Political Economy of Transparency

Raphael Galvão
University of Pennsylvania
rgalvao@sas.upenn.edu

November 20, 2017

Abstract

This paper develops a model where short-term reputation concerns guide the public disclosure of information and affect welfare in a coordination environment. Entrepreneurs use public information to make investment decisions, and there is complementarity in their actions. There are and high and low states that determine the productivity in the economy, and the high state is more likely if the government is *efficient* rather than *inefficient*. Governments know the state and make public reports with the objective to be perceived as efficient. Entrepreneurs form beliefs about the government based on the public report and the realized productivity, a noisy signal of the state. I find that the inefficient government is never completely truthful in equilibrium. When the efficient government is truthful, the inefficient government sends false reports of a high state with positive probability. This creates uncertainty following the report of a high state: if the true state is high, productivity is underestimated; if the true state is low, productivity is overestimated. This bias reduces welfare in the high state, but there is a tradeoff in the low state: marginal entrepreneurs lose from overestimating productivity; all entrepreneurs gain from a higher aggregate investment. I show that when the trust in the government's report is low, the inefficient government can improve welfare in the low state by sending false reports that increase investment. However, as the trust in the false reports rises, the bias in entrepreneurs' beliefs becomes large and welfare decreases (there is too much investment).

1 Introduction

I develop a model where short-term reputation concerns determine the public disclosure of information about the state of the economy, and then analyze its welfare effects in a coordination environment. When governments are privately informed about a payoff-relevant state, concerns for reputation might prevent them from truthfully disclosing the information. If the distribution of states is related to the government's type, the public information can be biased towards the state that is more likely under the agent's preferred type. The disclosure policy thus creates a bias in the agents' beliefs about the state, which affects their actions in equilibrium. When there is complementarity in the actions, it is possible that biased beliefs actually improve welfare in certain states.

There are *efficient* and *inefficient* governments, with privately known types, and both want to maximize their reputation for being efficient. The two types differ in their ability to generate the high and low productivity states. The high state is more likely when the government is efficient rather than inefficient. There is an underlying assumption that an unobservable and costly action can be taken to increase the probability of the high state, and only the efficient governments are willing to take that action. Governments learn the state and report it to entrepreneurs through a public signal. The reports are said to be *truthful* if they match the state, and they are *false* otherwise. The entrepreneurs rely on public information – the reports about the state and the realized productivity – to update their beliefs about the government.

Each period, entrepreneurs in the model can borrow in a competitive credit market to invest in a new venture, and there is complementarity in investment. Ventures face a common probability of failure, and entrepreneurs receive private signals about it. Conditional on success, the productivity of the ventures depends on the state of the economy. In equilibrium, given the public signal about the state, entrepreneurs follow a cutoff rule and invest if their private signals are high enough. The extent to which investment decisions respond to the public report depends on the government's reputation and on how truthful the public disclosure policy is. The more entrepreneurs believe that the state is high, the higher is their equilibrium cutoff given the public signal, and the more likely they are to invest.

In any equilibrium, the government's reputation evolves gradually over time, and entrepreneurs are never certain about the government's type – the distribution of productivity has the same support in both states. There is no equilibrium where the inefficient government follows a full disclosure policy. If the inefficient government were always truthful, the efficient government would respond by making false reports to distinguish itself from the inefficient type. This creates incentives for the inefficient government to deviate from full disclosure to be perceived as the efficient type. I focus on the equilibrium where the efficient government follows a full disclosure policy. In this equilibrium, the inefficient government is too optimistic: it is truthful in the high state, but in the low state it randomizes between true and false reports. The inefficient government's reports are thus biased toward the high state, which is more likely under the efficient type.

When the government reports a low state, entrepreneurs are certain that the state is indeed low, and their beliefs about the expected productivity are not biased. Following a report of a high state, entrepreneurs are not sure about the true state and their beliefs are biased: they overestimate productivity in the low state, and underestimate it in the high state. The higher is the trust in the government's announcement, the higher is the equilibrium level of investment when a report of a high state is sent. If the true state is high, welfare is increasing in the entrepreneurs' trust in the government, and welfare is maximized when entrepreneurs are sure of the state. However, in the low state there is a tradeoff when the inefficient government sends a false report: there are complementarity gains to all entrepreneurs from a higher level of investment, and potential losses to the marginal entrepreneurs due to the overestimation of productivity. As long as entrepreneurs do not place too much trust in the government's report, the bias is small enough and the net effect is positive for welfare. When the trust in the public signal increases, false reports induce too much investment and reduce welfare in the low state.

Related literature.

This paper relates to the literature in which, due to reputation concerns, agents with privately known types may modify their actions to affect other agents' beliefs about their type (see Mailath and Samuelson (2001)). Here, I focus on the government's incentives to send optimistic signals to be perceived as an efficient type, even if that results in lower welfare. In contrast to what is commonly assumed in the literature of government reputation (see, for example, Barro and Gordon (1983) and Phelan (2006)), the government here cannot take actions that directly affect the agents' payoffs; it can only disclose information about payoff-relevant states, and actions cannot reveal the government's type. [1]

This paper is closely related to Herrera et al. (2015). They show that, for emerging economies, the rise in governments' popularity is a better predictor of financial crises than other better-known indicators, such as credit booms (see, for example, Mendoza and Terrones (2012) and Schularick and Taylor (2012)). The paper argues that governments in emerging economies are more concerned with their reputation and choose to enjoy the short-term popularity benefits of weak credit booms rather than implementing costly policies that would reduce the probability that such booms end in crises. They develop a model where booms can be good (sustainable) or bad (unsustainable), and the policy that maximizes welfare is the regulation of bad booms, and no regulation of good booms. There is a good government, which always acts optimally, and a bad government that is strategic and wants to maximize its reputation for being good. Since the good boom is more likely under the good government, the bad governments will not always regulate bad booms, as

[1] Here, the government is not trying to convince agents that it will not behave *opportunistically* and take an action that negatively affects their payoffs (such as increasing capital taxes). Instead, the government is trying to show the agents that it can generate the high productivity states more often.

regulation reveals that the boom is not sustainable and it negatively impacts the government's reputation. In my model, I assume that the government cannot directly affect the outcome in the economy; the welfare effects of the public disclosure policy depend on how agents respond to it. As in Herrera et al. (2015), a large increase in reputation can be a bad sign for welfare in my model. When the inefficient government sends false reports with high probability, agents do not trust reports of a high productivity state. Thus, if the government sends a false report, it is not believed and there is only a small rise in reputation, followed by a small increase in credit and investment, which is welfare improving because of the complementarity in investment. However, when the inefficient government is not likely to make false reports, agents trust the public signal. In this case, following a false report in the low productivity state, there is large increase in reputation and a high level of investment, which decreases welfare. If entrepreneurs are required to borrow in order to invest, this results in a high probability of default, which can be interpreted as a credit crisis.

The paper also relates to the literature on pandering. For instance, Maskin and Tirole (2004) analyze a model where politicians might have the same preferences as the electorate or not, and their type is privately known. They show that when a politician has strong motives to remain in office, she always takes the popular action (the *ex ante* optimal action for the voters), even if she knows that the action is not optimal, and regardless of her type. The politician thus panders to public opinion because she wants to build a reputation for being the type that has the same preferences as the voters. In a different setting, a similar result is presented in Brandenburger and Polak (1996). They show that when a manager is concerned with the market's perception about his actions, he will distort his investment decision toward an investment that the market believes is *ex ante* more likely to succeed. In my model, instead of a privately informed government making decisions, I have agents that choose their actions based a public signal, and the government's type affects the distribution of payoff relevant states. The disclosure policy follows the same logic of pandering: when the efficient government is truthful, the inefficient government sends signals that are biased toward the state that agents believe is more likely when the government is efficient.

Finally, the paper is related to the literature on global games of regime change. The model can be interpreted as having two regimes: the default is a low productivity regime; and if the level of investment is high enough, there is a switch to a high productivity regime. By investing, entrepreneurs are attacking the low productivity regime, and the probability of failure is assumed to affect the success of the attack. In each period, the game between entrepreneurs is similar to the one in Morris and Shin (1998), who study a model of self-fulfilling currency attacks when the fundamentals that determine payoffs are not common knowledge among entrepreneurs. The equivalent to their state of the fundamentals in my model is the venture's probability of failure.

As in Morris and Shin (1998), the game between entrepreneurs has a unique equilibrium when the noise in the entrepreneurs' private information is small enough, and the equilibrium investment strategies also follows a cutoff rule based on their private signals. Deviation from common knowledge is key for the uniqueness of equilibrium. My model departs from Morris and Shin (1998) by introducing another state variable that affects the payoffs in case of a regime change, and a government that sends public signals about that variable (the state of the economy in the current paper, which affects the productivity of the ventures). The introduction of public policy in such coordination environments, and its signaling effects, have been extensively studied in the literature (see, for example, Angeletos et al. (2006), Angeletos and Pavan (2007, 2009) and Angeletos and Pavan (2013)). Breaking the uniqueness result in Morris and Shin (1998), Angeletos et al. (2006) point out that policy interventions that convey some information about the fundamentals may lead to multiple equilibria. In contrast to Angeletos et al. (2006), the public policy in my model does not lead to multiplicity. This is the case because there is no public information about the state that affects the success of an attack, only about the state that determines payoffs conditional on the regime change. The public signal only affects the entrepreneurs' cutoff rule: the cutoff is increasing in the probability that entrepreneurs assign to the high productivity state.

Structure of the paper. The remainder of this paper is divided as follows. Section 2 presents the model and the equilibrium disclosure policy for the government is characterized in Section 3. Section 4 analyzes the entrepreneurs' equilibrium investment strategies, and the welfare effects of the public policy. Section 5 concludes the paper and discusses extensions to the model. Appendix A analyzes the model when entrepreneurs are require to borrow in a competitive credit market to star a new venture. Furthermore, it presents conditions under which the two models have the same equilibrium investment strategies for the entrepreneurs. The proofs that are omitted in the main text are presented in Appendix B.

2 The Model

2.1 Actions and payoffs

Time is discrete and indexed by $t \in \{1, 2, ...\}$. There is a continuum of entrepreneurs of measure one, who are indexed by i and uniformly distributed on $[0, 1]$. They are infinitely-lived, risk-neutral profit maximizers. Each period, entrepreneurs have an endowment of one unit of labor, which can be used to start up a new, risky, venture, or to work for a fixed wage w.[2] For simplicity, there is no capital in the model, only labor. Appendix A analyzes the model when new ventures also require

[2] The wage w can be seen as the payoff from choosing a safe rather than a risky venture.

one unit of capital, which is borrowed in a perfectly competitive credit market. In the model with capital, there is an equilibrium where the investment decisions are the same as in the baseline model without capital. [3]

The ventures have a common probability of failure θ_t, which is drawn every period from a uniform distribution on $\Theta = [\theta_{\min}, \theta_{\max}]$. The total number of ventures in period t is denoted by n_t. In case of success, at the end of period t the venture pays

$$v, \quad \text{if } n_t < N(\theta_t),$$

and

$$v + \delta_t, \quad \text{if } n_t \geq N(\theta_t),$$

where $N(\cdot) < 1$ is weakly increasing in θ, with a continuous derivative $N'(\cdot)$. The productivity of the ventures is thus increased by $\delta_t > 0$ if the aggregate investment is high enough. Failed ventures are assumed to pay nothing.

Each period, the productivity parameter δ depends on a state variable $s \in S = \{H, L\}$. Given s_t, δ_t is follows a distribution with probability density function f_{s_t} and mean δ_{s_t}. It is assumed that

$$\operatorname{supp} f_H = \operatorname{supp} f_L = \Delta = [\delta_{\min}, \delta_{\max}],$$

in which case the realization of δ never reveals the state. State H is associated with higher productivity, as described in the assumption below.

Assumption 1. *The likelihood ratio $\lambda(\delta) \equiv f_H(\delta)/f_L(\delta)$ is continuously differentiable, increasing in δ, and it is strictly increasing for $\delta \in (\delta_1, \delta_2) \subseteq [\delta_{min}, \delta_{max}]$, where $\delta_1 < \delta_2$.*

Assumption 1 implies that $\delta_H > \delta_L$.

There is also a government in this economy, which can be *efficient* (type E) or *inefficient* (type I), and the types are private information. It is assumed that the type is permanent and the same government remains in power forever. The types only differ in their ability to generate the high productivity state H. Each period, high productivity states are more likely when the government is efficient:

$$\pi_E \equiv Pr(s_t = H|E) > \pi_B \equiv Pr(s_t = H|I), \quad \text{for all } t.$$

The government knows the state and can report it through a public signal $y_t \in Y = \{h, l\}$.[4] We say that the report is *truthful* if either $y_t = h$ when $s_t = H$, or $y_t = l$ when $s_t = L$, and it is *false* otherwise.

[3] For that result, the opportunity cost of a venture must be the same in both models. Without capital, the opportunity cost is w, the cost of labor. With capital, the opportunity cost is $1 + r + \tilde{w}$, the cost of labor plus capital, where r and $\tilde{w}$ are the risk-free rate and the wage in the model with capital. Therefore, we need that $w = 1 + r + \tilde{w}$.

[4] There are two interpretations for the disclosure policy. One is that the government observes s_t and sends a

The government's reputation at the beginning of period t is denoted by μ_t, which is the probability that entrepreneurs assign to the efficient type E. The government's payoff at period t is given by μ_{t+1}, the updated reputation at the end of the period, after entrepreneurs observe y_t and δ_t. Both types, E and I are strategic, and their goal in each period is to maximize the expected value of μ_t. The governments are assumed to be myopic and only care about their reputation at the end of the period.[5]

2.2 Timing and information

At period $t = 1$, nature draws the government's type from $\{E, I\}$ Entrepreneurs enter period $t = 1$ with a common prior μ_1 about the government's type. At the beginning of period t, nature draws the probability of failure $\theta_t \in \Theta$ and the state $s_t \in \{H, L\}$. The government observes s and sends a public signal $y_t \in \{h, l\}$ about the state. Entrepreneurs then form beliefs about the state and the expected value of δ_t. The expected value of δ_t is $\bar{\delta}_t = \mathbb{P}(s = H|\mu_t, y_t)\delta_H + \mathbb{P}(s = L|\mu_t, y_t) \in [\delta_L, \delta_H]$. Entrepreneur i also receives a private signal $x_{t,i}$ about θ_t. After observing the private and public signals, entrepreneurs simultaneously decide whether to invest or to work. Given s_t, nature draws the productivity parameter δ from a distribution with probability density function f_{s_t}. At the end of the period, the outcomes of all ventures are publicly observed, payoffs are received, and the government's reputation is updated to μ_{t+1}. The structure of the game is assumed to be common knowledge.

Given θ_t, entrepreneur i receives a private signal $x_{t,i} \in X = [\theta_{\min} - \varepsilon, \theta_{\max} + \varepsilon]$, where

$$x_{t,i} = \theta_t + \varepsilon_{t,i}.$$

The idiosyncratic noise $\varepsilon_{t,i}$ is drawn from a distribution with probability density function $g(\cdot)$, and cumulative distribution function $G(\cdot)$. Each $\varepsilon_{t,i}$ is independently and identically distributed across entrepreneurs and independent of θ_t. I assume that $\text{supp}(g) = [-\varepsilon, \varepsilon]$, with $\varepsilon > 0$, and $2\varepsilon < \min\{\underline{\theta} - \theta_{\min}, \theta_{\max} - \bar{\theta}_{\delta_H}\}$. Function $g(\cdot)$ is differentiable on $(-\varepsilon, \varepsilon)$, and its derivative, $g'(\cdot)$, is

(possibly random) public signal $y(s_t)$. Another interpretation is that the government follows an information acquisition procedure and, if the state is s_t, the outcome is a (possibly random) public signal $y(s_t)$. The latter is in line with the Bayesian persuasion literature (see, for example, Kamenica and Gentzkow (2011)). In this case, the government is not more informed than the entrepreneurs when the public signal is sent.

[5] The government in this model can be seen as a party (efficient or inefficient) that is perpetually in power. Each period there is a different member of the party who runs the government (the current president). She only cares maximizing the reputation of the party while she is charge, and future reputation is not a concern. Extensions where the government can be replaced and care about future reputation are discussed Section 5.

assumed to be bounded and such that[6]

$$\text{if } g'(\tilde{\varepsilon}) < 0, \text{ then } g'(\hat{\varepsilon}) \leq 0 \quad \forall \hat{\varepsilon} \in (\tilde{\varepsilon}, \varepsilon). \tag{1}$$

The posterior distribution of θ given private signal x has probability density function $\phi(\theta|x)$, where

$$\phi(\theta|x) = \frac{g(x-\theta)}{G(x-\theta_{\min}) - G(x-\theta_{\max})}. \tag{2}$$

The derivation of $\phi(\theta|x)$ is presented in Appendix B.[7]

At the end of the period, entrepreneurs might observe the realization of δ and use it to update their beliefs about the government. There are two alternative frameworks.

Assumption 2-A. *At the end of the period, the realization of δ is always publicly observed.*

Assumption 2-B. *The realization of δ is publicly observed when $n \geq N(\theta)$, in which case successful ventures pay $v + \delta$. If $n < N(\theta)$, entrepreneurs do not observe δ.*

In what follows, the results are true under both Assumption 2-A or Assumption 2-B, unless the required assumption is clearly specified.

2.3 Equilibrium

I restrict attention to *Markov* strategies: for any t, t', if $\mu_t = \mu_{t'}$, the government and the entrepreneurs follow the same strategies in periods t and t'. In other words, conditional on the current beliefs about the government, the strategies are independent of the history of actions, states, and outcomes that lead to those beliefs.

Remark: In this paper, the link between periods is the evolution of entrepreneurs' beliefs about the government. The per-period payoffs are independent of past and future actions, states and outcomes, the government maximizes its expected reputation at the end of each period, and I limit attention to equilibria in Markov strategies. I chose this highly stylized dynamic game rather than a static one to capture the evolution of the government's reputation, and how it affects entrepreneurs' strategies. In Section 5 I discuss possible extensions that would make the dynamic game more realistic.

[6] The assumptions on the structure of private signals are based on a previous work Galvao and Shalders (2017). They guarantee that, conditional on the public signal, the game between entrepreneurs in each period has a unique equilibrium.

[7] There is a pair of values of x that fully reveals θ. If $x = \theta_{\max} + \varepsilon$, we have $\mathbb{P}(\theta = \theta_{\max}|x = \theta_{\max} + \varepsilon) = 1$; likewise, when $x = \theta_{\min} - \varepsilon$, then $\mathbb{P}(\theta = \theta_{\min}|x = \theta_{\min} - \varepsilon) = 1$. For all other values of x, the conditional density of θ is given by (2).

The efficient government's strategy for period t is denoted by $p_E : [0,1] \times S \to [0,1]$, where $p_E(\mu_t, s)$ is the probability that the efficient government sends a signal $y = h$, given that the prior reputation is μ_t and the current state is s. [8] Similarly, the inefficient government's strategy for period t is denoted by $p_I : [0,1] \times S \to [0,1]$. Entrepreneurs beliefs about the productivity parameter δ are given by $\bar{\delta} : [0,1] \times Y \to [\delta_L, \delta_H]$, where $\bar{\delta}(\mu_t, y)$ is the expected value of δ given that a government with reputation μ_t has sent a public signal y. Entrepreneur i's strategy for period t is given by $a_i : [\delta_L, \delta_H] \times X \to \{0,1\}$, where $a_i(\bar{\delta}, x_i) = 1$ represents investing and $a_i(\bar{\delta}, x_i) = 0$ represents working, given a private signal x_i and $\bar{\delta}$.[9] The government's reputation at the end of period t is given by $\mu_{t+1} : [0,1] \times Y \times \Theta \times \Delta$, where $\mu_{t+1}(\mu_t, y, \theta, \delta)$ is the probability that entrepreneurs assign to the efficient type if a government of reputation μ_t sends a signal y, and the observed productivity is δ.

The equilibrium concept here is perfect Bayesian equilibrium (PBE). Given a common prior μ_1, a PBE consists of entrepreneurs' beliefs μ_t, strategies for types E, p_E, for type I, p_I, and for the continuum of entrepreneurs, $\{a_i\}_{i \in [0,1]}$, such that beliefs are updated using Bayes rule whenever possible[10] and, given the beliefs, no player has an incentive to deviate.

3 Optimal Disclosure Policy

This section characterizes the equilibrium disclosure policies at period t. At the beginning of period t, the prior reputation is given by μ_t, the probability that entrepreneurs assign to the efficient type at the end of period $t-1$. The efficient government's strategy for period t is given by $p_E(\mu_t, s)$, which denotes the probability that type E sends a signal $y = h$ given μ_t and state s. Similarly, the inefficient type I's strategy is given by $p_I(\mu_t, s)$. Both types follow disclosure policies that maximize their expected reputation at the end of the period, μ_{t+1}.

An equilibrium profile for time t consists of strategies p_E and p_I for types E and I, and beliefs and strategies for the entrepreneurs, such that beliefs are obtained using Bayes rule whenever possible[11] and, given the beliefs, no player has an incentive to deviate. This section characterizes the entrepreneurs' beliefs and the equilibrium strategies for the government. The equilibrium strategies for the entrepreneurs are characterized in Section 4.

There exist the trivial equilibria in which both types send either $y = h$ or $y = l$ regardless of the state. These equilibria are supported by the belief that the government is inefficient whenever

[8] Given the restriction to Markov strategies, I drop the subscript t, except for the reputation μ_t.

[9] It is assumed that entrepreneurs invest whenever indifferent, thus $a_i \in \{0,1\}$.

[10] In this setting, government's deviations from equilibrium are only observable if, for a prior reputation μ_t, both types send either $y_t = h$ or $y_t = l$ with probability 1, regardless of the true state s_t. Apart from the case of observable deviations, entrepreneurs use Bayes rule to update their beliefs.

[11] See footnote 10

a deviation is observed. There is no equilibrium in which the inefficient type I follows a full disclosure policy, i.e., where the reports are always truthful: $y = h$ in state H, and $y = l$ in state L. This result is formalized in Lemma 7, in Appendix B. Intuitively, if the inefficient government were always truthful, the efficient government would respond by making false reports to distinguish itself from the inefficient type. This creates incentives for the inefficient government to deviate from full disclosure to be perceived as the efficient type.

There exist equilibria where the efficient government follows a full disclosure policy. In what follows, I restrict attention to such equilibria. First, the efficient government is assumed to follow

$$p_E(\mu_t, H) = 1 - p_E(\mu_t, L) = 1, \quad \text{for all } \mu_t.$$

Then the best response of the inefficient government is characterized. Finally, I check whether this is an equilibrium strategy profile for period t.

3.1 Reputation

Given μ_t and the governments' strategies, entrepreneurs update beliefs using Bayes rule. First, entrepreneurs form intermediate beliefs following the public signal y and make investment decisions. Then, conditional on observing a realization of δ, entrepreneurs update the reputation to μ_{t+1}. If the government sends a public signal $y = h$, the entrepreneur's intermediate update about the government's reputation is

$$\mu^h(\mu_t) = \frac{\pi_E \mu_t}{\pi_E \mu_t + [\pi_I p_I(\mu_t, H) + (1 - \pi_I) p_I(\mu_t, L)]\,(1 - \mu_t)}. \tag{3}$$

If $y = l$, the intermediate update is

$$\mu^l(\mu_t) = \frac{(1 - \pi_E)\mu_t}{(1 - \pi_E)\mu_t + [\pi_I(1 - p_I(\mu_t, H)) + (1 - \pi_I)(1 - p_I(\mu_t, L))]\,(1 - \mu_t)}. \tag{4}$$

Given the public signal y, entrepreneurs form beliefs about the expected value of δ. If $y = h$, the expectation of δ is

$$\begin{aligned}\bar{\delta}(\mu_t, h) = &\left[\mu^h(\mu_t) + (1 - \mu^h(\mu_t)) \frac{\pi_I p_I(\mu_t, H)}{\pi_I p_I(\mu_t, H) + (1 - \pi_I) p_I(\mu_t, L)}\right] \delta_H \\ &+ \left[(1 - \mu^h(\mu_t)) \frac{(1 - \pi_I) p_I(\mu_t, L)}{\pi_I p_I(\mu_t, H) + (1 - \pi_I) p_I(\mu_t, L)}\right] \delta_L,\end{aligned} \tag{5}$$

and if $y = l$,

$$\bar{\delta}(\mu_t, l) = \left[(1-\mu^l(\mu_t))\frac{\pi_I(1-p_I(\mu_t,H))}{\pi_I(1-p_I(\mu_t,H))+(1-\pi_I)(1-p_I(\mu_t,L))}\right]\delta_H + \left[\mu^l(\mu_t)+(1-\mu^l(\mu_t))\frac{(1-\pi_I)(1-p_I(\mu_t,L))}{\pi_I(1-p_I(\mu_t,H))+(1-\pi_I)(1-p_I(\mu_t,L))}\right]\delta_L. \quad (6)$$

After investment decisions are made and the outcomes of all ventures are observed, entrepreneurs might observe δ. If $y = h$ and δ is observed, the government's updated reputation is

$$\mu^h_\delta(\mu_t) = \frac{\pi_E\mu_t}{\pi_E\mu_t + \left[\pi_I p_I(\mu_t,H) + \frac{f_L(\delta)}{f_H(\delta)}(1-\pi_I)p_I(\mu_t,L)\right](1-\mu)}. \quad (7)$$

From Assumption 1, the likelihood ratio $f_H(\delta)/f_L(\delta)$ is increasing in δ, thus μ^h_δ is also increasing in δ. The higher is δ, the more likely it is that the true state is H and that the report $y = h$ is truthful. Since type E is always truthful, the reputation increases in δ.

If $y = l$ and δ is observed, the updated reputation is

$$\mu^l_\delta(\mu_t) = \frac{(1-\pi_E)\mu_t}{(1-\pi_E)\mu_t + \left[\frac{f_H(\delta)}{f_L(\delta)}\pi_I(1-p_I(\mu_t,H)) + (1-\pi_I)(1-p_I(\mu_t,L))\right](1-\mu_t)}. \quad (8)$$

From Assumption 1, μ^l_δ is decreasing in δ. As δ increases, it less likely that the true state is L and that the report $y = l$ is truthful, therefore the reputation decreases.

Under Assumption 2-A, the realization of δ is always observed at the end of the period. In this case, the government's reputation at the end of the period is given by

$$\mu_{t+1}(\mu_t, y, \theta, \delta) = \mu^y_\delta(\mu_t), \quad \text{for all } \mu_t, y, \theta, \delta. \quad (9)$$

Given prior reputation μ_t and state s, the expected reputation by sending signal y is

$$\bar{\mu}_t(\mu_t, s, y) = \mathbb{E}_\delta[\mu_{t+1}(\mu_t, y, \theta, \delta)|s]. \quad (10)$$

Under Assumption 2-B, the realization of δ is only observed when the number of ventures is greater than $N(\theta)$, in which case successful ventures pay $v + \delta$. After observing y, entrepreneurs compute the expected value of δ, $\bar{\delta}(\mu_t, y)$, observe their private signals x_i and make their investment decisions, which are characterized in Section 4. From Proposition 2 in Section 4, the higher is the entrepreneurs expectation of δ, the higher is the equilibrium number of ventures, n, and the higher is the probability that δ is observed. It follows from Proposition 2 in Section 4 that, under

Assumption 2-B, the government's reputation at the end of the period is given by

$$\mu_{t+1}(\mu_t, y, \theta, \delta) = \begin{cases} \mu_\delta^y(\mu_t), & \text{if } \theta \leq \theta^*(\bar{\delta}(\mu_t, y)) \\ \mu^y(\mu_t), & \text{if } \theta > \theta^*(\bar{\delta}(\mu_t, y)) \end{cases}. \tag{11}$$

The probability of δ being observed is $\mathbb{P}(\theta \leq \theta^*(\bar{\delta}(\mu_t, y))) \equiv P^*(\mu_t, y)$. Given a prior μ_t and a state s, the government's expected reputation from sending a signal y is

$$\bar{\mu}_t(\mu_t, s, y) = P^*(\mu_t, y)\mathbb{E}_\delta[\mu_\delta^y(\mu_t)|s] + [1 - P^*(\mu_t, y)]\mu^y(\mu_t). \tag{12}$$

The government's objective is to maximize $\bar{\mu}_t$. The expected payoff gain from being truthful in state H and sending a signal h rather than a signal l is given by

$$G_H = \bar{\mu}_t(\mu_t, H, h) - \bar{\mu}_t(\mu_t, H, l). \tag{13}$$

The gain from being truthful in state L is given by

$$G_L = \bar{\mu}_t(\mu_t, L, l) - \bar{\mu}_t(\mu_t, L, h). \tag{14}$$

3.2 Equilibrium policy

In any equilibrium where the efficient type follows a full disclosure policy, the inefficient type will truthfully disclose the high productivity state H, as stated in the lemma below.

Lemma 1. *Let $\mu_t = \mu \in (0, 1)$. If the efficient government follows full disclosure, then the inefficient government is truthful when $s = H$:*

$$p_I(\mu, H) = 1.$$

The proof is in Appendix B. Intuitively, there are two reasons for the inefficient government to be truthful in state H when the efficient government is always truthful. When entrepreneurs observe a signal h, they believe that it is more likely that the government is efficient, since state H is more likely when the government is efficient. The second reason is that, if the government sends $y = l$ and entrepreneurs observe a realization of δ that is more likely under state H, they will assign a high probability to a false report, which only happens if the government is inefficient. Thus, by sending a signal h, the inefficient government increases both its reputation prior to the realization of δ and the expected reputation conditional on δ being observed.

Since the inefficient government is truthful in the high productivity state, there can only be false reports in the low productivity state. In what follows, denote by p_μ the probability that the inefficient government sends a signal h in state L ($p_\mu \equiv p_I(\mu, L)$). We can write the gain from making truthful reports in state L, given by (14), as a function $G_L(\mu, p_\mu)$.

Lemma 2. *$G_L(\mu, p_\mu)$ has the following properties:*

(i) $G_L(0, p) = G_L(1, p) = 0, \quad$ *for all* $p \in [0, 1]$.

(ii) $G_L(\mu, 0) < 0$, *for all* $\mu \in (0, 1)$.

(iii) $G_L(\mu, 1) > 0$, *for all* $\mu \in (0, 1)$.

(iv) Under Assumption 2-A, $G_L(\mu, 0)$ *is strictly convex in* μ.

From part (*i*) of Lemma 2, when entrepreneurs are sure about the government's type, the inefficient government has no incentives to make false reports in the low productivity state. However, from part (*ii*), incentives arise when there is uncertainty about the government's type. Part (*iii*), shows that the incentives to lie disappear when the probability of false reports, p_μ, becomes too high. Finally, parts (*i*), (*ii*), and (*iv*) imply that, under Assumption 2-A, the gain from always being truthful in state L, $G_L(\mu, 0)$, is U-shaped in μ: starting from $G_L(0, 0) = 0$, $G_L(\mu, 0)$ first decreases in μ, then it increases to reach $G_L(1, 0) = 0$. The incentives for the inefficient government to make false reports are thus highest for intermediate values of the prior reputation.

From Lemma 2 we get the following result.

Lemma 3. *Let* $\mu_t = \mu \in (0, 1)$. *If the efficient type follows full disclosure, then the inefficient government sends* $y = l$ *with positive probability in state L:*

$$p_\mu \in (0, 1),$$

where p_μ *is such that* $G_L(\mu, p_\mu) = 0$. *If Assumption 2-A holds, there exists a unique* $p^*_\mu \in (0, 1)$ *that solves* $G_L(\mu, p_\mu) = 0$.

Given the inefficient government's response to the efficient government's full disclosure policy, it is left to show that the efficient government has no incentives to deviate, and that the strategy profile is indeed an equilibrium for period t. This result is described in the following proposition.

Proposition 1. *Let* $\mu_t = \mu \in (0, 1)$. *There exist an equilibrium where, in period t, the efficient government follows a full disclosure policy and the inefficient government sets*

$$p_I(\mu_t, H) = 1,$$

and

$$p_I(\mu, L) = p_\mu \in (0, 1),$$

where p_μ solves $G_L(\mu, p_\mu) = 0$.

If Assumption 2-A holds, there exists a unique p^*_μ that solves $G_L(\mu, p_\mu) = 0$.

4 Investment and Welfare

This section analyzes the entrepreneurs' equilibrium strategies for period t, and the equilibrium levels of investment and welfare. Given a public signal y and prior reputation μ_t, entrepreneurs form expectations about δ, as described in (5) and (6), to make investment decisions. Here, I fix the expected value of δ at $\bar{\delta}(\mu_t, y) = \bar{\delta}$.

4.1 Investment

As mention in Section 2.3, given the restriction to Markov strategies, the entrepreneur i's strategy only depends on the current private signal x_i and on $\bar{\delta}$. Hence, conditional on $\bar{\delta}$, the game between the entrepreneurs in each period is similar to the one in Morris and Shin (1998). In their paper, entrepreneurs decide whether to attack a currency or not based on their private signals about the fundamentals of the economy. In the current paper, given $\bar{\delta}$, entrepreneurs decide whether to invest or to work given their private signals about the venture's probability of failure. For a given probability of failure θ, and a given $\bar{\delta}$, an entrepreneur's expected payoff from investing is

$$(1 - \theta)v, \quad \text{if } n < N(\theta),$$

and

$$(1 - \theta)(v + \bar{\delta}), \quad \text{if } n \geq N(\theta).$$

Denote by $\underline{\theta}$ the value of θ that solves $(1 - \theta)v = w$. If $\theta < \underline{\theta}$, it is optimal to invest even if no other entrepreneur is investing. Denote by $\bar{\theta}(\bar{\delta})$ the value of θ that solves $(1 - \theta)(v + \bar{\delta}) = w$. If $\theta > \bar{\theta}(\bar{\delta})$, it is not optimal to invest even if all entrepreneurs are investing. To simplify the notation, let $\bar{\theta}_H \equiv \bar{\theta}(\delta_H)$, and $\bar{\theta}_L \equiv \bar{\theta}(\delta_L)$.

When there is common knowledge about the probability of failure, Θ can be divided in three intervals[12], as is standard in the literature of self-fulfilling equilibria:[13]

[12] It is assumed that

- $v > w$;
- $\underline{\theta} = 1 - w/v > \theta_{\min}$;
- $\bar{\theta}_H = 1 - w/[v + \delta_H] < \theta_{\max}$.

[13] See, for example, Obstfeld (1996) and Morris and Shin (1998) in the case of self-fulfilling currency attacks.

- if $\theta \in [\theta_{\min}, \underline{\theta})$: it is always profitable to invest;
- if $\theta \in (\underline{\theta}, \bar{\theta}(\bar{\delta}))$: coordinated investment is profitable and, if entrepreneurs coordinate on not investing, investment is not profitable;
- if $\theta \in (\bar{\theta}(\bar{\delta}), \theta_{\max}]$: it is never profitable to invest.

As the expected value of δ, $\bar{\delta}$, increases, the threshold $\bar{\theta}(\bar{\delta})$ also increases. This means that there are more values of θ for which coordinated investment is profitable (the middle interval grows to the right), and there are fewer values of θ that prevent investment from being profitable (the upper interval shrinks).

Now we turn to the equilibrium with private information about θ. Conditional on $\bar{\delta}$, an equilibrium for the game between the entrepreneurs in period t consists of strategies such that no entrepreneur has an incentive to deviate. For a given profile of strategies for the entrepreneurs, the measure of entrepreneurs who invest given $\bar{\delta}$ and a private signal x is denoted by $\eta(\bar{\delta}, x)$. Given a probability of failure θ, the number of ventures is then

$$n(\bar{\delta}, \theta, \eta) = \int_{\theta-\varepsilon}^{\theta+\varepsilon} \eta(\bar{\delta}, x) g(x-\theta)\, dx. \tag{15}$$

Conditional on success, the expected productivity of a venture is increased by $\bar{\delta}$ when

$$n(\bar{\delta}, \theta, \eta) \geq N(\theta). \tag{16}$$

Thus, the event where a venture's expected payoff is $v + \bar{\delta}$ is given by

$$A(\bar{\delta}, \eta) = \{\theta : n(\bar{\delta}, \theta, \eta) \geq N(\theta)\}. \tag{17}$$

After observing x_i, entrepreneur i's expected payoff from investing is:

$$u(\bar{\delta}, x_i, \eta) = v \int_{x_i-\varepsilon}^{x_i+\varepsilon} (1-\theta)\phi(\theta|x_i) d\theta + \bar{\delta} \int_{[x_i-\varepsilon, x_i+\varepsilon]\cap A(\bar{\delta},\eta)} (1-\theta)\phi(\theta|x_i) d\theta, \tag{18}$$

where ϕ is given by (2). Entrepreneur i invests in equilibrium if:

$$u(\bar{\delta}, x_i, \eta) \geq w. \tag{19}$$

The following proposition characterizes the unique equilibrium of the game played by the entrepreneurs at time t, conditional on $\bar{\delta}$.

Proposition 2. *Given $\bar{\delta}$, the equilibrium of the game between entrepreneurs in period t is unique. The equilibrium strategy for the entrepreneurs is to invest if and only if their private signal is*

$$x \leq x^*(\bar{\delta}).$$

The equilibrium number of ventures n is thus decreasing in θ. $n \geq N(\theta)$ if and only if the probability of failure is

$$\theta \leq \theta^*(\bar{\delta}).$$

Both $x^*(\bar{\delta})$ and $\theta^*(\bar{\delta})$ are increasing in $\bar{\delta}$.

The proof of Proposition 2 is in Appendix B. Entrepreneurs follow a cutoff rule and invest if their private signal is below $x^*(\bar{\delta})$. Since $x^*(\bar{\delta})$ is increasing, for every θ the number of ventures is increasing in the entrepreneurs' expectation of δ. The cutoff rule leads to a threshold probability of failure $\theta^*(\bar{\delta})$, below which the total number of ventures is greater than $N(\theta)$, and the successful ventures pay $v + \delta$ instead of v. Since the threshold $\theta^*(\bar{\delta})$ is also increasing, the higher is the entrepreneurs' expectation of δ, the higher is the probability that ventures pay $v + \delta$ instead of v. The entrepreneurs' equilibrium strategy is thus

$$a_i(\bar{\delta}, x_i) = a^*(\bar{\delta}, x_i) = \begin{cases} 1, & \text{if } x_i \leq x^*(\bar{\delta}) \\ 0, & \text{if } x_i > x^*(\bar{\delta}) \end{cases}. \tag{20}$$

where $x^*(\bar{\delta})$ solves

$$u(\bar{\delta}, x^*(\bar{\delta}), a^*) = w, \tag{21}$$

Equation (21) is the indifference condition for the entrepreneur who receives the cutoff signal $x^*(\bar{\delta})$. In equilibrium, the total number of ventures is given by

$$n(\bar{\delta}, \theta, a^*) = \mathbb{P}(x \leq x^*(\bar{\delta})|\theta) = G(x^*(\bar{\delta}) - \theta).$$

4.2 Welfare

In state s, the mean value of δ is δ_s. If $\bar{\delta} \neq \delta_s$, the entrepreneurs' expectation of the productivity parameter is biased. The entrepreneurs' expected welfare in state s is given by[14]

$$W_s(\bar{\delta}) = (v + \delta_s) \int_{\theta_{\min}}^{\theta^*(\bar{\delta})} (1 - \theta) G(x^*(\bar{\delta}) - \theta) d\theta + v \int_{\theta^*(\bar{\delta})}^{x^*(\bar{\delta})+\varepsilon} (1 - \theta) G(x^* - \theta) d\theta$$

[14] θ is uniformly distributed on $[\theta_{\min}, \theta_{\max}]$, therefore the density is constant at $[1/(\theta_{\max} - \theta_{\min})]$. For simplicity, I multiplied the welfare function by $[\theta_{\max} - \theta_{\min}]$.

$$+ w\left[\int_{x^*(\bar{\delta})-\varepsilon}^{x^*(\bar{\delta})+\varepsilon}(1 - G(x^*(\bar{\delta}) - \theta))d\theta + \int_{x^*(\bar{\delta})+\varepsilon}^{\theta_{\max}} d\theta\right]. \tag{22}$$

From Proposition 1, both types are truthful when the state is H, but the inefficient government sends false reports with positive probability in state L. Following a signal $y = l$, entrepreneurs are sure that the true state is L, and there is no distortion in the entrepreneurs' expectation about δ: $\bar{\delta}(\mu, l) = \delta_L$ for all $\mu \in (0, 1)$. However, when the government sends a signal $y = h$, there is a distortion: $\bar{\delta}(\mu, l) \in (\delta_L, \delta_H)$ for all $\mu \in (0, 1)$. Entrepreneurs overestimate δ when the true state is L, and underestimate δ when the state is H. The higher is the entrepreneurs' trust in the public signal − their belief that the report is truthful, and the state is H − the higher is $\bar{\delta}$. Hence, in state L the distortion increases with the entrepreneurs' trust a signal h ($\bar{\delta}$ gets further away from δ_L), while in state H the distortion decreases with the entrepreneurs' trust ($\bar{\delta}$ gets closer to δ_L).

If the true state is H, welfare is increasing in the entrepreneurs' expectation of δ, and it is maximized at $\bar{\delta} = \delta_H$ (i.e., when the expectation is unbiased). This result is stated in the following lemma.

Lemma 4. *$W_H(\bar{\delta})$ in increasing in $\bar{\delta}$, for all $\bar{\delta} \leq \delta_H$.*

The more entrepreneurs believe that the government is being truthful when sending $y = h$, the higher is their expectation of δ and the more they are willing to invest. Lemma 4 thus implies that, in the high productivity state, welfare is increasing in the entrepreneurs' trust in the public signal.

In the low productivity state L, welfare increases if the entrepreneurs' expectation of δ is slightly biased. Starting at $\bar{\delta} = \delta_L$, a marginal increase in the $\bar{\delta}$ increases W_L. This result is formalized in the following lemma.

Lemma 5. *$\frac{\partial W_L(\bar{\delta})}{\partial \bar{\delta}} > 0$, at $\bar{\delta} = \delta_L$.*

Lemma 5 shows that entrepreneurs might benefit from having biased expectation of δ in state L. Biased expectations induce entrepreneurs to be more aggressive in their investment strategies and receive the complementarity gain δ more often. Complementarity in investment is thus key to this result. However, as the bias increases, welfare might start to decrease. This is the case when $x^*(\delta_H) > \bar{\theta}_L + \varepsilon$, which is true if $(\delta_H - \delta_L)$ is large enough.[15] This result is presented in the following lemma.

Lemma 6. *Suppose that $x^*(\delta_H) > \bar{\theta}_L + \varepsilon$. Then, there exists $\tilde{\delta} \in (\delta_L, d_H)$ such that $\frac{\partial W_L(\bar{\delta})}{\partial \bar{\delta}} < 0$, for $\bar{\delta} \geq \tilde{\delta}$.*

The intuition for Lemmas 5 and 6 is the following. When $\bar{\delta}$ increases, entrepreneurs expected payoff from investing also increases. This raises the equilibrium cutoff signal for investing, $x^*(\bar{\delta})$,

[15] For example, if $\delta_H > \frac{2\varepsilon(v+\delta_L)^2}{w-2\varepsilon(v+\delta_L)}$.

which in turn raises the threshold $\theta^*(\bar{\delta})$, below which entrepreneurs receive the productivity gain δ. When the true state is L, there is a tradeoff from raising the cutoff: the marginal investors are worse off due to their biased expectation of δ; while all entrepreneurs gain from the a higher level of investment. In equilibrium, there is more investment when the probability of failure is low ($\theta < \bar{\theta}_L$), and it is optimal to invest, but there there is also more investment when the probability of failure is high ($\theta > \bar{\theta}_L$), and it is optimal to work. If the entrepreneurs' expectation is biased, but close enough to $\bar{\theta}_L$, the positive effect dominates, and raising the cutoff increases welfare W_L. However, when the entrepreneurs' expectation of δ is too biased, such that $x^*(\bar{\delta}) > \bar{\theta}_L + \varepsilon$, the tradeoff disappears and only the negative effect on W_L remains: raising the cutoff only increases investment when $\theta > \bar{\theta}_L$, and it is optimal to work.

Thus, when the true state is L, if entrepreneurs assign a small probability to state H, there is a small increase in investment, which is welfare improving. As entrepreneurs become more convinced that the state H when it is in fact L, welfare starts to decrease because there is too much investment when the probability of failure is high, and working is optimal. This means that, when entrepreneurs have little trust in the government's report of $y = h$, the inefficient government increases welfare by making a false report in state L. As the trust in the false report increases, welfare will start to decrease. The welfare results are summarized in the following proposition.

Proposition 3. *In state H, welfare is increasing in the entrepreneurs' trust in the public signal. In state L, the inefficient government can increase welfare by making false reports if the trust in the public signal is low. As the trust in the public signal grows, welfare will start to decrease.*

5 Concluding Remarks

This paper analyzed the effects of short-term reputation concerns in the disclosure of public information in a coordination environment.

In equilibrium, when the efficient government is truthful, the inefficient government sends signals that are too optimistic, making false reports of a high productivity state with positive probability to be perceived as efficient. This creates a distortion in the entrepreneurs' beliefs about the productivity of investment. I find that false reports can increase welfare in the low productivity state. Following a false report, entrepreneurs overestimate the productivity of new venture and have more aggressive investment strategies. Since there is complementarity, entrepreneurs benefit from a higher level of aggregate investment. When agents distrust the government, the bias in the entrepreneurs' beliefs is small and welfare improving: the potential losses caused by overestimation of productivity are offset by the complementarity gains. As the trust in the false reports increases, there is too much investment and welfare starts to decrease. In the high productivity state however, welfare is increasing in the entrepreneurs' trust in the government. When the entrepreneurs do not trust a true report of a high productivity state, they underestimate the productivity of a new venture, there is less investment, and welfare is reduced.

There are two interesting extensions to the model: including a concern for welfare in the government's utility function; and introducing the concern for future reputation and the possibility of replacement. When welfare is taken into account, the efficient government might depart from a truthful policy to increase welfare in the low productivity state. If the government cares about the discounted value of being in office, it is possible to explore the tradeoff between current and future reputation. With the introduction of replacement, this framework can be used to analyze policy experiments concerning the frequency of elections. For example, if the government wants to maximize its reputation every T periods, when elections are held, we can see how the choice of T affects welfare. We can also analyze how the strength of the government (or institutions) affects the incentives to disclose information. Suppose that whenever the reputation falls below a threshold $\underline{\mu}$, the incumbent is replaced, and the stronger the government, the lower is $\underline{\mu}$. In this case, weaker governments will place a higher weight on short-term reputation. This is equivalent to introducing the possibility of recall at every period.

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Appendices

A Credit Market

This section drops the assumption that only labor is necessary to start a new venture. Now a venture also requires one unit of capital, which is borrowed in a perfectly competitive credit market. There exists an equilibrium for the model with capital where the investment decisions are the same as the equilibrium decisions in the model without capital, as described in Proposition 2. In this equilibrium, the welfare results from Section 4 still hold.

There are two types of agents: entrepreneurs and lenders. The agents' problem in each period is now similar to the one in Veldkamp (2005).[16] In each period, the entrepreneurs now have to borrow one unit of capital to invest in a new venture. An entrepreneur that does not invest works for a fixed wage $\tilde{w}$. There is a continuum of lenders, who are index by j and uniformly distributed on $[0, J]$, with $J > 1$.[17] As the entrepreneurs, lenders are infinitely-lived, risk-neutral profit maximizers. At the beginning of each period, lenders can either use one indivisible unit of capital to buy a risk-free bond which pays a return of $(1 + r)$ at the end of the period, or they can lend capital to an entrepreneurs. The risk-free rate is exogenous and constant. The lender receives $(1 + \rho)$ at the end of the period if the venture is successful, and nothing otherwise. The market lending rate is endogenous and depends on the expected rate of default. It is assumed that, when entrepreneur i and a lender j meet, the lender can perfectly observe the entrepreneur's private signal about the probability of failure, x_i.

A Markov strategy for lender j is $\rho_j : [0, 1] \times Y \times X \to \mathbb{R}$, where $\rho_j(\mu_t, y, x)$ is the interest rate that lender j charges from an entrepreneur who received a signal x, conditional on (μ_t, y). Given a reputation μ_t and a public signal y, agents form beliefs about the state and lenders announce a pricing function $\rho_j(\mu_t, y, x)$. Entrepreneurs can choose which lenders to borrow from, but lenders cannot commit to an interest rate. Once lender j observes x_i, he can decide not to lend to entrepreneur i. In this case, the lender buys the risk-free bond, while the entrepreneur can search for another lender. Interest rate $\rho_j(\mu_t, y, x_i)$ is only credible if lender j's expected payoff conditional on (μ_t, y, x_i) is greater than $(1 + r)$.

Apart from the introduction of the lenders and the requirement that one unit of capital must be borrowed to start a new venture, the model is the same as in Section 2. The timing in period t is as follows:

1. Reputation starts at μ_t.

2. Nature draws $s \in \{H, L\}$.

3. The government observes s and sends a signal $y \in \{h, l\}$.

4. Agents form beliefs about the state and lenders announce pricing functions $\{\rho_j(\mu_t, y, \cdot)\}_{j=\in[0,J]}$.

[16] In her paper, there is a finite number of entrepreneurs and lenders, who are infinitely lived, risk-neutral, and profit maximizers. There are more lenders than entrepreneurs, and the credit market is perfectly competitive. In each period, entrepreneurs can either borrow one unit of capital to invest in a new venture, or work for a fixed wage. Successful ventures pay v_i to entrepreneur i. The probability of success in each period is the same for all new ventures, and it depends on an unobservable and persistent state variable. Lenders can either invest one unit of capital in a risk-free bond that pays $(1 + r)$, or lend it to potential borrowers, who pay $(1 + \rho)$ in case of success, and nothing otherwise. In equilibrium, since lenders are perfectly competitive, the expected return from lending is the risk-free rate: $\mathbb{P}(success)(1 + \rho) = 1 + r$.

[17] There are more lenders than entrepreneurs.

5. Nature draws the probability of failure θ.

6. Entrepreneurs observe interest rates and private signals about θ, and decide whether or not borrow.

7. If entrepreneur i and lender j agree on a loan, i borrows at rate $\rho_j(\mu_t, y, x_i)$.

8. Lenders not matched with borrowers invest in the risk-free bond. Entrepreneurs that do not invest receive a wage $\tilde{w}$.

9. The outcomes of all ventures are publicly observed, payoffs are received, and the reputation is updated to μ_{t+1}.

Let $\bar{\delta}(\mu_t, y) = \bar{\delta}$, and let the measure of entrepreneurs who invest, given $\bar{\delta}$ and a private signal x, be denoted by $\eta(\mu_t, y, x)$. The number of ventures is characterized in (15), and the event where ventures pay $(v + \delta)$ is given by $A(\mu_t, y, \eta)$, described in (17). Lender $j's$ expected payoff from lending to an entrepreneur who receives private signal x is thus

$$\begin{aligned} R_j(\mu_t, y, x, \eta) =& \min\{1 + \rho_j(\mu_t, y, x), v\} \int_{[x-\varepsilon, x+\varepsilon]/A(\mu_t, y, \eta)} (1-\theta)\phi(\theta|x)d\theta \\ &+\mathbb{E}_\delta[\min\{1 + \rho_j(\mu_t, y, x), v + \delta\}|\mu_t, y] \int_{[x-\varepsilon, x+\varepsilon]\cap A(\mu_t, y, \eta)} (1-\theta)\phi(\theta|x)d\theta. \qquad (23) \end{aligned}$$

In equilibrium, lender j enters into a contract with an entrepreneurs who receives a signal x if

$$R_j(\mu_t, y, x) \geq 1 + r.$$

The interest rate is only credible if $\rho_j(\mu_t, y, x)$ is such that $R_j(\mu_t, y, x) \geq 1 + r$. If $R_j(\mu_t, y, x) < 1 + r$, entrepreneurs that receive a signal x know that lender j will renege on the interest rate $\rho_j(\mu_t, y, x)$ once he observes a signal x.

A.1 Equilibrium

The opportunity cost of a starting a new venture in the model without capital is w, the cost of labor. With the introduction of capital, the opportunity cost of a venture is now $1 + r + \tilde{w}$, the cost of labor plus capital. If $w = 1 + r + \tilde{w}$, there is an equilibrium in the model with capital that features the same investment strategies for the entrepreneurs as in the the baseline model from Section 2.

The expected surplus from a venture is given by

$$S(\mu_t, y, x, \eta) = v \int_{x-\varepsilon}^{x+\varepsilon} (1-\theta)\phi(\theta|x)d\theta + \bar{\delta}(\mu_t, y) \int_{[x-\varepsilon, x+\varepsilon]\cap A(\mu_t, y, \eta)} (1-\theta)\phi(\theta|x)d\theta - (1 + r + \tilde{w}), \quad (24)$$

which is the venture's expected payoff given (μ_t, y, x, η), minus the opportunity cost of capital and labor. Consider the following strategy for lenders: if $S(\mu_t, y, x, \eta) \geq 0$, lender j sets $\rho_j(\mu_t, y, x)$ such that $R_j(\mu_t, y, x) = 1 + r$; otherwise set $\rho_j(\mu_t, y, x)$ so high that no entrepreneur would borrow from j.[18] Consider the following rule for entrepreneurs to choose a lender: if entrepreneur i decides to borrow, only choose lender j if $\rho_j(\mu_t, y, x_i)$ such that $R_j(\mu_t, y, x_i) \leq 1 + r$. The pricing strategy for lenders and the rule for borrowers are part of an equilibrium. No lender has an incentive to deviate: if j sets $\rho_j(\mu_t, y, x')$ such that $R_j(\mu_t, y, x') > 1 + r$, no entrepreneur who observes x' borrows from j; if j sets $\rho_j(\mu_t, y, x')$ such that $R_j(\mu_t, y, x') < 1 + r$, the interest rate is not credible and no entrepreneur who observes x' borrows from j. No borrower has an incentive to deviate: entrepreneur i is better off by rejecting any lender j who sets $R_j(\mu_t, y, x_i) > 1 + r$, given that there are $J > 1$ lenders who are charging lower interest rates.

In such an equilibrium, after observing x_i, entrepreneur i's expected payoff from borrowing to invest is

$$\tilde{u}(\mu_t, y, x_i, \eta) = v \int_{x_i-\varepsilon}^{x_i+\varepsilon} (1-\theta)\phi(\theta|x_i)d\theta + \bar{\delta}(\mu_t, y) \int_{[x_i-\varepsilon, x_i+\varepsilon]\cap A(\bar{\delta},\eta)} (1-\theta)\phi(\theta|x_i)d\theta - (1+r). \tag{25}$$

Compared to the payoff in the model without capital, given by u in equation (18), we have

$$\tilde{u}(\mu_t, y, x_i, \eta) = u(\bar{\delta}(\mu_t, y), x_i, \eta) - (1+r), \qquad \text{for all } \mu_t, y, x_i, \eta.$$

Entrepreneur i invests in equilibrium if

$$\tilde{u}(\mu_t, y, x_i, \eta) \geq \tilde{w} \Leftrightarrow u(\bar{\delta}(\mu_t, y), x_i, \eta) \geq 1 + r + \tilde{w}. \tag{26}$$

Condition (26) is the same as condition (19) when $w = 1 + r + \tilde{w}$. In this case, the entrepreneurs' equilibrium investment strategies are the same as in the model with no capital, and Proposition 2 applies, with $\bar{\delta}(\mu_t, y) = \bar{\delta}$.

The agents' expected welfare in state s is thus given by

$$\begin{aligned}\tilde{W}_s(\bar{\delta}) =& (v + \delta_s) \int_{\theta_{\min}}^{\theta^*(\bar{\delta})} (1-\theta)G(x^*(\bar{\delta}) - \theta)d\theta + v \int_{\theta^*(\bar{\delta})}^{x^*(\bar{\delta})+\varepsilon} (1-\theta)G(x^*(\bar{\delta}) - \theta)d\theta \\ &+ (1 + r + \tilde{w}) \left[\int_{x^*(\bar{\delta})-\varepsilon}^{x^*(\bar{\delta})+\varepsilon} (1 - G(x^*(\bar{\delta}) - \theta))d\theta + \int_{x^*(\bar{\delta})+\varepsilon}^{\theta_{\max}} d\theta \right] + (J-1)(1+r). \end{aligned} \tag{27}$$

[18] For example, $\rho_j(\mu_t, y, x) = v + 2\delta_{\max}$.

The welfare in the model without capital, W_s, is described in (22). If $w = 1 + r + \tilde{w}$, we have

$$\tilde{W}_s(\bar{\delta}) = W_s(\bar{\delta}) + (J-1)(1+r).$$

Thus, the welfare results in Section 4 still hold. Lemmas 4, 5, and 6 also hold for the welfare function $\tilde{W}_s$, and so does Proposition 3.

In the model with credit, there are two types of default: default is *total* if the venture fails; and default is *partial* if the payoff from a successful venture is less than $1+\rho$. In the equilibrium above, given their beliefs, lenders are indifferent between lending of buying risk-free bonds. In the low productivity state L, when the inefficient government makes a false report $y = h$, the agents' beliefs are biased towards the high productivity state H. Lenders thus underestimate the probability of partial default, and charges interest rates that are too low. The more agents' trust the false report h, the higher is the probability of partial default in state L, and the lower is the lenders' payoff.

B Proofs

B.1 Posteriors

For any pair of continuous random variables A and B, let g_{AB} denote their joint pdf. Let g_A and g_B denote the marginal pdfs, and let $g_{A|B}$ denote the pdf of A conditional on B. Finally, denote the cdfs by G_A and G_B. Following the main text, we denote the pdf of the idiosyncratic noise by g, and its cdf by G, omitting the subscripts.

For $x \in (\theta_{\min} - \varepsilon, \theta_{\max} + \varepsilon)$:

$$\begin{aligned} g_{\theta|x}(\theta|x) &= \frac{g_{\theta x}(\theta, x)}{g_x(x)} = \frac{g(x-\theta)g_\theta(\theta)}{\int_{-\infty}^{+\infty} g(x-\tilde{\theta})g_\theta(\tilde{\theta})d\tilde{\theta}} \\ &= \frac{g\,(x-\theta)}{G\,(x-\theta_{\min}) - G\,(x-\theta_{\max})} \quad \text{, if } \theta \text{ is uniform on } [\theta_{\min}, \theta_{\max}]. \end{aligned}$$

For $x \in \{-\varepsilon, 1+\varepsilon\}$: $\mathbb{P}(\theta = 0 | x = -\varepsilon) = 1$; $\mathbb{P}(\theta = 1 | x = 1+\varepsilon) = 1$.

B.2 Equilibrium Policy

Before proving the results in Section 3, I first present some auxiliary results. For $\mu_t \in (0,1)$:

Claim 1. *Given Assumption 1, $F_H(\delta) < F_L(\delta)$, for $\delta \in (\delta_{min}, \delta_{max})$.*

Proof: Define $\lambda(\delta) \equiv f_H(\delta)/f_L(\delta)$, for all δ. First, notice that $\lambda(\delta_{\min}) < 1$, otherwise

$$\begin{aligned} \lambda(\delta_{\min}) &\geq 1, \quad \text{for } \delta \in (\delta_{\min}, \delta_1) \\ \Rightarrow \lambda(\delta_{\min}) &> 1, \quad \text{for } \delta > \delta_1, \end{aligned}$$

which implies that, for $\delta < \delta_1$

$$F_L(\delta) = \int_{\delta_{\min}}^{\delta} f_L(\tilde{d})d\tilde{\delta} \leq \int_{\delta_{\min}}^{\delta} f_H(\tilde{d})d\tilde{\delta} = F_H(\delta),$$

and for $\delta > \delta_1$

$$\begin{aligned} F_L(\delta) &= F_L(\delta_1) + \int_{\delta_1}^{\delta} f_L(\tilde{d})d\tilde{\delta} \\ &< F_H(\delta_1) + \int_{\delta_1}^{\delta} f_L(\tilde{d})d\tilde{\delta} \\ &= F_H(\delta), \end{aligned}$$

therefore $1 = F_L(\delta_{\max}) < F_H(\delta_{\max}) = 1$, a contradiction.

Define $\underline{\delta} = \inf\{\delta|\lambda(\underline{\delta}) = 1\}$. From Assumption 1, $\underline{\delta}$ is well defined and $\underline{\delta} < \delta_{\max}$, otherwise $\lambda(\delta) < 1$, for all $\delta < \delta_{\max}$, and

$$1 = F_H(\delta_{\max}) = \int_{\delta_{\min}}^{\delta_{\max}} f_H(\delta)d\delta < \int_{\delta_{\min}}^{\delta_{\max}} f_L(\delta)d\delta = F_L(\delta_{\max}) = 1,$$

a contradiction.

Finally, there exists $\bar{\delta} < \delta_{\max}$, such that $\lambda(\bar{\delta}) > 1$. If this is not the case, then $\lambda(\delta) = 1$ for all $\delta \in (\underline{\delta}, \delta_{\max}]$, therefore

$$\begin{aligned} 1 = F_H(\delta_{\max}) &= F_H(\delta_{\min}) + \int_{\delta_{\min}}^{\delta_{\max}} f_H(\delta)d\delta \\ &= F_H(\delta_{\min}) + \int_{\delta_{\min}}^{\delta_{\max}} f_L(\delta)d\delta \\ &< F_L(\delta_{\min}) + \int_{\delta_{\min}}^{\delta_{\max}} f_L(\delta)d\delta \\ &= F_L(\delta_{\max}) = 1, \end{aligned}$$

a contradiction.

Thus $f_H(\delta) < f_L(\delta)$, for $\delta \in [\delta_{\min}, \underline{\delta})$; $f_H(\delta) \geq f_L(\delta)$, for $\delta \in (\underline{\delta}, \bar{\delta}]$; and $f_H(\delta) > f_L(\delta)$, for $\delta \in (\bar{\delta}, \delta_{\max}]$. For $\delta \leq \underline{\delta}$, it is clear that $F_H(\delta) < F_L(\delta)$. Suppose that $F_H(\hat{\delta}) = F_L(\hat{\delta})$, for $\hat{\delta} \in (\underline{\delta}, \delta_{\max})$. Then

$$\begin{aligned}
1 = F_L(\delta_{\max}) &= F_L(\hat{\delta}) + \int_{\hat{\delta}}^{\bar{\theta}} f_L(\delta)d\delta + \int_{\bar{\theta}}^{\theta_{\max}} f_L(\delta)d\delta \\
&= F_H(\hat{\delta}) + \int_{\hat{\delta}}^{\bar{\theta}} f_L(\delta)d\delta + \int_{\bar{\theta}}^{\theta_{\max}} f_L(\delta)d\delta \\
&< F_L(\delta_{\min}) + \int_{\delta_{\min}}^{\delta_{\max}} f_H(\delta)d\delta \\
&= F_H(\delta_{\max}) = 1,
\end{aligned}$$

a contradiction. This proves the claim. □

Claim 2. *Given Assumption 1, for $\mu_t \in (0,1)$:*

(i) $\mathbb{E}_\delta[\mu^h_\delta(\mu_t)|H] \geq \mathbb{E}_\delta[\mu^h_\delta(\mu_t)|L]$, *with strict inequality if* $p_I(\mu_t, L) > 0$.

(ii) $\mathbb{E}_\delta[\mu^l_\delta(\mu_t)|H] \leq \mathbb{E}_\delta[\mu^l_\delta(\mu_t)|L]$, *with strict inequality if* $p_I(\mu_t, H) < 1$.

Proof: When $p_I(\mu_t, L) > 0$, and the inefficient government sends signal h with positive probability in state L, the updated reputation following a report $y = h$ and the observation of δ, given by $\mu^h_\delta(\mu_t)$ in (7), is strictly increasing in the likelihood ratio $\lambda(\delta) = f_H(\delta)/f_L(\delta)$, and it is constant if $p_I(\mu_t, L) = 0$. Given Assumption 1, if $p_I(\mu_t, L) > 0$

$$\begin{aligned}
&\int_\Delta \mu^h_\delta(\mu_t) f_H(\delta)d\delta - \int_\Delta \mu^h_\delta(\mu_t) f_L(\delta)d\delta \\
=&\mu^h_\delta(\mu_t)[F_H(\delta) - F_L(\delta)]\Big|^{\delta_{\max}}_{\delta_{\min}} - \int_\Delta \frac{\partial \mu^h_\delta(\mu_t)}{\partial \delta}[F_H(\delta) - F_L(\delta)]d\delta \\
=&-\int_{\Delta/(\delta_1,\delta_2)} \frac{\partial \mu^h_\delta(\mu_t)}{\partial \delta}[F_H(\delta) - F_L(\delta)]d\delta - \int_{\delta_1}^{\delta_2} \frac{\partial \mu^h_\delta(\mu_t)}{\partial \delta}[F_H(\delta) - F_L(\delta)]d\delta, \\
>&0
\end{aligned}$$

where the inequality comes from the fact that $[F_H(\delta) - F_L(\delta)] < 0$ for $\delta \in (\delta_{\min}, \delta_{\max})$, and because $\lambda(\delta)$ is strictly increasing for $\delta \in (\delta_1, \delta_2)$, and so is $\mu^h_\delta(\mu_t)$. This result implies that the expected value of $\mu^h_\delta(\mu_t)$ is strictly larger in state H than in state L. In other words, the government's expected reputation after a signal h is higher when the the report is truthful and the state is H.

Similarly, if $p_I(\mu_t, H) < 1$, and the government sends signal l with positive probability in state H, the updated reputation $\mu^l_\delta(\mu_t)$ in (8) is strictly decreasing in the likelihood ratio $\lambda(\delta)$, and it is constant if $p_I(\mu_t, H) = 1$. Given Assumption 1, $p_I(\mu_t, H) < 1$

$$\int_\Delta \mu_\delta^l(\mu_t) f_H(\delta) d\delta - \int_\Delta \mu_\delta^l(\mu_t) f_L(\delta) d\delta < 0,$$

which means that the expected updated reputation after a signal l is higher when the true state is L instead of H. □

Claim 3. *Given Assumption 1, for $\mu_t \in (0,1)$:*

(i) $\bar{\mu}_t(\mu_t, H, h) > \bar{\mu}_t(\mu_t, L, h)$, with strict inequality if $p_I(\mu_t, L) > 0$.

(ii) $\bar{\mu}_t(\mu_t, L, l) > \bar{\mu}_t(\mu_t, H, l)$, with strict inequality if $p_I(\mu_t, H) < 1$.

Proof: Under Assumption 2-A, the realization of δ is always observed and, from (10), $\bar{\mu}_t(\mu_t, s, y) = \mathbb{E}_\delta[\mu_\delta^y(\mu_t)|s]$. In this case, therefore the result follows immediately from Claim 2.

Under Assumption 2-B, the realization of δ is only observed if $n \geq N(\theta)$, from (12)

$$\bar{\mu}_t(\mu_t, s, y) = P^*(\mu_t, y)\mathbb{E}_\delta[\mu_\delta^y(\mu_t)|s] + [1 - P^*(\mu_t, y)]\mu^y(\mu_t),$$

and the result also follows from Claim 2. □The intuition behind Claim 3 is the following. Since the efficient government is always truthful, whenever the realization of δ is such that a false report is likely, the entrepreneurs revise their beliefs about the government toward a lower reputation. Hence, if the government send a signal h (l), the expected reputation is lower if true state is L instead of H (H instead of L).

B.2.1 Proof of Lemma 1

Let $\mu_t \in (0,1)$. It is sufficient to show that $G_H > 0$, where G_H is the gain from truthful disclosure in state H, given by (13). Suppose that $G_H \leq 0$. Then,

$$\bar{\mu}_t(\mu_t, L, l) \geq \bar{\mu}_t(\mu_t, H, l) \geq \bar{\mu}_t(\mu_t, H, h) \geq \bar{\mu}_t(\mu_t, L, h), \tag{28}$$

where the first and last inequalities come from Claim 3, and the second one follows from $G_H \leq 0$. If $p_I(\mu_t, L) > 0$, from Claim 3 the last inequality in (28) is strict, therefore $\bar{\mu}_t(\mu_t, L, l) > \bar{\mu}_t(\mu_t, L, h)$. This implies that G_L, given by (14), is strictly positive, and therefore the government only sends signal l in state L, a contradiction with $p_I(\mu_t, L) > 0$. Hence $p_I(\mu_t, L) = 0$, and from (7)

$$\mu_\delta^h(\mu_t) = \frac{\pi_E \mu_t}{\pi_E \mu_t + \pi_I p_I(\mu_t, H)(1-\mu)} = \mu^h(\mu_t),$$

where $\mu^h(\mu_t)$ is given by (3). From (10) and (12), it follows that $\bar{\mu}(\mu_t, H, h) = \mu^h(\mu_t)$.

To get a contradiction, I need to show that $\bar{\mu}(\mu_t, H, l) < \bar{\mu}(\mu_t, H, h)$, which implies that $G_H > 0$. Notice that $\mu^h(\mu_t)$ is strictly decreasing in $p_I(\mu_t, H)$, and from (4) and (8), both $\mu^l(\mu_t)$ and $\mu^l_\delta(\mu_t)$ are strictly increasing in $p_I(\mu_t, H)$, and so is $\bar{\mu}(\mu_t, H, l)$. It suffices to show that, for $p_I(\mu_t, H) = 1$, $\bar{\mu}(\mu_t, H, l) < \bar{\mu}(\mu_t, H, h) = \mu^h(\mu_t)$.

If $p_I(\mu_t, H) = 1$

$$\bar{\mu}(\mu_t, H, h) = \mu^h(\mu_t) = \frac{\pi_E \mu_t}{\pi_E \mu_t + \pi_I(1-\mu)},$$

and from (4), (8), (10) and (12)

$$\bar{\mu}(\mu_t, H, l) = \mu^l(\mu_t) = \frac{(1-\pi_E)\mu_t}{(1-\pi_E)\mu_t + (1-\pi_I)(1-\mu_t)}.$$

Then

$$\begin{aligned}
&\bar{\mu}(\mu_t, H, l) < \bar{\mu}(\mu_t, H, h) \\
\Leftrightarrow \frac{\pi_E \mu_t}{\pi_E \mu_t + \pi_I(1-\mu)} > \frac{(1-\pi_E)\mu_t}{(1-\pi_E)\mu_t + (1-\pi_I)(1-\mu_t)}& \\
&\Leftrightarrow \pi_E[(1-\pi_E)\mu_t + (1-\pi_I)(1-\mu_t)] > (1-\pi_E)[\pi_E\mu_t + \pi_I(1-\mu_t)] \\
&\Leftrightarrow \pi_E(1-\pi_I) > (1-\pi_E)\pi_I \\
&\Leftrightarrow \frac{(1-\pi_I)}{(1-\pi_E)} > \pi_I \pi_I,
\end{aligned}$$

which is true, since $\pi_E > \pi_I$. Thus $G_H > 0$, a contradiction.

B.2.2 Proof of Lemma 2

Let $p_\mu \equiv p_I(\mu, L)$. From Lemma 1, $p_I(\mu_t, H, h) = 1$.

(*i*). From (3),(4),(7),(8), (10) and (12), $\bar{\mu}(0, s, y) = 0$, for all s and y, and $\bar{\mu}(1, s, y) = 1$, for all s and y. Thus $G(0, p) = G(1, p) = 0$.

(*ii*).

$$G_L(\mu, 0) = \frac{(1-\pi_E)\mu}{(1-\pi_E)\mu_t + (1-\pi_I)(1-\mu)} - \frac{\pi_E \mu}{\pi_E \mu_t + \pi_I(1-\mu)}, \tag{29}$$

then

$$G_L(\mu, 0) < 0 \Leftrightarrow \frac{(1-\pi_I)}{(1-\pi_E)} > \pi_I \pi_I,$$

which holds, since $\pi_E > \pi_I$.

(*iii*). If $p_\mu = 1$, then the inefficient government always sends $y = h$. In this case, entrepreneurs

are sure that the government is efficient when $y = l$, but are uncertain about the type when $y = h$. Thus $\bar{\mu}(\mu, s, L) = 1$ and $\bar{\mu}(\mu, L, h) < 1$, which implies that $G_L(\mu, 1) > 0$.
(*iv*). From (29)

$$\frac{\partial}{\partial \mu} G_L(\mu, 0) = \frac{(1-\pi_E)(1-\pi_I)}{[(1-\pi_E)\mu_t + (1-\pi_I)(1-\mu)]^2} - \frac{\pi_E \pi_I}{[\pi_E \mu_t + \pi_I(1-\mu)]^2},$$

and

$$\frac{\partial^2}{\partial \mu^2} G_L(\mu, 0) = 2\frac{(1-\pi_E)(1-\pi_I)(\pi_E - \pi_I)}{[(1-\pi_E)\mu_t + (1-\pi_I)(1-\mu)]^3} + 2\frac{\pi_E \pi_I(\pi_E - \pi_I)}{[\pi_E \mu_t + \pi_I(1-\mu)]^3} > 0.$$

B.2.3 Proof of Lemma 3

Let $\mu_t \in (0, 1)$. From Lemma 2 part (*ii*), if entrepreneurs believe that $p_\mu = 0$, then the government is strictly better off by deviating and sending signal $y = h$. From Lemma 2 part (*iii*), if entrepreneurs believe that $p_\mu = 1$, then the government is strictly better off by deviating and sending signal $y = l$ in state L. If an equilibrium exists, then $p_\mu \in (0, 1)$, and the government must be indifferent between sending signals h and l when the state is L, which implies that $G_L(\mu, p_\mu) = 0$. From Lemma 2 parts (*ii*) and (*iii*), and from the continuity of $G_L(\mu, p)$ in p, there exists $p_\mu \in (0, 1)$ such that $G_L(\mu, p_\mu) = 0$, therefore an equilibrium exists.

Under Assumption 2-A,

$$G_L(\mu, p_\mu) = \frac{(1-\pi_E)\mu}{(1-\pi_E)\mu_t + (1-\pi_I)(1-p_\mu)(1-\mu)} - \mathbb{E}_\delta \left[\frac{\pi_E \mu_t}{\pi_E \mu + \left[\pi_I + \frac{f_L(\delta)}{f_H(\delta)}(1-\pi_I)p_\mu\right](1-\mu)} \middle| L \right]$$

, thus $G_L(\mu, p_\mu)$ is strictly increasing in p_μ. In this case, there exists a unique $p^*_\mu \in (0, 1)$ that solves $G_L(\mu, p_\mu) = 0$.

B.2.4 Proof of Proposition 1

Let $\mu_t \in (0, 1)$. From Lemma 3, if an equilibrium where the efficient government follows a full disclosure policy exists, the inefficient government's strategy for period t in such an equilibrium is given by $p_I(\mu_t, H) = 1$ and $p_I(\mu, L) = p_\mu \in (0, 1)$, where p_μ solves $G_L(\mu, p_\mu) = 0$. It is left to show that given the inefficient government's strategy and the entrepreneurs' beliefs, it is indeed optimal for the efficient government to be truthful. If entrepreneurs believe that the efficient government is truthful, then:(1) in the proof of Lemma 1 I show that $G_H > 0$; (2) and from Lemma 3, the inefficient government chooses p_μ such that $G_L(\mu, p_\mu) = 0$. From $G_H > 0$, the efficient government strictly prefers to be truthful in state H, and from $G_L = 0$, the efficient government is indifferent in state

L. Thus an equilibrium where the efficient government is always truthful exists. Furthermore, if Assumption 2-A holds, from Lemma 3, the equilibrium is unique, since there exists a unique p_μ^* that solves $G_L(\mu, p_\mu) = 0$.

B.2.5 Proof that there is no equilibrium where type *I* follows a full disclosure policy

Lemma 7. *Let $\mu_t \in (0,1)$. In equilibrium, the inefficient government never follows a full disclosure policy in period t. There is no equilibrium where*

$$p_I(\mu_t, H) = 1 - p_I(\mu_t, L) = 1.$$

Proof: If the inefficient government is always truthful, then

$$\mu_\delta^h(\mu_t) = \frac{\left[\pi_E p_E(\mu_t, H) + \frac{f_L(\delta)}{f_H(\delta)}(1-\pi_E)p_E(\mu_t, L)\right]\mu_t}{\left[\pi_E p_E(\mu_t, H) + \frac{f_L(\delta)}{f_H(\delta)}(1-\pi_E)p_E(\mu_t, L)\right]\mu_t + \pi_I(1-\mu)},$$

and

$$\mu_\delta^l(\mu_t) = \frac{\left[\frac{f_H(\delta)}{f_L(\delta)}\pi_E(1-p_E(\mu_t, H)) + (1-\pi_E)(1-p_E(\mu_t, L))\right]\mu_t}{\left[\frac{f_H(\delta)}{f_L(\delta)}\pi_E(1-p_E(\mu_t, H)) + (1-\pi_E)(1-p_E(\mu_t, L))\right]\mu_t + (1-\pi_I)(1-\mu_t)},$$

therefore $\mu_\delta^h(\mu_t)$ is strictly decreasing in $\lambda(\delta) = f_H(\delta)/f_L(\delta)$ if $p_E(\mu_t, L) > 0$, and constant otherwise; $\mu_\delta^l(\mu_t)$ is strictly increasing in $\lambda(\delta)$ if $p_E(\mu_t, H) < 0$, and constant otherwise. For $\mu_t \in (0,1)$, following similar arguments to those in Claim 3, Assumption 1 implies:

(A) $\bar{\mu}_t(\mu_t, H, h) < \bar{\mu}_t(\mu_t, L, h)$, with strict inequality if $p_E(\mu_t, L) > 0$.

(B) $\bar{\mu}_t(\mu_t, L, l) < \bar{\mu}_t(\mu_t, H, l)$, with strict inequality if $p_E(\mu_t, H) < 1$.

This means that if the efficient government is the only type that might not be truthful, the government's reputation increases whenever the realization of δ is such that a false report is likely. If the government send a signal h (l), the expected reputation is higher if the true state is L instead of H (H instead of L).

If the inefficient government is truthful, then $G_H \geq 0$, which implies that

$$\bar{\mu}_t(\mu_t, L, h) \geq \bar{\mu}_t(\mu_t, H, h) \geq \bar{\mu}_t(\mu_t, H, l) \geq \bar{\mu}_t(\mu_t, L, l), \tag{30}$$

where the first and last inequalities come from (A) and (B) above, and the second one follows from $G_H \geq 0$. If either $p_E(\mu_t, L) > 0$ or $p_E(\mu_t, H) < 1$, from (A) and (B), either the first or the third inequalities in (30) are strict, therefore and $\bar{\mu}_t(\mu_t, L, l) > \bar{\mu}_t(\mu_t, L, h)$. This implies that both $G_H > 0$

and $G_L > 0$, thus the inefficient government is always truthful. However, from Lemma 3, there is no equilibrium in which both types of government are always truthful, thus there is no equilibrium where the inefficient government is truthful. □

B.3 Equilibrium of the game between entrepreneurs

In this section, I characterize the equilibrium of the game between entrepreneurs, conditional on an expected value of δ given by $\bar{\delta}$. I provide results that will be used to prove Proposition 2. The results in this section are based on Galvao and Shalders (2017).

Lemma 8. *For a given public signal y, if $\pi(x,y) \geq \pi'(x,y)$ for all x, then $u_y(x,\pi) \geq u_y(x,\pi')$ for all x.*

Proof:

$$\eta(\bar{\delta},x) \geq \eta'(\bar{\delta},x)\forall x \Rightarrow n(\bar{\delta},\theta,\eta) \geq n(\bar{\delta},\theta,\eta')\forall\theta \Rightarrow A(\bar{\delta},\eta) \supseteq A(\bar{\delta},\eta') \Rightarrow u(\bar{\delta},x,\eta) \geq u(\bar{\delta},x,\eta').$$

□

For $k \in [\theta_{\min} - \varepsilon, \theta_{\max} + \varepsilon]$, let the indicator function I_k be defined as

$$I_k(x) = \begin{cases} 1, & \text{if } x \leq k \\ 0, & \text{if } x > k \end{cases}. \tag{31}$$

Suppose that the investment strategies are given by $a_i(\bar{\delta}, x_i) = I_k(x_i)$, for all i: entrepreneurs follow a cutoff strategy, investing if and only if $x_i \leq k$. The number of ventures is thus given by

$$n(\bar{\delta},\theta,I_k) = G(k-\theta). \tag{32}$$

Note that $n(\bar{\delta},\theta,I)$ is strictly decreasing in θ for $\theta \in (k-\varepsilon, k+\varepsilon)$, and constant otherwise. Let

$$t_k \equiv \sup\{\theta | n(\bar{\delta},\theta,I) \geq N(\theta)\},$$

and let $\theta_k = \min\{t_k, \theta_{\min}\}$. If the probability of failure is below t_k when entrepreneurs follow I_k, then the number of ventures is large enough so that the successful ventures pay $v+\delta$. If $k \in (\theta_{\min} - \varepsilon, \theta_{\max} + \varepsilon)$, there is a unique θ such that $n(\bar{\delta},\theta,I) = G(k-\theta) = N(\theta)$, and therefore $\theta_k = k - G^{-1}(N(\theta_k))$.

Let $\psi(k) = \theta_k - k$. The following lemma characterizes θ_k and $\psi(k)$.

Lemma 9. *(i) The function $\psi(\cdot)$ is continuous and decreasing, with $\psi(k) \in [-\varepsilon, \varepsilon]$, for all k.*

(ii) For $k \in (\theta_{min} + \varepsilon, \theta_{max} - \varepsilon)$, $\psi(\cdot)$ is differentiable, with derivative $\psi'(k) > -1$.

(iii) θ_k *is increasing in k, for all k.*

Proof: Let $\underline{k}$ solve $G(\underline{k}-\theta) = N(\theta_{\min})$. Then $\underline{k} = G^{-1}(N(\theta_{\min})) + \theta_{\min} \in (\theta_{\min} - \varepsilon, \theta_{\min} + \varepsilon)$. If $k < \underline{k}$, then for all θ

$$N(\theta) \geq N(\theta_{\min}) = G(\underline{k} - \theta) \geq G(k - \theta) \Rightarrow \theta_k = \theta_{\min} \in (k - \varepsilon, k + \varepsilon).$$

Let $\bar{k}$ solve $G(\bar{k} - \theta_{\max}) = N(\theta_{\max})$. Then $\bar{k} = G^{-1}(N(\theta_{\max})) + \theta_{\max} \in (\theta_{\max} - \varepsilon, \theta_{\max} + \varepsilon)$. If $k > \bar{k}$, then for all θ

$$G(k - \theta_{\max}) \geq G(\bar{k} - \theta_{\max}) = N(\theta_{\max}) \Longrightarrow \theta_k = \theta_{\max} \in (k - \varepsilon, k + \varepsilon).$$

For $k \in (\underline{k}, \bar{k})$, we have $\theta_k = k - G^{-1}(N(\theta_k)) \in (k + \varepsilon, k + \varepsilon)$. The function $\psi(k) = \theta_k - k$ is then given by

$$\psi(k) = \begin{cases} \theta_{\max} - k, & \text{if } k < \underline{k} = \theta_{\min} + G^{-1}(N(\theta_{\min})) \\ -G^{-1}(N(\theta_k)), & \text{if } \underline{k} \leq k \leq \bar{k} \\ \theta_{\min} - k, & \text{if } k > \bar{k} = \theta_{\max} + G^{-1}(N(\theta_{\max})) \end{cases}. \tag{33}$$

From (33), it is clear that $\psi(k)$ is continuous in k. Since $N(\theta)$ is increasing in θ, then θ_k is increasing in k, which implies that $\psi(k)$ is decreasing in k. Since $k \in (\theta_k - \varepsilon, \theta_k + \varepsilon)$, then $\psi(k) \in (-e, +\varepsilon)$, and part (*i*) is proved. If $k \in (\theta_{\min} + \varepsilon, \theta_{\max} - \varepsilon) \subseteq (\underline{k}, \bar{k})$,

$$\psi(k) = -G^{-1}(N(\theta_k)) \Rightarrow \psi'(k) = -\frac{N'(k + \theta_k)}{g(G^{-1}(N(k + \theta_k)))}(\psi'(k)+1) = -\frac{N'(k + \theta_k)}{N'(k + \theta_k) + g(G^{-1}(N(k + \theta_k)))} \in (-1, 0],$$

which proves part (*ii*). Finally, for $k \in (\underline{k}, \bar{k})$, θ_k is differentiable, with derivative $1 - \psi'(k) > 0$, and it is constant otherwise. This proves part (*iii*). □

From 18 and the definition of ψ, the expected payoff for the entrepreneur who observed the cutoff signal k is given by

$$u(\bar{\delta}, k, I_k) = v \int_{k-\varepsilon}^{k+\varepsilon} (1 - \theta)\phi(\theta|k)d\theta + \bar{\delta} \int_{k-\varepsilon}^{k+\psi(k)} (1 - \theta)\phi(\theta|k)d\theta. \tag{34}$$

Since $\phi(\cdot|k)$ and the limits of integration in (34) are continuous in k (because $\psi(\cdot)$ is continuous), $u(\bar{\delta}, k, I_k)$ is continuous in the cutoff k.

Lemma 10. *For* $k \in (\theta_{min} + \varepsilon, \theta_{max} - \varepsilon)$*, the payoff function* $u(\bar{\delta}, k, I_k)$ *is strictly decreasing in k.*

Proof: From (2) and (34), the payoff function is given by

$$u(\bar{\delta}, k, I_k) = v \int_{k-\varepsilon}^{k+\varepsilon} (1-\theta) \frac{g(k-\theta)}{G(k-\theta_{\min}) - G(k-\theta_{\max})} d\theta + \bar{\delta} \int_{k-\varepsilon}^{k+\psi(k)} (1-\theta) \frac{g(k-\theta)}{G(k-\theta_{\min}) - G(k-\theta_{\max})} d\theta. \tag{35}$$

From (33), $\psi(\cdot)$ is differentiable in k for $k \in (\theta_{\min} + \varepsilon, \theta_{\max} - \varepsilon)$, and so is $u(\bar{\delta}, k, I_k)$. Differentiating $u(\bar{\delta}, k, I_k)$ with respect to k and using the fact that $G(k - \theta_{\max}) = g(k - \theta_{\max}) = 0$, for $k < \theta_{\max} - \varepsilon$, yield

$$\begin{aligned}
&\frac{d}{dk} u(\bar{\delta}, k, I_k) \\
=&\frac{v}{G(k-\theta_{\min})} \Bigg[(1-k-\varepsilon) g(-\varepsilon) - (1-k+\varepsilon) g(\varepsilon) + \int_{k-\varepsilon}^{k+\varepsilon} (1-\theta) g'(k-\theta) d\theta \\
&\qquad - \int_{k-\varepsilon}^{k+\varepsilon} (1-\theta) \frac{g(k-\theta) g(k-\theta_{\min})}{G(k-\theta_{\min})} d\theta \Bigg] \\
&\frac{\bar{\delta}}{G(k-\theta_{\min})} \Bigg[(1-k-\psi(k)) g(-\psi(k))(1+\psi'(k)) - (1-k+\varepsilon) g(\varepsilon) + \int_{k-\varepsilon}^{k+\psi(k)} (1-\theta) g'(k-\theta) d\theta \\
&\qquad - \int_{k-\varepsilon}^{k+\psi(k)} (1-\theta) \frac{g(k-\theta) g(k-\theta_{\min})}{G(k-\theta_{\min})} d\theta \Bigg] \\
\leq&\frac{v}{G(k-\theta_{\min})} \Bigg[(1-k-\varepsilon) g(-\varepsilon) - (1-k+\varepsilon) g(\varepsilon) + \int_{k-\varepsilon}^{k+\varepsilon} (1-\theta) g'(k-\theta) d\theta \Bigg] \\
&\frac{\bar{\delta}}{G(k-\theta_{\min})} \Bigg[(1-k-\psi(k)) g(-\psi(k)) - (1-k+\varepsilon) g(\varepsilon) + \int_{k-\varepsilon}^{k+\psi(k)} (1-\theta) g'(k-\theta) d\theta \Bigg],
\end{aligned}$$

where the inequality comes from $\psi'(k) \leq 0$, and from the fact that the second and fourth integrals on the RHS of the equality are positive. Define $\underline{\tilde{\varepsilon}}$ as

$$\underline{\tilde{\varepsilon}} = \inf\{\tilde{\varepsilon} \in [-\varepsilon, \varepsilon] : g'(\hat{\varepsilon}) \leq 0 \quad \forall \hat{\varepsilon} > \tilde{\varepsilon}\}.$$

From (1), $\underline{\tilde{\varepsilon}}$ is well defined. Furthermore, $g'(\tilde{\varepsilon}) \geq 0$, for $\tilde{\varepsilon} \leq \underline{\tilde{\varepsilon}}$, and $g'(\tilde{\varepsilon}) \leq 0$, for $\tilde{\varepsilon} > \underline{\tilde{\varepsilon}}$. Define $\underline{\tilde{\theta}}$ as

$$\underline{\tilde{\theta}} = k - \underline{\tilde{\varepsilon}}.$$

Hence $\underline{\tilde{\theta}} \in [k-\varepsilon, k+\varepsilon]$. We then have

$$\int_{k-\varepsilon}^{k+\varepsilon} (1-\theta) g'(k-\theta) d\theta \leq (1-\underline{\tilde{\theta}}) \int_{k-\varepsilon}^{\tilde{\theta}} g'(k-\theta) d\theta + (1-\underline{\tilde{\theta}}) \int_{\underline{\tilde{\theta}}}^{k+\varepsilon} g'(k-\theta) d\theta$$

$$= (1 - \tilde{\underline{\theta}})[g(\varepsilon) - g(-\varepsilon)],$$

and

$$\begin{aligned}\int_{k-\varepsilon}^{k+\psi(k)} (1-\theta)g'(k-\theta)d\theta \leq& (1 - \min\{\tilde{\underline{\theta}}, k+\psi(k)\}) \int_{k-\varepsilon}^{\min\{\tilde{\underline{\theta}}, k+\psi(k)\}} g'(k-\theta)d\theta \\ &+ (1 - \min\{\tilde{\underline{\theta}}, k+\psi(k)\}) \int_{\min\{\tilde{\underline{\theta}}, k+\psi(k)\}}^{k+\psi(k)} g'(k-\theta)d\theta \\ =& (1 - \min\{\tilde{\underline{\theta}}, k+\psi(k)\})[g(\varepsilon) - g(-\psi(k))].\end{aligned}$$

Hence

$$\begin{aligned}&\frac{d}{dk}u(\bar{\delta}, k, I_k) \\ \leq& \frac{v}{G(k-\theta_{\min})}\left[(1-k-\varepsilon)g(-\varepsilon) - (1-k+\varepsilon)g(\varepsilon) + (1-\tilde{\underline{\theta}})[g(\varepsilon) - g(-\varepsilon)]\right] \\ & \frac{\bar{\delta}}{G(k-\theta_{\min})}\left[(1-k-\psi(k))g(-\psi(k)) - (1-k+\varepsilon)g(\varepsilon) + (1-\min\{\tilde{\underline{\theta}}, k+\psi(k)\})[g(\varepsilon) - g(-\psi(k))]t\right] \\ =& \frac{v}{G(k-\theta_{\min})}\left[g(-\varepsilon)[\tilde{\underline{\theta}} - (k+\varepsilon)] - g(\varepsilon)[\tilde{\underline{\theta}} - (k-\varepsilon)]\right] \\ & \frac{\bar{\delta}}{G(k-\theta_{\min})}\left[g(-\psi(k))[\min\{\tilde{\underline{\theta}}, k+\psi(k)\} - (k+\psi(k))] - g(\varepsilon)[\min\{\tilde{\underline{\theta}}, k+\psi(k)\} - (k-\psi(k))]\right] \\ <& 0,\end{aligned}$$

which implies that $u(\bar{\delta}, k, I_k)$ is strictly decreasing. □

B.3.1 Proof of Proposition 2

Using Lemma 10, the proof of existence and uniqueness of equilibrium in the game between entrepreneurs is analogous to the one in Morris and Shin (1998), Theorem 1. Entrepreneurs follow a cutoff rule in their private signal given by $I_{x^*(\bar{\delta})}$, where $x^*(\bar{\delta})$ is such that

$$u(\bar{\delta}, x^*(\bar{\delta}), I_{x^*(\bar{\delta})}) = w, \tag{36}$$

which means that the entrepreneur that receives the cutoff signal is indifferent between investing and working. Since $2\varepsilon < \min\{\underline{\theta} - \theta_{\min}, \theta_{\max} - \bar{\theta}_{\delta_H}\}$, then $x^*(\bar{\delta}) \in (\theta_{\min} + \varepsilon, \theta_{\max} - \varepsilon)$. The equilibrium number of ventures is

$$n(\bar{\delta}, \theta, I_{x^*(\bar{\delta})}) = G(x^*(\bar{\delta}) - \theta),$$

which is decreasing in θ. The threshold θ below which $n(\bar{\delta}, \theta, I_{x^*(\bar{\delta})}) \geq N(\theta)$ is given by $\theta^*(\bar{\delta}) \equiv \theta_{x^*(\bar{\delta})}$. From (34), it is clear that $u(\bar{\delta}, k, I_k)$ is strictly increasing in $\bar{\delta}$, for all k. Lemma 10 and (36) thus imply that $x^*(\bar{\delta})$ is strictly increasing in δ. Finally, from Lemma 9, part (*iii*), $\theta^*(\bar{\delta})$ is also strictly increasing in $\bar{\delta}$.

B.4 Welfare function

This section presents properties of the welfare function and establishes results used to prove Lemmas 4, 5, and 6.

Since $x^*(\bar{\delta}) \in (\theta_{\min} + \varepsilon, \theta_{\max} - \varepsilon)$, then $G(x^*(\bar{\delta}) - \theta_{\min}) \geq G(\varepsilon) = 1$, and $G(x^*(\bar{\delta}) - \theta_{\max}) \leq G(-\varepsilon) = 0$. From (35), the expected payoff after observing $x^*(\bar{\delta})$ can be written as

$$u(\bar{\delta}, x^*(\bar{\delta}), I_{x^*(\bar{\delta})}) = v \int_{x^*(\bar{\delta})-\varepsilon}^{x^*(\bar{\delta})+\varepsilon} (1-\theta) g(x^*(\bar{\delta}) - \theta) d\theta + \bar{\delta} \int_{x^*(\bar{\delta})-\varepsilon}^{x^*(\bar{\delta})+\psi(x^*(\bar{\delta}))} (1-\theta) g(x^*(\bar{\delta}) - \theta) d\theta,$$

and the indifference condition (36) implies

$$\begin{aligned} & v \int_{x^*(\bar{\delta})-\varepsilon}^{x^*(\bar{\delta})+\varepsilon} (1-\theta) g(x^*(\bar{\delta}) - \theta) d\theta + \bar{\delta} \int_{x^*(\bar{\delta})-\varepsilon}^{x^*(\bar{\delta})+\psi(x^*(\bar{\delta}))} (1-\theta) g(x^*(\bar{\delta}) - \theta) d\theta = w \\ \Rightarrow & \int_{x^*(\bar{\delta})-\varepsilon}^{x^*(\bar{\delta})+\varepsilon} [(1-\theta)v - w] g(x^*(\bar{\delta}) - \theta) d\theta = -\bar{\delta} \int_{x^*(\bar{\delta})-\varepsilon}^{x^*(\bar{\delta})+\psi(x^*(\bar{\delta}))} (1-\theta) g(x^*(\bar{\delta}) - \theta) d\theta. \end{aligned} \tag{37}$$

For $s \in \{H, L\}$, define the function $V_s(x^*)$ as

$$\begin{aligned} V_s(x^*) =& (v + \delta_s) \int_{\theta_{\min}}^{x^*-\varepsilon} (1-\theta) d\theta + \int_{x^*-\varepsilon}^{x^*+\psi(x^*)} [(1-\theta)(v+\delta_s) - w] G(x^* - \theta) d\theta \\ & + \int_{x^*-\varepsilon}^{x^*+\psi(x^*)} [(1-\theta)v - w] G(x^* - \theta) d\theta + w \int_{x^*-\varepsilon}^{\theta_{\max}} d\theta. \end{aligned}$$

Thus $V_s(x^*(\bar{\delta})) = W_s(\bar{\delta})$, for all $\bar{\delta}$, where $W_s(\bar{\delta})$ is the welfare function given by (22).

From Lemma 9, part (*ii*), $\psi(k)$ is differentiable at $x^*(\bar{\delta})$, and $\psi'(x^*(\bar{\delta})) > -1$. Hence $V_s(x^*)$ differentiable:

$$\begin{aligned} & \frac{\partial}{\partial x^*} V_s(x^*) = \\ & (v + \delta_s) \{1 - x^* + \varepsilon + [1 - x^* - \psi(x^*)][1 + \psi'(x^*)] G(-\psi(x^*)) - (1 - x + \varepsilon) G(\varepsilon)\} \\ & + v \{(1 - x^* - \varepsilon) G(-\varepsilon) - [1 - x^* - \psi(x^*)][1 + \psi'(x^*)] G(-\psi(x^*))\} \\ & - w \{G(-\psi(x^*))[1 + \psi'(x^*)] - G(\varepsilon) - G(-\varepsilon) - G(-\psi(x^*))[1 + \psi'(x^*)] + 1\} \end{aligned}$$

$$+\int_{x^*-\varepsilon}^{x^*+\psi(x^*)}[(1-\theta)(v+\delta_s)-w]g(x^*-\theta)d\theta+\int_{x^*-\varepsilon}^{x^*+\psi(x^*)}[(1-\theta)v-w]g(x^*-\theta)d\theta$$

$$=\int_{x^*-\varepsilon}^{x^*+\varepsilon}[(1-\theta)v-w]g(x^*-\theta)d\theta$$

$$+\delta_s\left\{[1-x^*-\psi(x^*)][1+\psi'(x^*)]G(-\psi(x^*))+\int_{x^*-\varepsilon}^{x^*+\psi(x^*)}(1-\theta)g(x^*-\theta)d\theta\right\}. \tag{38}$$

Using (37),

$$\frac{\partial}{\partial x^*}V_s(x^*)\bigg|_{x^*=x^*(\bar{\delta})}=(\delta_s-\bar{\delta})\int_{x^*-\varepsilon}^{x^*+\psi(x^*)}(1-\theta)g(x^*-\theta)d\theta+\delta_s[1-x^*-\psi(x^*)][1+\psi'(x^*)]G(-\psi(x^*)). \tag{39}$$

B.4.1 Proof of Lemma 5

From (39)

$$\frac{\partial}{\partial x^*}V_H(x^*)\bigg|_{x^*=x^*(\bar{\delta})}=(\delta_H-\bar{\delta})\int_{x^*-\varepsilon}^{x^*+\psi(x^*)}(1-\theta)g(x^*-\theta)d\theta+\delta_H[1-x^*+\psi(x^*)][1+\psi'(x^*)]G(-\psi(x^*)).$$

Since $\delta_H \geq \bar{\delta}$ (with strict inequality when entrepreneurs assign a positive probability to L), and $\psi'(x^*(\bar{\delta}))>-1$, then

$$\frac{\partial}{\partial x^*}V_H(x^*)\bigg|_{x^*=x^*(\bar{\delta})}\geq 0,$$

with strict inequality if $\bar{\delta}<\delta_H$. From $V_H(x^*(\bar{\delta}))=W_H(\bar{\delta})$, and since $x^*(\bar{\delta})$ is strictly increasing, it follows that

$$\frac{\partial}{\partial\bar{\delta}}W_H(\bar{\delta})=\frac{\partial}{\partial x^*}V_H(x^*)\bigg|_{x^*=x^*(\bar{\delta})}\frac{\partial x^*(\bar{\delta})}{\partial\bar{\delta}}\geq 0,$$

with with strict inequality if $\bar{\delta}<\delta_H$.

B.4.2 Proof of Lemma 6

From (39)

$$\frac{\partial}{\partial x^*}V_L(x^*)\bigg|_{x^*=x^*(\bar{\delta})}=(\delta_L-\bar{\delta})\int_{x^*-\varepsilon}^{x^*+\psi(x^*)}(1-\theta)g(x^*-\theta)d\theta+\delta_L[1-x^*-\psi(x^*)][1+\psi'(x^*)]G(-\psi(x^*)).$$

Since $[1-(x^*+\psi(x^*))] > (1-\delta)$, for $\theta > x^*+\psi(x^*)$, and

$$G(-\psi(x^*)) = \int_{x^*+\psi(x^*)}^{x^*+\varepsilon} g(x^*-\theta)d\theta, \tag{40}$$

then

$$\frac{\partial}{\partial x^*}V_L(x^*)\bigg|_{x^*=x^*(\delta_L)} \geq \delta_L[1+\psi'(x^*(\delta_L))]\int_{x^*(\delta_L)+\psi(x^*(\delta_L))}^{x^*(\delta_L)+\varepsilon}(1-\theta)g(x^*(\delta_L)-\theta)d\theta > 0.$$

From $V_L(x^*(\bar{\delta})) = W_L(\bar{\delta})$, and since $x^*(\bar{\delta})$ is strictly increasing, it follows that

$$\frac{\partial}{\partial\bar{\delta}}W_L(\bar{\delta})\bigg|_{\bar{\delta}=\delta_L} = \frac{\partial}{\partial x^*}V_L(x^*)\bigg|_{x^*=x^*(\delta_L)}\frac{\partial x^*(\bar{\delta})}{\partial\bar{\delta}}\bigg|_{\bar{\delta}=\delta_L} > 0.$$

B.4.3 Proof of Lemma 7

From (38) and (40)

$$\begin{aligned}
&\frac{\partial}{\partial x^*}V_L(x^*)\bigg|_{x^*=x^*(\delta_H)}\\
=&\int_{x^*-\varepsilon}^{x^*+\psi(x^*)}[(1-\theta)(v+\delta_l)-w]g(x^*-\theta)d\theta+\int_{x^*+\psi(x^*)}^{x^*+\varepsilon}[(1-\theta)v-w]g(x^*-\theta)d\theta\\
&+\delta_L[1-(x^*+\psi(x^*))][1+\psi'(x^*)]\int_{x^*+\psi(x^*)}^{x^*+\varepsilon}(1-\theta)g(x^*-\theta)d\theta\\
<&\int_{x^*+\psi(x^*)}^{x^*+\varepsilon}[(1-\theta)v-w]g(x^*-\theta)d\theta+\delta_L[1-(x^*+\psi(x^*))][1+\psi'(x^*)]\int_{x^*+\psi(x^*)}^{x^*+\varepsilon}(1-\theta)g(x^*-\theta)d\theta\\
\leq&\int_{x^*+\psi(x^*)}^{x^*+\varepsilon}[(1-(x^*+\psi(x^*)))v-w]g(x^*-\theta)d\theta+\delta_L[1-(x^*+\psi(x^*))][1+\psi'(x^*)]\int_{x^*+\psi(x^*)}^{x^*+\varepsilon}(1-\theta)g(x^*-\theta)d\theta\\
=&\int_{x^*+\psi(x^*)}^{x^*+\varepsilon}[(1-(x^*+\psi(x^*)))(v+\delta_L)-w]g(x^*-\theta)d\theta+\delta_L[1-(x^*+\psi(x^*))]\psi'(x^*)\int_{x^*+\psi(x^*)}^{x^*+\varepsilon}(1-\theta)g(x^*-\theta)d\theta\\
<&\int_{x^*+\psi(x^*)}^{x^*+\varepsilon}[(1-\bar{\theta}_L)(v+\delta_L)-w]g(x^*-\theta)d\theta+\delta_L[1-(x^*+\psi(x^*))]\psi'(x^*)\int_{x^*+\psi(x^*)}^{x^*+\varepsilon}(1-\theta)g(x^*-\theta)d\theta\\
=&\delta_L[1-(x^*+\psi(x^*))]\psi'(x^*)\int_{x^*+\psi(x^*)}^{x^*+\varepsilon}(1-\theta)g(x^*-\theta)d\theta \qquad < 0.
\end{aligned}$$

The first inequality follows from $(1-\theta)(v+\delta_L) < w$, for $\theta > \bar{\theta}_L$ and $x^*(\delta_H)-\varepsilon > \bar{\theta}_L$. The third inequality is obtained from $x^*(\delta_H)+\psi(x^*(\delta_H)) > x^*(\delta_H)-\varepsilon > \bar{\theta}_L$. The fourth inequality follows from the definition of $\underline{\theta}_L$: $(1-\underline{\theta}_L)(v+\delta_L) = w$. Finally, the last inequality follows from $\psi'(x^*(\delta_H)) < 0$.

From $V_L(x^*(\bar{\delta})) = W_L(\bar{\delta})$, and since $x^*(\bar{\delta})$ is strictly increasing, it follows that

$$\frac{\partial}{\partial \bar{\delta}} W_L(\bar{\delta})\bigg|_{\bar{\delta}=\delta_H} = \frac{\partial}{\partial x^*} V_L(x^*)\bigg|_{x^*=x^*(\delta_H)} \frac{\partial x^*(\bar{\delta})}{\partial \bar{\delta}}\bigg|_{\bar{\delta}=\delta_H} < 0.$$

From the continuity of W_L, there exists $\tilde{\delta}$ such that $\frac{\partial}{\partial \bar{\delta}} W_L(\bar{\delta}) < 0$, for $\bar{\delta} > \tilde{\delta}$.