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“Mediocracy”
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by

Andrea Mattozzi and Antonio Merlo

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Mediocracy∗

Andrea Mattozzi and Antonio Merlo†

Abstract

We study the recruitment of individuals in the political sector. We propose an equilibrium model of political recruitment by two political parties competing in an election. We show that political parties may deliberately choose to recruit only mediocre politicians, in spite of the fact that they could select better individuals. Furthermore, we show that this phenomenon is more likely to occur in proportional than in majoritarian electoral systems.

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†California Institute of Technology, Pasadena, CA 91125, <andrea@hss.caltech.edu>; University of Pennsylvania, Philadelphia, PA 19104, <merloa@econ.upenn.edu>
We'd all like to vote for the best man, but he is never a candidate. F. McKinney Hubbard

Our current political system ensures not that the worst will get on top – though they often do – but that the best will never even apply. Paul Jacob

1 Introduction

The quality of politicians has long been an issue of great concern in all democracies. A widespread sentiment summarized by the opening quotes above is that by and large the political class is typically not the best a country has to offer. Several recent studies have also documented that the quality of politicians varies significantly across countries, and that part of this variation is related to differences in the electoral system. For example, Persson, Tabellini, and Trebbi (2006) find that in a sample of 80 democracies, corruption of elected officials is higher in political systems with proportional representation than in majoritarian systems. Gagliarducci, Nannicini, and Naticchioni (2008) find that Italian politicians elected under proportional representation have higher absenteeism rates than their counterparts elected under plurality rule.¹

In this paper, we provide a novel explanation for these phenomena by focusing on the recruitment of individuals in the political sector and studying the effects of different electoral systems on the incentives of political parties to select good politicians. We propose an equilibrium model of political recruitment by two political parties competing in an election. We show that competing parties may deliberately choose not to recruit the best politicians both in proportional and majoritarian electoral systems. However, a mediocre equilibrium selection is more likely to arise in proportional systems.

In most countries, relatively few individuals start off their political careers by running for a public office. More frequently, they first test their political aspirations by holding positions within party organizations, which represent “breeding grounds” from which the vast majority of elected officials come from. The role of party service as an essential

¹Galasso, Landi, Mattozzi, and Merlo (2009) also document that the fraction of legislators without a high school degree is significantly larger in the Italian Parliament (which is elected under proportional representation), than in the United States Congress (which is elected with a majoritarian system). However, this is not the case in the general population, where the fraction of high school dropouts in the two countries is comparable (see, e.g., Checchi, Ichino, and Rustichini (1999)).
qualification for pursuing a political career is especially important in countries with a strong party system, such as, for example, Australia, Germany, Italy, Japan, the Netherlands, Sweden, and the U.K.\(^2\) In these countries, the individuals who are recruited by political parties determine the quality of the pool of potential electoral candidates.\(^3\)

As pointed out, for example, by Strom (1990), among others, political parties are “going concerns” and “successful political parties require extensive organizational capabilities [...] to meet the different needs faced by aspiring politicians under competitive circumstances” (p. 575). While the success of political parties ultimately depends on their electoral success, the very existence and survival of party organizations hinge on the willingness of their members to exert their best effort on the party’s behalf and perform a variety of services including gathering and disseminating information, organizing and mobilizing supporters, and raising funds. Given the limited availability of direct monetary compensation, the main incentive a party has to offer to reward such effort is the party electoral nomination. We show that these considerations entail a fundamental trade-off which may play an important role in a party’s recruiting decisions. On the one hand, recruiting the best possible individuals may enhance the party’s electoral prospects in a competitive electoral environment (\textit{competition effect}). On the other hand, recruiting a relatively “mediocre” but homogeneous group of individuals may maximize their collective effort on behalf of the party since the presence of “superstars” may discourage other party members and induce them to shirk (\textit{discouragement effect}). In equilibrium, there will either be “mediocracy” if parties choose not to recruit the best politicians, or “aristocracy” if they do.\(^4\)

\(^2\)Norris and Lovenduski (1995) document that in the 1992 British general election, about 95% of Labour candidates and 90% of Conservative candidates had held a position within the party. Rydon (1986) and Cotta (1979) suggest similar levels of party involvement among members of parliament in Australia and in Italy, respectively. See also Best and Cotta (2000). In other countries, like for example, Canada, Finland, and the U.S., party service is not necessarily a pre-requisite for advancement in political careers. Even in these countries, however, the fraction of party professionals in the political sector has grown considerably over the years. See, e.g., Norris (1997).

\(^3\)“Competitive democratic elections offer citizens a choice of alternative parties, governments and policies. [...] Which candidates get on the ballot, and therefore who enters legislative office, depends on the prior recruitment process. [...] In most countries recruitment usually occurs within political parties, influenced by party organizations, rules and culture.” Norris (1997) (pp. 1-14).

\(^4\)According to the Webster’s Third New International Dictionary of the English language, mediocracy is defined as: “rule by the mediocre.” Aristocracy, from the Greek word aristokratiā, is defined as: “the government of the best.”
case, parties never recruit the worst politicians. Because of their winner-takes-all nature, 
majoritarian electoral systems are more competitive than proportional systems, thus mak-
ing the electoral returns to candidates’ quality relatively higher and hence mediocracy less 
likely.\(^5\)

Before describing our model of political recruitment, it is important to stress that 
political ability is a rather vague concept, which is very difficult to define, let alone quantify. 
While there is little doubt that competence, honesty, and integrity should all represent 
positive traits of a politician, there is no obvious way to define unambiguously what it 
takes to be a good politician. In this paper, we adopt a fairly general approach and define 
political ability as the marginal cost of exerting effort in the political sector. We believe 
that this definition captures several characteristics that jointly define political ability.\(^6\) 
Furthermore, we assume that political ability is observable by parties. Indeed, people 
who are potentially interested in becoming politicians typically begin their involvement in 
politics by engaging in a variety of voluntary political activities that are organized and 
monitored by political parties (e.g., student political organizations, campaign teams, party 
internships). These activities thus provide opportunities for a political party to observe the 
political skills of individuals it may be potentially interested in recruiting.

The remainder of the paper is organized as follows. In Section 2, we review the related 
literature. In Section 3, we present the model. In Section 4, we analyze a simplified version

\(^5\)In his survey on political selection, Besley (2005) suggests that electoral competition may discourage 
a party from selecting a bad candidate: “Candidates are typically chosen by political parties. This fact 
raises the question of why a party would ever put a bad candidate up for election. One possibility is that if 
rents are earned by parties as well as successful candidates, and protection of those rents is dependent on 
selecting bad politicians with little public service motivation, then the party may have an interest in putting 
up bad candidates. The problem that parties face in making this choice arises from the risk that voters 
will choose the other party” (p. 55). Our analysis identifies a fundamental trade-off between electoral 
and organizational concerns of political parties and shows how the competitiveness of elections affects the 
parties’ recruitment decisions and ultimately the quality of elected representatives.

\(^6\)For example, a high-ability politician is most probably successful in raising funds on behalf of the 
party. Also, a high-ability politician will effectively contribute in shaping the party’s electoral platform. 
Furthermore, if nominated as an electoral candidate, a high-ability politician will most likely be able to run 
a successful campaign and attract votes for his party. As Besley (2005) argues: “the idea that potential 
politicians differ in their competence is no different from a standard assumption in labor market models 
that individual have specific skills so that they will perform better or worse when matched in certain jobs” 
(p. 48). This line of research has been pursued by Mattozzi and Merlo (2008) in their study of the careers 
of politicians.
of the model where elections are uncontested. This allows us to abstract from electoral competition and illustrate the discouragement effect. In Section 5, we introduce electoral competition and present our main results. We conclude in Section 6 with a discussion of possible extensions. The proofs are in the Appendix.

2 Related Literature

Our paper is related to the literature on the endogenous selection of politicians (see, e.g., the survey by Besley (2005)). Acemoglu, Egorov, and Sonin (2009) study the dynamic selection of governments under alternative political institutions (i.e., democratic vs non-democratic societies) and show that any deviation from perfect democracy may lead to an incompetent government in office being a stable and persistent outcome because of the dynamics of government formation. Caselli and Morelli (2004), Mattozzi and Merlo (2008) and Messner and Polborn (2004) focus on majoritarian elections, provide alternative explanations for why bad politicians may be elected to office, and analyze the relationship between the salary of elected officials and their quality. Caillaud and Tirole (2002), Carrillo and Mariotti (2001), Castanheira, Crutzen, and Sahuguet (2008), Jackson, Mathevet, and Mattes (2007) and Snyder and Ting (2002) study the internal organization of parties and the selection of electoral candidates within parties. None of these contributions, however, studies the issue of political recruitment or the effect of alternative electoral systems on the recruiting decisions of political parties.

Our work also relates to the theoretical literature on all-pay contests. In particular, we build on results by Baye, Kovenock, and de Vries (1993), Baye, Kovenock, and de Vries (1996), and Hillman and Riley (1989) that study all-pay auctions with complete information. Also, a recent paper by Kaplan and Sela (2008) studies two-stage political contests with private entry costs. They analyze a primary election where there is an entry stage and a campaigning stage and show that low-ability contestants (those with a higher marginal cost of exerting effort) may enter more often than high-ability contestants. Contrary to our paper, however, in their model the party does not choose contestants (i.e., there is no recruitment), since individuals can choose whether or not to participate in the contest at a (private) cost and, more importantly, there is no electoral competition.
3 The Model

There are two political parties competing in an election and two identical pools of potential recruits, one for each party. Potential recruits are heterogeneous with respect to their marginal cost of exerting effort in the political sector or political ability. A politician’s ability is observable by parties and affects his performance both as a party member and as an electoral candidate. Parties serve the role of gatekeepers: individuals can only run for public office if they are members of a party and are nominated by their party.

After each party has selected its members (the recruitment phase), the new recruits exert costly effort that benefits the party (the organizational phase), and the politician who exerts the highest effort for each party is rewarded by being selected to be the party’s electoral candidate. In the electoral phase, the two candidates (one for each party) then compete by exerting costly effort in the form of campaign activities, which affect the electoral outcome. In a majoritarian (winner-takes-all) system, the candidate who exerts the highest level of campaign effort wins the election. In a proportional system, the probability that each candidate wins the election is proportional to his campaign effort.

Each party benefits from the total effort of its members during the organizational phase, and also receives an additional benefit if its candidate wins the election. A party member obtains a positive payoff if he is selected by his party as the electoral candidate, and enjoys an additional benefit if he wins the election. We model both the organizational phase and the electoral phase as all-pay contests. The equilibrium of the model determines the ability

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7 We ignore inter-party competition in the recruitment of politicians and assume that each party can select its members from identical pools of recruits. In general, inter-party competition for potential politicians seems of secondary importance, as ideological preferences are more likely to draw individuals toward specific parties. In fact, the lack of within-sector competition for sector-specific skills is a distinctive feature of the political sector.

8 The restrictions applied to candidacy vary a lot across countries with a strong party system, and they sometimes call for additional requirements other than party membership. For example, according to Obler (1974), a potential candidate in the Belgian Socialist Party must: “(1) have been a member at least five years prior to the primary; (2) have made annual minimum purchases from the Socialist co-op; (3) have been a regular subscriber to the party’s newspaper; (4) have sent his children to state rather than Catholic schools; and (5) have his wife and children enrolled in the appropriate women’s and youth organizations” (p. 180).

of the politicians each party recruits, the effort exerted by the parties’ members in the organizational phase, the ability and the campaign effort of the electoral candidates, and the ability of the elected politician.

Formally, we consider two competing political parties indexed by $h = \{L, R\}$, and two identical populations of individuals seeking public office.\(^{10}\) Abusing notation, we use the same index $h$ for a party and its pool of recruits. Each population $h$ is composed of $N$ individuals. Each individual $i$ of population $h$ is endowed with a characteristic $\theta_{ih} \geq 0$ representing his political ability. We assume that political abilities are strictly ordered, that is, $\theta_{1L} = \theta_{1R} > \theta_{2L} = \theta_{2R} > \cdots > \theta_{NL} = \theta_{NR}$. The individual cost of exerting effort $e \geq 0$ in the political sector is equal to $e/\theta_{ih}$ (i.e., the higher is political ability the smaller is the marginal cost of exerting effort).

The game has three stages. In Stage 0 (the recruitment phase), parties simultaneously select their members at a fixed hiring cost $\nu > 0$ per party member. Let $K_h$ be the set of party $h$ members, where $|K_h| \leq N$. An individual who is not selected by a party earns a payoff of zero.\(^{11}\)

In Stage 1 (the organizational phase), party members exert effort $e_{1,ih}$ which benefits the party (where the first subscript denotes the stage) at a cost equal to $e_{1,ih}/\theta_{ih}$. The party member who exerts the highest effort is nominated to be the party’s electoral candidate, which we denote by $i_{ih}^*$ (accordingly, $e_{1,i_h^*}$ denotes the highest effort exerted in the organizational phase), and he earns a payoff equal to $\beta \in (0, 1)$. Hence, $\beta$ is the value of being the party’s nominee.\(^{12}\) We define “non active” a party member who chooses not to exert effort in Stage 1 ($e_{1,ih} = 0$).

In Stage 2 (the electoral phase), the two candidates nominated by their parties compete in an election. The electoral outcome is a function of the effort exerted by candidates in the electoral campaign, and the properties of this function depend on the electoral system. Specifically, in a majoritarian electoral system (FPP), $i_{ih}^*$ is elected if and only if $e_{2,i_{ih}^*} > e_{2,j_{ih}^*}$, where $e_{2,i_{h}^*}(e_{2,j_{h}^*})$ is Stage 2 effort of party $h(-h)$’s nominee, and ties are

\(^{10}\)We discuss the assumption of two exogenously given political parties in Section 6.

\(^{11}\)In general, the value of the outside option can be itself a function of political ability. See, e.g., Mattozzi and Merlo (2008). Here, we abstract from this possibility.

\(^{12}\)In Section 6, we consider the case in which $\beta$ is endogenous.
broken randomly. In a proportional electoral system (PR), \( i_h^* \) is elected with probability \( e_{2,i_h^*}/(e_{2,i_h^*} + e_{2,j_h^*}) \). The elected politician earns a payoff normalized to 1. The individual cost of campaigning in the election phase is equal to \( e_{2,i_h^*}/\theta_{i_h} \).\(^{13}\)

Since behavior is invariant to affine transformations, for convenience we consider an equivalent specification where the effort cost function is the identity function (i.e., \( c(e) = e \)), and the value of nomination and election equal \( \beta \theta_{i_h} \) and \( \theta_{i_h} \), respectively. According to this equivalent interpretation, a high-ability politician is an individual who values the political job more or has a larger public service motivation.

Formally, by letting \( e_t = (e_{t,K}, e_{t,K^c}) \) denote the effort profile in stage \( t = \{1, 2\} \), the payoff of individual \( i \) in party \( h \) in a majoritarian electoral system is equal to

\[
\Pi_{i_h}^{FPP}(e_1, e_2) = \begin{cases} 
0 & \text{if } i_h \not\in K_h \\
\frac{\theta_{i_h}(1+\beta)e_{2,i_h}}{|Z_h|} - e_{1,i_h} & \text{if } e_{1,i_h} \geq \max_{j_h \in K_h} \{e_{1,j_h}\} \text{ and } e_{2,i_h} > e_{2,j_h^*} \\
\frac{\theta_{i_h}(\frac{1+\beta}{2}e_{2,i_h})}{|Z_h|} - e_{1,i_h} & \text{if } e_{1,i_h} \geq \max_{j_h \in K_h} \{e_{1,j_h}\} \text{ and } e_{2,i_h} = e_{2,j_h^*} \\
\frac{\theta_{i_h}(\frac{1+\beta}{2}e_{2,i_h})}{|Z_h|} - e_{1,i_h} & \text{if } e_{1,i_h} \geq \max_{j_h \in K_h} \{e_{1,j_h}\} \text{ and } e_{2,i_h} < e_{2,j_h^*} \\
-\frac{\theta_{i_h}e_{1,i_h}}{|Z_h|} & \text{otherwise},
\end{cases}
\]

where \( Z_h \equiv \{ j_h \in K_h : e_{1,j_h} = \max_{i_h \in K_h} \{ e_{1,i_h} \} \} \). Similarly, the payoff of individual \( i \) in party \( h \) in a proportional electoral system is equal to

\[
\Pi_{i_h}^{PR}(e_1, e_2) = \begin{cases} 
0 & \text{if } i_h \not\in K_h \\
\frac{\theta_{i_h}(\frac{e_{2,i_h}}{2e_{1,i_h}} + \beta)}{|Z_h|} - e_{1,i_h} & \text{if } e_{1,i_h} \geq \max_{j_h \in K_h} \{e_{1,j_h}\} \\
-\frac{\theta_{i_h}e_{1,i_h}}{|Z_h|} & \text{otherwise},
\end{cases}
\]

and if \( e_{2,i_h} = e_{2,j_h^*} = 0 \) each candidate is elected with equal probability.

We assume that party \( h \) selects its members in order to maximize the following objective

\[
V^s(e_{2,i_h^*}, e_{2,j_h^*}) + E(\sum_{i_h \in K_h} e_{1,i_h}) - |K|\nu,
\]

where the last two terms represent the party’s expected payoff from the recruiting and organizational phases (i.e., the expected total effort of party members in the organizational

\(^{13}\)Assuming that the cost of exerting effort is the same across stages is not necessary for our results.
phase net of hiring costs), and \( V^s(\cdot, \cdot) \), \( s \in \{PR, FPP\} \), is the party’s expected payoff from the electoral phase. In particular,

\[
V^{FPP}(e_2, i_h^*, e_2, j_{-h}^*) = \begin{cases} 
\gamma & \text{if } e_2, i_h^* > e_2, j_{-h}^* \\
\frac{\gamma}{2} & \text{if } e_2, i_h^* = e_2, j_{-h}^* \\
0 & \text{otherwise,}
\end{cases}
\]

\[
V^{PR}(e_2, i_h^*, e_2, j_{-h}^*) = \gamma \frac{e_2, i_h^*}{e_2, i_h^* + e_2, j_{-h}^*},
\]

where \( \gamma \geq 0 \) is the party’s benefit of winning the election and \( V^{PR}(e_2, i_h^*, e_2, j_{-h}^*) = \gamma/2 \) if \( e_2, i_h^* = e_2, j_{-h}^* = 0 \).

In the next two sections, we characterize the subgame perfect equilibrium of the game where the profile of effort choices in the electoral phase is a Nash equilibrium of the all-pay contest between candidates, and the profile of effort choices in the organizational phase and the recruiting strategy of the party are optimal given subsequent play. We focus on the case of arbitrarily small hiring cost per party member (i.e., \( \nu \rightarrow 0 \)). We say that there is “mediocrity” if parties choose not to recruit the best (i.e., the individual with the highest political ability) nor the worst individuals. On the other hand, we say that there is “aristocracy” if parties choose to recruit the best individuals.

4 The Case of a Safe Seat

In order to disentangle the various forces at work behind our results, we begin by considering a simplified version of the model where electoral competition is absent: the case of a safe seat or an uncontested election. In this case, the recruiting decisions of the two parties are completely independent and do not depend on the electoral system. Hence, we can focus without loss of generality on a situation in which there is only one party that can recruit politicians and a single population of \( N \) individuals seeking office.

Consider, as before, that political ability \( \theta_i \geq 0 \) is perfectly observable and such that \( \theta_1 > \theta_2 > \cdots > \theta_N \). Since the election is uncontested, the party’s nominee is elected with probability one and earns a payoff normalized to 1.\(^{14}\) An individual who is not selected to

\( ^{14}\)In the absence of electoral competition, distinguishing between the payoff of winning the nomination
be a party member earns a payoff of zero. Considering the equivalent specification where
the effort cost function is the identity function and the payoff from being elected equals \( \theta_i \)
and letting \( \mathbf{e}_K \) denote the effort profile, we have that the payoff of individual \( i \) is equal to

\[
\Pi_i(\mathbf{e}_K) = \begin{cases} 
0 & \text{if } i \not\in \mathcal{K} \\
\frac{\theta_i}{|Z|} - e_i & \text{if } e_i \geq \max_{j \in \mathcal{K}} \{e_j\} \\
-e_i & \text{otherwise,}
\end{cases}
\]

where \( Z \equiv \{ j \in \mathcal{K} : e_j = \max_i \in \mathcal{K} \{e_i\} \} \) represents the set of party members winning the
nomination (ties are resolved with equal probability). The party selects its members in
order to maximize their expected total effort on behalf of the party net of hiring costs: that is, the party’s payoff is equal to \( E(\sum_{i \in \mathcal{K}} e_i) - |\mathcal{K}| \nu \), and we restrict attention to the case of \( \nu \) being arbitrarily small.

We assume the following condition throughout the rest of the paper:

**Condition A**

\[
\left( 1 + \frac{\theta_2}{\theta_1} \right) \frac{\theta_2}{2} < \left( 1 + \frac{\theta_3}{\theta_2} \right) \frac{\theta_3}{2}.
\]

This condition guarantees that there is “enough” heterogeneity between the highest-ability
politician and the second-highest. Given this assumption, we can now state our first result:

**Proposition 1** *If condition A holds then mediocracy is an equilibrium.*

Since the organizational phase is equivalent to an all-pay auction with complete informa-
tion and strictly ordered valuations equal to \( \theta_i \), to prove the result we can use Theorem
1 and Lemma 1 of Baye, Kovenock, and de Vries (1993), which builds on a previous result
by Hillman and Riley (1989), and conclude that expected total effort of party members in
equilibrium equals

\[
E(\sum_{i \in \mathcal{K}} e_i) = \left( 1 + \frac{\theta_{\max_{\mathcal{K}+1}}}{\theta_{\max_{\mathcal{K}}}} \right) \frac{\theta_{\max_{\mathcal{K}+1}}}{2},
\]

(2)

where \( \theta_{\max_{\mathcal{K}}} \) and \( \theta_{\max_{\mathcal{K}+1}} \) denote the abilities of the best politician in the party and of the
second best politician in the party, respectively. Hence, under Condition A, the party has
an incentive not to select the highest-ability individual (i.e., \( \theta_1 \)). Furthermore, since the
and the payoff of winning the election is inconsequential.
“prize” (i.e., the party nomination) cannot be shared, in the unique equilibrium, only the two highest-ability politicians selected by the party will be active (i.e., will be choosing to exert positive effort). As a result, the party never selects the worst available individuals.

The intuition for the result is simple. Suppose that Condition A holds: that is, the distribution of individual characteristics is such that there is only one outstanding potential politician (technically, the ratio of $\theta_2$ and $\theta_1$ is sufficiently smaller than the ratio of $\theta_3$ and $\theta_2$). In the unique equilibrium of the organizational phase of the game, the two best politicians recruited by the party (i.e., the party recruits with the two highest values of $\theta$) randomize over the same interval of effort levels. However, while the highest-ability one randomizes uniformly over the interval, the second-highest’s equilibrium strategy has a mass point on zero effort. In other words, the two best politicians selected by the party will almost mimic each other, but the “underdog” will shirk with some positive probability. When the difference in ability between the best party member and the second best is relatively large, the chances that the latter wins the party nomination are relatively low. This implies that the second-best party member will shirk more often in equilibrium. We refer to this as the 

 discouragement effect: the presence of a “superstar” discourages individuals of lesser ability from exerting high levels of effort. As a consequence, competition within the party will be relatively low and hence expected total effort by all party members will be low as well. By excluding the potential recruit with the highest ability, and recruiting mediocre but relatively homogenous politicians, the party can increase intra-party competition (i.e., reduce the discouragement effect), and hence the collective effort of its recruits on behalf of the party, which maximizes its payoff. This result is an application of the “exclusion principle” for all-pay auctions with complete information discovered by Baye, Kovenock, and de Vries (1993). In the next section, we introduce electoral competition and study how the interaction between intra-party and inter-party competition affects the equilibrium selection of politicians.

5 Electoral Competition

Consider now the general environment described in Section 3 where the two parties compete in an election. In a competitive electoral environment, having a high-ability candidate
improves a party’s electoral prospects \textit{(competition effect)}. Hence, a mediocre selection of politicians negatively affects a party’s chances of winning a contested election. This situation entails an interesting trade-off between two opposing effects: the competition effect (due to inter-party competition) and the discouragement effect (due to intra-party competition). The specifics of this trade-off depend on the electoral system, which affects the competitiveness of elections.

We begin by providing a characterization of the subgame perfect equilibrium of the game. To simplify the analysis, we assume the following condition throughout the rest of the paper:

\textbf{Condition B} For all $k > 3$

\[
\left(1 + \frac{\theta_3}{\theta_2}\right) \frac{\theta_3}{2} > \left(1 + \frac{\theta_k}{\theta_{k-1}}\right) \frac{\theta_k}{2}.
\]

This condition guarantees that in the recruitment phase of the game the optimal selection for each party is either the two highest-ability individuals ($\theta_1$ and $\theta_2$) or the second and the third highest-ability individuals ($\theta_2$ and $\theta_3$).\(^{15}\) Under Condition A and Condition B we obtain the next result.

\textbf{Theorem 1} For each electoral system $s = \{FPP, PR\}$ there exists a threshold $\bar{\gamma}_s$ such that mediocrity is an equilibrium if and only if $\gamma < \bar{\gamma}_s$.

Theorem 1 completely characterizes the equilibrium of the model. The proof of this result, which is provided in the Appendix, constructs the subgame perfect equilibrium for each electoral system. In equilibrium, both parties will either select the two highest-ability individuals (aristocracy) or the second and the third highest-ability individuals (mediocrity). The reason why the existence of mediocrity depends on the value of $\gamma$ is rather intuitive. When $\gamma$ is small, parties care relatively more about the expected total effort of their members in the organizational phase than about winning the election. Hence, the discouragement effect is more important than the competition effect. In this case, a mediocre selection provides the best incentives for all party members to exert effort on

\(^{15}\)This immediately follows from equation (2) in the proof of Proposition 1. Notice that, while it simplifies our analysis, Condition B is not necessary for our results.
their party’s behalf in the organizational phase. On the other hand, as $\gamma$ becomes larger, the payoff from winning elections increases and having mediocre but hard working party members may no longer be optimal from the party’s perspective, since a mediocre candidate will most probably run an unsuccessful campaign.

Next, we investigate the effects of changing the incentives of party members in the organizational phase (i.e., varying the value $\beta$ of obtaining the party nomination) on the likelihood that mediocracy arises in equilibrium. An increase in $\beta$ has two opposite effects on $\bar{\gamma}^s$: it decreases the parties’ gains in the recruitment phase from excluding the highest-ability individual (the discouragement effect is less severe), which leads to a decrease in $\bar{\gamma}^s$; but, it also increases the probability of winning the election following a downward deviation in the recruitment phase (the competition effect is weaker), which leads to an increase in $\bar{\gamma}^s$. The former effect is due to intra-party competition and is very intuitive: an increase in the value of winning the nomination increases intra-party competition and hence reduces the discouragement effect. The latter effect is more subtle and pertains to the interaction between intra-party and inter-party competition.

Suppose that party $L$ is selecting the two highest-ability individuals as its members. The incentives for party $R$ to do the same rather than opt for a mediocre selection are given by the consequences of such a choice on its expected probability of winning the election. In particular, the electoral incentives are stronger the higher is the probability that party $L$’s nomination process will lead to the candidacy of the highest-ability individual. Since the nomination is awarded to the party member who exerts the highest level of effort, and in equilibrium the two party members with the highest values of $\theta$ will randomize continuously on an interval of efforts levels, an increase in the value of winning the nomination leads the less able politician in party $L$ to behave more aggressively. Hence, it is more likely that the less able politician becomes party $L$’s electoral candidate. But this benefits party $R$ since its chances of winning election with a mediocre selection actually increase (i.e., the competition effect is watered down). For a distribution of types that most favors mediocracy in equilibrium, this latter effect is the dominant one. Indeed, when $\theta_3$ approaches $\theta_2$, $\bar{\gamma}^s$ is increasing in $\beta$, that is the higher is the value of winning the nomination, the higher is the
likelihood that mediocrity is an equilibrium.\textsuperscript{16}

It is interesting to point out that having a positive value of winning the party nomination (i.e., $\beta > 0$) is a necessary condition for mediocrity only in the case of majoritarian elections. Indeed, when $\beta$ approaches zero $\tilde{\gamma}^{FPP}$ vanishes. On the contrary, there exist type profiles such that $\tilde{\gamma}^{PR}$ is always bounded away from zero for all values of $\beta$ (including when $\beta = 0$).\textsuperscript{17} Hence, we have the following corollary to Theorem 1:

**Corollary 1**

- **Necessary and sufficient conditions for mediocrity to be an equilibrium in majoritarian elections are that 1) politicians are sufficiently valuable for the party even if they do not win elections, and that 2) candidates are rewarded even if they do not win elections.**

- **A necessary condition for mediocrity to be an equilibrium in proportional elections is that politicians are sufficiently valuable for the party even if they do not win elections. Furthermore, there exist type profiles such that this condition is also sufficient.**

To provide some intuition for the result, let us focus on majoritarian elections and note that the winner-takes-all nature of this electoral system makes the equilibrium continuation value of being an electoral candidate very steep (in fact, discontinuous) in $\theta$. Indeed, when $\beta$ approaches zero, and hence nomination has almost no value per se, the equilibrium continuation value of being party $h$’s candidate is strictly positive if and only if $\theta_{i^*} > \theta_{j^*}$ (i.e., party $h$’s candidate has a strictly higher ability than his opponent in the general election), and it is equal to zero otherwise.\textsuperscript{18} Hence, there is no gain from working hard as a party member in the organizational phase unless there is a positive chance of i) becoming the electoral candidate and ii) facing a “weak” (low $\theta$) challenger in the general election. As a result, if elections are majoritarian, the party cannot react to the discouragement

\textsuperscript{16}Note that it is easier to satisfy Conditions A and B when $\theta_3$ approaches $\theta_2$. In the Appendix, at the end of the proof of Theorem 1, we discuss the case in which $\theta_3$ is exactly equal to $\theta_2$.

\textsuperscript{17}The proof of this result is part of the proof of Theorem 1 in the Appendix.

\textsuperscript{18}Recall that in the unique equilibrium of a two-bidders all-pay auction with valuations $\theta_1 > \theta_2$, the expected equilibrium payoff of bidder 1 is equal to $\theta_1 - \theta_2$, while the second bidder completely dissipates his rents.
effect if nomination has no value, and it gains nothing from selecting mediocre individuals irrespective of the value of $\gamma$. On the other hand, since in proportional elections the equilibrium continuation value of being an electoral candidate is always positive, increasing, and smooth in $\theta$, a mediocre selection can be effective in counteracting the discouragement effect for all values of $\beta$. In the proof of Theorem 1, we show that this is indeed the case when $\theta_3$ is close to $\theta_2$.

As Corollary 1 suggests, the conditions for mediocracy to be an equilibrium are more demanding in the case of a majoritarian electoral system than in a proportional one. Next, we investigate whether electoral systems can be ranked in terms of their performance in selecting high-ability politicians. This ranking is particularly relevant when political talent is relatively scarce as in the case of $\theta_3$ approaching $\theta_2$. In this case, the second- and third-best political talents are similar and there is only one outstanding politician. It turns out that $\theta_3$ approaching $\theta_2$ is a sufficient condition to rank electoral systems independently of the level of $\beta$. We state this result in the next proposition.

**Proposition 2** When $\theta_3$ approaches $\theta_2$ mediocracy is more likely to arise in proportional elections than in majoritarian elections: that is, $\bar{\gamma}^{FPP} < \bar{\gamma}^{PR}$.

The main force driving this result is that a majoritarian system is fundamentally more competitive than a proportional system, because of its winner-takes-all nature. This implies that a politician’s continuation value of winning the nomination is flatter in proportional elections, and the gains to the party from excluding high-ability politicians are larger. To understand why this is the case, consider a downward deviation of one party in the recruitment phase. A deviation toward a mediocre selection has two consequences: First, it increases intra-party competition for nomination and therefore it reduces the discouragement effect. This represents the benefit from the deviation. Second, it reduces the probability of winning the general election, which is the cost of deviating. The latter is higher in majoritarian than in proportional elections since the probability of winning the general election with a mediocre selection is lower in a majoritarian electoral system than in a proportional one. On the other hand, comparing the benefit of deviating across electoral systems is less immediate.
The benefit of deviating depends itself on two intertwined components: i) the homogeneity of the deviating party’s recruits after the deviation, and ii) how big is the continuation value of being the electoral candidate for the worst party recruit, which is related to his likelihood of winning the general election. While the first component affects the level of competition in the organizational phase (the size of the discouragement effect), the second determines an upper bound on individual effort within the party. When $\theta_3$ is close to $\theta_2$ the first component is similar across electoral systems. On the contrary, the maximal effort exerted by politicians in the organizational phase is higher in proportional elections. The reason for this is that in majoritarian elections, the equilibrium continuation value of being the electoral candidate (net of $\beta$) is equal to zero for every party member but the very best, while in proportional elections, it is strictly positive even for mediocre politicians. Hence, the party has a stronger incentive to select mediocre politicians in proportional elections than in majoritarian elections which implies that $\bar{\gamma}^{FPP}$ must be smaller than $\bar{\gamma}^{PR}$.

Figure 1 represents the equilibrium selection of politicians in the space $(\beta, \gamma)$ for a given value of $\theta_2/\theta_1$, and the arrows describe the effect of an increase in $\theta_2/\theta_1$ on the boundaries of the regions.\(^\text{19}\) If we interpret the two parameters of our model, $\beta$ and $\gamma$, as capturing the politicians’ and the parties’ weights between objectives, Figure 1 provides several intuitive

\(^{19}\)The boundaries are depicted as straight lines only as an illustration, but in general are not linear.
insights. First, the likelihood of mediocracy being an equilibrium increases when party service is more important than electoral success (as one moves southwest in Figure 1). Second, for fixed $\gamma$ and $\beta$, a proportional electoral system, by weakening the link between political ability and electoral performance, “endogenously” shifts parties’ focus from inter-party competition to intra-party competition and it therefore makes a worse selection of politicians more likely. Finally, the less the best politician stands out with respect to the next best alternative ($\theta_2/\theta_1$ increases), the more likely it is to have a mediocre selection of politicians in equilibrium.

Proposition 2 focuses on the relative performance of alternative electoral systems in selecting the highest-ability individuals into politics. The next proposition compares their performance in electing the highest-ability politician (i.e., a type $\theta_1$), when it is a party member and hence a potential candidate under both electoral systems.

**Proposition 3** Let $\gamma > \bar{\gamma}^{PR}$, so that there is aristocracy in both electoral systems. There exists $q^*(\beta) \in [0, 1)$ such that the probability of electing the highest-ability politician is higher in majoritarian elections than in proportional elections if $\theta_2/\theta_1 > q^*(\beta)$. Further, there exists $\beta^* \in (0, 1)$ such that $q^*(\beta) > 0$ if $\beta < \beta^*$.

When parties recruit the best available politicians in both electoral systems (i.e., $\gamma > \bar{\gamma}^{PR}$), Proposition 3 establishes that the highest-ability politician is elected more often in a majoritarian system than in a proportional system if either the value of winning the party nomination is large or when the distribution of political talent is such that “there is no superstar” (i.e., $\theta_2/\theta_1$ is relatively large). Furthermore, it can be shown numerically that the sufficient conditions of Proposition 3 are also necessary, so that the highest-ability politician may be more likely to be elected in a proportional system than in a majoritarian system when $\beta$ and $\theta_2/\theta_1$ are relatively small. Hence, while we have established that parties are more likely to select better politicians under a majoritarian system, the comparison between the two systems is less clear when we focus on their “electing performance”: that is, their relative performance in electing the highest-ability individual given the same initial selection of politicians.

When $\beta$ is small, a majoritarian system elects the highest-ability politician more often than a proportional system when it is less needed: i.e., when the difference between the
two best politicians is small and therefore the next best alternative is relatively close to the best available option. On the other hand, a proportional system may outperform a majoritarian system in its electing performance when it matters the most: i.e., when the highest-ability politician is much better than the next best alternative. This suggests that it may be useful to compare the two electoral systems according to the average quality of the politicians elected under each system (maintaining fixed the initial selection of party members). While the two systems cannot be ranked, the results of numerical simulations indicate that, given the same initial selection of politicians, the difference in the average quality of elected politicians in majoritarian and proportional systems is quantitatively negligible for all values of $\beta$ and $\theta_2/\theta_1$. Hence, the two systems are very similar with respect to their electing performance when they both induce political parties to select the best politicians. The main difference between the two electoral systems is in their relative propensity to induce a mediocre selection by the parties.

By combining the results of Propositions 2 and 3, our analysis highlights the importance of taking into account the effects of different electoral systems on the initial recruitment of politicians. In this respect, our findings tilt the comparison between electoral systems in favor of majoritarian elections. We conclude the analysis by assessing which system provides the best incentives to exert effort in the general election, taking into account the equilibrium of the recruitment and the organizational phases.

**Proposition 4** The expected total campaign effort of electoral candidates is always greater in majoritarian elections than in proportional elections.

The ranking of Proposition 4 also extends to expected average campaign effort and the intuition for these results comes from the uniformly steeper incentives provided by majoritarian elections and their effects on the selection of party members and electoral candidates.

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20 The reason why a proportional system performs better than a majoritarian system in electing the best politician when $\theta_2/\theta_1$ is relatively small is due to the fact that the unique equilibrium of the organizational phase is in mixed strategies. In particular, when the underdog politician is much worse than the best one, to preserve indifference, he has to exert zero effort with higher probability (and hence is less likely to obtain the party nomination and become an electoral candidate), in a proportional system than in a majoritarian system.
6 Concluding Remarks

In this paper, we have proposed a novel approach to study the effects of alternative electoral systems on the quality of politicians. By focusing on the recruitment of individuals in the political sector, we have identified a fundamental trade-off between organizational and electoral concerns of political parties that may lead to a mediocre selection of politicians. The main driving force behind this result is what we have called the discouragement effect: that is, the tendency of individual members of an organization (e.g., a political party), who are of lesser ability, to get discouraged from the presence of superstars in a competitive environment and hence exert little effort on behalf of the organization. By excluding superstars, and selecting instead a mediocre but relatively homogenous group of individuals, a political party can maximize the collective effort of the group, but at the cost of possibly losing its competitive edge in the electoral arena. Electoral rules determine the competitiveness of the electoral environment: ceteris paribus, the more competitive the electoral environment, the less appealing a mediocre selection.

We have proposed an equilibrium model that formalizes these general ideas and naturally casts them in an all-pay auction environment. To keep the analysis tractable and focused on the main ideas, the model is deliberately simple and stylized. Nevertheless, it can be extended in several directions. Here, we briefly discuss three possible generalizations.

First, in our model the value of being nominated as an electoral candidate, $\beta$, is exogenous. Suppose, on the other hand, that $\beta$ is endogenous. For example, suppose that $\beta$ can be optimally chosen by parties at cost $c(\beta)$, and different electoral systems may lead to a different optimal $\beta^*$. Clearly, if parties can increase $\beta$ at no cost (i.e. $c(\beta) = 0$), they will do so in both electoral systems and $\beta^*_{PR} = \beta^*_{FPP} = 1$. In this case, our results about the relative desirability of majoritarian elections both in terms of selection and election of good politicians are reinforced. If instead $c(\beta)$ is increasing and convex, it can be shown that there exists a threshold $t$ such that $\beta^*_{PR} > \beta^*_{FPP}$ if and only if $\theta_2/\theta_1 > t$. If $\beta^*_{PR} > \beta^*_{FPP}$, our ranking of electoral systems in terms of both the selection and election of high-ability politicians is preserved. On the other hand, if $\beta^*_{PR} < \beta^*_{FPP}$, the relative performance of

\footnote{We believe that this is a rather general concept which may also apply to other contexts.}
alternative electoral systems may also depend on the convexity of $c(\beta)$.\footnote{Preliminary analysis suggests however that this additional component is of second-order importance. For example, in the case of a quadratic cost function, when $\theta_2/\theta_1 < t$ and hence $\beta_{FPP} < \beta_{PR}$, the ratio $\beta_{FPP}/\beta_{PR}$ is approximately equal to 1 and therefore treating $\beta$ as exogenous is inconsequential.}

Second, our common-value environment departs from the standard Downsian approach and abstracts from policy preferences. However, policy preferences can be introduced in our model in a relatively straightforward way. For example, suppose that the two political parties have observable policy positions $\{x_L, x_R\} \in [-y, y]^2$ that are perfectly implemented by their candidates if elected and, to preserve the symmetry of the model, are such that $x_L = -x_R$. Further, assume that policy and campaign effort enter the voters’ utility in an additively separable fashion, and that voters’ policy preferences are distributed symmetrically on the interval $[-y, y]$. It is easy to show that in this model the key mechanism leading to mediocrity and our results on the comparison between alternative electoral systems will be preserved. This is also true if political parties choose their policy positions. The additional predictions of the extended model concern the policy outcome, which will be more or less polarized depending on whether the parties are assumed to be policy or office motivated and on the specific extensive-form of the game.

Finally, we focus on two exogenously given political parties. In the case of majoritarian electoral systems, both theory and empirical evidence suggest that this assumption is to a large extent plausible.\footnote{There is a large theoretical literature providing a formalization of the well-known Duverger’s law, namely that majoritarian elections lead to a two-party system. See, e.g., Iaryczower and Mattozzi (2008) and references therein.} This is not necessarily the case for proportional electoral systems. However, for any number of parties in proportional elections, the marginal impact of individual campaign effort on the probability of winning the election will always be bounded. On the contrary, the winner-takes-all nature of majoritarian elections entails that an increase in campaign effort just above the competitors’ levels will lead to a discrete jump in the probability of winning. This suggests that the probability of electing the best candidate will always be higher in majoritarian elections than in proportional elections for any number of candidates.\footnote{For example, it can be shown that the probability that the best candidate wins a proportional election when he is facing two competitors is always bounded above by his probability of winning when he is facing only one competitor.}
7 Appendix

Proof of Theorem 1

We first analyze the subgame perfect equilibrium of the game with a FPP electoral system. We proceed by backward induction. First, note that election phase of the game is an all-pay auction between the two nominees with valuations $\theta^*_h$ and $\theta^*_j - h$, respectively. Without loss of generality, assume that $\theta^*_h \geq \theta^*_j - h$. Using well-known equilibrium properties of all-pay auctions, we have that the equilibrium is unique. Furthermore, we have two possible situations:

1. If $\theta^*_h = \theta^*_j - h$, the equilibrium is symmetric and both candidates randomize continuously on $[0, \theta^*_h]$. Their expected payoff is zero.

2. If $\theta^*_h > \theta^*_j - h$, candidate $i^*_h$ randomizes continuously on $[0, \theta^*_j - h]$, and earns an expected equilibrium payoff of $\left(\theta^*_h - \theta^*_j - h\right)$. Candidate $j^*_h$ randomizes continuously on $\left(0, \theta^*_j - h\right)$; he places an atom of size at $\left(\theta^*_h - \theta^*_j - h\right) / \theta^*_h$ at zero, and earns a payoff of zero.

We now move to the organizational phase of the game and define by $\theta_{max_L}^h$ and $max_K^h$, the highest quality among politicians selected in party $h$ and the identity of the highest quality politician selected in party $h$, respectively. In order to save notation let $\theta_{max_L}^h \equiv \theta_{max}^h$ and $max_K^h \equiv max_h$. We consider two cases:

Case 1: $\theta_{max_L} = \theta_{max_R}$

Consider the following strategy profile: in each party $h$ the highest quality politician randomizes continuously on $[0, \beta \theta_{max_L}^h + 1]$. The second highest quality politician randomizes continuously on $(0, \beta \theta_{max_L}^h + 1]$ and places an atom of size $\alpha_h$ at zero. All other politicians are not active. Note that, if politicians in party $L$ follow this profile, the expected value of participating in the election for party $R$'s politicians is zero (net of the nomination prize) for all potential candidates with less than highest quality, and it is equal to

$$(\theta_{max_R} - \theta_{max_L} + 1) \left(1 - \frac{\alpha_L}{2}\right)$$
for the highest quality politician ($\theta_{\text{max}}^R$). By defining

$$v_{1R} \equiv \beta \theta_{\text{max}}^R + (\theta_{\text{max}}^R - \theta_{\text{max}}^{L+1}) \left(\frac{1 - \alpha_L}{2}\right) > \beta \theta_{\text{max}}^{R+1}$$

and

$$v_{jR} \equiv \beta \theta_{\text{max}}^{R+j-1} \text{ for all } j = \{2, \ldots, |K^R|\},$$

it follows that the strategy profile described above is the unique best response for party $R$’s politicians since they are playing an all-pay auction with complete information and valuations $v_{jR}, j = \{1, \ldots, |K^R|\}$ defined above. Finally, we can pin down the unique value of $\alpha_h$ by using the fact that the highest quality candidate must be indifferent within his mixed-strategy support, and that his expected payoff must equal $v_{1h} - v_{2h}$. This implies that if a politician with quality $\theta_{\text{max}}^{h+1}$ exerts effort $e$ according to the distribution function $F_{\text{max}}^{h+1}$, it must be that $v_{1h}F_{\text{max}}^{h+1}(e) - e = v_{1h} - v_{2h}$ for all $e \in [0, \beta \theta_{\text{max}}^{h+1}]$. Hence, by solving

$$F_{\text{max}}^{h+1}(0) = 1 - \frac{v_{2h}}{v_{1h}(\alpha_h)} = \alpha_h,$$

and letting $z = \theta_{\text{max}}^{h+1}/\theta_{\text{max}}^h$, we obtain that

$$\alpha_h = 1 - \frac{\sqrt{\beta^2 + 2\beta z(1 - z)} - \beta}{1 - z}, \quad (3)$$

which is decreasing in $\beta$ and $z$.

Case 2: $\theta_{\text{max}}^L > \theta_{\text{max}}^R$

For simplicity we focus on the case where $\theta_{\text{max}}^{L+1} = \theta_{\text{max}}^R$. Other cases can be analyzed in a similar way. Consider the following strategy profile: In party $R$ the highest quality politician randomizes continuously on $[0, \beta \theta_{\text{max}}^{R+1}]$. The second highest quality politician randomizes continuously on $(0, \beta \theta_{\text{max}}^{R+1}]$ and places an atom of size $\alpha'_R$ at zero. In party $L$ the highest quality politician randomizes continuously on $[0, x]$, where

$$x = \beta \theta_{\text{max}}^{L+1} + (\theta_{\text{max}}^{L+1} - \theta_{\text{max}}^{L+2}) \left(\frac{1 - \alpha'_L}{2}\right).$$

The second highest quality politician randomizes continuously on $(0, x]$ and places an atom of size $\alpha'_L$ at zero. All other politicians are not active. Note that, if politicians in party $L$ follow the candidate profile, the expected value of participating in the election for all party
R’s politicians is zero (net of the nomination prize), which implies that by redefining $v'_{jR} \equiv \beta \theta_{\text{max}R+j}$ for all $j = \{1, \ldots, |K_R|\}$, their strategy profile is optimal. On the other hand, if politicians in party $R$ follow the candidate profile, the expected value of participating in the election for party $L$’s politicians is zero (net of the nomination prize) for all potential candidates with less than second highest quality, and it is equal to

$$\left(\theta_{\text{max}L} - \theta_{\text{max}R}\right) \left(\frac{1 + \alpha'_R}{2}\right) + \left(\theta_{\text{max}L} - \theta_{\text{max}R+1}\right) \left(\frac{1 - \alpha'_R}{2}\right) =$$

$$\left(\theta_{\text{max}L} - \theta_{\text{max}L+1}\right) \left(\frac{1 + \alpha'_R}{2}\right) + \left(\theta_{\text{max}L} - \theta_{\text{max}L+2}\right) \left(\frac{1 - \alpha'_R}{2}\right)$$

for the highest quality politician, and equal to

$$\left(\theta_{\text{max}L+1} - \theta_{\text{max}R}\right) \left(\frac{1 - \alpha'_R}{2}\right) = \left(\theta_{\text{max}L+1} - \theta_{\text{max}L+2}\right) \left(\frac{1 - \alpha'_R}{2}\right)$$

for the second highest quality politician. By redefining

$$v'_1L = \beta \theta_{\text{max}L} + \left(\theta_{\text{max}L} - \theta_{\text{max}L+1}\right) \left(\frac{1 + \alpha'_R}{2}\right) + \left(\theta_{\text{max}L} - \theta_{\text{max}L+2}\right) \left(\frac{1 - \alpha'_R}{2}\right),$$

$$v'_2L = \beta \theta_{\text{max}L+1} + \left(\theta_{\text{max}L+1} - \theta_{\text{max}L+2}\right) \left(\frac{1 - \alpha'_R}{2}\right)$$

and

$$v'_{jL} = \beta \theta_{\text{max}L+j-1} \text{ for all } j = \{3, \ldots, |K_R|\},$$

and letting

$$\alpha'_L = 1 - \frac{v'_2L}{v'_1L} = 1 - \frac{\beta \theta_{\text{max}L+1} + \left(\theta_{\text{max}L+1} - \theta_{\text{max}L+2}\right) \left(\frac{1 - \alpha'_R}{2}\right)}{\beta \theta_{\text{max}L} + \left(\theta_{\text{max}L} - \theta_{\text{max}L+1}\right) \left(\frac{1 + \alpha'_R}{2}\right) + \left(\theta_{\text{max}L} - \theta_{\text{max}L+2}\right) \left(\frac{1 - \alpha'_R}{2}\right)},$$

and

$$\alpha'_R = 1 - \frac{\theta_{\text{max}R+1}}{\theta_{\text{max}R}},$$

it follows that the strategy profile described above is the unique best response for party $L$’s politicians.

In order to show that this is the unique equilibrium of the organizational phase, suppose that party $R$’s members play any strategy $\sigma_j : \theta_j \rightarrow \Delta[0,b_j], j = \{\text{max}R, \ldots, |K_R|\}$, where
\( \Delta[0,b_j] \) denotes a probability distribution on the interval \([0,b_j]\) and \( b_j < B < \infty \). The profile \( \sigma = (\sigma_{\max R}, \cdots, \sigma_{|K_R|}) \) generates a probability of winning party \( R \)'s nomination \( q_j(\sigma) \in [0,1] \) for \( j = \{1, \cdots, |K_R|\} \) such that \( \sum_j q_j(\sigma) = 1 \) and, if \( \max R > 1 \), \( q_j(\sigma) = 0 \) for \( j = \{1, \cdots, \max R\} \). The expected value of winning the nomination in party \( L \) is therefore

\[
\hat{v}_j = \beta \theta_{\max L} + j - 1 + \frac{|K_R|}{\sum_{s=\max L+j-1}^{\max L} q_s(\sigma)(\theta_{\max L} + j - 1 - \theta_s)},
\]

for \( j = \{1, \cdots, |K_L|\} \). Furthermore,

\[
\hat{v}_j - \hat{v}_{j+1} = \left( \beta + \sum_{s=\max L+j}^{\max L+j-1} q_s(\sigma) \right) (\theta_{\max L+1} - \theta_{\max L+j}) > 0.
\]

Hence, for any strategy profile \( \sigma = (\sigma_{\max R}, \cdots, \sigma_{|K_R|}) \) of party \( R \)'s members, the organizational phase of the game for party \( L \)'s members is an all-pay auction with complete information and strictly ordered expected valuations \( \hat{v}_j \) defined above, which has a unique equilibrium.

We now move to the recruitment phase of the game and show that there exists a \( \bar{\gamma}^{FPP} \) such that a necessary and sufficient condition to have a mediocracy equilibrium is \( \gamma < \bar{\gamma}^{FPP} \). In order to show this, suppose that we want to support a symmetric selection profile where aristocracy arises in equilibrium, i.e., each party in the recruitment phase selects only \( \{\theta_{1h}, \theta_{2h}\}, h = \{R, L\} \). Note that condition B guarantees that the selection that maximizes expected total effort in each party is either \( \{\theta_{2h}, \theta_{3h}\} \) or \( \{\theta_{1h}, \theta_{2h}\} \). Since the probability of winning the election decreases by selecting worst politicians, it follows that it is enough to check that a party does not want to deviate to a selection \( \{\theta_{2h}, \theta_{3h}\} \).

The expected payoff of each party \( h \) in an aristocracy equilibrium is

\[
\frac{\gamma}{2} + \left( 1 + \frac{v_{2h}}{v_{1h}} \right) \frac{v_{2h}}{2}
\]

where

\[
v_{1h} = \beta \theta_{1h} + (\theta_{1h} - \theta_{2h}) \left( \frac{1 - \alpha}{2} \right) \quad \text{and} \quad v_{2h} = \beta \theta_{2h},
\]

and, using (3) and suppressing the party index,

\[
\alpha = 1 - \frac{\sqrt{\beta^2 \theta_1^2 + 2 \beta \theta_2 (\theta_1 - \theta_2) - \beta \theta_1}}{\theta_1 - \theta_2}.
\]
By deviating to \(\{\theta_{2h}, \theta_{3h}\} \) (without loss of generality let \(h\) be the deviating party), party \(h\)'s payoff is

\[
\gamma P_h + \left(1 + \frac{v_{3h}}{v_{2h}} \right) \frac{v_{3h}}{2},
\]

where \(v_{ih} = \beta \theta_{ih}\), and \(P_h < 1/2\) is the probability that party \(h\) wins the election. Hence, a necessary and sufficient condition for party \(h\) not to deviate is

\[
\gamma > \bar{\gamma}_{FPP} \equiv \frac{\left(1 + \frac{v_{3h}}{v_{2h}} \right) v_{3h} - \left(1 + \frac{v_{2h}}{v_{1h}} \right) v_{2h}}{1 - 2P_h}.
\]

(4)

Furthermore, by defining

\[
\rho_1 = \Pr(e_{1,2h} < e_{1,3h}) = \Pr(e_{2,2-h} < e_{2,3h}) = \frac{1}{2} \frac{\theta_3}{\theta_2}
\]

\[
\rho_2 = \Pr(e_{1,1-h} < e_{1,2-h}) = \frac{1}{2} \beta \frac{\theta_2}{\theta_1} + \left(\frac{\theta_3}{\theta_1} - \frac{\theta_2}{\theta_1}\right) \frac{\theta_3}{\theta_2}
\]

\[
\rho_3 = \Pr(e_{2,1-h} < e_{2,2h}) = \frac{1}{2} \frac{\theta_2}{\theta_1} \quad \text{and} \quad \Pr(e_{2,1-h} < e_{2,3h}) = \frac{1}{2} \frac{\theta_3}{\theta_1} = 2 \rho_1 \rho_3,
\]

where Condition A implies that \(\rho_2 < \rho_3 < \rho_1\), we obtain that \(P_h\) equals

\[
P_h = \left(1 - \rho_1 (1 - 2\rho_1)\right) \left(\rho_2 \frac{1}{2} + (1 - \rho_2) \rho_3\right) \in (\rho_3, \rho_1),
\]

(5)

which is increasing in \(\beta\) since \(\rho_2\) is increasing in \(\beta\) and \(\rho_3 < 1/2\). Further, it is immediate to see that \(P_h > \rho_3\), while condition A and tedious algebra delivers that \(P_h\) is increasing in \(\theta_3\) and that \(P_h < \rho_1\). In a similar fashion it can be shown that a necessary and sufficient condition to support a symmetric selection profile where each party in the recruitment phase selects only \(\{\theta_{2h}, \theta_{3h}\}\), \(h = \{R, L\}\) is \(\gamma < \bar{\gamma}_{FPP}\).

Since \(P_h < 1/2\), the denominator of (4) is always positive. Further, since the numerator vanishes as \(\beta\) approaches zero, we have that \(\lim_{\beta \to 0} \bar{\gamma}_{FPP} = 0\). When \(\gamma\) vanishes, mediocrity arises if and only if

\[
\left(1 + \frac{\theta_3}{\theta_2}\right) \theta_3 > \left(1 + \frac{v_2}{v_1}\right) \theta_2,
\]

and condition A is a sufficient condition for the above inequality to hold since \(v_2/v_1 < \theta_2/\theta_1\).
We now analyze the subgame perfect equilibrium of the game with a PR electoral system. Consider first the election phase of the game in a PR electoral system. In this case, in the unique equilibrium, the nominees will choose
\[ \hat{\theta}^{e}_{2,i} = \frac{\theta_{h}^{2} \theta_{j-h}^{*}}{(\theta_{i}^{*} + \theta_{j-h}^{*})^2} \] and
\[ \hat{\theta}^{e}_{2,j} = \frac{\theta_{j-h}^{2} \theta_{i}^{*}}{(\theta_{i}^{*} + \theta_{j-h}^{*})^2}. \]
Furthermore, \( i_{h}^{*} \) and \( j_{-h}^{*} \) will earn payoffs
\[ \theta_{i}^{3}/(\theta_{i}^{*} + \theta_{j-h}^{*})^2 \] and
\[ \theta_{j-h}^{3}/(\theta_{i}^{*} + \theta_{j-h}^{*})^2, \] respectively.

We now move to the organizational phase of the game. Consider the following strategy profile: in each party the highest quality politician randomizes continuously on \([0, w_{\text{max}_{h}+1}]\). The second highest quality politician randomizes continuously on \((0, w_{\text{max}_{h}+1}]\) and places an atom of size \( \delta_{h} \) at zero. All other politicians are not active. Note that, if politicians in party \(-h\) follow this profile, the expected value of participating in the election for a party \( h \) politician with quality \( \theta_{ih} \) is
\[ \frac{1 + \delta_{-h}}{2} \frac{\theta_{ih}^{3}}{(\theta_{i} + \theta_{\text{max}_{-h}})^2} + \frac{1 - \delta_{-h}}{2} \frac{\theta_{ih}^{3}}{(\theta_{i} + \theta_{\text{max}_{-h}+1})^2}. \]
By defining
\[ w_{ih} = \beta \theta_{ih} + \frac{1 + \delta_{-h}}{2} \frac{\theta_{ih}^{3}}{(\theta_{i} + \theta_{\text{max}_{-h}})^2} + \frac{1 - \delta_{-h}}{2} \frac{\theta_{ih}^{3}}{(\theta_{i} + \theta_{\text{max}_{-h}+1})^2}, \]
and noticing that \( w_{ih} \) is strictly increasing in \( \theta_{ih} \), it follows that the strategy profile described above is the unique best response for party \( h \) politicians. We can pin down the equilibrium value of \( \delta_{h} \) solving the system
\[ \delta_{h} = 1 - \frac{w_{\text{max}_{h}+1}(\delta_{-h})}{w_{\text{max}_{h}}(\delta_{-h})}, \] for \( h \in \{L, R\} \).
Since each equation of the system in (6) is a continuous function of \( \delta_{-h} \) that maps the unit interval into itself, a solution always exists. If \( max_{L} = max_{R} \), (6) has trivially a unique solution where \( \delta_{h} = \delta_{-h} = \delta^{*} \), and it is easy to show that \( \delta^{*} \) is decreasing in \( \beta \) and decreasing in \( \theta_{\text{max}_{h}+1}/\theta_{\text{max}_{h}} \). If instead \( max_{L} \neq max_{R} \), it must be the case that \( \delta_{h} \neq \delta_{-h} \), and tedious but straightforward algebra shows that the solution is still unique.
In order to show that this is the unique equilibrium of the organizational phase, we apply the same argument as before and suppose that party R’s members play any strategy \( \sigma_j : \theta_j \rightarrow \Delta[0, b_j], j = \{\text{max}_R, \ldots, |K_R|\} \), where \( \Delta[0, b_j] \) denotes a probability distribution on the interval \([0, b_j]\) and \( b_j < B < \infty \). The profile \( \sigma = (\sigma_{\text{max}_R}, \ldots, \sigma_{|K_R|}) \) generates a probability of winning party R’s nomination \( q_j(\sigma) \in [0, 1] \) for \( j = \{1, \ldots, |K_R|\} \) such that \( \sum_j q_j(\sigma) = 1 \) and, if \( \text{max}_R > 1 \), \( q_j(\sigma) = 0 \) for \( j = \{1, \ldots, \text{max}_R\} \). The expected value of winning the nomination in party L is therefore

\[
\hat{w}_j = \beta \theta_{\text{max}_L}^{j+1} + \sum_{s=1}^{|K_R|} q_s(\sigma) \frac{\theta_{\text{max}_L}^3}{(\theta_{\text{max}_L} + \theta_s)^2},
\]

for \( j = \{1, \ldots, |K_L|\} \). Furthermore,

\[
\hat{w}_j - \hat{w}_{j+1} = \beta (\theta_{\text{max}_L}^{j+1} - \theta_{\text{max}_L}^j) + \sum_{s=1}^{|K_R|} q_s(\sigma) \left( \frac{\theta_{\text{max}_L}^3}{(\theta_{\text{max}_L} + \theta_s)^2} - \frac{\theta_{\text{max}_L}^3}{(\theta_{\text{max}_L} + \theta_s)^2} \right) > 0.
\]

Hence, for any strategy profile \( \sigma = (\sigma_{\text{max}_R}, \ldots, \sigma_{|K_R|}) \) of party R’s members, the organizational phase of the game for party L’s members is an all-pay auction with complete information and strictly ordered expected valuations \( \hat{w}_j \) defined above, which has a unique equilibrium.

We now move to the recruitment phase of the game and show that there exists a \( \bar{\gamma}^{PR} \) such that a necessary and sufficient condition to have a mediocracy equilibrium is \( \gamma < \bar{\gamma}^{PR} \).

In order to support a symmetric selection profile where aristocracy arises in equilibrium, i.e., \( \{\theta_1, \theta_2\}, h = \{R, L\} \), it is enough to check that a party does not want to deviate to a selection \( \{\theta_2, \theta_3\} \) (condition B).

The expected payoff of party h in an aristocracy equilibrium is

\[
\frac{\gamma}{2} + \left(1 + \frac{w_{2,h}(\delta^*)}{w_{1,h}(\delta^*)}\right) \frac{w_{2,h}(\delta^*)}{2},
\]

where

\[
w_{ih}(\delta) = \beta \theta_{ih} + \frac{1 + \delta}{2} \left(\frac{\theta_{ih}^3}{(\theta_{ih} + \theta_1)^2} + \frac{1 - \delta}{2} \frac{\theta_{ih}^3}{(\theta_{ih} + \theta_2)^2}\right),
\]

and \( \delta^* \) is the unique solution to (6) when \( \text{max}_h = \text{max}_{-h} = 1 \). By deviating to \( \{\theta_2, \theta_3\} \) party h’s payoff is

\[
\gamma \hat{P}_h + \left(1 + \frac{w_{3,h}(\delta_{-h}^*)}{w_{2,h}(\delta_{-h}^*)}\right) \frac{w_{3,h}(\delta_{-h}^*)}{2},
\]

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where \( \hat{P}_h < 1/2 \), and \( (\delta^*_h, \delta^*_{-h}) \) solve (6) when \( \max_{-h} = 1 \) and \( \max_h = 2 \). Hence, a necessary and sufficient condition for party \( h \) not to deviate is

\[
\gamma > \gamma^{PR} \equiv \frac{1}{1 - 2\hat{P}_h} \left( 1 + \frac{w_{3h}(\delta^*_{-h})}{w_{2h}(\delta^*_{-h})} \right) w_{3h}(\delta^*_{-h}) - \left( 1 + \frac{w_{2h}(\delta^*)}{w_{1h}(\delta^*)} \right) w_{2h}(\delta^*).
\] (7)

By letting

\[
\hat{\rho}_1 = Pr(e_{1,2_h} < e_{1,3_h}) = \frac{1}{2} \frac{w_{3h}(\delta^*_{-h})}{w_{2h}(\delta^*_{-h})} = \frac{1}{2} (1 - \delta^*_h) < \rho_1,
\]

and

\[
\hat{\rho}_3 = Pr(e_{1,1_{-h}} < e_{1,2_{-h}}) = \frac{1}{2} \frac{w_{2h}(\delta^*_{-h})}{w_{1h}(\delta^*_{-h})} = \frac{1}{2} (1 - \delta^*_{-h}) < \rho_3,
\]

it follows that

\[
\hat{P}_h = \hat{\rho}_1 \left( \frac{\hat{\rho}_3 \frac{\theta_3}{\theta_2 + \theta_3} + (1 - \hat{\rho}_3) \frac{\theta_3}{\theta_1 + \theta_3}}{1 - \hat{\rho}_1} \right) + (1 - \hat{\rho}_1) \left( \frac{\hat{\rho}_3}{2} + (1 - \hat{\rho}_3) \frac{\theta_2}{\theta_1 + \theta_2} \right) < \frac{1}{2}.
\] (8)

In a similar fashion it can be shown that a necessary and sufficient condition to support a symmetric selection profile where each party in the first stage selects only \( \{\theta_{2_h}, \theta_{3_h}\} \), \( h = \{R, L\} \) is \( \gamma < \gamma^{PR} \). Since \( \hat{P}_h < 1/2 \), the denominator of (7) is always positive. Further, when \( \theta_1 > \theta_2 \) and \( \theta_3 \) approaches \( \theta_2 \), \( w_{3h}(\delta^*_{-h}) \) approaches \( w_{2h}(\hat{\delta}^*_{-h}) \), where \( \hat{\delta}^*_{-h} \equiv \lim_{\theta_3 \rightarrow \theta_2} \delta^*_{-h} \), and the numerator of (7) simplifies to

\[
2w_{2h}(\hat{\delta}^*_{-h}) - \left( 1 + \frac{w_{2h}(\delta^*)}{w_{1h}(\delta^*)} \right) w_{2h}(\delta^*) = 2w_{2h}(\hat{\delta}^*_{-h}) - (2 - \delta^*) w_{2h}(\delta^*).
\]

The last expression is strictly positive since tedious algebra shows that it is increasing in \( \beta \), \( w_{2h}(\hat{\delta}^*_{-h}) < w_{2h}(\delta^*) \) if and only if \( \hat{\delta}^*_{-h} > \delta^* \), and there exists a \( \bar{\beta} > 0 \) such that \( \hat{\delta}^*_{-h} > \delta^* \) if and only if \( \beta < \bar{\beta} \). Note that contrary to the case of \( \theta_2 > \theta_3 \), when \( \theta_1 > \theta_2 \) and \( \theta_2 \) is exactly equal to \( \theta_3 \), the equilibrium of the organizational phase of the game is not unique anymore (Baye, Kovenock, and de Vries (1993)). Here, we focus on the limit of the unique equilibrium described above, i.e., when \( \theta_3 - \theta_2 < \epsilon \) for \( \epsilon > 0 \) positive and small. It is worth mentioning that even in the case of \( \theta_3 = \theta_2 \) the equilibrium that we described above exists and it is the one that maximizes expected effort in the organizational phase, see Baye, Kovenock, and de Vries (1993). In conclusion, mediocrity arises in PR if and only if \( \gamma < \gamma^{PR} \) and, when \( \theta_1 > \theta_2 \) and \( \theta_3 \) approaches \( \theta_2 \), \( \gamma^{PR} \) is strictly positive for all values of \( \beta \). Q.E.D.
Proof of Proposition 2
Using equations (4) and (7), let \( Q(\beta, \theta_2/\theta_1) \) denote the ratio \( \bar{\gamma}_{PR}/\bar{\gamma}_{FPP} \) when \( \theta_3 \) approaches \( \theta_2 \). Then, tedious algebra delivers that \( Q(\beta, \theta_2/\theta_1) \) is decreasing in \( \beta \) and therefore \( Q(\beta, \theta_2/\theta_1) > Q(1, \theta_2/\theta_1) \geq 1 \), where the last inequality follows from the fact that \( Q(1, \theta_2/\theta_1) \geq Q(1, 0) = 1 \). Q.E.D.

Proof of Proposition 3
Let \( Z_s \) be the probability of electing a type \( \theta_1 \) in electoral system \( s \in \{ \text{FPP}, \text{PR} \} \). Then in the case of FPP we have that

\[
Z_{FPP} = \frac{(1 + \alpha)^2}{4} + \frac{1 - \alpha^2}{2} \left(1 - \frac{q}{2}\right),
\]

where \( \alpha(\beta, q) = 1 - \frac{\sqrt{\beta^2 + 2\betaq(1-q) - \beta}}{1-q} \in (0, 1) \), and \( q = \frac{\theta_2}{\theta_1} \).

In the case of PR we have that

\[
Z_{PR} = \frac{(1 + \delta)^2}{4} + \frac{1 - \delta^2}{2} \left(\frac{1}{1+q}\right),
\]

where \( \delta(\beta, q) \in (0, 1) \) is the unique solution to (6) when \( \max L = \max R = 1 \). Note that since \( Z_{FPP} \) is increasing in \( \alpha \) and \( 1 - q/2 > 1/(1+q) \), then if \( \alpha \geq \delta \) it immediately follows that \( Z_{FPP} > Z_{PR} \). Since \( \alpha(\beta, 0) = \delta(\beta, 0) = 1 \) and \( \alpha(\beta, 1) = \delta(\beta, 1) = 0 \) and by definition

\[
\delta = 1 - \frac{q \left(2\beta + \frac{1}{4} + \frac{q^2}{(1+q)^2}\right) - \delta q \left(\frac{1}{4} - \frac{q^2}{(1+q)^2}\right)}{2\beta + \frac{1}{4} + \frac{1}{(1+q)^2} - \delta \left(\frac{1}{(1+q)^2} - \frac{1}{4}\right)}
\]

\[
\alpha = 1 - \frac{2\beta q}{2\beta + (1-q)(1-\alpha)},
\]

we have that when \( q \in (0, 1) \), \( \alpha = \delta = x \) if and only if

\[
\frac{2\beta + \frac{1}{4} + \frac{q^2}{(1+q)^2} - qx \left(\frac{1}{4} - \frac{q^2}{(1+q)^2}\right)}{2\beta + \frac{1}{4} + \frac{1}{(1+q)^2} - x \left(\frac{1}{(1+q)^2} - \frac{1}{4}\right)} = \frac{2\beta}{2\beta + (1-q)(1-x)}.
\]

The last expression is quadratic in \( x \), it admits two solutions, and it can be verified that only one solution is strictly smaller than 1. Therefore there exist a unique \( \bar{q}(\beta) \in (0, 1) \) such that \( \alpha(\beta, \bar{q}(\beta)) = \delta(\beta, \bar{q}(\beta)) \). Further, since

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\[
\frac{\partial \alpha (\beta, q)}{\partial q} = - \frac{2 \beta (2 \beta + 1 - \alpha)}{(2 \beta + (1 - q) (1 - \alpha))^2 + 2 \beta q (1 - q)} < 0
\]
\[
\frac{\partial \alpha (\beta, q)}{\partial q} |_{q=0} = -1 \quad \text{and} \quad \frac{\partial \alpha (\beta, q)}{\partial q} |_{q=1} = -1 - \frac{1}{2 \beta},
\]
and
\[
\frac{\partial \delta (\beta, q)}{\partial q} = - \frac{2 \beta + \frac{1}{4} (1 - \delta) + \frac{3 - q}{(1+q)^2} q^2 (1 + \delta) + (1 - \delta)^2 \frac{2}{(1+q)^3}}{2 \beta + \frac{1}{2} + 2 (1 - \delta) \left( \frac{1}{(1+q)^2} - \frac{1}{4} \right) - q \left( \frac{1}{4} - \frac{q^2}{(1+q)^2} \right)} < 0
\]
\[
\frac{\partial \delta (\beta, q)}{\partial q} |_{q=0} = - \frac{4 \beta}{1 + 4 \beta} > \frac{\partial \alpha (\beta, q)}{\partial q} |_{q=0} \quad \text{and} \quad \frac{\partial \delta (\beta, q)}{\partial q} |_{q=1} = -1 - \frac{1}{2 + 8 \beta} > \frac{\partial \alpha (\beta, q)}{\partial q} |_{q=1},
\]
we have that \( \alpha > \delta \) if and only if \( q > \bar{q} (\beta) \). Hence, we can conclude that when \( q > \bar{q} (\beta) \) the probability of electing a type \( \theta_1 \) is higher in FPP than in PR. Finally, since
\[
\lim_{q \to 0} \frac{Z_{FPP}}{Z_{PR}} |_{\beta=1} \geq 1 \geq \lim_{q \to 0} \frac{Z_{FPP}}{Z_{PR}} |_{\beta=0},
\]
there exist an \( q^* (\beta) \) such that \( Z_{FPP} \geq Z_{PR} \) if \( q \geq q^* (\beta) \), and there exist a \( \beta \in (0, 1) \) such that \( q^* (\beta) > 0 \) if \( \beta < \beta^* \). Q.E.D.

**Proof of Proposition 4**

Consider first the case of \( \gamma \leq \min \{ \tilde{\gamma}^{PR}, \tilde{\gamma}^{FPP} \} \) or \( \gamma \geq \max \{ \tilde{\gamma}^{PR}, \tilde{\gamma}^{FPP} \} \) and let \( q \equiv \theta_{\max + 1}/\theta_{\max} \) and let \( \Pr (\theta_x, \theta_y) \) denote the equilibrium probability that the election is contested between politicians of quality \( \theta_x \) and \( \theta_y \). Then, the expected total campaign effort of electoral candidates in FPP is equal to
\[
\Pr (\theta_{\max}, \theta_{\max}) \theta_{\max} + \Pr (\theta_{\max + 1}, \theta_{\max + 1}) \theta_{\max + 1} + 2 \Pr (\theta_{\max}, \theta_{\max + 1}) \frac{\theta_{\max + 1}}{2} \left( 1 + \frac{\theta_{\max + 1}}{\theta_{\max}} \right) =
\]
\[
\theta_{\max} \left( \frac{(1 + \alpha)^2}{4} + \frac{(1 - \alpha)^2}{4} q + (1 - \alpha^2) \frac{q (1 + q)}{4} \right) > \frac{\theta_{\max}}{2},
\]
where the last inequality follows from the fact that the term in parentheses is increasing in \( \alpha \), and \( \alpha = \left( 1 - q - \sqrt{\beta^2 + 2q\beta (1-q) + \beta} \right) / (1-q) \) is decreasing in \( \beta \). Hence,
\[
\frac{(1 + \alpha)^2}{4} + \frac{(1 - \alpha)^2}{4} q + (1 - \alpha^2) \frac{q (1 + q)}{4} >
\]
\[
\frac{\theta_{\max}}{2} \left( \frac{(1 + \alpha_{|\beta=1})^2}{4} + \frac{(1 - \alpha_{|\beta=1})^2}{4} q + \frac{(1 - \alpha^2_{|\beta=1}) q (1 + q)}{4} \right),
\]
and the last expression is only a function of $q$ and it is always bigger than $1/2$. On the other hand, the expected total campaign effort of electoral candidates in PR is equal to

$$\Pr(\theta_{\text{max}}, \theta_{\text{max}}) \frac{\theta_{\text{max}}}{2} + \Pr(\theta_{\text{max}+1}, \theta_{\text{max}+1}) \frac{\theta_{\text{max}+1}}{2} + 2 \Pr(\theta_{\text{max}}, \theta_{\text{max}+1}) \frac{\theta_{\text{max}} \theta_{\text{max}+1}}{\theta_{\text{max}} + \theta_{\text{max}+1}} =$$

$$\frac{\theta_{\text{max}}}{2} \left( \frac{(1 + \delta)^2}{4} + \frac{(1 - \delta)^2}{4} q + (1 - \delta^2) \frac{q}{1 + q} \right) < \frac{\theta_{\text{max}}}{2},$$

since

$$\frac{(1 + \delta)^2}{4} + \frac{(1 - \delta)^2}{4} q + (1 - \delta^2) \frac{q}{1 + q} < \left( (1 + \delta^2) + (1 - \delta^2) \right) \frac{1}{2} = 1.$$

Finally, since when $\theta_3$ is relatively close to $\theta_2$ the only case left is $\gamma \in (\gamma^{FPP}, \gamma^{PR})$, and in this case it is immediate to check that the expected total campaign effort of electoral candidates is higher in FPP than in PR, we are done. Q.E.D.
References


