Penn Institute for Economic Research
Department of Economics
University of Pennsylvania
3718 Locust Walk
Philadelphia, PA 19103-6297

pier@ssc.upenn.edu
http://www.econ.upenn.edu/pier

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"Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks?"

by

Stephen Morris and Hyun Song

Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks *

Stephen Morris
Department of Economics
University of Pennsylvania
Philadelphia
PA19104, U.S.A.

Hyun Song Shin
Nuffield College, Oxford
OX1 1NF, England

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Abstract

Even though self-fulfilling currency attacks lead to multiple equilibria when fundamentals are common knowledge, we demonstrate the *uniqueness* of equilibrium when speculators face a small amount of noise in their signals about the fundamentals. This unique equilibrium depends not only on the fundamentals, but also on financial variables, such as the quantity of hot money in circulation and the costs of speculative trading. In contrast to multiple equilibrium models, our model allows analysis of policy proposals directed at curtailing currency attacks.

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1. Introduction

The self-fulfilling nature of the belief in an imminent speculative attack has been a dominant theme in the currency crisis literature. Such self-fulfilling beliefs are driven by multiple equilibria. If speculators believe that a currency will come under attack, their actions in anticipation of this attack precipitate the crisis itself, while if they believe that a currency is not in danger of imminent attack, their inaction spares the currency from attack, thereby vindicating their initial beliefs[1]. A number of recent currency crises have refocused attention on the self-fulfilling beliefs scenario. The turmoil in the Exchange Rate Mechanism (ERM) of the European Monetary System in 1992 occurred in a system where it had been widely believed that political forces would sustain existing parities, and in which there had been no recent change in the fundamentals (Eichengreen and Wyplosz (1993)). In Mexico following the December 1994 crisis, U.S. policy was explicitly based on the premise that the attack outcome was just one possible equilibrium outcome (Summers (1995)).

However, the multiplicity of equilibria associated with the scenario of self-fulfilling attacks has limited the usefulness of theoretical models of currency crises[2]. First, the theory has little to offer in assessing alternative policy proposals for curbing speculative attacks. For example, it is often argued that increased capital mobility (induced by lower transaction costs) increases the likelihood of currency crises (Eichengreen, Tobin and Wyplosz (1993)), and that judicious 'throwing of sand' into the excessively well-oiled wheels of international finance will play a role in curbing speculative attacks. However, a multiple equilibrium model is of limited use in this debate, since the question of which equilibrium will transpire is beyond the scope of such a model. Any argument for or against such a proposal has to resort to forces outside the model.

Second, the multiplicity of equilibria makes it impossible to explain the actual *onset* of currency crises when they occur. By most accounts, both the ERM and the Mexican peso were "ripe" for attack for a long time before the crises that brought them down - at least two years in Europe and perhaps a year in Mexico (Dornbusch and Werner (1994)). At any point in those periods, concerted selling by speculators would have raised the costs of maintaining the exchange rate

[1]This theme is explored in Flood and Garber (1984) and Obstfeld (1986, 1994, 1995).

[2]Postlewaite and Vives (1987) discuss the problems of multiple equilibrium models in the closely related context of bank runs. See also Jovanovic (1989).

sufficiently high that the abandonment of the parity would have been forced on the monetary authorities. Once they were abandoned even temporarily, governments would not have had the incentives to restore the pre-crisis parities. Thus attacks would have created profits. Why did the attacks not happen sooner? Conventional accounts have been forced to resort to forces which operate outside the theoretical model in trying to explain the shift of expectations which precipitated the attack. Some of these informal accounts are more persuasive than others, but none can be fully compelling as a theoretical model of the onset of a currency crisis.

We argue in this paper that the apparent multiplicity of equilibria is the consequence of rather strong informational assumptions which imply common knowledge of the fundamentals. A more realistic modelling of the information structure underlying speculative situations eliminates the multiplicity of equilibria. Moreover, we can say something about how this unique outcome depends on the parameters of the problem, such as the costs of speculation, the underlying 'strength' of the economy, and the size of the pool of 'hot money'.

We develop our argument in a model in which a government defends a fixed exchange rate peg, while interacting strategically with a group of speculators. In the benchmark model where there is common knowledge of the state of the fundamentals, we have a tripartite partition of the fundamentals[3]. When the fundamentals are sufficiently favorable, the government will defend the exchange rate (even if all speculators attack it). When the fundamentals are sufficiently unfavorable, the government will abandon the exchange rate (even if no speculator attacks). However, we are most interested in an intermediate zone of values of fundamentals when we say the exchange rate is *ripe for attack*. This is the case where, in the absence of an attack by investors, the government is prepared to incur the costs of defending the currency; but if enough investors sell, the cost of intervention is too large for the government to bear. In this case, there is an equilibrium where the speculators do not attack and the exchange rate is sustained. But there is another where there is a self-fulfilling attack.

We next consider what happens when there is a failure of common knowledge of the state of fundamentals. In particular, we assume that each speculator observes an independent, noisy, signal of the state of fundamentals. With the introduction of the noisy signal, there is a unique equilibrium of the model. We demonstrate that there is a critical state of fundamentals, strictly within the ripe for attack

[3]Obstfeld (1996) argues that this tripartite distinction lies at the heart of the literature on self-fulfilling attacks.

Thus the speculator's payoff is

$$e^* - f(\theta) - t.$$

If the government defends the peg, then the speculator pays the transaction cost, but has no capital gain. Thus his payoff is

$$-t.$$

If the speculator chooses not to attack the currency, his payoff is zero.

The government derives a value $v > 0$ from defending the exchange rate at pegged level, but also faces costs of doing so. The cost of defending the peg depends on the state of the fundamentals, as well as the proportion of speculators who attack the currency. We denote by $c(\alpha, \theta)$ the cost of defending the peg if proportion α of the speculators attack the currency at the state θ. The government's payoff to abandoning the exchange rate is thus zero while the payoff to defending the exchange rate is

$$v - c(\alpha, \theta).$$

We assume that c is continuous and is increasing in α while decreasing in θ. In particular, to make the problem economically interesting we will impose the following assumptions on the cost function and the floating exchange rate $f(\theta)$.

- $c(0,0) > v$. In the worst state of fundamentals, the cost of defending the currency is so high that it exceeds the value v even if no speculators attack.
- $c(1,1) > v$. If all the speculators attack the currency, then even in the best state of the fundamentals, the cost of defending the currency exceeds the value.
- $e^* - f(1) < t$. In the best state of the fundamentals, the floating exchange rate $f(1)$ is sufficiently close to the pegged level e^* such that any profit from the depreciation of the currency is outweighed by the transactions cost t.

Figures 1 and 2 illustrate these features.

[Figures 1 and 2 here]

Let us denote by $\underline{\theta}$ the value of θ which solves $c(0, \theta) = v$. In other words, $\underline{\theta}$ is the value of θ at which the government is indifferent between defending the peg and abandoning it in the absence of any speculative selling. When $\theta < \underline{\theta}$, the cost of defending the currency exceeds the value, even if no speculators attack the currency (see figure 1). At the other end, denote by $\bar{\theta}$ the value of θ at which $f(\theta) = e^* - t$, so that the floating exchange rate is below the peg by the amount of the cost of attack. When $\theta > \bar{\theta}$, then the floating exchange rate is sufficiently close to the peg that a speculator cannot obtain a positive payoff by attacking the currency (see figure 2). Using the two benchmark levels of the state of fundamentals $\underline{\theta}$ and $\bar{\theta}$, we can classify the state of fundamentals under three headings, according to the underlying strategic situation.

2.1. Tripartite Classification of Fundamentals

Assuming that $\underline{\theta} < \bar{\theta}$, we can partition the space of fundamentals into three intervals[4].

- In the interval $[0, \underline{\theta}]$, the value of defending the peg is outweighed by its cost irrespective of the actions of the speculators. The government then has no reason to defend the currency. For this reason, we say that the currency is *unstable* if $\theta \in [0, \underline{\theta}]$.

- In the interval $\left(\underline{\theta}, \bar{\theta}\right)$, the value of defending the currency is greater than the cost, provided that sufficiently few speculators attack the currency. In particular, if none of the speculators attack, then the value of defending the currency is greater than the cost, and so the government will maintain the peg, which in turn justifies the decision not to attack. However, it is also the case that if all the speculators attack the currency, then the cost of defending the currency is too high, and the government will abandon the peg. Moreover, since $\bar{\theta}$ is the right end point of this interval, a speculator will make a positive profit if the government were to abandon the peg at any state θ in the interval $\left(\underline{\theta}, \bar{\theta}\right)$, so that if a speculator believes that the currency peg will be abandoned, then attacking the currency is the rational action. For this reason, we say that the currency is *ripe for attack* if $\theta \in \left(\underline{\theta}, \bar{\theta}\right)$.

[4]This assumption will hold if v is large and t is small.

- Finally, in the interval $[\bar{\theta}, 1]$, although the speculators can force the government to abandon the peg, the resulting depreciation of the currency is so small that they cannot recoup the cost of attacking the currency. Thus, even if a speculator were to believe that the currency will fall, the rational action is to refrain from attacking it. In other words, it is a dominant action not to attack. For this reason, we say that the currency is *stable* if $\theta \in [\bar{\theta}, 1]$.

The economically interesting range of fundamentals is the 'ripe for attack' region. Suppose that the government's decision on whether or not to defend the currency is determined purely by weighing up the costs and benefits, and that it makes its decision once all the speculators have made their decisions. Then, if all the speculators have perfect information concerning the realization of θ, the 'ripe for attack' region gives rise to the standard case of multiple equilibria due to the self-fulfilling nature of the speculators' beliefs. If the speculators believe that the currency peg will be maintained, then it is rational not to attack, which in turn induces the government to defend the currency, thereby vindicating the speculators' decisions not to attack the currency. On the other hand, if the speculators believe that the currency peg will be abandoned, the rational action is to attack the currency, which in turn induces the government to abandon the peg, vindicating the decision to attack. Given this multiplicity of equilibria, no definitive prediction can be made as to whether the currency will come under attack or not. We will now see, however, that the situation is very different when the speculators face a small amount of uncertainty concerning the fundamentals. Each state of fundamentals gives rise to a unique outcome.

2.2. Game with Imperfect Information of Fundamentals

Our paper is concerned with the case where the speculators each have a signal concerning the state of fundamentals, but where there is a small amount of idiosyncratic noise in the signal. The extensive form can be described as follows.

- Nature chooses the state of fundamentals θ according to the uniform density over the unit interval.

- When the true state is θ, a speculator observes a signal x which is drawn uniformly from the interval $[\theta - \varepsilon, \theta + \varepsilon]$, for some small[5] $\varepsilon > 0$. Conditional

[5] In particular, we assume that $2\varepsilon < \min\{\underline{\theta}, 1 - \bar{\theta}\}$.

on θ, the signals are identical and independent across individuals. Based on the signal observed, a speculator decides whether or not to attack the currency.

- The government observes the realized proportion of speculators who attack the currency, α, and observes θ.

The payoffs of the game follow from the description of the model above. We assume that if a speculator is indifferent between attacking and not attacking, he will refrain from attacking and that if the government is indifferent between defending the peg and abandoning it, it will choose to abandon it[6].

An equilibrium for this game consists of strategies for government and for the continuum of speculators such that no player has an incentive to deviate. We can simplify the analysis of this game by solving out the government's strategy at the final stage of the game, to define a reduce-form game between the speculators only. To do this, consider the critical proportion of speculators needed to trigger the government to abandon the peg at state θ. Let $a(\theta)$ denote this critical mass. In the 'unstable' region $a(\theta) = 0$, while elsewhere $a(\theta)$ is the value of α which solves $c(\alpha, \theta) = v$. Figure 3 depicts this function, which is continuous and strictly increasing in θ where it takes a positive value, and is bounded above by 1.

[Figure 3]

The unique optimal strategy for the government is then to abandon the exchange rate only if the observed fraction of sellers, α, is greater than or equal to the critical mass $a(\theta)$ in the prevailing state θ.

Taking as given this optimal strategy for the government, we can characterize the payoffs in the reduced form game between the speculators. For a given profile of strategies of the speculators, we denote by

$$\pi(x)$$

the proportion of speculators who attack the currency when the value of the signal is x. We denote by $s(\theta, \pi)$ the proportion of speculators who end up attacking

[6] Nothing substantial hinges of these assumptions, which are made for purposes of simplifying the statement of our results.

the currency when the state of fundamentals is θ, given aggregate selling strategy π. Since signals are uniformly distributed over $[\theta - \varepsilon, \theta + \varepsilon]$ at θ, we have

$$s(\theta, \pi) = \int_{\theta-\varepsilon}^{\theta+\varepsilon} \pi(x) dx. \tag{2.1}$$

Now we denote by $A(\pi)$ the event where the government abandons the currency peg if the speculators follow strategy π and the government follows its optimal strategy:

$$A(\pi) = \{\theta | s(\theta, \pi) \geq a(\theta)\}. \tag{2.2}$$

Using this definition of the event $A(\pi)$, we can define the payoffs of a reduced form game between the speculators. The payoff to a speculator of attacking the currency at state θ when aggregate short sales are given by π is

$$h(\theta, \pi) \equiv \begin{cases} e^* - f(\theta) - t & \text{if } \theta \in A(\pi) \\ -t & \text{if } \theta \notin A(\pi) \end{cases} \tag{2.3}$$

However, a speculator does not observe θ directly. The payoff to attacking the currency must be calculated from the posterior distribution over the states conditional on the signal x. The expected payoff to attacking the currency conditional on the signal x is given by the expectation of (2.3) conditional on x. Denoting this by $u(x, \pi)$, we have

$$\begin{aligned} u(x, \pi) &= \frac{1}{2\varepsilon} \int_{x-\varepsilon}^{x+\varepsilon} h(\theta, \pi) d\theta \\ &= \frac{1}{2\varepsilon} \left[\int_{A(\pi) \cap [x-\varepsilon, x+\varepsilon]} (e^* - f(\theta)) \, d\theta \right] - t. \end{aligned} \tag{2.4}$$

Since a speculator can guarantee a payoff of zero by refraining from attacking the currency, the rational decision conditional on signal x depends on whether $u(x, \pi)$ is positive or negative. Thus if the government follows its unique optimal strategy, π is an equilibrium of the first period game if $\pi(x) = 1$ whenever $u(x, \pi) > 0$ and $\pi(x) = 0$ whenever $u(x, \pi) \leq 0$.

3. Unique Equilibrium

We now state the main result of our paper, noting the contrast between the multiplicity of possible outcomes when there is perfect information of the fundamentals against the uniqueness of outcome when there is a small amount of noise.

Theorem 1. There is a unique θ^* such that, in any equilibrium of the game with imperfect information, the government abandons the currency peg if and only if $\theta \leq \theta^*$.

The argument for our result can be presented in several steps. We start with the following intuitive result.

Lemma 1. If $\pi(x) \geq \pi'(x)$ for all x, then $u(x, \pi) \geq u(x, \pi')$ for all x.

In other words, if we compare two strategy profiles π and π', where π entails a greater proportion of speculators who attack for any message x, then the payoff to attacking the currency is greater given π than when it is given by π'. Thus speculators' decisions to attack the currency are strategic complements.

Proof of Lemma 1. Since $\pi(x) \geq \pi'(x)$, we have $s(\theta, \pi) \geq s(\theta, \pi')$ for every θ, from the definition of s given by (2.1). Thus, from (2.2),

$$A(\pi) \supseteq A(\pi').$$

In other words, the event in which the currency peg is abandoned is strictly larger under π. Then, from (2.4) and the fact that $e^* - f(\theta)$ is non-negative,

$$\begin{aligned} u(x, \pi) &= \tfrac{1}{2\varepsilon}\left[\int_{A(\pi)\cap[x-\varepsilon, x+\varepsilon]} (e^* - f(\theta))\, d\theta\right] - t \\ &\geq \tfrac{1}{2\varepsilon}\left[\int_{A(\pi')\cap[x-\varepsilon, x+\varepsilon]} (e^* - f(\theta))\, d\theta\right] - t \\ &= u(x, \pi'). \end{aligned}$$

which proves the lemma.

For the next step in our argument, we introduce a simple class of strategies for a speculator that6 will play a prominent part in our analysis. Consider the strategy profile where every speculator attacks the currency if and only if the message x is greater than some fixed number k. Then, aggregate short sales π will be given by the indicator function I_k, defined as

$$I_k(x) = \begin{cases} 1 & \text{if} \quad x < k \\ 0 & \text{if} \quad x \geq k \end{cases} \tag{3.1}$$

When speculators follow this simple rule of action, the expected payoff to attacking the currency satisfies the following property.

Lemma 2. $u(k, I_k)$ is continuous and strictly decreasing in k.

In other words, when aggregate short sales is governed by I_k, and we consider the payoff to attacking the currency given the marginal message k, this payoff is decreasing as the fundamentals of the economy become stronger. Put another way, when the fundamentals of the economy are stronger, the payoff to attacking the currency is lower for a speculator on the margin of switching from attacking to not attacking. Such a property would be a reasonable feature of any model of currency attacks where the government is able to resist speculators better when the fundamentals are stronger.

The proof of lemma 2 is simple but involves some algebraic manipulation, and hence is presented separately in the appendix. Taking lemma 2 as given, we can then prove the following result.

Lemma 3. There is a unique x^* such that, in any equilibrium of the game with imperfect information of the fundamentals, a speculator with signal x attacks the currency if and only if $x < x^*$.

In proving this result, we begin by establishing that there is a unique value of k at which

$$u(k, I_k) = 0.$$

From lemma 2, we know that $u(k, I_k)$ is continuous and strictly decreasing in k. If we can show that it is positive for small values of k and negative for large values, then we can guarantee that $u(k, I_k) = 0$ for some k. When k is sufficiently small (i.e. $k \leq \underline{\theta} - \varepsilon$), the marginal speculator with message k knows that the true state of fundamentals is in the 'unstable' region, since such a message is consistent only with a realization of θ in the interval $[0, \underline{\theta}]$. Since the payoff to attacking the currency is positive at any θ in this interval, we have $u(k, I_k) > 0$. Similarly, when k is sufficiently large (i.e. $k \geq \bar{\theta} + \varepsilon$), the marginal speculator with message k knows that the true state of fundamentals is in the 'stable' region. Since the payoff to attacking is negative at every state in this region, we have $u(k, I_k) < 0$. Hence, there is a unique value of k for which $u(k, I_k) = 0$, and we define the value

$$x^*.$$

as this unique solution to $u(k, I_k) = 0$.

Now, consider any equilibrium of the game, and denote by $\pi(x)$ the proportion of speculators who attack the currency given message x. Define the numbers $\underline{x}$ and $\bar{x}$ as

$$\begin{aligned} \underline{x} &= \inf\{x|\pi(x) < 1\} \\ \bar{x} &= \sup\{x|\pi(x) > 0\} \end{aligned}$$

Now $\bar{x} \geq \sup\{x|0 < \pi(x) < 1\} \geq \inf\{x|0 < \pi(x) < 1\} \geq \underline{x}$, so that

$$\underline{x} \leq \bar{x}. \tag{3.2}$$

When $\pi(x) < 1$, there are some speculators who are not attacking the currency. This is only consistent with equilibrium behaviour if the payoff to not attacking is at least as high as the payoff to attacking given message x. By continuity, this is true at $\underline{x}$ also. In other words,

$$u(\underline{x}, \pi) \leq 0. \tag{3.3}$$

Consider the payoff

$$u(\underline{x}, I_{\underline{x}}).$$

Clearly, $I_{\underline{x}} \leq \pi$, so that lemma 1 and (3.3) imply

$$u(\underline{x}, I_{\underline{x}}) \leq u(\underline{x}, \pi) \leq 0.$$

Thus, $u(\underline{x}, I_{\underline{x}}) \leq 0$. Since we know from lemma 2 that $u(k, I_k)$ is decreasing in k and x^* is the unique value of k which solves $u(k, I_k) = 0$, we have

$$\underline{x} \geq x^*. \tag{3.4}$$

When $\pi(x) > 0$, there are some speculators who are attacking the currency given message x. This implies that the payoff to attacking is at least as high as that from not attacking. By continuity, this is true at $\bar{x}$ also. That is,

$$u(\bar{x}, \pi) \geq 0. \tag{3.5}$$

Now, consider the payoff $u(\bar{x}, I_{\bar{x}})$. Since $I_{\bar{x}} \geq \pi$, lemma 1 and (3.5) imply

$$u(\bar{x}, I_{\bar{x}}) \geq u(\bar{x}, \pi) \geq 0.$$

Thus, $u(\bar{x}, I_{\bar{x}}) \geq 0$. Since $u(k, I_k)$ is decreasing in k, and $u(x^*, I_{x^*}) = 0$, we have

$$\bar{x} \leq x^*. \tag{3.6}$$

Thus, from (3.4) and (3.6), we have $\underline{x} \geq x^* \geq \bar{x}$. However, we know from (3.2) that this implies

$$\underline{x} = x^* = \bar{x}.$$

Thus, the equilibrium π is given by the step function I_{x^*}, which is what lemma 3 states. Hence, this proves lemma 3.

From this, it is a short step to the proof of the main theorem itself. Given that equilibrium π is given by the step function I_{x^*}, the aggregate short sales at the state θ is given by

$$s(\theta, I_{x^*}) = \begin{cases} 1 & \text{if } \theta < x^* - \varepsilon \\ \frac{1}{2} - \frac{1}{2\varepsilon}(\theta - x^*) & \text{if } x^* - \varepsilon \leq \theta < x^* + \varepsilon \\ 0 & \text{if } \theta \geq x^* + \varepsilon \end{cases}$$

Aggregate short sales $s(\theta, I_{x^*})$ is decreasing in θ when its value is strictly between 0 and 1, while $a(\theta)$ is increasing in θ for this range.

[Figure 4 here]

We know that $x^* > \underline{\theta} - \varepsilon$, since otherwise attacking the currency is a strictly better action, contradicting the fact that x^* is a switching point. Thus, $s(\theta, I_{x^*})$ and $a(\theta)$ cross precisely once. Define θ^* to be the value of θ at which these two curves cross. Then, $s(\theta, I_{x^*}) \geq a(\theta)$ if and only if $\theta \leq \theta^*$, so that the government abandons the currency peg if and only if $\theta \leq \theta^*$. This is the claim of our main theorem.

4. Comparative Statics and Policy Implications

Our formal model showed how adding noise to a simple model of self-fulfilling attacks led to a unique equilibrium. The model was static. But if we imagine the model being repeated through time (with a new draw of θ each period), comparative static analysis tells us how outcomes depend on exogenous features of the model. This in turn allows us to evaluate the implications of various policies for coping with currency attacks.

In this section, we consider how the information structure, the size of transaction costs and the volume of hot money affect the equilibrium we identified. Each of these exercises corresponds to a debated policy question, and we draw out the policy implications of our analysis.

4.1. Changes in the Information Structure

When there is no noise, there are multiple equilibria throughout the 'ripe for attack' region of fundamentals. But when there is positive noise, there is a unique equilibrium with critical value θ^*. The value of θ^* is always strictly within the 'ripe for attack' regions. In the limit as ε tends to zero, θ^* has a particularly simple characterization.

Theorem 2. In the limit as ε tends to zero, θ^* is given by the unique solution to the equation

$$f(\theta^*) = e^* - 2t.$$

The proof is in the appendix. A rough intuition for this result can be gained by considering the marginal speculator who observes message $x = \theta^*$. With ε small, this tells the speculator that the true θ is close to θ^*. Since the government abandons the peg if and only if θ is less than θ^*, he attaches equal probability to the currency being abandoned and defended. So, the expected payoff to attacking is $\frac{1}{2}(e^* - f(\theta^*))$, while the cost is t. For the marginal speculator, these are equal, leading to the equation in theorem 2.

Why does noise have such a big role in characterizing equilibrium? If there is no noise, there is common knowledge of the value of θ among the speculators. But with noise, it is never common knowledge in our model that the fundamentals are consistent with the government maintaining the currency peg, (i.e. $\theta \geq \underline{\theta}$). This is so however small the noise is, and however large your signal is. Suppose that you observe a signal x which is much greater than $\underline{\theta}$. If you observe a signal greater than $\underline{\theta} + \varepsilon$, then you can *know* that $\theta \geq \underline{\theta}$, since your message has the margin of error of ε. Now, when do you know that *everyone* knows that $\theta \geq \underline{\theta}$? In other words, when do you know that everyone has observed a signal greater than $\underline{\theta} + \varepsilon$? Since others' signals can differ from yours by at most 2ε, this will be true if you observe a signal greater than $\underline{\theta} + 3\varepsilon$. This argument is clearly one which will iterate. Proceeding in this way, we can see that there is "nth order knowledge" that $\theta \geq \underline{\theta}$ (i.e. everyone knows that everyone knows... (n times) that everyone knows it) exactly if everyone has observed a signal greater than or equal to $\underline{\theta} + (2n-1)\varepsilon$. But, by definition, there is common knowledge that $\theta \geq \underline{\theta}$ if and only if there is nth order knowledge for every n. But for any fixed ε and signal x, there will be some level n at which nth order iterated knowledge fails. Thus it is never common knowledge that θ is not in the unstable region.

Common knowledge allows the multiplicity of equilibria within the 'ripe for attack' region. Our analysis showed that in noisy environments, the breakdown of common knowledge allowed a unique prediction[7]. One interpretation we may put on the case of noisy information is that the recipients of the differential information learn of the true underlying fundamentals of the economy with little error, but that there are small discrepancies in how these messages are interpreted by the recipients. This interpretation has implications both for explaining the timing of currency attacks and for government informational policy.

Our analysis suggests that if there is common knowledge that fundamentals are consistent with the government maintaining the exchange rate policy, an exchange rate peg may be sustainable even within the 'ripe for attack' zone. When looking for a cause or a trigger for a currency attack, we should look for the arrival of noisy information, i.e. news events that are not interpreted in exactly the same way by different speculators. The informational events which matter may be quite subtle. A 'grain of doubt,' allowing that others may believe that the economy is, in fact, unstable will lead to a currency crises even if everyone knows that the economy is not unstable. In predicting when crises will occur, average opinion or even extreme opinion need not precipitate a crisis. Rather, what matters is the higher order beliefs of some participants who are apprehensive about the beliefs of others, concerning the beliefs of yet further individuals, on these extreme opinions.

This interpretation sheds light on some accounts of recent currency crises. Rumors of political trouble in Chiapas province were widely cited as a cause of the 1994 Mexico crisis (New York Times (1994)); uncertainty about 'Maastricht,' German unification and Bundesbank pronouncements were argued to be important in the 1992 European Monetary System crises. Our analysis suggests that such 'informational events' might precipate a crisis even if no investor thought they they conveyed real information about fundamentals themselves. It is enough that the announcements remove common knowledge that the fundamentals were sustainable.

[7] Game theorists have explored this theme in some detail for two player two action games (Carlsson and van Damme (1993) and Morris, Rob and Shin (1995)).

The subtle role of the information structure in currency crises suggests an important role for public announcements by the monetary authorities, and more generally, the transparency of the conduct of monetary policy and its dissemination to the public. If it is the case that the onset of currency crises may be precipitated by higher order beliefs, even though participants believe that the fundamentals are sound, then the policy instruments which will stabilize the market are those which aim to restore transparency to the situation, in an attempt to restore common knowledge of the fundamentals. The most effective means towards this would be a prominent public announcement which is commonly known to convey information to all relevant participants. The canonical case of a communication arrangement which would be conducive to achieving common knowledge is the 'town hall meeting' in which an announcement is made to an audience gathered in a single room, where everyone can observe that all other participants are in an identical position. In contrast, if the audience is fragmented, and must communicate in small groups, common knowledge is extremely difficult to achieve.[8] The Clinton administration's announcement of the $40 billion dollar rescue package for the Mexican peso can be seen as an attempt to restore the sort of transparency referred to above. Its effectiveness derived more from its very public nature, rather than the actual sum of money involved. This suggests a crucial role for the timely and effective dissemination of information on the part of policy makers, and the smooth functioning of a reliable and transparent set of communication channels between market participants and the policy makers, as well as between the market participants themselves.

4.2. Changes in Transaction Costs

How does the critical level of fundamentals as the transaction cost t varies? For simplicity, we analyze what happens in the limit as ε becomes small (qualitatively similar results hold for large ε). Drawing on theorem 2, which determines θ^* in the limit as ε becomes small, we can totally differentiate the equation $f(\theta^*) = e^* - 2t$ to obtain

$$\frac{d\theta^*}{dt} = -\frac{2}{f'(\theta^*)}.$$

[8]Rubinstein's (1989) e-mail game is a case in point, and Chwe (1996) has suggested some of the relevant factors which would allow us to address this issue in a more general context.

Thus increasing transaction costs prevents currency crises, since it reduces the range of fundementals where an attack occurs. The size of this effect depends on the slope of f: when the f' is small, an increase in the cost of speculation has a large effect on the switching point θ^*.

This suggests that the imposition of small transactions costs as advocated by several commentators will have a large impact on the prevalence of speculation precisely when the *consequences* of such speculative attacks is small. If speculative attacks are predicted to lead to drastic effects (i.e. when $f(\theta)$ is a steep function of θ), then the imposition of a small additional cost is unlikely to have a large effect on the incidence of currency attacks.

4.3. Changes in aggregate wealth

So far in this paper, we have kept the level of aggregate wealth of the speculators constant. Let us now consider how our analysis is affected when this aggregate wealth varies. The international flow of so-called 'hot money' would be one factor in determining this aggregate wealth, as well as changes in the numbers of speculators themselves. The main effect of a change in aggregate wealth of the speculators is a change in the function $a(\theta)$, which indicates the critical proportion of speculators needed to attack the currency in order to induce the government to abandon the currency peg. When the aggregate wealth of the speculators increases, then this critical proportion of speculators falls, since the government's decision is based on the absolute *level* of short sales.

As can be seen from figure 4, a downward shift in the $a(\theta)$ function has the effect of enlarging the set of states at which the government abandons the exchange rate peg. In other words the event

$$A(\pi) = \{\theta | s(\theta, \pi) > a(\theta)\}$$

is strictly larger with a lower $a(.)$ function. Since the payoff to speculation is given by

$$\int_{A(\pi)\cap[x-\varepsilon,x+\varepsilon]} (e^* - f(\theta))\, d\theta - t,$$

the enlargement of the event $A(\pi)$ has an unambiguous effect in increasing the payoff to attacking the currency at any value of the signal x. Thus, the benchmark value θ^* is shifted to the right, and the incidence of speculative attacks increases.

Note, however, from figure 4 that the effect of an increase in the $a(.)$ function depends on the size of the noise ε. The effect is largest when ε is also large. In the limiting case when ε tends to zero, the equilibrium $s(\theta, I_{x^*})$ becomes the step function I_{x^*}, so that a shift in the $a(.)$ function has no effect on the switching point θ^*. Thus, our analysis suggests that changes in the aggregate wealth of speculators need not have a large impact on the incidence of currency attacks when the speculators have fairly precise information concerning the fundamentals. It is when the noise is large that shifts in wealth have a big impact.

This suggests that the imposition of direct capital controls work best when there is a lack of 'transparency' of the economic fundamentals, in the sense that observers differ widely in their interpretation of the economic fundamentals. When the fundamentals are relatively transparent to all (corresponding to a small ε), direct capital controls seem far less effective. Under such circumstances, strategic considerations dominate any uncertainty concerning the fundamentals.

5. Conclusion

Existing models of currency attacks which focus on fundamentals ignore the role of speculators' beliefs about other speculators' behavior. Existing self-fulfilling beliefs models of currency attacks assume that speculators *know* (in equilibrium) exactly what other speculators will do. Neither feature is realistic. Our model takes neither extreme. Because there is some uncertainty about equilibrium, speculators are uncertain as to exactly what other speculators will do; but their behavior depends non-trivially on what they believe they will do. Because our model of self-fulfilling currency attacks is consistent with unique equilibrium, we are able to analyze the impact of alternative policies.

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Appendix

Proof of lemma 2. Consider the function $s(\theta, I_k)$, which gives the proportion of speculators who attack the currency at θ when the aggregate short sales is given by the step function I_k. Since x is uniformly distributed over $[\theta-\varepsilon, \theta+\varepsilon]$, we have

$$s(\theta, I_k) = \begin{cases} 1 & \text{if } \theta \leq k-\varepsilon \\ \frac{1}{2} - \frac{1}{2\varepsilon}(\theta - k) & \text{if } k-\varepsilon \leq \theta \leq k+\varepsilon \\ 0 & \text{if } \theta \geq k+\varepsilon \end{cases} \tag{A1}$$

If aggregate short sales are given by I_k, there is a unique θ (which depends on k) where the mass of speculators attacking equals the mass of speculators necessary to cause the government to abandon the exchange rate (where $s(\theta, I_k) = a(\theta)$). Write $\psi(k)$ for the amount that θ must exceed k in order for this to be true. In other words, $\psi(k)$ is the unique value of ψ solving $s(k+\psi, I_k) = a(k+\psi)$. Observe that $\psi(k) = \varepsilon$ if $k \leq \underline{\theta} - \varepsilon$, while if $k > \underline{\theta} - \varepsilon$, then $-\varepsilon < \psi(k) < \varepsilon$ and is the value of ψ solving $\left(\frac{1}{2} - \frac{\psi}{2\varepsilon}\right) = a(k+\psi)$.

Since the government abandons the currency peg if and only if θ lies in the interval $[0, k+\psi(k))$, the payoff function $u(k, I_k)$ is given by

$$\frac{1}{2\varepsilon}\left[\int_{k-\varepsilon}^{k+\psi(k)} (e^* - f(\theta))\, d\theta\right] - t. \tag{A2}$$

Since $e^* - f(\theta)$ is strictly decreasing in θ, if we can show that $\psi(k)$ is weakly decreasing in k, this will be sufficient to show that $u(k, I_k)$ is strictly decreasing in k.

To demonstrate that $\psi(k)$ is weakly decreasing in k, totally differentiate the equation $\left(\frac{1}{2} - \frac{\psi}{2\varepsilon}\right) = a(k+\psi)$ with respect to k, to obtain $-\frac{1}{2\varepsilon}\psi'(k) = a'(\theta)(1+\psi'(k))$. Hence,

$$\psi'(k) = -\frac{a'(\theta)}{a'(\theta) + \frac{1}{2\varepsilon}} \leq 0,$$

which is sufficient for $u(k, I_k)$ to be strictly decreasing in k. Finally, the continuity of $u(k, I_k)$ follows immediately from the fact that it is an integral in which the limits of integration are themselves continuous in k. This completes the proof of lemma 2.

Proof of theorem 2. Consider the switching point x^*, which is the solution to the equation

$$u(x^*, I_{x^*}) = 0. \tag{A3}$$

Then, writing $F(\varepsilon) = \int_{A(I_{x^*})\cap[x^*-\varepsilon, x^*+\varepsilon]} (e^* - f(\theta))\, d\theta$, we can express this equation as

$$\frac{F(\varepsilon)}{2\varepsilon} - t = 0.$$

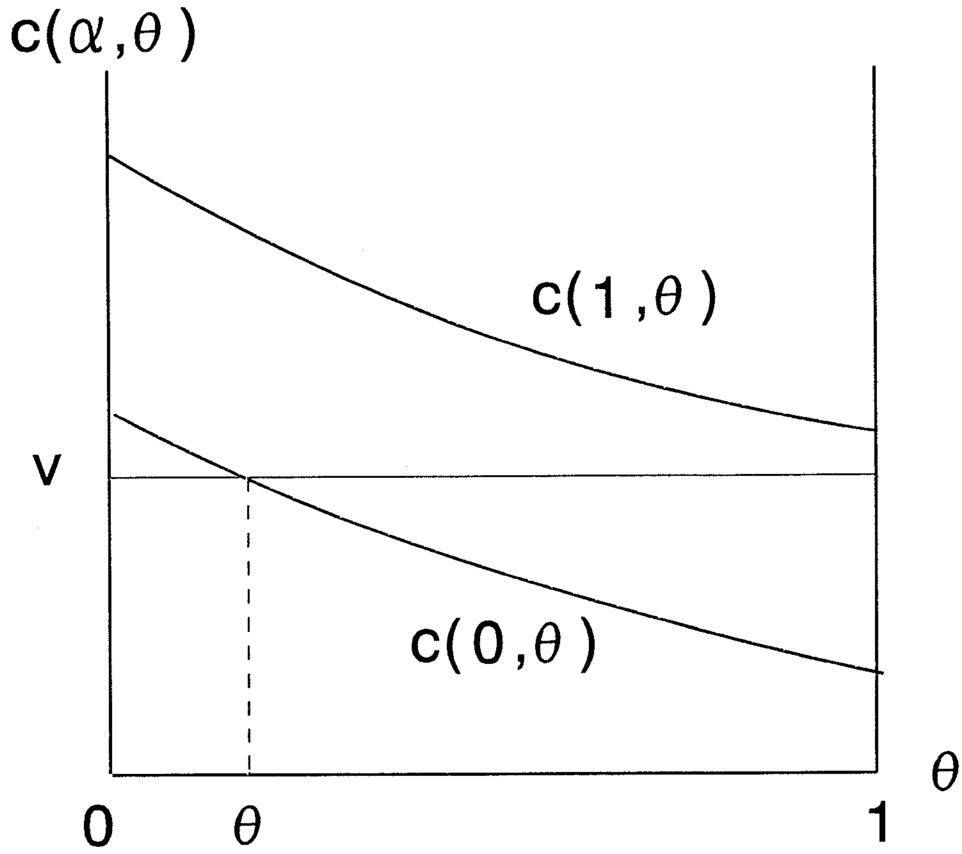
By using L'Hôpital's rule,

$$\lim_{\varepsilon\to 0} \frac{F(\varepsilon)}{2\varepsilon} = \frac{F'(0)}{2} = \frac{e^* - f(x^*)}{2}.$$

Thus, in the limit as $\varepsilon \to 0$, equation (A3) yields:

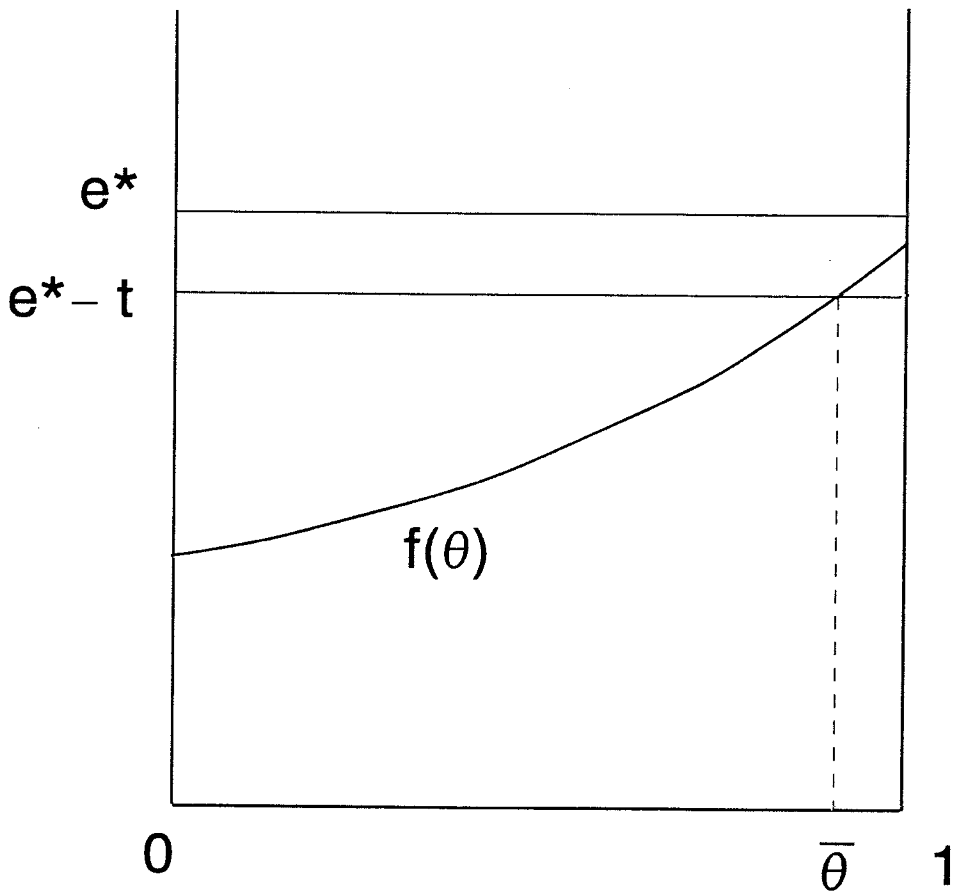
$$f(x^*) = e^* - 2t.$$

Finally, we note that x^* converges to θ^* when ε tends to zero, since in the limit, $s(\theta, I_{x^*}) = I_{x^*}$. This completes the proof of theorem 2.



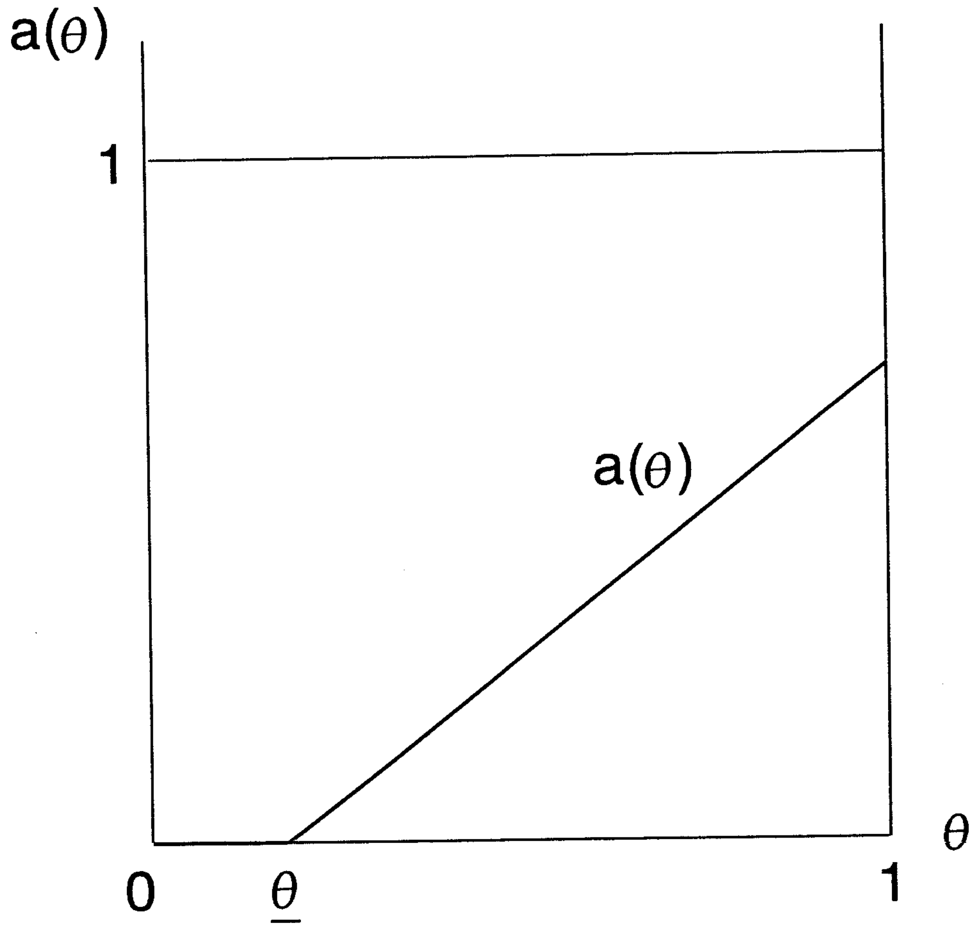
[Figure 1]

Figure 1 for "Unique Equilibrium in a Model of Self-fulfilling Currency Attacks" by Stephen Morris and Hyun Song Shin



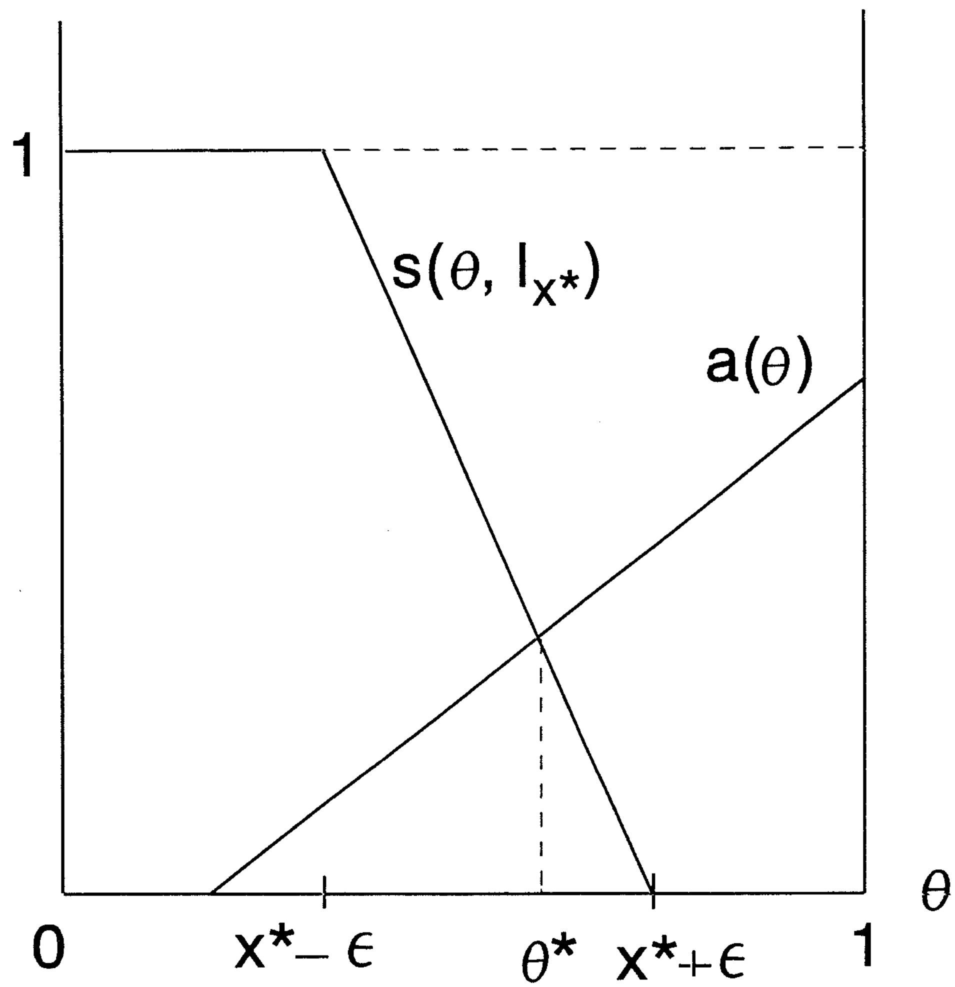
[Figure 2]

Figure 2 for "Unique Equilibrium in a Model of Self-fulfilling Currency Attacks" by Stephen Morris and Hyun Song Shin



[Figure 3]

Figure 3 for "Unique Equilibrium in a Model of Self-fulfilling Currency Attacks" by Stephen Morris and Hyun Song Shin



[Figure 4]

Figure 4 for "Unique Equilibrium in a Model of Self-fulfilling Currency Attacks" by Stephen Morris and Hyun Song Shin