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"Homework in Labor Economics: Household Production and Intertemporal Substitution"

by

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Homework in Labor Economics: Household Production and Intertemporal Substitution*

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1 Introduction

Following the seminal work of Lucas and Rapping (1969), the intertemporal substitution hypothesis has played a key role in equilibrium business cycle theory. Based on the "balanced growth" observation that over long horizons wages and other income have increased substantially while hours worked have not, modern business cycle models typically make the assumption that in the long run the income and substitution effects on hours worked of an increase in wages and other income must offset (see, e.g., any number of papers in the volume edited by Cooley 1995). Hence, these models generate fluctuations in hours worked only to the extent that individuals are willing to substitute leisure at one date for leisure at other dates in response to transitory wage changes, and so the extent of this willingness is a critical factor in determining the performance of the models.

The single largest source of information on individual willingness to intertemporally substitute hours comes from the analysis of male labor supply over the life cycle. The clear consensus in the profession is that the evidence does *not* support a large intertemporal substitution elasticity.[1] The point of this paper is to argue that previous analyses have potentially underestimated individual willingness to intertemporally substitute hours of work over the business cycle because of their failure to account for the fact that individuals have more uses for their time than labor and leisure. In particular, we argue for incorporating nonmarket work, a category of time use that we like to call *household production*, into both the theoretical and empirical model.

The intuition is simple. Loosely speaking, studies based on the life cycle derive information about intertemporal substitution by measuring the extent to which hours of market work increase in step with market wages. There is an implicit but important assumption that other factors influencing hours

[1] See, for instance, the well-known surveys in MaCurdy (1985) and Pencavel (1986); there are, however, some dissenting opinions in the literature, including Kennan (1988), Kimmel and Kneisner (1993), and Mulligan (1995), for example.

of work are not highly correlated with changes in wages over the life cycle. Changes in productivity in nonmarket activities are potentially a significant violation of this assumption. The phase of the life cycle in which wages are high may also be the period in which individuals have the greatest productivity (or the greatest demands on their time) in household production activities like, for example, child rearing. To the extent that this is so, the change in market hours that would result from a given change in wages, holding all else constant, would presumably be much larger than implied by existing estimates based on life cycle data. This is significant for macroeconomics because there is little reason to believe that increases in market productivity are correlated with increases in household productivity over the business cycle in the same way that they may be over the life cycle.

To pursue these issues we generalize the standard life cycle model of labor supply, as studied by Ghez and Becker (1975), MaCurdy (1981), Altonji (1986), or Browning, Deaton and Irish (1986), for example, to explicitly incorporate household production. The model generates a simple relationship between total (market plus home) work and wages that allows us to identify empirically the extent to which individuals are willing to substitute hours intertemporally. The extent to which our results differ from those of earlier studies without household production will depend on how hours spent in home work vary over the life cycle, and, in particular, how they vary in relation to the wage. We use data from the Michigan Time Use Survey to construct a synthetic cohort, and find that home hours and wages are positively correlated. We then estimate the structural model under a variety of assumptions regarding functional forms and for different age groups.

There are two main findings. First, even without taking home production into account, for many of our specifications we estimate elasticities that are large relative to those typically found in the literature. For example, using a sample of men aged 22-62 with positive hours of market work, we obtain estimates in the neighborhood of one, more than double those obtained by

Ghez and Becker (1975) using similar techniques on a similar sample.[2] Second, and more importantly from our perspective, we find that ignoring home production in life cycle models can lead to a large negative bias in estimates of the amount of intertemporal substitution. That is, incorporating household production into the model substantially increases our prediction of the change in hours that results from a cyclical wage increase.

The rest of the paper is outlined as follows. Section 2 lays out the life cycle model with home production. Section 3 describes how we construct a synthetic cohort. Section 4 describes the data. Section 5 discusses how to compare home production and non-home production models. Section 6 provides estimates. Section 7 discusses the implications for business cycles. Section 8 concludes.

2 The Model

We begin with a brief review of the standard labor supply model, as analyzed by Ghez and Becker (1975), MaCurdy (1981), or Altonji (1982), for example. Consider an individual who solves the problem

$$\max \sum_{t=1}^{T} \beta^t U(c_{mt}, h_{mt})$$

$$\begin{aligned} s.t. \sum_{t=1}^{T}(1+r)^{-t}c_{mt} &\leq A_0 + \sum_{t=1}^{T}(1+r)^{-t}w_t h_{mt} \\ h_{mt} &\leq H, \end{aligned}$$

where c_{mt}, h_{mt}, and w_t denote market consumption, hours of market work, and the real wage, respectively, at time t (one may interpret t as an index for the age of the individual). Also, A_0 denotes initial asset holdings, β the

[2] Mulligan (1995) reports similar findings.

subjective discount factor, r the interest rate, and H the per period time endowment.[3]

The usual monotonicity and convexity assumptions concerning the utility function U are imposed. Then the first order conditions for an interior solution are $\beta^t U_1(t) = \lambda(1+r)^{-t}$ and $-\beta^t U_2(t) = \lambda(1+r)^{-t} w_t$, where λ is the multiplier on the lifetime budget constraint and $U(t)$ indicates that U is being evaluated at arguments as of time t. The latter condition implies

$$\log[-U_2(t)] = \log\lambda - t\log\beta(1+r) + \log w_t. \tag{1}$$

If, as is typical in the literature, we assume $U = u(c_{mt}) - v(h_{mt})$, then c_{mt} does not appear in (1), and we have a simple relation between market hours and wages.[4]

For example, if $v(h_{mt}) = \phi h_{mt}^{\gamma}$, where $\gamma \geq 1$ is required for concavity, then (1) becomes

$$(\gamma - 1)\log h_{mt} = \log\frac{\lambda}{\gamma\phi} - t\log\beta(1+r) + \log w_t. \tag{2}$$

Alternatively, if $v(h_{mt}) = -\phi(H - h_{mt})^{\gamma}$, where now $\gamma \leq 1$ for concavity, we get the same relationship except that $(\gamma-1)\log(H-h_{mt})$ appears on the left hand side. And if $v(h_{mt}) = \phi\exp[-\gamma(H-h_{mt})]$, we get the same relationship with $-\gamma(H - h_{mt})$ on the left hand side. Given assumptions about how ϕ is distributed (across individuals or time), each of these functional form assumptions yields a regression equation that allows us to identify, among other things, the parameter γ.

The above specification assumes that there are only two ways to spend one's time: market work and leisure. Recent research on household production suggests that this dichotomous characterization masks some important

[3] Although this model is deterministic and assumes complete markets, in the Appendix we outline a version with uncertainty and a sequence of budget constraints.

[4] Even though it has no effect on (1), one still may want to impose restrictions on utility to be consistent with the "balanced growth" observations. Here, this means either: (a) $U(c_{mt}, h_{mt}) = (c_{mt}^{\eta}/\eta)\, v(H - h_{mt})$ for some function $v(h_{mt})$, where $1-\eta$ is the coefficient of relative risk aversion with $\eta < 1$ and $\eta \neq 0$; or (b) $U(c_{mt}, h_{mt}) = \log(c_{mt}) - v(h_{mt})$ for some function $v(h_{mt})$.

features of how time is allocated across activities (see, e.g., Becker 1988 and Greenwood, Rogerson and Wright 1995 for discussions in the context of macroeconomics; see Becker 1965 and Gronau 1986 for earlier discussions in the context of labor economics). To capture this, we extend the model to allow for three uses of time, market work (h_{mt}), nonmarket work or home production (h_{nt}), and leisure ($H - h_{mt} - h_{nt}$). Hours of home work are combined with home capital (k_{nt}) to produce a nontradable home consumption good (c_{nt}), according to the home production function $g_t(h_{nt}, k_{nt})$. This function depends on t to allow for the possibility that productivity in home production varies over the life cycle, which one may view as the nonmarket analogue of wages changing over the life cycle.

The maximization problem solved by our individual is now:

$$\max \sum_{t=1}^{T} \beta^t U(c_{mt}, c_{nt}, h_{mt}, h_{nt})$$

$$\begin{aligned} s.t. \ \sum_{t=1}^{T}(1+r)^{-t}(c_{mt}+i_{nt}) &\leq A_0 + \sum_{t=1}^{T}(1+r)^{-t}w_t h_{mt} \\ c_{nt} &\leq g_t(h_{nt}, k_{nt}) \\ k_{nt+1} &= (1-\delta)k_{nt} + i_{nt} \\ H &\geq h_{mt} + h_{nt}. \end{aligned}$$

All of the notation has been defined earlier with the exception of i_{nt}, investment in home capital, and δ, the depreciation rate.[5] The above framework is similar to that used in general equilibrium macro models in Benhabib, Rogerson and Wright (1992) and McGrattan, Rogerson and Wright (forthcoming). Accumulation of home capital will actually play no role in the empirical analysis that follows, but we include it here because it is important to note that the results are not based on a specification that precludes the

[5] Again, this is a deterministic model with complete markets, but in the Appendix we outline a version with uncertainty and a sequence of budget constraints.

accumulation of home capital.[6]

It is straightforward to obtain first order conditions for the above problem. We focus on the condition for h_{mt}, which, assuming an interior solution, is $-\beta^t U_3(t) = \lambda(1+r)^{-t} w_t$, where λ is the multiplier on the lifetime budget constraint. This leads to

$$\log[-U_3(t)] = \log\lambda - t\log\beta(1+r) + \log w_t, \tag{3}$$

which is very similar to (1). For example, if $U(c_{mt}, c_{nt}, h_{mt}, h_{nt}) = u(c_{mt}, c_{nt}) - v(h_{mt}, h_{nt})$ and $v(h_{mt}, h_{nt}) = \phi(h_{mt} + h_{nt})^\gamma$ we have

$$(\gamma - 1)\log(h_{mt} + h_{nt}) = \log\frac{\lambda}{\gamma\phi} - t\log\beta(1+r) + \log w_t. \tag{4}$$

Comparing this with (2), the only difference that arises from the introduction of home production is in the left hand side variable. The same thing happens for versions using the other functional forms discussed above, $v(h_{mt}, h_{nt}) = -\phi(H - h_{mt} - h_{nt})^\gamma$ and $v(h_{mt}, h_{nt}) = \phi\exp[-\gamma(H - h_{mt} - h_{nt})]$. Effectively, adding home production simply means redefining time spent working: it should be the sum of time spent working in the market and the home, not simply time spent working in the market.

As mentioned above, one might want to restrict U to be consistent with the "balanced growth" observations, which imply that the income and substitution effects of a permanent increase in wages and other income must offset. Given $U(c_{mt}, c_{nt}, h_{mt}, h_{nt}) = u(c_{mt}, c_{nt}) - v(h_{mt}, h_{nt})$, this means that $u(c_{mt}, c_{nt}) = \log c_t$, where c_t is a homothetic function of the two consumption goods, c_{mt} and c_{nt}.[7] This specification implies that if initial assets, market wages, and the home production function are all scaled by a common factor, then the solution to the individual's time allocation problem is unchanged

[6] Rupert, Rogerson and Wright (1995) study a similar framework, but there individuals are not allowed to invest in home capital.

[7] For example, a specification that is often used in the literature is the CES aggregator, $c_t = [ac_{mt}^\sigma + (1-a)c_{nt}^\sigma]^{1/\sigma}$.

whereas c_{mt} and c_{nt} are scaled by the same factor. Although such a restriction on utility does not affect the equation that we actually estimate in the empirical work, it does play a role when we argue that use of synthetic cohort data is justified.

Although equation (3) was derived under the assumption of complete markets and no uncertainty, we show in the Appendix that essentially the same result holds in a stochastic model with incomplete markets (the only complication is that λ now depends on t, but this can be dealt). To close this section, we also point out that the same results hold when the basic decision making unit is the family rather than a single individual. To illustrate, assume for example that the family consists of two members and that the instantaneous utility function is

$$U = u(c_{1mt}, c_{1nt}, c_{2mt}, c_{2nt}) - v(h_{1mt} + h_{1nt}) - v(h_{2mt} + h_{2nt}),$$

where the subscripts 1 and 2 refer to the two individuals.

Assume that the family maximizes the discounted present value of U, subject to

$$\begin{aligned}
\sum_{t=1}^{T}(1+r)^{-t}(c_{1mt} + c_{2mt} + i_{nt}) &\leq A_0 + \sum_{t=1}^{T}(1+r)^{-t}(w_{1t}h_{1mt} + w_{2t}h_{2mt}) \\
c_{1nt} + c_{2nt} &\leq g_t(h_{1nt}, h_{2nt}, k_{nt}, t) \\
k_{nt+1} &= (1-\delta)k_{nt} + i_{nt} \\
H &\geq h_{1mt} + h_{1nt} \\
H &\geq h_{2mt} + h_{2nt}.
\end{aligned}$$

The first order conditions for this problem can be used to generate an equation like (3) for each individual, just as we derived (3) for the single individual's problem. Therefore, for a particular functional form, we get an estimating equation like (4) for each individual. This is significant when we take the theory to the data because we can thereby justify looking at married, and not only single, individuals. Moreover, note that as long as the two labor inputs into home production are not perfect substitutes, it does not follow that one individual j must be at a corner for h_{jt} whenever $w_{1t} \neq w_{2t}$.

3 Synthetic Cohort Construction

It is instructive to begin with a brief review of attempts to estimate the intertemporal elasticity of substitution using micro data. Ghez and Becker (1975) use census data to construct a synthetic cohort – that is, they turn a cross section of individuals who vary with respect to age into a life cycle profile for a hypothetical representative agent – and used this data to estimate (2). MaCurdy (1981) estimates (2) using individual level panel data, rather than aggregating to construct a synthetic life cycle profile for a representative agent. Altonji (1982) shows how one could also obtain estimates from individual level data if it included information on consumption. Mulligan (1995) contains a discussion of the merits of the different procedures, and ends up arguing in favor of the synthetic cohort method.[8]

In our case, choice of procedures is dictated by data availability. The highest quality data on the allocation of time is the Michigan Time Use Survey (see Juster and Stafford 1991 for a detailed discussion). However, the Michigan Time Use Survey is not a panel, and does not include information on individual consumption.[9] Hence, we follow Ghez and Becker and construct a synthetic cohort from the cross section. As we outline below, under certain assumptions, the resulting data corresponds to the life cycle wage and hours profiles for a representative individual, and hence permit us to carry out

[8]There is also a literature that uses aggregate data to estimate paramaters of a representative agent utility function. See, for example, Eichenbaum, Hansen and Singleton (1988).

[9]The PSID does provide panel data on both home and market hours, as well as market consumption, which is what we used in Rupert et al. (1995) to estimate the elasticity of substitution between c_{mt} and c_{nt}. But there seems to be little doubt that the time use survey data provides much more reliable information on hours, and this seems more important for estimating intertemporal labor supply elasticities. The empirical work reported below has been redone using the PSID, but the results were not particularly satisfactory.

structural estimation.

We assume that individuals differ along several dimensions. First, they may be born at different dates. Second, holding age constant, they may face different lifetime economic opportunities (e.g., wage profiles), which will imply a different value for the multiplier λ. Finally, we assume that preferences, as parameterized by ϕ, may vary across agents. We index our individuals by the pair (a, i), where a is the individual's age (i.e., cohort) and i indexes heterogeneity in preferences and opportunities. Although we only have data for individuals at a single point in time, each individual is solving a life cycle problem as formulated in Section 2. Hence, if for each individual (a, i), $v_{ai}(h_{mai}, h_{nai}) = \phi_{ai}(h_{mai} + h_{nai})^{\gamma}$ for example, where γ is constant, the following equation holds:

$$(\gamma - 1)\log(h_{mai} + h_{nai}) = \log\lambda_{ai} - \log\gamma - \log\phi_{ai} - a\log\beta(1+r) + \log w_{ai}. \quad (5)$$

Consider a cohort of individuals who are of the same age but possibly different i types. Adding equation (5) over these individuals and dividing by the number in the cohort, we have

$$(\gamma - 1)\hat{h}_a = \hat{\lambda}_a - \log\gamma - \hat{\phi}_a - a\log\beta(1+r) + \hat{w}_a, \quad (6)$$

where $\hat{h}_a$, $\hat{w}_a$, $\hat{\phi}_a$, and $\hat{\lambda}_a$ are the averages of $\log(h_{mai} + h_{nai})$, $\log w_{ai}$, $\log\phi_{ai}$ and $\log\lambda_{ai}$ for individuals of age a. In order to derive an estimable equation, we need to make some assumptions regarding $\hat{\phi}_a$ and $\hat{\lambda}_a$. Regarding the former, we simply assume that ϕ_{ai} is log-normally distributed across i, with the same mean and variance for all a. This implies that $\hat{\phi}_a = \hat{\phi}_0 + \xi_a$, where ϕ_0 is a constant and ξ_a is normal.

Regarding the latter, $\hat{\lambda}_a$, we could assume that λ_{ai} is log-normally distributed across i for a given a, but it is hard to justify the assumption that the mean is the same for all a because of the fact that (holding age constant) real wages increased steadily over the relevant period. The issue is easily handled, however, by assuming that wages grow at a constant rate x each period for a worker of given age and type, and that the home production function and initial assets are also scaled upward each period at this same rate. If preferences are consistent with the "balanced growth" observations, as discussed above, then the only difference between successive ages for a given type i is that consumption will be higher by a factor $1+x$ for younger cohorts, and the multiplier for cohort a will be decreasing at rate x: $\lambda_{ai} = \lambda_{0i}(1+x)^{-a}$, where λ_{0i} does not depend on a and is log-normally distributed across i. Taking logs and averaging across i, we have $\hat{\lambda}_a = \hat{\lambda}_0 - xa + \zeta_a$, where $\hat{\lambda}_0$ is a constant and ζ_a is normal.

Inserting these results into equation (6), we arrive at the regression equation

$$\hat{h}_a = \alpha_0 + \alpha_1 a + \alpha_2 \hat{w}_a + \mu_a, \tag{7}$$

where α_0 is a constant, $\alpha_1 = [\log \beta(1+r) - x]/(\gamma - 1)$, $\alpha_2 = 1/(\gamma - 1)$, and μ_a is a normally distributed error term. This equation can be estimated, either in levels or first differences.[10] The intertemporal elasticity of substitution is given by α_2, while α_1 tells us how hours move over time for a given wage. Given that wage and hours profiles are concave, α_1 determines where they reach their relative peaks. For example, if $\alpha_1 = 0$ then wages and hours always move in the same direction, while if $\alpha_1 < 0$ the effect is to add a positively sloped component to hours, making them peak after wages. Intuitively, this positively sloped component occurs when a high interest rate increases the return to work early compared to later in life.

Exactly the same procedure can be used with other functional forms. Thus, $v(h_{mt}, h_{nt}) = -\phi(H - h_{mt} - h_{nt})^{\gamma}$ yields an equation just like (7), except on the left hand side $\hat{h}_a$ will be the average of $\log(H - h_{mai} + h_{nai})$ across i within a cohort. In this case, α_2 measures the elasticity of leisure with respect to wages. Also, $v(h_{mt}, h_{nt}) = \phi \exp[-\gamma(H - h_{mt} - h_{nt})]$ yields the same thing except $\hat{h}_a$ will be the average of $H - h_{mai} - h_{nai}$ across i

[10] Alternative distributional assumptions concerning λ and ϕ are needed to justify first difference estimation; in any case, our level and first difference results are very similar.

within a cohort, and $-1/\gamma$ replaces $1/(\gamma-1)$ in the definitions of α_1 and α_2. In this case, α_2 is not an elasticity, but tells us the total increase in hours that results from a 1 percent increase in wages.

We close this section with a few remarks about the specification. First, our formulation assumes that preferences do not change in a systematic way over the life cycle. One might argue that changes in preferences are a natural way to captures changes in time use associated with the arrival of children. However, if one allows preferences for work to vary systematically over the life cycle, we must effectively give up hope of recovering information about substitution elasticities from this data. In our specification, all such effects are captured by changes in productive opportunities; hence, for example, having a baby does not change your preferences for spending time with children, it changes the opportunities to do so. The advantage of this interpretation is that it allows one to maintain that preferences do not change systematically over time.[11]

The other important feature of our specification is that we do not need a measure of how home production opportunities change over the life cycle. Nor do we require any assumptions about the substitutability of time and capital in the home production function. To obtain information about the substitutability of hours over time, it is sufficient to know how many hours individuals spend in home and market work. The key here is that optimization implies that time spent in the market and in the home have the same value at the margin – namely, the wage rate w_t. If one wanted to match the actual profile of home hours over the life cycle, it may be necessary to be more specific about the nature of home production opportunities; but for our purposes, it is not.

[11]For the reader familiar with home production theory, recall that these models do generate reduced form preferences over leisure and market consumption where shifts in the home production technology show up as shifts in preferences. See Greenwood et al. (1995).

4 Data

We require a data set that includes detailed information about time use at the individual level. Our data are drawn from the Michigan Time Use Survey for the years 1975-76, which is the most comprehensive time use data available (see Juster and Stafford 1985, 1991 for additional discussion). The survey had individuals record their time use for four days over a twelve month period. The four days are comprised of two weekdays, a Saturday, and a Sunday, and the data are combined to reflect average weekly values (allowing for the composition of weekdays and weekends). The base sample consists of 332 men ages 22 to 65. However, since our theoretical results assume an interior solution, we restrict our sample to those who report positive hours of market work. For older workers, this may cause sample selection problems.[12] We deal with this issue by considering two subsamples: men between the ages of 22 and 45, and men between 22 and 62, in each case restricting attention to those who report positive hours.

In the time diaries, respondents account for the use of their time in 15 minute segments. Each segment is allocated to one of ten major categories, each of which is subdivided into several subcategories. The ten major categories are market work, house/yard work, child care, services/shopping, personal care, education, organizations, social entertainment, active leisure, and passive leisure. In our empirical work, we use the market work category as our measure of h_{mt}, and the sum of the house/yard work, child care, and services/shopping categories as our measure of h_{nt}. The units are hours per week. For regressions that have leisure, $H - h_{mt} - h_{nt}$, on the left hand side, there is an issue concerning the value of H.[13] We try several options, including setting H to a fixed number of hours, setting H_t in each period to

[12] For the sample as a whole, about 90 percent of the men report positive market hours, while for the 63-65 age group, only 40 percent report positive market hours.

[13] Because the appropriate value of H is unclear, Browning, Deaton and Irish (1985) argue for using the specification with labor on the left hand side.

a fixed number of hours minus reported sleep and personal care (which are not constant), and using a direct measure of reported leisure.

Wage data in the Time Use Survey are not good (among other things, there are lots of missing observations). Hence, we use data in the 1975 PSID on average hourly earnings for male workers between the ages of 22 and 65 to construct a measure of w_t for our synthetic cohort. Presumably, the economically relevant variable is the after-tax wage. To take account of this in a relatively simple way, we use a measure of the tax rate for a given cohort adapted from Mulligan's (1995) work. He computed the statutory tax rate for each individual in his sample; we simply take the average of his tax rates for each given age. We report results below using both the before- and after-tax wage.

For the purpose of looking at the data it is useful to smooth the series, which we do by regressing each variable on a cubic in age. For the age 22-62 sample, the correlation matrices for the smoothed series, in both levels and first differences, are:[14]

	$\log h_m$	$\log h_n$	$\log(h_m + h_n)$	$\log w$
$\log h_m$	1.000			
$\log h_n$	−0.456	1.000		
$\log(h_m + h_n)$	0.753	0.211	1.000	
$\log w$	−0.060	0.881	0.500	1.000

	$\Delta \log h_m$	$\Delta \log h_n$	$\Delta \log(h_m + h_n)$	$\Delta \log w$
$\Delta \log h_m$	1.000			
$\Delta \log h_n$	0.562	1.000		
$\Delta \log(h_m + h_n)$	0.804	0.943	1.000	
$\Delta \log w$	0.988	0.682	0.886	1.000

The following facts emerge. Market hours and home hours are negatively correlated in levels but positively correlated in differences. Market hours are

[14]These results are for the sample that only includes men who report positive market hours, and for the before-tax wage. The results are bascially the same using the after-tax wage. In the 22-65 sample, and also in the 22-45 sample, wages and market hours are positively correlated in levels as well as growth rates.

uncorrelated with the wage in levels but almost perfectly correlated in differences. Home hours and total hours are both positively correlated with the wage. The positive correlation between home hours and wages suggests that productivity in the home increases along with productivity in the market, something to which we alluded in the introduction.

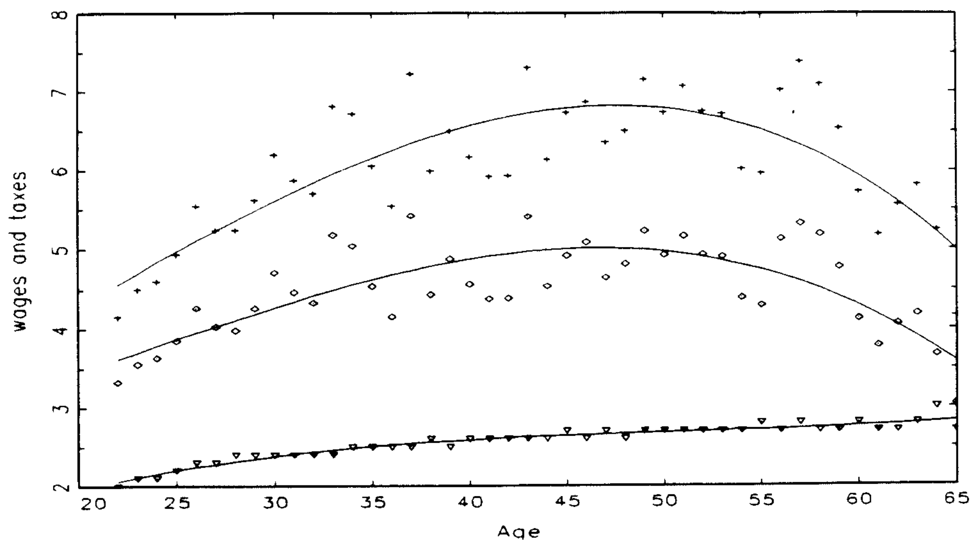

Figure 1: Wage and Tax Rates

Figures 1 and 2 graph the wage and hours series, in levels rather than logs, where the symbols indicate raw data and the solid curves smoothed data. The curves in Figure 1 are, from north to south, before-tax wages, after-tax wages, and tax rates. The curves in Figure 2 are total (market plus home) hours, market hours and home hours. Wages and market hours have the familiar concave shapes, but home hours do not. The total hours profile is concave. The tax rate rises with age, from about 20 to 30 percent, and the after-tax wage is less variable than the before-tax wage. Notice that home hours and wages are both increasing over the ages 22-45, which is what led us to believe in the first place that previous estimates of intertemporal

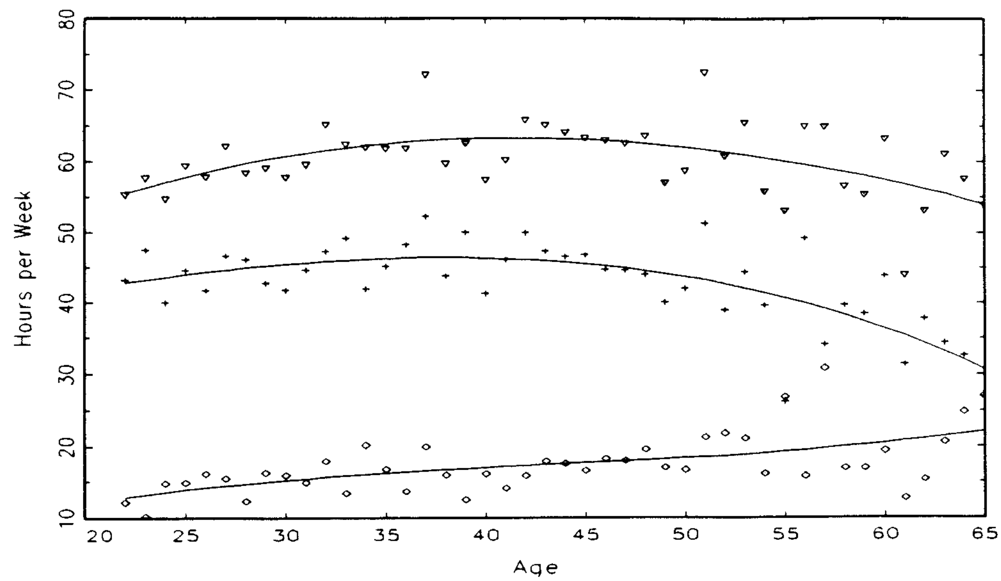

Figure 2: Home, Market and Total Hours (Men, Workers Only)

substitution elasticities are biased downward. Also note that home hours peak at a later date than market hours. The wage peaks in between the two hours series, at about the same time as total hours.

Figure 3 shows the market, home and total hours profiles for all men, including those who report zero hours of market work (we only need to assume interior solutions in order to perform structural estimation, not to look at the data). The basic pattern for market hours is similar, although declines more rapidly towards the end of the sample due to fact that the number of individuals working with zero market hours increases with age. The pattern for home hours here differs from than in Figure 2 due to the fact that older men who are not working in the market spend a substantial amount of time in home production. The total hours profile is still concave.

For the sake of interest, Figure 4 shows hours for women (including those who report zero market hours). The pattern for total hours of work for

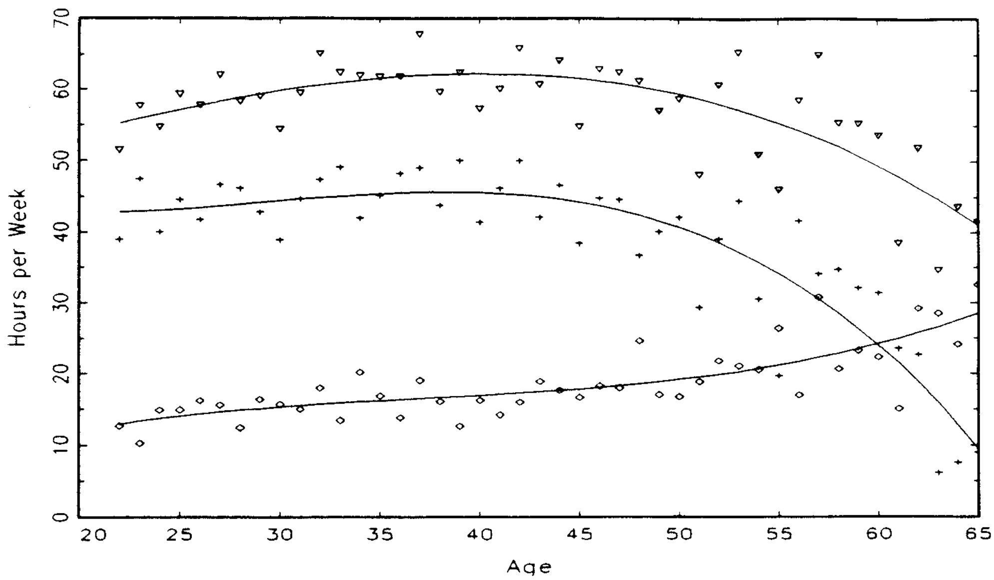

Figure 3: Market, Home and Total Hours (All Men)

women is remarkably similar to the pattern for men: both are concave, and the levels similar. The components of time use, however, differ significantly: for women, hours of market work are actually decreasing until about 30, after which they follow a concave pattern, and home hours are increasing until the mid thirties, after which they follow a convex pattern. As was seen for men in Figure 3, in the later part of the life cycle women increase hours spent working at home. A similar pattern for married couples is observed in Figure 5. Although women and couples are worthy of more detailed study, in order to focus our results, and to avoid potentially severe sample selection issues with women, estimation here is performed only on men. Recall, however, that we can use both married and single men, since the same estimating equation arrises out of the single-agent and family decision problems.

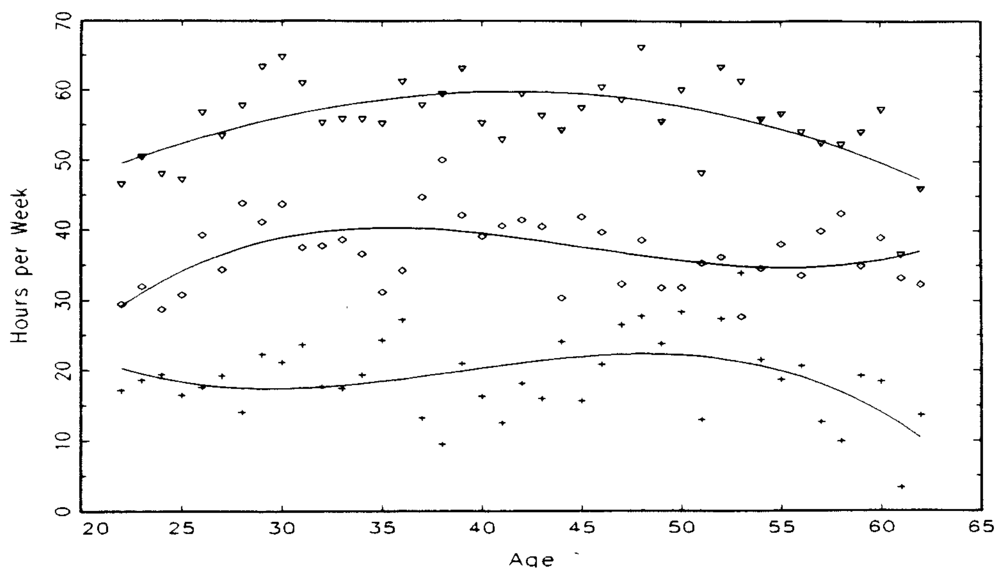

Figure 4: Home, Market and Total Hours (All Women)

5 Comparing Results Across Models

Before turning to the results, in this section we discuss some issues that need to be considered when comparing estimates across models. Consider $v(h_{mt}, h_{nt}) = \phi(h_{mt} + h_{nt})^{\gamma}$; then α_2 in equation (7) tells us the percent increase in total hours as the wage increases by 1 percent along the life cycle wage profile. In the standard model without home production, one would be led to a version of (7) with only market hours (instead of total hours) on the left hand side. The essential point we want to make here is that, with this specification, even *if* we obtain the same estimate for the value of α_2 with and without home production, the model with home production entails different predictions. The reason is that in a model with home production the total amount of work is greater (as the sum of both home and market work). As a result, even if measured elasticities are the same, a given percentage increase in the market wage implies a greater increase in total hours of work

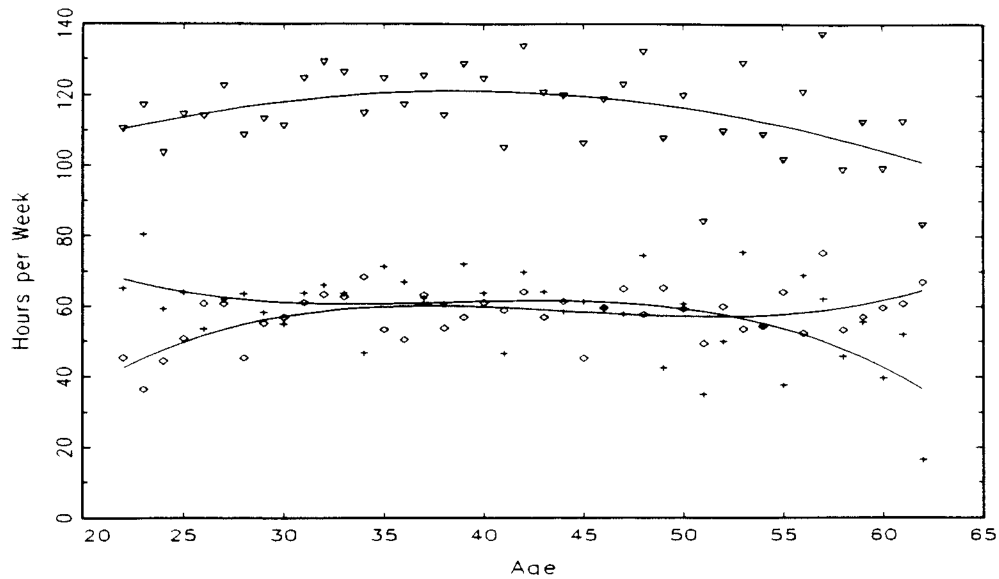

Figure 5: Home, Market, and Total Hours (All Families)

in the home production model.

This is of particular interest for business cycle analysis. To be concrete, assume that on average h_{mt} is 40 hours per week and h_{nt} is 20 hours per week (not far from the averages in our sample), and suppose for the sake of argument that we estimate the elasticity to be the same number α_2 using the models with and without home production. In the non-home production model, this implies that a 1% increase in the wage along the life cycle profile increases h_{mt} by $0.4\alpha_2$ hours per week while in the home production model it increases $h_{mt} + h_{nt}$ by $0.6\alpha_2$ hours per week. At business cycle frequencies, when the wage increases presumably most of the increase in total hours will show up in the market sector; indeed market hours will increase by more than $0.6\alpha_2$ if individuals reduce their home production during booms in market activity. Even making the conservative assumption that home work does not decline during booms, so that market hours increase by exactly $0.6\alpha_2$, we see that the home production model predicts the percentage increase in h_{mt} will be 50 percent greater than predicted by the non-home production model, given the same α_2.

Now, obviously, there is no reason to expect the same estimate of α_2 in the models with and without home production; one cannot simply take the α_2 estimated in a non-home production model and use it in the context of a home production model. The main goal of this paper is to examine whether introducing home production affects our measurement of this parameter. The point of the above example is simply that even if the elasticity were estimated to be similar, it does not follow that the two models have the same implications for macroeconomic analysis.

The opposite points apply to the functional form $v(h_{mt}, h_{nt}) = -\phi(H - h_{mt} - h_{nt})^{\gamma}$; since the base hours of leisure are lower in the home production model, given the same estimate of α_2, it implies a smaller change in leisure. In the case of $v(h_{mt}, h_{nt}) = \phi \exp[-\gamma(H - h_{mt} - h_{nt})]$, the coefficient α_2 is not an elasticity, but equals the total increase in hours in response to a one percent wage increase. Hence, given the same α_2, under the assumption that home work does not change during booms, the two models would predict the same change in h_{mt}. But, again, there is no reason to expect that the estimate of α_2 will be the same in the home production and non-home production models.

6 Results

In this section we report the results of estimating equation (7) for the three different functional forms discussed above. Table 1 gives the results for the case $v(h_{mt} + h_{nt}) = \phi(h_{mt} + h_{nt})^{\gamma}$, which means $\log(h_{mt} + h_{nt})$ is the dependent variable (Tables 1-4 can be found at the end of the paper). We report estimates from the non-home production (NHP) model, which means $\log h_{mt}$ is the dependent variable, in addition to the home production (HP) model.

In each case, we estimate the model on both subsamples, ages 22-45 and ages 22-62, and we estimate the model in both levels and first differences. We also estimate the models using before-tax wages w (see the column labeled BTW) and after-tax wages $w(1-\tau)$ (see the column labeled ATW). Standard errors are in parentheses.[15]

We begin by looking at α_1, the coefficient on t. In both the HP and NHP models, estimates of α_1 are basically zero for the 22-45 sample, whether we use levels or first differences and before- or after-tax wages. This is not too surprising, perhaps, since over these ages both hours and wages are monotone increasing, and intuitively one might expect this parameter to be identified from the turning points of the series (recall the discussion following equation (7) in Section 3). When we use the 22-62 sample, α_2 is negative and statistically significant in all instances. Note that it is greater in absolute value for the non-home production model, consistent with a later turning point for total hours as compared to market hours.

We now move to α_2, the coefficient on the wage. First, by way of comparison with previous studies, consider the point estimates from the NHP model. Using after-tax wages, for example, we obtain estimates for α_2 between 0.160 and 0.323 for the 22-45 sample, and between 0.730 and 0.977 for the 22-62 sample. Using data from the 1960 census, Ghez and Becker (1975) obtained estimates on the order of 0.4 for their 22-62 sample, lower than our number. However, our estimates are similar to those in Mulligan (1995), who runs similar regressions under various alternative assumptions. In particular, using hours of market work from the 1975 Time Use Survey and wages from the 1979 CPS, obtains estimates of 0.25 for men age 25-55 and 0.97 for men age 20-64.[16]

[15] We use raw hours data and smoothed wage data, so in effect we are using a polynomial in age to instrument for wages.

[16] Mulligan's procedure is not identical to ours. He uses three-year moving averages for both hours and wages, and instruments for wages. Additionally, for some reason, in his synthetic cohort construction he uses logs of averages rather than averages of logs.

Hence, differences between our results and those in Ghez and Becker (1975) are due at least in part to the fact that the Time Use Survey implies greater variations in hours. Also notice that using after-tax wages increases the estimate of α_2 in every case, which makes perfect sense given that the progressivity of the tax system implies that after-tax wages do not vary as much as pre-tax wages. Although both of these results are interesting, they are not the effects on which we want to focus (see Mulligan for additional discussion). Rather, we are most interested in the impact of explicitly incorporating home production, given a single set of data for hours and wages.

For the case of individuals aged 22-45, Table 1 shows that the point estimate for the intertemporal elasticity parameter α_2 is higher in the home production model, whereas in the 22-62 sample it is slightly lower. For example, using after-tax wages and estimation based on levels, incorporating home production increases the point estimate of α_2 from 0.160 to 0.316, in the shorter sample, and decreases it slightly from 0.730 to 0.710 in the longer sample. Similar results obtain when we use before-tax wages and/or first difference estimation. However, as argued in Section 4 and discussed further below, for this specification, even given similar estimates of α_2 the models have different predictions for the volatility of market hours over the business cycle.

We now turn to results for the specification $v(h_{mt}+h_{nt}) = -\phi(H - h_{mt} - h_{nt})^{\gamma}$, which means that $\log(H - h_{mt} - h_{nt})$ is the dependent variable, or $\log(H - h_{mt})$ in the model that ignores home production. Given that the left hand side is interpreted as (the log of) leisure, as discussed above, there are several ways that one could in principle measure it. Table 2 presents estimates when we use 112 minus hours worked per week. Table 3 presents estimates when we use 168 minus sleep minus personal care minus hours worked per week. We also estimated the model using several other measures of H, as well as using directly reported measures of leisure for the dependent variable; the results were similar, and for brevity are not all reported here.

With this specification, intertemporal substitution implies a negative value of α_2, because $H - h_{mt} - h_{nt}$ must decrease when the wage increases (holding λ constant). On the age group 22-45, however, we estimate α_2 to be positive in the NHP model for most of the specifications. In contrast, in the HP model, α_2 is negative in all instances, although not always significant. On the age group 22-62, α_2 is negative in both the NHP and HP models, but is larger in absolute value in the HP model.

In Table 4 we report results for the specification $v(h_{mt}, h_{nt}) = \phi \exp[-\gamma(H - h_{mt} - h_{nt})]$. In this case, $H - h_{mt} - h_{nt}$ appears on the left hand side (i.e., hours rather than the log of hours), or $H - h_{mt}$ in the NHP model. In fact, we can move H to the right hand side and subsume it in the constant, so that we end up with $h_{mt} + h_{nt}$ as the dependent variable, or h_{mt} in the NHP model. Recall that in this case α_2 measures the change in hours in response to a wage increase: e.g., if the wage doubles, work increases by α_2 hours (not α_2 percent).

For the age group 22-45, estimates using the NHP model imply that a wage increase generates a very small increase, and maybe even a decrease, in hours worked, depending on the specification. The results are much different using the HP model; for example, if the wage doubles, $h_{mt} + h_{nt}$ increases by between 12.4 to 15.8 hours per week. For the age group 22-62, the difference between the HP and NHP estimates is less dramatic, but still there. The difference in this case is less dramatic not because α_2 is small in the home production model, but because it is fairly big in the non-home production model.

7 Implications

As emphasized earlier, the main issue from a macroeconomic perspective is not the size of the intertemporal substitution elasticity *per se*, but what the model implies for fluctuations in h_m in response to various types of shocks.

In this section we do two things. First, we calculate the effect on market hours of a temporary change in the real wage based on the various estimates for the different models and samples in the previous section. Second, we report the results of simulating a full blown real business cycle model using the functional form and parameter values implied by our estimates.

In the first exercise, we initially make the conservative assumption that the intertemporal substitution response is the only response to a wage change; that is, we abstract from the potential contribution of *intratemporal* substitution between home and market activities, by assuming h_n does not change when the wage increases. As discussed in Benhabib et al. (1991), if one imposes standard "balanced growth" restrictions, then in an NHP model there is no within period substitution while in the HP model the movement of hours between market and nonmarket work can generate sizable intratemporal effects. Therefore, we also calculate the response of h_m under the assumption that h_n falls by an amount corresponding to what we know about the size of this intratemporal response.

Table 5 presents the change in h_m that results from a 1% increase in w under the assumption that h_n is constant.[17] We use the values of α_2 estimated using levels and after-tax wages, and we report the results for both age samples, for both HP and NHP models, using estimates from the various functional form specifications in Tables 1, 2, and 4 (3 is similar to 2 and hence is not reported). The important point for our purposes is the relative magnitudes of the numbers across models. For the 22-45 group, the HP estimates predict a much greater response in market hours on average,

[17] Recall that our estimates are based on the assumption that the marginal utility of consumption λ is constant. In the life cycle model this means that the change in w was either anticipated or fully insured. Strictly speaking, in a standard macroeconomic context, like in equilibrium business cycle theories, the relevant shocks to productivity are neither anticipated nor insurable, but are small enough relative to lifetime wealth that λ is approximately constant. In some equilibrium business cycle models with labor contracts, shocks to productivity are insurable, and so λ is literally constant; see Wright (1988), Greenwood and Gomme (1993), or Boldrin and Horvath (1995).

although not necessarily for all specifications. For the 22-62 group, the HP estimates imply substantially greater increases in h_M in all cases.

Table 5 Business Cycle Implications With $\Delta h_n = 0$

5a. Ages 22-45 ($h_m = 45.6$, $h_n = 15.4$)						
Based on:	Table 1		Table 2		Table 4	
	NHP	HP	NHP	HP	NHP	HP
Δh_m	0.073	0.193	0.100	0.090	0.036	0.158
$\%\Delta h_m$	0.160	0.258	0.218	0.198	0.079	0.346

5b. Ages 22-62 ($h_m = 43.6$, $h_n = 16.9$)						
Based on:	Table 1		Table 2		Table 4	
	NHP	HP	NHP	HP	NHP	HP
Δh_m	0.318	0.430	0.315	0.420	0.188	0.305
$\%\Delta h_m$	0.730	0.985	0.723	0.963	0.431	0.700

In Table 6, we report the changes in h_m under the assumption that h_n falls with an increase in w by the amount predicted by the estimates in McGrattan et al. (forthcoming). Those estimates imply that a 1% increase in w results in about a 1% decline in h_n. By definition this does not affect the results based on the NHP model, but adds to the change in h_m in the HP model. Given that h_n is roughly 1/3 of h_m, the elasticity of h_m with respect to changes in w increases by about 0.33. The bottom line is that the estimates based on the HP model indicate a much larger net response of h_m with respect to changes in w. In fact, for the 22-62 age group, the average elasticity is 1.27.

Table 6 Business Cycle Implications With $\%\Delta h_n = -1 \times \%\Delta w$

6a. Ages 22-45 ($h_m = 45.6$, $h_n = 15.4$)						
Based on:	Table 1		Table 2		Table 4	
	NHP	HP	NHP	HP	NHP	HP
Δh_m	0.073	0.347	0.100	0.244	0.036	0.312
$\%\Delta h_m$	0.160	0.596	0.218	0.536	0.079	0.684

6b. Ages 22-62 ($h_m = 43.6$, $h_n = 16.9$)						
Based on:	Table 1		Table 2		Table 4	
	NHP	HP	NHP	HP	NHP	HP
Δh_m	0.318	0.599	0.315	0.589	0.188	0.474
$\%\Delta h_m$	0.730	1.38	0.723	1.35	0.431	1.09

Our second exercise is to study a standard real business cycle model calibrated using our parameter estimates. We will not describe the model in detail, since it is identical to that in Benhabib et al. (1991) except that preferences are given by

$$U(c_{mt}, c_{nt}, h_{mt}, h_{nt}) = \log(c_t) - \phi(h_{mt} + h_{nt})^\gamma,$$

where c_t is a CES aggregate of home and market consumption,

$$c_t = [a_m c_{mt}^\sigma + (1 - a_m)c_{nt}^\sigma]^{1/\sigma}.$$

Two key parameters are γ and σ, since γ determines the amount of intertemporal substitution, and σ determines the amount of intratemporal substitution (because it measures how much agents are willing to move between market and nonmarket activity).[18]

[18] Except for preferences, the model is calibrated following the same procedure as Benhabib et al. (1991). In partilcuar, we match the following steady-state observations: market capital to output ratio equal to 8.5, home capital to output ratio equal to 1, annual real interest rate equal to 4%, fraction of time spent in market work equal to 0.33 and fraction of time spent in home work equal to 0.25. Here, we assume that technology shocks to the market sector are the only source of randomness, and that they follow an AR(1) process with persistence parameter equal to 0.95 and standard deviation of innovations equal to 0.007. The basic message is not overly sensitive to these numbers.

One observation on which people often focus when they evaluate business cycle models is the standard deviation of market hours relative to the standard deviation of output, which for Hodrick-Prescott filtered US data is roughly between 0.8 and 1.0, depending on which market hours series one uses (see Hansen and Wright 1992). How close does one come to this number using our estimated labor supply elasticities?

In Table 7 we report the standard deviation of market hours relative to the standard deviation of output generated by the model for various combinations of γ and σ. Suppose we take our estimate of γ to be between 2 and 2.5 (see Table 1, ages 22-62, after-tax wages). The column corresponding to $\sigma = 0$ indicates what happens when there is no intratemporal substitution (see Benhabib et al. for a discussion), which implies a relative standard deviation of market hours just over one half. However, given the estimates of σ in McGrattan, Rogerson and Wright (forthcoming) and Rupert, Rogerson and Wright (1995) of approximately 0.45, the relative standard deviation is closer to two-thirds. Hence, while the estimated parameters do not imply that the model accounts for *all* of the variability in hours, it does account for a large fraction.

Table 7: Relative Volatility of Market Hours to Output

$\gamma \backslash \sigma$	0	0.45	0.8
1.25	0.67	0.74	0.87
1.50	0.62	0.69	0.83
2.0	0.55	0.64	0.80
2.5	0.52	0.62	0.79
3.0	0.50	0.60	0.78

8 Conclusion

In this paper we have integrated household production into the standard life cycle model. We have argued that studies that ignore home production may generate biased estimates of some key parameters, including the intertemporal substitution elasticity. We find that estimates based on home production models are indeed larger than those based on non-home production models. Based on our analysis, it does not seem difficult to obtain elasticities around unity, as used in the real business cycle literature. One qualification should be kept in mind in interpreting the results. The real business cycle model that we calibrated in the previous section treats an infinitely-lived household as the basic unit of analysis, and does not incorporate households with multiple members, while our estimated elasticities come from data on males only. A more detailed analysis would incorporate both men and women into an equilibrium business cycle model and have them finite lived. We leave this for future work.

References

Altonji, J., "Intertemporal Substitution in Labor Supply: Evidence from Micro Data," *Journal of Political Economy* 94 (1986).

Becker, G., "Family Economics and Macro Behavior," *American Economic Review* 78 (1988), 1-13.

Benhabib, J., R. Rogerson and R. Wright, "Homework in Macroeconomics: Household Production and Aggregate Fluctuations," *Journal of Political Economy* 99 (1991), 1166-87.

Boldrin, M. and M. Horvath, "Contracts in a Real Business Cycle Model," *Journal of Political Economy* 103 (1995).

Browning, M., A. Deaton, and M. Irish, "A Profitable Approach to Labor Supply and Commodity Demands Over the Life Cycle," *Econometrica* 53 (1985), 503-544.

Cooley, T.F. *Frontiers in Business Cycle Theory*, Princeton: Princeton University Press, 1975.

Eichenbaum, M., L.P. Hansen and K. Singleton, "Time Series Analysis of Representative Agent Models of Consumption and Leisure under Uncertainty," *Quarterly Journal of Economics* 103 (1988), 51-78.

Ghez, G. and G. Becker, *The Allocation of Time and Goods Over the Life Cycle*, New York, NBER, 1975.

Greenwood, J. and P. Gomme, "On the Cyclical Allocation of Risk," *Journal of Economic Dynamics and Control* (1993).

Greenwood, J., R. Rogerson, and R. Wright, "Household Production in Real Business Cycle Theory," in *Frontiers in Business Cycle Theory*, edited by T.F. Cooley, Princeton: Princeton University Press, 1975.

Gronau, R. "Home Production: A Survey," in *Handbook of Labor Economics*, edited by O. Ashenfelter and R. Layard, Amsterdam, North-Holland, 1986.

Hansen, G.D. and R. Wright, "The Labor Market Real Business Cycle Theory," *Federal Reserve Bank of Minneapolis Quarterly Review*, Sept. 1992.

Kimmel, J. and T.J. Kniesner, "The Intertemporal-Substitution Hypothesis is Alive and Well and Hiding in the Data," Indiana University Working Papers in Economics Number 93-104.

Kennan, J. *NBER Macro Annual* 1988

Lucas, R.E., and L. Rapping, "Intertemporal Substitution of Leisure in the Theory of Labor Supply," *Journal of Political Economy* 77 (1969).

Juster, F.T. and F.P. Stafford, *Time, Goods and Well-Being*, Ann Arbor, 1985

Juster, F.T. and F.P. Stafford, "The Allocation of Time: Empirical Findings, Behavioral Models, and Problems of Measurement," *Journal of Economic Literature* 29 (1991), 471-522.

MaCurdy, T., "An Empirical Model of Labor Supply in a Life Cycle Setting," *Journal of Political Economy* 89 (1981), 1059-1085.

MaCurdy, T., "Interpreting Empirical Models of Labor Supply in an Intertemporal Framework with Uncertainty," in *Longitudinal Analysis of Labor Market Data*, edited by B. Singer and J. Heckman, New York, Cambridge University Press, 1985.

McGrattan, E., R. Rogerson and R. Wright, "An Equilibrium Model of the Business Cycle with Household Production and Fiscal Policy," *International Economic Review*, forthcoming.

Mulligan, C., "Intertemporal Substitution: What Does the Evidence Say?", mimeo, 1995

Pencavel, J., "Labor Supply of Men," in *Handbook of Labor Economics*, edited by O. Ashenfelter and R. Layard, Amsterdam, North-Holland, 1986.

Rupert, P., R. Rogerson, and R. Wright, "Using Panel Data to Estimate Substitution Elasticities in Household Production Models," *Economic Theory* 6 (1995), 179-193.

Wright, R. "The Observational Implications of Labor Contracts in a Dynamic General Equilibrium Model," *Journal of Labor Economics* 6 (1988), 530-551.

Appendix: Uncertainty and Incomplete Markets

Here we show how to incorporate uncertainty and incomplete markets. The maximization problem is:

$$\max E \sum_{t=1}^{T} \beta^t U(c_{mt}, c_{nt}, h_{mt}, h_{nt}, s_{1t}) + W_T(A_T)$$

$$\begin{aligned} s.t.\ c_{mt} + i_{nt} + A_t &\leq A_{t-1}(1+r) + w_t(s_{2t})h_{mt} \\ c_{nt} &\leq g_t(h_{nt}, k_{nt}, s_{3t}) \\ k_{nt+1} &= (1-\delta)k_{nt} + i_{nt} \\ H &\geq h_{mt} + h_{nt}, \end{aligned}$$

where s_{it}, $i = 1, 2, 3$, index stochastic shocks in period t to preferences, wages, and the home technology. To simplify notation, let $s_t = (s_{1t}, s_{2t}, s_{3t})$ and allow u, w, and g to all depend on s_t. The constraints hold for $t = 1, 2, \ldots T$ and all realizations of s. In particular, the individual faces a sequence of budget constraints and not a single lifetime budget equation (which is relevant to the extent that there is uncertainty as we do not include state contingent securities). The function $W_T(A_T)$ captures the terminal value of assets at the end of one's life.

We set this up as a dynamic programming problem. Let the state and control variables be $z_t = (A_t, k_{nt}, s_t)$ and $a_t = (h_{mt}, h_{nt}, A_{t+1}, k_{t+1})$. Bellman's equation is

$$V_t(z_t) = \max_{a_t}\{U[c_{mt}(z_t, a_t), g_t(h_{nt}, k_{nt}, s_t), h_{mt}, h_{nt}, s_t] + \beta E_t V_{t+1}(z_{t+1})\}$$

where $c_{mt}(z_t, a_t) = A_t(1+r) + w_t(s_t)h_{mt} + (1-\delta)k_{nt} - k_{nt+1} - A_{t+1}$. The first order conditions (for an interior solution) are:

$$U_1(t)w(t) + U_3(t) = 0 \tag{8}$$

$$U_2(t)g_1(t) + U_4(t) = 0 \tag{9}$$

$$U_1(t) = \beta E_s V_1(t+1) \tag{10}$$

$$U_1(t) = \beta E_s V_2(t+1). \tag{11}$$

Differentiating the value function implies:

$$V_1(t) = U_1(t)(1+r) \tag{12}$$

$$V_2(t) = U_1(t)(1-\delta) + U_2(t)g_2(t). \tag{13}$$

Define $\lambda_t = U_1(t)$. Then (8) implies

$$\log[-U_3(t)] = \log \lambda_t + \log w_t. \tag{14}$$

Unfortunately, λ_t is not constant in this model. However, we can proceed as follows. Equations (10) and (12) imply the Euler equation

$$\lambda_t = \beta(1+r)E_t\lambda_{t+1}. \tag{15}$$

Define $\epsilon_t = \log \lambda_t - E_{t-1} \log \lambda_t$. Since $\lambda_t = \exp(\log \lambda_t)$, it follows that

$$E_{t-1}\lambda_t = \exp(E_{t-1} \log \lambda_t)E_{t-1}(\exp \epsilon_t). \tag{16}$$

Rearranging, we have

$$\lambda_t = \frac{E_{t-1}\lambda_t}{E_{t-1} \exp \epsilon_t}\epsilon_t. \tag{17}$$

Now (15) and (17) yield

$$\lambda_t = \frac{1}{\beta(1+r)} \frac{\lambda_{t-1} \exp \epsilon_t}{E_{t-1}(\exp \epsilon_t)}, \tag{18}$$

which in turn gives

$$\log \lambda_t = -\log \beta(1+r) - \log E_{t-1}(\exp \epsilon_t) + \log \lambda_{t-1} + \epsilon_t \tag{19}$$

By definition of an expectation, ϵ_t must have mean zero conditioned on all information available as of $t-1$ and be uncorrelated with all ϵ_s for $s \leq t-1$. Iteration on equation (19) gives:

$$\log \lambda_t = \log \lambda_0 - t \log \beta(1+r) - \sum_{j=1}^{t} \log E_{j-1}(\exp \epsilon_j).$$

Inserting this into (14) yields

$$\log[-U_3(t)] = \log \lambda_0 - t \log \beta(1+r) - \sum_{j=1}^{t} \log E_{j-1}(\exp \epsilon_j) + \log w_t.$$

This is our regression equation. Recall, however, that we can use both married and single men, since the same estimating equation arrises out of the single-agent and family decision problems.

Table 1. Parameter Estimates for $v(h) = \phi h^\gamma$

1a. log(*hours*) on log(*wages*)								
	22-45				22-62			
	NHP		HP		NHP		HP	
	BTW	ATW	BTW	ATW	BTW	ATW	BTW	ATW
t	.001	.002	.001	.001	-.012	-.011	-.006	-.005
	(.004)	(.004)	(.002)	(.002)	(.003)	(.002)	(.002)	(.001)
$\log w$	.145	—	.299	—	.676	—	.673	—
	(.203)	—	(.125)	—	(.241)	—	(.131)	—
$\log w(1-\tau)$	—	.160	—	.316	—	.730	—	.710
	—	(.210)	—	(.129)	—	(.254)	—	(.139)
R^2	.115	.117	.444	.451	.330	.336	.422	.418

1b. log(*hours*) on smoothed log(*wages*)								
	22-45				22-62			
	NHP		HP		NHP		HP	
	BTW	ATW	BTW	ATW	BTW	ATW	BTW	ATW
t	-.001	-.000	.000	.001	-.013	-.012	-.006	-.005
	(.005)	(.001)	(.003)	(.003)	(.003)	(.003)	(.002)	(.002)
$\log w$	.286	—	.335	—	.875	—	.728	—
	(.306)	—	(.187)	—	(.332)	—	(.179)	—
$\log w(1-\tau)$	—	.323	—	.374	—	.972	—	.816
	—	(.342)	—	(.208)	—	(.369)	—	(.201)
R^2	.117	.092	.443	.445	.318	.320	.420	.408

1c. smoothed log(*hours*) on smoothed log(*wages*), first differences								
	22-45				22-62			
	NHP		HP		NHP		HP	
	BTW	ATW	BTW	ATW	BTW	ATW	BTW	ATW
Δt	-.001	-.001	-.000	.000	-.012	-.011	-.008	-.007
	(.001)	(.001)	(.000)	(.000)	(.000)	(.000)	(.001)	(.001)
$\Delta \log w$	.225	—	.319	—	.881	—	.713	—
	(.046)	—	(.012)	—	(.022)	—	(.061)	—
$\Delta \log w(1-\tau)$	—	.258	—	.360	—	.977	—	.807
	—	(.050)	—	(.011)	—	(.030)	—	(.063)
R^2	.536	.560	.972	.980	.976	.965	.779	.805

Table 2. Parameter Estimates for $v(h) = \phi(112 - h)^{\gamma}$

2a. log(*leisure*) on log(*wages*)								
	22-45				22-62			
	NHP		HP		NHP		HP	
	BTW	ATW	BTW	ATW	BTW	ATW	BTW	ATW
t	-.004	-.004	-.002	-.003	.005	.004	.003	.003
	(.004)	(.003)	(.004)	(.003)	(.001)	(.001)	(.001)	(.001)
$\log w$	-.114	—	-.164	—	-.331	—	-.441	—
	(.176)	—	(.191)	—	(.114)	—	(.123)	—
$\log w(1-\tau)$	—	-.150	—	-.177	—	-.461	—	-.816
	—	(.182)	—	(.198)	—	(.164)	—	(.130)
R^2	.232	.241	.189	.191	.267	.270	.258	.251

2b. log(*leisure*) on smoothed log(*wages*)								
	22-45				22-62			
	NHP		HP		NHP		HP	
	BTW	ATW	BTW	ATW	BTW	ATW	BTW	ATW
t	-.010	-.001	-.000	-.000	.006	.005	.004	.003
	(.005)	(.005)	(.005)	(.005)	(.002)	(.001)	(.002)	(.001)
$\log w$	.304	—	-.290	—	-.469	—	-.566	—
	(.296)	—	(.288)	—	(.159)	—	(.117)	—
$\log w(1-\tau)$	—	.341	—	-.325	—	-.523	—	-.633
	—	(.339)	—	(.322)	—	(.164)	—	(.192)
R^2	.027	.022	.172	.170	.239	.233	.237	.216

2c. smoothed log(*leisure*) on smoothed log(*wages*), first differences								
	22-45				22-62			
	NHP		HP		NHP		HP	
	BTW	ATW	BTW	ATW	BTW	ATW	BTW	ATW
Δt	-.010	-.010	-.000	-.001	.006	.005	.005	.004
	(.001)	(.001)	(.001)	(.000)	(.000)	(.000)	(.000)	(.000)
$\Delta \log w$	.266	—	-.267	—	-.465	—	-.559	—
	(.029)	—	(.017)	—	(.014)	—	(.029)	—
$\Delta \log w(1-\tau)$	—	.301	—	-.302	—	-.521	—	-.629
	—	(.031)	—	(.018)	—	(.030)	—	(.029)
R^2	.803	.821	.917	.930	.966	.976	.905	.923

Table 3. Parameter Estimates for $v(h) = \phi(168 - sleep - pc - h)^\gamma$

3a. log(*leisure*) on log(*wages*)

	22-45				22-62			
	NHP		HP		NHP		HP	
	BTW	ATW	BTW	ATW	BTW	ATW	BTW	ATW
t	-.006	-.005	-.006	-.006	.003	.003	.002	.001
	(.002)	(.002)	(.004)	(.003)	(.001)	(.001)	(.002)	(.002)
$\log w$	.069	—	-.224	—	-.259	—	-.611	—
	(.097)	—	(.081)	—	(.102)	—	(.145)	—
$\log w(1-\tau)$	—	.060	—	-.247	—	-.274	—	-.641
	—	(.101)	—	(.180)	—	(.108)	—	(.155)
R^2	.386	.382	.479	.485	.185	.184	.336	.330

3b. log(*leisure*) on smoothed log(*wages*)

	22-45				22-62			
	NHP		HP		NHP		HP	
	BTW	ATW	BTW	ATW	BTW	ATW	BTW	ATW
t	-.007	-.006	-.008	-.008	.004	.004	.003	.002
	(.003)	(.002)	(.005)	(.004)	(.001)	(.001)	(.002)	(.002)
$\log w$	.143	—	-.082	—	-.451	—	-.762	—
	(.146)	—	(.264)	—	(.145)	—	(.201)	—
$\log w(1-\tau)$	—	.154	—	-.096	—	-.505	—	-.856
	—	(.165)	—	(.293)	—	(.164)	—	(.228)
R^2	.369	.356	.463	.467	.108	.086	.317	.296

3c. smoothed log(*leisure*) on smoothed log(*wages*), first differences

	22-45				22-62			
	NHP		HP		NHP		HP	
	BTW	ATW	BTW	ATW	BTW	ATW	BTW	ATW
Δt	-.006	-.006	-.008	-.008	.005	.005	.006	.005
	(.002)	(.002)	(.001)	(.001)	(.000)	(.000)	(.001)	(.001)
$\Delta \log w$	.224	—	-.024	—	-.444	—	-.744	—
	(.060)	—	(.044)	—	(.027)	—	(.076)	—
$\Delta \log w(1-\tau)$	—	.244	—	-.032	—	-.501	—	-.845
	—	(.069)	—	(.049)	—	(.028)	—	(.063)
R^2	.396	.373	.013	.019	.873	.893	.713	.741

Table 4. Parameter Estimates for $v(h) = \phi \exp[(H-h)^\gamma]$

4a. *hours* on log(*wages*)								
	22-45				22-62			
	NHP		HP		NHP		HP	
	BTW	ATW	BTW	ATW	BTW	ATW	BTW	ATW
t	.143	.141	.099	.131	-.290	-.259	-.213	-.160
	(.140)	(.126)	(.134)	(.120)	(.070)	(.065)	(.063)	(.058)
$\log w$	2.69	—	14.7	—	17.6	—	29.0	—
	(6.86)	—	(6.58)	—	(6.12)	—	(5.48)	—
$\log w(1-\tau)$	—	3.61	—	15.8	—	18.8	—	30.5
	—	(7.09)	—	(6.76)	—	(6.47)	—	(5.82)
R^2	.180	.184	.490	.500	.314	.318	.427	.422

4b. *hours* on smoothed log(*wages*)								
	22-45				22-62			
	NHP		HP		NHP		HP	
	BTW	ATW	BTW	ATW	BTW	ATW	BTW	ATW
t	.185	.183	.119	.142	-.348	-.307	-.245	-.194
	(.185)	(.169)	(.177)	(.161)	(.081)	(.074)	(.072)	(.065)
$\log w$	.112	—	13.4	—	26.7	—	34.1	—
	(10.3)	—	(9.83)	—	(8.60)	—	(7.56)	—
$\log w(1-\tau)$	—	.281	—	15.0	—	29.7	—	38.2
	—	(11.4)	—	(10.8)	—	(9.62)	—	(8.55)
R^2	.175	.176	.489	.499	.275	.267	.414	.396

4c. smoothed *hours* on smoothed log(*wages*), first differences								
	22-45				22-62			
	NHP		HP		NHP		HP	
	BTW	ATW	BTW	ATW	BTW	ATW	BTW	ATW
Δt	.166	.158	.113	.134	-.351	-.305	-.319	-.263
	(.057)	(.055)	(.020)	(.017)	(.002)	(.001)	(.033)	(.029)
$\Delta \log w$	-2.58	—	12.4	—	26.6	—	33.6	—
	(2.00)	—	(.699)	—	(.092)	—	(1.98)	—
$\Delta \log w(1-\tau)$	—	-2.65	—	14.0	—	29.7	—	37.9
	—	(2.26)	—	(.713)	—	(.047)	—	(2.01)
R^2	.073	.071	.938	.949	.999	.999	.884	.903