PIER Working Paper 10-040

"Information, Liquidity, Asset Prices and Monetary Policy"
Second Version

by

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http://ssrn.com/abstract=1727602

Information, Liquidity, Asset Prices

and Monetary Policy*

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December 16, 2010

Abstract

What determines which assets are used in transactions? We develop a framework where the extent to which assets are recognizable determines the extent to which they are acceptable in exchange – i.e., their liquidity. We analyze the effects of monetary policy on asset markets. Recognizability and liquidity are endogenized by allowing agents to invest in information. There can be multiple equilibria with different transaction patterns. These transaction patterns are not invariant to policy. We show small changes in information may generate large responses in prices, allocations and welfare. We also discuss issues in international economics, including exchange rates and dollarization.

Aringosa walked back to his black briefcase, opened it, and removed one of the bearer bonds. He handed it to the pilot. "What's this?" the pilot demanded. "A ten-thousand-euro bearer bond drawn on the Vatican Bank." The pilot looked dubious. "It's the same as cash." "Only cash is cash," the pilot said, handing the bond back. Dan Brown, *The Da Vinci Code*.

1 Introduction

We study economies in which assets are valued for both their rate of return and liquidity, by which we mean their usefulness in the transactions process. In our model, at least some trades are conducted in markets where certain frictions, including limited commitment and incomplete record keeping, make credit imperfect. Sellers in these markets are unwilling to give buyers unsecured loans, and this makes assets essential for trade: buyers must either hand over assets to sellers directly, or use them to collateralize debt. Hence, assets facilitate exchange. This much is standard in modern monetary theory, what the recent surveys by Williamson and Wright [54], [55] and the book by Nosal and Rocheteau [44] call New Monetarist Economics. The novel feature here is that some assets are not as good as others at facilitating transactions due to asymmetric information.

In particular, it can be difficult for some agents to distinguish or recognize good- and bad-quality versions of certain assets, which makes them reluctant to accept them either as a means of payment or as collateral. We develop a general framework, with arbitrary numbers of real and monetary assets, in which these information frictions are made explicit, and use it to discuss applications in finance and monetary economics. Many of the basic ideas go back a long way. Classic discussions can be found in Jevons [22] and Menger [43], and the idea that the intrinsic properties of objects make them more or less well suited for use in payments can be found in many textbooks (see Nosal and Rocheteau [44]). These properties include portability, storability, divisibility and recognizability. It is recognizability that we emphasize here.[1]

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[1] Other work on information and liquidity includes Alchian [2], Brunner and Meltzer [7], Freeman [11], and Banerjee and Maskin [6]. We are closer to search models with information frictions, as surveyed in Nosal and Rocheteau [44] and Williamson and Wright [55]. An alternative approach to liquidity going back to Glosten and Milgrom [16] and Kyle [32] also considers exchange between asymmetrically-informed agents, but has little to say

As a simple example, it is typically thought that currency, or at least domestic currency, is recognizable to virtually everyone active in the economy, while alternatives are less so. These alternatives include foreign currency, bonds like those Aringosa tried to pass in the epigraph, less exotic claims like T-bills, equity shares, and so on. There can also be recognizability differences across these alternatives, potentially making, say, some bonds more liquid than others, or making bonds more or less liquid than equity. Historically, it was difficult for sellers to recognize the quality (i.e., the weight and purity) of coins. Even in modern economies, similar problems can arise with respect to legitimate and counterfeit currencies. Most recently, volatility in the real estate market has made it increasingly difficult to value complicated bundles of assets like mortgage-backed securities. In general, in any situation where buyers and sellers are asymmetrically informed about the values of assets, exchange is hindered.

Since recognizability is vital for liquidity, it is desirable to model it endogenously. To this end we allow agents to invest in information – to acquire the knowledge, or perhaps the technology, to distinguish high- and low-quality versions of some assets. This leads to coordination issues that can generate multiple equilibria. Related results have been discussed previously, but in contrast to that work, multiplicity here is due to explicit general equilibrium asset market effects. For instance, there is a literature studying payment methods, like credit cards, that correctly emphasizes that what sellers accept depends on what buyers carry and vice versa (see Hunt [20] and references therein). While it is not hard to get multiple equilibria by assuming the benefit to using one type of instrument goes up, or the cost goes down, when others also use it, the results here are more subtle. In our model, when more sellers recognize a particular asset, it becomes more liquid, and hence more useful in the exchange process. This makes buyers want more of the asset, and this increases its price. When the asset is more valuable, sellers are more willing to pay to be able to differentiate high- and low-quality versions of the asset. It is this complementarity that implies multiplicity may arise.

about the substantive issues addressed here. There are several recent papers that, like us, study finance with search frictions; rather than listing individual contributions we refer again to the above-mentioned surveys.

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Once liquidity is incorporated in a model, it is apparent that assets generally can be valued for more than their rate of return. The leading example is fiat money, an asset with a perfectly predictable permanent dividend of 0, and hence one that should have a price of 0 according to standard finance theory. In monetary economics, however, agents may value fiat currency, even if it is dominated in return by other assets, because it provides transactions services. The value of fiat money can be interpreted as a liquidity premium. Once this is understood, it must be acknowledged that any asset can bear a liquidity premium, which means that its price can exceed the fundamental price, defined by the present value of its dividend stream. All else equal, if it is harder to trade using asset a_1 than asset a_2, the latter will have a higher price and a lower return than the former. This much is obvious. What is perhaps more interesting is the way we endogenize liquidity, based on recognizability, and endogenize recognizability by allowing agents to invest in information.

We present several applications of the general framework. First, in our baseline model, we specify the environment so that agents who are not informed about a particular asset simply refuse to accept it in exchange. This is technically convenient because it allows us to use standard bargaining theory to determine the terms of trade: since agents only exchange objects that they recognize, they never bargain under asymmetric information. The advantage of modeling information frictions in this way is that it allows us to emphasize liquidity differentials without overly complicating the analysis of the terms of trade. However, in the interest of generality, we also show how one can relax the assumptions in the baseline model so that assets that are not recognized are still accepted up to a point. Having understood this, for other applications we usually revert to the baseline model where assets are only accepted by those who recognize them. One application focuses on the case of two assets, equity and fiat currency, with exogenous differentials in liquidity. This allows us to highlight some interesting connections between monetary policy and asset pricing. In particular, we show inflation makes people want to shift their portfolios out of currency and into alternative assets, increasing the price of, and lowering

the return on, these alternatives.[2]

Much more emerges when we endogenize information. For instance, since the proximate effect of inflation is to decrease the demand for money and increase demand for alternatives, it raises the market value of alternative assets and therefore the incentive to acquire information. Hence inflation increases the liquidity of alternative assets. One implication is that the share of transactions where cash is apparently required is endogenous: in the case of multiple equilibria, it is not uniquely determined by fundamentals; and even if equilibrium is unique, it is not invariant to policy. This calls into question the practice of assuming exogenous transactions patterns, as in cash-in-advance models, or at the opposite extreme, cashless exchange as in New Keynesian Economics. As always, the validity of any approach depends on the issues at hand, but it would be hard to argue that in monetary economics the transactions process should not be endogenous. In a related application, we use the model to think about the recent crises, and show how relatively small changes in fundamentals, including the information structure, can generate large responses in liquidity, prices, output and welfare.

Another application concerns two fiat monies. Suppose we interpret easily-recognizable assets as local currency, like pesos in Latin America, and the alternative as US dollars, which constitute a better store of value. Consistent with much experience, when peso inflation is not

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[2]The basic ideas in terms of monetary policy also go back a long way. A concise statement is contained in Wallace's [50] analysis of OLG (overlapping generations) models:

> Of course, in general, fiat money issue is not a tax on all saving. It is a tax on saving in the form of money. But it is important to emphasize that the equilibrium rate-of-return distribution on the equilibrium portfolio does depend on the magnitude of the fiat money-financed deficit ... [T]he real rate-of-return distribution faced by individuals in equilibrium is less favorable the greater the fiat money-financed deficit. Many economists seem to ignore this aspect of inflation because of their unfounded attachment to Irving Fisher's theory of nominal interest rates. (According to this theory, (most?) real rates of return do not depend on the magnitude of anticipated inflation.) The attachment to Fisher's theory of nominal interest rates accounts for why economists seem to have a hard time describing the distortions created by anticipated inflation. The models under consideration here imply that the higher the fiat money-financed deficit, the less favorable the terms of trade – in general, a distribution – at which present income can be converted into future income. This seems to be what most citizens perceive to be the cost of anticipated inflation.

We think these words ring true, but many questions arise. How can the Fisher Equation not hold? Why do different assets bear different returns in the first place? In the OLG models Wallace mentions, it is *not* differences in liquidity. This is where more modern monetary theory helps.

too high, locals in Latin America tend to use pesos as a means of payment, and dollars do not circulate widely nor are they universally recognized. If the peso inflation rate increases, however, transacting in the local currency becomes more costly and more agents learn to recognize and use US currency. This is dollarization. Notice, however, that if peso inflation later subsides, we should not expect dollars to fall into disuse, because once agents learn to recognize them for transactions purposes they do not quickly forget. This imparts a natural hysteresis effect into dollarization, as has often been discussed (see Section 6 for references), but not formalized in this way. We also discuss exchange rates.

The rest of the paper is organized as follows. Section 2 describes the assumptions, defines equilibrium and presents a rudimentary example. Section 3 studies the effects of monetary policy when information is exogenous. Section 4 endogenizes information. In these applications, we make assumptions implying sellers never accept assets they do not recognize, the key assumption being that low-quality assets can be produced for free. Section 5 relaxes this by making low-quality assets costly to produce. Section 6 discusses international issues. Section 7 provides a general discussion of the approach and the results. Section 8 concludes.

2   The Model

The framework is based on the benchmark New Monetarist model, which is itself based on Lagos and Wright [34]. A defining feature of this class of models is that agents trade with each other, and not merely against their budget equations. A useful assumption in the model, though not a defining feature, is that in each period of discrete time, a $[0, 1]$ continuum of infinitely-lived agents participate in two distinct markets: a frictionless centralized market CM, as in standard general equilibrium theory; and a decentralized market DM, where buyers and sellers meet and trade bilaterally, as in search theory. These alternating markets are useful because we can impose interesting frictions in the DM, while the presence of the CM keeps the analysis tractable by helping to reduce the dimensionality of the state space. To make exchange interesting, it is

assumed that in DM meetings sellers can produce something buyers want but buyers cannot reciprocate.

In principle, buyers could promise to pay sellers in the next meeting of the CM, but standard assumptions imply that they could renege without fear of repercussion. These assumptions, sometimes packaged under the label *anonymity*, are that there is limited commitment, so that promises are not perfectly credible, and a lack of monitoring or record keeping that makes the use of trigger strategies as a punishment device difficult (see Kocherlakota [27], Wallace [51], or Aliprantis et al. [3] for formal treatments). Hence, unsecured credit is not available in the DM, and assets have a role in facilitating exchange. Buyers in the DM can either hand over assets directly, or use them as collateral. In the second case, in the next CM, either the loan is repaid or sellers keep the collateral – a matter of indifference to both parties in equilibrium. In the first case, there is finality when the assets are handed over, while in the second there is delayed settlement, but aside from this detail the two interpretations are equivalent, as we further discuss in Section 7.

At each date agents first trade in the DM and then the CM. In the CM there is a consumption good X that all agents can produce one-for-one using labor H, and utility is $U(X) - H$. In the DM there is another good q that agents value according to $u(q)$ and can be produced at disutility cost $c(q)$. Goods are nonstorable. We assume $u' > 0$, $u'' < 0$, $c' > 0$, $c'' > 0$, $u(0) = c(0) = c'(0) = 0$, and $U'(0) = u'(0) = \infty$. Also, let X^* and q^* solve $U'(X^*) = 1$ and $u'(q^*) = c'(q^*)$. In any bilateral DM meeting, each agent has an equal probability of being a buyer or a seller, so if we normalize the probability of meeting anyone to 2λ, then λ is the probability of being a buyer. Although we do not do so here, one can allow barter or unsecured credit to be available in some meetings without changing the results; it is only necessary that barter or unsecured credit is not available in all meetings.

Assets are indexed by $j = 1, ... n$, and a portfolio is $\mathbf{a} \in \mathbb{R}^n_+$. As in the standard Lucas [41] model, each asset j can be interpreted as a claim to (an equity share in) a tree j, yielding a

dividend in terms of fruit δ_j in units of good X each period in the CM. As a special case, asset j may be a fiat object, like outside money. By definition a fiat object is "intrinsically worthless" (Wallace [50]), which in this context means $\delta_j = 0$. To keep the environment stationary, for any real asset j with $\delta_j > 0$, we fix the supply at A_j. If j is a fiat object, however, we can let the supply change according to $A_j = (1+\gamma_j)A_j^-$, where A^- is the value of (any variable such as) A in the previous period, without changing real resources because $\delta_j = 0$. Thus, we allow government to issue or retire currency but not to cultivate or cut down fruit-bearing trees. Changes in the supply of fiat objects are accomplished in the CM by lump sum transfers if $\gamma_j > 0$ or taxes if $\gamma_j < 0$. Let ϕ_j be the price of asset j. Stationarity implies $\phi_j A_j = \phi_j^- A_j^-$, which means that γ_j is the inflation rate measured in asset j prices – a version of the Quantity Theory. We assume $\gamma_j > \beta - 1$, where β is the discount factor, but also consider the limit as $\gamma_j \to \beta - 1$, which is the Friedman Rule (there is no equilibrium if $\gamma_j < 1 - \beta$).

We introduce qualitative uncertainty concerning assets, as in Akerlof's [1] lemons model, by assuming that any asset can be of high or low quality, where for simplicity a low-quality asset is completely useless in the sense that it bears no dividend. More generally, the value of any security can be random and agents may have asymmetric information about the probability distribution; the possibility that it may be totally worthless is the special case in which its value may be 0. One interpretation of a worthless asset is that it is *a bad claim to a good tree* – a counterfeit. Another interpretation is that it is *a good claim to a bad tree* – a lemon (tree). For instance, a seller could be offered a bogus equity claim on a profitable company, or he could be offered a legitimate share in a company that was once a going concern but, unbeknownst to him, now has future profit stream of 0. This distinction does not matter for what we do.

In the baseline model, agents can produce worthless assets at any time at cost $k = 0$. This makes worthless assets different from fiat money, even though both have 0 dividends. Fiat money may be valued in equilibrium only if agents cannot costlessly produce passable counterfeit facsimiles of it themselves. So, a bad claim, if recognized as such, will never be accepted, even

though agents may accept fiat money when they recognize that it has a 0 dividend, because they cannot produce genuine currency themselves (for details, see Wallace [52]). One reason $k = 0$ simplifies the analysis is that it implies no seller ever accepts assets he cannot recognize, because, if he did, buyers would simply produce and hand over worthless paper.[3]

The assumption $k = 0$ is extreme, but it is also extremely useful: when sellers reject outright assets that they cannot evaluate, we can use simple bargaining theory in the DM. Since unrecognized assets are not even on the table, negotiations always occur under full information. In this way informational frictions help determine liquidity, but we avoid well-known problems with bargaining under asymmetric information. However, in Section 5, we allow $k > 0$ and show the main results are robust. When $k > 0$, sellers accept assets then cannot evaluate, but only up to a point: they will produce $q^k > 0$ for a^k units of the asset, but q^k and a^k may be less than their values be under full information. In this case, illiquidity means that only a small amount of an asset is acceptable in DM trades. As we show below, $q^k \to 0$ as $k \to 0$.

Only in the DM is there a problem distinguishing high- and low-quality assets, since the CM is frictionless (one story, stepping outside the formal model, is that there are banks or related institutions freely available only in the CM to certify quality). In any bilateral DM meeting a seller may be informed or uninformed about the quality of any given asset. Subsequently we endogenize the information structure; for now it is taken as given. Index any DM meeting by $S \in \mathcal{P}$, where $\mathcal{P}$ is the power set of $\{1, 2, ...n\}$, indicating the subset of assets that the seller recognizes. Let ρ_S be the probability of a type S meeting, or meeting a type S seller. Also, let $\mathcal{P}_j = \{S \in \mathcal{P} : j \in S\}$ be the set of meetings where the seller recognizes asset j. In a type S meeting, a buyer with portfolio $\mathbf{a}$ has liquid or recognizable wealth $y_S(\mathbf{a}) = \sum_{j \in S} (\delta_j + \phi_j) a_j$,

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[3]This is different from work on information-based monetary theory going back to Williamson and Wright [53]. In those models, agents make ex ante choices to bring good or bad assets to the market, and sellers always accept assets with positive probability even if they cannot recognize them. The logic is simple. Suppose there are some informed and uninformed sellers. Informed sellers never accept low-quality assets. If uninformed sellers never accept them, then buyers with such assets cannot trade, and no one brings them to the market. But then uninformed sellers have no reason to reject. The difference here is that buyers can produce worthless assets on the spot. See Lester et al. [37] for details.

and payment to the seller $p_S(\mathbf{a})$ is constrained by $p_S(\mathbf{a}) \leq y_S(\mathbf{a})$. In general, liquid wealth is less than total wealth, $y_S(\mathbf{a}) \leq y(\mathbf{a}) = \sum_{j=1}^{n} \left(\delta_j + \phi_j \right) a_j$.

Let $V(\mathbf{a})$ be the value function for an agent in the DM. In the CM, where all assets are recognized, all that matters for an individual is total wealth $y(\mathbf{a})$, and we write the value function as $W[y(\mathbf{a})]$. Since technology imples the equilibrium real wage is 1, the CM problem is

$$
\begin{aligned}
W(y) &= \max_{X,H,\hat{\mathbf{a}}} \{U(X) - H + \beta V(\hat{\mathbf{a}})\} \\
\text{s.t. } X &= H + y - \sum_j \phi_j \hat{a}_j + T,
\end{aligned}
\tag{1}
$$

where $\hat{\mathbf{a}} \in \mathbb{R}_+^n$ is the portfolio taken into the next DM, while T is a transfer to accommodate potential changes in the supply of fiat objects. There may be an additional constraint $H \in [0, \bar{H}]$, but assuming it is not binding (see Lagos and Wright [34]), we eliminate H to write

$$
W(y) = U(X^*) - X^* + y + T + \max_{\hat{\mathbf{a}}} \left\{ -\sum_j \phi_j a_j + \beta V(\hat{\mathbf{a}}) \right\}.
\tag{2}
$$

It is immediate from (2) that W is linear, $W'(y) = 1$, and $\hat{\mathbf{a}}$ is independent of y and hence $\mathbf{a}$. This reduces the dimensionality of the state space substantially because we do not have to track the distribution of $\mathbf{a}$ across agents in the DM, since we can restrict attention to the case where they all choose the same $\hat{\mathbf{a}}$. If two assets are perfect substitutes, like a ten dollar bill and two fives, agents may hold different portfolios, but they have the same value. Hence, we focus on a symmetric choice for $\hat{\mathbf{a}}$, satisfying the FOC

$$
-\phi_j + \beta \frac{\partial V(\hat{\mathbf{a}})}{\partial \hat{a}_j} \leq 0, \quad = \text{ if } \hat{a}_j > 0 \text{ for } j = 1, ... n.
\tag{3}
$$

These look like conditions one might see in many old, and some not-so-old, models where assets are inserted directly into utility functions. It is important to emphasize, however, that for us $V(\cdot)$ is *not* a primitive – it is the continuation value of participating in the DM.

To characterize the DM terms of trade, a variety of mechanisms can be and have been used in the literature, but for tractability, in this paper we use Kalai's [23] proportional bargaining

solution.[4] In this class of models, proportional bargaining guarantees that $V(\cdot)$ is concave, and that each agent's surplus increases monotonically with the match surplus, neither of which is guaranteed with Nash bargaining (Aruoba et al. [5]). A very useful implication of monotonicity here is that agents have no incentive to "hide" some of their asset holdings, as they do with Nash bargaining (Lagos and Rocheteau [33]; Geromichalos et al. [14]). One can deal with these technicalities, but proportional bargaining avoids them, easing the presentation considerably. The theory is robust, however, in the sense that one can derive qualitatively similar results using generalized Nash (Lester et al. [36]) or Walrasian pricing (Guerierri [17]).

To apply proportional bargaining, note that the surplus of a buyer who gets q for payment p is $u(q)+W(y-p)-W(y) = u(q)-p$, using the linearity of $W(\cdot)$. Similarly, the surplus of the seller is $p-c(q)$. Consider a type S meeting where the buyer has liquid wealth $y_S(\mathbf{a}) = \sum_{j\in S} \left(\delta_j + \phi_j\right) a_j$. The proportional solution is given by a payment $p = p_S(\mathbf{a})$ and quantity $q = q_S(\mathbf{a})$ solving

$$\max_{p,q} \{u(q) - p \} \text{ s.t. } u(q) - p = \theta\left[u(q) - c(q)\right] \text{ and } p \le y_S(\mathbf{a}),$$

where θ is the buyer's bargaining power. Notice $p_S(\mathbf{a})$ and $q_S(\mathbf{a})$ depend on the portfolio of the buyer $y_S(\mathbf{a})$ and information of the seller S, but not on the portfolio of the seller or information of the buyer. Define

$$z(q) = \theta c(q) + (1-\theta)u(q), \tag{4}$$

and let $y^* = z(q^*)$, where $u'(q^*) = c'(q^*)$. The next result says that if liquid wealth exceeds y^* the buyer pays y^* and gets q^*, but if it is below y^* he hands it all over and gets $q < q^*$.[5]

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[4]Other options in the literature include Nash bargaining, price taking, price posting, auctions, and pure mechanism design (see the surveys cited above for references). Proportional bargaining has several advantages in these models, and for this reason it is being used in many recent applications; see Aruoba et al. [5], Aruoba [4], Rocheteau and Wright [47], and Geromichalos and Simonovska [15].

[5]The proof is omitted as it is the same as Lagos and Wright [34], even though they use generalized Nash bargaining. With generalized Nash, Lemma 1 below holds as stated if we redefine

$$z(q) = \frac{\theta u'(q)}{\theta u'(q) + (1-\theta)c'(q)}c(q) + \frac{(1-\theta)c'(q)}{\theta u'(q) + (1-\theta)c'(q)}u(q).$$

In both cases $z(q)$ is a convex combination of $u(q)$ and $c(q)$, but with Nash bargaining the weights depend on q. Comparing this with (4), one can see how proportional bargaining simplifies the algebra – but, again, the results are robust.

Lemma 1. *If $y_S(\mathbf{a}) \geq y^*$ then $p_S(\mathbf{a}) = y^*$ and $q_S(\mathbf{a}) = q^*$; if $y_S(\mathbf{a}) < y^*$ then $p_S(\mathbf{a}) = y_S(\mathbf{a})$ and $q_S(\mathbf{a}) < q^*$ solves $z(q) = y_S(\mathbf{a})$.*

The value of entering the DM can now be written

$$V(\mathbf{a}) = W[y(\mathbf{a})] + \lambda \sum_{S \in \mathcal{P}} \rho_S \{u[q_S(\mathbf{a})] - p_S(\mathbf{a})\} + K. \tag{5}$$

The first term on the RHS is the value of proceeding to the CM with one's portfolio $\mathbf{a}$ intact. The second term is the probability of being a buyer λ multiplied by the expected trade surplus across types of meetings. The final term K is the expected surplus from being a seller, which as shown above does not depend on $\mathbf{a}$. Differentiation leads to[6]

$$\frac{\partial V}{\partial a_j} = \delta_j + \phi_j + \lambda \sum_{S \in \mathcal{P}_j} \rho_S \left(\delta_j + \phi_j\right) \left\{ \frac{u'[q_S(\mathbf{a})]}{z'[q_S(\mathbf{a})]} - 1 \right\}, \tag{6}$$

using $\partial p / \partial a_j = \delta_j + \phi_j$ and $\partial q / \partial a_j = (\delta_j + \phi_j) u'(q)/z'(q)$ if $y < y^*$, while $\partial p / \partial a_j = \partial q / \partial a_j = 0$ otherwise, by Lemma 1.

We can rewrite (6) as

$$\frac{\partial V}{\partial a_j} = \left(\delta_j + \phi_j\right) \left\{ 1 + \lambda \sum_{S \in \mathcal{P}_j} \rho_S \ell[q_S(\mathbf{a})] \right\}, \tag{7}$$

by introducing the *liquidity premium*

$$\ell(q) = \frac{u'(q)}{z'(q)} - 1 = \frac{\theta[u'(q) - c'(q)]}{\theta c'(q) + (1 - \theta)u'(q)}, \tag{8}$$

where the second equality derives $z'(q)$ from (4). This premium is the payoff from a marginal unit of wealth that is liquid, in the sense that it can be used to acquire more q in the DM, as opposed to simply carrying it through to the next CM. From Lemma 1 we have $q \in [0, q^*]$, and from (8) we have $\ell'(q) < 0$ over this range, with $\ell(q) > 0$ if $q < q^*$ and $\ell(q^*) = 0$. For future reference, we say that agents are *satiated in liquidity* at $\mathbf{a}$ when $\ell[q_S(\mathbf{a})] = 0$, or equivalently $q_S(\mathbf{a}) = q^*$, for all S.

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[6]Note that in the summation, for any S such that $y_S(\mathbf{a}) \geq y^*$, the term in braces is 0. So the summation is only positive if $y_S(\mathbf{a}) < y^*$ for some S with $\rho_S > 0$. Also, the summation runs only over $S \in \mathcal{P}_j$ since an asset only helps if the seller recognizes it.

Combining (7) and (3), being careful with the timing, we get

$$-\phi_j^- + \beta \left(\delta_j + \phi_j\right) \left\{ 1 + \lambda \sum_{S \in \mathcal{P}_j} \rho_S \ell\left[q_S\left(\mathbf{a}\right)\right] \right\} \leq 0, \ = \ \text{if } \hat{a}_j > 0, \ j = 1, ...n. \qquad (9)$$

where the first term ϕ_j^- is the price of asset j in the previous period's CM while all variables in the second term are in the current period. For any real asset with $\delta_j > 0$, in equilibrium $\phi_j > 0$ and $a_j = A_j > 0$, so (9) holds with equality. For a fiat asset a_j, we can have $\phi_j = 0$, but as long as a_j is valued (9) also holds with equality. Hence, it holds with equality in all relevant situations. Using this and market clearing $\hat{\mathbf{a}} = \mathbf{A}$, we arrive at

$$\phi_j^- = \beta \left(\delta_j + \phi_j\right) \left\{ 1 + \lambda \sum_{S \in \mathcal{P}_j} \rho_S \ell\left[q_S\left(\mathbf{A}\right)\right] \right\}, \ j = 1, ...n. \qquad (10)$$

Equilibrium asset prices are given by any sequence $\{\boldsymbol{\phi}_t\}_{t=0}^{\infty}$ satisfying (10) that is nonnegative and bounded,[7] from which we can easily determine all of the other endogenous variables. In particular, given asset prices, we can compute $y_S(\mathbf{a})$ in any DM meeting, and then $q_S\left(\mathbf{a}\right)$ and $p_S\left(\mathbf{a}\right)$ follow from Lemma 1.

Here we focus mainly on stationary equilibria. First, suppose $\lambda = 0$ (the DM is shut down). Then there are no liquidity considerations, and (10) reduces to $\phi_j^- = \beta\left(\delta_j + \phi_j\right)$. The stationary solution to this is $\phi_j = \beta\delta_j/(1 - \beta) = \phi_j^*$, where ϕ_j^* is the *fundamental price* of asset j, defined as the present value of its dividend stream. For a fiat object, notice, if $\lambda = 0$ then $\phi_j^* = 0$. Now suppose $\lambda > 0$. Let $\bar{y}_S\left(\mathbf{A}\right) = \sum_{j \in S} A_j \delta_j/(1 - \beta)$ be liquid wealth in meeting S when all assets are priced fundamentally. Agents are satiated in liquidity when $\bar{y}_S\left(\mathbf{A}\right) \geq y^*$ with probability 1, and in this case the unique equilibrium also has assets priced fundamentally. But if $\bar{y}_S\left(\mathbf{A}\right) < y^*$ for some $S \in \mathcal{P}_j$ with positive probability and some j, then agents are not satiated in liquidity, and ϕ_j can exceed the fundamental price.[8]

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[7]Boundedness follows from transversality conditions in this type of model (Rocheteau and Wright [47]).

[8]When assets are priced fundamentally, all equilibria are stationary – e.g., in the case $\lambda = 0$ we have $\phi_j^- = \beta\left(\delta_j + \phi_j\right)$, and the *only* solution to this difference equation that is nonnegative and bounded is the constant (fundamental) solution $\phi_j = \phi_j^*$. When assets bear liquidity premia, there may exist nonstationary equilibria in related models(Rocheteau and Wright [47]); we relegate analyses of nonstationary outcomes to future work.

Although we are interested mainly in economies with multiple assets, consider an example with $n = 1$. Dropping the subscript (e.g., $a_1 = a$), (10) becomes

$$\frac{\phi}{\beta(\delta + \phi)} - 1 = \lambda \rho \ell [q(a)], \tag{11}$$

where ρ is the probability a is recognized, and the bargaining solution implies

$$q(a) = \begin{cases} z^{-1}(y) & \text{if } y < y^* \\ q^* & \text{if } y \geq y^* \end{cases}. \tag{12}$$

Inserting $q(a)$ from (12) into (11) implicitly defines the demand for a as a function of the price ϕ. Figure 1 shows the (inverse) demand curve as continuous, and decreasing, until it becomes flat at $A^* = y^*/(\delta + \phi)$.[9] The market-clearing price corresponds to the intersection of demand with the supply curve, which is vertical at $a = A$.

<center>INSERT FIGURE 1 ABOUT HERE</center>

For the sake of illustration, suppose a is a T-bill, which is only a slight stretch since one can easily rewrite the model in terms of pure-discount bonds rather than dividend-bearing securities. Then this example constitutes a structural model of the T-bill demand function Krishnamurthy and Vissing-Jorgensen [31] take to the data, derived indirectly from the premise that T-bills are useful as a means of payment or as collateral, rather than sticking them directly into the utility function. More generally, our theory delivers an interrelated asset demand system for $\mathbf{a} \in \mathbb{R}^n_+$ as a function of $\boldsymbol{\phi} \in \mathbb{R}^n_+$, analogous to the standard interrelated factor demand systems that are often estimated for firms as functions of input prices. In principle one can estimate this asset demand system jointly – instead of the usual procedure of estimating in isolation the demand for money, the demand for T-bills, etc., but this is obviously beyond the scope of the current project. In any case, we summarize the results for $n = 1$ as follows:

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[9]To verify this, differentiate to get

$$\frac{d\phi}{da} = \begin{cases} \beta \lambda \rho \ell'(q)(\delta + \phi)^3/z'(q)\delta & \text{if } a < A^* \\ 0 & \text{if } a \geq A^* \end{cases}$$

Since $\ell' < 0$ and $z' > 0$, we have $d\phi/da < 0$ when $a < A^*$. Note that $a < A^*$ iff $q < q^*$ iff $\ell(q) > 0$. From (11), $\ell(q) > 0$ iff $\phi > \phi^*$. Therefore, demand is infinitely elastic at ϕ^*.

<center>13</center>

Proposition 1. *Define $A^* = y^*(1 - \beta)/\delta$. (i) If $A \geq A^*$ then there exists a unique equilibrium with $\phi = \phi^*$, $p = y^* = (\phi^* + \delta)A^*$, and $q = q^*$. (ii) If $A < A^*$, then $\phi < \phi^*$ satisfies (11) with $a = A$, $p = y = (\phi + \delta)A < y^*$, and $q = z^{-1}(y) < q^*$.*

In terms of economics, in case (i) of the Proposition agents are satiated in liquidity and there is no premium: $\phi = \phi^*$, and DM buyers use A^* assets for trade and hold the remaining $A - A^*$ purely as a store of value. In case (ii), however, liquidity is scarce and the asset price bears a premium: $\phi > \phi^*$. To capture fiat money, as a special case, set $\delta = 0$, and increase A at rate γ. In stationary equilibrium the inflation rate is $\phi/\phi^- = 1 + \gamma$. Now (10) implies

$$\frac{1 + \gamma - \beta}{\beta} = \lambda \rho \ell(q). \tag{13}$$

Letting $\gamma \to \beta - 1$, which is the Friedman Rule, we get $\ell(q) = 0$ and $q = q^*$; for any $\gamma > \beta - 1$, there is a liquidity premium, and $q < q^*$.[10]

Define the real and nominal interest rates r and i as the returns on real and nominal bonds traded between one CM and the next CM, under the proviso that these bonds are illiquid in the sense that they cannot be traded in the DM (say, because agents cannot verify their authenticity in the DM). It should be obvious that $1 + r = 1/\beta$ and $1 + i = (1 + r)(1 + \gamma)$, the latter being the standard Fisher Equation. Given this, (13) becomes $i = \lambda \rho \ell(q)$, which implies $\partial q/\partial i < 0$. The Friedman Rule can be stated as $i = 0$, and in this model it delivers $q = q^*$. The Friedman Rule is of course optimal in many monetary models; something that is somewhat here new is that, given $i > 0$, the distortion is greater when information problems are more severe – i.e., q is smaller when ρ is smaller.

3   Monetary Policy and Asset Prices

Consider $n = 2$ assets: $a_1 = m$ is fiat currency with $\delta_1 = 0$ and $A_1 = M$; $a_2 = a$ is a real asset with $\delta_2 = \delta > 0$ and $A_2 = A$. We write their prices as $\phi_1 = \phi$ and $\phi_2 = \psi$, and focus on

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[10]This is a difference between proportional and generalized Nash bargaining: the latter implies $q \to q^*$ as $\gamma \to \beta - 1$ iff $\theta = 1$; the former $q \to q^*$ as $\gamma \to \beta - 1$ for all θ.

stationary monetary equilibrium where $\phi^-/\phi = M/M^- = 1 + \gamma$. For this exercise we abstract from counterfeit currency considerations and assume agents always recognize money: $m \in S$ with probability 1. But only a fraction recognize real assets: $a \in S$ with probability $\rho \in (0, 1)$. Call the event $S_1 = \{m\}$, in which case a seller in the DM does not recognize a, a *type 1 meeting*. The buyer's liquid wealth in such a meeting is $y_{S_1}(m, a) = y_1 = \phi m$ and the terms of trade are (p_1, q_1), as given in Lemma 1. Similarly, call $S_2 = \{m, a\}$ a *type 2 meeting*. Liquid wealth in such a meeting is $y_{S_2}(m, a) = y_2 = \phi m + (\psi + \delta)a$ and the terms of trade are (p_2, q_2).

Taking ρ as given for now, the DM value function is the special case of (5) given by

$$V(m, a) = W(y_2) + \lambda_1 [u(q_1) - p_1] + \lambda_2 [u(q_2) - p_2] + K, \tag{14}$$

where $\lambda_1 = \lambda(1 - \rho)$ and $\lambda_2 = \lambda\rho$, while (10) reduces to

$$\phi^- = \beta\phi [1 + \lambda_1\ell(q_1) + \lambda_2\ell(q_2)] \tag{15}$$

$$\psi^- = \beta(\psi + \delta) [1 + \lambda_2\ell(q_2)] . \tag{16}$$

The bargaining solution and market clearing imply $z(q_1) = \phi M$ and $z(q_2) = \phi M + (\psi + \delta)A$ as long as these yield $q_j < q^*$; otherwise $q_j = q^*$. Using this to eliminate (q_1, q_2), (15)-(16) becomes a system of equations in asset prices, as in the general case. It is easier in this application, however, to work with quantities rather than prices.

Using $\psi A = z(q_2) - z(q_1)$, $1 + i = (1 + \gamma)/\beta$ and $\beta = 1/(1 + r)$, we can reduce (15)-(16) to

$$i = \lambda_1\ell(q_1) + \lambda_2\ell(q_2) \tag{17}$$

$$(1 + r)A\delta = [z(q_2) - z(q_1)][r - \lambda_2\ell(q_2)]. \tag{18}$$

A stationary monetary equilibrium is summarized by a positive solution (q_1, q_2) to (17)-(18), as long as $q_j < q^*$. Notice that $q_1 < q_2$ and $q_j = q^*$ iff $\ell(q_j) = 0$ iff $y_j \geq y^*$. Also, if $i > 0$ then $y_1 < y^*$ and $q_1 < q^*$, intuitively, because a small reduction in q near q^* has a negligible effect (the envelope theorem) while there is a first-order cost to carrying money balances. It is possible to have $y_2 \geq y^*$, whence liquidity in a type 2 meeting suffices to purchase q^*. In this

case, $q_2 = q^*$, $\ell(q_2) = 0$, and $q_1 = \tilde{q} < q^*$ solves $\lambda_1 \ell(\tilde{q}) = i$. But it is also possible to have $y_2 < y^*$ and $q_2 < q^*$. Either case may obtain, depending on parameters, and in particular depending on whether the real asset is relatively abundant or scarce.

Proposition 2. *Define A^* by*

$$\frac{A^* \delta}{r} = \frac{z(q^*) - z(\tilde{q})}{1 + r} > 0.$$

(i) If $A \geq A^*$ there exists a unique stationary monetary equilibrium, with $(q_1, q_2) = (\tilde{q}, q^*)$, $\phi = z(\tilde{q})/M$ and $\psi = \delta/r$. (ii) If $A < A^*$ there exists a unique stationary monetary equilibrium, where (q_1, q_2) solves (17)-(18), $\phi = z(q_1)/M$ and $\psi = [z(q_2) - z(q_1)]/A - \delta > \delta/r$.

The proof is in the Appendix, but the results should be clear from Figure 2, which displays functions $q_1 = \mu(q_2)$ and $q_2 = \alpha(q_1)$ defined by (17) and (18). It is easy to show $\mu(\cdot)$ is decreasing and $\alpha(\cdot)$ increasing, and they intersect for some $q_1 \in [0, q^*]$. For $A \leq A^*$, this intersection determines the equilibrium $(q_1, q_2) \in [0, q^*]^2$. For $A > A^*$, the intersection occurs at $q_2 > q^*$, but since $q_2 > q^*$ is not possible, equilibrium is $(q_1, q_2) = (\tilde{q}, q^*)$. Thus, the unique (stationary monetary) equilibrium is given by the intersection of $q_1 = \mu(q_2)$ and $q_2 = \bar{\alpha}(q_1) = \min\{\alpha(q_1), q^*\}$.

<center>INSERT FIGURE 2 ABOUT HERE</center>

When $A < A^*$, the asset bears a liquidity premium.[11] Table 1 shows the effects of changing the rate of monetary expansion γ and the recognizability parameter ρ in this case (derivations are in the Appendix). Increasing γ increases inflation and lowers the return on perfectly liquid money, $R_m = \phi/\phi^-$. It has no effect on the return of illiquid real bonds, $R_r = 1 + r = 1/\beta$, while the return on illiquid nominal bonds $R_n = 1 + i = (1 + \gamma)/\beta$ increases one-for-one with inflation (Fisher's theory). For our *partially* liquid asset a, an increase in γ shifts the μ curve southwest and leaves α unchanged, reducing q_1, q_2 and the return $R_a = (\psi + \delta)/\psi$. Intuitively, as inflation

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[11]To get a simple formula, after some algebra, we can write

$$\psi = \frac{\delta}{r} \left[1 + \frac{(1 + r)\lambda_2 \ell(q_2)}{r - \lambda_2 \ell(q_2)} \right].$$

Since (18) implies $r > \lambda_2 \ell(q_2)$, ψ exceeds the fundamental price δ/r.

<center>16</center>

increases, agents try to economize on cash, reducing the CM price ϕ and DM value of money $q_1 = z^{-1}(\phi M)$. As agents desire fewer money balances, they endeavor to shift into real assets, which are (imperfect) substitutes for cash. With a fixed supply A, this raises the price ψ, lowers the dividend-price ratio, and hence lowers the return R_a.[12]

ζ	$\dfrac{\partial q_1}{\partial \zeta}$	$\dfrac{\partial q_2}{\partial \zeta}$	$\dfrac{\partial \phi}{\partial \zeta}$	$\dfrac{\partial \psi}{\partial \zeta}$	$\dfrac{\partial R_m}{\partial \zeta}$	$\dfrac{\partial R_a}{\partial \zeta}$
γ	$-$	$-$	$-$	$+$	$-$	$-$
ρ	$-$	$?$	$-$	$+$	0	$-$

Table 1: Effects of parameters when $A < \bar{A}$.

Figure 3 plots ψ against i for an example, for different values of ρ. As ρ increases, so does $\partial \psi / \partial i$. Intuitively, as a becomes more liquid, the effect on ψ of i becomes stronger, because a is a better substitute for m. In terms of changes in ρ, more generally, Table 1 indicates that when the recognizability of a goes up, at the margin agents desire a reallocation of their portfolios out of m and into a. This drives ϕ down and ψ up, with ambiguous effects on q_2. All these results are for the case $A \leq \bar{A}$. When $A > \bar{A}$, $q_2 = q^*$ and $q_1 = \tilde{q}$, where $\ell(\tilde{q}) = i/\lambda_1$. In this case, an increase in γ or ρ reduces q_1 and ϕ, with no effect on q_2 or ψ, which are pinned down by fundamentals. Again, it clearly matters whether A is above or below A^*, as we discuss further in Section 7.

INSERT FIGURE 3 ABOUT HERE

Before endogenizing information, we mention one more application with ρ fixed. This shows how monetary policy affects not only agents or markets that use money, but also those that do not. Consider two distinct decentralized markets, call them DM_1 and DM_2. After each meeting of the CM, some agents go to DM_1 and others to DM_2, say because different goods

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[12]This sounds like routine discussions in monetary policy circles, claiming that easing monetary policy reduces interest rates, which is hard to understand in terms of Fisher's theory. It also helps us makes sense of the remarks by Wallace [50] in footnote 2: assets bear different returns because of liquidity differentials, and Fisher's theory may fail for liquid assets because they are partial substitutes for cash. See Geromichalos et al. [14] and references therein for related results. See Li and Li [38] for a model that delivers the opposite result: increases in inflation decrease the price and increase the return on real assets, because they are complements for cash (they are used to collateralize loans of currency, not real loans).

are traded there. In DM$_2$ all sellers accept both a and m, which means no one brings m to DM$_2$. Meanwhile, in DM$_1$ only a fraction ρ of sellers accept a, so agents bring a portfolio (m, a) as in the benchmark model. One can show (Lester et al. [36]) that an increase in γ reduces consumption and welfare in *both* markets, even though there is no money in DM$_2$. Intuitively, as γ increases, agents going to DM$_1$ shift out of m and into a, driving up ψ. This lowers the return on a, as well as the amount held by agents in DM$_2$. This lowers consumption and utility for agents in DM$_2$ even though they never use money.[13]

4   Endogenous Information

We now utilize the results in Proposition 2 to endogenize ρ in the model with one real asset a and one fiat object m. Assume that agent $h \in [0, 1]$ has an ex ante choice whether to acquire at cost $\kappa(h)$ the information, or perhaps the technology, that allows him to recognize the quality of the real asset (here we maintain the assumption that agents can recognize money at zero cost). Without loss of generality, label agents so that $\kappa(h)$ is increasing. Agent h accepts a in the DM iff he pays $\kappa(h)$, since this is the only way to distinguish asset quality, and if an uninformed seller were to accept a a buyer would hand over worthless facsimiles. The fraction of agents that incur the cost of becoming informed therefore determines the fraction that accept a in the DM, ρ. For now we assume that $\kappa(h)$ is a flow cost that h must pay each period to accept a; later we consider a one-time cost.

One can imagine several interpretations of κ. It is typically thought to be costly to learn to use a new medium of exchange, for a variety of reasons, some of which are discussed in Lotz and Rocheteau [40] and, in the context of dollarization, the references in Section 6. Historically, it was difficult to distinguish the weight and fineness of coins without devices like scales and touchstones. It would have been even harder in those days to distinguish real from bogus paper claims, especially for the many who could not read. Although literacy has improved since then,

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[13]This extension was motivated by those who say monetary policy is irrelevant (to them?) because they never use cash. The logic here shows that this position is, in addition to being egocentric, incorrect.

distinguishing low- from high-quality assets remains a costly endeavor – a contemporary financial institution that wants to trade pools of asset-backed securities, say, must set up a department with analysts to ascertain their values. Other costs may be technological, as in the case of debit cards, where sellers must buy a machine to transfer funds from one account to another. As these all seem potentially relevant, we are agnostic about the exact nature of κ.

Conditional on a fraction $\rho \in [0, 1]$ of others being informed, let $\Pi(\rho)$ denote the benefit of information for any given individual. To determine this, let $\Sigma_1(\rho) = z[q_1(\rho)] - c[q_1(\rho)]$ denote a seller's surplus in a type 1 meeting, where $z(q)$ defined in (4) gives the real value of the payment to the seller as a function of q, and $q_1(\rho)$ is the equilibrium outcome characterized in Proposition 2. Similarly define $\Sigma_2(\rho)$ for a type 2 meeting. Then $\Pi(\rho) = \beta\lambda [\Sigma_2(\rho) - \Sigma_1(\rho)]$. The following useful result is proved in the Appendix.

Lemma 2. $\Pi(\rho)$ *is increasing.*

Equilibrium is defined by (q_1, q_2) satisfying (17)-(18) from the model with ρ fixed, plus a measure of informed agents ρ satisfying one of the following configurations: $\rho = 0$ and $\Pi(0) \leq \kappa(0)$; $\rho = 1$ and $\Pi(1) \geq \kappa(1)$; or $\rho \in (0, 1)$ and $\Pi(\rho) = \kappa(\rho)$. Consider the CDF, $F : \mathbb{R} \to [0, 1]$ where $F(\bar{\kappa})$ is the fraction of agents for whom $\kappa \leq \bar{\kappa}$. Then define the mapping $T : [0, 1] \to [0, 1]$ by $T(\rho) = F[\Pi(\rho)]$. Equilibrium ρ is summarized by a fixed point of T, from which we then get (q_1, q_2) from Proposition 2. The following is now straightforward.[14]

Proposition 3. *There exists an equilibrium* (ρ, q_1, q_2).

Proof: From Lemma 2, $T(\rho)$ is monotonically increasing on $[0, 1]$. So a fixed point is guaranteed by Tarski's theorem. The rest is obvious. ∎

Assume that $(1 + r)\delta A < rz(q^*)$, which implies a is sufficiently scarce to guarantee there is a monetary equilibrium even if $\rho = 1$ (this did not come up before because we always had $\rho < 1$).

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[14]The proof uses Tarski's theorem, although we only need a special case since our T maps $[0, 1]$ into itself. But this is still nice because Tarski does not require continuity, as most fixed-point theorems do, so we need no assumptions on $F(\tilde{\kappa})$ (e.g., the result covers discrete and continuous distributions).

Consider the payoff to an individual from being informed when all others are uninformed. Since $\rho = 0$ implies $q_1 = \hat{q} = \ell^{-1}(i/\lambda)$ and $\psi = \delta/r$, if a single individual were to be informed, instead of accepting only m in exchange for q_1, he could accept m and a for $q_2 = \min\{q^*, \bar{q}\}$, where $\bar{q}$ solves $(1+r)A\delta = r\left[z\left(\bar{q}\right) - z\left(\hat{q}\right)\right]$. Since $\Pi(\rho)$ is increasing, $\Pi(0)$ is the minimum possible benefit to information. Now consider the benefit when $\rho = 1$. Since $(1+r)\delta A < rz(q^*)$, equilibrium exists with $q_2 = \hat{q} > q_1 = \underline{q} > 0$, where $(1+r)A\delta = r\left[z\left(\hat{q}\right) - z\left(\underline{q}\right)\right]$. Given this, $\Pi(1)$ is the maximum possible benefit to being informed. Equilibrium with $\rho^* = 0$ exists if $\kappa(0) \geq \Pi(0)$, and it is unique if $\kappa(0) > \Pi(1)$. This is more likely when buyer's bargaining power θ is high, the matching probability λ is low, or information is expensive. Similarly, equilibrium with $\rho^* = 1$ exists if $\Pi(1) \geq \kappa(1)$, and it is unique when $\Pi(0) > \kappa(1)$. It is also easy to get $\rho^* \in (0, 1)$, a sufficient condition being $\kappa(0) < \Pi(0)$ and $\kappa(1) > \Pi(1)$.

When $\rho^* = 0$, the DM looks like a cash-in-advance market, as in Lucas and Stokey [42]. When $\rho^* = 1$, a is perfectly liquid, and it circulates concurrently with m. The reason m is needed at all is that a is scarce by the condition $(1+r)\delta A < rz(q^*)$. Without this condition, $\rho^* = 1$ implies m is not valued, and the DM looks like the limiting cashless economy studied by Woodford [56]. Our point, however, is *not* to provide microfoundations for any particular *ad hoc* assumption on transaction patterns, like money is always used or is never used. To the contrary, as soon as one endogenizes the set of transactions where a particular instrument is used for payments, or equivalently, as collateral, it becomes all too evident that in general this set is neither uniquely determined nor structurally invariant. Equilibrium is given by $\rho^* = T(\rho^*)$, but there need not be a unique solution, and in any case there is no reason to suppose ρ^* does not depend on policy or other parameters.[15]

Figure 4 illustrates the possibility of multiple equilibria. Multiplicity here arises from general equilibrium asset market effects that to our knowledge are new. When more sellers can

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[15]In models that impose a cash-in-advance constraint on some goods and not others, changes in parameters can generate adjustment on the intensive margin (i.e., on quantities consumed of so-called cash goods and so-called credit goods), but, with few exceptions, there is no adjustment along the extensive margin determining which trades use which instruments.

discriminate between high- and low-quality assets, these assets become more liquid. This increases demand for the assets and hence their price in the CM. When assets are more valuable, sellers are more interested in being able to accept them (the trade surplus increases) in the DM. Hence, they are more inclined to pay for requisite information. A related points is that small changes in the environment, including the cost of information, can have large effects. Consider changing $F_1(\kappa)$ to $F_2(\kappa)$, where the latter first-order stochastically dominates. This shifts $T(\rho)$ down for all ρ. In the case of a unique interior equilibrium this unambiguously reduces ρ^*. More dramatically, in Figure 4 the shift in $T(\rho)$ changes the cardinality of the equilibrium set: starting in equilibrium with the highest ρ, a small increase in the cost of verifying asset quality leads to a drastic change in asset liquidity, with commensurate changes in prices, allocations and payoffs.

INSERT FIGURE 4 ABOUT HERE

Although we do not want to push this too far, we think that it is interesting to view modern financial crises through the lens of our theory. Even if a change in the cost of information does not alter the structure of the equilibrium set, the model generates internal feedback or multiplier effects. Suppose there is a unique equilibrium $\rho \in (0, 1)$. When the cost of information goes up, the number of informed agents ρ falls holding other variables constant. If we may be permitted to indulge in pseudo-dynamics, for a moment, the fall in ρ leads to a decrease in the value of the asset, so even fewer agents are willing to bear the cost of information. This continues as the initial increase in cost leads to further and further declines in asset prices. In terms of observations, in the case of mortgage-backed securities, for instance, when housing prices fell the value of the assets became difficult to discern, and counterparties became increasing unwilling to accept them. The haircut on agency mortgage-backed securities went from 2.5 % in the spring of 2007 to 8.5 % in the fall of 2008 (Krishnamurthy [30]).

A similar multiplier effect holds for an increase in inflation. Initially, this lowers the value of money ϕ, which increases ρ, which further decreases ϕ, and so on. On this point, however, a

note of caution is warranted: increasing inflation may well increase the liquidity of some assets, but one should not think inflation is therefore desirable for welfare. As mentioned earlier, the efficient outcome here is achieved when inflation is minimized, at the Friedman Rule. This policy not only satiates agents in liquidity, resulting in $q = q^*$ in all DM meetings, it additionally allows us to save on the cost of information aquisition, given the maintained assumption that all agents recognize cash without having to invest – although it could also be interesting to proceed without this assumption.

Related to this last point, to close this section we sketch how one can use the framework to think about choices over alternative candidate media of exchange in historical contexts. Consider the case of two real assets, say a_1 is gold and a_2 is silver, to capture in a stylized way the long and contentious debate over bimetallism in US history (see Friedman [12] for a readable synopsis and references). Assume without loss in generality that the two metals have the same dividend $\delta > 0$, by appropriately choosing units. Also, assume $\max\{A_1, A_2\} < A^* = ry^*/(1+r)\delta$, so that neither is individually plentiful enough to support q^*. To ease the presentation, let the cost of recognizing them be the same, $\kappa_1(h) = \kappa_2(h) = \kappa(h)$.[16]

Consider a candidate equilibrium in which everyone becomes informed about gold but not silver. Let the CM prices be ϕ_1^G and ϕ_2^G and the DM quantity $q^G = z^{-1}\left[(\phi_1^G + \delta)A_1\right]$ in this scenario, where gold is the unique medium of exchange since it is uniquely recognizable. By the usual reasoning,

$$\frac{\phi_1^G}{\beta(\phi_1^G + \delta)} - 1 = \lambda\ell(q^G) \text{ and } \phi_2^G = \delta/r,$$

so that gold bears a liquidity premium while silver is priced fundamentally. Suppose an individual deviates and becomes informed about silver *in addition to* gold, and let $\tilde{q}^G$ be the amount he produces in a DM meeting. If $(\phi_1^G + \delta)A_1 + (\phi_2^G + \delta)A_2 > y^*$ then $\tilde{q}^G = q^*$; otherwise $\tilde{q}^G < q^*$ solves $z\left(\tilde{q}^G\right) = (\phi_1^G + \delta)A_1 + (\phi_2^G + \delta)A_2$. Also, let $\widehat{q}^G = z^{-1}\left[(\phi_1^G + \delta)A_2\right]$ be the quantity

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[16]This means that a household has to pay $2\kappa(h)$ to be able to evaluate both gold and silver, which may not be the most realistic scenario (particularly if an agent only needs to buy a scale to evaluate both). But for the purpose of illustration this assumption seems fine.

he produces if he deviates by becoming informed about silver *instead of* gold. To support this equilibrium we need: (i) no agent wants to learn to accept silver as well as gold; (ii) no agent wants to learn to accept silver instead of gold; and (iii) all agents want to learn to accept gold. These conditions all hold when

$$q^G \geq \widehat{q}^G; \; \kappa(0) > \beta\lambda \left[z\left(\tilde{q}^G\right) - c\left(\tilde{q}^G\right) - z\left(q^G\right) + c\left(q^G\right) \right]; \; \kappa(1) < \beta\lambda \left[z\left(q^G\right) - c\left(q^G\right) \right]. \quad (19)$$

Symmetrically, an equilibrium where silver is the unique medium of exchange, with CM prices ϕ_1^S and ϕ_2^S and DM quantity q^S, exists when (19) holds after changing the superscript from G to S. Both equilibria exist for some parameters (e.g., when A_1 and A_2 are not too different), but one may dominate the other. If $A_1 < A_2 < A^*$, so that gold is more scarce than silver, a gold equilibrium can exist even though a silver equilibrium would support more DM trade. It may be better still to have both gold and silver used as money, of course, but it is important not to neglect the cost $\kappa(h)$ in this calculation.[17] We leave further analysis to future work. The point here is to show that once exchange patterns and transaction instruments are endogenous, one can study a variety of questions in historical and contemporary economics.

5 Costly Counterfeiting

Tractability is enhanced by the property that sellers who do not recognize an asset reject it outright, since this avoids complications associated with bargaining under private information. This property follows from the assumption that the cost of producing worthless facsimiles of an asset is $k = 0$. We now consider $k > 0$. As we show, this implies sellers may accept an asset that they do not recognize, but only up to a point: they only produce q^k for a^k units of the asset, where generally $a^k < a$ when k is small. Such restrictions on asset transferability are often imposed exogenously – Kiyotaki and Moore [25], [26] and Holmstrom and Tirole [19] providing some of the best examples – but here the restriction is an equilibrium outcome. This means it is

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[17]It is also clear that the choice of gold, silver or both has an impact not only on liquidity and DM trade, but on the payoffs of those who mine the two metals, in the model, and as reflected in the bimetalism debates.

not generally invariant to parameter changes, of course, and in particular, we show $a^k \to 0$ and $q^k \to 0$ as $k \to 0$, which means that our baseline model can be considered a useful limiting case.

To demonstrate this formally, we need some assumptions, especially concerning bargaining under asymmetric information. Our setup is based on Rocheteau [46] and Li and Rocheteau [39], and in fact, the analysis is sufficiently similar that we only sketch the results, refering readers to those papers for details. First, we assume that buyers in DM meetings make take-it-or-leave-it offers (as Li and Rocheteau [39] assume in their baseline analysis, although they also show how to relax this). Consider an economy with two assets, m and a, where again everybody recognizes m and no one a, so that $\rho = 0$; this is without much loss in generality, since if any seller were to recognize a, bargaining could proceed under full information. Now agents in the DM can produce any number of worthless facsimiles of a, at zero marginal cost, iff they pay a fixed cost $k > 0$ each period in the CM. Following the literature, bad assets fully depreciate after one period (e.g., imagine that in the CM they are identified, confiscated and destroyed, the way counterfeit currency is usually handled).

Let χ denote the probability that an agent pays cost k to acquire the counterfeiting technology. Agents in the CM also choose a portfolio $[m(\chi), a(\chi)]$, then proceed to the DM. With probability λ, each agent is a buyer in a match and makes a take-it-or-leave-it offer (q, d, s), asking for q in exchange for d dollars and s shares of the asset, with no stipulation that these shares are real or counterfeit, since the seller cannot tell the difference. Any offer must be feasible, in the sense that a buyer must have the dollars and (genuine or counterfeit) shares at hand. In response, the seller chooses the probability of accepting, denoted ξ. Unfortunately, as is common in models with bargaining under asymmetric information, there are many equilibria in this game. Specifically, if the choice of χ occurs before the offer (q, d, s), the seller has to formulate beliefs about χ based on the offer. Since sequential equilibrium imposes little discipline on these beliefs, many equilibria are possible.

Alternatively, following In and Wright [21], consider the *reverse-order game* in which buyers

first choose (q, d, s) and then χ. Changing the timing in this way does not affect the payoffs or information of the seller; indeed, the two games have the same reduced normal form. This approach is in the spirit of the invariance condition required for strategic stability in Kohlberg and Mertens [29], which requires that a solution to a game should also be a solution to any other game with the same reduced normal form. With the new timing, however, the game described above has proper subgames, and thus sequential equilibrium imposes strict discipline: following any offer, the seller's belief about χ must be consistent with equilibrium strategies. Li and Rocheteau [39] show that there is a unique equilibrium of this game, which is also an equilibrium of the original game for specific reasonable beliefs, and so the approach can be thought of as a natural refinement on the original game.

In this reverse-order game, take any offer (q, d, s), and consider the subgame that follows in a meeting: the buyer chooses χ, and the seller chooses ξ. Working backwards, let $\tilde{\chi}$ denote the seller's belief that the buyer is a counterfeiter. The optimal choice of ξ is

$$\xi = 1 \text{ or } 0 \text{ or } [0,1] \text{ as } -c(q) + \phi d + \tilde{\chi}(\psi + \delta)s \text{ is } > 0 \text{ or } < 0 \text{ or } = 0.$$

In words, sellers weigh the cost $c(q)$ against the expected value of the payment $p = \phi d + \tilde{\chi}(\psi + \delta)s$, where the latter is based on beliefs about the probability $\tilde{\chi}$. Similarly, buyers choose not to counterfeit if

$$-\phi^- d - \psi^- s + \beta \left\{ \lambda \xi [u(q) - \phi d - (\psi + \delta)s] + \phi d + (\psi + \delta)s \right\} \geq$$
$$-k - \phi^- d + \beta \left\{ \lambda \xi [u(q) - \phi d] + \phi d \right\}.$$

The LHS is the payoff from acquiring d dollars and s genuine units of the asset, while the RHS is the payoff from acquiring d dollars and paying k for the ability to counterfeit.[18] Therefore

$$\chi = 1 \text{ or } 0 \text{ or } [0,1] \text{ as } k - s\left[\psi - \beta(\psi + \delta)(1 - \lambda \xi)\right] \text{ is } > 0 \text{ or } < 0 \text{ or } = 0.$$

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[18]If the buyer did not acquire genuine assets in the previous CM, he saved the cost $\psi^- s$, but also lost the benefit of having genuine assets in the event that he does not trade in the DM. Here we use the result that agents bring no genuine assets to the DM if $\chi = 1$.

Given (q, d, s), $(\chi, \xi, \tilde{\chi})$ is an equilibrium in the subgame if $\chi(q, d, s)$ and $\xi(q, d, s)$ are best responses and sellers' beliefs are consistent $\chi(q, d, s) = \tilde{\chi}(q, d, s)$. Now, moving back in time, the equilibrium offer (q, d, s) maximizes an agent's payoff given $\chi(q, d, s)$ and $\xi(q, d, s)$. As in Li and Rocheteau [39], there is a unique such (q, d, s) and it solves

$$\max \left\{ -i\phi d - r(\psi - \psi^*)s + \lambda[u(q) - \phi d - (\psi + \delta)s] \right\} \tag{20}$$

$$\text{s.t. } c(q) \leq \phi d + (\psi + \delta)s \text{ and } k \geq s\left[\psi - \beta(1 - \alpha\lambda)(\psi + \delta)\right],$$

where in equilibrium $\chi(q, d, s) = 0$ and $\xi(q, d, s) = 1$. The objective function in (20) is expected utility in the CM. The first constraint is the incentive condition, or IC, ensuring $\xi = 1$; the second is the no-counterfeiting condition, or NC, ensuring $\chi = 0$. Hence, a buyer's offer maximizes his payoff subject to the seller accepting and, critically, subject to the credibility of the belief that s is genuine. Credibility here means that s is not too big, because the seller can believe that buyers would not find it worthwhile to pay k to pass $(\psi + \delta)s$ worth of worthless assets.

The NC condition is the bound on asset transferability mentioned above. Related to our comments on cash-in-advance and other *ad hoc* transaction patterns, it is apparent that this constraint ought to be endogenous and is not generally invariant to policy or other interventions. Moreover, it is immediate from NC that $s \to 0$ as $k \to 0$. But we can say more, by characterizing equilibrium for all $k \geq 0$. The result is algebraically tedious (see the Appendix), but most of the economics can be seen from Figure 5, which partitions the parameter space into regions in which different types of equilibria exist.

Proposition 4. *Define q_m and q_a by*

$$\frac{i}{\lambda} + 1 = \frac{u'(q_m)}{c'(q_m)} \text{ and } (1 + r)A\delta = c(q_a)\lambda\left[\frac{r}{\lambda} + 1 - \frac{u'(q_a)}{c'(q_a)}\right].$$

Depending on A, there are three types of monetary equilibrium. In each of them, $q = q_m$, $d = M$ and $\phi = [c(q_m) - (\psi + \delta)s]/M > 0$, while ψ and s are given by:

1. $A\delta < \min\left\{\frac{c(q_m)(r-i)}{1+r}, \frac{k(r-i)}{i+\lambda}\right\} \Rightarrow s = A$, $\psi = \frac{\delta(1+i)}{r-i} > \psi^*$ and NC not binding.

2. $\frac{k(r-i)}{i+\lambda} < A\delta < \min\left\{\frac{c(q_m)(r+\lambda)-(1+r)k}{1+r}, \frac{rk}{\lambda}\right\} \Rightarrow s = A, \psi = \frac{(1+r)k+(1-\lambda)\delta A}{(r+\lambda)A} > \psi^*$ *and NC*

binding.

3. $A\delta > \frac{rk}{\lambda}$ *and* $\frac{c(q_m)\lambda}{1+r} > k \Rightarrow s = \frac{rk}{\delta\lambda} < A, \psi = \psi^*$, *and NC binding.*

Also, there are four types of nonmonetary equilibrium, given by:

4. $\frac{c(q_m)(r-i)}{1+r} < A\delta < \min\left\{\frac{c(q^*)r}{1+r}, \frac{c(q_a)(r+\lambda)-(1+r)k}{1+r}\right\} \Rightarrow s = A, q = q_a, \psi = \frac{c(q_a)}{A} - \delta > \psi^*$ *and*

NC not binding.

5. $\frac{c(q_m)(r+\lambda)-(1+r)k}{1+r} < A\delta < \min\left\{\frac{rk}{\lambda}, \frac{c(q_a)(r+\lambda)-(1+r)k}{1+r}\right\} \Rightarrow s = A, \psi = \frac{(1+r)k+(1-\lambda)\delta A}{(r+\lambda)A} > \psi^*$,

$c(q) = (\psi + \delta)A$ *and NC binding.*

6. $\frac{rk}{\lambda} < A\delta$ *and* $\frac{c(q_m)\lambda}{1+r} < k < \frac{c(q^*)\lambda}{1+r} \Rightarrow s = \frac{rk}{\lambda} < A, \psi = \psi^*, c(q) = \frac{(1+r)k}{\lambda}$ *and NC binding.*

7. $A\delta > \frac{c(q^*)r}{1+r}$ *and* $k > \frac{c(q^*)\lambda}{1+r} \Rightarrow s = \frac{c(q^*)r}{(1+r)\delta} < A, \psi = \psi^*, q = q^*$ *and NC not binding.*

INSERT FIGURE 5 ABOUT HERE

In type 1, 4 and 7 equilibria, k is large, so NC is not binding and a is perfectly liquid. In type 1 equilibrium, A is small and hence m is essential. As A grows, since it is perfectly liquid and dominates m in rate of return, it eventually drives m out. However, as k decreases, NC eventually binds and a is no longer perfectly liquid. When A is small, as in type 2 and 5 equilibria, a decrease in k tightens NC, inhibiting the use of assets in transactions and decreasing ψ. In the type 2 monetary equilibrium, this decrease in the liquidity of a increases demand for m, and hence ϕ. Also, when k is small and A is big, as in type 3 and 6 equilibria, a is priced fundamentally. In equilibria where NC binds, again, only a fraction of one's assets can be traded to an uninformed seller. In general, assets become less liquid as k decreases, and when $k \to 0$ we recover the benchmark model.

6   International Economics

Consider the case of an economy with two fiat objects, M_1 and M_2, as in a Latin American country with both dollars and pesos. Let M_j grow at rate γ_j. Let ρ_1 be the probability sellers

recognize only M_1, ρ_2 the probability they only recognize M_2, and ρ_{12} the probability they recognize both. With a slight abuse of notation, let $\lambda_1 = \lambda\rho_1$, $\lambda_2 = \lambda\rho_2$, and $\lambda_{12} = \lambda\rho_{12}$ denote the probabilities that an agent is a buyer in the respective meetings. We begin with an exogenous information structure, where λ_1, λ_2 and λ_{12} are fixed. In any equilibrium in which both currencies are valued, the quantities traded in DM meetings are $q_1 = z^{-1}(\phi_1 M_1)$, $q_2 = z^{-1}(\phi_2 M_2)$ and $q_{12} = z^{-1}(\phi_1 M_1 + \phi_2 M_2)$. For each currency, $1 + \pi_j = \phi_j^-/\phi_j$ is the inflation rate, which may or may not equal $1 + \gamma_j$ in this application, as we will see. Using the Fisher Equation, as before, $1 + i_j = \phi_j^-/\phi_j\beta$ is the interest rate on an illiquid nominal bond denominated in currency j.

If both currencies are valued, the equilibrium conditions are

$$i_1 = \lambda_1 \ell(q_1) + \lambda_{12} \ell(q_{12}) \tag{21}$$

$$i_2 = \lambda_2 \ell(q_2) + \lambda_{12} \ell(q_{12}). \tag{22}$$

There are several cases of interest. The simplest is when some sellers recognize currency 1 and others recognize currency 2, but no one recognizes both: $\lambda_1 > 0$, $\lambda_2 > 0$, $\lambda_{12} = 0$. In this case we can solve independently for q_1 and q_2 from $\ell(q_j) = i_j/\lambda_j$, and here $\pi_j = \gamma_j$. Both currencies are valued, even if one is being issued at a higher rate and hence has a higher inflation and nominal interest rate, since M_1 and M_2 are each essential for some meetings. It is immediate that $\partial q_j/\partial \lambda_j > 0$ and $\partial q_j/\partial i_j < 0$.[19]

For the next case suppose $\rho_1 = 0$, $\rho_2 > 0$ and $\rho_{12} > 0$, which captures the scenario where all sellers accept pesos, and some also accept dollars. The equilibrium conditions are

$$i_1 = \lambda_{12} \ell(q_{12})$$

$$i_2 = \lambda_2 \ell(q_2) + \lambda_{12} \ell(q_{12}),$$

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[19]Notice that λ_1 and i_1 do not affect q_2, and vice versa, but this result is not especially robust. If there were an asset a_3, agents would hold all three. Then an increase in i_1 makes them want to shift out of m_1 and into a_3, driving up ϕ_3. This makes them want to shift out of a_3 and into m_2. This increased demand for m_2 affects ϕ_2 and q_2. So, even if i_1 does not affect m_2 directly, there are equilibrium effects via a_3.

where $q_2 = z^{-1}(\phi_2 M_2)$ and $q_{12} = z^{-1}(\phi_1 M_1 + \phi_2 M_2)$ and again $\pi_j = \gamma_j$. Combining these implies $i_2 - i_1 = \lambda_2 \ell(q_2)$, which implies that both currencies are valued only if $i_2 > i_1$. Since dollars are strictly dominated by pesos in terms of liquidity (i.e., anyone who accepts dollars also accepts pesos), the former needs a better return if anyone is to hold them. Among other results, Table 2 shows that increasing the rate at which we issue dollars γ_1 increases π_1 and i_1, which decreases ϕ_1, increases ϕ_2 and obviously increases the exchange rate $e = \phi_2/\phi_1$.

ζ	$\dfrac{\partial q_{12}}{\partial \zeta}$	$\dfrac{\partial q_2}{\partial \zeta}$	$\dfrac{\partial \phi_1}{\partial \zeta}$	$\dfrac{\partial \phi_2}{\partial \zeta}$	$\dfrac{\partial e}{\partial \zeta}$
i_1	$-$	$+$	$-$	$+$	$+$
i_2	0	$-$	$+$	$-$	$-$

Table 2: Effects of policy changes.

Instead of taking recognizability to be exogenous, to discuss dollarization, we now allow locals to invest in order to recognize foreign currency. For this application we change the interpretation of $\kappa(h)$ to a one-time, not a per-period, cost.[20] Now the decision to acquire the information needed to recognize US dollars is based on a comparison of $\kappa(h)$ with the expected benefit of accepting both pesos and dollars forever. This is given by

$$\frac{\Pi(\rho_{12})}{1-\beta} = \frac{\beta\lambda(1-\theta)}{1-\beta}\left[u(q_{12}) - c(q_{12}) - u(q_2) + c(q_2)\right],$$

where it is understood that q_{12} and q_2 depend on ρ_{12}. One can show that $\Pi(\rho_{12})$ is increasing as $\Pi(\rho)$ was in Section 4. Intuitively, when the measure of locals who accept dollars increases, ϕ_1 goes up and this gives other locals more incentive to learn to trade in dollars. As before, this can generate multiple equilibria, some where dollars circulate widely and are highly valued, and others where this is not the case.

Moreover, $\Pi(\rho_{12})$ shifts up when we raise γ_2. Assuming there is a unique equilibrium $\rho_{12} \in (0,1)$, this leads to an increase in i_2 and hence dollarization. This effect can be dramatic in practice. As Guidotti and Rodriguez [18] report, dollarization in Bolivia went from close to 0 in

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[20]Discussion of why it might be costly to learn to use a new medium of exchange, with a particular focus on dollarization, is contained in Uribe [49], Guidotti and Rodriguez [18], and Dornbusch et al. [9].

1985 to nearly 50% in 1987. Note the asymmetry here when $\kappa(h)$ is a one-time cost: although an increase in peso inflation leads to dollarization, a subsequent decrease does *not* reverse the process. This is because once locals learn how to trade using dollars, they do not forget when peso inflation comes down. This captures the notion of hysteresis in dollarization that has been discussed in the literature but not formalized in this way. The theory also predicts ρ_{12} increases if we lower i_1, instead of raising i_2. Thus, dollars are more likely to circulate in Latin America when either peso inflation is high or dollar inflation is low, consistent with conventional wisdom.[21]

The next case we consider is $\rho_1 = \rho_2 = 0$ and $\rho_{12} > 0$, so that the two currencies are perfect substitutes. To reduce notation, assume $\rho_{12} = 1$. Then, in any equilibrium in which both currencies are valued, we have

$$i_1 = \lambda \ell\left(q\right) \ \text{and} \ i_2 = \lambda \ell\left(q\right),$$

where $q = q_{12}$. A necessary condition for this type of equilibrium is that $i_1 = i_2$, but note that, in this case, this does *not* necessarily mean the growth rates of M_1 and M_2 have to be the same (see below). This case is studied by Kareken and Wallace [24] in an OLG model, where they prove the exchange rate is indeterminate, and argue that this is natural since $e = \phi_2/\phi_1$ is the relative price of two fiat objects. We think this is worth re-examining in our model, not just to show yet another application, but because their result seems to be under-appreciated.[22]

There are two versions of Kareken-Wallace indeterminacy: one in a stationary and the other in a non-stationary context. To begin with the stationary result, which is much easier, suppose $\gamma_1 = \gamma_2 = \gamma$, and consider an equilibrium where $i_1 = i_2 = i$ and q solves $i = \lambda \ell(q)$. This implies $z(q) = \phi_1 M_1 + \phi_2 M_2$ is constant as well. Now consider any arbitrary constant exchange rate $e > 0$, and let $M = M_1 + eM_2$ denote the aggregate supply of money measured in units of

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[21]We are obviously being a little loose in this paragraph about expectations of policy changes, which matter for one-time investment decisions. For the sake of discussion, assume that both increases and decreases in inflation come as complete surprises.

[22]Perhaps this is because, as they themselves put it, "some economists are more than a little doubtful about all OLG models." Again, this is where modern monetary theory may help.

currency 1, which is growing at rate γ. Since $\phi_1 A = z(q)$ is constant, ϕ_1 decreases at rate γ, as does ϕ_2. An equilibrium is comprised of time-invariant values for q and e, plus paths for ϕ_1 and ϕ_2, such that q satisfies $i = \lambda \ell(q)$, ϕ_1 and ϕ_2 decrease at rate γ, and $z(q) = \phi_1 [M_1 + eM_2]$. Consider this last condition at date $t = 0$. There are two free variables – the exchange rate e and the initial price ϕ_1 – that must satisfy a single equality. This makes e indeterminate.

This initially surprising result has an intuitive interpretation. Suppose there are two fiat objects, blue notes and red notes, both of which are being issued at the same rate and are equally liquid. If $e = 2$, we could call blue and red notes 5 and 10 dollar bills, say, while if $e = 4$ we could call them 5 and 20 dollar bills. Prices at time $t = 0$ will adjust, which can affect the total amount of money measured in units of blue notes, but this does not affect q. Still, due to fiscal considerations, the choice of e is not neutral. Suppose naturally that lump sum transfers of new notes go to different subsets of agents: Americans get dollars and Mexicans get pesos. For group 1, the value of the transfer in terms of CM numeraire is $T_1 = \gamma M_1 \phi_1$, while for group 2 it is $T_2 = \gamma M_2 \phi_1 e$. When we increase e, ϕ_1 and ϕ_2 both fall, while T_1 falls and T_2 rises. Since the transfers are lump sum they do not affect q, but they do affect CM hours worked and hence welfare for the two groups. Therefore e matters.

To briefly sketch the nonstationary Kareken-Wallace result, suppose the two currencies are being issued at different rates γ_1 and γ_2 (both still constant over time). Consider an arbitrary $e > 0$. Given the stocks of each currency at $t = 0$, $M_{1,0}$ and $M_{2,0}$, for all future t we have $M_{j,t} = (1 + \gamma_j)^t M_{j,0}$. Letting $M_t = M_{1,t} + eM_{2,t}$, the growth rate of this aggregate $1 + \bar{\gamma}_t = M_t/M_{t-1}$ satisfies

$$1 + \bar{\gamma}_t = (1 + \gamma_1) \left(\frac{1}{1 + \omega_t} \right) + (1 + \gamma_2) \left(\frac{1}{1 + 1/\omega_t} \right),$$

where $\omega_t = e(1 + \gamma_2)^{t-1} M_{2,0}/(1 + \gamma_1)^{t-1} M_{1,0}$, and $\bar{\gamma}_t \to \max\{1 + \gamma_1, 1 + \gamma_2\}$ as $t \to \infty$. In equilibrium with both currencies valued,

$$(1 + r)\phi_{j,t-1}/\phi_{j,t} = 1 + \lambda \ell(q_t). \tag{23}$$

Then, using $z(q_t) = \phi_{1,t} M_t$ for all t, we have

$$\frac{\phi_{j,t-1}}{\phi_{j,t}} = \bar{\gamma}_t \frac{z(q_{t-1})}{z(q_t)}. \tag{24}$$

Combining (23) and (24) yields a first order difference equation,

$$(1+r)\bar{\gamma}_t \frac{z(q_{t-1})}{z(q_t)} = 1 + \lambda \ell(q_t). \tag{25}$$

As in Kareken-Wallace, for any e there exists a unique solution to this difference equation satisfying the nonnegativity and boundedness requirements. This equilibrium has a time-varying supply of real balances, which makes it hard to characterize, but again nothing pins down e. Note that deriving this result within the context of our model potentially allows us to move beyond the original Kareken-Wallace result: while they assumed that both currencies are perfectly liquid, in our model one could allow them to have different liquidities, either exogenously or endogenously. More work on these issues may be interesting, but is beyond the scope of this paper.

7   Discussion

The basic ideas here are not new. Samuelson [48], for one, put it this way:

> It is true that in a world involving no transaction friction and no uncertainty, there would be no reason for a spread between the yield on any two assets, and hence there would be no difference in the yield on money and on securities... In fact, in such a world securities themselves would circulate as money and be acceptable in transactions... Of course, the above does not happen in real life, precisely because uncertainty, contingency needs, non-synchronization of revenues and outlay, transactions frictions, etc., etc. are all with us.

Samuelson and his and contemporaries, however, did not write down explicit models incorporating the relevant frictions.[23] On informational frictions, consider Jevons [22], who defined a quality called *cognizability* as follows:

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[23] Nor did they say much about why these frictions, if they were in a model, would be expected to generate a role for money or liquid securities – as opposed to credit or record keeping.

By this name we may denote the capability of a substance for being easily recognized and distinguished from all other substances. As a medium of exchange, money has to be continually handed about, and it will occasion great trouble if every person receiving currency has to scrutinize, weigh, and test it. If it requires any skill to discriminate good money from bad, poor ignorant people are sure to be imposed upon... Precious stones, even if in other respects good as money, could not be so used, because only a skilled lapidary can surely distinguish between true and imitation gems.

A century later, Alchian [2] argued "Any exchange proposed between two parties with two goods will be hindered... the less fully informed are the two parties about the true characteristics of the proffered goods." Like us, he assumed "interpersonal differences exist in degrees of knowledge about different goods – either by fortuitous circumstance or by deliberate development of such knowledge," and that assets or goods "differ in the costs of determining or conveying to others their true qualities and attributes." Alchian concluded, "It is not the absence of a double coincidence of wants, nor of the costs of searching out the market of potential buyers and sellers of various goods, nor of record keeping, but the costliness of information about the attributes of goods available for exchange that induces the use of money in an exchange economy." Modern theory tries to be explicit about how this works, but to do so, one has to make assumptions and abstractions. Here we discuss some issues that come up.

First, agents in our model sometimes use real assets to purchase goods directly, which, one might say, does not resemble actual markets. As noted earlier, however, an alternative interpretation that may be more realistic is that agents use assets as collateral.[24] Also, as Ravikumar and Shao [45] point out, sometimes one can write checks on mutual funds, and

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[24]As David Andofatto put it in a recent blog: "On the surface, these two methods of payment look rather different. The first entails immediate settlement, while the second entails delayed settlement. To the extent that the asset in question circulates widely as a device used for immediate settlement, it is called money (in this case, backed money). To the extent it is used in support of debt, it is called collateral. But while the monetary and credit transactions just described look different on the surface, they are equivalent in the sense that capital is used to facilitate transactions that might not otherwise have taken place."

even if not, purchasing power may be easily transferrable into bank accounts on which one can. Although we do not actually model transfers from equity to checking accounts, for many applications we are still happy with the idea that assets help consumers make payments. For investors or firms, this may be even more relevant. In OTC (over-the-counter) markets trades often boil down to an exchange of securities, such as swaps between fixed- and variable-rate assets. One might also object to our dividends generating utility, although this has been standard since Lucas (1978), and seems reasonable for assets that pay dividends or interest. In fact, if one prefers, in our model dividends can be paid in the CM in units of numeraire, or in cash.

One might also question our DM specification more broadly, featuring as it does search and bargaining. In OTC markets, however, these features are quite realistic, as sophisticated products like derivatives or complex securitized loans are traded by dealers who must search for counterparties and bargain over the terms of trade.[25] We do not claim that our DM constitutes the definitive model of such activity; only that search and bargaining are not unnatural in finance generally. Random matching is also a convenient way to model agents meeting and trading with each other, rather than against their budget equations. By explicitly modeling trade in this way, one can start to ask whether agents use barter, credit or other means, and if they use media of exchange to ask which assets play that role. But one does not have to take it literally, and we could replace random with directed search, or replace search entirely with preference and technology shocks (again we refer to the surveys mentioned above).

Many results hinge on whether assets are scarce or plentiful, i.e., $A < A^*$ or $A > A^*$. In the former but not the latter case we get a liquidity premium, the Fisher Equation fails, easy money can reduce some real returns, etc. Which is the empirically relevant case? Caballero [8] advocates the position that there is indeed a dearth of financial assets in the world. The

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[25] As Duffie et al. [10] put it, "Many assets, such as mortgage-backed securities, corporate bonds, government bonds, US federal funds, emerging-market debt, bank loans, swaps and many other derivatives, private equity, and real estate, are traded in [OTC] markets. Traders in these markets search for counterparties, incurring opportunity or other costs. When counterparties meet, their bilateral relationship is strategic; prices are set through a bargaining process that reflects each investor's alternatives to immediate trade."

idea is that the demand for such assets is huge. Geanakoplos and Zame [13] document that "the total [value] of collateralized lending is enormous: the value of U.S. residential mortgages alone exceeds $9.7 trillion (only slightly less than the $10.15 trillion total capitalization of S&P 500 firms)." To understand why there is so much demand for liquidity relative to GDP, note that many final or deferred-but-collateralized payments must be made on transactions that do not add to net output. A prime example is clearing and settlement among banks and related institutions. The average daily values of transactions on the two big settlement systems in the US, Fedwire and CHIPS, are currently around $2.3 trillion and $1.4 trillion – meaning that the equivalent of annual output flows throughout the system used every 4 days.[26]

Although we use clearing and settlement to help motivate the demand for liquid assets, in the model taken literally, assets are used only to finance consumption. Models in this general class have been used to study payments across financial institutions (Koeppl, Monnet and Temzilides [28]; Lester [35]), but there is much more to be done. Also, mitigating this demand for liquidity is the fact that one can sometimes use the same assets as collateral many times, as well as the fact that intermediaries with illiquid assets issue liquid liabilities. But it is important to understand recognizability when thinking about the quantity of available liquidity. While the total value of assets may be large, especially if one includes various types of intangible or human capital, to the extent that these are difficult to use in transactions or as collateral, liquidity may still be scarce. The issue is not whether there is a shortage of wealth; it is whether it is easy to pull together enough assets with desirable properties – recognizability, transferability, etc. – to accommodate the need for liquidity.

8   Conclusion

There is a long tradition arguing that informational frictions are central for understanding money. We are convinced that they are central for understanding liquidity generally. In our

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[26]For more details, see www.frbservices.org/operations/fedwire/fedwire_funds_services_statistics.html and www.chips.org/docs/000652.pdf.

framework, real and fiat assets can differ in terms of recognizability, and this can give rise to liquidity premia. We analyzed implications for monetary policy, showing how the Fisher Equation holds for some but not other assets, and how inflation affects even those who never use cash. We endogenized recognizability and liquidity by letting agents invest in information, proved existence, and discussed the possibility of multiplicity. One lesson is that small changes in fundamentals may have big effects. Another is that models with exogenous restrictions, like cash in advance, or cashless models, or those with constraints on asset transferability, are problematic because generally transaction patterns are neither uniquely determined nor invariant to interventions. Relatedly, we advocate models where the role of assets in exchange is modeled explicitly, as opposed to sticking them directly into utility function, because it imposes more discipline and structure.

Several assumptions kept the analysis tractable. One is that bad assets can be produced at cost $k = 0$, which implies agents who cannot verify an asset's authenticity never accept it. This allows us to use recognizability as a determinant of liquidity while avoiding technical problems assoociated with bargaining under private information. However, we also characterized equilibrium for $k > 0$, and showed that as $k \to 0$ the outcomes converge to the those in the benchmark. Another reason for tractability is that we use proportional bargaining, but the theory is robust in that similar results hold with alternative solution concepts, including generalized Nash and Walrasian pricing. Still, more could be done in terms of studying different mechanisms. Other outstanding issues include considering versions where firms or investors, not only consumers, are subject to liquidity concerns, and bringing intermediation into the picture. It would be interesting to add a secondary market – convening, say, between the CM and DM – where agents swap assets before trading for goods. One could also pursue empirical implications, as discussed in the context of our interrelated asset demand system. We leave all this to future work.

Appendix

Results in Table 1: Let Δ denote the determinant of the following matrix:

$$\begin{bmatrix} \lambda_1\ell'(q_1) & \lambda_2\ell'(q_2) \\ [\lambda_2\ell(q_2) - r]\,z'(q_1) & [r - \lambda_2\ell(q_2)]z'(q_2) - [z(q_2) - z(q_1)]\lambda_2\ell'(q_2) \end{bmatrix}$$

Since $q_2 \geq q_1$ implies $r \geq \lambda_2\ell(q_2)0$, we have $\Delta < 0$. Then it is easy to compute:

$$\frac{\partial q_1}{\partial i} = \frac{[r - \rho\lambda\ell(q_2)]z'(q_2) - [z(q_2) - z(q_1)]\rho\lambda\ell'(q_2)}{\Delta} < 0$$

$$\frac{\partial q_2}{\partial i} = \frac{[r - \rho\lambda\ell(q_2)]z'(q_1)}{\Delta} < 0$$

$$\frac{\partial q_1}{\partial \rho} = \frac{\lambda[\ell(q_1) - \ell(q_2)][r - \rho\lambda\ell(q_2)]z'(q_2) - [z(q_2) - z(q_1)]\rho\lambda\ell'(q_2)\lambda\ell(q_1)}{\Delta} < 0$$

$$\frac{\partial q_2}{\partial \rho} = \frac{\lambda[\ell(q_1) - \ell(q_2)][r - \rho\lambda\ell(q_2)]z'(q_1) + [z(q_2) - z(q_1)](1 - \rho)\lambda\ell'(q_1)\lambda\ell(q_2)}{\Delta}.$$

Given $z(q_1) = \phi M$ and $z(q_2) - z(q_1) = (\psi + \delta)A$, we have

$$\frac{\partial \psi}{\partial i} = \frac{[z(q_2) - z(q_1)]\rho\lambda\ell'(q_2)z'(q_1)}{A\Delta} > 0$$

$$\frac{\partial \phi}{\partial i} = \frac{z'(q_1)}{M}\frac{\partial q_1}{\partial i} < 0$$

$$\frac{\partial \psi}{\partial \rho} = \frac{[z(q_2) - z(q_1)][(1 - \rho)\lambda\ell'(q_1)\lambda\ell(q_2)z'(q_2) + \rho\lambda\ell'(q_2)\lambda\ell(q_1)z'(q_1)]}{\Delta} > 0$$

$$\frac{\partial \phi}{\partial \rho} = \frac{z'(q_1)}{M}\frac{\partial q_1}{\partial \rho} < 0.$$

To see that $\lim_{\rho \to 0} \partial\psi/\partial i = 0$, note that $q_1 \to \hat{q} = \ell^{-1}(i/\lambda) \in (0, q^*)$ as $\rho \to 0$, while $q_2 \to \min\{q^*, \bar{q}\}$, where $\bar{q}$ satisfies $(1 + r)\delta A = r\left[z(\bar{q}) - z(\hat{q})\right]$. Hence $\Delta \to \lambda\ell'(q_1)rz'(q_2)$, which is strictly negative and bounded, so that $\partial\psi/\partial i \to 0$ as $\rho \to 0$. ∎

Proof of Proposition 2: Suppose $A < A^*$. We claim there exists a unique (q_1, q_2) with $q_1 > 0$ and $q_2 < q^*$ that satisfies the equilibrium conditions (17)-(18). Let μ and α denote the implicit functions (17)-(18), mapping q_1 into q_2. We have

$$\mu'(q_1) = -\frac{\lambda_1\ell'(q_1)}{\lambda_2\ell'(q_2)} < 0$$

$$\alpha'(q_1) = \frac{z'(q_1)\{1 - \beta[\lambda_2\ell(q_2) + 1]\}}{z'(q_2)\{1 - \beta[\lambda_2\ell(q_2) + 1]\} - [z(q_2) - z(q_1)]\beta\lambda_2\ell'(q_2)} > 0.$$

Let $\breve{q}$ satisfy $\ell(\breve{q}) = (\gamma - \beta)/\beta\lambda_1 + \lambda_2/\lambda_1$, with $0 < \breve{q} < \tilde{q} \leq q^*$. Since $\ell'(q) < 0$ and $\lim_{q \to \infty} \ell(q) = -1$, it is easy to see that $\lim_{q_1 \to \breve{q}^+} \mu(q_1) = \infty$. Moreover, we claim $\lim_{q_1 \to \breve{q}^+} \alpha(q_1) < \infty$. Suppose not, so that $\lim_{q_1 \to \breve{q}^+} \alpha(q_1) = \infty$. Then

$$A\delta = \lim_{q_1 \to \breve{q}^+}[z(\alpha(q_1)) - z(q_1)][1 - \beta + \beta\lambda_2].$$

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This implies $A\delta \geq [z(q^*) - z(\breve{q})][1 - \beta + \beta\lambda_2]$, which implies $A > A^*$, a contradiction. Therefore $\lim_{q_1 \to \breve{q}^+} \mu(q_1) > \lim_{q_1 \to \breve{q}^+} \alpha(q_1)$. Now consider the function μ at $q_1 = q^*$, so that $\gamma/\beta = \lambda_2 \ell(q_2) + 1$. This implies $\ell(q_2) > 0$, so that $\mu(q^*) < q^*$. Now consider α at $q_2 = q^*$, which implies $A\delta = [z(q^*) - z(q_1)](1 - \beta)$. Since $\frac{A\delta}{1-\beta} > 0$, $\alpha^{-1}(q^*) < q^*$. Since $\alpha' > 0$, $\alpha(q^*) > q^* \geq \mu(q^*)$. Since μ and α are continuous, $\mu' < 0$ and $\alpha' > 0$, $\mu(q') > \alpha(q')$ for some $q' < q^*$, and $\alpha(q^*) \geq \mu(q^*)$, we conclude that there exists a unique pair (q_1, q_2) with $q_1 > 0$ and $q_2 \leq q^*$ that satisfies (17)-(18).

Now suppose $A > A^*$. We claim that there is no pair (q_1, q_2) with $q_2 < q^*$ satisfying (17)-(18). To see this, let $\hat{q}$ be the value of q_1 such that $\alpha(q_1) = q^*$. It is easy to show that $A > A^* \Rightarrow \hat{q} < \tilde{q}$, so $\mu(\hat{q}) > q^*$, so there does not exist a $q_1 < \hat{q}$ satisfying $\mu(q_1) = \alpha(q_1)$. Therefore, $q_2 = q^*$. Then $q_1 = \tilde{q}$, and the rest follows immediately. ∎

Proof of Lemma 2: Substituting $z(q)$ into $\Pi(\rho) = \beta\lambda [\Sigma_2(\rho) - \Sigma_1(\rho)]$, we have

$$\Pi(\rho) = \beta\lambda(1 - \theta)[(u_2 - c_2) - (u_1 - c_1)],$$

where $u_1 = u[q_1(\rho)]$ and so on. Therefore,

$$\Pi' \approx (u_2' - c_2')\partial q_2/\partial \rho - (u_1' - c_1')\partial q_1/\partial \rho,$$

where $\approx$ means the two expressions have the same sign. Inserting $\partial q_j/\partial \rho$, we get

$$
\begin{aligned}
\Pi' &\approx (u_2' - c_2')\left[-\lambda(1 - \rho)\ell_1'\ell_2(z_2 - z_1) + (\ell_2 - \ell_1)(r - \lambda\rho\ell_2) z_1'\right] \\
&\quad -(u_1' - c_1')\left[(\ell_2 - \ell_1)(r - \lambda\rho\ell_2) z_2' + \lambda\rho\ell_2'\ell_1(z_2 - z_1)\right] \\
&= (\ell_2 - \ell_1)(r - \lambda\rho\ell_2)\left[z_1'(u_2' - c_2') - z_2'(u_1' - c_1')\right] \\
&\quad -\lambda(z_2 - z_1)\left[\rho\ell_1\ell_2'(u_1' - c_1') + (1 - \rho)\ell_1'\ell_2(u_2' - c_2')\right].
\end{aligned}
$$

Since $z_2 > z_1$ and $u_i' \geq u_i'$ for all $q_j \leq q^*$, the second term is positive. Since $\ell_2 < \ell_1$ and $r > \lambda\rho\ell_2$, a sufficient condition for $\Pi' > 0$ is therefore

$$
\begin{aligned}
0 &\geq z_1'(u_2' - c_2') - z_2'(u_1' - c_1') \\
&= [\theta c_1' + (1 - \theta)u_1'](u_2' - c_2') - [\theta c_2' + (1 - \theta)u_2'](u_1' - c_1') \\
&= c_1' u_2' - u_1' c_2'.
\end{aligned}
$$

Since c is convex and u concave, the proof is complete. ∎

Proof of Proposition 4: For each case we find necessary and sufficient conditions for equilibrium. First, for monetary equilibria, we know from standard results (Lagos and Wright 2005)

that $q = q_m$ and $d = M$. Also, let ω and η be the multipliers on the IC and NC constraints, respectively. Then we have the following:

Case 1 ($s = A$, NC not binding): Given NC does not bind, FOC reduce to

$$q \quad : \quad \lambda u'(q) - \omega c'(q) = 0 \tag{26}$$

$$d \quad : \quad \phi(-i - \lambda + \omega) = 0 \tag{27}$$

$$s \quad : \quad -r(\psi - \psi^*) + (\omega - \lambda)(\psi + \delta) = 0. \tag{28}$$

Using $q = q_m$ in (28) yields $\psi = \delta(1+i)/(r-i)$. It remains to confirm $\phi > 0$ and $k > \beta\lambda(\psi+\delta)s$. Given $s = A$,

$$\phi M = c(q) - (\psi + \delta)A = c(q_m) - \frac{\delta(1 + r)A}{r - i},$$

so $\phi > 0$ iff $A\delta < c(q_m)(r - i)/(1 + r)$. Moreover, NC is not binding iff

$$k > A\left[\psi - \frac{1 - \lambda}{1 + r}(\psi + \delta)\right] = A\delta\left(\frac{i + \lambda}{r - i}\right),$$

or $A\delta < k(r - i)/(i + \lambda)$.

Case 2 ($\phi > 0$, $s = A$, NC binding): When NC binds, the FOC with respect to s is

$$-r(\psi - \psi^*) + (\omega - \lambda)(\psi + \delta) - \eta\left[\psi - \frac{1 - \lambda}{1 + r}(\psi + \delta)\right] = 0. \tag{29}$$

NC binding implies $\psi = [(1 + r)k + (1 - \lambda)\delta A] / (r + \lambda) A$. We require $q = q_m$ and $\phi > 0$, which holds iff $c(q_m) > (\psi + \delta)A$, or $A\delta(1+r) < c(q_m)(r + \lambda) - (1+r)k$. We also have to check that $\eta \geq 0$. Solving (29), $\eta \approx \psi(i - r) + \delta(1 + i) \geq 0$ iff $A\delta > k(r - i)/(i + \lambda)$. Finally, $\psi \geq \psi^*$ iff $A\delta \leq rk/\lambda$.

Case 3 ($\phi > 0$, $s < A$, NC binding): If $s < A$ and NC binding, $\psi = \psi^*$. Then NC binding implies $s = rk/\delta\lambda$. Therefore, $s < A$ iff $A\delta > rk/\lambda$, $\phi > 0$ iff $k < c(q_m)\lambda/(1 + r)$ and $\eta \geq 0$ for all $i > 0$.

Case 4 ($\phi = 0$, $s = A$, NC not binding): Now q and ψ satisfy

$$c(q) = (\psi + \delta)A$$

$$r(\psi - \psi^*) = \lambda(\psi + \delta)\left[\frac{u'(q)}{c'(q)} - 1\right].$$

Combining these, we get

$$A\delta = \frac{c(q)\lambda}{1 + r}\left[\frac{r}{\lambda} + 1 - \frac{u'(q)}{c'(q)}\right].$$

There is at most one solution to this; call it q_a. For this equilibrium: (i) $s = A$ requires $q_a \leq q^*$; (ii) $\phi = 0$ requires $q_a > q_m$; and (iii) NC not binding requires $k > A\left[\psi - \frac{1-\lambda}{1+r}(\psi + \delta)\right]$.

The condition $q_a \leq q^*$ can be expressed as

$$A\delta \leq \frac{c(q^*)\lambda}{1+r}\left[\frac{r}{\lambda} + 1 - \frac{u'(q)}{c'(q)}\right].$$

Since $u'(q^*) = c'(q^*)$, this reduces to $A\delta \leq c(q^*)r/(1+r)$. Similarly, to write $q_m < q_a$ as a condition on $A\delta$ and k, first write

$$A\delta > \frac{c(q_m)}{1+r}\left[r + \lambda - \lambda\frac{u'(q_m)}{c'(q_m)}\right].$$

Since $i/\lambda + 1 = u'(q_m)/c'(q_m)$, by construction, this reduces to $A\delta > c(q_m)(r-i)/(1+r)$. Also, NC is not binding when $A\delta(1 + r) < c(q_a)(r + \lambda) - (1+r)k$. Notice that

$$\psi + \delta = \frac{c(q_a)}{A} = \frac{(1+r)\delta}{r - \lambda\left[\frac{u'(q)}{c'(q)} - 1\right]} \geq \frac{(1+r)\delta}{r} = \psi^* + \delta.$$

Case 5 ($\phi = 0$, $s = A$, NC binding): Equilibrium is now characterized by

$$(\psi + \delta)A - c(q) = 0 \tag{30}$$

$$-r(\psi - \psi^*) + \lambda\left[\frac{u'(q)}{c'(q)} - 1\right](\psi + \delta) - \eta\left[\psi - \frac{1-\lambda}{1+r}(\psi + \delta)\right] = 0 \tag{31}$$

$$k - A\left[\psi - \frac{1-\lambda}{1+r}(\psi + \delta)\right] = 0. \tag{32}$$

For this equilibrium we require: (i) $q > q_m$; (ii) $\psi \geq \psi^*$; and (iii) $\eta \geq 0$. Using (30) and (32), $q > q_m$ is equivalent to $A\delta(1 + r) > c(q_m)(r + \lambda) - (1 + r)k$. Then $\psi \geq \psi^*$ iff $A\delta \leq rk/\lambda$. Finally, $\eta \geq 0$ requires $q \leq q_a$, or $A\delta(1 + r) \leq c(q_a)(r + \lambda) - (1 + r)k$, and $q \leq q^*$ is guaranteed by $q_a \leq q^*$.

Case 6 ($\phi = 0$, $s < A$, NC binding): Now $\psi = \psi^*$, and NC binding implies $s = rk/\lambda\delta$. Then $s < A$ reduces to $A\delta > rk/\lambda$. We have $c(q) = (\psi + \delta)s = (1 + r)k/\lambda$, so $k = c(q)\lambda/(1 + r)$. To ensure $\phi = 0$, we need $q > q_m$, or $k > c(q_m)\lambda/(1 + r)$. Likewise, NC binding implies $q \leq q^*$, so that $k \leq c(q^*)\lambda/(1 + r)$.

Case 7 ($\phi = 0$, $s < A$, NC not binding): We need $\psi = \psi^*$ and $q = q^*$. Thus, $s = c(q^*)r/(1+r)\delta$, so that $s < A$ iff $A\delta > c(q^*)r/(1 + r)$. Moreover, NC not binding requires $k > c(q^*)\lambda/(1 + r)$.

This exhausts the possibilities and completes the proof. ∎

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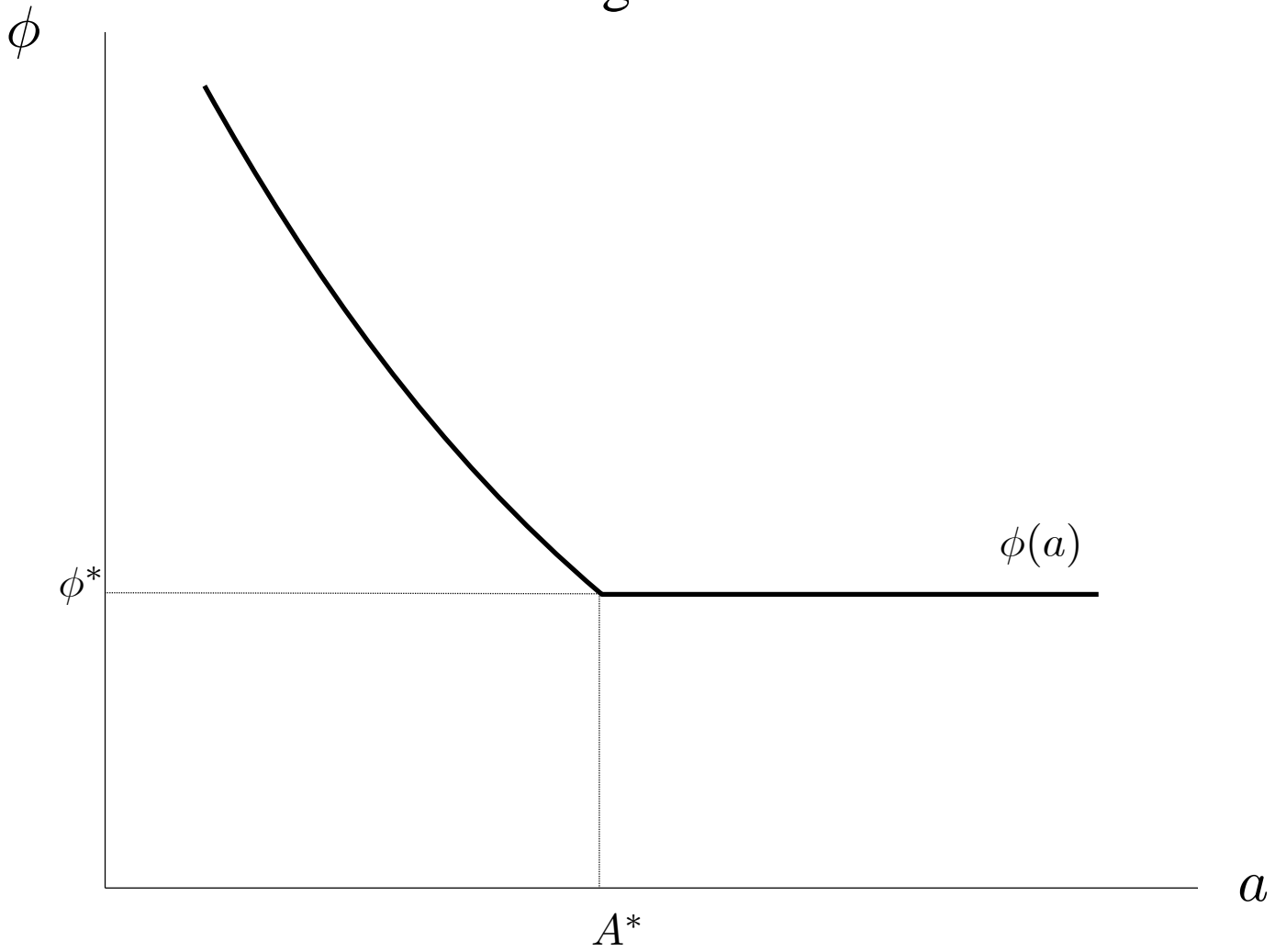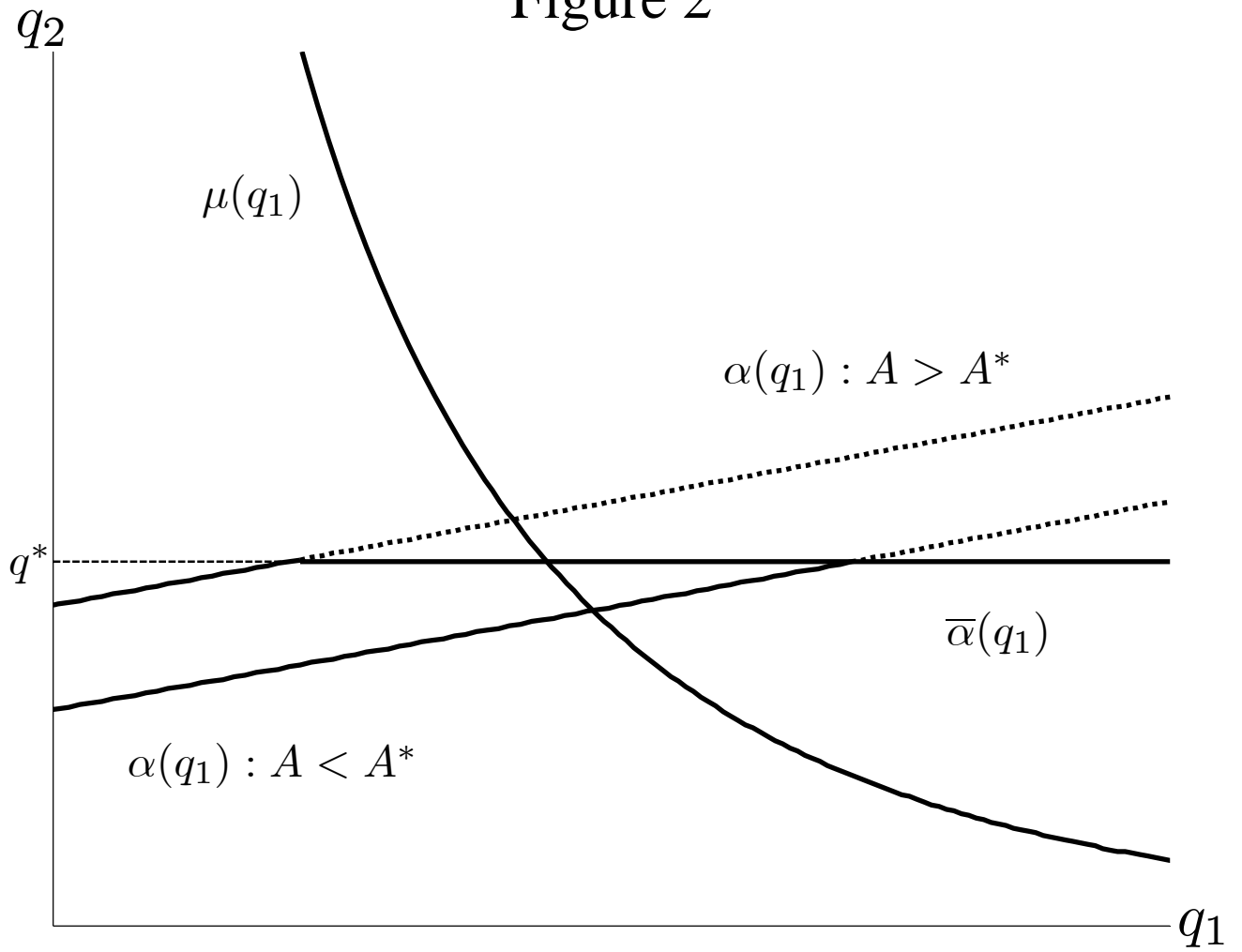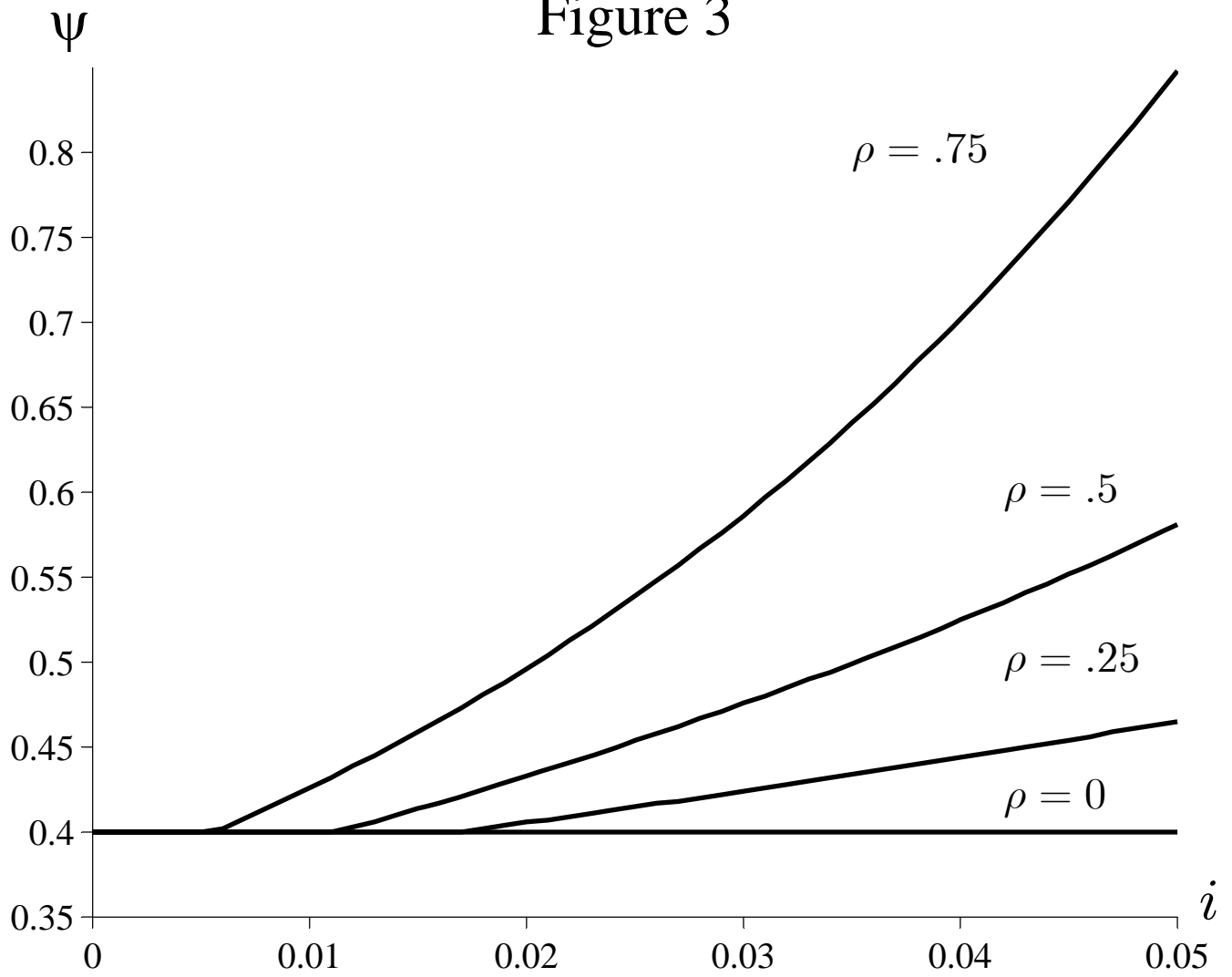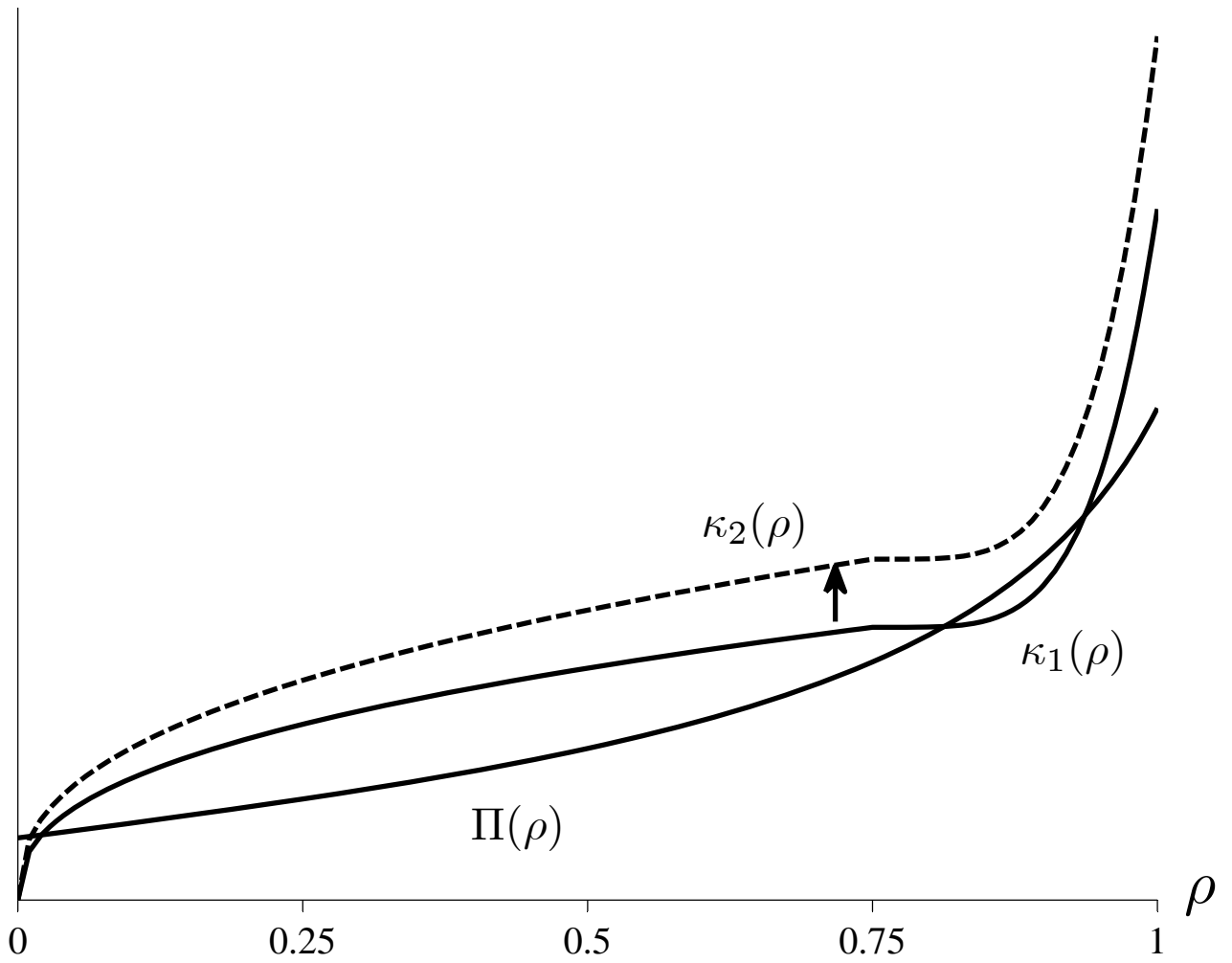
Figure 1

Figure 2

Figure 3

Figure 4

Figure 5