k-1}{\sigma}\right) = g\left(\frac{k-1}{\sigma}\right) = 0$, for $k < 1 - \varepsilon$, yield

$$\begin{aligned} & \frac{d}{dk} u(k, I_k) \\ &= [e^* - f(k + \psi(k))](1 + \psi'(k)) \frac{\frac{1}{\sigma} g\left(\frac{-\psi(k)}{\sigma}\right)}{G\left(\frac{k}{\sigma}\right)} - [e^* - f(k - \varepsilon)] \frac{\frac{1}{\sigma} g\left(\frac{\varepsilon}{\sigma}\right)}{G\left(\frac{k}{\sigma}\right)} \end{aligned}$$

$$\begin{aligned}
& + \int_{k-\varepsilon}^{k+\psi(k)} [e^* - f(\theta)] \frac{\frac{1}{\sigma^2} g' \left(\frac{k-\theta}{\sigma} \right)}{G \left(\frac{k}{\sigma} \right)} d\theta - \int_{k-\varepsilon}^{k+\psi(k)} [e^* - f(\theta)] \frac{1}{\sigma^2} \frac{g \left(\frac{k-\theta}{\sigma} \right) g \left(\frac{k}{\sigma} \right)}{G \left(\frac{k}{\sigma} \right)^2} d\theta \\
& \leq [e^* - f(k + \psi(k))] \frac{\frac{1}{\sigma} g \left(\frac{-\psi(k)}{\sigma} \right)}{G \left(\frac{k}{\sigma} \right)} - [e^* - f(k - \varepsilon)] \frac{\frac{1}{\sigma} g \left(\frac{\varepsilon}{\sigma} \right)}{G \left(\frac{k}{\sigma} \right)} + \int_{k-\varepsilon}^{k+\psi(k)} [e^* - f(\theta)] \frac{\frac{1}{\sigma^2} g' \left(\frac{k-\theta}{\sigma} \right)}{G \left(\frac{k}{\sigma} \right)} d\theta,
\end{aligned}$$

where the inequality comes from $\psi'(k) \leq 0$, and from the fact that the last term on the RHS of the equality is positive. Define $\underline{\varepsilon}$ as

$$\underline{\varepsilon} = \inf \left\{ \varepsilon \in \left[\frac{-\psi(k)}{\sigma}, \frac{\varepsilon}{\sigma} \right] : g'(\varepsilon) \leq 0 \quad \forall \varepsilon > \underline{\varepsilon} \right\}.$$

From (1), $\underline{\varepsilon}$ is well defined. Furthermore, $g'(\varepsilon) \geq 0$, for $\varepsilon \leq \underline{\varepsilon}$, and $g'(\varepsilon) \leq 0$, for $\varepsilon > \underline{\varepsilon}$. Define $\underline{\theta}$ as

$$\underline{\theta} = k - \sigma \underline{\varepsilon}.$$

Hence $\underline{\theta} \in [k - \varepsilon, k + \psi(k)]$. We then have

$$\begin{aligned}
& G \left(\frac{k}{\sigma} \right) \left[\frac{d}{dk} u(k, I_k) \right] \\
& \leq [e^* - f(k + \psi(k))] \frac{1}{\sigma} g \left(\frac{-\psi(k)}{\sigma} \right) - [e^* - f(k - \varepsilon)] \frac{1}{\sigma} g \left(\frac{\varepsilon}{\sigma} \right) \\
& \quad + \int_{k-\varepsilon}^{\min\{\underline{\theta}, k + \psi(k)\}} [e^* - f(\theta)] \frac{1}{\sigma^2} g' \left(\frac{k-\theta}{\sigma} \right) d\theta + \int_{\min\{\underline{\theta}, k + \psi(k)\}}^{k + \psi(k)} [e^* - f(\theta)] \frac{1}{\sigma^2} g' \left(\frac{k-\theta}{\sigma} \right) d\theta \\
& \leq [e^* - f(k + \psi(k))] \frac{1}{\sigma} g \left(\frac{-\psi(k)}{\sigma} \right) - [e^* - f(k - \varepsilon)] \frac{1}{\sigma} g \left(\frac{\varepsilon}{\sigma} \right) \\
& \quad + [e^* - f(\min\{\underline{\theta}, k + \psi(k)\})] \int_{k-\varepsilon}^{\min\{\underline{\theta}, k + \psi(k)\}} \frac{1}{\sigma^2} g' \left(\frac{k-\theta}{\sigma} \right) d\theta \\
& \quad + [e^* - f(\min\{\underline{\theta}, k + \psi(k)\})] \int_{\min\{\underline{\theta}, k + \psi(k)\}}^{k + \psi(k)} \frac{1}{\sigma^2} g' \left(\frac{k-\theta}{\sigma} \right) d\theta \\
& = [e^* - f(k + \psi(k))] \frac{1}{\sigma} g \left(\frac{-\psi(k)}{\sigma} \right) - [e^* - f(k - \varepsilon)] \frac{1}{\sigma} g \left(\frac{\varepsilon}{\sigma} \right) \\
& \quad + [e^* - f(\min\{\underline{\theta}, k + \psi(k)\})] \left[\frac{1}{\sigma} g \left(\frac{\varepsilon}{\sigma} \right) - \frac{1}{\sigma} g \left(\frac{k - \min\{\underline{\theta}, k + \psi(k)\}}{\sigma} \right) \right] \\
& \quad + [e^* - f(\min\{\underline{\theta}, k + \psi(k)\})] \left[\frac{1}{\sigma} g \left(\frac{k - \min\{\underline{\theta}, k + \psi(k)\}}{\sigma} \right) - \frac{1}{\sigma} g \left(\frac{-\psi(k)}{\sigma} \right) \right]
\end{aligned}$$

$$= \frac{1}{\sigma} \left\{ g \left(\frac{-\psi(k)}{\sigma} \right) \left[f(\min\{\tilde{\theta}, k + \psi(k)\}) - f(k + \psi(k)) \right] + g \left(\frac{\varepsilon}{\sigma} \right) \left[f(k - \varepsilon) - f(\min\{\tilde{\theta}, k + \psi(k)\}) \right] \right\},$$

$$< 0,$$

which implies that $u(k, I_k)$ is strictly increasing in k . \square

B.6 Proof that Assumption 1 holds for concave or normal distributions

Here we show two assumptions on the distribution of the idiosyncratic noise that guarantee that Assumption 1 is satisfied. The first assumption is that the probability density function g is concave on $[-\varepsilon, \varepsilon]$. The second assumption is that the noise follows a truncated normal distribution on $[-\varepsilon, \varepsilon]$.

Without loss of generality, assume that $\sigma = 1$. In this case, $\varepsilon = \bar{\varepsilon}$. Let the public signal be y and consider two private signals x_1 and x_2 , with $x_1 < x_2$. There are five possible cases:

1. $\theta \leq x_1 - \varepsilon$: $\Phi_y(\theta|x_1) = \Phi_y(\theta|x_2) = 0$;
2. $\theta \geq x_2 + \varepsilon$: $\Phi_y(\theta|x_1) = \Phi_y(\theta|x_2) = 1$;
3. $\theta \in (x_1 - \varepsilon, x_2 - \varepsilon)$: $\Phi_y(\theta|x_1) > \Phi_y(\theta|x_2) = 0$;
4. $\theta \in (x_1 + \varepsilon, x_2 + \varepsilon)$: $1 = \Phi_y(\theta|x_1) > \Phi_y(\theta|x_2)$;
5. $\theta \in [x_2 - \varepsilon, x_1 + \varepsilon]$.

To prove that Assumption 1 is satisfied, it is left to show that $\Phi_y(\theta|x_2) \leq \Phi_y(\theta|x_1)$ for all $\theta \in [x_2 - \varepsilon, x_1 + \varepsilon]$. Note that, in this case, $x_2 < x_1 + 2\varepsilon$.

B.6.1 Concave distribution

Let the probability density function of the idiosyncratic noise, g , be concave on $[-\varepsilon, \varepsilon]$. The density of θ conditional on a public signal $y = y_n$ and a private signal x is given by

$$\phi_y(\theta|x) = \frac{g(x - \theta)}{G(x - m_{n-1}) - G(x - m_n)}.$$

Consider two private signals x_1 and x_2 , with $x_1 < x_2$ and $x_2 < x_1 + 2\varepsilon$, and define $\delta = x_2 - x_1 < 2\varepsilon$.

For $\theta \in [x_2 - \varepsilon, x_1 + \varepsilon]$

$$\frac{\phi_y(\theta|x_1)}{\phi_y(\theta|x_2)} = \frac{g(x_1 - \theta)}{g(x_1 - \theta + \delta)} \bar{c},$$

where $\bar{c} = [G(x_2 - m_{n-1}) - G(x_2 - m_n)] / [G(x_1 - m_{n-1}) - G(x_1 - m_n)]$.

To prove that Assumption 1 holds for $\theta \in [x_2 - \varepsilon, x_1 + \varepsilon]$, two results are needed.

Claim 1. *If g is concave on $[-\varepsilon, \varepsilon]$, then $\phi_y(\cdot|x_1)$ crosses $\phi_y(\cdot|x_2)$ at most once for $\theta \in [x_2 - \varepsilon, x_1 + \varepsilon]$.*

Proof: Suppose that there exists θ_1 and θ_2 in $[x_2 - \varepsilon, x_1 + \varepsilon]$, with $\theta_1 < \theta_2$, such that

$$\frac{\phi_y(\theta_1|x_1)}{\phi_{y_n}(\theta_1|x_2)} = \frac{\phi_y(\theta_2|x_1)}{\phi_y(\theta_2|x_2)} = 1.$$

Define $\varepsilon_H = x_1 - \theta_1$ and $\varepsilon_L = x_1 - \theta_2$. Hence

$$\begin{aligned}\bar{c}g(\varepsilon_H) &= g(\varepsilon_H + \delta), \\ \bar{c}g(\varepsilon_L) &= g(\varepsilon_L + \delta).\end{aligned}$$

There are three cases:

1. $\bar{c} > 1$:

$$\begin{aligned}g(\varepsilon_H) &< g(\varepsilon_H + \delta), \\ g(\varepsilon_L) &< g(\varepsilon_L + \delta).\end{aligned}$$

It must be true that $g(\varepsilon_H) > g(\varepsilon_L)$, otherwise g would decrease or be constant somewhere between ε_L and ε_H , and then increase somewhere between ε_H and $\varepsilon_H + \delta$, a contradiction with the concavity of g . The slope of the line segment that connects points $(\varepsilon_L, g(\varepsilon_L))$ and $(\varepsilon_L + \delta, g(\varepsilon_L + \delta))$ is given by

$$S_L = (\bar{c} - 1) \frac{g(\varepsilon_L)}{\delta},$$

and the slope of the line segment that connects $(\varepsilon_H, g(\varepsilon_H))$ and $(\varepsilon_H + \delta, g(\varepsilon_H + \delta))$ is given by

$$S_H = (\bar{c} - 1) \frac{g(\varepsilon_H)}{\delta}.$$

Since $g(\varepsilon_H) > g(\varepsilon_L)$, it follows that $S_H > S_L$, a contradiction with the concavity of g .

2. $\bar{c} < 1$:

$$\begin{aligned}g(\varepsilon_H) &> g(\varepsilon_H + \delta), \\ g(\varepsilon_L) &> g(\varepsilon_L + \delta).\end{aligned}$$

It must be true that $g(\varepsilon_L) > g(\varepsilon_H)$, otherwise $g(\varepsilon_L) \leq g(\varepsilon_H)$ and $g(\varepsilon_L + \delta) = \bar{c}g(\varepsilon_L) \leq \bar{c}g(\varepsilon_H) = g(\varepsilon_H + \delta)$. In this case, g would decrease between ε_L and $\varepsilon_L + \delta$, and then increase or be

constant somewhere between $\varepsilon_L + \delta$ and $\varepsilon_H + \delta$, a contradiction with the concavity of g . The slope of the line segment that connects points $(\varepsilon_L, g(\varepsilon_L))$ and $(\varepsilon_L + \delta, g(\varepsilon_L + \delta))$ is given by

$$S_L = (\bar{c} - 1) \frac{g(\varepsilon_L)}{\delta},$$

and the slope of the line segment that connects $(\varepsilon_H, g(\varepsilon_H))$ and $(\varepsilon_H + \delta, g(\varepsilon_H + \delta))$ is given by

$$S_H = (\bar{c} - 1) \frac{g(\varepsilon_H)}{\delta}.$$

Since $g(\varepsilon_L) > g(\varepsilon_H)$ and $\bar{c} < 1$, it follows that $S_H > S_L$, a contradiction with the concavity of g .

3. $c = 1$:

$$\begin{aligned} g(\varepsilon_H) &= g(\varepsilon_H + \delta), \\ g(\varepsilon_L) &= g(\varepsilon_L + \delta), \end{aligned}$$

thus it must be the case that $g(\varepsilon_H) = g(\varepsilon_L)$ and g is flat between ε_L and $\varepsilon_H + \delta$.

□

Claim 2. *If g is concave on $[-\varepsilon, \varepsilon]$, then $\phi_y(\cdot|x_2)$ can only cross $\phi_y(\cdot|x_1)$ from below for $\theta \in [x_2 - \varepsilon, x_1 + \varepsilon]$.*

Proof: Suppose that $\phi_y(\theta|x_2)$ crosses $\phi_y(\theta|x_1)$ from above in $[x_2 - \varepsilon, x_1 + \varepsilon]$. Then, there exist θ_1 and θ_2 in $[x_2 - \varepsilon, x_1 + \varepsilon]$, with $\theta_1 < \theta_2$, such that:

$$\begin{aligned} \phi_y(\theta_1|x_2) &= \phi_y(\theta_1|x_1), \\ \phi_y(\theta_2|x_2) &< \phi_y(\theta_2|x_1). \end{aligned}$$

Define $\varepsilon_H = x_1 - \theta_1$ and $\varepsilon_L = x_1 - \theta_2$. The inequalities above imply

$$\begin{aligned} \bar{c}g(\varepsilon_H) &= g(\varepsilon_H + \delta), \\ \bar{c}g(\varepsilon_L) &> g(\varepsilon_L + \delta). \end{aligned}$$

Denote the slope of the line segment that connects points $(\varepsilon_I, g(\varepsilon_I))$ and $(\varepsilon_I + \delta, g(\varepsilon_I + \delta))$ by S_I , for $I \in \{L, H\}$. There are three cases:

1. $\bar{c} = 1$: then $S_L < 0$ and $S_H, 0$, a contradiction with the concavity of g .
2. $\bar{c} > 1$: then $g(\varepsilon_H) < g(\varepsilon_H + \delta)$. It follows that $g(\varepsilon_L) \leq g(\varepsilon_H)$, otherwise g would decrease between ε_L and ε_H , and then increase between ε_H and $\varepsilon_H + \delta$, a contradiction with the

concavity of g . Thus

$$S_L < (\bar{c} - 1) \frac{g(\varepsilon_L)}{\delta} \leq (\bar{c} - 1) \frac{g(\varepsilon_H)}{\delta} = S_H,$$

a contradiction with the concavity of g .

3. $\bar{c} < 1$: then $g(\varepsilon_L) > g(\varepsilon_L + \delta)$. It follows that $g(\varepsilon_L) > g(\varepsilon_H)$, otherwise

$$g(\varepsilon_H + \delta) = \bar{c}g(\varepsilon_H) \geq \bar{c}g(\varepsilon_L) > g(\varepsilon_L + \delta),$$

therefore g would decrease between ε_L and $\varepsilon_L + \delta$, and then increase between $\varepsilon_L + \delta$ and $\varepsilon_H + \delta$, a contradiction with the concavity of g . Thus

$$S_L < -(1 - \bar{c}) \frac{g(\varepsilon_L)}{\delta} \leq -(1 - \bar{c}) \frac{g(\varepsilon_H)}{\delta} = S_H,$$

a contradiction with the concavity of g .

□

Claims 1 and 2 imply that, if there exists $\theta \in [x_2 - \varepsilon, x_1 + \varepsilon]$, such that $\phi_y(\theta|x_2) > \phi_y(\theta|x_1)$, then $\phi_y(\tilde{\theta}|x_2) \geq \phi_y(\tilde{\theta}|x_1)$ for all $\tilde{\theta} > \theta$. This implies that, if there exists $\theta \in [x_2 - \varepsilon, x_1 + \varepsilon]$, such that $\Phi_y(\theta|x_2) > \Phi_y(\theta|x_1)$, then $\Phi_y(\tilde{\theta}|x_2) > \Phi_y(\tilde{\theta}|x_1)$, for all $\tilde{\theta} > \theta$. In particular, $\Phi_y(x_1 + \varepsilon|x_2) > \Phi_y(x_1 + \varepsilon|x_1) = 1$, a contradiction. Thus $\Phi_y(\theta|x_2) \leq \Phi_y(\theta|x_1)$, for all $\theta \in [x_2 - \varepsilon, x_1 + \varepsilon]$, and Assumption 1 holds.

B.6.2 Normal distribution

Suppose that the idiosyncratic noise follows a truncated normal distribution on $[-\varepsilon, \varepsilon]$, with the originating distribution having mean μ and variance ν^2 . Let the public signal be y and consider two private signals x_1 and x_2 , with $x_1 < x_2$ and $x_2 < x_1 + 2\varepsilon$. To prove that $\Phi_y(\theta|x_2) \leq \Phi_y(\theta|x_1)$ for $\theta \in [x_2 - \varepsilon, x_1 + \varepsilon]$, it suffices to show that the following monotone likelihood ratio holds

$$\frac{\phi_y(\theta_2|x_2)}{\phi_y(\theta_1|x_2)} \geq \frac{\phi_y(\theta_2|x_1)}{\phi_y(\theta_1|x_1)}, \quad \forall \theta_1 < \theta_2.$$

Let $\theta_1, \theta_2 \in [x_2 - \varepsilon, x_1 + \varepsilon]$. Then

$$\frac{\phi_y(\theta_2|x_2)}{\phi_y(\theta_1|x_2)} \geq \frac{\phi_y(\theta_2|x_1)}{\phi_y(\theta_1|x_1)}$$

$$\begin{aligned}
& \Leftrightarrow \frac{\exp\left(-\frac{(x_2-\theta_2-\mu)^2}{2\nu^2}\right)}{\exp\left(-\frac{(x_2-\theta_1-\mu)^2}{2\nu^2}\right)} \geq \frac{\exp\left(-\frac{(x_1-\theta_2-\mu)^2}{2\nu^2}\right)}{\exp\left(-\frac{(x_1-\theta_1-\mu)^2}{2\nu^2}\right)} \\
& \Leftrightarrow \exp\left(\frac{-(x_2-\theta_2-\mu)^2 + (x_2-\theta_1-\mu)^2}{2\nu^2}\right) \geq \exp\left(\frac{-(x_1-\theta_2-\mu)^2 + (x_1-\theta_1-\mu)^2}{2\nu^2}\right) \\
& \Leftrightarrow -(x_2-\theta_2-\mu)^2 + (x_2-\theta_1-\mu)^2 \geq -(x_1-\theta_2-\mu)^2 + (x_1-\theta_1-\mu)^2 \\
& \Leftrightarrow -(x_2-\theta_2)^2 + 2\mu(\theta_1-\theta_2) + (x_2-\theta_1)^2 \geq -(x_1-\theta_2)^2 + 2\mu(\theta_1-\theta_2) + (x_1-\theta_1)^2 \\
& \Leftrightarrow -x_2^2 + 2x_2\theta_2 - \theta_2^2 + x_2^2 - 2x_2\theta_1 + \theta_1^2 \geq -x_1^2 + 2x_1\theta_2 - \theta_2^2 + x_1^2 - 2x_1\theta_1 + \theta_1^2 \\
& \Leftrightarrow 2x_2(\theta_2 - \theta_1) \geq 2x_1(\theta_2 - \theta_1) \\
& \Leftrightarrow \theta_1 \leq \theta_2,
\end{aligned}$$

which completes the proof.

B.7 Lemma 9

Lemma 9. *Suppose that $y = y_n$ and that speculators follow I_k , for $k \in X_{y_n}$. When a speculator receives the private signal $x = k$, the payoff from attacking, $u_{y_n}(k, I_k)$, is continuous in both m_{n-1} and m_n . The payoff $u_{y_n}(k, I_k)$ is decreasing in m_{n-1} for $k < m_{n-1} + \varepsilon$, and constant otherwise. It is also decreasing in m_n for $k > m_n - \varepsilon$, and constant otherwise.*

Proof: Without loss in generality, let $y = y_2$. Then,

$$u_{y_2}(k, I_k) = \int_{\max\{k-\varepsilon, m_1\}}^{\max\{\min\{k+\psi(k), m_2\}, m_1\}} [e^* - f(\theta)] \frac{\frac{1}{\sigma} g\left(\frac{k-\theta}{\sigma}\right)}{D(k, m_1, m_2)} d\theta - t,$$

where

$$D(k, m_1, m_2) = G\left(\frac{k - m_1}{\sigma}\right) - G\left(\frac{k - m_2}{\sigma}\right).$$

The limits of integration are continuous in m_{n-1} and m_n , and, since G is a continuous function, D is continuous in all of its arguments. Hence $u_{y_2}(k, I_k)$ is continuous in m_{n-1} and m_n .

If $m_1 - \psi(k) \geq k$, then $u_{y_2}(k, I_k) = -t$, which is constant in m_1 . If $k > m_1 + \varepsilon$ then the limits of integration above are constant in m_1 and so is $D(k, m_1, m_2)$,²⁷ therefore $u_{y_2}(k, I_k)$ is constant in m_1 . Now consider the case $m_1 - \psi(k) < k \leq m_1 + \varepsilon$:

$$u_{y_2}(k, I_k) = \int_{m_1}^{\min\{k+\psi(k), m_2\}} [e^* - f(\theta)] \frac{\frac{1}{\sigma} g\left(\frac{k-\theta}{\sigma}\right)}{D(k, m_1, m_2)} d\theta - t,$$

²⁷For all m_1 such that $m_1 < k - \varepsilon$: $\frac{k-m_1}{\sigma} > \bar{\varepsilon} \Rightarrow G\left(\frac{k-m_1}{\sigma}\right) = 1$

then

$$\begin{aligned}
\frac{\partial}{\partial m_1} u_{y_2}(k, I_k) &= -[e^* - f(m_1)] \frac{\frac{1}{\sigma} g\left(\frac{k-m_1}{\sigma}\right)}{D(k, m_1, m_2)} \\
&\quad + \left[\frac{\frac{1}{\sigma} g\left(\frac{k-m_1}{\sigma}\right)}{D(k, m_1, m_2)} \right] \int_{m_1}^{\min\{k+\psi(k), m_2\}} [e^* - f(\theta)] \frac{\frac{1}{\sigma} g\left(\frac{k-\theta}{\sigma}\right)}{D(k, m_1, m_2)} d\theta \\
&< -[e^* - f(m_1)] \frac{\frac{1}{\sigma} g\left(\frac{k-m_1}{\sigma}\right)}{D(k, m_1, m_2)} \left[1 - \int_{m_1}^{\min\{k+\psi(k), m_2\}} \frac{\frac{1}{\sigma} g\left(\frac{k-\theta}{\sigma}\right)}{D(k, m_1, m_2)} d\theta \right] \\
&= -[e^* - f(m_1)] \frac{\frac{1}{\sigma} g\left(\frac{k-m_1}{\sigma}\right)}{D(k, m_1, m_2)} \left[1 - \frac{G\left(\frac{k-m_1}{\sigma}\right) - G\left(\frac{k-\min\{k+\psi(k), m_2\}}{\sigma}\right)}{D(k, m_1, m_2)} \right] \\
&\leq 0,
\end{aligned}$$

which completes the proof for m_1 .

Let $k \leq m_2 - \varepsilon$. Then

$$u_{y_2}(k, I_k) = \int_{\max\{k-\varepsilon, m_1\}}^{\max\{k+\psi(k), m_1\}} [e^* - f(\theta)] \frac{\frac{1}{\sigma} g\left(\frac{k-\theta}{\sigma}\right)}{D(k, m_1, m_2)} d\theta - t,$$

Since the limits of integration above are constant in m_2 , and so is $D(k, m_1, m_2)$,²⁸ then $u_{y_2}(k, I_k)$ is constant in m_2 . The same is true for $k \leq m_2 - \psi(k)$.

Now suppose $m_2 < k + \psi(k)$. The payoff becomes

$$u_{y_2}(k, I_k) = \int_{\max\{k-\varepsilon, m_1\}}^{m_2} [e^* - f(\theta)] \frac{\frac{1}{\sigma} g\left(\frac{k-\theta}{\sigma}\right)}{D(k, m_1, m_2)} d\theta - t,$$

then

$$\begin{aligned}
\frac{\partial}{\partial m_2} u_{y_2}(k, I_k) &= [e^* - f(m_2)] \frac{\frac{1}{\sigma} g\left(\frac{k-m_2}{\sigma}\right)}{D(k, m_1, m_2)} \\
&\quad - \left[\frac{\frac{1}{\sigma} g\left(\frac{k-m_2}{\sigma}\right)}{D(k, m_1, m_2)} \right] \int_{\max\{m_1, k-\varepsilon\}}^{m_2} [e^* - f(\theta)] \frac{\frac{1}{\sigma} g\left(\frac{k-\theta}{\sigma}\right)}{D(k, m_1, m_2)} d\theta \\
&< [e^* - f(m_2)] \frac{\frac{1}{\sigma} g\left(\frac{k-m_2}{\sigma}\right)}{D(k, m_1, m_2)} \left[1 - \int_{\max\{m_1, k-\varepsilon\}}^{m_2} \frac{\frac{1}{\sigma} g\left(\frac{k-\theta}{\sigma}\right)}{D(k, m_1, m_2)} d\theta \right]
\end{aligned}$$

²⁸For all m_2 such that $m_2 \geq k + \varepsilon$: $\frac{k-m_2}{\sigma} \leq -\bar{\varepsilon} \Rightarrow G\left(\frac{k-m_2}{\sigma}\right) = 0$

$$\begin{aligned}
&= [e^* - f(m_2)] \frac{\frac{1}{\sigma} \mathcal{G}\left(\frac{k-m_2}{\sigma}\right)}{D(k, m_1, m_2)} \left[1 - \frac{G\left(\frac{k-\max\{m_1, k-\varepsilon\}}{\sigma}\right) - G\left(\frac{k-m_2}{\sigma}\right)}{D(k, m_1, m_2)} \right] \\
&= 0.
\end{aligned}$$

The last inequality comes from the fact that

$$\begin{aligned}
G\left(\frac{k-\max\{m_1, k-\varepsilon\}}{\sigma}\right) &= \min\left\{G\left(\frac{k-m_1}{\sigma}\right), G\left(\frac{\varepsilon}{\sigma}\right)\right\} = \min\left\{G\left(\frac{k-m_1}{\sigma}\right), 1\right\} \\
&= G\left(\frac{k-m_1}{\sigma}\right) \\
\Rightarrow G\left(\frac{k-\max\{m_1, k-\varepsilon\}}{\sigma}\right) - G\left(\frac{k-m_2}{\sigma}\right) &= D(k, m_1, m_2).
\end{aligned}$$

□

B.8 Proof of Theorem 2

First, we need to compare $u_y(k, I_k)$ and $u(k, I_k)$. Note that for $k \leq m - \varepsilon$, from (12) and (17), the limits of integration are²⁹

$$a_{y_l} = a \quad , \quad b_{y_l} = b,$$

and the density functions are the same, which implies that $u_{y_l}(k, I_k)$ equals the payoff function $u(k, I_k)$. For $k > m + \varepsilon$, from (12) and (17),

$$a_{y_h} = a \quad , \quad b_{y_h} = b,$$

and the density functions are the same, which implies that $u_{y_h}(k, I_k)$ equals the payoff function $u(k, I_k)$. From Lemma 2, the continuity of $u_{y_h}(k, I_k)$ in k implies that $u_{y_h}(k, I_k) = u(k, I_k)$ for $k = m + \varepsilon$. The comparison between $u_y(k, I_k)$ and $u(k, I_k)$ when $k \in (m - \varepsilon, m + \varepsilon)$ is analyzed in the two following lemmas.

Lemma 10. *If the public signal is $y = y_l$, then $u_{y_l}(k, I_k) > u(k, I_k)$ for all $k \in (m - \varepsilon, m + \varepsilon)$.*

Proof:

$$u_{y_l}(k, I_k) - u(k, I_k)$$

²⁹ Here $y_l = y_1$, $y_h = y_2$, and $m = m_1$.

$$\begin{aligned}
&= \int_{\max\{k-\varepsilon, 0\}}^{\min\{k+\psi(k), m\}} [e^* - f(\theta)] \phi_{y_l}(\theta|k) d\theta - \int_{\max\{k-\varepsilon, 0\}}^{k+\psi(k)} [e^* - f(\theta)] \frac{\frac{1}{\sigma} g\left(\frac{k-\theta}{\sigma}\right)}{G\left(\frac{k}{\sigma}\right) - G\left(\frac{k-1}{\sigma}\right)} d\theta \\
&= \left(\frac{1}{G\left(\frac{k}{\sigma}\right) - G\left(\frac{k-m}{\sigma}\right)} - \frac{1}{G\left(\frac{k}{\sigma}\right) - G\left(\frac{k-1}{\sigma}\right)} \right) \int_{\max\{k-\varepsilon, 0\}}^{\min\{k+\psi(k), m\}} [e^* - f(\theta)] \frac{1}{\sigma} g\left(\frac{k-\theta}{\sigma}\right) d\theta \\
&\quad - \int_{\min\{k+\psi(k), m\}}^{k+\psi(k)} [e^* - f(\theta)] \frac{\frac{1}{\sigma} g\left(\frac{k-\theta}{\sigma}\right)}{G\left(\frac{k}{\sigma}\right) - G\left(\frac{k-1}{\sigma}\right)} d\theta
\end{aligned}$$

If $k + \psi(k) \leq m$, the last integral equals zero, therefore $u_{y_l}(k, I_k) > u(k, I_k)$. For $k + \psi(k) > m$,

$$\begin{aligned}
&[u_{y_l}(k, I_k) - u(k, I_k)] \left[G\left(\frac{k}{\sigma}\right) - G\left(\frac{k-1}{\sigma}\right) \right] \\
&= \left(\frac{G\left(\frac{k-m}{\sigma}\right) - G\left(\frac{k-1}{\sigma}\right)}{G\left(\frac{k}{\sigma}\right) - G\left(\frac{k-m}{\sigma}\right)} \right) \int_{\max\{k-\varepsilon, 0\}}^m [e^* - f(\theta)] \frac{1}{\sigma} g\left(\frac{k-\theta}{\sigma}\right) d\theta \\
&\quad - \int_m^{k+\psi(k)} [e^* - f(\theta)] \frac{1}{\sigma} g\left(\frac{k-\theta}{\sigma}\right) d\theta \\
&> [e^* - f(m)] \left[\left(\frac{G\left(\frac{k-m}{\sigma}\right) - G\left(\frac{k-1}{\sigma}\right)}{G\left(\frac{k}{\sigma}\right) - G\left(\frac{k-m}{\sigma}\right)} \right) \int_{\max\{k-\varepsilon, 0\}}^m \frac{1}{\sigma} g\left(\frac{k-\theta}{\sigma}\right) d\theta - \int_m^{k+\psi(k)} \frac{1}{\sigma} g\left(\frac{k-\theta}{\sigma}\right) d\theta \right] \\
&= [e^* - f(m)] \left\{ \left(\frac{G\left(\frac{k-m}{\sigma}\right) - G\left(\frac{k-1}{\sigma}\right)}{G\left(\frac{k}{\sigma}\right) - G\left(\frac{k-m}{\sigma}\right)} \right) \left[G\left(\frac{k - \max\{k-\varepsilon, 0\}}{\sigma}\right) - G\left(\frac{k-m}{\sigma}\right) \right] \right. \\
&\quad \left. - \left[G\left(\frac{k-m}{\sigma}\right) - G\left(\frac{-\psi(k)}{\sigma}\right) \right] \right\} \\
&= [e^* - f(m)] \left\{ \left(\frac{G\left(\frac{k-m}{\sigma}\right) - G\left(\frac{k-1}{\sigma}\right)}{G\left(\frac{k}{\sigma}\right) - G\left(\frac{k-m}{\sigma}\right)} \right) \left[G\left(\frac{k}{\sigma}\right) - G\left(\frac{k-m}{\sigma}\right) \right] \right. \\
&\quad \left. - \left[G\left(\frac{k-m}{\sigma}\right) - G\left(\frac{-\psi(k)}{\sigma}\right) \right] \right\} \\
&= [e^* - f(m)] \left[G\left(\frac{-\psi(k)}{\sigma}\right) - G\left(\frac{k-1}{\sigma}\right) \right] \\
&\geq 0,
\end{aligned}$$

where the last inequality comes from $k + \psi(k) \leq 1$. This implies that $u_{y_l}(k, I_k) > u(k, I_k)$, which completes the proof. \square

Lemma 11. *If the public signal is $y = y_h$, then $u_{y_h}(k, I_k) < u(k, I_k)$ for all $k \in (m - \varepsilon, m + \varepsilon)$.*

Proof: First note that if $k + \psi(k) \leq m$, $u_{y_h}(k, I_k) = -t < u(k, I_k)$. If $k + \psi(k) > m$,

$$\begin{aligned}
& [u(k, I_k) - u_{y_h}(k, I_k)] \left[G\left(\frac{k}{\sigma}\right) - G\left(\frac{k-1}{\sigma}\right) \right] \\
&= \int_{\max\{k-\varepsilon, 0\}}^{k+\psi(k)} [e^* - f(\theta)] \frac{1}{\sigma} g\left(\frac{k-\theta}{\sigma}\right) d\theta \\
&\quad - \left[G\left(\frac{k}{\sigma}\right) - G\left(\frac{k-1}{\sigma}\right) \right] \int_m^{k+\psi(k)} [e^* - f(\theta)] \phi_{y_h}(\theta|k) d\theta \\
&= \int_{\max\{k-\varepsilon, 0\}}^m [e^* - f(\theta)] \frac{1}{\sigma} g\left(\frac{k-\theta}{\sigma}\right) d\theta \\
&\quad - \left(\frac{G\left(\frac{k}{\sigma}\right) - G\left(\frac{k-m}{\sigma}\right)}{G\left(\frac{k-m}{\sigma}\right) - G\left(\frac{k-1}{\sigma}\right)} \right) \int_m^{k+\psi(k)} [e^* - f(\theta)] \frac{1}{\sigma} g\left(\frac{k-\theta}{\sigma}\right) d\theta \\
&< [e^* - f(m)] \left[\int_{\max\{k-\varepsilon, 0\}}^m \frac{1}{\sigma} g\left(\frac{k-\theta}{\sigma}\right) d\theta - \left(\frac{G\left(\frac{k}{\sigma}\right) - G\left(\frac{k-m}{\sigma}\right)}{G\left(\frac{k-m}{\sigma}\right) - G\left(\frac{k-1}{\sigma}\right)} \right) \int_m^{k+\psi(k)} \frac{1}{\sigma} g\left(\frac{k-\theta}{\sigma}\right) d\theta \right] \\
&= [e^* - f(m)] \left\{ G\left(\frac{k - \max\{k - \varepsilon, 0\}}{\sigma}\right) - G\left(\frac{k-m}{\sigma}\right) \right. \\
&\quad \left. - \left(\frac{G\left(\frac{k}{\sigma}\right) - G\left(\frac{k-m}{\sigma}\right)}{G\left(\frac{k-m}{\sigma}\right) - G\left(\frac{k-1}{\sigma}\right)} \right) \left[G\left(\frac{k-m}{\sigma}\right) - G\left(\frac{-\psi(k)}{\sigma}\right) \right] \right\} \\
&\leq [e^* - f(m)] \left[G\left(\frac{k - \max\{k - \varepsilon, 0\}}{\sigma}\right) - G\left(\frac{k}{\sigma}\right) \right] \\
&= 0,
\end{aligned}$$

which implies that $u(k, I_k) < u_{y_h}(k, I_k)$, therefore the proof is complete. \square

Now we can prove Theorem 2:

Theorem 2. *Fix m . If $m = \theta^*$, there is a unique equilibrium, in which speculators follow the public signal. If $m \neq \theta^*$, the equilibrium may not be unique. There are bounds $\underline{x}^* \geq x^*$ and $\bar{x}^* \leq x^*$ such that, in any equilibrium, $\pi(x, y_l) \geq I_{\underline{x}^*}(x)$ and $\pi(x, y_h) \leq I_{\bar{x}^*}(x)$ for all x . The equilibria are as follows:*

- i. *if $m < \theta^*$: speculators always attack the currency and the peg is abandoned if $y = y_l$; moreover, if $m \in (x^* - \varepsilon, \theta^*)$, then $\bar{x}^* < x^*$;*
- ii. *if $m > \theta^*$: the currency is not attacked and the peg defended if $y = y_h$; moreover, if $m \in (\theta^*, x^* + \varepsilon)$, then $\underline{x}^* > x^*$.*

Proof: Now consider any possible strategy profile for the speculators. For $y \in \{y_l, y_h\}$, let $\pi(x, y)$ denote the proportion of speculators who attack the currency given a private signal x . Define \underline{x}_y and \bar{x}_y as

$$\underline{x}_y = \inf\{x \in X_y : \pi(x, y) < 1\}, \quad \text{and} \quad \bar{x}_y = \sup\{x \in X_y : \pi(x, y) > 0\}.$$

Note that $\underline{x}_y \leq \bar{x}_y$. If $\underline{x}_y \in X_y$, then from Lemma 1

$$u_y(\underline{x}_y, I_{\underline{x}_y}) \leq u_y(\underline{x}_y, \pi) \leq 0, \quad (18)$$

and if $\bar{x}_y \in X_y$

$$u_y(\bar{x}_y, I_{\bar{x}_y}) \geq u_y(\bar{x}_y, \pi) \geq 0. \quad (19)$$

Using Lemma 3, the proof of existence and uniqueness of equilibrium in the game without public signal is analogous to the one in Morris and Shin (1998). The speculators follow a cutoff strategy I_{x^*} , such that $u(x^*, I_{x^*}) = 0$, with $x^* \in (\varepsilon, 1 - \varepsilon)$. Since $u(k, I_k) > 0$ for $k \leq \varepsilon$, and $u(k, I_k) < 0$ for $k \geq 1 - \varepsilon$, it follows from Lemma 3 that $u(k, I_k) > 0$ for $k < x^*$, and that $u(k, I_k) < 0$ for $k > x^*$.

First, let $\theta \in y_l = [0, m]$. From Lemma 10, $u_{y_l}(k, I_k) \geq u(k, I_k)$, with strict inequality for $k \in (m - \varepsilon, m + \varepsilon)$. If $x^* \notin X_{y_l} = [-\varepsilon, m + \varepsilon]$, then $u_{y_l}(k, I_k)$ is strictly positive for all k . From (18) all speculators attack the currency for $\theta \in y_l$, therefore the peg is always abandoned. If $x^* \in X_{y_l}$, then

$$u_{y_l}(x^*, I_{x^*}) \geq u(x^*, I_{x^*}) = 0,$$

with strict inequality for $m \in (x^* - \varepsilon, x^* + \varepsilon)$. Hence, either every speculator attacks for all $x \in X_{y_l}$, in which case $\underline{\theta}^* = m$, or $\underline{x}_{y_l} \geq x^*$, with strict inequality if $m \in (x^* - \varepsilon, x^* + \varepsilon)$. In the latter case, all speculators attack the currency for $x < \underline{x}_{y_l}$. This guarantees the existence of $\underline{\theta}^* \in [\theta^*, m]$ such that the government always abandons the currency peg for all $\theta \leq \underline{\theta}^*$.³⁰ Furthermore, if $m \in (x^* - \varepsilon, x^* + \varepsilon)$ then $\underline{\theta}^* > \theta^*$.

Now let $\theta \in y_h = (m, 1]$. From Lemma 11, $u_{y_h}(k, I_k) \leq u(k, I_k)$, with strict inequality for $m \in (x^* - \varepsilon, x^* + \varepsilon)$. If $x^* \notin X_{y_h} = (m - \varepsilon, 1 + \varepsilon]$, then $u_{y_h}(k, I_k)$ is strictly negative for all k . From (19) the currency is not attacked for any $\theta \in y_h$, and the government finds it optimal to keep the peg. If $x^* \in X_{y_h}$, then

$$u_{y_h}(x^*, I_{x^*}) \leq u(x^*, I_{x^*}) = 0,$$

with strict inequality for $m \in (x^* - \varepsilon, x^* + \varepsilon)$. Hence, either $\bar{x}_{y_h} \leq x^*$, with strict inequality if

³⁰Take for example $\underline{\theta}^* = \theta_{\underline{x}_{y_l}}$, the value of θ that makes the government indifferent when speculators follow $I_{\underline{x}_{y_l}}$ ($s(\theta_{\underline{x}_{y_l}}, I_{\underline{x}_{y_l}}) = a(\theta_{\underline{x}_{y_l}})$).

$m \in (x^* - \varepsilon, x^* + \varepsilon)$, or the currency is never attacked, in which case $\bar{\theta}^* = m$. In the former case, no speculator attacks the currency for $x > \bar{x}_{y_h}$, therefore there exists $\bar{\theta}^* \in [m, \theta^*]$ such that the currency peg is never abandoned for $\theta > \bar{\theta}^*$.³¹ Furthermore, if $m \in (x^* - \varepsilon, x^* + \varepsilon)$ then $\bar{\theta}^* < \theta^*$.

For $m = \theta^*$, since $\underline{\theta}^* \in [\theta^*, m]$ and $\bar{\theta}^* \in [m, \theta^*]$, it must be the case that $\underline{\theta}^* = \bar{\theta}^* = m$. Thus the unique equilibrium involves coordination on the public signal.

For $m < \theta^*$, then the currency is always attacked on $y_l = [0, m]$ and $\underline{\theta}^* = m$, otherwise $\theta^* \leq \underline{\theta}^* \leq m$, a contradiction. Since $m < \theta^*$, then $x^* \in X_{y_h}$. Hence, if $x^* \notin (m - \varepsilon, m + \varepsilon)$, then $\bar{\theta}^* \in [m, \theta^*]$, and if $x^* \in (m - \varepsilon, m + \varepsilon)$, then $\bar{\theta}^* \in [m, \theta^*)$. In the latter case, $\bar{x}_{y_h} < x^*$.

For $m > \theta^*$, then the currency is never attacked on $y_h = (m, 1]$ and $\bar{\theta}^* = m$, otherwise $m \leq \bar{\theta}^* \leq \theta^*$, a contradiction. Since $m > \theta^*$, then $x^* \in X_{y_l}$. Hence, if $x^* \notin (m - \varepsilon, m + \varepsilon)$, then $\underline{\theta}^* \in [\theta^*, m]$, and if $x^* \in (m - \varepsilon, m + \varepsilon)$, then $\underline{\theta}^* \in (\theta^*, m]$. In the latter case, $\underline{x}_{y_l} > x^*$.

B.9 Lemma 12

Lemma 12. *Suppose that Assumption 1 is satisfied. Then, $\underline{m} < \bar{\theta}$.*

Proof: We need to find $m < \bar{\theta}$ such that $u_{(m,1]}(k, I_k) < 0$ for all k . Consider the partition $P^{\bar{\theta}}$ and let \bar{k} solve $\theta_{\bar{k}} = \bar{\theta}$.³²

We claim that $u_{(\bar{\theta},1]}(k, I_k) \leq \delta < 0$ for all $k \in (\bar{\theta} - \varepsilon, 1 + \varepsilon)$. To see this, let $k \leq \bar{k}$. If speculators follow I_k , then the threshold for the government to abandon the peg is $\theta_k \leq \bar{\theta}$, which means that the government does not abandon the peg on $(\bar{\theta}, 1]$. Hence $u_{(\bar{\theta},1]}(k, I_k) = -t$ for any $k \leq \bar{k}$. For $k > \bar{k}$

$$u_{(\bar{\theta},1]}(k, I_k) \leq u_{(\bar{\theta},1]}(k, I_{1+\varepsilon}) \leq u_{(\bar{\theta},1]}(\bar{k}, I_{1+\varepsilon}) \equiv \delta < 0,$$

where the first inequality comes from Lemma 1, the second inequality comes from Lemma 4, and the last inequality comes from the fact that it is never profitable to attack when $y = (\bar{\theta}, 1]$. Since $\delta \geq -t$, we have that $u_{(\bar{\theta},1]}(k, I_k) \leq \delta$ for all k .

Define l_m^1 and l_m^2 as

$$l_m^1 = \lim_{k \downarrow \bar{k}} u_{(m,1]}(k, I_{1+\varepsilon}),$$

and

$$l_m^2 = \lim_{k \downarrow \bar{\theta} - \varepsilon} u_{(m,1]}(k, I_{\bar{k}}).$$

Since $u_{(\bar{\theta},1]}(k, I_{1+\varepsilon}) \leq \delta$ for all $k > \bar{k}$, continuity implies that $l_{\bar{\theta}}^1 \leq \delta$. Since $u_{(\bar{\theta},1]}(k, I_{\bar{k}}) \leq \delta$ for

³¹Take for example $\bar{\theta}^* = \theta_{\bar{x}_{y_h}}$, the value of θ that makes the government indifferent when speculators follow $I_{\bar{x}_{y_h}}$ ($s(\theta_{\bar{x}_{y_h}}, I_{\bar{x}_{y_h}}) = a(\theta_{\bar{x}_{y_h}})$).

³² $a(\bar{\theta}) = s(\bar{\theta}, I_{\bar{k}})$, that is, if speculators follow the cutoff rule $I_{\bar{k}}$, the government is indifferent between defending the currency and abandoning the peg at $\theta = \bar{\theta}$.

$k \in (\bar{\theta} - \varepsilon, \bar{k}]$, continuity also implies that $l_{\bar{\theta}}^2 \leq \delta$. From Lemmas 1 and 4, $l_m^1 \geq u_{(m,1]}(k, I_k)$ for $k > \bar{k}$, and $l_m^2 \geq u_{(m,1]}(k, I_k)$ for $k \in (\bar{\theta} - \varepsilon, \bar{k}]$. Then $l_m \equiv \max\{l_m^1, l_m^2\} \geq u_{(m,1]}(k, I_k)$ for $k > \bar{\theta} - \varepsilon$. From Lemma 9, l_m^1 and l_m^2 are continuous in m , and so is l_m . Hence, there exists $m' < \bar{\theta}$ such that $l_{m'} < l_{\bar{\theta}} - \delta/2 \leq \delta/2 < 0$. This implies that $u_{(m',1]}(k, I_k) \leq \delta/2$ for $k > \bar{\theta} - \varepsilon$. In this case, either $u_{(m',1]}(k, I_k) < 0$ for all $k \in (m' - \varepsilon, \bar{\theta} - \varepsilon]$, or there exists $k' = \sup\{k \in (m' - \varepsilon, \bar{\theta} - \varepsilon] : u_{(m',1]}(k, I_k) \geq 0\}$. From Lemma 6, either there is no attack on $(m', 1]$, thus $m' \in M$, or, in the worst equilibrium for the government, speculators follow $I_{k'}$ after observing $(m', 1]$. In the latter case, the government abandons the peg for $\theta \leq \theta_{k'} \in (m', \bar{\theta})$. Consider the partition $P^{\theta_{k'}}$. From Lemma 9, $u_{(\theta_{k'},1]}(k, I_k) < 0$ for all $k \in X_{(\theta_{k'},1]}$, and, from Lemma 6, there is no attack on y_h . This means that $\theta_{k'} \in M$. Thus, either $\bar{\theta} > m' \in M$ or $\bar{\theta} > \theta_{k'} \in M$, which implies that $\underline{m} < \bar{\theta}$. \square

B.10 Proof of Theorem 3

Theorem 3. *Suppose that Assumption 1 is satisfied. For every partition P , $V(P) \leq \bar{V}$, where*

$$\bar{V} = \lim_{m \downarrow \underline{m}} V(P^m) = \sup_{m \in M} V(P^m).$$

Then

- i. if $\underline{m} \in M$, the government's equilibrium payoff is \bar{V} . In equilibrium, when $\theta > \underline{m}$, there are no attacks and the peg is maintained; and when $\theta \leq \underline{m}$, every speculator attacks the currency and the peg is abandoned. The government can achieve the payoff \bar{V} with the two-interval partition $P^{\underline{m}} = \{0, \underline{m}, 1\}$.
- ii. if $\underline{m} \notin M$, no equilibrium exists. However, the government can achieve a payoff arbitrarily close to \bar{V} .

Proof: For any two partitions A and B , if $V(A) > V(B)$, then A is said to be preferred to B . From Lemma 12, we know that $\underline{m} < \bar{\theta}$.

Suppose that the partition P is optimal. From Theorem 1, we can assume that $P = \{0, m, 1\}$.

- i. a. Suppose that $m > \underline{m}$. In this case, there exists $m' \in [\underline{m}, m) \cap M$. If $m \leq \theta^*$, from Theorem 2, it follows that the peg is abandoned if and only if $\theta \in [0, m]$. Since $m' \in M$ and $m' < \theta^*$, the peg is abandoned if and only if $\theta \in [0, m']$. Hence the partition $\{0, m', 1\}$ is preferred to P . If $m > \theta^*$, from Theorem 2, partition $\{0, \theta^*, 1\}$ is preferred to P , a contradiction with the optimality of P . Hence $m \leq \underline{m} < \bar{\theta}$.
- b. Suppose that $m < \underline{m}$. Since $m < \bar{\theta}$, the peg is abandoned for $\theta \in [0, m]$. From Lemma 7, in the worst equilibrium for the government, speculators follow a cutoff rule I_{k_h} after observing y_h , where $k_h = \sup\{k \in X_{y_h} : u_{y_h}(k, I_k) \geq 0\}$. Given the speculators' strategy, there

exists $\theta_{k_h} > m$ such that the peg is abandoned if and only if $\theta \leq \theta_{k_h}$. From Lemma 9, increasing m would (weakly) decrease the cutoff signal k , which would (weakly) decrease the threshold state θ_k . This implies that, with partition $P' = \{0, \theta_{k_h}, 1\}$, no one attacks if $\theta \in (\theta_{k_h}, 1]$. Thus, P' is preferred to P , a contradiction with the optimality of P . We have that $m = \underline{m}$.

- ii. From *a.*, *b.*, if P is an optimal partition, then $m = \underline{m}$. If $\underline{m} \in M$, partition $P^{\underline{m}} = \{0, \underline{m}, 1\}$ is optimal. If $\underline{m} \notin M$, there is no equilibrium, but the government can achieve a payoff arbitrarily close to $\bar{V} = \lim_{m \downarrow \underline{m}} V(P^m)$.

□