

## ECON 4150 (Fall 2023) Mathematical Economics

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**Lectures:** Two 1.5 hour lectures a week.

**Office Hours:** I will hold one scheduled hour of office hours each Monday at 10 am (some of these will be conducted via Zoom). If you plan to attend *please let me know in advance* so as to manage congestion. If you are unable to make this time, feel free to arrange an appointment at a mutually convenient time.

**Assessment:** The grade will depend upon 7 homeworks, one open book, open notes take-home exam and one brief oral exam. In the oral exam I will ask you to go over your solution to one of the homework problems and explore minor variations of it.

### Description

Mathematics plays a fundamental role in Economics. There are three basic mathematical questions that arise with unerring regularity in economic analysis: Given a function  $f$  and a set  $S$ ,

- find an  $x$  such that  $f(x)$  is in  $S$ . This is the **feasibility** question.
- Find an  $x$  in  $S$  that optimizes  $f(x)$ . This is the problem of **optimality**.
- Find an  $x$  in  $S$  such that  $f(x) = x$ ; this is the **fixed point** problem.

The three problems are, in general, quite difficult. However, if one is prepared to make assumptions about the nature of the underlying function (say it is linear, convex or continuous) and the nature of the set  $S$  (convex, compact etc.) it is possible to provide answers and very nice ones at that.

This semester long course is about the answers, the relationships between them and the role they play in economics. This will be illustrated with a host of examples, for instance, input/output systems, no-arbitrage theorems, auctions, marriage problems, Arrow's Impossibility theorem, the mini-max theorem, the principal agent problem, Nash and correlated equilibria and existence of the core.

1. Week 1- 3: The theorem of the alternative and its applications to co-operative games, option pricing and auctions.

2. Week 4-6: Convexity and the duality theorem of linear programming and its applications to the zero-sum games, assignment model and revealed preference.
3. Week 7-9: Non-linear programs and applications to market games and the principal agent problem.
4. Week 10-12: Fixed points and applications to equilibrium existence.
5. Week 13-15: Lattices, supermodularity with applications to stable matchings and financial networks.

The grade for the course will depend upon seven homeworks an oral exam and a single take home final exam.

Much of what will be covered can be found in the Mathematical Appendix of **Microeconomic Theory** by Mas-Collel, Whinston and Green, Rangarajan Sundaram's **A First Course in Optimization Theory** or **Advanced Mathematical Economics** by your humble servant.

CANVAS is used to post announcements, homework assignments, video recordings and other important materials. You are responsible for regularly checking, downloading and reading materials posted on the site, as they form an integral part of the class.

**Prerequisites:** Ideally, Math 360, Math 314, Econ 212 and Econ 101. However, one or more can be waived with permission. What is crucial is that interested students have exposure to and comfort with the 'theorem-proof' style of mathematics.

**Grade:** It will depend on seven Homeworks (60 % of grade) and one oral exam (10 % of grade) one final exam (30% of grade). No scores are dropped.

**Final Exam:** It will be given as a 48 hour *open book* take home on date TBD. No assistance may be given or received during an exam.<sup>1</sup> You are expected to abide by the Code of Academic Integrity in the completion of assignments, papers and exams.

**Homework:** Homework assignments to be submitted via CANVAS as a single PDF file on the due date. Use a scanning app like Dropbox or Genius to create

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<sup>1</sup>The Economics Department Course Policies, which include rules about exam attendance, make-up exams, grading appeals, etc., are available at: <http://economics.sas.upenn.edu/undergraduate-program/course-information/guidelines/policies>

the single PDF. **No late work is accepted.** There are *no* make-up homeworks. Students can be excused from at most two homeworks. The weights on the *subsequent* homeworks and final exam will be adjusted upwards to account for missing them.

**Rules Regarding Assistance:** Write-ups must be your original work. The use of materials containing solutions or partial solutions to the assignments (including solutions prepared by current or former students) would be contrary to Penn's code of academic integrity.<sup>2</sup> If your solutions contains information from outside sources, you should properly acknowledge them.

While you are required to complete the assignments individually, I don't wish to discourage learning from one's peers. This leaves room for ambiguity, so I will try to make expectations as clear as possible. In brief:

1. Discussing the *general* ideas behind the problems is permitted.
2. Writing formal solutions should be *completely individual*, done in the equivalent of separate rooms.

As discussions of general ideas gradually become more specific, some judgment is unavoidable, but here's the kind of interaction I have in mind: If a peer conveys an idea which seems central to the solution, *do not write it down....immediately.* Approach the problem again on your own as if afresh, influenced by however much of their idea you remember. If you can re-create it without notes, you have mastered it, and I'm happy to give you credit. In this way we can let everyone help each other learn, while steering a wide berth around simple copying.

I also don't wish to discourage the use of computer assistance such as WolframAlpha or ChatGPT is permitted, with a "Disclose and Detail" policy of citing external assistance and showing work.

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<sup>2</sup><https://catalog.upenn.edu/pennbook/code-of-academic-integrity/>