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PIER Working Paper 97-014

"Policy Analysis in Search-Based Models of Money"

by

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Policy Analysis in Search-Based Models of Money*

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November 21, 1996

*For helpful comments or suggestions, we thank Tim Kehoe, Warren Weber, Neil Wallace, and Alex Taber, as well as participants in seminars at the Federal Reserve Bank of Cleveland, the NBER Summer Institute, and the conference on Recent Developments in Monetary Theory and the Implications for Policy organized by the University of Miami and the Federal Reserve Bank of Atlanta. The NSF provided financial support.

1 Introduction

It is a venerable notion that what government accepts in payment in its transactions can have an impact on what private agents do. Smith (1776; 1963), for instance, argued that "A prince, who should enact that a certain proportion of his taxes should be paid in a paper money of a certain kind, might thereby give a certain value to this paper money; even though the term of its final discharge and redemption should depend altogether upon the will of the prince." Lerner (1947) further suggested that "The modern state can make anything it chooses generally acceptable as money and thus establish its value quite apart from any connection, even of the most formal kind, with gold or with backing of any kind. It is true that a simple declaration that such and such is money will not do, even if backed with the most convincing constitutional evidence of the state's absolute sovereignty. But if the state is willing to accept the proposed money in payment of taxes and other obligations to itself the trick is done."

Can the government really make "anything it chooses" generally acceptable as money? Or, does the answer depend on the importance and influence of the government in the economy? Does it depend on the nature of the proposed money, or on the existence and efficacy of potential substitutes, as in economies where there may coexist money and barter or two types of money (say, commodity and fiat money, or domestic and foreign currency)? And exactly how does it depend on the nature of the government's policy?

Historically, it is not uncommon for governments to adopt policies designed influence whether different objects circulate, and at what relative prices, such as policies that favor some object by accepting only it in payment for taxes or in other transactions, by announcing its legal tender status, and so on.[1] These policies have sometimes worked and other times failed. Friedman (1994) describes a case that worked: France's success in maintaining a stable bimetalic commodity money system from 1785-1873. He argues that this success was due to the combination of France's economic importance in the world and the large amounts of metal in the country at that time, and concludes that "These two factors made France a major participant in the market for silver and gold, an important enough participant to be able to peg the price ratio despite major changes in the relative production of silver and gold."

Fukujiro (1935) describes an example of a policy that failed. During the early stage of the colonial period in Taiwan (circa 1895), the Japanese colonial government required that taxes had to be paid in Japanese money, rather than the copper coins that were being used in private transactions at the time, with the purpose of establishing Japanese currency as medium of exchange. The effect was that people stopped paying taxes, decreasing revenue by so much that the policy had to be abandoned! Comparing this to

[1]The legal tender status of National Bank notes was established by declaring that these notes "shall be received at par in all parts of the United States in payment of taxes, excises, public lands, and all other dues to the United States, except duties on imports; and also for all salaries and other debts and demands owing by the United States to individuals, corporations, and associations within the United States, except interest on the public debt, and in redemption of the national currency" (Sec. 5182, p. 397 of Huntington and Mawhinney 1910).

the example in the previous paragraph, it seems that the Japanese government was not such a major participant in the local economy at the time, and were therefore not able to exert enough influence, to determine which objects were used as money. Later, when their presence became more important in the Taiwanese economy, the money favored by the Japanese government did begin to circulate.

The objective of this paper is to study the effects of government policies like those described above in a search-theoretic (or random matching) model of money. A search model is the right tool for the job because it generates an endogenous transactions pattern that allows one to determine in equilibrium which objects are accepted in which trades, and at what relative prices. Our framework will allow one to analyze the effects of government transactions policies on these outcomes, and to make precise how these effects depend on things like the size and influence of the government in the economy.

In order to pursue these issues, one needs to introduce government into the framework in a way that preserves the spatial, temporal, and informational frictions that are inherent in the search model and that lead to an interesting role for a medium of exchange in the first place. The view adopted here is that the government is nothing more nor less than a subset of the agents in the economy, who are subject to the same random matching technology and other constraints as private agents, but behave in a specific way. For example, they may always accept a particular money, they may accept only money (i.e., they may refuse to barter), they may accept money with some arbitrary probability, they may accept money but only at a particular exchange rate, and so on. The objective is to see how their trading policies, which we specify exogenously, affect the trading strategies that private agents choose endogenously.

Some of the results can be summarized as follows. We show that a policy whereby the government accepts money can guarantee the existence and uniqueness of an equilibrium where money is universally accepted if and only if the government is sufficiently big (i.e., involved in a sufficiently big proportion of economic activity). Just how big depends on several factors, including the quantity of money, its properties, and the availability and efficacy of substitutes like direct barter or foreign currency. It also depends on other aspects of policy, such as whether the government also accept other objects, and if they do, at what prices. Note that a sufficiently big government can establish the existence of a monetary equilibrium even when one does not exist without intervention, although this may entail the government running a deficit (producing more than it consumes).

Also, by refusing to accept a particular money, like a foreign currency, or by only accepting it at very unfavorable terms, the government can rule out equilibria where that money has value if and only if the government is sufficiently big. Moreover, even if government policy does not affect the number of equilibria, it will generally affect equilibrium prices. For example, the government can affect the relative prices of foreign and domestic currency by setting an official rate at which it sells goods for each of the monies. Market exchange rates are influenced by these official exchange rates, although they typically are not equal.

The rest of the paper is organized as follows. Section 2 considers a relatively simple model, with indivisible commodities, in order to analyze when money will have value and how this depends on policy. Section 3 considers a version with divisible commodities and bargaining in order to additionally analyze the prices at which objects trade. Section 4 considers economies with two monies (e.g., domestic and foreign currencies). The analysis to that point focuses on steady state equilibria, but in Section 5 we also consider dynamic equilibria. Section 6 concludes.[2]

2 A Model with Indivisible Goods

In order to introduce the basic notation and assumptions, we begin by briefly reviewing a version of the model (with no government) in Kiyotaki and Wright (1993), where goods and money are indivisible and hence every trade

[2]There are several related papers in the literature. The closest is Aiyagari and Wallace (forthcoming), where monetary policy is also modeled by specifying a set of government agents and analyzing the effects of their transaction rules in a random matching framework. Aside from many technical differences, including the fact that they study a special subset of the policies analyzed here, the main difference is that they only consider a model with indivisible goods, which means that prices are exogenous. Aiyagari, Wallace and Wright (forthcoming) also include government agents in a search model of money, and allow them to issue debt. The model in Li (1994, 1995) has government agents who are allowed to tax private money holdings. Ritter (1995) studies a model where the role of government is to issue fiat money in the first place. Green and Weber (1996) study a model with government agents whose role is to detect and confiscate counterfeit notes. A major difference between the present framework and all of those papers (except Aiyagari, Wallace and Wright) is that they assume that prices are exogenous. An early analysis of the effects of a government that demands money for tax payments, although in a Walrasian model and not a model with explicit frictions, is contained in Starr (1974).

In addition to the goods described above, there is another indivisible but storable object that no one produces or consumes called *money.* A fraction $M \in [0,1]$ of the agents are endowed with money, while the rest are endowed with production opportunities. Unless he is initially endowed with a production opportunity, every agent i must consume one of the goods in S_i before producing. This means that every trade is either a direct barter trade or a monetary trade which involves an agent giving 1 unit of money to an agent with 0 units of money. Hence, there are always M agents with one unit of money, called *buyers*, and $1-M$ agents without money, called *sellers.*

The above assumptions imply that an agent is willing to accept a good in trade if and only if it is in S_i, and when he gets it he consumes it immediately. What remains to be determined is whether or not an agent accepts money. Let Π be the probability that a random agent in the economy accepts money, and let π be the best response of a maximizing individual. Let V_0 and V_1 denote the value functions of buyers and sellers. Then standard arguments lead to the dynamic programming equations

$$rV_0 = v_0 + \alpha(1-M)xy(U-C) + \alpha Mx \max_{\pi} \pi(V_1 - V_0 - C) \quad (1)$$

$$rV_1 = v_1 + \alpha(1-M)x\Pi(U + V_0 - V_1), \quad (2)$$

where r is the discount rate, v_1 is an instantaneous utility from holding money, and v_0 is an instantaneous utility from holding a production opportunity. The v_j's can be interpreted in a variety of ways; for example, if $v_1 < 0$ then money has a "storage cost" which one may want to loosely interpret in terms of inflation.[4]

The maximization problem in (1) implies:

$$\pi = \begin{cases} 0 & \text{if } V_1 - V_0 - C < 0 \\ [0,1] & \text{if } V_1 - V_0 - C = 0 \\ 1 & \text{if } V_1 - V_0 - C > 0 \end{cases} \tag{3}$$

In what follows we normalize $\alpha x = 1$, with no loss in generality as long as we adjust r and v_j, by defining units of time appropriately. Then (1) and (2) imply

$$V_1 - V_0 - C = \frac{(1-M)(U-C)(\Pi - y) - (rC + v_0 - v_1)}{r + (1-M)\Pi + M\pi}.$$

Therefore, $V_1 - V_0 - C$ takes the same sign as

$$\Delta = (1-M)(\Pi - y) - K, \tag{4}$$

where $K = (rC + v_0 - v_1) \,/\, (U - C)$.

An equilibrium can now be defined as a value of Π satisfying

$$\Delta < 0 \Rightarrow \Pi = 0; \quad \Delta > 0 \Rightarrow \Pi = 1; \quad \Pi \in (0,1) \Rightarrow \Delta = 0, \tag{5}$$

where Δ is given by (4). If $\Pi = 0$ the equilibrium is called nonmonetary, and

[4] Equation (1) says that the flow value to being a seller, rV_0, is the sum of three terms. The first is the instantaneous utility v_0. The second is the probability of meeting a seller, $\alpha(1-M)$, times the probability he wants what you produce and you want what he produces, xy, times the gain from trading. The final term is the probability of meeting a buyer, αM, times the probability he wants what you produce, x, times the gain from trading with probability π where π is chosen optimally. Equation (2) has a similar interpretation.

if $\Pi > 0$ the equilibrium is called monetary.[5] In Figure 1, we depict the best response correspondence $\pi = \pi(\Pi)$ for the case $0 < \hat{\Pi} < 1$, where

$$\hat{\Pi} = y + \frac{K}{1 - M}. \tag{6}$$

When $0 < \hat{\Pi} < 1$, as shown in the figure, there are exactly three equilibria: $\Pi = 0$, $\Pi = 1$, and $\Pi = \hat{\Pi}$. However, it could be that $\hat{\Pi} > 1$, in which case the only equilibrium is $\Pi = 0$; or it could be that $\hat{\Pi} < 0$, in which case the only equilibrium is $\Pi = 1$.

The model illustrates how there are two critical requirements for money to circulate. First, fundamentals have to be right. Here, this means $\hat{\Pi} \leq 1$, which means y, M, r, $v_0 - v_1$, or $C - U$ cannot be too big (barter cannot be too easy, money too plentiful, individuals too impatient, money too costly to store, or production too expensive). Second, if $\hat{\Pi} \in (0, 1)$, then even if there exists a monetary equilibrium there also exists a nonmonetary equilibrium; hence, even if fundamentals are right, individuals have to "believe in" money in order for it to be accepted as a medium of exchange.

In terms of welfare, it is easy to see that V_0 and V_1 are both increasing

[5] In principle, one also has to check the incentive constraints that make buyers and sellers willing to trade and the participation constraints $V_j \geq 0$. The incentive constraint for buyers is $U + V_0 \geq V_1$, which one can show holds in equilibrium if and only if

$$1 + r - (1 - M)(\Pi - y) + K \geq 0;$$

a sufficient condition for this is that $v_1 - v_0$ is not too big (if v_1 is too big, e.g., then it is better to hoard money than to spend it). The incentive constraints for sellers are never binding. The participation constraint $V_j \geq 0$ holds as long as v_j is not too big a negative number. Hence, these constraints all hold as long as $|v_0|$ and $|v_1|$ are not too big.

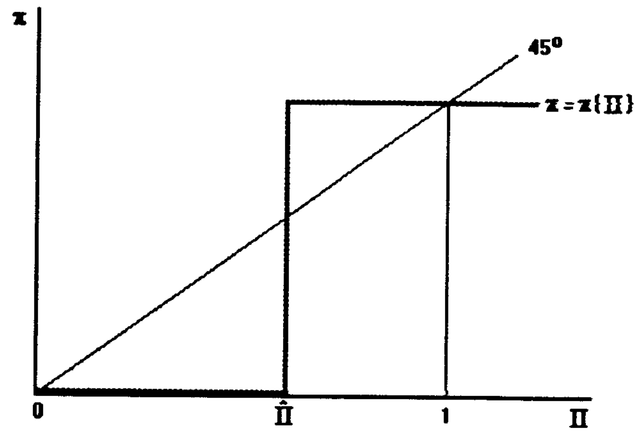

Figure 1: Equilibria in the Fixed Price Model

in Π. Hence, $\Pi = 1$ Pareto dominates $\Pi \in (0,1)$ and $\Pi \in (0,1)$ Pareto dominates $\Pi = 0$. Note that this statement is about comparing utility across values of Π irrespective of whether these values constitute equilibria (e.g., the outcome implied by $\Pi = 1$ dominates the outcome implied by $\Pi = 0$ even if $\Pi = 1$ is not an equilibrium). Of course, a special case of this result is that when the equilibria coexist they can be Pareto ranked.

We are interested in knowing whether policy can be used to guarantee the existence of the equilibrium with $\Pi = 1$, and also whether policy can be used to rule out the existence of the other equilibria. To this end, we introduce government as follows: Assume that a fraction of the population γ constitutes a special class of agents called *government agents.* They are in all respects exactly like private agents except that they adopt exogenous trading rules rather than strategies based on maximizing behavior.[6] Private agents continue to use the individually-maximizing trading strategies described above: type i trades his production good for a good in S_i with probability 1; trades money for a good in S_i with probability 1; and trades his production good for money with probability π. But a government agent trades his production good for a good in S_i with probability T_{gg}; trades money for a good in S_i with probability T_{mg}; and trades his production good for money with probability T_{gm}. A government trading policy is specified by $T = (T_{gg}, T_{mg}, T_{gm})$.

We need to keep track of who holds the money. Let $m = (m_p, m_g)$, where m_p is the fraction of private agents and m_g the fraction of government agents with cash. Given the normalization $\alpha x = 1$, the rate at which a private agent switches from buyer to seller is $\gamma(1-m_g)T_{gm} + (1-\gamma)(1-m_p)\Pi$, and the rate at which he switches back is $\gamma m_g T_{mg}\Pi + (1-\gamma)m_p\Pi$. Combining these and using the identity $\gamma m_g + (1-\gamma)m_p = M$, the steady state value of m_p solves

$$\dot{m}_p = \Pi T_{mg} M - \left[\Pi T_{mg} + (T_{gm} - \Pi T_{mg})(\gamma - M)\right] m_p - (T_{gm} - \Pi T_{mg})(1-\gamma) m_p^2 = 0. \tag{7}$$

The dynamic programming equations for a private agent can now be written

$$rV_0 = v_0 + A_1(U - C) + A_2 \max_{\pi} \pi(V_1 - V_0 - C) \tag{8}$$

[6]One should interprert γ as capturing the importance of government in the economy, or the frequency with which private agents interact with the public sector, rather than measuring number of individuals who "work for the government". Indeed, Aiyagari and Wallace (forthcoming) interpret their government agents as "vending machines" whose role is simply to store and trade various objects.

the condition for the existence of the $\Pi = 0$ equilibrium is

$$\Delta = (\gamma - M)(1 - yT_{gg}) - (1 - \gamma)y - K \leq 0.$$

This is violated if and only if

$$\gamma > \gamma_1 = \frac{K + y + M(1 - yT_{gg})}{1 + y(1 - T_{gg})}$$

If $K < (1 - M)(1 - yT_{gg})$ then $\gamma_1 < 1$, and there is an nonempty interval such that $\gamma \in (\gamma_1, 1)$ rules out the $\Pi = 0$ equilibrium under a policy $T_{gm} = 1$. And note that as long as $K \geq -y(1 - M)$ the $\Pi = 0$ equilibrium always exists without government. Also note that the nonmonetary equilibrium is less likely to exist under $T_{gm} = 1$ when we make T_{gg} small; that is, a policy that favors the use of money is more effective in combination with a policy that disfavors barter.

Continuing with the analysis of policy $T_{gm} = 1$, the next question to ask is this: when will $\Pi = 1$ be an equilibrium under this policy? For simplicity, consider the case where we also set $T_{mg} = 1$, so that government agents not only accept money but also spend it whenever they can. This implies that in steady state $m_p = m_g = M$, and the condition for the existence of the $\Pi = 1$ equilibrium is

$$\Delta = (1 - M)[\gamma(1 - yT_{gg}) + (1 - \gamma)(1 - y)] - K \geq 0.$$

If $K \leq 0$ then $\Delta \geq 0$ for all $\gamma \geq 0$; but if $K > 0$ then $\Delta \geq 0$ if and only if

$$\gamma \geq \gamma_2 = \frac{K - (1 - M)(1 - y)}{(1 - M)y(1 - T_{gg})}.$$

If $K < (1 - M)(1 - yT_{gg})$, then $\gamma_2 < 1$, and there is an nonempty interval such that $\gamma \in (\gamma_2, 1)$ guarantees the $\Pi = 1$ equilibrium exists under the policy

$T_{gm} = 1$. And note that as long as $K \geq (1-M)(1-y)$ the $\Pi = 1$ equilibrium does not exist without government. Note also that reducing T_{gg} makes $\Delta \geq 0$ more likely; again, a policy that favors the use of money is more effective in combination with a policy that disfavors barter. We conclude that a policy of $T_{gm} = 1$ can guarantee the existence of monetary equilibrium, as well as rule out the existence of nonmonetary equilibrium, if and only if the government is sufficiently big relative to K, y, M, and T_{gg}.

We turn now to the second policy, $T_{gm} = 0$. One question to ask is this: can $\Pi = 1$ still be an equilibrium? That is, can money still circulate among private agents if the government does not accept it? If $\Pi = 1$ then under this policy the steady state is:

$$(m_p, m_g) = \begin{cases} (\frac{M}{1-\gamma}, 0) & \text{if } \gamma \leq 1 - M \\ (1, \frac{M+\gamma-1}{\gamma}) & \text{if } \gamma > 1 - M \end{cases} \tag{12}$$

Consider the case $\gamma \leq 1 - M$ (the other case is similar). The condition for the existence of the $\Pi = 1$ equilibrium is

$$\Delta = (1 - M - \gamma)(1 - y) - \gamma y T_{gg} - K \geq 0,$$

which is violated if and only if

$$\gamma > \gamma_3 = \frac{(1 - M)(1 - y) - K}{1 - y(1 - T_{gg})}.$$

If $K > -M(1 - y) - yT_{gg}$ then $\gamma_3 < 1$, and there is a nonempty interval such that $\gamma \in (\gamma_3, 1)$ eliminates the $\Pi = 1$ equilibrium under the policy $T_{gm} = 0$. And note that as long as $K \leq (1-M)(1-y)$ the $\Pi = 1$ equilibrium always exists without government. Also notice that a larger value of T_{gg} makes it more likely that the monetary equilibrium will not exist.

We can also analyze polices where the government accepts money with some probability $T_{gm} \in (0, 1)$. For simplicity, consider the case where $T_{gg} = T_{mg} = 1$, so that the only policy instrument is the probability with which government agents accept money. Then one can show $\partial\hat{\Pi}/\partial T_{gm} < 0$, which means that an increase in the probability with which government agents accept money shifts the best response correspondence in Figure 1 to the left. If government is big enough, then as T_{gm} increases eventually $\hat{\Pi}$ becomes negative.[8] Hence, by increasing T_{gm} towards 1 the government makes it more likely that there is a unique equilibrium and it is $\Pi = 1$.

We conclude this section with a summary of the results.

Proposition 1 *If and only if γ is big enough, a policy whereby government agents accept money with high probability can eliminate equilibria with $\Pi < 1$ that exist without government intervention, and can guarantee the existence of a unique equilibrium with $\Pi = 1$. This is true even if the $\Pi = 1$ equilibrium does not exist without government intervention. Such a policy is more effective if T_{gg} is small. Also, if and only if γ is big enough, a policy whereby government agents accept money with low probability can eliminate a monetary equilibrium that exists without intervention. This policy is more effective if T_{gg} is big.*

[8]To be more precise, suppose T_{gm} is given. Then one can show the following: if $K < (T_{gm} - y)(1 - M)$ then $\hat{\Pi} < 0$ if and only if $\gamma > [K + MyT_{gm} + (1 - M)y]/T_{gm}$; and if $K > (T_{gm} - y)(1 - M)$ then $\hat{\Pi} < 0$ if and only if $\gamma > [K + MyT_{gm} + (1 - M)y](1 - M)/[K + y(1 - M)]$.

3 A Model with Bargaining

In this section we relax the assumption of indivisible goods and introduce bargaining, as in Shi (1995) and Trejos and Wright (1995), so that we can discuss prices. Otherwise, everything is the same as in the previous section. In particular, we still assume that money is indivisible and that agents need to consume in order to produce, so that they always have either 1 or 0 units of currency. We continue to concentrate for now on steady state equilibria, postponing dynamics until Section 5. And, as in the previous section, we begin with the economy without government as a benchmark.

When agent i consumes q units of a good in S_i he enjoys utility $u(q)$, and when he produces q units of his production good he suffers disutility $c(q)$. We normalize $c(q) = q$ with no loss of generality. Assume $u(0) = 0$, $u'(q) > 0$ and $u''(q) < 0$ for all $q > 0$. Also, there is a $\hat{q} > 0$ such that $u(\hat{q}) = \hat{q}$. If a unit of money buys q units of output the nominal price level is $p = 1/q$.

When agents meet and there are potential gains from trade, they bargain. When two sellers meet and there is a double coincidence of wants, we assume that they bargain according to the symmetric Nash solution, which implies that each produces q^* for the other where q^* satisfies $u'(q^*) = c'(q^*)$. When a buyer and seller meet, we assume for simplicity that the former gets to make a take-it-or-leave-it offer to the latter, which allows him to extract the entire surplus from the trade. Note that these bargaining outcomes can be interpreted as the equilibria of explicit strategic bargaining games (see Coles and Wright 1995 for details). In any case, other bargaining solutions generate similar results, although the assumption of take-it-or-leave-it offers when a buyer and seller meet turns out to simplify the analysis considerably.[9]

For comparing the results with those in the previous section, it is instructive to analyze the model as follows. All agents know that there is a going market price that implies a dollar buys Q units of output, and when any individual buyer and seller meet the former chooses q taking Q as given (Q is analogous to Π and q is analogous to π in the model with indivisible goods). Then the value functions satisfy

$$rV_0 = v_0 + (1-M)y[u(q^*) - q^*] + M \max_{\pi} \pi[V_1 - V_0 - Q] \quad (13)$$

$$rV_1 = v_1 + (1-M)\Pi \max[u(Q) + V_0 - V_1, 0]. \quad (14)$$

Notice that the maximization problem in (14) implies the buyer chooses whether to spend his money or not when he meets a seller. However, in any monetary equilibrium, take-it-or-leave-it offers implies $Q = V_1 - V_0$, which means $u(Q) + V_0 - V_1 = u(Q) - Q \geq 0$ for all $Q \leq \hat{q}$. Here we assume parameter values are such that $Q < \hat{q}$ in equilibrium, and so buyers are always willing to spend their money. Also notice that, given $Q \in [0, \hat{q}]$, a buyer's take-it-or-leave-it offer will be $q = q(Q) = \max[D(Q), 0]$, where $D(Q) = V_1 - V_0$. In equilibrium, then, either $Q = V_1 - V_0 > 0$, or $V_1 \leq V_0$ and $Q = 0$, which implies $\pi = 0$. In either case, the last term in (13) vanishes: $\pi[V_1 - V_0 - Q] = 0$.

A steady state equilibrium is a fixed point of $q(Q)$. If $D(Q) < 0$ then the

[9] Nothing really depends on using the symmetric Nash solution in barter exchange, and imposing another bargaining solution when two sellers meet does not affect the results at all. Indeed, we can eliminate barter altogether by setting $y = 0$ and the results are qualtitativey exactly the same.

equilibrium is nonmonetary and $\Pi = 0$; if $D(Q) > 0$ then the equilibrium is monetary and $\Pi = 1$.

Assuming we are in a monetary steady state, manipulation of (13) and (14) yields

$$D(Q) = \frac{(1-M)u(Q) + MQ - k}{1+r}$$

where $k = (1-M)y[u(q^*) - q^*] + v_0 - v_1$. To reduce the number of cases, we assume here that $k > 0$ (but see below). Then $D(0) < 0$, and since $D(Q)$ is increasing and concave the best response correspondence is as depicted in Figure 2. There is always a nonmonetary steady state. Whether there exist other equilibria depends on parameters: there is a critical $\tilde{k}$ such that $k > \tilde{k}$ implies $D(Q)$ is below the 45^o line for all Q and there are no monetary equilibria; and $k < \tilde{k}$ implies $D(Q)$ intersects the 45^o line twice and there are two monetary equilibria, as shown in Figure 2. In both monetary equilibria money is accepted with probability 1, but one has a low q_l and hence a high nominal price level while the other has a high q_h and hence a low nominal price level.[10]

In terms of welfare, when multiple equilibria coexist, it is easy to show that the low price equilibrium Pareto dominates the high price equilibrium and both dominate the nonmonetary equilibrium. However, even the best

[10]For completeness, we report the following results for $k < 0$. First, $k < 0$ implies all equilibria are monetary. Then there are two cases to consider. On the one hand, if $0 > k > -r\hat{q}$ then $D(0) > 0$ and $D(\hat{q}) < \hat{q}$, so there is a unique intersection of $q(Q)$ with the 45^o line and a unique monetary equilibrium. On the other hand, if $k < -r\hat{q}$ then $D(Q)$ lies above the 45^o line for all $Q \in [0, \hat{q}]$. This means that sellers are willing to accept money, but buyers prefer to hoard their cash and enjoy lifetime utility v_1/r.

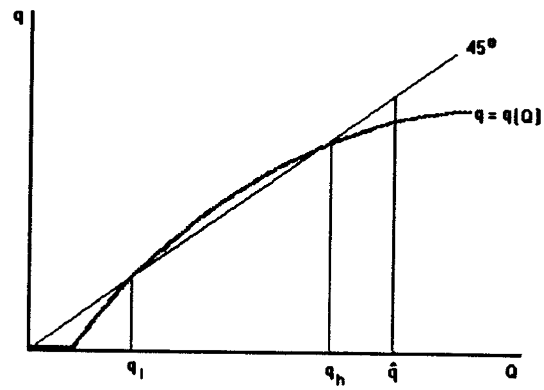

Figure 2: Equilibria in the Bargaining Model ($0 < \mathrm{k} < \tilde{\mathrm{k}}$)

equilibrium is not necessarily efficient. For example, suppose that a planner can impose any q. Then the q that maximizes welfare, as given by $W = MV_1 + (1-M)V_0$, is the q^* that solves $u'(q^*) = c'(q^*)$. In general, equilibrium q may be bigger or smaller than q^*.

We now re-introduce government. For now, set $T = (1, 1, 1)$, and consider first a policy that fixes the quantity of goods that government agents supply in exchange for money, $q^s \in (0, \hat{q})$. This implies

$$rV_0 = v_0 + (1-M)y[u(q^*) - q^*] \tag{15}$$

$$\begin{aligned} rV_1 = {} & v_1 + (1-\gamma)(1-m_p)\Pi \max[u(Q) + V_0 - V_1, 0] \\ & + \gamma(1-m_g)\max[u(q^s) + V_0 - V_1, 0]. \end{aligned} \tag{16}$$

Notice that (15) is identical to (13) from the model without government, but (16) differs from (14) because q^s can differ from Q. In principle, buyers may reject trades with either private sellers or government sellers. However, $u(Q) + V_0 - V_1 > 0$ in equilibrium as long as $Q \in [0, \hat{q})$, and we only consider policies where $u(q^s) + V_0 - V_1 > 0$ in equilibrium.

Suppose we are in an equilibrium with $\Pi = 1$. Then in steady state $m_p = m_g = M$, and manipulation of (15) and (16) yields

$$D(Q) = \frac{(1-M)[(1-\gamma)u(Q) + \gamma u(q^s)] + MQ - k}{1+r}.$$

Suppose that without government intervention there exist two monetary equilibria, which we now denote (q_l^o, q_h^o). As shown in Figure 3, an increase in γ rotates $D(Q)$ clockwise around the point $[q^s, D(q^s)]$. For $\gamma > k/(1-M)u(q^s)$ we have $D(0) > 0$, and therefore $q(Q)$ has a unique fixed point q^γ, as shown.[11]

We conclude from this that, when there are multiple monetary steady state equilibria, a big government can eliminate all but one of them. Similarly, a big government can eliminate the nonmonetary equilibrium. In particular, suppose there are initially two monetary steady state equilibria without government intervention, (q_l^o, q_h^o), and set $q^s = q_h^o$. Then $\gamma \geq k/(1-M)u(q_h^o)$ implies that $q = q_h^o$ is the unique monetary steady state equilibrium. Notice that in this case government agents consume and produce the same amount; more generally, however, $q^s > q$ implies running a deficit and $q^s < q$ implies

[11]Now that we have q^γ, we still have to check $u(q^s) > V_1 - V_0 = q^\gamma$ to verify that private buyers want to trade with government sellers, as assumed above. There are three cases. First, suppose $q^s \leq q_l^o$. Then $q^\gamma \in (0, q^s]$ and therefore $u(q^s) \geq q^\gamma$, as assumed. Now suppose $q^s \geq q_h^o$. Then $q^\gamma \in [q_h^o, q^s]$ and $u(q^s) \geq q^\gamma$ once again. Finally, suppose $q_l^o < q^s < q_h^o$. Then $q^\gamma \in (q^s, q_h^o)$ and we cannot be sure that $u(q^s) \geq q^\gamma$ is satisfied; however, as long as q^s is close to q_h^o it will be.

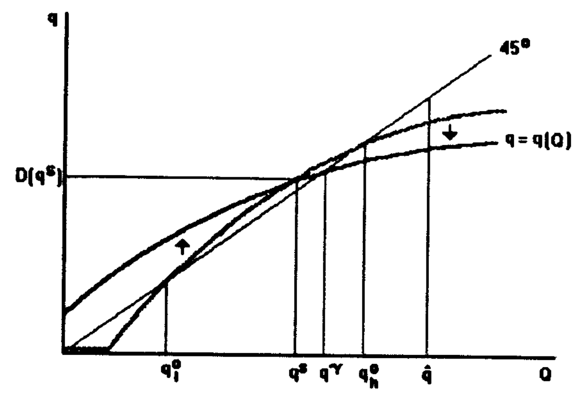

Figure 3: Equilibrium with Government Policy

a surplus, in the sense that government agents consume more or less than they produce.[12]

Now suppose that k is so big that with $\gamma = 0$ monetary equilibria do not exist. It is still the case that $\gamma > k/(1-M)u(q^s)$ implies $q(Q)$ has a (unique) fixed point $q^\gamma \in (0, q^s]$. Hence, a q^s policy not only can eliminate equilibria, it can also establish a monetary equilibrium that would not have existed without intervention. Intuitively, private agents know they can always get q^s for money from government agents, and, if they interact with them frequently enough, money will be valued in private transactions. But for such a result

[12]Although we do not explcitly model the government agents choice problem, notice also that as long as q^s is no greater than the equilibrium q they will have no incentive to defect and play some other strategy; i.e., it is incentive compatible to ask the government agents to use q^s.

we must have $q^s > q^\gamma$, which implies that the government is producing more than it is consuming. Thus, it is necessary to run a deficit to get a money to circulate when it otherwise could not.

We now consider a different policy, whereby government agents demand some exogenously fixed quantity q^d in exchange for a dollar and act in all other respects just like private agents. In particular, we now assume that the government accepts money with whatever probability private agents choose in equilibrium: $T_{gm} = \Pi$. Then

$$\begin{aligned} rV_0 &= v_0 + (1-M)y[u(q^*) - q^*] + \gamma m_g \max(V_1 - V_0 - q^d, 0) \\ rV_1 &= v_1 + (1-M)\Pi[u(Q) + V_0 - V_1]. \end{aligned}$$

If we are initially in an equilibrium without government intervention where the value of money is $q \in \{0, q_l^o, q_h^o\}$, then $q^d > q$ implies that private agents will reject offers from government buyers (since q is the most that a seller will accept, given take-it-or-leave-it offers). However, in equilibrium a seller receives zero surplus whether he accepts an offer q from private buyer or rejects the government demand of q^d; therefore, V_0 is unaffected by a policy with $q^d \geq q$. Hence, this policy cannot affect the number of equilibria or the equilibrium prices when $q^d \geq q$.

Alternatively, suppose we set q^d below the equilibrium value of q (although one should note that such a policy entails a deficit). For example, suppose we are initially in the q_h^o monetary equilibrium without government intervention and we set $q^d < q_h^o$. Since an increase in γ rotates $D(Q)$ clockwise around $[q^d, D(q^d)]$, the bigger is γ the lower is the new equilibrium value of q_h. In fact, if γ is big enough, and if q^d is small enough, this policy can eliminate monetary equilibria. Intuitively, if government buyers demand a very low q^d for a dollar, sellers will trade with private buyers only at a very low q. But if q is too low private buyers may prefer to hoard their money. Hence, by committing to a very low q^d, government could drive money out of circulation.

If government is not so big, the choice of q^d or q^s will not affect the number of equilibria but can affect the equilibrium prices. It is easy to derive the effects of these policies on prices simply by rotating the best response function $D(Q)$, and we do not report all the possible results. Instead, we conclude this section by summarizing the main results as follows.

Proposition 2 *If and only if γ is big enough, a policy whereby government agents supply q^s in exchange for money can eliminate both a low q monetary equilibrium and the nonmonetary equilibrium, and can guarantee the existence and uniqueness of a high q monetary equilibrium. This is true even if a monetary equilibrium does not exist without intervention, although then establishing a monetary equilibrium implies the government must run a deficit. If there exist monetary equilibria without government, for big γ, setting $q^s = q_h^0$ implies $q = q_h^0$ is the unique equilibrium and entails no deficit or surplus. Also, a policy whereby government agents demand q^d in exchange for money can eliminate monetary equilibria that exist without intervention if q^d is low and γ big enough.*

4 Multiple Currencies

As discussed in the Introduction, there are many examples of economies with more than one potential medium of exchange, including coins of different metals, heavy and light coins of the same metal, commodity and fiat monies, and domestic and foreign currencies, and sometimes governments try to adopt policies that favor one. In this section we study how such policies affect whether a given money circulates and at what price. As in previous sections, we begin with the economy with no government as a benchmark.

Suppose there are two types of monies in the economy, called money 1 and money 2, with stock M_i of currency i and $M = M_1 + M_2 < 1$. All agents take as given the probability Π_i that money i will be accepted as well as the market price as given by $p_i = 1/Q_i$. Let the value function for an agent holding money i be V_i. Then we have

$$\begin{aligned} rV_0 &= v_0 + (1-M)y[u(q^*) - q^*] \\ rV_1 &= v_1 + (1-M)\Pi_1 \max[u(Q_1) + V_0 - V_1, 0] \\ rV_2 &= v_2 + (1-M)\Pi_2 \max[u(Q_2) + V_0 - V_2, 0]. \end{aligned}$$

There is no currency exchange in this model, because we assume that money holders do not meet each other in the matching process. This gives the model a simple recursive structure: we can solve for the value of each money independently.

Given $Q_i \in (0, \hat{q})$, a buyer with money i will demand $q_i = q_i(Q_i) = \max[D_i(Q_i), 0]$, where $D_i(Q_i) = V_i - V_0$. In an equilibrium with $\Pi_1 = \Pi_2 = 1$, we have

$$D_i(Q_i) = \frac{(1-M)u(Q_i) - k_i}{1+r}$$

where $k_i = (1-M)y[u(q^*) - q^*] + v_0 - v_i$, $i = 1, 2$. Depending on parameter values (k_1 and k_2 in particular), $q_i(Q)$ can have zero, one, or two fixed points in $(0, \hat{q})$. Each possible combination of q_1 and q_2 is a different equilibrium.

Various policies can be considered, but here we assume that government accepts currency i with probability T_{gi}, and, if it accepts it, supplies q_i^s in exchange. The value functions with government then satisfy

$$\begin{aligned}
rV_0 &= v_0 + (1-M)y[u(q^*) - q^*] \\
rV_1 &= v_1 + (1-\gamma)(1-m_{p1}-m_{p2})\Pi_1 \max[u(Q_1) + V_0 - V_1, 0] \\
&\quad +\gamma(1-m_{g1}-m_{g2})T_{g1} \max[u(q_1^s) + V_0 - V_1, 0] \\
rV_2 &= v_2 + (1-\gamma)(1-m_{p1}-m_{p2})\Pi_2 \max[u(Q_2) + V_0 - V_2, 0] \\
&\quad +\gamma(1-m_{g1}-m_{g2})T_{g2} \max[u(q_2^s) + V_0 - V_2, 0]
\end{aligned}$$

Following the previous section, we assume that parameter values are such that $Q_i < \hat{q}$, and that when $T_{gi} > 0$ q_i^s is such that private agents do not reject trades with government sellers.

Consider first the case where government always accepts both monies at the exogenously fixed terms q_1^s and q_2^s. When $T_{gi} = \Pi_i = 1$, in steady state we have $m_{pi} = m_{gi} = M_i$, for $i = 1, 2$, and

$$D_i(Q_i) = \frac{(1-M)[(1-\gamma)u(Q_i) + \gamma u(q_i^s)] - k_i}{1+r}$$

Following the analysis in the previous section, we can eliminate nonmonetary and low q monetary equilibria if and only if the government is big enough.

In particular, if we set $(q_1^s, q_2^s) = (q_{1h}^o, q_{2h}^o)$, where q_{ih}^o is the high value of money i without government intervention, $(q_1, q_2) = (q_{1h}^o, q_{2h}^o)$ is the unique equilibrium if and only if

$$\gamma > \max\left[\frac{k_1}{(1-M)u(q_1^s)}, \frac{k_2}{(1-M)u(q_2^s)}\right].$$

One can also think about using policy to drive one of the currencies out of circulation. Suppose, for example, that the government rejects one of them, say money 2. Thus, set $(T_{g1}, T_{g2}) = (1, 0)$ and $q_1^s = q_{1h}^o$ (obviously, q_2^s is irrelevant when $T_{g2} = 0$). By the usual reasoning, we can rule out equilibria with $\Pi_2 = 1$ if γ is sufficiently big. Then the only possible equilibria involve $\Pi_2 = 0$, so we are essentially back to an economy with only one money, and there will again be unique equilibrium with $\Pi_1 = 1$ if γ is big enough. We conclude that a sufficiently big government can establish a unique circulating currency with $(T_{g1}, T_{g2}) = (1, 0)$.

Policy can also be used to affect the market exchange rate, as defined by $e = q_1/q_2$. For example, suppose that we start in an equilibrium with $(q_1, q_2) = (q_{1h}^o, q_{2h}^o)$, and the government announces the official values of the monies to be $q_1^s = q_{1h}^o$ and $q_2^s = q_{2h}^o - \varepsilon$ for $\varepsilon > 0$. Then the equilibrium value of q_1 stays the same while q_2 falls, which moves the exchange rate e in favor of money 1. Notice, however, there will be a difference between the official and the market exchange rates, because the equilibrium q_2 exceeds q_2^s.

5 Dynamics

In this section we analyze dynamic equilibria in the model with divisible goods and one fiat money. As in the earlier sections, we start with the model without government. Also, as above, we assume that buyers get to make take-it-or-leave-it offers, and that sellers always produce q^* for each other in all barter trades (both of these assumptions can be shown to be equilibrium outcomes of explicit strategic bargaining games, even outside of steady state).

Let V_{0t} and V_{1t} denote expected lifetime utilities for a seller and a buyer at date t and let $D_t = V_{1t} - V_{0t}$. Then, the generalizations of (13) and (14) are

$$rV_{0t} = v_0 + (1-M)y[u(q^*) - q^*] + \dot{V}_{0t} \tag{17}$$

$$rV_{1t} = v_1 + (1-M)\Pi_t \max[u(Q_t) - D_t, 0] + \dot{V}_{1t}. \tag{18}$$

In a monetary equilibrium, take-it-or-leave-it offers imply $Q_t = D_t$, which requires $Q_t \leq \hat{q}$ in order for buyers to be willing to spend their money.

We can reduce the dimensionality of the problem by rearranging (17) and (18) to yield the dynamical system $\dot{D} = \varphi(D_t)$, where

$$\varphi(D_t) = k + rD - (1-M)\Pi \max[u(Q_t) - D_t, 0].$$

A (perfect foresight) equilibrium is a bounded time path for D_t satisfying $\dot{D}_t = \varphi(D_t)$.[13] There is no initial condition in the definition of equilibrium

[13] For an equilibrum D_t must be bounded, because V_{0t} and V_{1t} are bounded by the following argument. At any date, instantaneous utility for sellers cannot exceed $v_0 + (1-M)y[u(q^*) - q^*]$, and for buyers cannot exceed $\max[v_1, u(\hat{q})]$ because $q_t \leq \hat{q}$ (if in

because D_0 can assume any value, as long as the implied time path from this initial value stays bounded.

Note that $\varphi(0) = k$, $\varphi'(D_t) = r$ for $D_t \leq 0$, $\varphi'(0^+) = -\infty$ and $\varphi''(D_t) > 0$ for $D_t > 0$. As we already know from previous analysis, if we assume $k \in (0, \tilde{k})$, then there exist two monetary steady states, denoted (q_l^o, q_h^o), as well as the nonmonetary steady state. This case is shown in Figure 4, from which the following is clear: the set of equilibria includes the three steady states, plus a continuum of non-steady state equilibria indexed by initial beliefs, in the sense that for any initial belief $D_0 \in (-k/r, q_h^o)$ there exists an equilibrium, with $D_t \to q_l^o$. In particular, there exist paths where D starts negative, so that money is initially not accepted, but then D becomes positive and converges to q_l^o.

We now re-introduce government. We set $T = (1,1,1)$ and consider only the policy $q^s \in (0, \hat{q})$, where we suppose that in a monetary equilibrium $q^s < \hat{q}$ and $u(q^s) < D_t$ (of course, this has to be checked once we find a candidate equilibrium). The value functions are

$$\begin{aligned} rV_{0t} &= v_0 + (1-M)y[u(q^*) - q^*] + \dot{V}_{0t} \\ rV_{1t} &= v_1 + (1-\gamma)(1-m_{pt})\Pi \max[u(Q_t) - D_t, 0] \\ &\quad + \gamma(1-m_{gt})\max[u(q^s) - D_t, 0] + \dot{V}_{1t}. \end{aligned}$$

Following the above analysis, we can define a dynamical system as $\dot{D}_t = \psi(D_t)$ where

$$\psi(D_t) = k + rD_t - (1-\gamma)(1-m_{pt})\Pi \max[u(Q_t) - D_t, 0] \quad (19)$$

equilibrium $q_t = D_t > \hat{q}$ then buyers will not trade). Since instantaneous utility is bounded and agents discount, the value functions must be bounded.

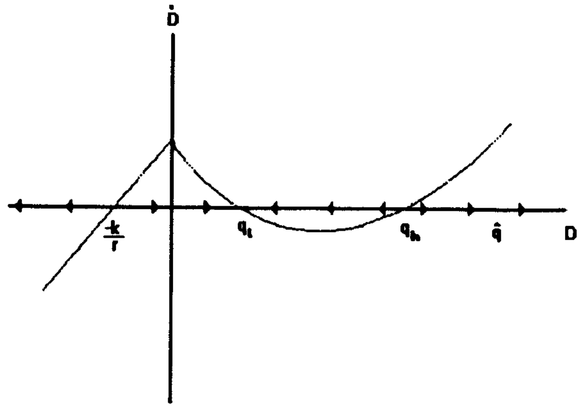

Figure 4: Dynamic Equilibrium without Government

$$-[\gamma - M + (1-\gamma)m_{pt}]\max[u(q^s) - D_t, 0].$$

The model is now complicated by the fact that the measure of private agents holding money can change over time. Using (7), we have

$$\dot{m}_p = \begin{cases} M - m_p & \text{if } \Pi = 1 \\ -[1 - M - (1-\gamma)(1-m_p)]m_p & \text{if } \Pi = 0 \end{cases} \tag{20}$$

(ignoring the time subscripts from now on when there is no risk of confusion). Note that m_p must lie in an interval $[\underline{m}, \overline{m}]$ for all t.[14] An equilibrium is defined to be a bounded path for (D_t, m_{pt}) satisfying (19) and (20), plus an initial condition $m_{p0} \in [\underline{m}, \overline{m}]$, which is given at time 0.

[14]The interval $[\underline{m}, \overline{m}]$ is defined as follows. If $\gamma < 1/2$ then: (1) $[\underline{m}, \overline{m}] = [0, \frac{M}{1-\gamma}]$ if $M < \gamma$; (2) $[\underline{m}, \overline{m}] = [\frac{M-\gamma}{1-\gamma}, \frac{M}{1-\gamma}]$ if $\gamma < M < 1-\gamma$; and (3) $[\underline{m}, \overline{m}] = [\frac{M-\gamma}{1-\gamma}, 1]$ if $1-\gamma < M$. If $\gamma > 1/2$ then: (1) $[\underline{m}, \overline{m}] = [0, \frac{M}{1-\gamma}]$ if $M < 1-\gamma$; (2) $[\underline{m}, \overline{m}] = [0, 1]$ if $1-\gamma < M < \gamma$; and (3) $[\underline{m}, \overline{m}] = [\frac{M-\gamma}{1-\gamma}, 1]$ if $\gamma < M$. Note that $\underline{m} < M < \overline{m}$ in all cases.

Suppose for now that there are two monetary steady states, denoted (q_l, q_h), in addition to the nonmonetary steady state. We start with a standard local analysis by linearizing (19) and (20). The determinant and trace of the Jacobian matrix evaluated at a monetary steady state is

$$\begin{aligned} \det &= (1-\gamma)(1-M)u'(D_t) - (r+1-M) \\ \mathrm{tr} &= r - M - (1-\gamma)(1-M)u'(D_t). \end{aligned}$$

One can show that at q_h we have $\det < 0$, so q_h is a saddle point; and at q_l we have $\det > 0$ and $\mathrm{tr} < 0$, so q_l is a sink. The determinant evaluated at the nonmonetary steady state is

$$\det = -[1 - M - (1-\gamma)(1-2\underline{m})][r + \gamma - M + (1-\gamma)\underline{m}] < 0.$$

Hence, the nonmonetary steady state is a saddle point.

The locus of points in (m_p, D) space along which $\dot{m}_p = 0$ is given by $m_p = M$ if $D > 0$ and $m_p = \underline{m}$ if $D < 0$. The locus of points along which $\dot{D} = 0$ depends on q^s. Figures 5 and 6 show the case where $q^s > q_h^0$ and the case where $q_l^0 < q^s < q_h^0$ (other cases are similar except for the slopes of the different branches of $\dot{D} = 0$). In any case, it can be seen that for any initial m_p, there is a unique initial D on the saddle path leading to the steady state with high D; there is a unique initial D on the saddle path leading to the nonmonetary steady state; and for any initial D between the two saddle paths the system converges to the steady state with low D. All of these paths constitute equilibria. Any initial D that is not between the two saddles paths generates a path that is unbounded and hence not an equilibrium.

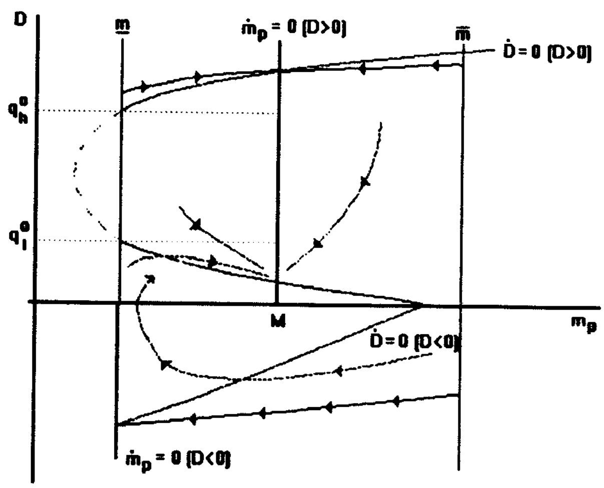

Figure 5: Dynamic Equilibria with Government: $q^s > q_h^0$.

In particular, in the case where the policy is $q^s = q_h^0$. Then the high D steady state is given by $D = q_h^0$, and the saddle path leading to it corresponds to the $\dot{D} = 0$ locus and is horizontal at $D = q_h^0$. Hence, for any initial m_p, there exists an equilibrium where $q_t = q_h^0$ for all t, and $m_p \to M$ monotonically.

As is known from the analysis in the previous sections, if the government is big enough then all steady states except the one with the highest D will disappear. In this case, for all initial m_p, there is a unique equilibrium with D starting on the saddle path converging to the unique steady state (all other choices for starting values of D generate unbounded paths). This is shown in Figure 7, where we set $q^s = q_h^o$, which implies that the saddle path is horizontal at $D = q_h^o$, and the unique equilibrium has constant prices. Notice

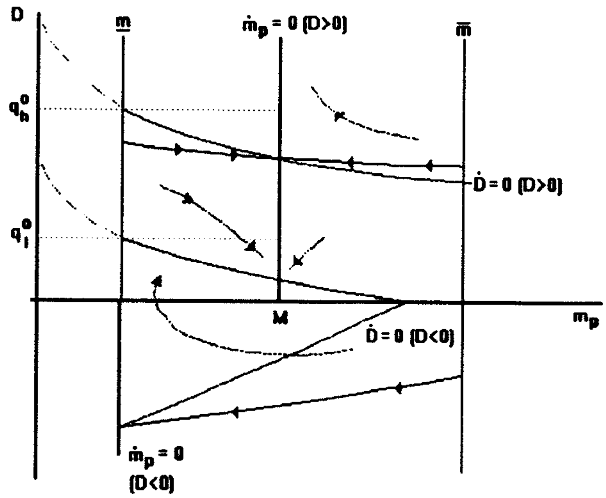

Figure 6: Dynamic Equilibria with Government: $q_l^0 < q^s < q_h^0$.

that in this case each government agent consumes in trades with private agents the same amount that he produces in trades with private agents. But if we start with $m_p > M$ then out of steady state there will be more private buyers than government buyers, and so total government production exceeds total government consumption, and if we start with $m_p < M$ then the opposite is true.

We summarize these results on dynamic equilibria in the following proposition.

Proposition 3 *Suppose there are two monetary steady states and a non-monetary steady state. Then, given policy q^s and any initial condition for m_p, we have the following: there is an equilibrium starting with D on the saddle path converging to (M, q_h); there is an equilibrium starting with D*

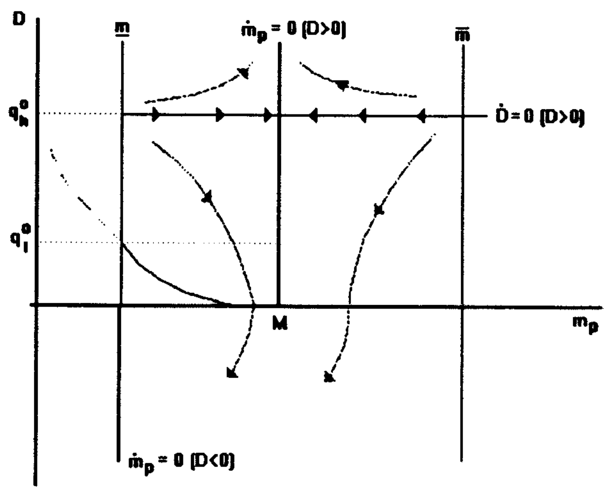

Figure 7: Unique Dynamic Equilibrium with Government

on the saddle path converging to the nonmonetary steady state; and there is a continuum of equilibria starting with any D between the two saddle paths and converging to (M, q_l). *If* γ *is big enough that* (M, q_h) *is the unique steady state, then given any initial condition for* m_p*, the unique equilibrium involves starting on the saddle path converging to* (M, q_h). *In particular, if* $q^s = q_h^0$*, then the unique equilibrium has* $q_t = q_h^0$ *for all* t.

6 Conclusion

This paper has studied the effects of policies that specify what types of transactions the government makes and at what prices. It has been established that when a sufficiently big government accepts a certain money with a high probability and at a favorable price, it can guarantee the existence and the uniqueness of an equilibrium where this money is universally accepted in private transactions, whether or not such an equilibrium exists without intervention. How big is sufficiently big depends on several factors, including properties of the money, the presence of alternative means of payment, and other aspects of policy. Also, by refusing to accept a certain money, a sufficiently big government can preclude an equilibrium where it circulates. Even when government is not so big, so that these policies do not affect the number of equilibria, they will generally affect equilibrium prices and exchange rates.

We did not spend much time discussing why the government might choose to follow particular policies. We simply wanted to describe the correspondence from exogenous government trading strategies to the endogenous strategies of the private sector. Moreover, we did not necessary insist on balancing the budget, in the sense of constraining government agents to consume and produce the same amount, although we did point out when a given policy leads to deficit or surplus. For example, if there is no monetary equilibrium without intervention, the government may be able to use policy to establish one but only by producing more for private agents than private agents produce for the government. Similarly, if they want to influence market exchange rates, this may also entail a deficit. It would be interesting to consider maximizing some objective function, subject to some government budget constraint, by choosing policies designed to get a particular set of objects to circulate as media of exchange, and to influence the prices charged in private transactions. We leave this to future research.

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