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"Toward a Theory of International Currency: A Step Further"

by

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Toward a Theory of International Currency: A Step Further*

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Abstract

We generalize the two-country, two-currency, search-based monetary model of Matsuyama, Kiyotaki and Matsui to resolve two "shortcomings" in their approach. First, we endogenize prices and exchange rates. Second, we discuss some policy considerations. We use the model to address several questions related to the determination of the realms of circulation and relative values of different monies.

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1 Introduction

Matsuyama, Kiyotaki and Matsui (1993) develop a two-country, two-money, search-theoretic model of the world economy and use it to study various issues related to international currency.[1] Their framework provides an important first step toward a microfoundation for international monetary economics, and allows one to answer many questions that could not even be raised in the context of earlier models.[2] The authors themselves acknowledge, however, two "shortcomings" in their approach. First, in their model, as in all of the early search-based models of money, every exchange is assumed to be a one-for-one trade, "which makes it impossible to talk about the determination of prices and exchange rates" (p. 304). Second, national governments or monetary authorities are not explicitly incorporated, "which makes it impossible to discuss any policy issue" (p. 303).

The purpose of this paper is to remedy these two shortcomings. First, in order to determine nominal prices and exchange rates endogenously, we

[1]To be precise, an *international currency* is one that circulates abroad as a medium of exchange. For instance, it is not uncommon for the US dollar to be used for transactions in Latin American countries, including transactions between locals. The same was true of various coins at different times in Medieval Europe; e.g., Lopez (1951) and Cippolla (1956) document the international circulation at different times of the Byzantine nomisma, Moslem dinar, Fiorentine fiorino and Venetian ducato. The role of a money as an international currency is to be distinguished from its role as a *vehicle currency*, which means it is used as a medium of exchange in money-for-money interbank transactions; from its role as a *reserve currency*, which means it is held as a store of value by foreign central banks or private agents; and from its role as an *intervention currency*, which means foreign monetary authorities trade it to affect prices and exchange rates.

[2]Examples include the following: What features of a country make it likely that its currency will become an international currency? When can local currencies survive in the presence of a universally accepted international currency? Does an international currency emerge naturally as economies become integrated? What are the costs and benefits to a country of having its currency serve as an international medium of exchange? See Zhou (1996) for additional applications of their model.

adopt a version of the bargaining approach introduced into search models of money by Shi (1995) and Trejos and Wright (1995). Second, we incorporate governments in such a way that they face nontrivial policy options and seignorage opportunities, depending not only on their own actions but also on those of the other country. These extensions allow us to address a whole range of new issues, especially those concerning the purchasing powers of the different currencies.

The basic model can be described as follows. There are two countries, each of which issues its own currency. There is a bilateral matching technology that generates potential trading opportunities, some of which are between agents from the same country and some of which are between agents from different countries. When a seller and a buyer with a particular money meet they must decide whether or not to trade, and this determines the realms of circulation of the two currencies. If they decide to trade, the terms of trade are determined using bargaining theory, and this determines the values of the two currencies.

As in Matsuyama et al., we show that three distinct regimes (or types of equilibria) can exist, in which both monies circulate locally and either zero, one, or both of the monies circulate internationally. Sometimes multiple equilibria coexist. We characterize how the existence of each equilibrium depends on parameter values. For the most general version of the model, we display the possibilities numerically. Then we provide a set of simplifying assumptions that leads to a much more tractable special case in which we are able to derive analytic results about when the different equilibria exist and about their qualitative properties.

Some examples of these results are the following. Prices in a country increase with monetary expansion by that country in all the regimes, and also with monetary expansion by the other country in all regimes except the one where both currencies circulate only locally. Other things being equal, prices in a country are higher when foreign currency circulates at home and lower when it's currency circulates abroad. When one money circulates internationally and the other circulates locally the former has greater purchasing power. Also, an international currency typically commands higher purchasing power at home than abroad. The host country may benefit from using an international currency, since it helps to facilitate trade; but may also suffer since the use of an international currency increases prices denominated in the local currency. Seignorage in a country increases when its money is used abroad and decreases when foreign money circulates at home.

We then endogenize policy by assuming that governments are interested either in maximizing the welfare of their citizens or maximizing seignorage. If governments choose policies independently, the outcome is inefficient. For instance, if at least one money circulates internationally and governments are interested in maximizing seignorage, both can raise more revenue when they cooperate. We find that, other things being equal, if only one money circulates internationally then citizens from the country that issues it are unambiguously better off than their foreign counterparts. The model also displays the following feature: although the highest welfare can potentially be achieved in a regime where both currencies circulate internationally, this regime also generates the strongest incentives for governments to behave non-cooperatively. This suggests that deriving the benefits from a unified currency also requires a unified central bank.

Some of our results are reminiscent of discussions in the literature on the various aspects of currency substitution (see the survey by Giovannini and Turtelboom 1992). For instance, our model implies the possibility of multiple equilibria and of hysteresis in the determination of monetary regimes. Also, the model predicts international currencies tend to be issued by larger economies, as discussed by Swoboda (1969), and by Krugman (1980, 1984) in the context of vehicle currencies. Also, although our model does not easily accommodate inflation, we incorporate a tax on the holdings of currency as a proxy for inflation, and show that high tax rates on one currency reduce the likelihood that it circulates abroad and increase the likelihood that foreign currency circulates locally, as is consistent with much of the discussion on inflation and currency substitution.

Relative to most of the previous literature, however, there is a substantive difference in approach. The goal here, as in Matsuyama et al., is to study the endogenous determination of the media of exchange and their realms of circulation (and not to impose these things exogenously, as, for example, in the cash-in-advance approach). This is what distinguishes the present framework from most previous work on currency substitution. And, to reiterate, determining prices and exchange rates endogenously, as well as discussing certain policy issues, is what distinguishes this paper from Matsuyama et al.

2 The Model

Time is continuous and unbounded. There are two countries, labeled $i = 1$, 2. Each country starts with a continuum of agents, and both populations grow at the same rate $\gamma \geq 0$. The time-invariant fraction of individuals from country i is N_i, with $N_1 + N_2 = 1$. Once an agent is born, he lives forever. There are many goods, and the economy is specialized in the sense that each individual is only able to produce some goods and consume others. Without going into any more detail, we need only assume the following: for any two individuals picked at random, given that the first is able to produce something desired by the second, the probability that the second can also produce something desired by the first (Jevons' "double coincidence of wants") is given by y.[3]

If an agent produces q units of output for a consumer of that good, the latter enjoys utility $u(q)$ while the former suffers disutility $c(q)$. We assume $u'(q) > 0$, $c'(q) > 0$, $u''(q) < 0$ and $c''(q) \geq$ for all $q > 0$. Also, $u(0) = c(0) = 0$, $u'(0) = \infty$, $c'(0) = 0$, and there is a $\hat{q} > 0$ such that $u(\hat{q}) = c(\hat{q})$.

[3] See Kiyotaki and Wright (1989, 1991) for more detailed discussions of specialization and how it relates to the difficulty of barter versus monetary exchange.

There is no centralized market or auctioneer in this model. Rather, agents meet according to a bilateral matching process in which all trade is quid pro quo. The consumption goods are nonstorable so as to rule out commodity money. Also, although not essential for the results, we assume in this paper that $y = 0$ so that direct barter is, at best, a measure zero event. Hence, all exchange will necessarily involve some form of fiat money. However, we do not impose which particular money is used in which transactions, as in models with particular cash-in-advance constraints, because the goal here is to determine this endogenously.

The two fiat monies are introduced by the governments of the two countries, as follows: country j issues one unit of currency j to some fraction $M_j \in (0,1)$ of its newborn citizens, at each point in time, in exchange for consumption goods at the going price. Following most of the related literature, in the interest of tractability, we make the following assumptions regarding money. First we assume that an agent with a unit of currency always spends it all at once, so that no one ever holds less than one unit. Second we assume that an agent with a unit of currency does not acquire a second unit before spending what he has, so that no one ever holds more than one unit. Hence, at each point in time, the population can be partitioned into a group of agents called *buyers* each with one unit of money, and a disjoint group called *sellers* with zero units of money. The fraction of agents from country i with currency j is denoted m_{ij}, and the fraction of agents from country i with no money is $m_{i0} = 1 - m_{i1} - m_{i2}$.

The matching process is fully described by four Poisson parameters, α_{ij}, $i, j = 1, 2$, where α_{ij} is the rate at which an individual from country i meets a producer of his consumption good from country j, and also, by symmetry, the rate at which an individual from country i meets a consumer of his production good from country j. One can interpret α_{ii} as a measure of the size or level of potential economic activity of country i, and α_{ij} as the degree of economic integration between countries i and j. We assume that $\alpha_{ii} \geq \alpha_{ji}$, which means that, for example, it is at least as easy for a Mexican buyer to meet Mexican sellers as it is for an American buyer to meet Mexican sellers.[4]

If an individual with money meets a producer of his consumption good without money, the parties first choose simultaneously if they want to enter a bargaining process, or, alternatively, continue searching for other potential trading partners. Each party incurs a transactions cost $\varepsilon > 0$ from deciding to enter the bargaining process (regardless of the choice of the other agent). We focus here on the limiting case where $\varepsilon \to 0$; thus, agents enter the bargaining process if and only if they assign positive probability to the event that it will lead to an agreement which generates a positive surplus.

When a buyer from country j with money k meets a seller from country i, let $\sigma^j_{ik} = 1$ if the seller chooses to bargain and $\sigma^j_{ik} = 0$ otherwise, and let $\beta^j_{ik} = 1$ if the buyer chooses to bargain and $\beta^j_{ik} = 0$ otherwise. When both parties agree to bargain they enter a bargaining game along the lines of that in Rubinstein (1982). As is well known, as long as we concentrate on steady states, different versions of that bargaining game generate outcomes that can be represented as Nash bargaining solutions with different bargaining power and threat points. Here we will simply adopt the Nash representation of the bargaining outcome without explicit reference to the underlying game; the reader is referred to Coles and Wright (1995) for explicit details of the derivation in the context of a closely related model. Given a buyer from country j with money k and a seller from country i agree to trade, the bargaining solution will generate some amount of output q^j_{ik}, and hence the nominal price $p^j_{ik} = 1/q^j_{ik}$. Notice that in principle the price can depend on the nationalities of the two parties as well as the type of currency.

[4]Matsuyama et al. analyze the special case where $\alpha_{ii} = N_i$ and $\alpha_{ij} = AN_j$, $A \leq 1$. In any case, the α_{ij}'s must satisfy the identity $N_i\alpha_{ij} = N_j\alpha_{ji}$, although this will play no explicit role in our analysis.

3 Equilibrium

Define a money to be a national currency if it is only used in transactions where the buyer, seller and money all come form the same country (for instance, pesos are a national currency if they only change hands when both the buyer and the seller are Mexican). Define a money to be an international currency if it is used in every possible kind of transaction (for instance, dollars are an international currency if we observe both American and Mexican buyers using dollars to trade with both American and Mexican sellers). More formally, currency j is an international currency when $\beta^h_{kj}\sigma^h_{kj} = 1 \ \forall h, k$, and currency j is a national currency when $\beta^j_{jj}\sigma^j_{jj} = 1$ and $\beta^j_{ij}\sigma^j_{ij} = 0$ for $j \neq i$. In this paper we consider regimes, or types of equilibria, where both currencies circulate at home, and either none, one, or both circulate internationally.[5]

Let V_{ij} be the value function for a buyer from country i with currency j and V_{i0} the value function for a seller from country i. Let r be the common discount rate. Then the value functions satisfy the standard dynamic programming equations from search theory, which (in steady state) can be written:

$$
\begin{aligned}
rV_{ii} &= \alpha_{ii}m_{i0}\beta^i_{ii}\sigma^i_{ii}[u(q^i_{ii}) + V_{i0} - V_{ii}] + \alpha_{ij}m_{j0}\beta^i_{ji}\sigma^i_{ji}[u(q^i_{ji}) + V_{i0} - V_{ii}] \\
rV_{ij} &= \alpha_{ii}m_{i0}\beta^i_{ij}\sigma^i_{ij}[u(q^i_{ij}) + V_{i0} - V_{ij}] + \alpha_{ij}m_{j0}\beta^i_{jj}\sigma^i_{jj}[u(q^i_{jj}) + V_{i0} - V_{ij}] \qquad (1) \\
rV_{i0} &= \alpha_{ii}\sum_{k=1}^{2} m_{ik}\beta^i_{ik}\sigma^i_{ik}[-c(q^i_{ik}) + V_{ik} - V_{i0}]
\end{aligned}
$$

[5]In principle, other regimes – where at least one of the monies is neither national nor international in the sense defined above – are possible, including for example cases where one money does not circulate at all, or circulates abroad but not at home. We focus here only on these three regimes, which seem the most interesting, and are also the analogues of the three types of equilibria emphasized in Matsuyama et al.

$$+\alpha_{ij} \sum_{k=1}^{2} m_{jk} \beta_{ik}^{j} \sigma_{ik}^{j} [-c(q_{ik}^{j}) + V_{ik} - V_{i0}],$$

for $i = 1, 2$ and $j \neq i$.

The first equation in (1) sets the flow payoff for a buyer from country i holding money i, rV_{ii}, equal to the sum of two terms. The first term is the rate at which he meets sellers from his own country, $\alpha_{ii} m_{i0}$, times the probability both want to trade, $\beta_{ii}^{i} \sigma_{ii}^{i}$, times his surplus from trading, consuming and switching from buyer to seller, $u(q_{ii}^{i}) + V_{i0} - V_{ii}$. The second term is the rate at which he meets foreign sellers, times the probability both want to trade, times his surplus. The second equation similarly describes the payoff for a buyer from country i holding money j. The final equation describes the payoff of a seller from country i. The first term represents the expected payoff to selling to a local buyer with either local or foreign money, and the second term represents the expected payoff to selling to a foreign buyer with either local or foreign money.

When a seller from country h and a buyer from country i with money j both choose to bargain, the outcome is given by the generalized Nash bargaining solution,

$$q_{hj}^{i} = \arg\max_{q} \, [-c(q) + V_{hj} - V_{h0}]^{\theta} [u(q) + V_{i0} - V_{ij}]^{1-\theta} \qquad (2)$$

where θ is the bargaining power of the seller.[6] Of course, they will choose not to bargain, and therefore they will not trade, if there is no surplus to be had. Since the most the buyer could get before the seller would refuse to trade is the q that satisfies $c(q) = V_{hj} - V_{h0}$, they therefore both choose to bargain if and only if the utility of this q exceeds $V_{ij} - V_{i0}$; that is,

$$\beta_{hj}^{i} = \sigma_{hj}^{i} = 1 \Longleftrightarrow u[c^{-1}(V_{hj} - V_{h0})] > V_{ij} - V_{i0}. \qquad (3)$$

[6]Note that the threat points are given by the value functions V_{h0} and V_{ij}, which is appropriate under the assumption that agents continue to search while bargaining in the strategic game, as discussed in Coles and Wright (1995) in the context of a closely related model.

We now determine the steady state values of m_{ij}. First note that trades between a buyer and seller of the same nationality cannot alter m_{ij}, since the two agents simply switch states and leave the aggregate distribution unchanged. Therefore, the steady state equations are

$$\begin{aligned} \dot{m}_{ii} &= \alpha_{ij} m_{i0} m_{ji} \beta^{j}_{ii} \sigma^{j}_{ii} - \alpha_{ij} m_{ii} m_{j0} \beta^{i}_{ji} \sigma^{i}_{ji} + \gamma(M_i - m_{ii}) = 0 \\ \dot{m}_{ij} &= \alpha_{ij} m_{i0} m_{jj} \beta^{j}_{ij} \sigma^{j}_{ij} - \alpha_{ij} m_{ij} m_{j0} \beta^{i}_{jj} \sigma^{i}_{jj} - \gamma m_{ij} = 0 \end{aligned} \tag{4}$$

for $i, j = 1, 2$, $i \neq j$. Consider the first equation in (4). The first term says that m_{ii} increases when a seller from country i meets a buyer from country j with currency i and they trade. The second term says that m_{ii} decreases when a buyer from country i with currency i meets a seller from country j and they trade. The final term says that m_{ii} increases when the fraction of the newborn in country i who receive currency from their government, M_i, exceeds m_{ii}. The second equation has a similar interpretation.

The solutions to the steady state equations for each of the three types of equilibria considered here are given in the Appendix. From these solutions, one can show that m_{10} and m_{20} are decreasing in both M_1 and M_2 in all regimes, except the regime with two national monies, where m_{i0} is independent of M_j for $j \neq i$. Also, for given values of M_i, m_{10} is lower and m_{20} is higher in the regime where money 1 is national and money 2 is international than in the other two regimes. These results will be used below in discussing the characteristics of the various equilibria.

Before proceeding to the analysis, we define an equilibrium formally. Let $q = (q_{ij})$, $V = (V_{ij})$, $\beta = (\beta^{k}_{ij})$, $\sigma = (\sigma^{k}_{ij})$ and $m = (m_{ij})$. Since (3) implies we do not have to keep track of both β and σ, an equilibrium is defined as a list (q, V, β, m) satisfying (1)-(4). The method for analyzing equilibria is to choose a regime (that is, a value for β since this determines where the different monies circulate), solve for V, q and m using (1), (2) and (4), and then determine the set of parameters for which (3) is satisfied. Deriving analytical results regarding the existence of the different types of equilibria and their properties is difficult at best in this general version of the model. In the next section, therefore, we provide a set of simplifying assumptions that lead to a special case for which one can derive analytic results. However, it seems worthwhile to first present some results based on numerical calculations.

Let $u(q) = \sqrt{q}$ and $c(q) = q$, and take as benchmark parameters $\theta = 1/2$, $\gamma = 0.02$, $r = 0.05$, $M_1 = M_2 = 0.4$, $\alpha_{11} = 1$, $\alpha_{12} = 0.08$, $\alpha_{21} = 0.05$, $\alpha_{22} = 1.5$. These values of α correspond to country 1 being small and open relative to country 2. In the top panel of Figure 1 we vary (M_1, M_2) holding the other parameters constant and display the values for which each of the regimes is an equilibrium. In the bottom panel of the figure we do the same thing in $(\alpha_{11}, \alpha_{12})$ space. One salient finding is the possibility of multiple equilibria: for many parameter values, more than one regime is an equilibrium and, indeed, there are parameter values for which all regimes are equilibria.[7] To a large extent, money is a convention, and there can be alternative self-fulfilling expectations regarding what circulates where.

However, fundamentals are also important: it is not the case that all regimes are equilibria for all parameter values. For instance, if we want an equilibrium where a money is an international currency, the meeting rates for the two countries must not be too different. The intuition is that very different arrival rates imply very different price levels in the two countries, which means that agents in the low price country will not want to spend their money abroad. Also, if we want an equilibrium where money j is a national currency, we need α_{ii} to be high relative to α_{ij} and α_{jj}. The intuition is that, even if one expects foreign money to only be used by foreigners, a buyer with foreign money and a local seller will still trade if the latter has frequent enough opportunities to interact with foreigners (which happens when α_{ij}

[7] Not only can different regimes coexist as equilibria, there can also be multiple equilibria of a given type. In the simplified version to be studied in the next section, however, there is always a unique equilibrium of each type, although there can still coexist equilibria of different types.

is high relative to α_{ii}), or if foreign prices are sufficiently low relative to domestic prices (which happens when α_{jj} is high relative to α_{ii}). Other similar observations can be made from the figure.

4 Simplification

In this section we do several things that simplify the analysis considerably. First, we normalize $c(q) = q$. Second, we focus on the limiting case $\theta \to 0$, where sellers have no bargaining power and buyers effectively make take-it-or-leave-it offers.[8] This means that when a buyer from country h with money i meets a seller from country j, if the buyer wants to trade at all he demands $q_{ji}^h = V_{ji} - V_{j0}$ since this makes the seller indifferent between accepting and rejecting. Note that this quantity does not depend on the buyer's nationality, so that we can ignore the superscript on q_{ji} for now. Moreover, given $q_{ji} = V_{ji} - V_{j0}$, (1) implies $V_{i0} = 0$, and therefore

$$q_{ij} = V_{ij}. \tag{5}$$

The other thing we do in this section is to assume that the nationality of the buyer cannot be observed by the seller. This means, in particular, that the seller must make the same decision whenever he encounters any buyer with a given money, and therefore $\sigma_{ik}^1 = \sigma_{ik}^2 \equiv \sigma_{ik}$. The equilibrium condition for σ is no longer the same as the condition (3) that determines β, but is replaced by

$$\sigma_{ik} = 1 \Leftrightarrow \beta_{ik}^1 m_{1k} + \beta_{ik}^2 m_{2k} > 0. \tag{6}$$

This says that a seller is willing to bargain if and only if he assigns positive probability to the event that the buyer will want to do so as well.[9]

[8] To be precise, we will look at the limiting case as θ and ε both go to zero but ε goes to zero at a faster rate; thus, even though the net payoff for the seller is negligible, it can still be a best response on his part to enter the bargaining process.

[9] If $\beta_{ik}^1 m_{1k} + \beta_{ik}^2 m_{2k} = 0$ then there are no buyers with money k who want to bargain

The case of $\theta \to 0$ simplifies things considerably by eliminating the dependence of q_{ij} on the nationality of the buyer, guaranteeing $V_{i0} = 0$, and identifying the other value functions with the quantities, as shown by (5), thereby reducing the number of variables we need to determine. Inserting (5) into (1), the dynamic programming equations reduce to

$$
\begin{aligned}
rq_{ii} &= \alpha_{ii} m_{i0} \beta^i_{ii} \sigma^i_{ii} [u(q_{ii}) - q_{ii}] + \alpha_{ij} m_{j0} \beta^i_{ji} \sigma^i_{ji} [u(q_{ji}) - q_{ii}] \\
rq_{ij} &= \alpha_{ii} m_{i0} \beta^i_{ij} \sigma^i_{ij} [u(q_{ij}) - q_{ij}] + \alpha_{ij} m_{j0} \beta^i_{jj} \sigma^i_{jj} [u(q_{jj}) - q_{ij}].
\end{aligned}
\tag{7}
$$

Moreover, the conditions determining whether a country h buyer with money j wants to bargain with a country i seller reduce to

$$
\beta^i_{hj} = 1 \Leftrightarrow u(q_{hj}) > q_{ij}. \tag{8}
$$

Why do we impose the assumption regarding the inability of sellers to recognize a buyer's nationality? Without it, in this simplified version there does not exist an equilibrium where currency i is not used to buy from a seller from country $j \neq i$. That is, under our simplifying assumptions the regimes with zero or one international money cannot be equilibria, unless one makes this extra assumption.

To see why, consider for example the regime where both monies are national (a related argument applies to the regime where one money is national). This regime is defined by $\beta^1_{11} = \beta^2_{22} = 1$ and $\beta^1_{21} = \beta^2_{12} = 0$. By virtue of (8), for this to be an equilibrium we require $u(q_{11}) \geq q_{11} \geq u(q_{21})$ (so that country 1 buyers spend money 1 on country 1 sellers but not country 2 sellers), and $u(q_{22}) \geq q_{22} \geq u(q_{12})$ (so that country 2 buyers spend money 2 on country 2 sellers but not country 1 sellers). These conditions imply, among other things, that

$$
q_{11} \geq q_{21} \text{ and } q_{22} \geq q_{12}. \tag{9}
$$

with sellers from country i. Hence, the seller's best response is to set $\sigma_{ik} = 0$, since there is no reason to incur the transaction cost ε if he believes that the other party will not enter the bargaining process.

Note that we at this point can say nothing about β^1_{12} and β^2_{21} – i.e., about what happens when a local buyer with foreign money meets a local seller (which is off the equilibrium path, since local agents never acquire foreign currency in this regime, but we still have to say what happens in such an event).

In the regime in question, the dynamic programming equations in (7) for citizens of country 1 become

$$\begin{aligned} rq_{11} &= \alpha_{11}m_{10}[u(q_{11}) - q_{11}] \\ rq_{12} &= \alpha_{11}m_{10}\beta^1_{12}[u(q_{12}) - q_{12}] + \alpha_{12}m_{20}[u(q_{22}) - q_{12}]. \end{aligned} \tag{10}$$

The first equation in (10) can be solved uniquely for $q_{11} > 0$, and it implies $q_{11} < \hat{q}$, where $\hat{q} = u(\hat{q})$. Now (9) implies $q_{21} < \hat{q}$, which means $u(q_{12}) > q_{12}$ and hence $\beta^1_{12} = 1$; that is, if a citizen of country 1 ever found himself with money 2 in a meeting with a fellow country 1 seller, they would trade. But then the second equation in (10) implies $q_{12} > q_{11}$. Now (9) implies $q_{22} > q_{11}$. However, by symmetry, the same argument will imply $q_{11} > q_{22}$. This is a contradiction.

The intuition for the above result is as follows. Suppose, for example, that we want dollars to be an exclusively national currency (i.e., to not circulate, in Mexico). This must be because Americans expect that Mexicans do not value dollars very much, and so an American with a dollar prefers waiting to meet a fellow American rather than trading with a Mexican. Why would Mexicans not value dollars very much? It must be because they believe that other Mexicans do not value dollars very much, and so they themselves, in the event they have dollars, must wait until they meet an American to be able to trade dollars at an acceptable rate.

But such a belief cannot be an equilibrium belief. As we showed above, when a Mexican with a dollar meets a Mexican seller he will trade (i.e., we showed $\beta^1_{12} = 1$). Indeed, the less valuable a dollar is to a Mexican buyer the more willing he is to trade, and our simplifying assumptions imply that the seller is always willing to trade since his equilibrium payoff is $V_{10} = 0$ in any event. Thus, even if q_{12} is very low – or, more precisely, *especially* if q_{12} is very low – Mexicans with dollars always trade with Mexican sellers. This would not necessarily be so if sellers had some bargaining power, in which case $V_{10} > 0$, and they might not want to give up their status as seller in exchange for dollars.[10]

Assuming that the nationality of a buyer cannot be observed by a seller, a seller and buyer will not always make the same decision about whether they want to enter the bargaining process, as the latter has some information about which the former can only make an inference. Thus, for example, in a regime where Mexicans never carry dollars, sellers who meet buyers with dollars must infer that they are American. It is quite possible that Americans with dollars do not want to buy from Mexican sellers, even though as we argued above Mexicans with dollars will always trade with Mexican sellers. Now the Mexican seller who sees a buyer with a dollar and infers that he is American will not even negotiate, because he rationally believes that the buyer is American and hence there is zero probably of trade. Since $\sigma_{ij} = 1$ only if the seller assigns positive probability to trade, Mexican sellers will never acquire dollars in equilibrium, and dollars can potentially end up an exclusively national currency.

[10]As an alternative to $\theta > 0$, we can also assume $y > 0$ (a positive probability of direct barter), which also implies $V_{i0} > 0$. Alternatively, we can assume a fixed cost, $c(q) > \delta > 0$ for all $q > 0$, in which case it might not be worth producing a very small q_{12} in exchange for dollars. It is the combined absence of the sellers' bargaining power, barter, and fixed cost that makes it impossible to have an equilibrium where currency j circulates only locally. Given that we want to also study equilibria of this type, it turns out to be easier to introduce private information about the buyer's nationality than to relax any of these other assumptions.

Inserting $\sigma_{ik}^j \equiv \sigma_{ik}$ into (7), we have

$$\begin{aligned} rq_{ii} &= \alpha_{ii} m_{i0} \beta_{ii}^i \sigma_{ii} [u(q_{ii}) - q_{ii}] + \alpha_{ij} m_{j0} \beta_{ji}^i \sigma_{ji} [u(q_{ji}) - q_{ii}] \\ rq_{ij} &= \alpha_{ii} m_{i0} \beta_{ij}^i \sigma_{ij} [u(q_{ij}) - q_{ij}] + \alpha_{ij} m_{j0} \beta_{jj}^i \sigma_{jj} [u(q_{jj}) - q_{ij}]. \end{aligned} \tag{11}$$

An equilibrium for the modified version of the model can now be defined as a list (q, β, σ, m) satisfying (11), (8), (6) and (4). In the rest of this section, we consider in turn each of the three types of equilibria and discuss the conditions under which they exist, as well as some of their properties.

4.1 Two National Monies

Consider first the regime with no international money, which means $\sigma_{12} = \sigma_{21} = 0$, and $\sigma_{11} = \sigma_{22} = \beta_{11}^1 = \beta_{22}^2 = 1$.[11] For this regime to be an equilibrium, the conditions determining whether buyers want to bargain (8) require

$$u(q_{ii}) \geq q_{ii} \geq u(q_{ji}), \tag{12}$$

for $i = 1, 2$, $i \neq j$, which says that country i buyers with money i spend it on country i sellers but not on country j sellers.

In the regime in question, (11) becomes

$$\begin{aligned} rq_{ii} &= \alpha_{ii} m_{i0} [u(q_{ii}) - q_{ii}] \\ rq_{ij} &= \alpha_{ij} m_{i0} [u(q_{jj}) - q_{ij}] \end{aligned} \tag{13}$$

which imply unique values for q_{ii} and q_{ij}, for $i = 1, 2$, $j \neq i$. Given this, the first inequality in (12), which tells us that currencies circulate in their own country, can easily be seen to hold for all parameter values. The second

[11] The other values of β_{jk}^i are not pinned down in this equilibrium. Thus, for example, whether or not an American with pesos wants to trade with an American seller does not matter as long as the seller will not trade with him.

inequality, which tells us that currencies do not circulate abroad, can be rewritten

$$u\left[\frac{\alpha_{ji}}{\alpha_{ii}}\frac{r+\alpha_{ii}(1-M_i)}{r+\alpha_{ji}(1-M_i)}q_{ii}\right]\leq q_{ii}$$

using the steady state result that $m_{i0}=1-M_i$ in this regime. This condition is satisfied if and only if α_{ji} is small compared to α_{ii}. Intuitively, under the belief that foreign money does not circulate at home, foreigners will hold on to their money rather spend it on locals if and only if locals interact sufficiently infrequently with foreigners.

As long as this equilibrium exists it is easy to show that q_{ii} and q_{ji} are both decreasing in M_i, and independent of M_j for $j \neq i$. Also, $q_{11} > q_{22}$ if and only if $\alpha_{11}(1-M_1) > \alpha_{22}(1-M_2)$. Since one unit of currency i buys q_{ii} units of real output, we can say that 1 unit of currency 1 is worth $e = q_{11}/q_{22} = p_{22}/p_{11}$ units of currency 2; this is the exchange rate that would be implied by purchasing power parity.[12] One can show that e falls with M_1 and rises with M_2. Also, e rises with α_{11} and falls with α_{22}. Other real factors that would also affect the exchange rate include international differences in utility and production functions, but we have kept these the same across countries for notational simplicity.

4.2 Two International Monies

Now turn to the regime with two international currencies. Here the inequalities determining whether trade takes place are given by $u(q_{ij}) \geq q_{kj}\ \forall i,j,k$. We claim in this regime that $q_{ii} = q_{ij} = Q_i$; that is, the two monies are *perfect substitutes* in the sense that they purchase the same amount from a

[12]Suppose one peso buys q_{11} units of output in Mexico and one dollar buys q_{22} units of output in America, and suppose that there is a market in which currencies can be traded at the nominal exchange rate e (i.e., one peso buys e dollars). Then one peso could be used to buy eq_{22} units of output in America using dollars. Purchasing power parity holds if a peso buys the same amount directly at home and using dollars abroad: $q_{11} = eq_{22}$.

given seller. To see this, first note in this regime (11) implies the subsystem

$$\begin{aligned} rq_{11} &= \alpha_{11}m_{10}[u(q_{11}) - q_{11}] + \alpha_{12}m_{20}[u(q_{21}) - q_{11}] \\ rq_{21} &= \alpha_{21}m_{10}[u(q_{11}) - q_{21}] + \alpha_{22}m_{20}[u(q_{21}) - q_{21}], \end{aligned} \tag{14}$$

which can be solved for q_{11} and q_{21} independently of q_{12} and q_{22}. It is shown in the Appendix that (14) has a unique solution $(q_{11}, q_{21}) = (Q_1, Q_2) \in (0, \hat{q})^2$. Similarly, (11) also implies a subsystem that can be solved for q_{12} and q_{22} independent of q_{11} and q_{21}. Inspection reveals that this subsystem is identical to (14), and so $(q_{12}, q_{22}) = (Q_1, Q_2)$ as well.

To see when this regime constitutes an equilibrium, we need to check when the conditions $u(Q_1) \geq Q_2$ and $u(Q_2) \geq Q_1$ hold, since the other conditions can be shown to hold for all parameter values. If the countries are symmetric, in the sense that $M_1 = M_2$, $\alpha_{ii} = \alpha_{jj}$ and $\alpha_{ij} = \alpha_{ji}$, it follows that $Q_1 = Q_2$, and then the relevant conditions hold with strict inequalities. By continuity, these conditions hold when the parameters are near symmetric. But, if the two countries are very different, the relevant conditions will not hold. Thus, an equilibrium with two international currencies exists if and only if the two countries are sufficiently similar.

Since $q_{ij} = Q_i$ does not depend on which currency the buyer is using, if there were a market in which agents can trade the two monies, the market clearing price would have to be 1. But the purchasing power parity exchange rate, as defined above, is given by $e = Q_1/Q_2$. In general e differs from unity in this regime, and so purchasing power parity does not hold, because, even though the terms of trade do not depend on which currency the buyer is using, they do depend on the nationality of the seller. In the Appendix we show that Q_1 and Q_2 are both decreasing in M_1 and M_2. Thus, an increase in either money supply increases the price level in both countries. One can also show that, as long as the two countries are not too different in terms of α_{ij} and M_j, the effect of a change in M_1 is stronger in country 1 than in country 2 and vice-versa. Thus, e is decreasing in M_1 and increasing in M_2.

In the symmetric case with α_{ij} small, this equilibrium coexists with the equilibrium that has two national currencies. And, if M_1 and M_2 are similar, this regime Pareto dominates because having internationally accepted money facilitates international trade. However, if M_1 is much larger than M_2, then country 2 can be better off in the regime with two national currencies. We will discuss this in more detail in the section on policy.

4.3 One National and One International Money

We now turn to the regime where citizens of country 1 accept money 2 but not vice-versa. Thus, we need to check $u(q_{12}) \geq \max\{q_{12}, q_{22}\}$ (so that all buyers spend money 2 on country 1 sellers); $u(q_{22}) \geq \max\{q_{12}, q_{22}\}$ (so that all buyers spend money 2 on country 2 sellers); $u(q_{11}) \geq q_{11}$(so that country 1 buyers spend money 1 on country 1 sellers); and $u(q_{21}) < q_{11}$ (so that country 1 buyers do not spend money 1 on country 2 sellers).

In this regime, the equations in (11) can be written[13]

$$\begin{aligned} rq_{11} &= \alpha_{11} m_{10}[u(q_{11}) - q_{11}] \\ rq_{21} &= \alpha_{21} m_{10}[u(q_{11}) - q_{21}] \\ rq_{12} &= \alpha_{11} m_{10}[u(q_{12}) - q_{12}] + \alpha_{12} m_{20}[u(q_{22}) - q_{12}] \\ rq_{22} &= \alpha_{21} m_{10}[u(q_{12}) - q_{22}] + \alpha_{22} m_{20}[u(q_{22}) - q_{22}]. \end{aligned} \tag{15}$$

The first and second equations in (15) determine the values of the national currency, q_{11} and q_{21}, independent of q_{12} and q_{22}. In fact, these two equations are qualitatively identical to the equations in (13) for q_{ii} and q_{ij} in the regime with two national monies. However, since m_{10} is lower in this regime, q_{11} and q_{21} are lower when money 1 is national and money 2 is international than when both monies are national. Intuitively, the influx of foreign money inflates prices denominated in the domestic currency. Also, q_{11} and q_{21} are decreasing in both M_1 and M_2, because m_{10} is.

[13]In writing (15) we use $u(q_{11}) \geq q_{21}$, which can be shown to hold always.

Now the third and fourth equations in (15) determine the purchasing power of the international currency, q_{12} and q_{22}, independent of q_{11} and q_{21}. In fact, these two equations are qualitatively identical to the equations in (14) from the regime with two international monies, and the same analysis implies that there exists a unique solution and both q_{12} and q_{22} are decreasing in M_1 and M_2. Moreover, $q_{12} > q_{11}$. Hence, citizens of country 1 value the international currency more than their own domestic currency. In general, q_{12} can be greater or less than q_{22}, depending on M_i and α_{ij}; in the symmetric case, however, one can show unambiguously that $q_{22} > q_{12}$. Hence, an international currency tends to have higher purchasing power at home than abroad.

We still need to check when this equilibrium exists. It turns out that the binding conditions that need to be verified are $u(q_{22}) \geq V_{12}$, $u(q_{12}) \geq V_{22}$ and $u(q_{21}) < V_{11}$ (the other conditions hold always). Each of these conditions looks qualitatively like one that has been encountered earlier and holds under similar circumstances: we need country 2 to be isolated enough for money 1 to be national, and the two countries similar enough for money 2 to be international.

4.4 Summary

We summarize what we know analytically in the simplified version of the model. First, under the assumptions in this section, there is at most one equilibrium of each type (i.e., each regime implies a unique m and q), although different types of equilibria may well coexist. Other things being equal, the value of a money is higher if it circulates internationally, and lower if the other money circulates internationally. Other properties of the different regimes include:

- two national monies (dollars circulate only in America, pesos circulate only in Mexico) $\Longrightarrow$

1. $q_{11} > q_{22}$ iff $\alpha_{11}(1 - M_1) > \alpha_{22}(1 - M_2)$;
2. q_{ii} decreasing in M_i and independent of M_j;
3. this is an equilibrium iff the two countries are relatively isolated (α_{ij} is small relative to α_{ii}).

- two international monies (dollars and pesos both circulate in both countries) $\Longrightarrow$

  1. $q_{i1} = q_{i2} = Q_i$ (dollars and pesos are perfect substitutes);
  2. Q_i is decreasing in M_i and M_j, and $e = Q_1/Q_2$ is decreasing in M_1/M_2 if the two countries are not too dissimilar;
  3. this is an equilibrium iff the two countries are not too dissimilar.

- one national money, one international money (dollars circulate in both countries, pesos circulate only in Mexico) $\Longrightarrow$

  1. $q_{12} > q_{11}$ (Mexicans value dollars more than Mexicans value pesos), and $q_{22} > q_{12}$ (Americans value dollars more than Mexicans value dollars) if the two countries are not too dissimilar;
  2. q_{ij} decreasing in M_i and M_j for all i, j;
  3. this is an equilibrium iff one country is relatively isolated and the two countries are not too dissimilar..

5 Policy

In this section we consider several aspects of international monetary policy. First we introduce a tax on currency holdings as a proxy for inflation. Then, we analyze the effects of exogenous changes in (M_1, M_2). Finally, we endogenize (M_1, M_2) by modeling the objective functions for the two governments and the rules for their strategic interaction.

5.1 An Inflation-Like Tax

It has been observed in many countries that the use of foreign money can be triggered by an episode of high inflation in local money. It is not so easy to introduce inflation into the model, so in lieu of this we introduce a proxy for inflation by taxing currency holdings, as in the work of Li (1994, 1995). We then consider the equilibrium with two national monies, and ask if differences in our inflation proxy can preclude the existence of such an equilibrium, by leading locals either to embrace the foreign currency or to abandon the national currency. In this subsection we set $\theta = 1/2$, and $c(q) = q$.

Assume that with arrival rate π_i any buyer with money i meets a government agent who taxes away his money, so that the buyer has to become a seller without consuming. Also, with arrival rate π_i^*, a seller from country i meets a government agent who issues money i in exchange for output at the going price. If we assume $\pi_i^* = \pi_i M_i/(1 - M_i)$, then the per capita supply of money i does not change over time. One can think of π_i as a proxy for the inflation rate, in the sense that it is a tax on holding money i, and (unlike a tax on expenditures) the extent of the tax increases with the length of time that the money is held.

In the regime with two national monies, the value functions can now be written

$$\begin{aligned} rV_{ii} &= \alpha_{ii}(1 - M_i)[u(q_{ii}^i) + V_{i0} - V_{ii}] + \pi_i(V_{i0} - V_{ii}) \\ rV_{ij} &= \alpha_{ij}(1 - M_j)[u(q_{jj}^i) + V_{i0} - V_{ij}] + \pi_j(V_{i0} - V_{ij}) \\ rV_{i0} &= \alpha_{ii}M_i[V_{ii} - V_{i0} - q_{ii}^i] + \pi_i^*(V_{ii} - V_{i0}). \end{aligned} \tag{16}$$

One can solve things sequentially. First, use (16) to derive the V_{ii}, V_{i0}, and q_{ii}^i for each country i. Then, use those values to derive how much could one buy from a foreign seller with his money, that is, the values q_{jj}^i, $i \neq j$. Then solve, again using (16), for V_{ij}, $i \neq j$, and then for q_{ji}^i. Then check the existence conditions for this equilibrium: $u(q_{ii}^i) \geq V_{ii} - V_{i0} \geq q_{ii}^i$ (each money circulates in its own country); and there is no q^i_{ji} such that $u(q^i_{ji}) \geq V_{ii} - V_{i0}$ and $V_{ji} - V_{j0} \geq q^i_{ji}$, $i \neq j$ (holders of local currency do not spend it abroad).

We have analyzed the effects numerically and, in every case we tried, the results confirm the following basic intuition. First, a high π_1, which is a proxy for high Mexican inflation, always reduces the pesos' purchasing power, q^1_{11}. Also, it reduces the value of acquiring pesos for Americans, $V_{21} - V_{20}$, and increases the value of acquiring dollars for Mexicans, $V_{12} - V_{10}$. This means that a higher π_1 increases the potential gains from trade between an American buyer and a Mexican seller, and thus makes it more likely that dollars begin to circulate in Mexico. In fact, at a high enough value of π_1, not only do dollars start circulating but pesos stop circulating in Mexico. Also, when π_1 is high it is less likely that pesos circulate in America. A final result is that, given $\pi_1 = \pi_2$, this regime exists when the common inflation rate is low but not when it is high: it is not only the difference in inflation rates but also their absolute level what matters.

To the extent to which π_j is a proxy for inflation, the model therefore can be used to articulate the observed relationships between inflation rates and the degree of "dollarization" or currency substitution. It can be also used to account for the hysteresis apparently observed in the determination of monetary regimes: a high inflation may instigate a transition away from the regime with two national currencies, but maintaining that high inflation is not necessary to keep an economy in the new "dollarized" regime once it has emerged. Thus, continuing peso inflation is not necessary to maintain the foreign circulation of dollars, because the very belief that dollars circulate at home has an important effect on whether they will.

5.2 Changes in $\mathbf{M}_j$

Here we consider changes in M_1 and M_2. Analyzing numerical examples, we discovered several results which we could not shown analytically. For example, prices are more sensitive to local than foreign currency changes;

this makes sense, intuitively, because governments introduce new money by spending it on their own citizens, and this implies that in every regime m is skewed toward country i agents holding more of currency i. Also, when money 1 circulates internationally its purchasing power is less responsive to M_1; intuitively, this is because country 1 is able to export high prices abroad when its currency circulates abroad.

We also studied numerically the purchasing power parity exchange rate, $e = q_{11}/q_{22}$, in each of the regimes, as a function of M_1. We found that, even though both currencies lose value as M_1 increases, currency 1 loses value faster so that e decreases with M_1. This effect is particularly strong in the regime with two national currencies, where country 1 is not able to export high prices because its currency is not accepted in country 2, and particularly weak in regime with two international ones, where it is easiest to export high prices abroad.

We are also interested in the way that seignorage depends on M_1 and M_2. Since government i is assumed to purchase goods from a fraction M_i of its newborn citizens, per capita seignorage revenue in real terms is given by $S_i = \gamma M_i q_{ii}$. One can show that S_1 first increases and then decreases with M_1, and always decreases with M_2. Also, given that multiple equilibria exist, S_1 is greater when currency 1 is international than when it is not, and lower when currency 2 is international than when it is not. Finally, given M_2, S_1 is maximized at different levels of M_1 in the different regimes, depending on the extent to which country 1 is able to export high prices abroad, with the seignorage-maximizing level of M_1 being higher when currency 1 is international and lower when currency 2 is international.

We are also interested in welfare in each country i, which we define here as the average (or steady state) utility of its citizens:

$$W_i = m_{i0}V_{i0} + m_{i1}V_{i1} + m_{i2}V_{i2}.$$

For given values of M_1 and M_2, it is ambiguous which regime is best. For instance, there is a tendency for welfare to be higher in country i in the regimes where money i circulates internationally, as this makes money i more liquid and more valuable. At the same time, if M_i is too low, this tendency is dominated by the fact that money is already too scarce in country i, which makes local trade very difficult. In this case W_i is highest in the regime where currency i is national and the other currency is international.[14]

There is also a tendency for international circulation of either currency to be good for both because it facilitates extracting gains from international trade. However, this is not necessarily so when the two countries are very different. For instance, two international monies may or may not dominate two national monies; if M_2 is very low or very high then W_1 is highest in the regime with no international money. Intuitively, if Mexico is running a particularly bad monetary policy then the US will be better off under complete isolation.

5.3 Endogenous Policy

The next step is to endogenize (M_1, M_2). Throughout this subsection we assume that governments take the regime as given, and restrict their choices to policies that allow for the existence of that regime as an equilibrium (i.e., we ignore policies aimed at changing a currency's realm of circulation). We also restrict attention to policies that do not change over time, and to steady state comparisons.

One thing we do is look for Nash equilibria when each government seeks to maximize the welfare of its own citizens; we denote this policy pair by (M_1^W, M_2^W). We also look for Nash equilibria when each government seeks to maximize seignorage; we denote this by (M_1^S, M_2^S). Given that they seek to maximize seignorage, we also consider the possibility of international policy coordination by letting the governments choose (M_1, M_2) jointly. One way

[14]This result is more than a theoretical curiosity. Historically, countries have got into currency shortages when their money began to circulate abroad, which led to attempts to make currency export illegal (see, e.g., Wallace and Zhou 1996).

to do this is to assume that seignorage is freely transferable across countries, in which case they maximize $S_1 + S_2$; we denote this outcome by (M_1^T, M_2^T). Or, we can assume seignorage is nontransferable, in which case we use Nash's cooperative bargaining solution with threat points given by the noncooperative solution; we denote this by (M_1^N, M_2^N). Hence, (M_1^N, M_2^N) solves $\max[S_1 - S_1^S][S_2 - S_2^S]$, where S_j^S is seignorage in country j when policy is given by (M_1^S, M_2^S).

One result is that if foreign money circulates in country i, independently of whether money i circulates abroad, then the welfare maximizing level of M_i is lower than the level that maximizes seignorage. Another result is that, starting from the Nash equilibrium where governments maximize seignorage, reducing the amount of money in both countries increases welfare in both countries. Furthermore, and more interestingly, reducing the amount of money in both countries increases seignorage for both governments. As neither government takes into account the effect it has on the other, the noncooperative equilibrium is characterized by too much money. The cooperative equilibrium (M_1^N, M_2^N) involves less money and more seignorage.

For the simplified model in Section 4, the above results hold in general. We now describe some additional results from numerical experiments. First, while the noncooperative solutions are always inside the frontier in (S_1, S_2) space, this is especially so in regime with two international monies, where it is easiest to shift abroad the pressure towards higher prices. In other words, although this regime entails the highest potential welfare, this regime also generates the strongest incentives for governments to behave non-cooperatively. Also, with one international money, when transfers are allowed the cooperative solution is to concentrate seignorage collection in the country that issues it. That is, if they were to cooperate the governments would print more of the international currency and less of the national currency, in addition to printing less money in total.

We also found that when both governments try to maximize seignorage, one of them may actually end up with less seignorage than when both are trying to maximize welfare. And, symmetrically, citizens in one country may be worse off in terms of welfare when both governments are trying to maximize welfare than when both governments are concerned with seigniorage. Finally, we found that in the regime with one international money, the country that issues it is better off. Recall that this was not necessarily true when (M_1, M_2) was exogenous, because in that case a currency shortage may arise at home once a money becomes international. A currency shortage cannot happen when (M_1, M_2) is endogenous because the government in question would, assuming it is feasible, simply print more money.

6 Conclusion

This paper has developed an extended version of the international currency model in Matsuyama, Kiyotaki and Matsui (1993). The key extensions are to endogenize prices and exchange rates, which the earlier model took to be fixed, and also to introduce some policy considerations. A few of the findings are the following: other things being equal, an international currency has a higher purchasing power than a national one; an international money purchases more at home than abroad; and monetary expansion in one country tends to produces upward pressure on prices in both countries. We also found a tendency for governments to print too much money, especially in the regime where both currencies circulate internationally. There is, then, a role for cooperation. For example, if they were to cooperate to maximize seignorage they would issue more of an international currency and less of a national one. We also found that a country could potentially be worse off when its currency begins to circulate abroad if the money stock is exogenous, but not if it is endogenous.

Although the model is obviously very abstract and contains many extreme assumptions, we think that is also has some virtues relative to other models. Namely, search-based models have explicit descriptions of frictions that lead to explicit, endogenous, determination of the realms of circulation of the different monies. Although endogenizing the use and choice of media of exchange would seem to be a virtue in any monetary theory, it may be particularly important to analyze models with explicit frictions in an international context because such frictions generate departures from purchasing power parity in a natural way. In particular, the fact that the value of money is determined here in part by its endogenous liquidity allows us to potentially discuss new interpretations of some interesting empirical relationships.

One example is the fact that the US dollar has higher purchasing power domestically than predicted by cross-country data.[15] The explanation suggested by this paper is that the liquidity advantage of international circulation raises its value beyond what would be predicted by fundamentals, and therefore an international currency should buy more at home than it would in an otherwise identical foreign country. There are potentially many other empirical or policy relevant issues that could profitably be analyzed in the context of this or related models, but we leave further investigation of these applied topics to future research. The main goal here was to show how one can determine prices and exchange rates endogenously in the two-country, two-currency, search-based monetary model.

[15] Much empirical work has documented significant deviations from purchasing power parity; alternative theories by Balassa (1964), Samuelson (1964), Kravis and Lipsey (1983) and Bhagwati (1984) predict that those deviations would be related to national incomes and capital/labor ratios. Cross-country data confirms the existence of this relationship, but the US dollar appears as a significant outlier (see Rogoff [1996] for an extended discussion of the issues). For instance, if one regresses PPP deflators on national incomes and capital/labor ratios using post-1975 cross-country data, one finds the predicted relationship fits well, but each year overestimates the US dollar deflator by about 25% (significant at the 99% confidence level).

7 Appendix

7.1 Steady States

Here we solve the steady state equations for each of the regimes considered in the paper. In the regime with two national and no international monies, $m_{ii} = M_i$ and $m_{ij} = 0$, $i = 1, 2$, $j \neq i$, since only agents of country i hold money i. In the regime with two international monies, routine algebra yields

$$m_{10} = 1 - \frac{(\gamma + \alpha_{21})M_1 + \alpha_{12}M_2}{\gamma + \alpha_{12} + \alpha_{21}}$$

$$m_{11} = \frac{\gamma(\gamma + \alpha_{12} + \alpha_{21}) + \alpha_{21}[(\gamma + \alpha_{21})(1 - M_1) + \alpha_{12}(1 - M_2)]}{(\gamma + \alpha_{12} + \alpha_{21})[\gamma + \alpha_{21}(1 - M_1) + \alpha_{12}(1 - M_2)]}$$

for citizens of country 1. For citizens of country 2, reverse the subscripts. Finally, consider the regime where money 2 is international and money 1 is national. Since money 1 is not held by citizens of country 2, $m_{11} = M_1$ and $m_{21} = 0$. The distribution of money 2 holdings is given by

$$m_{12} = \frac{\alpha_{12}(1 - M_1)M_2}{\gamma + \alpha_{12} + \alpha_{21}(1 - M_1)}$$

$$m_{22} = \frac{(\gamma + \alpha_{12})M_2}{\gamma + \alpha_{12} + \alpha_{21}(1 - M_1)}.$$

7.2 Two International Money Regime

Here we verify some statements made in the text about the regime with two international monies, in the simplified version of the model in Section 4. First we show that (14) has a unique solution $(Q_1, Q_2) \in (0, \hat{q})^2$. To this end, use the two equations of (14) to define two continuous functions $q_{21} = g(q_{11})$ and $q_{21} = h(q_{11})$. Solutions are given by intersections in the (q_{11}, q_{21}) plane of the graphs of g and h, in the interval $(0, \hat{q})^2$. It is straightforward to verify that $g(q) = 0$ for some $q > 0$, that $g' > 0$, and that $g(\hat{q}) > \hat{q}$. Also, $h(0) > 0$,

$h' > 0$, and $h(\hat{q}) < \hat{q}$. Hence, g and h intersect in $(0, \hat{q})^2$. Moreover, $g' > h'$ whenever $g = h$, and so g and h intersect no more than once.

We now verify the comparative static results. To this end, insert $(q_{1j}, q_{2j}) = (Q_1, Q_2)$ into (14), differentiate, and simplify to get

$$\begin{bmatrix} A_{11} & -\alpha_{12}m_{20}u_2' \\ -\alpha_{21}m_{10}u_1' & A_{22} \end{bmatrix} \begin{bmatrix} dQ_1 \\ dQ_2 \end{bmatrix} = \begin{bmatrix} \alpha_{11}(u_1 - Q_1) \\ \alpha_{21}(u_1 - Q_2) \end{bmatrix} dm_{10},$$

where $u_i = u(Q_i)$ and $A_{ii} = \alpha_{ii}m_{i0}(\frac{u_i}{Q_i} - u_i') + \alpha_{ij}m_{j0}\frac{u_j}{Q_i}$. The determinant of the square matrix is

$$\Delta = \alpha_{11}\alpha_{22}m_{10}m_{20}(\tfrac{u_1}{Q_1} - u_1')(\tfrac{u_2}{Q_2} - u_2') + \alpha_{11}\alpha_{21}m_{10}^2(\tfrac{u_1}{Q_1} - u_1')\tfrac{u_1}{Q_2}$$
$$+\alpha_{22}\alpha_{12}m_{20}^2(\tfrac{u_2}{Q_2} - u_2')\tfrac{u_2}{Q_1} + \alpha_{12}\alpha_{21}m_{10}m_{20}(\tfrac{u_1}{Q_1}\tfrac{u_2}{Q_2} - u_1'u_2'),$$

which is positive since $\frac{u_i}{Q_i} > u_i'$ by concavity. Hence,

$$\Delta\tfrac{\partial Q_1}{\partial m_{10}} = \alpha_{11}\alpha_{22}m_{20}(u_1 - Q_1)(\tfrac{u_2}{Q_2} - u_2') + \alpha_{11}\alpha_{21}m_{10}(u_1 - Q_1)\tfrac{u_1}{Q_2}$$
$$+\alpha_{21}\alpha_{12}m_{20}(u_1 - Q_2)u_2' \geq 0$$

$$\Delta\tfrac{\partial Q_2}{\partial m_{10}} = \alpha_{11}\alpha_{21}m_{10}(u_1 - Q_2)(\tfrac{u_1}{Q_1} - u_1') + \alpha_{12}\alpha_{21}m_{20}(u_1 - Q_2)\tfrac{u_2}{Q_1}$$
$$+\alpha_{11}\alpha_{21}m_{10}(u_1 - Q_1)u_1' \geq 0$$

since $u_i \geq Q_j$, $i, j = 1, 2$ in this regime. We conclude that Q_1 and Q_2 are both increasing in m_{10}. Similarly, Q_1 and Q_2 are both increasing in m_{20}. Since m_{10} and m_{20} are decreasing in M_1 and M_2, Q_1 and Q_2 are decreasing in M_1 and M_2. □

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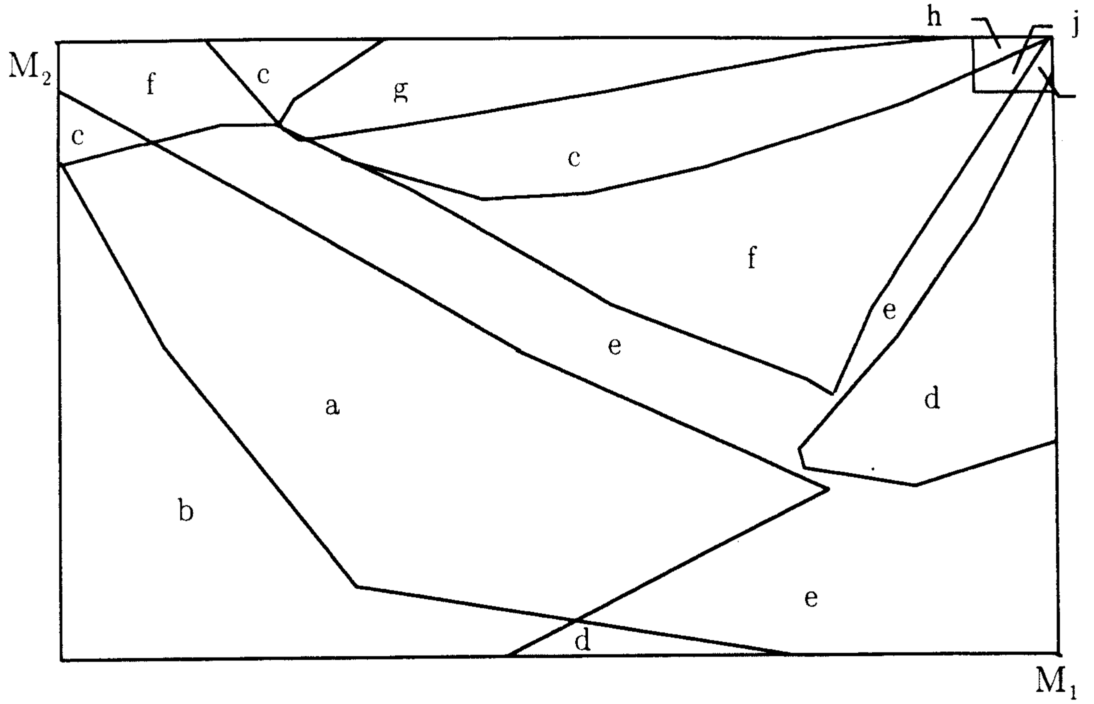
Figure 1: Areas in parameter space where the different regimes are equilibria



a=all regimes exist
b=all regimes but (1n,2n)
c=all regimes but (1i,2n)
d=only (1i,2n) and (1i,2i)
e=all regimes but (1n,2i)
f=only (1n,2n) and (1i,2i)
g=only (1n,2i) and (1i,2i)
h=only (1n,2n) and (1n,2i)
i=only (1n,2n) and (1i,2n)
j=only (1n,2n)
k=only (1i,2i)
m=only (1i,2n)