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"Information, Liquidity and Asset Prices"

by

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Information, Liquidity and Asset Prices*

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Abstract

We study economies with multiple assets that are valued both for their return and liquidity. Exchange occurs in decentralized markets with frictions making a medium of exchange essential. Some assets are better suited for this role because they are more liquid – more likely to be accepted in trade – even if they have a lower return. The reason assets are more or less likely to be accepted is modeled using informational frictions, or recognizability. While everyone understands e.g. what currency is and what it is worth, some might be less sure about other claims. In our model, agents who do not recognize assets do not accept them in trade. Recognizability is endogenized by letting agents invest in information, potentially generating multiple equilibria with different liquidity. We discuss implications for asset pricing and for monetary policy. In particular, we show explicitly that what may look like a cash-in-advance constraint is not invariant to policy interventions or other changes in the economic environment.

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Aringosa walked back to his black briefcase, opened it, and removed one of the bearer bonds. He handed it to the pilot. "What's this?" the pilot demanded. "A ten-thousand-euro bearer bond drawn on the Vatican Bank." The pilot looked dubious. "It's the same as cash." "Only cash is cash," the pilot said, handing the bond back. Dan Brown, *The Da Vinci Code*.

1 Introduction

It is by now well understood that in the absence of a double coincidence of wants it is natural and efficient for some object to emerge as a medium of exchange.[1] A key property of any object serving as a medium of exchange is that it is accepted by a large number of agents, who rationally expect that others will accept it in the future. Many assets could potentially serve this role, in principle, and agents may face a tradeoff between an asset's fundamental value (its rate of return or promise to deliver future dividends) and its liquidity (acceptability in exchange). In this paper, we set out a model in which this tradeoff is modeled explicitly, and use it to address a set of questions, including the following. Why do people transact with some assets, such as money, when there appear to be alternatives with higher returns? If it is due to liquidity considerations, what makes money more liquid than other assets? And, given the answers to those questions, how does monetary policy affect asset prices, liquidity, and subsequent allocations?

Although our framework can be used to analyze any vector of assets, including stocks, bonds, local or foreign currency, and mortgage-backed securities, consider as a leading example the case where there are two: money, and real claims like the claims to "trees" bearing "fruit" as dividends in standard Lucas [40] asset-pricing theory. To model acceptability we invoke the idea of recognizability via informational frictions: while everyone understands e.g. what currency is and what it is worth (at least if we abstract from inflation uncertainty), some people might

[1] To be accurate, it is also well known that the double coincidence problem has to be combined with two additional frictions, limited commitment and incomplete record keeping (or imperfect memory) to rule out credit-like arrangements that could render a tangible medium of exchange inessential. See Kocherlakota [32] for a formalization of this idea. See also Wallace [53], Corbae et al. [12], Araujo [4], and Aliprantis et al. [2], [3] for extended discussions.

not be so sure about other claims.[2] In our model, a seller who does not recognize an asset will not accept it – he won't give a buyer anything for it. One interpretation is that the seller is concerned a claim may be counterfeit, or a worthless lemon (a "lemon tree" as it were). More generally, a seller may worry the value of a claim is random, even if it is known to the buyer; the possibility that it may be worthless is simply a special case where the value may be 0.[3]

As a preliminary step, suppose we take liquidity differentials as given. That is, some exogenous fraction of agents recognize assets, and therefore accept them in trade, while others do not, so currency is essential for at least some transactions. We endogenize this below, but for now, we can ask what monetary policy does taking this scenario as given. The direct effect of monetary policy is to affect the return on cash. When this falls agents try to economize on its use – they substitute out of money into alternative assets, raising the prices and lowering the returns on these assets. Thus policy affects all asset returns and equilibrium allocations generally. As a special case, we show how inflation affects even individuals or markets that never use cash. This indicates monetary policy may continue to be relevant even if transactions are increasingly taking place without the use of currency, due to the development of alternatives like credit and debit cards, privately issued means of payment, including e-cash, and so on.

[2] Although the framework can be used to study any vector of assets, not only money and real claims, the idea in monetary economics goes back at least to Menger [41] that recognizability is a key property media of exchange might have. It is also consistent with Hicks' [26] "suggestion" for monetary economists to "look frictions in the face" to explain liquidity. Alchian [1], Brunner and Meltzer [8], Freeman [20], and Banerjee and Maskin [6] discuss the connection between money and information using varying degrees of formal modeling. Our approach is closer to search-based monetary theory, where informational frictions have been incorporated by Williamson and Wright [54], Trejos [48], [49], Li [39], Cuadras-Moreto [13], Kim [46], Velde et al. [51], and Berentsen and Rocheteau [7]. See also Ennis [17], Faig and Jerez [18], Nosal and Wallace [42], Cavalcanti and Nosal [9], Hu [27], Rocheteau [44], and Kim and Lee [30]. An alternative approach to modelling liquidity by e.g. Glosten and Milgrom [23] and Kyle [33] also considers bilateral transactions between asymmetrically informed agents; while similar in spirit, the models are different, and those papers have little to say about many of the substantive issues addressed here, including the role of money and monetary policy.

[3] We are aware that an agent might accept an asset in exchange even if he does not recognize it, as illustrated e.g. in Williamson and Wright [54]. As discussed below, we have to specify the details of the model somewhat carefully to be sure sellers simply refuse to take things they do not recognize. By way of preview, our story is that buyers can always come up with a worthless asset at 0 cost; hence sellers who cannot verify an asset's quality know they will get something worthless if they agree to accept it. Note carefully that worthless claims cannot be valued in equilibrium, even though fiat currency can, because by assumption agents can always produce their own worthless claims, but cannnot produce fiat money.

To go more deeply into the issues, we must endogenize the set of agents who recognize the alternative asset and hence accept it in transactions. To this end we postulate that anyone can, at a cost that potentially differs across agents, acquire the knowledge (or perhaps the technology) to identify this asset – that is, to distinguish a genuine real claim from a counterfeit or worthless lemon.[4] In equilibrium an endogenous fraction is willing to make this investment. There is an important complementarity at work here: if more agents become informed about the alternative asset, it will be more liquid, and hence the asset will be more valuable, increasing agents' incentive to invest in information. This can generate multiple equilibria with different liquidity properties, with interesting consequences. For example, when policy makes holding cash relatively undesirable through inflation, agents have greater incentive to transact using alternative assets, and more people may well acquire the relevant information. Consequently, monetary policy affects the liquidity of assets generally.

An implication of these results that is consistent with much experience comes from interpreting the easily recognizable asset as a local currency, such as the peso in Latin American, and the alternative asset as the US dollar, which does not literally pay real dividends, of course, but traditionally constitutes a better store of value due to lower inflation rates. When peso inflation is not too high, locals are relatively happy using the peso as a means of payment, so dollars do not circulate widely and hence may not be universally recognized. If the peso inflation rate increases, however, transacting in local currency becomes more costly, and at some point the economy dollarizes. Notice, however, that if peso inflation subsequently subsides, the dollar does fall into disuse as a medium of exchange; once the locals have learned to recognize and it and use it for transactions, they do not quickly forget. This imparts a natural hysteresis effect into dollarization, as has often been discussed in the literature, but has not been formalized in

[4]One may want to distinguish between counterfeit and worthless assets – i.e. "bad claims to good trees" and "good claims to bad trees" – for some purposes; here we remain agnostic, since it makes little difference for the present discussion.

this way.[5]

Some aspects of these policy implications can be traced back to earlier thought, including the portfolio theory of the demand for money by Tobin [47]. Also, in the more rigorous framework of overlapping generations models, Wallace [52] speaks directly to the issues:

> Of course, in general, fiat money issue is not a tax on all saving. It is a tax on saving in the form of money. But it is important to emphasize that the equilibrium rate-of-return distribution on the equilibrium portfolio does depend on the magnitude of the fiat money-financed deficit ... the real rate-of-return distribution faced by individuals in equilibrium is less favorable the greater the fiat money-financed deficit. Many economists seem to ignore this aspect of inflation because of their unfounded attachment to Irving Fisher's theory of nominal interest rates. (According to this theory, (most?) real rates of return do not depend on the magnitude of anticipated inflation.) The attachment to Fisher's theory of nominal interest rates accounts for why economists seem to have a hard time describing the distortions created by anticipated inflation. The models under consideration here imply that the higher the fiat money-financed deficit, the less favorable the terms of trade – in general, a distribution – at which present income can be converted into future income. This seems to be what most citizens perceive to be the cost of anticipated inflation.

These words ring true, but they are not easily formalized, and many questions arise. How can the Fisher equation not hold? Why do different assets bear different returns in the first place? In the models Wallace mentions, it is *not* differences in liquidity – notice he talks about "saving" and defines returns in terms of the rate "at which present income can be converted into future income" but there is no mention of a transactions or medium of exchange role. This

[5] See Uribe [50] for a discussion of the issues and literature. Uribe also provides a model of the phenomenon in question, but simply assumes exogenously that the cost of accepting foreign currency is decreasing in the fraction of other agents that accept it.

is where modern monetary theory comes in, with explicit descriptions of trading processes and liquidity. Early search-based models such as Kiyotaki and Wright [31] determine endogenously the acceptability of different objects in exchange, but are too crude to address the issues studied in this paper. Hence we use a multiple-asset version of the more recent model in Lagos and Wright [37]. While others have considered multiple assets in this environment, our focus is on differential liquidity and how this can be determined endogenously using informational frictions.[6]

The rest of the paper is organized as follows. In Section 2 we lay out a simple version of our model in which it is taken as given that money is essential for some trades because some agents do not recognize, and hence do not accept, alternative assets. In Section 3 agents choose whether or not to acquire the ability to recognize the alternative asset, thus endogenizing recognizability and liquidity. We conclude in Section 4. Before proceeding, we emphasize that many models simply assume differential liquidity between money and alternative assets, including the entire cash-in-advance literature. In those models, the set of goods (or trades) for which agents need money is exogenous; here it is endogenous and will respond e.g. to changes in inflation. More generally, we show explicitly that what may look like a cash-in-advance constraint is not invariant to changes in policy or other aspects of the environment.[7]

2 The Basic Model

Time is discrete and continues forever. There is a continuum of infinitely-lived agents. As in Lagos and Wright [37], hereafter LW, in each period agents participate in two distinct markets: a frictionless centralized market CM, and a decentralized market DM where agents meet bilaterally

[6] Lagos and Rocheteau [36] have two assets, money and capital, but they are equally liquid and hence bear the same return. Geromichalos, Licari and Suarez-Lledo [22] have money and claims to Lucas "trees" like we do, but again they are equally liquid and bear the same return. Differential liquidity was considered by Lagos [35] with two assets meant to resemble stocks and bonds, but this differential is exogenous, and he does not discuss money or monetary policy. To reiterate our goal, we want to make liquidity endogenous by taking recognizability seriously.

[7] Although our approach is novel, people have previously considered making cash-in-advance constraints endogenous or responsive to policy; see e.g. Shreft and Lacker [34] or Ireland [29]. A recent paper that does something similar, but more in the search tradition, is Dong [14], who also gives other references.

and anonymously. These meetings in the DM are characterized by a standard double coincidence problem, detailed below, which rules out direct barter. Since anonymity rules out credit in the DM, some tangible medium of exchange is essential for trade (see the references in footnote 1 for formal demonstrations). At each date in the CM there is a consumption good x that agents can produce using labor h according to $x = h$, and utility is $U(x) - h$. In the DM there is another good q that all agents value according to $u(q)$ and can be produced at disutility cost $c(q)$. Define x^* and q^* by $U'(x^*) = 1$ and $u'(q^*) = c'(q^*)$. Assume $u' > 0$, $u'' < 0$, $c' > 0$, $c'' > 0$, $u(0) = c(0) = c'(0) = 0$, and $U'(0) = u'(0) = \infty$.

We assume that there are two assets, although this can easily be generalized. The first is for now interpreted as fiat money; the second is a real asset like the claims to "trees" in the standard Lucas [40] asset-pricing model, yielding a dividend δ in terms of "fruit" in the next CM. We introduce informational frictions as follows. Generally, a buyer might have either a high- or low-quality asset; we focus on the limiting case in which the latter is worthless – say a pure lemon or counterfeit.[8] Genuine real claims can be recognized, or distinguished from worthless claims, by some but not all sellers; for ease of exposition we assume fiat money is universally recognized. One could easily include a potential recognizability problem with currency as well without changing the qualitative nature of our results as long as currency was recognized by a larger fraction of sellers than the second asset.[9]

Sellers who do not recognize the asset refuse to accept it. As this is a central feature of our model, it is worth discussing. The set up is based on the idea that agents can produce at 0 cost

[8] These are different interpretations of a worthless asset. Say e.g. a real asset is a stock certificate in the profitable company IBM. A counterfeit might be a photocopy of a certificate, which does not actually entitle the holder to anything. Alternatively, a worthless asset might be a stock certificate in the LPW company, which has zero profits. These two cases represent the "bad claims to good trees" and "good claims to bad trees" mentioned above. They are equivalent in our model, although perhaps not in every model (e.g. one with fraud laws).

[9] We could also make money harder to recognize, although this is probably not the most relevant case. Currency and coins have always been designed with recognizability in mind when they were stamped with the likeness of the monarch, governments have always made big efforts to reduce counterfeiting, and so on. Also, as Nobu Kiyotaki pointed out, in the olden days most people could recognize coins, although could not even read, let alone understand a piece of paper claiming some payoff in a future contingency. Literacy may have improved since then, but people still have trouble evaluating the worth of complicated financial instruments, including the recently relevant example of mortgage-backed securities.

worthless claims whenever they want. A seller who cannot verify whether a claim is genuine will therefore never accept it, since he knows the buyer can always slip him a worthless claim at the last minute. A seller without a technology for verifying a check, for instance, will not accept one if he knew a buyer could always dupe him with a fake (e.g. sign someone else's name). This scenario seems plausible, but we are aware that it is not the only way of modeling this kind of informational friction, and there are examples in the literature where agents accept assets with positive probability even when they cannot recognize them, including Williamson and Wright [54] and the related papers mentioned in footnote 2.

In those papers, agents make an ex ante choice to bring either good or bad assets to the market. As in our model, asset quality is recognized in a subset of bilateral meetings, and no one knowingly accepts a bad asset. Suppose there is an equilibrium where agents do not accept things they do not recognize. Then agents with bad assets cannot trade, so no one brings bad assets to the market. Hence it is not a best reply to reject assets you cannot recognize. In Lester et al. [38], we analyze in detail a class of related games, and show the crucial ingredient that assures sellers simply reject assets they do not recognize is the assumption made here that agents can produce worthless claims at any time, as opposed to committing before they enter the market. We also emphasize that worthless claims cannot be valued in equilibrium even though fiat currency, which is also an intrinsically worthless asset, in the sense that it is not a claim to any real dividend, can be valued. As discussed in detail by Cavalcanti and Wallace [11] and [10], as long as individuals can produce their own intrinsically worthless claims at 0 cost, there is no equilibrium where they are valued.[10]

In addition to being plausible, it simplifies the analysis considerably to have sellers who do not recognize assets refuse to accept them. We assume bilateral trade in the DM, as this is a standard (if not the only) way to generate a double coincidence problem, and the terms of trade

[10] In case it is not completely obvious, you would never give something up to acquire an object if you can produce something equivalent for free (e.g. if you can always issue stock on a new worthless company). A critical property of fiat currency of course is that you cannot produce it yourself.

are determined through bargaining. As we discuss below, there are alternatives, but bargaining is natural and standard in models with bilateral meetings. It is also simple, the way we do it, even though bargaining with asymmetric information is often intractable. In our set up, where a seller never accepts something he does not recognize, a buyer can only pay with assets the seller does recognize. Consequently, all bargaining occurs under full information. In this way informational frictions may be critical for determining acceptability and liquidity, but we avoid the usual problems with bargaining under asymmetric information.

Because we want to focus on steady state equilibria here, assume there is a fixed supply of "trees" denoted A, while the supply of money M grows according to $\hat{M} = \gamma M$ (for any variable z, $\hat{z}$ denotes its value next period). Changes in the money stock, $\hat{M} - M = (\gamma - 1)M$, are accomplished using lump sum transfers, or taxes if $\gamma < 1$, although it is equivalent for most purposes to assume the government uses new money to buy x in the CM, since utility is quasi-linear. In what follows, we assume $\gamma > \beta$ where β is the discount factor; we do consider the limit as $\gamma \to \beta$, which is the Friedman rule. Let ϕ be the CM price of money and ψ the CM price of the real asset, both in terms of x. In the CM all prices are taken parametrically, while as mentioned above, in the DM agents bargain over the terms of trade.

It is well known (see Lagos and Rocheteau [36] or Geromichalos et al. [22]) that, when the DM terms of trade are determined by bargaining, an agent who acquires a units of a real asset in the CM may not want to bring it all to the DM, since the terms of trade in the bargaining problem generally may depend on his asset holdings. Thus, an agent may acquire a large amount of a because it is a good store of value, given its return, but not bring it all to the DM. There is no similar effect on m since it has no dividend; the only use for m is a medium of exchange. Thus, agents in the CM choose a portfolio comprised of m units of money taken to the DM, a_1 units of the asset not taken to the DM, and a_2 units taken to the DM.

Let $V(m, a_1, a_2)$ be the value function of an agent in the DM with the portfolio (m, a_1, a_2), and $W(y)$ the value function in the CM with $y = \phi m + (\delta + \psi)(a_1 + a_2)$ (obviously in the frictionless CM only the value of the portfolio matters). The CM problem is

$$\begin{aligned} W(y) &= \max_{x,h,\hat{m},\hat{a}_1,\hat{a}_2} \{U(x) - h + \beta V(\hat{m}, \hat{a}_1, \hat{a}_2)\} \\ \text{s.t. } x &= h + y - \phi\hat{m} - \psi(\hat{a}_1 + \hat{a}_2) + T, \end{aligned}$$

where $T = (\gamma - 1)M$ is the transfer. Substituting for h, first order conditions are:

$$x \;:\; U'(x) = 1 \tag{1}$$

$$\hat{m} \;:\; \phi \geq \beta V_1(\hat{m}, \hat{a}_1, \hat{a}_2), \;=\; \text{if } \hat{m} > 0 \tag{2}$$

$$\hat{a}_1 \;:\; \psi \geq \beta V_2(\hat{m}, \hat{a}_1, \hat{a}_2), \;=\; \text{if } \hat{a_1} > 0 \tag{3}$$

$$\hat{a}_2 \;:\; \psi \geq \beta V_3(\hat{m}, \hat{a}_1, \hat{a}_2), \;=\; \text{if } \hat{a_2} > 0 \tag{4}$$

Notice that x and $(\hat{m}, \hat{a}_1, \hat{a}_2)$ do not depend on y, and the value function is linear $W'(y) = 1$; these results follow from quasi-linear utility, as in the basic LW model.[11]

In the DM, there is a probability λ of a meeting in which you are a buyer and an equal probability of a meeting in which you are a producer, or seller. We distinguish two types of DM meetings: with probability ρ it is a *type 2 meeting*, in which sellers accept either m or a_2; with probability $1 - \rho$ it is a *type 1 meeting*, in which sellers accept only m. We assume $0 < \rho < 1$ for now. Because this is based on the idea of recognizability, in a type 1 (type 2) meeting we call the seller uninformed (informed). For now, since recognizability is taken as given, one can think of the model as a random matching version of an otherwise standard cash-in-advance model, with some cash and some credit goods, or perhaps some cash and some credit meetings. However, in the next section we endogenize the set of goods or meetings that require cash by having sellers invest ex ante in information, which is not part of standard cash-in-advance theory.

[11] We assume a unique solution at least for $\hat{m}$ and $\hat{a}_2$ to the CM problem, as is necessarily true under assumptions discussed in footnote 14 below, although agents may be indifferent about how much a_1 to hold in some equilibria. We also assume an interior solution for h; see LW for conditions to guarantee that this is the case.

Next, we determine the terms of trade in the DM using the generalized Nash bargaining solution.[12] Note that in either type of meeting, a seller cares only about the total value of the assets that he receives. Now consider a type j meeting between a buyer with (m, a_1, a_2) and a seller with $(\tilde{m}, \tilde{a}_1, \tilde{a}_2)$. The former pays p_j to the latter for q_j units of the good, determined by

$$\max \ \left[u(q_j) + W(y - p_j) - W(y)\right]^{\theta} \left[-c(q_j) + W(\tilde{y} + p_j) - W(\tilde{y})\right]^{1-\theta} \tag{5}$$

subject to the constraint $p_j \leq y_j$, where y and $\tilde{y}$ are total wealth of the buyer and seller, and y_j describes the wealth the buyer can use in that meeting: $y_1 = \phi m$ and $y_2 = \phi m + (\psi + \delta)a_2$. This problem is essentially the same as that in LW, so we can use their solution:

Lemma 1. *The solution to (5) is*

$$q_j = \min\left\{z^{-1}(y_j), q^*\right\} \ \text{ and } p_j = \min\left\{y_j, y^*\right\},$$

where the function z is defined by

$$z(q) \equiv \frac{\theta u'(q)c(q) + (1-\theta)u(q)c'(q)}{\theta u'(q) + (1-\theta)c'(q)},$$

while q^* given by $u'(q^*) = c'(q^*)$ and $y^* = z(q^*)$.

The DM value function satisfies

$$V(m, a_1, a_2) = \lambda_1\left[u(q_1) + W(y - p_1)\right] + \lambda_2\left[u(q_2) + W(y - p_2)\right] + (1-\lambda)W(y) + k, \tag{6}$$

where $\lambda_1 = \lambda(1-\rho)$, $\lambda_2 = \lambda\rho$, and k is a constant unimportant for what follows.[13] Differentiating

[12] There are alternative approaches for determining the terms of trade in the DM. With symmetric information, the orginal LW model uses generalized Nash bargaining; Aruoba, Rocheteau and Waller [5] consider different bargaining solutions; Rocheteau and Wright [45] analyze price taking and price posting; Galenianos and Kircher [21] and Dutu et al. [16] use auctions in versions with some multilateral meetings. In a version with private information, Ennis [17] uses price or contract posting; see also Faig and Jerez [18]. In a comment on our paper, with private information, Guerrieri [24] uses competitive price taking, and derives some similar results at least for the case where information is exogenous. We like bargaining because it is simple, and seems natural, especially in the extended framework where information acquisition is endogenous.

[13] There are three relevant events described in (6): you are a buyer in a type 1 meeting; you are a buyer in a type 2 meeting; and you are not a buyer. In the third case, you may be a seller or you may not trade at all, and this effects your continuation value, but since Lemma 1 implies the terms of trade do not depend on the seller's state and $W(y)$ is linear, we can represent this as $W(y)$ plus a constant. We need not know what happens when you are a seller in order to determine your portfolio demand.

V and substituting the derivatives of q_j wrt (m, a_1, a_2),

$$V_1(m, a_1, a_2) = \phi\left[\lambda_1 \ell(q_1)\mathbf{1}\{y_1 < y^*\} + \lambda_2 \ell(q_2)\mathbf{1}\{y_2 < y^*\} + 1\right] \quad (7)$$

$$V_2(m, a_1, a_2) = \psi + \delta \quad (8)$$

$$V_3(m, a_1, a_2) = (\psi + \delta)\left[\lambda_2 \ell(q_2)\mathbf{1}\{y_2 < y^*\} + 1\right] \quad (9)$$

where $\mathbf{1}\{e\}$ is an indicator function equaling 1 iff e is true and $\ell(q) \equiv \frac{u'(q)}{z'(q)} - 1$. Note that $\ell(q)$ is a *liquidity premium* – the value of an additional unit of wealth available in a type j meeting, over and above its return if it were simply carried to the next CM. We assume $\ell'(q) < 0$, which is true under known conditions.[14] Combining (7)-(9) and (2)-(4), we arrive at the conditions determining portfolio demand:

$$m \;:\; \phi \geq \beta\hat{\phi}\left[\lambda_1 \ell(\hat{q}_1)\mathbf{1}\{\hat{y}_1 < y^*\} + \lambda_2 \ell(\hat{q}_2)\mathbf{1}\{\hat{y}_2 < y^*\} + 1\right], \; = \text{ if } \hat{m} > 0 \quad (10)$$

$$a_1 \;:\; \psi \geq \beta(\hat{\psi} + \delta), \; = \text{ if } \hat{a}_1 > 0 \quad (11)$$

$$a_2 \;:\; \psi \geq \beta(\hat{\psi} + \delta)\left[\lambda_2 \ell(\hat{q}_2)\mathbf{1}\{\hat{y}_2 < y^*\} + 1\right], \; = \text{ if } \hat{a}_2 > 0 \quad (12)$$

An equilibrium can now be defined in terms of time paths for asset holdings (m, a_1, a_2), asset prices (ϕ, ψ), the DM terms of trade (p_j, q_j), $j = 1, 2$, and the CM allocation (x, h), for every agent, satisfying the utility maximization conditions derived above, the bargaining solution, and market clearing. Given the other variables, we know that $x = x^*$ from (1) and can determine h from the budget equation, hence the CM allocation will be ignored in what follows. We focus on steady states. A steady state is an equilibrium in which the real variables (q_1, q_2) are constant over time, which implies ϕm and ψa_2 are constant, and hence, $\phi/\hat{\phi} = \hat{M}/M = \gamma$. We also focus on monetary steady states, where $\phi > 0$, $\hat{m} > 0$, $q_1 > 0$ and (10) holds with equality.

We now characterize steady state monetary equilibrium. To begin, notice from Lemma 1 that q_j is an increasing function of y_j and that $q_1 \leq q_2 \leq q^*$. It is easy to show $q_j \leq \bar{q}$, where $\bar{q}$

[14] As in LW, $\ell' < 0$ if θ is close to 1, or if c is linear and u displays decreasing absolute risk aversion. These conditions also guarantee a unique solution to the CM problem. The method in Wright [55] can be used to dispense with these side conditions and establish generic uniqueness of the CM solution even if ℓ is nonmonotone; rather than going through the details, here, to ease the presentation, we impose $\ell' < 0$.

maximizes the buyer's surplus $u(q) - p = u(q) - z(q)$, and $\bar{q} \leq q^*$, with strict inequality unless $\theta = 1$.[15] Also, notice that $\ell(\bar{q}) = 0$. We next claim that $a_2 > 0$ (assuming, as we do, $\lambda_2 > 0$). The proof is in Appendix A; intuitively, because it is costly to carry cash, agents do not bring enough to buy $\bar{q}$, and so want to bring at least some a_2 to the DM.

Lemma 2. *$a_2 > 0$.*

Thus, $a_2 > 0$, and $m > 0$ by definition in any monetary equilibrium. It remains to determine whether $a_1 = 0$ or $a_1 > 0$. To answer this, let $\tilde{q} < \bar{q}$ be defined by $\ell(\tilde{q}) = (\gamma - \beta) / \beta\lambda_1$, and let

$$\bar{A} = [z(\bar{q}) - z(\tilde{q})] (1 - \beta) / \delta > 0.$$

The next result, the proof of which is in Appendix B, demonstrates that $A \leq \bar{A}$ (the real asset is relatively scarce) implies $a_1 = 0$, and $A > \bar{A}$ (the real asset is plentiful) implies $a_1 > 0$. Notice $A > \bar{A}$ is more likely when γ or ρ is small and β, δ or λ is large.

Proposition 1. *(i) If $A \leq \bar{A}$ there exists a unique steady state monetary equilibrium, and in this equilibrium, (q_1, q_2) solves*

$$A\delta = [z(q_2) - z(q_1)] \{1 - \beta[\lambda_2\ell(q_2) + 1]\} \quad (13)$$

$$\gamma = \beta [\lambda_1\ell(q_1) + \lambda_2\ell(q_2) + 1], \quad (14)$$

prices are $\phi = z(q_1)/M$ and $\psi = [z(q_2) - z(q_1)] /A - \delta$, and the portfolio is $(m, a_1, a_2) = (M, 0, A)$.

(ii) If $A > \bar{A}$ there exists a unique steady state equilibrium, and in this equilibrium, $(q_1, q_2) = (\tilde{q}, \bar{q})$, prices are $\phi = z(\tilde{q})/M$ and $\psi = \beta\delta/ (1 - \beta)$, and $(m, a_1, a_2) = (M, A - \bar{A}, \bar{A})$.

To facilitate the economic discussion, imagine a hypothetical asset that costs 1 unit of x in the current CM and pays $1 + r$ units in the next CM, but cannot be traded in the DM (for

[15] See Geromichalos et al. [22] for the routine argument. Intuitively, if $\bar{y} = z(\bar{q}) \leq y^*$, the buyer's surplus is decreasing in y for $y > \bar{y}$ because the value of what he pays increases by more than what he gets. Furthermore, $\bar{y} < y^*$ unless $\theta = 1$ because, unless the buyer has all the bargaining power, the q that maximizes his surplus is not the q^* that maximizes total surplus.

example, it is merely a book entry, not a tangible asset). In equilibrium its real return satisfies $1+r=1/\beta$. Now imagine an asset that costs 1 dollar in the current CM and pays $1+i$ dollars in the next CM, and similarly cannot be traded in the DM. Its return, the nominal rate, satisfies $1+i=\phi/\hat{\phi}\beta$. Hence, $1+i=(1+r)\phi/\hat{\phi}$, which is a version of the Fisher equation that must always hold (it is a no-arbitrage condition). Given this, we can equivalently discuss monetary policy in terms of either the nominal interest rate i or the inflation rate $\phi/\hat{\phi}$ or the money growth rate γ. It is convenient to use i, and rewrite (13)-(14), the equilibrium conditions for the case $A<\bar{A}$, as

$$(1+r)A\delta = [z(q_2)-z(q_1)][r-\lambda_2\ell(q_2)] \tag{15}$$

$$i = \lambda_1\ell(q_1)+\lambda_2\ell(q_2). \tag{16}$$

Let $q_1=\mu(q_2)$ and $q_2=\alpha(q_1)$ denote the implicit functions characterized by (16) and (15). It is routine to demonstrate that $\mu(\cdot)$ is decreasing while $\alpha(\cdot)$ is increasing, and that they intersect for some $q_1\in[0,\bar{q}]$, as seen in the figures below. For $A\leq\bar{A}$, the intersection of α and μ determines the equilibrium $(q_1,q_2)\in[0,\bar{q}]^2$, from which we can determine the other endogenous variables using Proposition 1. For $A>\bar{A}$, the intersection of α and μ occurs at $q_2>\bar{q}$, and the equilibrium is $(q_1,q_2)=(\tilde{q},\bar{q})$. The unique steady state monetary equilibrium is therefore conveniently characterized by the intersection of $q_1=\mu(q_2)$ and $q_2=\bar{\alpha}(q_1)=\min\{\alpha(q_1),\bar{q}\}$. When $A\leq\bar{A}$, we have $q_2<\bar{q}$, and a bears a liquidity premium $\ell(q_2)>0$. In this case, (12) implies $\psi>\beta\delta/(1-\beta)=\delta/r$ and the price of the real asset a exceeds the present value of its dividend stream, because this price reflects not only fundamentals but, in addition, the value of the asset as a medium of exchange.

INSERT FIGURES 1 AND 2 ABOUT HERE

To be precise, in steady state (12) at equality yields

$$\psi=\frac{\beta\delta\left[1+\lambda_2\ell(q_2)\right]}{1-\beta\left[1+\lambda_2\ell(q_2)\right]}=\frac{\delta}{r}\left[1+\frac{(1+r)\lambda_2\ell(q_2)}{r-\lambda_2\ell(q_2)}\right],$$

which exceeds the fundamental price when $\lambda_2 > 0$ and $q_2 < \bar{q}$.[16] This is depicted in Figure 1. On the other hand, when $A > \bar{A}$, we have $q_2 = \bar{q}$, so that the real asset bears no liquidity premium and its price equals the fundamental value, $\psi = \delta/r$. When $A > \bar{A}$, we have $a_1 > 0$, and agents at the margin are indifferent between holding a for its dividend stream alone or using as a medium of exchange. It is also true that different agents may choose different a_1 when $A > \bar{A}$, since they are indifferent about how much to hold as a store of value when the real asset is priced fundamentally, even if equilibrium pins down all the other variables. This is depicted in Figure 2.

$x =$	i	A	δ	λ	ρ
$\frac{\partial q_1}{\partial x}$	$-$	$-$	$-$	$+$	$-$
$\frac{\partial q_2}{\partial x}$	$-$	$+$	$+$	$+$	?
$\frac{\partial \phi}{\partial x}$	$-$	$-$	$-$	$+$	$-$
$\frac{\partial \psi}{\partial x}$	$+$	$+$	$+$	$+$	$+$

Table 1: Effects of parameters when $A < \bar{A}$

Table 1 shows the effects (see Appendix D) of parameter changes when $A < \bar{A}$. These results show up graphically as shifts in the α and μ curves. Thus, an increase in i shifts the μ curve southwest and leaves α unchanged, reducing q_1 and q_2. Intuitively, as i increases agents try to economize on m, reducing its CM price ϕ and DM value $q_1 = z^{-1}(\phi M)$. Given this, agents want to hold more a, raising its CM price ψ but on net lowering $q_2 = z^{-1}(\phi M + \psi A)$. Notice that the observed return on a between meetings of the CM, $1 + \delta/\psi$, decreases with i. As emphasized in Geromichalos et al. [22], inflation reduces stock market returns. Another implication is that the Fisher equation *apparently* does not hold for a, since the observed return on a is not independent of nominal interest or inflation rates. This is because i affects the demand for m, which affects demand for a and hence its price and return. This would not happen if a were never traded in

[16] An alternative but equivalent way to price a in equilibrium with $A < \bar{A}$ comes from the bargaining solution, which says $z(q_1) = M\phi$ and $z(q_2) = A(\psi + \delta) + M\phi$, and hence implies $A\psi = z(q_2) - z(q_1) - \delta A$.

the DM; only when a bears a liquidity premium does its observed return depend on i.[17]

In terms of other parameters changes, an increase in δ or A shifts the μ curve northwest but leaves μ unchanged, leading to a fall in q_1 and rise in q_2, with resulting changes in ϕ and ψ. Intuitively, as dividends increase agents substitute into a out of m, which affects both their DM and CM values. Increasing λ shifts μ right and α left, but one can show the net effects on q_1 and q_2 are positive. Increasing ρ decreases q_1 but the effect on q_2 is ambiguous. One can show $\partial\phi/\partial\rho < 0$ and $\partial\psi/\partial\rho > 0$. All these results are for $A < \bar{A}$. When $A > \bar{A}$, we have $q_2 = \bar{q}$ and $q_1 = \tilde{q}$ where $\tilde{q}$ solves $\ell(\tilde{q}) = i/\lambda(1-\rho)$. In this case, $\partial\tilde{q}/\partial i < 0$, $\partial\tilde{q}/\partial\lambda > 0$, and $\partial\tilde{q}/\partial\rho < 0$, while neither A nor δ affect q_1, and none of these variables affects q_2. Also, when $A > \bar{A}$, ϕ is decreasing in i and ρ and increasing in λ, while as we already have remarked $\psi = \beta\delta/(1-\beta)$ is pinned down by fundamentals.

We summarize a few of the key results on asset pricing for the more interesting case $A < \bar{A}$ as follows:[18]

Proposition 2. *Assume* $0 < \rho < 1$. *When* $A < \bar{A}$, *a bears a liquidity premium* $\ell(q_2) > 0$ *and is priced above its fundamental value,* $\psi > \delta/r$. *In this case, an increase in i reduces the demand for m and hence* ϕ, *increasing the demand for a and hence* ψ, *and decreasing the observed return* $1 + \delta/\psi$.

To close this section, we sketch an extension to illustrate that a change in i can affect not only those agents or markets that use cash directly, but others as well, including agents, markets and goods which never involve money. To this end, suppose there are two distinct decentralized markets, call them markets B and C, where a fraction b and $1-b$ of the agents go between

[17] Thus, ψ and the return on a are independent of i when $A > \bar{A}$, and the real return on an asset that cannot be traded in the DM (like the hypothetical book entry mentioned above) is pinned down by $1+r = 1/\beta$ independent of i. So the Fisher equation holds in some circumstances, or for some assets, and not others

[18] Also, notice that $\tilde{q} < \bar{q}$ if $i > 0$, but $\tilde{q} \to \bar{q}$ as $i \to 0$. In fact, as $i \to 0$, $\bar{A} \to 0$, which means equilibrium entails $a_1 > 0$. This says that at the Friedman rule $i = 0$ we have $q_1 = q_2 = \bar{q}$ and all assets bear the same return $1 + r = 1/\beta$. Although the focus here is not on welfare, for completeness we mention that $i = 0$ is the optimal policy, but it does not give the first best $q = q^*$ unless $\theta = 1$, as is typical in related models.

meetings of the CM. Assume they are permanently assigned to one of the markets, although things are the same if agents are randomly assigned each period, as long as they know where they are going prior to choosing portfolios. In market B, sellers always accept *both* a and m, while in market C a fraction ρ of transactions require *cash* as in the baseline model. Since market C is is identical to the DM in that model, the first order conditions are (10)-(12).

To derive the first order conditions for agents going to the other market, for any variable z associated with market C write the analog associated with market B as z^B. Then

$$V^B(m^B, a_1^B, a_2^B) = (1-\lambda^B)W^B(y^B) + \lambda^B\left[u(q_2^B) + W^B\left(y^B - p_2^B\right)\right],$$

since all market B meetings are type 2 meetings. Differentiation yields the analogs of (10)-(12). Let us focus on the case where q_2, $q_2^B < \bar{q}$. One can show a_2, $a_2^B > 0$, and $m^B = 0$ (no one takes cash to market B). Also, the bargaining solutions are $z(q_1) = \phi m$, $z(q_2) = \phi m + (\psi + \delta)a_2$, and $z(q_2^B) = (\psi + \delta)a_2^B$, and market clearing implies $A = (1-b)a_2 + ba_2^B$. Given all this, routine manipulation allows us to describe (q_1, q_2, q_2^B) by

$$\begin{aligned}
i &= (1-\rho)\lambda\ell(q_1) + \rho\lambda\ell(q_2) \\
(1+r)A\delta &= \Big\{(1-b)[z(q_2) - z(q_1)] + bz(q_2^B)\Big\}[r - \rho\lambda\ell(q_2)] \\
(1+r)A\delta &= \Big\{(1-b)[z(q_2) - z(q_1)] + bz(q_2^B)\Big\}[r - \lambda^B\ell(q_2^B)].
\end{aligned}$$

In fact, since $q_2 = h(q_2^B) \equiv \ell^{-1}\left[\frac{\lambda^B}{\rho\lambda}\ell(q_2^B)\right]$, these reduce to two equations in (q_1, q_2^B):

$$\begin{aligned}
i &= (1-\rho)\lambda\ell(q_1) + \lambda^B\ell(q_2^B) \\
(1+r)A\delta &= \left\{(1-b)[z\left[h(q_2^B)\right] - z(q_1)] + bz(q_2^B)\right\}[r - \lambda^B\ell(q_2^B)].
\end{aligned}$$

Appendix C shows $\partial q_2^B/\partial i < 0$, as inflation causes agents in market C to shift out of m and into a, driving up ψ. Hence, agents in market B enjoy lower utility when i increases, even though they themselves *never use cash.* One implication is that *even if* all US dollars were abroad and all domestic payments were made in real assets, inflation can still affect equilibrium asset prices, consumption, and welfare at home, as long as it leads whoever it is holding the dollars to adjust their portfolios toward real assets. The general message is that monetary policy may continue to be relevant despite more and more transactions taking place without the use of currency.

3 Endogenous Liquidity

The probability that an agent is able to differentiate high- versus low-quality assets, and hence the probability he accepts assets in DM transaction, was an exogenous parameter ρ in the analysis above. We now endogenize this probability. Suppose that agent $i \in [0, 1]$ has the ex ante choice whether or not to acquire at cost $\kappa(i)$ the information or technology that allows him to recognize assets. We arrange agents so that $\kappa'(i) \geq 0$, assuming for simplicity that $\kappa(i)$ is differentiable. If he pays this cost, i can trade in assets, since he can distinguish genuine from worthless claims. The fraction of agents that incur the cost determines the fraction that is informed and, therefore, the fraction that accept assets in equilibrium.

One can imagine several interpretations of $\kappa(i)$. It is typically thought to be costly to learn how to use a new medium of exchange, for many reasons, as has been documented in episodes of dollarization by Uribe [50], Guidotti and Rodriguez [25], and Dornbusch et al. [15]. Stepping outside the formal model, a financial institution that wants to accept a pool of asset-backed securities in payment must hire a team of analysts to ascertain their value. Other costs may be technological, as in the case of debit or credit cards, for example, where sellers must buy a machine to verify buyers' credit, or to transfer funds from one financial institution to another. Agents as buyers choose to carry cash or alternatives that may yield a higher return but might not be accepted in all transactions, while agents as sellers choose whether to make an investment allowing them to accept these assets. Naturally, coordination will be central in determining equilibrium – a common theme in the literature on payment networks, even though the models are quite different (see Hunt [28] and Rochet and Tirole [43] for surveys).

Agents who choose to become informed can accept assets, in addition to cash, in the DM. Conditional on a fraction $\rho \in [0, 1]$ of other agents becoming informed, their benefit is

$$\Pi(\rho) \equiv \beta\lambda \{z[q_2(\rho)] - c[q_2(\rho)]\} - \beta\lambda \{z[q_1(\rho)] - c[q_1(\rho)]\} . \quad (17)$$

This benefit is the expected discounted surplus a seller gets in a type 2 meeting, over and above what he gets in a type 1 meeting. Note that $q_1(\rho)$ and $q_2(\rho)$ are well-defined objects in (17), since Proposition 1 fully characterizes equilibrium for any given ρ. The best response condition is obviously to acquire the relevant information if $\Pi(\rho) \geq \kappa$. An equilibrium is a fixed point ρ^* of this best response condition. Existence of ρ^* follows immediately from the standard fixed-point theorems. Given ρ^* all of the other endogenous variables follow, as in the previous section. Hence, it is possible to determine endogenously which objects are accepted in which trades, as opposed to simply assuming the outcome.

In terms of the kind of equilibria that may exist, there is always a trivial equilibrium with $\rho = 0$, in which no one invests and no one brings a to the DM, although sometimes this can be ruled out by assuming that some agents are exogenously informed. Of course, if the buyer has all the bargaining power, $\theta = 1$, then $\rho = 0$ is the only equilibrium, since there is no way a seller will invest when he gets none of the gains from trade. Similarly, if the costs were prohibitive – say, $\kappa(i) > u(q^*) - c(q^*)$ for all i – then $\rho = 0$ is obviously again the unique equilibrium. Whenever $\rho = 0$ the outcome in some sense looks like a cash-in-advance specification. On the other hand, if $\kappa(i)$ is low enough for all i, the natural outcome is that everyone invests, so $\rho = 1$, and there is no role for money as long as A is sufficiently big.[19] We are interested also in interior equilibria, $\rho \in (0, 1)$, where both m and a are used in some transactions. Obviously, such an equilibrium exists when some agents have sufficiently low and others sufficiently high costs of information acquisition.

[19] The case $\rho = 1$ is the one analyzed in Lagos and Rocheteau [36] and Geromichalos et al. [22]. Exactly as in those models, there is no essential role for money when $\rho = 1$ and A is large – but this does not conflict with our earlier results on monetary equilibrium since they depend on $\rho < 1$.

To make this more precise, consider the situation where all agents carry $m = 0$ and just enough a to purchase $\bar{q}$. Clearly $\Pi(1) \leq \bar{\Pi} = \beta\lambda[z(\bar{q}) - c(\bar{q})]$. Also, consider

$$\begin{aligned}
\lim_{\rho\to 0} q_1(\rho) &\equiv \hat{q}_1 = \ell^{-1}(i/\lambda) \\
\lim_{\rho\to 0} q_2(\rho) &\equiv \hat{q}_2 = z^{-1}\left[(1+r)A\delta/r + z(\hat{q}_1)\right] > \hat{q}_1,
\end{aligned}$$

and let

$$\underline{\Pi} = \lim_{\rho\to 0} \Pi_1(\rho) = \beta\lambda\left[z(\hat{q}_2) - c(\hat{q}_2)\right] - \beta\lambda\left[z(\hat{q}_1) - c(\hat{q}_1)\right] > 0.$$

Proposition 3. *If $\kappa(0) < \underline{\Pi}$ and $\kappa(1) > \bar{\Pi}$ then there exists an equilibrium $\rho^* \in (0,1)$ with $\kappa(\rho^*) = \Pi(\rho^*)$.*

Note that there may easily exist multiple interior equilibria, or there may coexist interior and other equilibria, as one might expect given the network nature of the game. The various cases are illustrated in Figures 3-6. The economics is straightforward, although perhaps somewhat more interesting than what one sees in simple network or coordination games meant to illustrate similar points, because here the result works through a general equilibrium asset market effect. Thus, when ρ is bigger it is easier to spend a in the DM, leading to an increase in demand for this asset. This bids up the CM price ψ, which makes agents more willing to pay the cost of information allowing them to trade a in the DM.

INSERT FIGURES 3-6 ABOUT HERE

Obviously policy can have a large impact here, since i affects the value of real assets and hence information acquisition and liquidity. The implications of Table 1 and the surrounding analysis permit us to determine how $\Pi(\rho)$ and thus the (set of) equilibrium values for ρ^* vary with parameters. Again, the meetings that require money are determined endogenously, and are certainly not invariant to changes in monetary policy. Consider for the sake of illustration an example with $u(q) = \sqrt{q}$, $c(q) = q$, and $\kappa(i) = ki$.[20] In Figure 7, we graph $\Pi(\cdot)$ when $i_1 = 0.01$

[20]For this example we use $\theta = 0.5$, $r = 0.01$, $A = 0.05$, $\delta = 0.01$, $\lambda = 0.2$, and $k = 0.025$.

and $i_2 = 0.06$. In this example there is a unique equilibrium ρ^* in either case, although as illustrated in Figure 8 monetary policy also could cause a shift to a region of parameter space with multiple equilibria. Higher inflation causes the price of a to increase and m to decrease, shifting Π up. Thus, inflation increases ρ^* and the acceptability of assets in this case, although, as always, when there is multiplicity the effects go in opposite directions in alternate equilibria.

INSERT FIGURES 7 AND 8 ABOUT HERE

This kind of prediction is consistent with experience in a variety of episodes. In many Latin American countries e.g. inflation has at times induced the adoption of an alternative medium of exchange with a better rate of return – namely, the US dollar (see Guidotti and Rodriguez [25] for a discussion of this in countries such as Bolivia, Mexico, Peru, and Uruguay). Similar episodes of currency substitution have been observed in Eastern Europe and the Middle East (Feige [19]). We emphasize that our model also generates the phenomenon of hysteresis in dollarization: when inflation goes up agents make an investment in information that entails increase in its use for payments; when inflation goes back down they can continue to use dollars as a medium of exchange, because they have already paid the fixed cost and do not forget the information right away. Although this phenomenon has been discussed at length, we think our way of modeling dollarization and hysteresis in this way will be useful for discussing the issues in greater depth in future work.

Finally, although as we said above the primary focus here is not on welfare, we want to mention one feature of the model. There is a *double holdup problem* at work here, as in many other bargaining models with ex ante investments. Thus, the buyer must get the entire surplus in DM trade to encourage him to make an efficient investment by bringing the right amount of money, while the seller must get the entire surplus to encourage him to make an efficient investment in information. In particular, if buyers make take it or leave it offers, $\theta = 1$, then sellers get no surplus and have no incentive to invest in information. But at the Friedman rule they do not need to make any investment in information, since money is a perfect means of payment when $i = 0$. Thus, $i = 0$ combined with $\theta = 1$ actually gives the first best outcome: $q = q^*$, and no resources are wasted learning to recognize other assets. If for reasons not specified here, however, policy sets $i > 0$, then it would be socially valuable to have agents invest in information to facilitate exchange, but equilibrium will typically be inefficient.

4 Conclusion

We developed a tractable framework to study asset pricing and exchange, where assets potentially differ in terms of their liquidity, based on informational frictions. Given the information structure, the model generates a unique steady state equilibrium in which there may or may not be a liquidity premium, depending in natural ways on parameters. Although the theory applies to any combination of assets, because recognizability has long been thought to be highly relevant for monetary economics, we discussed in some detail monetary economies and monetary policy. We showed how monetary policy affects asset prices and equilibrium allocations, generally, and in particular affects even individuals or markets that never use money. A difference between our model and many others is that we endogenize which assets are accepted in which transactions by allowing agents to invest in information. This highlights a natural complementarity: if more agents become informed, assets are more liquid and hence more valuable, which increases the incentive to invest. This can generate multiple equilibria with different liquidity properties.

Moreover, the decision to investment in information and begin accepting alternative assets in exchange depends on policy – it is not in general appropriate to take cash-in-advance constraints as given or invariant. The model generates several other predictions we find interesting, including results on dollarization and hysteresis. Much additional work remains to be done on informational frictions in models where exchange is modeled explicitly. In our set up, sellers who do not recognize an asset simply refuse to accept it. This is certainly convenient, since among other things it allows a simple solution to the bargaining problem, despite the asymmetric information friction, which is key for determining liquidity. But it is also certainly extreme, and one might like to see generalizations or alternative specifications where agents sometimes trade for and bargain over assets whose quality is unknown. This obviously would be interesting, if considerably more difficult. We think our analysis provides some preliminary steps in the right direction.

5 Appendix

A. Proof of Lemma 2: Suppose $a_2 = 0$. Then $q_1 = q_2 \equiv q_0$. Given $\gamma > \beta$, we know $q_0 < \bar{q} \leq q^*$ by standard results (when $a_2 = 0$ there are no claims traded in the DM and the model is equivalent to the baseline LW model). Since $q_0 < q^*$, from (10) at equality we have

$$(\lambda_1 + \lambda_2)\ell(q_0) + 1 = \phi/\beta\hat{\phi} = \gamma/\beta > 1,$$

which implies $\ell(q_0) > 0$. Since $a_2 = 0$, market clearing implies $a_1 = A > 0$, and (11) holds at equality. Thus, $\psi = \beta(\hat{\psi} + \delta)$. Then (12) implies $\lambda_2\ell(q_0) \leq 0$, a contradiction. ■

B. Proof of Proposition 1: Suppose $A \leq \bar{A}$. We first show that there exists a unique pair (q_1, q_2) that satisfy (13) and (14). We then show these conditions are equivalent to the necessary and sufficient conditions for equilibrium.

By the implicit function theorem:

$$\begin{aligned}
\mu'(q_1) &= -\frac{\beta\lambda_1\ell'(q_1)}{\lambda_2\ell'(q_2)} < 0 \\
\alpha'(q_1) &= \frac{-z'(q_1)\Big[1 - \beta[\lambda_2\ell(q_2) + 1]\Big]}{\beta\lambda_2 e'(q_2)(z(q_2) - z(q_1)) - z'(q_1)\Big[1 - \beta[\lambda_2\ell(q_2) + 1]\Big]} > 0
\end{aligned}$$

Let $\breve{q}$ satisfy $\ell(\breve{q}) = \frac{\gamma-\beta}{\beta\lambda_1} + \frac{\lambda_2}{\lambda_1}$, with $\breve{q} < \tilde{q} \leq \bar{q}$. Since $\ell'(q) < 0$ and $\lim_{q\to\infty}\ell(q) = -1$, it is easy to see that $\lim_{q_1\to\breve{q}^+}\mu(q_1) = \infty$. Moreover, we claim $\lim_{q_1\to\breve{q}^+}\alpha(q_1) < \infty$. Suppose not. That is, suppose $\lim_{q_1\to\breve{q}^+}\alpha(q_1) = \infty$. Then using (13) we have

$$A\delta = \lim_{q_1\to\breve{q}^+}[z\big(\alpha(q_1)\big) - z(q_1)][1 - \beta + \beta\lambda_2].$$

This implies $A\delta \geq [z(\bar{q}) - z(\breve{q})][1 - \beta + \beta\lambda_2]$, which implies $\frac{\delta}{1-\beta} > \frac{z(\bar{q})-z(\breve{q})}{A}$, a contradiction. Therefore, $\lim_{q_1\to\breve{q}^+}\mu(q_1) > \lim_{q_1\to\breve{q}^+}\alpha(q_1)$.

Now consider (14) with $q_1 = \bar{q}$, so that $\frac{\gamma}{\beta} = \lambda_2\ell(q_2) + 1$. This implies $\ell(q_2) \leq 0$, so that $\mu(\bar{q}) \leq \bar{q}$. Now consider (13). If $q_2 = \bar{q}$ then $A\delta = [z(\bar{q}) - z(q_1)](1 - \beta)$. Since $\frac{A\delta}{1-\beta} > 0$, $\alpha^{-1}(\bar{q}) < \bar{q}$. Since $\alpha' > 0$, $\alpha(\bar{q}) > \bar{q} \geq \mu(\bar{q})$. Since μ and α are continuous, $\mu' < 0$ and $\alpha' > 0$,

$\mu(q') > \alpha(q')$ for some $q' < \bar{q}$, and $\alpha(\bar{q}) \geq \mu(\bar{q})$, we conclude that there exists a unique pair (q_1, q_2) with $q_1 > 0$ and $q_2 \leq \bar{q}$ that satisfy (13) and (14).

It is left to show that (13) and (14) are equivalent to the necessary and sufficient conditions for an equilibrium with $m > 0$, $a_1 = 0$, and $a_2 > 0$. Since $m > 0$, (10) holds with equality in equilibrium. Since $\gamma = \phi/\phi'$, clearly (14) and (10) are equivalent. Since $a_2 > 0$, (12) must also hold with equality. We know that $a_1 = 0 \Rightarrow a_2 = A$. Also, $z(q_1) = \phi M$ and $z(q_2) = \phi M + (\psi + \delta)A$ implies the asset pricing equation

$$\psi = \frac{z(q_2) - z(q_1)}{A} - \delta. \tag{18}$$

Substituting this into (13) yields (12).

Now suppose $A > \bar{A}$. We claim that there does not exist a pair (q_1, q_2) with $q_2 < \bar{q}$ that satisfy (13) and (14). To see this, let $\tilde{\tilde{q}}$ be the value of q_1 such that $\alpha(q_1) = \bar{q}$. It is easy to show that $A > \bar{A} \Rightarrow \tilde{\tilde{q}} < \tilde{q}$, which implies that $\mu(\tilde{\tilde{q}}) > \bar{q}$, so there does not exist a $q_1 < \tilde{\tilde{q}}$ satisfying $\mu(q_1) = \alpha(q_1)$. Therefore, $q_2 = \bar{q}$. From (10), $q_1 = \tilde{q}$ and the corresponding prices follow immediately. ■

C. Results for the Cashless Market: We have

$$\begin{aligned}
\frac{\partial q_1^B}{\partial i} &= \frac{(1-b)\Big\{z'[h(q_2^B)]h'(q_2^B)[r - \lambda^B \ell(q_2^B)]\Big\} - \Big\{(1-b)[z\big(h(q_2^B)\big) - z(q_1)] + bz(q_2^B)\Big\}\lambda^B \ell'(q_2^B)}{\Psi} \\
\frac{\partial q_2^B}{\partial i} &= \frac{(1-b)z'(q_1)[r - \lambda^B \ell(q_2^B)]}{\Psi}
\end{aligned}$$

and so both take the sign of

$$\begin{aligned}
\Psi &= (1-\rho)\lambda_1 \ell'(q_1)\Big\{(1-b)\Big[z'[h(q_2^B)]h'(q_2^B)[r - \lambda^B \ell(q_2^B)]\Big] \\
&- \Big[(1-b)[z\big(h(q_2^B)\big) - z(q_1)] + bz(q_2^B)\Big]\lambda^B \ell'(q_2^B)\Big\} + (1-b)z'(q_1)[r - \lambda^B \ell(q_2^B)]\lambda^B \ell'(q_2^B) < 0.
\end{aligned}$$

D. Results in Table 1: Let Δ denote the determinant of the following matrix:

$$\begin{bmatrix} \lambda_1 \ell'(q_1) & \lambda_2 \ell'(q_2) \\ [\lambda_2 \ell(q_2) - r]\, z'(q_1) & [r - \lambda_2 \ell(q_2)]z'(q_2) - [z(q_2) - z(q_1)]\lambda_2 \ell'(q_2) \end{bmatrix}$$

From (15), an equilibrium with $q_2 \geq q_1$ requires $r - \lambda_2 \ell(q_2) \geq 0$, so $\Delta < 0$. Then we have

$$\begin{aligned}
\frac{\partial q_1}{\partial i} &= \frac{[r-\rho\lambda\ell(q_2)]z'(q_2) - [z(q_2)-z(q_1)]\rho\lambda\ell'(q_2)}{\Delta} < 0 \\
\frac{\partial q_2}{\partial i} &= \frac{[r-\rho\lambda\ell(q_2)]z'(q_1)}{\Delta} < 0 \\
\frac{\partial q_1}{\partial \delta} &= \frac{-(1+r)A\rho\lambda\ell'(q_2)}{\Delta} < 0 \\
\frac{\partial q_2}{\partial \delta} &= \frac{(1+r)A(1-\rho)\lambda\ell'(q_1)}{\Delta} > 0 \\
\frac{\partial q_1}{\partial A} &= \frac{-(1+r)\delta\rho\lambda\ell'(q_2)}{\Delta} < 0 \\
\frac{\partial q_2}{\partial A} &= \frac{(1+r)\delta(1-\rho)\lambda\ell'(q_1)}{\Delta} > 0 \\
\frac{\partial q_1}{\partial \rho} &= \frac{\lambda[\ell(q_1)-\ell(q_2)][r-\rho\lambda\ell(q_2)]z'(q_2) - [z(q_2)-z(q_1)]\rho\lambda\ell'(q_2)\lambda\ell(q_1)}{\Delta} < 0 \\
\frac{\partial q_2}{\partial \rho} &= \frac{\lambda[\ell(q_1)-\ell(q_2)][r-\rho\lambda\ell(q_2)]z'(q_1) + [z(q_2)-z(q_1)](1-\rho)\lambda\ell'(q_1)\lambda\ell(q_2)}{\Delta} \\
\frac{\partial q_1}{\partial \lambda} &= \frac{(1-\rho)\ell(q_1)[z(q_2)-z(q_1)]\rho\lambda\ell'(q_2) - [(1-\rho)\ell(q_1)+\rho\ell(q_2)][r-\rho\lambda\ell(q_2)]z'(q_2)}{\Delta} > 0 \\
\frac{\partial q_2}{\partial \lambda} &= \frac{(1-\rho)\ell(q_2)[z(q_2)-z(q_1)]\rho\lambda\ell'(q_1) - [(1-\rho)\ell(q_1)+\rho\ell(q_2)][r-\rho\lambda\ell(q_2)]z'(q_1)}{\Delta} > 0
\end{aligned}$$

Given $z(q_1) = \phi M$ and $z(q_2) - z(q_1) = (\psi + \delta)A$, we have

$$\begin{aligned}
\frac{\partial \psi}{\partial i} &= \frac{[z(q_2)-z(q_1)]\rho\lambda\ell'(q_2)z'(q_1)}{A\Delta} > 0 \\
\frac{\partial \phi}{\partial i} &= \frac{z'(q_1)}{M}\frac{\partial q_1}{\partial i} < 0 \\
\frac{\partial \psi}{\partial \delta} &= \frac{[z(q_2)-z(q_1)](1-\rho)\lambda\ell'(q_1)\rho\lambda\ell'(q_2) + [1+\rho\lambda\ell(q_2)][(1-\rho)\lambda\ell'(q_1)z'(q_2)+\rho\lambda\ell'(q_2)z'(q_1)]}{\Delta} \\
\frac{\partial \phi}{\partial \delta} &= \frac{z'(q_1)}{M}\frac{\partial q_1}{\partial \delta} < 0 \\
\frac{\partial \psi}{\partial A} &= \frac{[z(q_2)-z(q_1)](1-\rho)\lambda\ell'(q_1)\rho\lambda\ell'(q_2)}{A\Delta} < 0 \\
\frac{\partial \phi}{\partial A} &= \frac{z'(q_1)}{M}\frac{\partial q_1}{\partial A} < 0 \\
\frac{\partial \psi}{\partial \rho} &= \frac{[z(q_2)-z(q_1)][(1-\rho)\lambda\ell'(q_1)\lambda\ell(q_2)z'(q_2)+\rho\lambda\ell'(q_2)\lambda\ell(q_1)z'(q_1)]}{\Delta} > 0 \\
\frac{\partial \phi}{\partial \rho} &= \frac{z'(q_1)}{M}\frac{\partial q_1}{\partial \rho} < 0 \\
\frac{\partial \psi}{\partial \lambda} &= \frac{(1-\rho)[z(q_2)-z(q_1)][\ell(q_2)\ell'(q_1)z'(q_2)-\ell(q_1)\ell'(q_2)z'(q_1)]}{\Delta} \\
\frac{\partial \phi}{\partial \lambda} &= \frac{z'(q_1)}{M}\frac{\partial q_1}{\partial \lambda} > 0
\end{aligned}$$

References

[1] Alchian, A. Why Money? *Journal of Money, Credit and Banking 9*, 1 (1977), 133–40.

[2] Aliprantis, C., Camera, G., and Puzzello, D. Anonymous Markets and Monetary Trading. *Journal of Monetary Economics 54*, 7 (2007), 1905–1928.

[3] Aliprantis, C., Camera, G., and Puzzello, D. Contagion Equilibria in a Monetary Model. *Econometrica 75*, 1 (2007), 277–282.

[4] Araujo, L. Social Norms and Money. *Journal of Monetary Economics 51*, 2 (2004), 241–256.

[5] Aruoba, S., Rocheteau, G., and Waller, C. Bargaining and the Value of Money. *Journal of Monetary Economics 54* (2007), 2636–55.

[6] Banerjee, A., and Maskin, E. A Walrasian Theory of Barter and Exchange. *Quarterly Journal of Economics 111*, 4 (1996), 955–1005.

[7] Berentsen, A., and Rocheteau, G. Money and Information. *Review of Economic Studies 71*, 4 (2004), 915–944.

[8] Brunner, K., and Meltzer, A. The Uses of Money: Money in the Theory of an Exchange Economy. *American Economic Review 61*, 5 (1971), 784–805.

[9] Cavalcanti, R., and Nosal, E. Counterfeiting as Private Money in Mechanism Design. *working paper, Federal Reserve Bank of Cleveland* (2007).

[10] Cavalcanti, R., and Wallace, N. A Model of Private Bank-Note Issue. *Review of Economic Dynamics 2*, 1 (1999), 104–136.

[11] Cavalcanti, R., and Wallace, N. Inside and Outside Money as Alternative Media of Exchange. *Journal of Money, Credit & Banking 31*, 3 (1999).

[12] Corbae, D., Temzelides, T., and Wright, R. Directed Matching and Monetary Exchange. *Econometrica 71*, 3 (2003), 731–756.

[13] Cuadras-Morató, X. Commodity Money in the Presence of Goods of Heterogeneous Quality. *Economic Theory 4*, 4 (1994), 579–591.

[14] Dong, M. Money and Costly Credit. *manuscript, Simon Fraser University* (2008).

[15] Dornbusch, R., Sturzenegger, F., Wolf, H., Fischer, S., and Barro, R. J. Extreme Inflation: Dynamics and Stabilization. *Brookings Papers on Economic Activity 1990*, 2 (1990), 1–84.

[16] Dutu, R., Julien, B., and King, I. Competing Auctions in a Monetary Economy. *manuscript, University of Auckland* (2007).

[17] Ennis, H. Search, Money, and Inflation Under Private Information. *Journal of Economic Theory 138*, 1 (2008), 101–131.

[18] Faig, M., and Jerez, B. Inflation, Prices, and Information in Competitive Search. *Advances in Macroeconomics 6*, 1 (2006).

[19] Feige, E. Dynamics of Currency Substitution, Asset Substitution and De facto Dollarisation and Euroisation in Transition Countries. *Comparative Economic Studies 45*, 3 (2003), 358–383.

[20] Freeman, S. Fiat Money as a Medium of Exchange. *International Economic Review 30*, 1 (1989), 137–51.

[21] Galenianos, M., and Kircher, P. A Model of Money with Multilateral Matching. *Journal of Monetary Economics* (2008).

[22] Geromichalos, A., Licari, J., and Suárez-Lledó, J. Monetary Policy and Asset Prices. *Review of Economic Dynamics 10*, 4 (2007), 761–779.

[23] Glosten, L., and Milgrom, P. Bid, Ask, and Transaction Prices in a Specialist Market with Heterogeneously Informed Agents. *Journal of Financial Economics 14* (1985), 71–100.

[24] Guerrieri, V. Comment on "Information, Liquidity, and Asset Prices"prepared for Penn Econ - Philadelphia Fed Conference on Money and Macro Economics . *manuscript, University of Chicago, Graduate School of Business* (2008).

[25] Guidotti, P., and Rodriguez, C. Dollarization in Latin America: Gresham's Law in Reverse? *working paper, Universidad del CEMA* (1992).

[26] Hicks, J. A Suggestion for Simplifying the Theory of Money. *Economica 2*, 1 (1935), 1–19.

[27] Hu, T. Counterfeiting Currency Substitutes: a New Rationale for Prohibiting Trades with High Return Assets. *manuscript, Pennsylvania State University* (2008).

[28] Hunt, R. An Introduction to the Economics of Payment Card Networks. *Review of Network Economics 2* (2003).

[29] Ireland, P. Endogenous Financial Innovation and the Demand for Money. *Journal of Money, Credit & Banking 27*, 1 (1995), 107–23.

[30] Kim, Y., and Lee, M. Recognizability and Liquidity. *manuscript, Seoul National University* (2008).

[31] Kiyotaki, N., and Wright, R. On Money as a Medium of Exchange. *The Journal of Political Economy 97*, 4 (1989), 927.

[32] Kocherlakota, N. Money Is Memory. *Journal of Economic Theory 81*, 2 (1998), 232–251.

[33] Kyle, A. Continuous Auctions and Insider Trading. *Econometrica 53*, 6 (1985), 1315–1335.

[34] Lacker, J., and Schreft, S. Money and Credit as Means of Payment. *Journal of Monetary Economics 38*, 1 (1996), 3–23.

[35] Lagos, R. Asset Prices and Liquidity in an Exchange Economy. *working paper, Federal Reserve Bank of Minneapolis* (2006).

[36] Lagos, R., and Rocheteau, G. Money and Capital as Competing Media of Exchange. *Journal of Economic Theory 142*, 2 (2008), 247–258.

[37] Lagos, R., and Wright, R. A Unified Framework for Monetary Theory and Policy Analysis. *Journal of Political Economy 113*, 3 (2005), 463–484.

[38] Lester, B., Postlewaite, A., and Wright, R. Information and Liquidity. *manuscript, University of Pennsylvania* (2008).

[39] Li, Y. Commodity Money Under Private Information. *Journal of Monetary Economics 36*, 3 (1995), 573–592.

[40] Lucas, R. Asset Prices in an Exchange Economy. *Econometrica 46*, 6 (1978), 1429–1445.

[41] Menger, C. On the Origin of Money. *Economic Journal 2*, 6 (1892), 239–55.

[42] Nosal, E., and Wallace, N. A Model of (the Threat of) Counterfeiting. *Journal of Monetary Economics 54*, 4 (2007), 994–1001.

[43] Rochet, J., and Tirole, J. Externalities and Regulation in Card Payment Systems. *Review of Network Economics 5*, 1 (2006), 1–14.

[44] Rocheteau, G. Money and Competing Assets Under Private Information. *working paper, Federal Reserve Bank of Cleveland* (2008).

[45] Rocheteau, G., and Wright, R. Money in Search Equilibrium, in Competitive Equilibrium, and in Competitive Search Equilibrium. *Econometrica 73*, 1 (2005), 175–202.

[46] Sik Kim, Y. Money, Barter, and Costly Information Acquisition. *Journal of Monetary Economics 37*, 1 (1996), 119–142.

[47] Tobin, J. Money and Economic Growth. *Econometrica 33*, 4 (1965), 671–684.

[48] Trejos, A. Incentives to Produce Quality and the Liquidity of Money. *Economic Theory 9*, 2 (1997), 355–365.

[49] Trejos, A. Search, Bargaining, Money, and Prices Under Private Information. *International Economic Review 40*, 3 (1999), 679–696.

[50] Uribe, M. Hysteresis in a Simple Model of Currency Substitution. *Journal of Monetary Economics 40*, 1 (1997), 185–202.

[51] Velde, F., Weber, W., and Wright, R. A Model of Commodity Money, with Applications to Gresham's Law and the Debasement Puzzle. *Review of Economic Dynamics 2*, 1 (1999), 291–323.

[52] Wallace, N. The Overlapping Generations Model of Fiat Money. In *Models of Monetary Economies: Proceedings and Contributions from Participants of a December 1978 Conference Sponsored by the Federal Reserve Bank of Minneapolis* (1980), Federal Reserve Bank of Minneapolis.

[53] Wallace, N. Whither Monetary Economics? *International Economic Review 42*, 4 (2001), 847–869.

[54] Williamson, S., and Wright, R. Barter and Monetary Exchange Under Private Information. *American Economic Review 84* (1994), 104–123.

[55] Wright, R. A Uniqueness Proof for Monetary Steady State. *manuscript, University of Pennsylvania* (2008).

Figure 1: $A \leq \bar{A}$

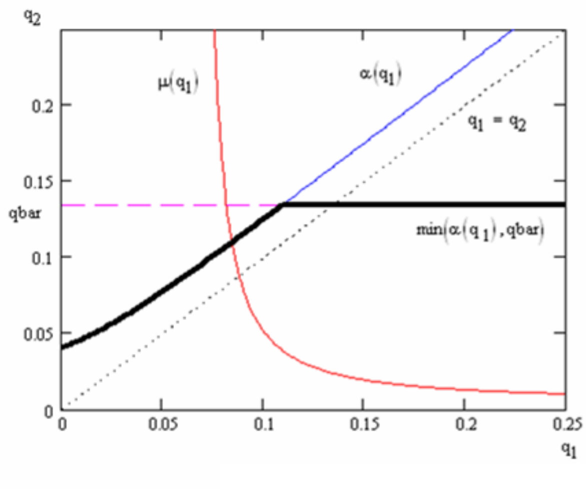

Figure 2: $A > \bar{A}$

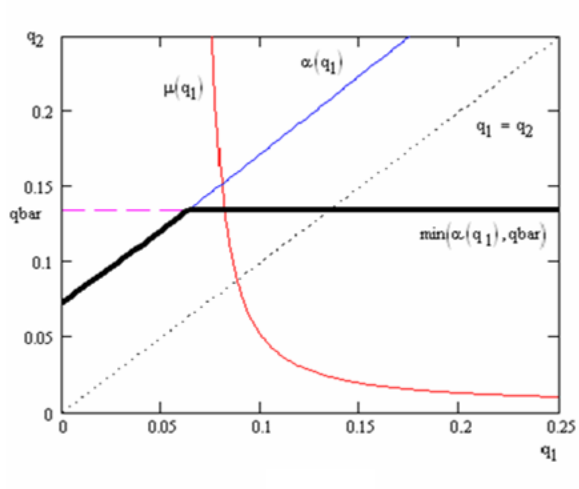

Figure 3: $\rho^* = 1$

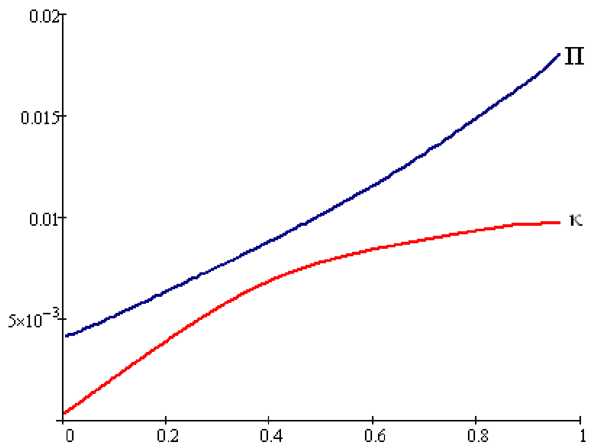

Figure 4: $\rho^* = 0$

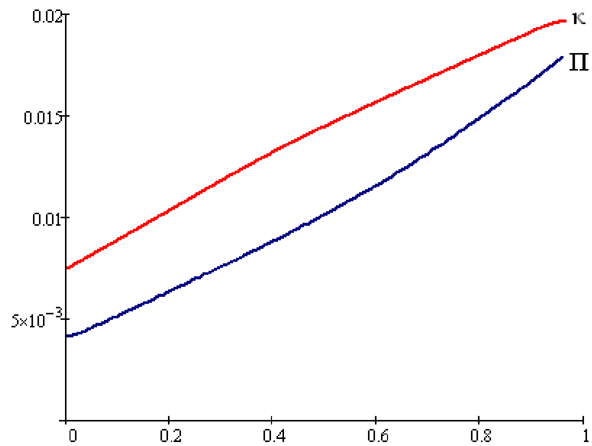

Figure 5: Unique $\rho^* \in (0, 1)$

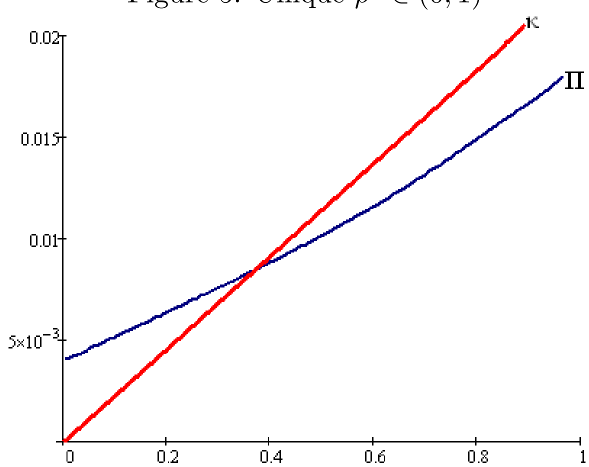

Figure 6: Multiple $\rho^* \in (0, 1)$

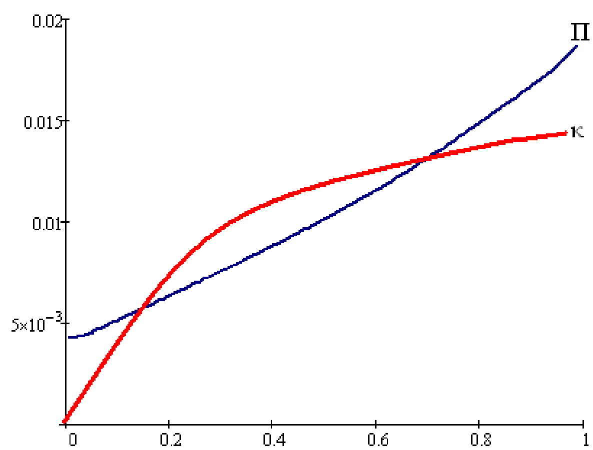

Figure 7: Unique Equilibria

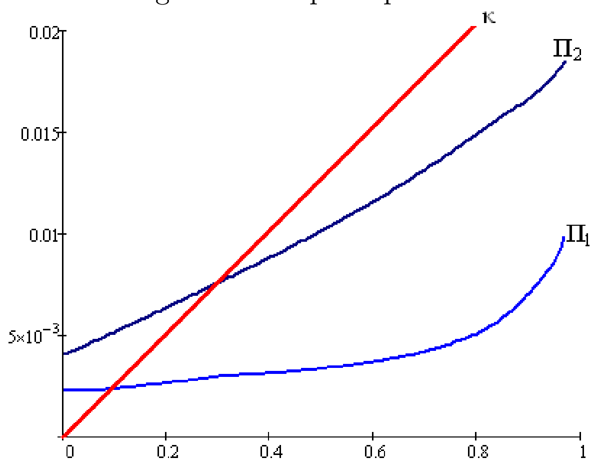

Figure 8: Shift to Multiple Equilibria