Economics 701B
Fall, 2017

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- This is graduate level introduction to general equilibrium, social choice and mechanism design. It is designed for first year Economics Ph.D. students, to be taken in conjunction with Economics 703. Anyone who is not a first year Econ. Ph.D. student must see the instructor.
- 701B is taught by Professor Andrew Postlewaite (room 458 McNeil building, phone: 898-7350, office hours: Friday 1:30-2:30 or by appointment), and 701A is taught by Professor Steven Matthews; teaching assistants are Joao Granja de Almeida and Shunyi Zhao. The class will meet Mondays and Wednesdays 1:30 - 3 p.m in McNeil 410. The teaching assistants' office hours will be as before and the T.A. session will also be as before.

Problem sets will be assigned at least once every two weeks and no more than once a week. The problem sets are important. You will not learn this material if you do not do these thoroughly. You are encouraged to work on them together but *not by dividing up the questions*. There may be occasional deviations from this schedule depending on the timing of lectures. Teaching assistants will provide answer sheets. While these will not formally be part of your grade, anyone who has not done the problems will not be treated sympathetically.

- There will a final exam that will cover only the material in 701B after classes end at the regularly scheduled time (12:00pm Thursday December 14). Your grade will be based 50% on the first half of the course and 50% on the second half. The schedule is subject to change.
- A tentative outline for the course is listed below. There will undoubtably be deviations, but the order of topics will be followed quite closely.

Readings
Texts for 701A are:

- Andreu Mas-Collel, Michael Whinston and Jerry Green [MWG]: *Microeconomic Analysis**. New York: Oxford University Press (1995).
- Geoffrey Jehle and Philip Reny [JR]: *Advanced Economic Theory*. London: Pearson (2011) (suggested)

Other books that may be helpful:

- Truman Bewley [B]: *General Equilibrium, Overlapping Generations Models, and Optimal Growth Theory.* Cambridge: Harvard University Press (2007).
- Salanie, Bernard [S]: *Microeconomics of Market Failures.* Cambridge, MA: MIT Press (2000).

1 Decision Theory

1. Introduction (B 1-2)
2. Decision Making under Certainty
   (a) MWG 1.B, 3.B, 3.C
   (b) JR 1.2

2 General Equilibrium

1. Definition
   (a) MWG 15.B
   (b) JR 5.1
   (c) Cobb-Douglas MWG 3.D

3 Welfare Theorems

1. MWG 10, 16
2. JR 5.2

4 Existence and Local Uniqueness of General Equilibrium

1. MWG 17
2. JR 5.2

5 Production

1. MWG 15.C, 17
2. JR 5.3

6 Equilibrium under Uncertainty

1. MWG 19
2. JR 5.4
3. Rational Expectations MWG 19.H

7 Incomplete Markets

1. Asset Markets MWG 19.E
2. Incomplete Markets MWG 19.F
3. Overlapping Generations MWG 20.D

8 Externalities and Public Goods

1. MWG 11, Example 16.G.3; S 5-6

9 Game Theoretic Foundations of Competitive Equilibria

1. Shapley-Shubik Models MWG 18.C.3
2. Core
   - (a) MWG 18.B
     (b) JR 5.5

10 Problems

The following is the tentative schedule of problem sets. The problem sets will definitely be due to turn it, but the dates are tentative. Many are the same as the problems from past years. Answers to the problems are probably available. It cannot be prevented but it is expected that the problems will be done without reference to these. I can assure you of two things: first, it is relatively easy to tell when someone is submitting answers that are taken from past answer sheets, and second, ultimately you will do better in everything that matters in this program if you do the problems without reference.

10.1 Problem Set I: Preference and Utility; Walrasian Equilibrium

1. MWG 15.B.1
2. MWG 15.B.2
3. Superscripts denote the individual, subscripts the commodity. Suppose there are 2 commodities and 2 agents. Each agent's consumption space is $X^h = R^2_+$. Suppose that agents' utility functions are Leontief with

$$u^1(x^1_1, x^1_2) = \min(x^1_1, x^1_2) \quad u^2(x^2_1, x^2_2) = \min(3x^2_1, x^2_2)$$

and suppose the endowments are given by

$$e^1 = (2, 1), \quad e^2 = (0, 2).$$

   (a) Compute all Walrasian equilibria.

4. MWG 15.B.6
5. MWG 15.B.10

10.2 Problem Set II Walrasian Equilibrium

1. Consider a standard Arrow Debreu exchange economy: There are H households and L commodities. Each household $h = 1, ..., H$ has a utility function which is continuous and increasing. Each household has strictly positive individual endowments $e^h \in I\!R^L_{++}$. Suppose that $\sum_{h \in \mathcal{H}} e^h_l = 1$ for all commodities $l = 1, ..., L$. Suppose that for each h $u^h(c) = \sum_{l=1}^L v^h(c_l)$, where $v^h : I\!R_+ \to I\!R$ is increasing, strictly concave, differentiable and satisfies $\lim_{x \to 0} v^{h\prime}(x) = \infty$.

   (a) Show that for any household h and any price p, if $p_i > p_j$, h demands more of commodity j.

   (b) b. Using the observation in a, prove that the economy has at most one competitive equilibrium.

2. Consider an economy with two households and two commodities. Assume household 1 has utility function

$$u^1(x_1, x_2) = \min(x_1, \frac{1}{4}x_2),$$

and household 2 has utility function

$$u^2(x_1, x_2) = \min(x_1, \frac{3}{4}x_2).$$

Assume initial endowments are $e^1 = (\alpha, 1)$ and $e^2 = (1 - \alpha, 1)$.

(a) Compute the equilibrium price correspondence as a function of α for all $\alpha \in (0,1)$.

(b) Explain why the correspondence is upper hemicontinuous, or show that it is not.

3. Bewley 4.7 Consider the Edgeworth box example where $e^A = (12,0)$, $e^B = (0,12)$ and $u^A(x_1,x_2) = u^B(x_1,x_2) = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}}$.

   (a) Calculate and draw accurately the offer curves for each consumer.

   (b) Find all competitive equilibria with the price of the first good equal to 1.

4. MWG 15.B.3

5. MWG 15.B.8

6. MWG 15.C.2

10.3 Problem Set III: Competitive Equilibria: Existence and Optimality

1. Consider an economy with two agents and two commodities, with endowments $e^1 = (\alpha, 1)$ and $e^2 = (1,1)$ and with utility functions $u^1(x) = x_2^1$ and $u^2(x) = x_1^2$.

   (a) Argue that for $\alpha > 0$ a Walrasian equilibrium always exists: Compute the equilibrium allocation and the equilibrium price as a function of α.

   (b) Show that for $\alpha = 0$ there is no Walrasian equilibrium. Explain why the standard existence proof fails.

2. Suppose now that individual endowments are

$$e^1 = (5,1) \quad e^2 = (1,1)$$

   and that utility functions are given by

$$u^1(x_1^1, x_2^1) = (x_1^1 - 1)^2 + (x_2^1 - 1)^2 \quad u^2(x_1^2, x_2^2) = x_1^2 + x_2^2$$

   (a) Does there exist a Walrasian equilibrium? Explain.

3. Possible failures of first welfare theorem.

   (a) Suppose that there are two agents and two commodities. Both agents have differentiable, strictly increasing and strictly concave utility, but there is a missing market for commodity 2 (that is, they cannot trade commodity 2). Define Walrasian equilibrium and show in an Edgeworth box (or analytically if you prefer) that typically Walrasian equilibria are not Pareto-efficient here.

(b) Suppose there is the following externality in the economy. Let $x^1 = (x_1^1, x_2^1)$ be household 1's consumption, let $x^2 = (x_1^2, x_2^2)$ be household 2's consumption. The two households have utility functions

$$u^1(x) = \log(x_1^1 + x_1^2) + \log(x_2^1)$$

and

$$u^2(x) = \log(x_1^2) + \log(x_2^2).$$

Suppose endowments are $e_1 = (1,2)$ $e_2 = (2,1)$. Agents behave competitively. Compute all Pareto-optimal allocations and compute one Walrasian equilibrium. Is it Pareto-optimal?

4. For each of the assumptions we made to ensure existence of Walrasian equilibria, give a counterexample to existence if that assumption fails but the other assumptions hold.

5. Consider a competitive economy with two firms and two consumers. Firm 1 is entirely owned by consumer 1. It produces guns from oil via the production function $g(x) = 2x$. Firm 2 is entirely owned by consumer 2; it produces butter from oil via the production function $b(x) = 3x$. Each consumer owns 10 units of oil. Consumers 1 and 2 have respective utilities $u^1(g,b) = g^{0.4}b^{0.6}$ and $u^2(g,b) = 10 + 0.5\ln g + 0.5\ln b$.

   (a) Find the market-clearing prices for guns, butter and oil.

   (b) How many guns and how much butter does each agent consume?

   (c) How much oil does each firm use?

10.4 Problem Set IV: Markets with Uncertainty

1. Three hunters will hunt for deer tomorrow in a game park with exactly two deer. Each hunter will catch at most one deer (by park regulations), and both deer will be caught. There are thus three states of the world tomorrow: state $s = 1,2,3$ represents the event that each hunter **except** hunter s catches a deer. Letting ω^i denote hunter i's initial endowment of contingent deer meat, we have

$$\omega^1 = (0,1,1),\ \omega^2 = (1,0,1),\ \omega^3 = (1,1,0).$$

Today (date $t = 0$) they arrange for how the meat from any deer caught tomorrow (date $t = 1$) will be shared. The utility function of hunter i is

$$U^i(x_i) = \Sigma_{s=1}^3 \pi_s^i u^i(x_s^i),$$

where x_s^i is his consumption of deer meat in state s, and π_s^i is his belief probability that state s will occur. Assume u^i is continuous, strictly concave, and strictly increasing.

(a) Suppose the hunters agree that the state probabilities are $(1/2, 1/4, 1/4)$. (Hunter 1 is believed to be twice as likely to not catch a deer as is either of the other two.) Show that at any interior Pareto efficient allocation, hunter 1 will consume the same amount of deer meat regardless of who catch the deer. (You can assume for this part that each u_i is differentiable.)

For the remaining parts, assume instead that each hunter is so self-confident that he believes he will surely catch a deer: $\pi^i_i = 0$ for each i. But he is not as confident about the others: $\pi^i_s > 0$ for $s \neq i$.

(b) Prove that if (x^{1*}, x^{2*}, x^{3*}) is a Pareto efficient allocation, then $x^{i*}_i = 0$ for each i.

(c) Prove that competitive equilibria exist, letting p_s denote the price at date 0 for contingent deer meat in state s at date 1. (You cannot simply quote the existence result proved in class, since here the preferences are **not** strongly monotone because each $\pi^i_i = 0$.)

2. B 7.8

3. B 7.10

4. B 7.12

5. MWG 19.C.1

6. MWG 19.C.3

7. MWG 19.C.4

8. MWG 19.C.5

9. MWG 19.H.6

10. MWG 19.H.7 Problem 19.H.7 has a typo. For consumer 2 in state 2 the multiplication sign should be replaced by a sum.

10.5 Problem Set V: Incomplete Markets and Core

1. Asset pricing. Consider a 2 period GEI model of an exchange economy with a single commodity per state. Suppose there are 5 states and 3 assets. The assets pay

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 0 & 3 \\ 1 & 3 & 0 \\ 0 & 0 & 1 \\ 2 & 1 & 2 \end{pmatrix}$$

Which of the following asset prices preclude arbitrage ?

(a) $q = (1, 2, 1)$

(b) $q = (2, 4, 1)$

(c) $q = (1, 1, 1)$

(d) $q = (6, 5, 7)$

(e) $q = (2.1, 1.2, 2.2)$

2. a) In the previous problem you figured out which of the prices precluded arbitrage. Suppose now that all agents have von Neumann-Morgenstern utility functions and markets are complete (there are 2 options in addition to the 3 assets above). Suppose that $\pi_1 = \pi_2 = \pi_3 = \pi_4 = \pi_5 = 0.2$ and that $e_5 > e_4 > e_3 > e_2 > e_1$. Which of the above prices 1-5 could be equilibrium prices for this economy (if any) ?
   b) Suppose now that all agents have quadratic utility functions and that in addition to the three assets above there is a bond in the economy. The price of the bond is 1. Suppose that aggregate endowments are $e = (2, 5, 1, 1, 4)$. Which of the above prices 1-5 could be equilibrium prices for this economy (if any)?

3. MWG 19.E.6

4. MWG 19.H.4

5. a) State assumptions under which a Walrasian equilibrium allocation is contained in the core, and prove the result.
   b) Give an example of an economy in which there is a Walrasian equilibrium allocation that is *not* in the core.

6. MWG 18.B.5