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PIER Working Paper 08-034

"Do Voters Vote Ideologically?"
Third Version

by

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Do Voters Vote Ideologically?*

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Revised, August 2008

ABSTRACT

In this paper we address the following question: To what extent is the hypothesis that voters vote "ideologically" (i.e., they always vote for the candidate who is ideologically "closest" to them) testable or falsifiable? We show that using data only on how individuals vote in a single election, the hypothesis that voters vote ideologically is irrefutable, regardless of the number of candidates competing in the election. On the other hand, using data on how the same individuals vote in multiple elections, the hypothesis that voters vote ideologically is potentially falsifiable, and we provide general conditions under which the hypothesis can be tested.

***JEL* D72, C12, C63; *Keywords*: voting, spatial models, falsifiability, testing.**

*We would like to thank three anonymous referees and the editor, Alessandro Lizzeri for their useful comments. Steve Coate, Aureo de Paula, George Mailath, Nolan McCarty, Robin Pemantle, Wolfgang Pesendorfer, Alvaro Sandroni, Ken Wolpin and conference and seminar participants at several institutions provided helpful suggestions. Degan acknowledges financial support from the Social Sciences and Humaniteis Research Council and Merlo from the National Science Foundation. This is a much revised and improved version of a paper that originally circulated under the title "Do Voters Vote Sincerely?" (PIER Working Paper 07-006).

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1 Introduction

Voting is a cornerstone of democracy and voters' decisions in elections and referenda are fundamental inputs in the political process that shapes the policies adopted by democratic societies. Hence, understanding observed patterns of voting represents an important step in the understanding of democratic institutions. Moreover, from a theoretical standpoint, voters are a fundamental primitive of political economy models. Different assumptions about their behavior have important consequences on the implications of these models and, more generally, on the equilibrium interpretation of the behavior of politicians, parties and governments they may induce.[1]

The *spatial theory of voting*, originally formulated by Downs (1957) and Black (1958) and later extended by Davis, Hinich and Ordershook (1970), Enelow and Hinich (1984) and Hinich and Munger (1994), among others, is a staple of political economy.[2] This theory postulates that each individual has a most preferred policy or "bliss point" and evaluates alternative policies or candidates in an election according to how "close" they are to her ideal. More precisely, consider a situation where at some date a group of voters is facing some contested elections (i.e., there is at least one election and two or more candidates in each election). Suppose that each voter has political views (i.e., their bliss point) that can be represented by a position in some common, multi-dimensional ideological (metric) space, and each candidate can also be represented by a position in the same ideological space. According to the spatial framework, in each election, each voter will cast her vote in favor of the candidate whose position is closest to her bliss point (given the positions of all the candidates in the election). If this is the case, we say that voters *vote ideologically*.[3]

An important question thus is whether in reality voters do vote ideologically based on their political views, or whether other factors (like for example instrumental considerations, or their assessment of candidates' personal characteristics) determine the way individuals

[1] See, e.g., the survey by Merlo (2006) for a general overview of the implications of alternative theories of voting in political economy.

[2] See, e.g., Hinich and Munger (1997).

[3] In this paper, we ignore the issue of abstention. For recent surveys of alternative theories of voter turnout see, e.g., Dhillon and Peralta (2002) and Merlo (2006).

vote. Clearly, this is an empirical question. Given the definition of ideological voting, it follows immediately that if the positions of all voters and candidates as well as the voting decisions of all voters were observable, we could then directly assess whether or not the behavior of each voter in any election is consistent with ideological voting. However, this is generally not the case. While there exist surveys containing information on how the same individuals vote in a number of simultaneous elections (e.g., the American National Election Study, the Canadian National Election Study and the British Election Survey), and data sets containing measures of the positions of politicians in the ideological space based on their observed behavior in a variety of public offices (e.g., Poole and Rosenthal (1997) and Hix, Nouri and Roland (2006)), the ideological positions of voters are not directly observable.[4] The relevant empirical question thus becomes: To what extent is the hypothesis that voters vote ideologically testable or falsifiable (in a Popperian sense)?[5] In other words, which kind of data on candidates' positions and individuals' voting behavior would allow a researcher to potentially falsify and hence possibly reject the hypothesis that voters vote ideologically? This is the question we address in this paper.

The first result of our analysis is that using data only on how individuals vote in a single isolated election, the hypothesis that voters vote ideologically is irrefutable, regardless of the number of candidates competing in the election. Given any configuration of distinct candidates' positions, any observed vote is consistent with a voter voting ideologically for some voter's ideological position. This result holds for any number of dimensions of the ideological space.

Second, we show that using data on how the same individuals vote in multiple simultaneous elections it is possible in principle to determine whether or not the behavior of

[4] Note that in order to directly assess whether the behavior of voters is consistent with ideological voting one would need a consistent set of observations on the ideological positions of all voters and candidates in the same metric space. Hence, measures of citizens' self-reported ideological placements that are contained in some surveys (like, for example, the variable contained in the American National Election Studies, where voters are asked to place themselves on a 7-point liberal-conservative scale), cannot be used for this purpose, since, for instance, different people may interpret the scale differently.

[5] See, e.g., Popper (1935).

voters is consistent with ideological voting. In other words, the hypothesis that voters vote ideologically in multiple elections is potentially falsifiable, and we provide conditions under which the hypothesis can be tested. We show that in general environments where individual voting decisions and candidates' positions are observable but voters' positions are not, the hypothesis that individuals vote ideologically in multiple elections with any number of candidates is falsifiable if the number of elections is greater than the number of dimensions of the ideological space. Given any configuration of distinct candidates' positions in two or more simultaneous elections, there always exists at least a voting profile (that is, a vector of votes by the same individual in all elections) that is not consistent with a voter voting ideologically in these elections based on any voter's ideological position.

Finally, we characterize the maximum number of voting profiles that are consistent with ideological voting as a function of the number of elections, the number of candidates in each election, and the number of dimensions of the ideological space. All our results are formally stated in Section 2. In Section 3, we consider several extensions of the basic theoretical framework and analyze the robustness of our results on the falsifiability of the ideological voting hypothesis. In Section 4, we then consider an application (voting in U.S. national elections) and illustrate how existing data can be used to quantify the extent to which, in environments where the hypothesis is falsifiable, the observed behavior of voters is not consistent with ideological voting.

Before turning attention to our analysis, some remarks are in order. The general approach we follow is based on a standard revealed preference argument according to which individual choices are the result of an optimization problem. Hence, at a general level, our work is related to the literature on revealed preferences which tries to determine the restrictions that observed behavior imposes on the structure of preferences, or alternatively the type of behavior which would represent a violation of basic tenets of the theory of choice. This literature is quite vast. It originated in the context of consumer theory with the work of Samuelson (1938, 1948), and was later developed by, among others, Houthakker (1950), Afriat (1967) and Varian (1982). Their goal is to find necessary and sufficient conditions for the observed consumer choice data to be the result of the maximization of some well-behaved utility function subject to a budget constraint. Afriat (1967) characterizes several equivalent conditions for the existence of a utility function that *rationalizes* a finite set of demand points and provides an algorithm to compute whether such utility function exists. Varian (1982) shows the equivalence between the Afriat conditions and the generalized axiom of revealed preferences (GARP), which provides much simpler conditions to verify existence. In more general settings, Arrow (1959) and Sen (1971) extend the notion of rationalizability to general choice sets with finitely many alternatives. A recent literature in decision theory has also addressed the issue of rationalizability of patterns of choices that may violate the Weak Axiom of Revealed Preferences (WARP).[6]

Our work is also related to the literature on characteristics models pioneered by Gorman (1956) and Lancaster (1966). According to these models goods can be described by a finite set of characteristics and consumers with monotonic preferences over these characteristics must choose between goods given their budget. The framework we consider can be interpreted as a characteristics model in that candidates (like goods) are characterized by a combination of characteristics (their positions on several ideological dimensions), and voters have preferences over these characteristics. Unlike the consumers in characteristics models, however, the voters in our framework have satiated preferences and, rather than having a standard budget set, in any election can only choose a candidate among a finite number of alternatives. While characteristics models have also been formulated in the context of a discrete choice framework (see, e.g., McFadden (1973, 1981), Berry, Levinsohn and Pakes (1995), and Berry and Pakes (2007)), the main focus of this literature has been the estimation of these models (and in particular the issue of recovering the consumers' marginal valuations of product attributes). Recently, Blow, Browning, and Crawford (2008) provide necessary and sufficient conditions under which data on consumers' behavior are non-parametrically consistent with the characteristics model. As explained above, the emphasis of our work is on the issue of falsifiability. In this respect, our paper is most closely related to the work of Chiappori and Donni (2006) who analyze the empirical content of Nash bargaining and derive sufficient conditions on the auxiliary assumptions of the model under which Nash bargaining generates testable predictions.

[6] See, e.g., Eliaz and Ok (2006) and Manzini and Mariotti (2007).

2 Baseline model

Consider a situation where a population of voters N is facing $m \geq 1$ simultaneous elections. Consistent with the spatial theory of voting, there is a common ideological space, Y, which is taken to be the k-dimensional Euclidean space (i.e., $Y = R^k$, $k \geq 1$). For any election $e \in \{1, ..., m\}$, let $q_e = |J^e| \in \{2, ..., \overline{q}\}$ denote the number of candidates competing in the election, where J^e is the set of candidates. Each candidate $j \in \{\cup_{e=1}^m J^e\}$ is characterized by a distinct position in the ideological space, $y^j \in Y$, which is known to the voters.[7]

Each voter $i \in N$ has an ideological position (or bliss point) $y^i \in Y$, and her preferences are characterized by indifference sets that are spheres in the k-dimensional Euclidean space (or k-spheres), centered around her bliss point.[8] It follows that voter i's preferences over candidates in election e can be summarized by the utility function

$$U_e^i(j_e) = u_e^i\left(d\left(y^i, y^{j_e}\right)\right), \tag{1}$$

where $u_e^i(\cdot)$ is a decreasing function which may differ across voters and elections and $d(\cdot, \cdot) \geq 0$ denotes the Euclidean distance (i.e., for any two points $x, z \in R^k$, $d(x, z) = \sqrt{\sum_{r=1}^{k} (x_r - z_r)^2}$). Other than monotonicity, we impose no additional restrictions on the $u_e^i(\cdot)$ functions, which are therefore left unspecified. Given these preferences, a voter i (strictly) prefers candidate j_e to candidate ℓ_e in election e if $d(y^i, y^{j_e}) < d(y^i, y^{\ell_e})$.

For each voter $i \in N$, let $v^i = (v_1^i, ..., v_m^i) \in V^m$ denote her voting profile, where $v_e^i \in J^e$ denotes her voting choice in election $e = 1, ..., m$, and V^m is the set of all possible distinct voting profiles in the m elections. Hence, i's voting profile contains the list of candidates she votes for in the m elections (one for each election). Let $v \in V^m$ denote a generic voting profile and note that the number of possible distinct voting profiles in the m elections is $|V^m| = \Pi_{e=1}^m q_e$.[9]

[7] In Section 3, we consider an environment where voters are uncertain about the candidates' positions.

[8] In one dimension, the restriction implies that each voter's utility function is single-peaked and symmetric. In Section 3, we consider more general specifications of preferences where the voters' indifference sets are ellipsoids in the k-dimensional Euclidean space. When $k \geq 2$, such preferences allow for the possibility that voters may evaluate different ideological dimensions using different weights.

[9] For example, if there are two elections, 1 and 2, with candidates a_1 and b_1 competing in election 1,

Definition 1: *Voter i votes ideologically in election e if she votes for the candidate whose position is closest to her bliss point (i.e., $d\left(y^i, y^{j_e}\right) < d(y^i, y^{\ell_e})$ for all $\ell_e \in J^e$, $\ell_e \neq j_e \in J^e$, implies that $v_e^i = j_e$). Voter i votes ideologically if she votes ideologically in all elections $e = 1, ..., m$.*

We are interested in determining the conditions under which the hypothesis that voters vote ideologically is falsifiable. In other words, the main goal of our analysis is to characterize which type of data a researcher would need to potentially falsify this hypothesis. Clearly, given individual-level data on voters' behavior, if a researcher could observe the positions of electoral candidates as well as the voters' preferences, the hypothesis that each voter votes ideologically in each election would be falsifiable. This situation, however, is unrealistic, since for all practical purposes there do not exist data containing all this information. Hence, suppose instead that a researcher has access only to limited information, and consider the best case scenario where the researcher observes the way some individuals vote in each of m simultaneous elections and the positions of all the candidates in these elections but does not know the voters' preferences. Is the hypothesis that voters vote ideologically falsifiable given such data?

To address this question, we begin by defining the notions of consistency of a voting profile with ideological voting and of falsifiability of the ideological voting hypothesis that we use throughout our analysis.

Definition 2: *A voting profile $v \in V^m$ is consistent with ideological voting if there exists some subset of the ideological space, $Y^v \subseteq Y = R^k$, such that if a voter i's ideological position is in that subset ($y^i \in Y^v$) and i votes ideologically, then her voting profile v^i is equal to v. If it exists, then Y^v is the ideological support of the voting profile v.*

Definition 3: *The hypothesis that voters vote ideologically is falsifiable if there exists at least a voting profile $v \in V^m$ that is not consistent with ideological voting.*

Using the terminology of the revealed preference literature we described in the Introduction, Definition 2 is equivalent to say that a voting profile v is rationalizable by ideological

and candidates a_2 and b_2 competing in election 2, the set of the four possible voting profiles is $V^2 = \{(a_1, a_2), (a_1, b_2), (b_1, a_1), (b_1, b_2)\}$.

voting if and only if we can find an ideological support for v. Following Popper, the notion of falsifiability we adopt is the logical possibility that an hypothesis can be shown false by an observation. Hence, as long as there exists a voting profile that is feasible, and thus could be observed in the data, but is not consistent with ideological voting, the hypothesis that voters vote ideologically can, in principle, be falsified.

In the analysis that follows, we first consider the case of two-candidate elections, and then investigate the general case of elections with any number of candidates. Two-candidate elections have a prominent role in political economy, since they are the norm in two-party political systems like, for example, the U.S. Furthermore, the analysis of two-candidate elections allows us to obtain some stronger results and at the same time it helps to clarify some of the issues that arise in the more complex environments with multiple candidates.

2.1 Two-candidate elections

Consider the case where there are $m \geq 1$ simultaneous two-candidate elections (i.e., $q_e = 2$ for all $e = 1, ..., m$). For each election $e \in \{1, ..., m\}$, let $y^{j_e}, y^{\ell_e} \in Y = R^k, y^{j_e} \neq y^{\ell_e}$, denote the ideological positions of the two candidates $j_e, \ell_e \in J^e$ in the election, and let $H^e = \left\{y \in Y : d\left(y, y^{j_e}\right) = d(y, y^{\ell_e})\right\}$ be the set of points in the ideological space Y that are equidistant from the candidates' positions.

Since $d\left(\cdot,\cdot\right)$ is the Euclidean distance, it follows that for each election e there exists a non-zero vector $\lambda^e = (\lambda_1^e, ..., \lambda_k^e) \in R^k$ and a scalar $\mu_e \in R$ such that

$$H^e = \{y \in Y : \lambda^e y' = \mu^e\}, \; e = 1, ..., m, \tag{2}$$

where y' denotes the transpose of $y = (y_1, ..., y_k)$.[10] Hence, each election $e = 1, ..., m$ implies a hyperplane H^e in R^k which partitions the ideological space Y into two regions (or half spaces),

$$Y^{j_e} = \{y \in Y : \lambda^e y' < \mu_e\}$$

and

$$Y^{\ell_e} = \{y \in Y : \lambda^e y' > \mu^e\},$$

[10]Note that λ^e and μ_e only depend on y^{j_e} and y^{ℓ_e}.

where Y^{j_e} (Y^{ℓ_e}) is the set of ideological positions that are closer to the position of candidate j_e (ℓ_e) than to the position of the other candidate, or equivalently, is the ideological support of voting for candidate j_e (ℓ_e) in election e.[11]

It follows that the collection of the m hyperplanes implied by the m elections, $\{H^1, ..., H^m\}$, partitions the ideological space Y into $r_m \leq 2^m$ convex regions, where each region is the ideological support of a distinct voting profile $v \in V^m$. Since in the case of two-candidate elections the number of possible distinct voting profiles is $|V^m| = 2^m$, it follows that the hypothesis that voters vote ideologically in two-candidate elections is falsifiable if and only if $r_m < 2^m$.

We can now state our first set of results.[12]

Proposition 1: *In two-candidate elections, the hypothesis that voters vote ideologically is falsifiable if the number of elections m is strictly larger than the number of dimensions of the ideological space k. Otherwise, the hypothesis is generically not falsifiable.*

Corollary 1: *The hypothesis that voters vote ideologically in a single election with two candidates is not falsifiable regardless of the number of dimensions of the ideological space (i.e., for all $k \geq 1$).*

Since each hyperplane H^e only depends on the positions of the candidates in election e and these positions are observable, we can calculate whether or not each voting profile is consistent with ideological voting. Hence, the conditions for falsifiability of the ideological voting hypothesis in Proposition 1 and Corollary 1 apply to each individual voter.

In order to illustrate the result that in two-candidate elections the hypothesis that voters vote ideologically is falsifiable only if the number of elections is larger than the number of dimensions of the ideological space, consider an example in the two-dimensional space, $Y = R^2$. In this case, each election implies a line that partitions the plane into two regions and generically the lines implied by any two elections must intersect.[13]

[11] Note that $Y^{j_e} \cap Y^{\ell_e} = \varnothing$ and $Y^{j_e} \cup Y^{\ell_e} \cup H^e = Y$.

[12] All proofs are contained in the Appendix. The proof of Corollary 1 follows directly from the proof of Proposition 1 and is therefore omitted.

[13] While there exist configurations of candidates' positions such that these lines would be parallel (a case that would occur, for example, if the pair of candidates' positions in one election is a linear transformation

Figure 1 depicts a situation where there are three elections $e = 1, 2, 3$, the set of candidates in each election is $J^e = \{a_e, b_e\}$, and the candidates' ideological positions y^{a_e} and y^{b_e} are such that the region to the left of each line H^e is closer to the position of a_e than to that of b_e for each election e. Several observations emerge from this figure. If we consider any single election $e \in \{1, 2, 3\}$ in isolation (i.e., $m = 1$), then it is obvious that each voting profile $v \in \{a_e, b_e\}$ is consistent with ideological voting (since the two half planes determined by H^e are the ideological supports of a_e and b_e, respectively). This is also true if we consider any pair of elections $e, f \in \{1, 2, 3\}$, $e \neq f$, (i.e., $m = 2$), since H^e and H^f partition the ideological space in four regions that represent the ideological supports of each of the four possible voting profiles (a_e, a_f), (a_e, b_f), (b_e, a_f), and (b_e, b_f). However, when we consider the three elections all together (i.e., $m = 3$), we see that H^1, H^2 and H^3 partition the ideological space in only seven regions, while there are eight possible voting profiles. In this example, there do not exist ideological positions such that the voting profile (a_1, b_2, a_3) is consistent with ideological voting (that is, there does not exist an ideological support for (a_1, b_2, a_3)).

It is should also be clear from the example that increasing the number of elections, while keeping the number of ideological dimensions constant, would increase the number of voting profiles that are inconsistent with ideological voting and, hence, the theoretical possibility of refuting the theory. The following proposition characterizes the upper bound on the number of voting profiles that are consistent with ideological voting (i.e., the number of regions r_m) as a function of the number of elections m and the number of dimensions of the ideological space k.[14]

Proposition 2: *In two-candidate elections, the maximum number of voting profiles that are consistent with ideological voting depends on the number of elections m and on the number*

of the pair of candidates' positions in another election), this case is non generic.

[14]The issue we are considering corresponds to the problem of counting the number of regions in arrangements of hyperplanes in k-dimensional Euclidean space. This problem has been extensively studied in computational and combinatorial geometry (see, e.g., Orlik and Terao (1992)), and Proposition 2 follows from a general result that was first proved by Buck (1943).

of dimension of the ideological space k and is equal to

$$\rho(m,k) = \sum_{t=0}^{k} \binom{m}{t}. \tag{3}$$

Note that if the number of elections m is smaller than or equal to the number of dimensions of the ideological space k, Proposition 2 implies that

$$\rho(m,k) = \sum_{t=0}^{m} \binom{m}{t} = 2^m$$

and this bound is generically attained (Proposition 1). If, on the other hand, $m > k$, then for example in a two-dimensional ideological space with three, four, and five elections, we have that $\rho(3,2) = 7$, $\rho(4,2) = 11$, and $\rho(5,2) = 16$, respectively. This implies that when there are three elections at most 7 out of the 8 possible voting profiles are consistent with ideological voting; when there are four elections at most 11 out of the 16 possible voting profiles are consistent with ideological voting; and when there are five elections the maximum number of voting profiles that are consistent with ideological voting is 16 out of 32 possible profiles.

2.2 Multi-candidate elections

Consider now the general case where the number of candidates may vary across elections and any election may have more than two candidates (i.e., $q_e \in \{2,...,\overline{q}\}$, $e = 1,...,m$). For each election $e \in \{1,...,m\}$, and position $y^{j_e} \in Y = R^k$ of a generic candidate $j_e \in J^e$ in the election, let $Y^{j_e} = \{y \in Y : d(y, y^{j_e}) < d(y, y^{\ell_e}), \forall \ell_e \in J^e,\ \ell_e \neq j_e\}$ be the set of points in the ideological space Y that are closer to y^{j_e} than to the position of any other candidate in the election.

Since $d(\cdot,\cdot)$ is the Euclidean distance, it follows that for each pair of candidates in election e, $j_e, \ell_e \in J^e$, the set of points in the ideological space Y that are equidistant from y^{j_e} and y^{ℓ_e} is a hyperplane H^{j_e,ℓ_e}, which partitions the ideological space Y into two regions (or half spaces), $Y^{j_e}_{\ell_e}$ and $Y^{\ell_e}_{j_e} = Y \backslash \{Y^{j_e}_{\ell_e} \cup H^{j_e,\ell_e}\}$, where $Y^{j_e}_{\ell_e}$ is the set of ideological positions that are closer to the position of candidate j_e than to the position of candidate ℓ_e and vice versa for the set $Y^{\ell_e}_{j_e}$. Hence, for each candidate $j_e \in J^e$, Y^{j_e} is the intersection of the half spaces determined by the $q_e - 1$ hyperplanes $\{H^{j_e,\ell_e}\}_{\ell_e \in J^e \backslash j_e}$ (i.e., $Y^{j_e} = \cap_{\ell_e \in J^e \backslash j_e} Y^{j_e}_{\ell_e}$). Note that for all candidates $j_e \in J^e$ and all elections $e \in \{1, ..., m\}$, Y^{j_e} is non empty and convex.[15] Hence, each election $e \in \{1, ..., m\}$ implies a partition T^e of the ideological space Y into q_e convex regions, $\{Y^{j_e}\}_{j_e \in J^e}$, where each region Y^{j_e} is the ideological support of voting for candidate j^e in election e.[16] For each election $e \in \{1, ..., m\}$, the set $T^e = \{Y^{j_e}\}_{j_e \in J^e}$ defines what in computational and combinatorial geometry is called a *Voronoi tessellation* of R^k and each region Y^{j_e}, $j_e \in J^e$, is a k-dimensional *Voronoi polyhedron*.[17] Figure 2 illustrates an example of the Voronoi tessellation that corresponds to an election with 5 candidates, $\{a, b, c, d, e\}$, with positions $\{y_a, y_b, y_c, y_d, y_e\}$ in the two-dimensional ideological space $Y = R^2$, and introduces some useful terms.

It follows that the collection of the m tessellations implied by the m elections, $\{T^1, ..., T^m\}$, partitions the ideological space Y into $r_m \leq \Pi_{e=1}^m q_e$ convex regions, where each region is the ideological support of a distinct voting profile $v \in V^m$. Since in the general case where the number of candidates may vary across elections the number of possible, distinct voting profiles is $|V^m| = \Pi_{e=1}^m q_e$, it follows that the hypothesis that voters vote ideologically is falsifiable if and only if $r_m < \Pi_{e=1}^m q_e$.

We can now state our second set of results.

Proposition 3: *The hypothesis that voters vote ideologically in a single election with any number of candidates is not falsifiable regardless of the number of dimensions of the ideological space (i.e., for all $k \geq 1$).*

Proposition 3 generalizes Corollary 1. In order to illustrate the result consider the following example in the two-dimensional space, $Y = R^2$. Figure 3 depicts a situation where there is a single election $e = 1$, and the set of candidates in the election is $J^1 = \{a_1, b_1, c_1\}$. Given the candidates' ideological positions, y^{a_1}, y^{b_1}, and y^{c_1}, for each $j_1 \in J^1$, Y^{j_1} is the ideological support of voting for candidate j_1 in the election. Hence, it follows immediately that each voting profile $v \in V^1 = \{a_1, b_1, c_1\}$ is consistent with ideological voting. In fact, it should be clear that this result holds for any number of candidates, any distinct candidates'

[15] Also, note that Y^{j_e} only depends on $\left(y^{1_e}, ..., y^{q_e}\right)$.

[16] Note that $Y^{j_e} \cap Y^{\ell_e} = \varnothing$ for all $j_e, \ell_e \in J^e$, $j_e \neq \ell_e$, and $\cup_{j_e \in J^e}\{Y^{j_e} \cup_{\ell_e \in J^e \setminus j_e} H_{\ell_e}^{j_e}\} = Y$.

[17] For a comprehensive treatment of Voronoi tessellations and their properties, see, e.g., Okabe et al. (2000).

positions, and any number of dimensions of the ideological space.

Proposition 4: *In elections with any number of candidates, the hypothesis that voters vote ideologically is falsifiable if the number of elections m is larger than the number of dimensions of the ideological space k.*

Note that each tesselation T^e only depends on the positions of the candidates in election e, which are observable. This implies that we can calculate whether or not each voting profile is consistent with ideological voting. Hence, as with Proposition 1, the condition for falsifiability in Proposition 4 applies to each individual voter.

Since the hypothesis that voters vote ideologically is always falsifiable when the number of elections is greater than the number of dimensions of the ideological space, regardless of the number of candidates in each election, Proposition 4 extends the result of the first part of Proposition 1. However, for the case where $1 < m \leq k$, while the hypothesis is generically not falsifiable when each election has two candidates, when there are more than two candidates in at least one election, this is no longer the case. In fact, there exist configurations of candidates' positions, $\{y^j\}_{j \in \{\cup_{e=1}^m J^e\}}$, such that the hypothesis that voters vote ideologically is falsifiable, and configurations such that the hypothesis is not falsifiable.

In order to illustrate this result consider the following example in the two-dimensional space, $Y = R^2$. Suppose that in addition to election 1 depicted in Figure 3, there is a second election with two candidates (i.e., $e \in \{1, 2\}$, $q_1 = 3$ and $q_2 = 2$). The set of candidates in election 2 is $J^2 = \{a_2, b_2\}$, and the candidates' ideological positions are such that for each $j_2 \in J^2$, Y^{j_2} is the ideological support of voting for candidate j_2 in election 2. Figures 4 and 5 depict two possible situations that correspond to different configurations of the positions of the two candidates in election 2. As we can see from Figure 4, of the six possible voting profiles in elections 1 and 2, (a_1, a_2), (a_1, b_2), (b_1, a_2), (b_1, b_2), (c_1, a_2), and (c_1, b_2), only five have an ideological support in Y. In this example, there do not exist ideological positions such that the voting profile (a_1, b_2) is consistent with ideological voting (that is, there does not exist an ideological support for (a_1, b_2)). However, this is not the case in Figure 5, where there exists an ideological support for each of the six possible voting profiles in the two elections. Each one of the two cases illustrated in Figures 4 and 5 is robust to small perturbations of the candidates' positions, and is therefore generic. Similar examples can be constructed for any combination of the number of candidates in two or more elections as long as there is at least one election with more than two candidates.

When the ideological space is either one- or two-dimensional (i.e., $k \leq 2$), we can also characterize the upper bound on the number of voting profiles that are consistent with ideological voting (i.e., the number of regions r_m), as a function of the number of elections m and the number of candidates in each election, $q_1, ..., q_m$.[18]

Proposition 5: *In elections with any number of candidates, the maximum number of voting profiles that are consistent with ideological voting depends on the number of candidates in each election; if the ideological space is uni-dimensional, it is equal to*

$$\tau_1(q_1, ..., q_m) = 1 + \sum_{e=1}^{m} (q_e - 1)\,; \tag{4}$$

if the ideological space is two-dimensional, it is equal to

$$\tau_2(q_1, ..., q_m) = 1 + \sum_{e=1}^{m} \left[(q_e - 1) \left(1 + \sum_{f=e+1}^{m} (q_f - 1) \right) \right]. \tag{5}$$

Note that if there is only one election, $\tau_1(q_1) = \tau_2(q_1) = q_1$, and if there are two elections, $\tau_1(q_1, q_2) = q_1 + q_2 - 1 < \tau_2(q_1, q_2) = q_1 q_2$. Furthermore, when there are more than two elections, $\tau_1(q_1, ..., q_m) < \tau_2(q_1, ..., q_m) < \Pi_{e=1}^{m} q_e$, and the number of voting profiles that are not consistent with ideological voting increases both with the number of elections and with the number of candidates in an election. For example, if the ideological space is two-dimensional, then if there are three elections and three candidates in each election, $\tau_2(3, 3, 3) = 19$ (i.e., at most 19 out of the 27 possible voting profiles are consistent with ideological voting); if there are four elections each with three candidates, $\tau_2(3, 3, 3, 3) = 33$ (i.e., at most 33 out of the 81 possible voting profiles are consistent with ideological voting); and if there are three elections, two of which have three candidates and one with four candidates, $\tau_2(3, 3, 4) = 24$ (i.e., at most 24 out of 36 possible voting profiles are consistent with ideological voting).[19]

[18] The issue we are considering corresponds to the problem of counting the number of regions in arrangements of Voronoi tessellations in k-dimensional Euclidean space. This problem has not yet been studied in computational and combinatorial geometry, and there are no known results in the literature.

[19] As in the case of two-candidate elections, given the configuration of candidates' positions, we can also determine which profiles are not consistent with ideological voting.

3 Extensions

In this section, we consider several extensions of the basic framework of Section 2 and analyze the robustness of our results on the falsifiability of the ideological voting hypothesis. We begin by generalizing the specification of voters' preferences. We then consider an environment where electoral candidates also differ with respect to (non-spatial) personal characteristics that are valued by the voters, and one where voters are uncertain about the candidates' positions.

3.1 Voters' preferences

The utility specification in the baseline model (equation (1)) assumes that voters evaluate the relative distance of candidates' positions from their bliss point according to the (simple) Euclidean distance. This implies that all the dimensions of the ideological space are equally important (or salient) in all elections for all voters. A more general specification is that the preferences of a generic voter i over candidates in election e are summarized by the utility function

$$U_e^i(j_e) = u_e^i\left(d^{W_e^i}\left(y^i, y^{j_e}\right)\right), \tag{6}$$

where $u_e^i(\cdot)$ is a decreasing function which may differ across voters and elections, and $d^{W_e^i}(\cdot,\cdot) \geq 0$ denotes the weighted Euclidean distance with weighting matrix W_e^i which may also differ across voters and elections (i.e., for any two points $x, z \in R^k$, $d^W(x,z) = \sqrt{(x-z)' W (x-z)}$, where W is a $k \times k$, symmetric and positive definite matrix).[20] According to the spatial theory of voting, the main diagonal elements of the weighting matrix W_e^i (*salience terms*) measure the relative importance of the ideological dimensions to voter i in election e, while the off-diagonal elements (*interaction terms*) describe the way in which i makes trade-offs between them (see, e.g., Hinich and Munger (1997)). As before, we impose no additional restrictions on the $u_e^i(\cdot)$ functions, which are therefore left unspecified.

Given these preferences (that are characterized by indifference sets that are ellipsoids in the k-dimensional Euclidean space, centered around the bliss point y^i), a voter i (strictly) prefers candidate j_e to candidate ℓ_e in election e if $d^{W_e^i}(y^i, y^{j_e}) < d^{W_e^i}(y^i, y^{\ell_e})$. Hence, i

[20] If the weighting matrix is equal to the identity matrix (i.e., $W = I$), the weighted Euclidean distance reduces to the (simple) Euclidean distance.

votes ideologically in election e if she votes for the candidate whose position is closest to her bliss point according to the distance $d^{W_i^e}$ (i.e., $d^{W_i^e}(y^i, y^{j_e}) < d^{W_i^e}(y^i, y^{\ell_e})$ for all $\ell_e \in J^e$, $\ell_e \neq j_e \in J^e$, implies that $v_e^i = j_e$).

For each election $e \in \{1, ..., m\}$ with any number of candidates $q_e \in \{2, ..., \overline{q}\}$, and for any weighting matrix W, let

$$Y^{j_e}(W) = \{y \in Y : d^W\left(y, y^{j_e}\right) < d^W(y, y^{\ell_e}), \forall \ell_e \in J^e, \ell_e \neq j_e\} \tag{7}$$

be the set of points in the ideological space Y that are closer to y^{j_e} than to the position of any other candidate in the election according to the distance d^W. Given the definition of d^W, similar to the case of the simple Euclidean distance considered in Section 2 above, it follows that for any given weighting matrix W, for each pair of candidates in election e, $j_e, \ell_e \in J^e$, there exists a non-zero vector $\lambda^{j_e,\ell_e}(W) = \left(\lambda_1^{j_e,\ell_e}(W), ..., \lambda_k^{j_e,\ell_e}(W)\right) \in R^k$ and a scalar $\mu_{j_e,\ell_e}(W) \in R$ such that the set of points in the ideological space Y that are equidistant from y^{j_e} and y^{ℓ_e} according to d^W is a hyperplane $H^{j_e,\ell_e}(W) = \left\{y \in Y : \lambda^{j_e,\ell_e}(W)\, y' = \mu_{j_e,\ell_e}(W)\right\}$. This hyperplane partitions the ideological space Y into two regions (or half spaces), $Y_{\ell_e}^{j_e}(W)$ and $Y_{j_e}^{\ell_e}(W) = Y \backslash \{Y_{\ell_e}^{j_e}(W) \cup H^{j_e,\ell_e}(W)\}$, where $Y_{\ell_e}^{j_e}(W)$ is the set of ideological positions that are closer to the position of candidate j_e than to the position of candidate ℓ_e according to d^W (vice versa for $Y_{j_e}^{\ell_e}(W)$). Hence, for each candidate $j_e \in J^e$, for any given weighting matrix W, $Y^{j_e}(W)$ is an intersection of the half spaces determined by the $q_e - 1$ hyperplanes $\{H^{j_e,\ell_e}(W)\}_{\ell_e \in J^e \backslash j_e}$ (i.e., $Y^{j_e}(W) = \cap_{\ell_e \in J^e \backslash j_e} Y_{\ell_e}^{j_e}(W)$), which is non empty and convex. Hence, for any W, each election $e \in \{1, ..., m\}$ implies a Voronoi tessellation $T^e(W)$ of the ideological space Y into q_e convex regions, $\{Y^{j_e}(W)\}_{j_e \in J^e}$, where each region $Y^{j_e}(W)$ is the ideological support of voting for candidate j^e in election e based on the distance d^W.

It follows that if the weighting matrices W_e^i's are allowed to differ across elections and across voters in an unrestricted fashion, the hypothesis that voters vote ideologically is falsifiable only if these matrices are known for all voters and all elections. In fact, if a researcher observes the W_i^e's, then Propositions 1-5 still apply. The only difference in the analysis is that from the point of view of the researcher (who still does not observe the voters' ideological positions y^i's), for each election, each voter is described by a different Voronoi tessellation. If, on the other hand, the weighting matrices W_e^i's are not known (as it is reasonable to assume since they are part of the voters' preferences), then the hypothesis that voters vote ideologically is not falsifiable, since it is always possible to find a weighting matrix for each election and a voter's ideological position such that any voting profile is consistent with ideological voting.

There exist, however, restrictions on the weighting matrices W_e^i's such that the ideological voting hypothesis is still falsifiable even when these matrices are not known. In particular, if the weighting matrices are allowed to differ across voters but are constant across elections (i.e., $W_e^i = W^i$ for all $e = 1, ..., m$), then Propositions 1-5 still apply. In fact, whether or not an ideological support exists for any given voting profile is based on the solution of a system of linear inequalities (see the proofs of Propositions 1-5 in the Appendix). Hence, if **all** the inequalities are multiplied by the **same** positive definite matrix the result about the existence or non existence of an ideological support for any voting profile remains the same and is therefore independent of the weighting matrix.

To illustrate this point, consider a generic voter with weighting matrix W. Since W is a symmetric, positive definite matrix, using the Cholesky decomposition we have that $W = LL'$, where L is a lower triangular matrix with strictly positive diagonal elements. Hence, given the definition in (7), the ideological support for voting for a generic candidate j^e in election e based on the distance d^W is equal to

$$\begin{aligned} Y^{j_e}(W) &= \left\{ y \in Y : \sqrt{(y - y^{j_e}) W (y - y^{j_e})'} < \sqrt{(y - y^{\ell_e}) W (y - y^{\ell_e})'}, \forall \ell_e \in J^e, \ell_e \neq j_e \right\} \\ &= \left\{ y \in Y : \left(y - y^{j_e}\right) LL' \left(y - y^{j_e}\right)' < \left(y - y^{\ell_e}\right) LL' \left(y - y^{\ell_e}\right)', \forall \ell_e \in J^e, \ell_e \neq j_e \right\} \\ &= \left\{ y \in Y : \left(yL - y^{j_e}L\right) \left(yL - y^{j_e}L\right)' < \left(yL - y^{\ell_e}L\right) \left(yL - y^{\ell_e}L\right)', \forall \ell_e \in J^e, \ell_e \neq j_e \right\}. \end{aligned}$$

If we let $\widetilde{y} = yL$, we have that the ideological support for voting for a generic candidate j^e in election e in the transformed space is

$$\begin{aligned} \widetilde{Y}^{j_e} &= \left\{ \widetilde{y} \in \widetilde{Y} : \left(\widetilde{y} - \widetilde{y}^{j_e}\right) \left(\widetilde{y} - \widetilde{y}^{j_e}\right)' < \left(\widetilde{y} - \widetilde{y}^{\ell_e}\right) \left(\widetilde{y} - \widetilde{y}^{\ell_e}\right)', \forall \ell_e \in J^e, \ell_e \neq j_e \right\} \\ &= \left\{ \widetilde{y} \in \widetilde{Y} : d\left(\widetilde{y}, \widetilde{y}^{j_e}\right) < d(\widetilde{y}, \widetilde{y}^{\ell_e}), \forall \ell_e \in J^e, \ell_e \neq j_e \right\}, \end{aligned}$$

where d is the (simple) Euclidean distance.[21] Since the weighting matrix for a voter is the same in all the elections, the same linear transformation applies to all the Voronoi

[21] Note that since L is a lower triangular matrix, $\widetilde{Y} = Y = R^k$.

tessellations that correspond to the m elections faced by the same voter. Furthermore, for any pair of candidates j_e and j_f in two different elections e and f, whether $Y^{j_e}(W) \cap Y^{j_f}(W) = \varnothing$ or $Y^{j_e}(W) \cap Y^{j_f}(W) \neq \varnothing$ does not depend on W.[22] It follows that the analysis of Section 2 extends directly to the case where the preferences over candidates in election e of a generic voter i are described by the utility function $U_e^i(j_e) = u_e^i\left(d^{W^i}(y^i, y^{j_e})\right)$ and the individual-specific weighting matrices W^i's are not known.

Another restriction on the weighting matrices W_e^i's, which allows us to obtain some useful (though weaker) results on the falsifiability of the ideological voting hypothesis when these matrices are not known, is to impose that they are constant across voters although they may differ across elections (i.e., $W_e^i = W_e$ for all $i \in N$). It should be clear from our previous discussion that most of the analysis of Section 2 also applies to this case. For any given W_e, each election implies a Voronoi tessellation $T(W_e)$ and if the number of elections m is larger than the number of dimensions of the ideological space k then for any set of weighting matrices $\{W_1, ..., W_m\}$ the collection of the m tessellations, $\{T^1(W_1), ..., T^m(W_m)\}$, always partitions the ideological space Y into fewer regions than the number of distinct voting profiles. However, if the weighting matrices $\{W_1, ..., W_m\}$ are not known we can no longer determine which profiles do not have a sincere support, since, unlike in the previous case, these profiles now depend on the weighting matrices.

To illustrate this point, consider the following example in R^2. Figures 6 depicts a situation where there are three elections $e = 1, 2, 3$, the set of candidates in each election is $J^e = \{a_e, b_e\}$, and $W_1 = W_2 = W_3 = I$. Figure 7 depicts the same situation except that $W_1 = \begin{bmatrix} 10 & 0 \\ 0 & 1 \end{bmatrix}$. As we can see from these figures, in both cases $H^1(W_1)$, $H^2(W_2)$ and $H^3(W_3)$ partition the ideological space in only seven regions, while there are eight possible distinct voting profiles. However, while in Figure 6 there does not exist an ideological support for (a_1, b_2, a_3), in Figure 7 the voting profile for which there does not exist an ideological support

[22] If it exists, the intersection of polyhedral convex sets is also a polyhedral convex set. Theorem 19.3 in Rockafellar (1970, p. 174) states that for any linear transformation A from R^n to R^m, AC is a polyhedral convex set in R^m for each polyhedral convex set C in R^n, and $A^{-1}D$ is a polyhedral convex set in R^n for each polyhedral convex set D in R^m.

is (b_1, a_2, b_3).

It follows that for the case where the preferences over candidates in election e of a generic voter i are described by the utility function $U_e^i(j_e) = u_e^i\left(d^{W_e}(y^i, y^{j_e})\right)$ and the election-specific weighting matrices W_e's are not known Propositions 2, 3 and 5 still hold. However, since in this environment we can only determine the maximum number of voting profiles that are consistent with ideological voting, but not which profiles are inconsistent with ideological voting, the hypothesis that each individual voter votes ideologically is not falsifiable. Instead, the only hypothesis that can be potentially falsified is that **all** voters vote ideologically, and the following results (which are weaker versions of Propositions 1 and 4) apply.[23]

Proposition 6: *In two-candidate elections, the hypothesis that **all** voters vote ideologically is falsifiable if the number of elections m is larger than the number of dimensions of the ideological space k. If $m \leq k$, the hypothesis is generically not falsifiable.*

Proposition 7: *In elections with any number of candidates, the hypothesis that **all** voters vote ideologically is falsifiable if the number of elections m is larger than the number of dimensions of the ideological space k.*

Note that simply observing more distinct voting profiles in the data than the maximum number of voting profiles that are consistent with ideological voting would prove the hypothesis that all voters vote ideologically false, but would give us no indication of which voting behavior is inconsistent with ideological voting or of the number of voters whose behavior is inconsistent with ideological voting.

3.2 Candidates' characteristics

Another important extension of our framework is to consider the possibility that electoral candidates differ not only with respect to their positions in the ideological space, but also with respect to (non-spatial) personal characteristics, such as "valence" or "charisma" which are valued equally by all voters. In particular, suppose that each candidate $j \in \{\cup_{e=1}^m J^e\}$ is characterized by a distinct position in the ideological space, $y^j \in Y$, and by a valence

[23] The proofs of Propositions 6 and 7 are straightforward and are therefore omitted. Note that here we are implicitly considering a situation where the data on voters' behavior contain at least as many observations as the number of possible distinct voting profiles. Otherwise, the hypothesis cannot be falsified.

parameter, $\theta^j \in R$, which are known to the voters, and that the preferences of a generic voter i over candidates in election e are summarized by the utility function

$$U_e^i(j_e) = -\left(d\left(y^i, y^{j_e}\right)\right)^2 + \theta^{j_e}. \tag{8}$$

This linear-quadratic specification is widely used in the political economy literature (see, e.g., Enelow and Hinich (1984)).

According to these preferences, the set of points in the ideological space such that a voter with ideological position in this set is indifferent between voting for a candidate with position y^{j_e} and valence θ^{j_e} or voting for a candidate with position y^{ℓ_e} and valence θ^{ℓ_e} is still a hyperplane which partitions the ideological space into two regions.[24] All voters with ideological positions in one region strictly prefer to vote for one candidate, and all voters with ideological positions in the other region strictly prefer to vote for the other candidate. This implies that the geometric representation of elections based on Voronoi tessellations which we used throughout our analysis also applies for the preferences given in (8).[25] It follows that Propositions 1-5 also extend to this case, in the sense that they provide general conditions for the falsifiability of the hypothesis that individuals "vote their preferences" as specified in (8), for each individual voter. This conclusion clearly hinges on the availability of data on candidates' valence. If, on the other hand, the researcher does not observe the valence parameters θ^j's for all $j \in \{\cup_{e=1}^m J^e\}$, the situation is analogous to the one in Section 3.1 where the weighting matrices in the voters' preferences differ across elections but not across voters, and the researcher does not know them. In this case, Propositions 2, 3, 5, 6 and 7 still hold, again in the sense that they provide general conditions for the falsifiability of the hypothesis that all individuals "vote their preferences" as specified in (8).

The analysis can also be extended to an environment where voters are uncertain about the candidates' positions. Suppose that in each election $e = 1, ..., m$, the voters' (common)

[24] Note that this result does not hold for other decreasing functions of the weighted Euclidean distance, since for general $u_e^i(\cdot)$ functions the indifference condition between any pair of candidates would not characterize a hyperplane in the ideological space.

[25] The literature on spatial tessellations refers to Voronoi tessellations under this alternative metric as "power diagrams" (see, e.g., Okabe et al. (2000, p. 128)).

perception of any candidate j_e's position, $j_e \in J^e$, on the r^{th} ideological dimension, $r = 1, ..., k$, is $\widehat{y}_r^{j_e} = y_r^{j_e} + \varepsilon_r^{j_e}$, where $y_r^{j_e}$ is the true position and $\varepsilon_r^{j_e}$ is an estimation (or perception) error with mean zero and variance $(\sigma_r^{j_e})^2$, and the preferences of a generic voter i over candidates in election e are summarized by the utility function

$$U_e^i(j_e) = -\left(d\left(y^i, y^{j_e}\right)\right)^2. \tag{9}$$

This environment has often been studied in the voting literature (see, e.g., Alvarez (1998)). In this framework, voter i casts her ballot in election e in favor of the candidate associated with the highest expected utility, where the expected utility of voting for candidate j_e is equal to

$$\begin{aligned} E[U_e^i(j_e)\,|\widehat{y}^{j_e}] &= -E\left[\sum_{r=1}^{k}\left(\widehat{y}_r^{j_e} - \varepsilon_r^{j_e} - y_r^i\right)^2 |\widehat{y}^{j_e}\right] = -\sum_{r=1}^{k}\left(\widehat{y}_r^{j_e} - y_r^i\right)^2 - \sum_{r=1}^{k}(\sigma_r^{j_e})^2 \\ &= -\left(d\left(y^i, \widehat{y}^{j_e}\right)\right)^2 + \rho^{j_e}, \end{aligned}$$

where $\rho^{j_e} = -\sum_{r=1}^{k}(\sigma_r^{j_e})^2$ is analogous to a negative valence parameter. It follows that if the ρ^{j_e}'s differ across the candidates in election e (i.e., the estimates of the positions of some candidates are more precise than those of others), then the analysis is identical to the one of the previous case where voters know the true candidates' positions and candidates also differ with respect to their valence. If, on the other hand, the distribution of perception errors is the same for all the candidates in the same election (i.e., $\rho^{j_e} = \rho^e$ for all $j_e \in J^e$), then this environment is equivalent to the baseline, and all the results of Section 2 apply directly.

4 An application: evidence from U.S. national elections

In the previous two sections, we have characterized general conditions under which the hypothesis that voters vote ideologically is in principle falsifiable. In this section, we provide an application of the theoretical framework, and illustrate how we can use existing data to assess empirically the extent to which, in environments where the hypothesis is falsifiable, the observed behavior of voters is not consistent with ideological voting.

Our goal is to analyze an individual-level data set on how a sample of individuals vote in a number of simultaneous elections, determine whether the behavior of each individual is consistent with ideological voting, and obtain an estimate of the lower bound of the fraction of voters who do not vote ideologically. Hence, in our illustrative application we assume that the preferences over candidates in election e of a generic voter i are described by the utility function $U_e^i(j_e) = u_e^i\left(d^{W^i}(y^i, y^{j_e})\right)$, and without further loss of generality let $W^i = I$ for all voters.

We focus on national elections in the United States between 1970 and 2000. It is important to stress, however, that the same analysis can also be replicated for other countries, or other types of elections, or other time periods for which there are available data. Since, as shown in Section 2, this empirical analysis is meaningful only if we have access to data on how individuals vote in multiple elections, we consider the situation faced by U.S. voters in a presidential election year (henceforth, an election year), where presidential and congressional elections occur simultaneously.[26] In any election year, U.S. voters elect the President and, at the same time, each voter faces an election that determines the representative of his or her district in the House of Representatives.[27] Some voters also face a Senate election in their state.[28] Each election is typically contested by two candidates belonging to the Democratic and the Republican party, respectively.[29]

Since the set of candidates competing for a seat in the House of Representatives is different

[26] In the United States, citizens are called to participate in national elections to elect the President and the members of Congress. While congressional elections occur every two years, the time between presidential elections is four years. We refer to an election year where both presidential and congressional elections occur simultaneously as a presidential election year.

[27] Citizens who reside in the District of Columbia do not elect a House representative but only a congressional delegate.

[28] Senate elections are staggered, and in any given election year, there are elections to the U.S. Senate in approximately one third of the states. In addition, many voters also face other local elections and referenda. Since data on how individuals vote in these elections is typically not available, we do not consider them here.

[29] In some elections a single candidate runs uncontested. Occasionally, a third, independent candidate also runs. However, data on the positions of independent candidates are not available and it is not clear how to assign an ideological position to a third party's candidate. In fact, the procedure we describe below to deal with missing positions of Democratic or Republican candidates cannot be used in this case due to the extremely limited number of candidates who are elected to Congress from third parties. In addition, the presence of such candidates, although feasible, would complicate the calculations. Given the illustrative nature of the exercise, we therefore restrict attention to Democratic and Republican candidates only.

in each congressional district, our unit of analysis is the district. In a generic election year t, a voter i residing in district $h \in \{1, ..., 435\}$ and state $s \in \{1, ..., 50\}$ faces a House election. Let J_t^h denote the set of candidates competing in the House election in congressional district h at time t. Like all other voters in the nation, voter i also faces a presidential election, and let J_t^p denote the set of presidential candidates at time t. If a Senate seat is up for election in state s at time t, then voter i also faces a Senate election, where the set of candidates is J_t^s. Hence, in any given district $h = 1, ..., 435$ in state $s = 1, ..., 50$, a voter i is facing either two or three simultaneous elections, and $v^i = (v_p^i, v_h^i)$ or $v^i = (v_p^i, v_h^i, v_s^i)$ denotes i's voting profile, where $v_e^i \in J_t^e$ indicates how voter i votes in election $e = p, h, s$.[30] For example, a voter facing three elections may vote for the Democratic candidate in each of the elections, or vote for the Democratic presidential candidate and the Republican candidates in the House and Senate elections, and so on.

The data we use for our empirical analysis come from two sources. The first source is the American National Election Studies (NES), which for each election year contains individual voting decisions in presidential and congressional elections of a nationally representative sample of the voting age population. In addition, the NES contains information on the congressional district where each individual resides, the identity of the Democratic and the Republican candidate competing for election in his or her congressional district, and, in the event that a Senate election is also occurring in his or her state, the identity of the candidates competing in the Senate race.[31]

The second source of data is the Poole and Rosenthal NOMINATE Common Space Scores. Using data on roll call voting by each member of Congress and support to roll call votes by each President, Poole and Rosenthal developed a methodology to estimate the

[30] Recall that here we are ignoring abstention, and only consider the way in which voters vote. For a recent study of the empirical implications of alternative models of voter turnout, see, e.g., Coate and Conlin (2004). In Degan and Merlo (2007), we structurally estimate a model of participation and voting in U.S. national elections.

[31] The NES is available on-line at http://www.umich.edu/~nes. For thorough discussions of potential limitations of the survey data in the NES see, e.g., Anderson and Silver (1986) and Wright (1993). Note, however, that the NES represents the best and most widely used source of individual-level data on electoral participation and voting in the U.S.

positions of all politicians who ever served either as Presidents or members of Congress, in a common two-dimensional ideological space (see, e.g., Poole (1998) and Poole and Rosenthal (1997, 2001)). These estimates, which are comparable across politicians and across time, are contained in their NOMINATE Common Space Scores data set.[32]

We restrict attention to the period 1970-2000, and consider seven election years: 1972, 1976, 1980, 1984, 1988, 1996, and 2000.[33] For each year, Table 1 contains the number of observations in the NES sample of individuals who reported how they voted in the presidential and House elections, as well as in the sub-sample of individuals who were also facing a senatorial election in their state, and reported how they voted in the presidential, House, and Senate elections.[34]

For each of the seven years we consider, we match each voter in the NES sample with the positions of the presidential candidates, as well as (when available) the positions of the House candidates running in his or her congressional district and the positions of the Senate candidates running in his or her state, if applicable. Consistent with the general environment described in Section 2, we assume that the voters know the positions of all candidates in all the elections they face. These positions, however, may or may not be observable to the econometrician. The NOMINATE datasets, for example, only contain estimates of the positions of politicians who have been elected to Congress. Hence, the positions we observe are those of all the incumbents and of the challengers who where either in Congress at some previous date or were eventually elected to Congress by 2006 (the last year for which the NOMINATE Scores are available).

[32] This data set is also available on-line at http://voteview.com. For a discussion of potential limitations of the methodology proposed by Poole and Rosenthal see, e.g., Heckman and Snyder (1997). For a comparison of alternative estimation procedures see Clinton, Jackman and Rivers (2004). Note, however, that none of the other procedures has been used to generate a comprehensive data set similar to the one by Poole and Rosenthal.

[33] The NES data for the election year 1992 contains a mistake in the variable that identifies the congressional district of residence of the individuals in the sample (see ftp://ftp.nes.isr.umich.edu/ftp/nes/studypages/1992prepost/int1992.txt). Hence, it cannot be used for the purpose of our analysis.

[34] Obviously, we only consider congressional elections that are contested, and observations for which the voters' district and state of residence are not missing.

For all presidential candidates, and all congressional candidates for whom we have an entry in the Poole and Rosenthal data set, we assume that their position is given by their NOMINATE score.[35] To determine the positions of all other congressional candidates for whom a NOMINATE score is not available we need to introduce further assumptions. The procedure we use is similar in spirit to the one used by Blow, Browning, and Crawford (2008) in the context of the estimation of a characteristics model. When faced with the problem of assigning a value for unobserved prices, they chose to treat missing prices as unknown parameters and search for values so that the constructed data satisfy the conditions of the model. While in their setting there is no a priori restriction on prices apart from non-negativity, in the application we consider we believe it is appropriate to let the possible values taken by the policy positions of candidates in Congress to differ by Chamber, party and region. Therefore, we assume that the position of a Democratic (Republican) candidate for the House (Senate) is restricted to be one of the NOMINATE scores of Democratic (Republican) members of the House (Senate) in the same election year, in the U.S. region where the candidate is competing.[36]

For the cases where we observe the positions of all the candidates competing in the elections faced by the voters residing in a district, we can directly assess whether each observed individual voting profile in those districts is consistent with ideological voting. For each case where we do not observe the position(s) of some candidate(s) competing in the elections faced by the voters residing in a district, we use the following procedure. If the

[35] Note that Michael Dukakis, the Democratic presidential candidate in 1988, who at the time was the governor of Massachusetts, is the only presidential candidate during the period we consider for whom there is no entry in the Poole and Rosenthal data set. Following Gaines and Segal (1988), we approximate Dukakis' position in the ideological space with that of the Democratic Massachusetts senator in 1988 (Ted Kennedy).

[36] We consider four different regions: Northeast, South, Midwest and West. Alternative ways of constructing the empirical distributions are also possible. Note, however, that it would be unfeasible to characterize a separate empirical distribution for each party in each state (let alone district) in each year, since the number of representatives or senators of either party in each state in any given year is too small. Alternatively, since the first dimension of the NOMINATE score is the liberal-conservative scale we could have only imposed that in any given election the Democratic candidate has a position on the first dimension of the ideological space to the left of the Republican.

district belongs to a state without a Senate election or with a Senate election in which both candidates positions are observed, we take as measure of the missing position of the House candidate the "best" position. That is, among all the admissible positions for the candidate we take the one that leads to the highest number of profiles that are consistent with ideological voting. If the district belongs to a state with a Senate election where we do not observe the position of one of the candidates in that election, say candidate j_s, we repeat the above procedure for all the districts in state s, for each possible value of candidate j_s's position y^{j_s}. We then take as measure of y^{j_s} the position that leads to the highest number of voting profiles that are consistent with ideological voting in the districts belonging to that state. Hence, our procedure provides an estimate of the lower bound of the fraction of voters whose observed voting behavior is not consistent with ideological voting.[37]

In order to perform these calculations, we need to specify the number of elections m we consider, and the number of dimensions of the ideological space k (where it has to be the case that $m > k$). We begin by ignoring Senate elections, and evaluate the extent to which the observed voting behavior of all individuals in the NES samples who voted in the presidential and House elections is consistent with ideological voting when we restrict attention to a unidimensional liberal-conservative ideological space.[38] We then take into consideration that while some voters only face the presidential and a House election, some voters also face a Senate election, and evaluate the extent to which the observed behavior of voters in presidential and congressional (House or House and Senate) elections is consistent with ideological voting, while still maintaining the assumption of a unidimensional ideological space. Finally, we restrict attention to the sub-samples of individuals in the NES who voted in three elections (presidential, House, and Senate), and perform our calculations for the

[37] An alternative procedure would be to assume that a candidate's position is a draw from the empirical distribution of positions of legislators of the same party, for the same office, for the same year and region. In that case, one would have to calculate the probability that a voting pattern is consistent with ideological voting by integrating over the relevant distribution of positions of the candidate.

[38] In particular, we only consider the first dimension of the Poole and Rosenthal NOMINATE scores. Note that according to Poole and Rosenthal (1997; p.5), "from the late 1970s onward, roll call voting became largely a matter of positioning on a single, liberal-conservative dimension."

case where the ideological space is two-dimensional.[39] Table 2 contains our results, where each column corresponds to one of the three scenarios.

As we can see from the first column in Table 2, ideological voting is consistent with most of the individual-level observations on voting behavior in presidential and House elections in the data. Its worst "failure" amounts to the inability of accounting for 5.1% of the observations in 1980. Overall, by combining all the samples in the seven election years we consider, we have that only 3.3% of the observed individual voting profiles are not consistent with ideological voting.[40]

Columns 2 and 3 in Table 2 help us to assess the robustness of these findings with respect to the choice of the number of elections and the number of dimensions of the ideological space. From the analysis in Section 2, we know that given the number of dimensions of the ideological space, an increase in the number of elections increases the number of voting profiles that cannot be rationalized by a voter voting ideologically in these elections. This increases the extent to which the hypothesis that voters vote ideologically may fail to explain the data. Consistent with this result, we find that increasing the number of elections while maintaining the dimensionality of the ideological space fixed, worsens the empirical performance of the ideological-voting hypothesis (Column 2). Nevertheless, under the maintained assumption that the ideological space is unidimensional, over 92% of the observed individual voting profiles in presidential and congressional (House or House and Senate) elections between 1970 and 2000 are still consistent with ideological voting. Moreover, in a two-dimensional ideological space, the hypothesis that voters vote ideologically in presidential and congressional (House and Senate) elections only fails to account for less than 1% of the observations in

[39] Recall that the hypothesis that voters vote sincerely in presidential and House elections only is not falsifiable if $k = 2$.

[40] Note that "errors" of this magnitude would be within the margin of tolerance if one were to allow for sampling (or measurement) error. One potential source of measurement error in the data, for example, is that individuals in the NES samples may be assigned to the wrong congressional district (a possibility that arises whenever the location where an individual is interviewed does not correspond to his or her permanent residence). Another source of measurement error consists of treating the NOMINATE scores (which are point estimates of legislators' positions) as the true positions. When standard errors of these estimates are available, it would be possible to bootstrap standard errors associated with the results reported in Table 2.

each of the seven election years we consider (Column 3). All the caveats we pointed out before not withstanding, we conclude that the observed behavior of voters in U.S. national elections can be interpreted as being mostly consistent with ideological voting.

5 Concluding Remarks

Do people vote based on ideological considerations? In this paper, we have provided general conditions under which the hypothesis that voters vote ideologically can be falsified. A key result of our analysis is that, when voters' ideological positions are not observed, falsifiability of the ideological-voting hypothesis hinges on the availability of data on how individuals vote in multiple elections. Furthermore, the number of elections has to be greater than the number of dimensions of the ideological space. Given the dimensionality of the ideological space, the larger the number of elections, the larger the number of voting profiles that are not consistent with a voter voting ideologically in these elections. Hence, the larger the number of elections for which there are data on how individuals vote in each election, the higher the possibility of "rejecting" the ideological voting hypothesis.

To conclude, it should be stressed that, as noted for example by Sproumont (2000) and Hausman (1992), some sort of stability of preferences is necessary for the analysis of the empirical content of any theory, which requires fixed preferences over changing choices. In our context, such stability amounts to assuming that voters' bliss points are constant across elections.

Appendix

Proof of Proposition 1: Let $q_e = 2$ for all $e \in \{1, ..., m\}$. We first show that for all $k \geq 1$ and $m \geq 1$, if $Y = R^k$ and $m \leq k$, then generically $r_m = 2^m$. The reason why the result is true is that if $m \leq k$ then the intersection of the m hyperplanes $H^1, ..., H^m$ defined in (2) is generically non-empty. Hence, each hyperplane H^e, $e \in \{1, ..., m\}$, partitions each of the 2^{m-1} regions in R^k given by the intersections of the half spaces determined by the other $m - 1$ hyperplanes in two.

Formally, the hyperplanes $H^1, ..., H^m$ in R^k define a system of m linear equations in k variables

$$\Lambda y' = \mu, \tag{10}$$

where

$$\Lambda = \begin{bmatrix} \lambda_1^1 & \cdots & \lambda_k^1 \\ \vdots & & \vdots \\ \lambda_1^m & \cdots & \lambda_k^m \end{bmatrix},$$

and

$$\mu = \begin{bmatrix} \mu_1 \\ \vdots \\ \mu_m \end{bmatrix}.$$

Since generically the vectors $\lambda^e = (\lambda_1^e, ..., \lambda_k^e)$, $e = 1, ..., m$, are linearly independent, the rank of Λ is equal to m. Hence, for $m \leq k$ a solution to the system of linear equations (10) exists and the dimension of the space of solutions is $k - m$. In particular, when $m = k$ the unique solution to (10) is a point in R^k where all the hyperplanes $H^1, ..., H^k$ intersect.

Next, we show that for all $k \geq 1$, if $Y = R^k$ and $m > k$, then $r_m < 2^m$. Given the m hyperplanes $H^1, ..., H^m$ defined in (2), consider an arbitrary collection containing k of these hyperplanes. From the previous part of the proof we know that generically a collection of k hyperplanes partitions R^k into 2^k regions. Since each hyperplane can at most partition each region in two, in order to prove that $r_m < 2^m$ it is enough to show that adding another hyperplane to the collection can never partition R^k into 2^{k+1} regions. In other words, an additional hyperplane cannot partition all of the 2^k regions given by the intersections of the half spaces determined by k other hyperplanes.

Without loss of generality, consider the collection of k hyperplanes, $H^1, ..., H^k$. Let $y^* \in R^k$ denote the intersection of $H^1, ..., H^k$, that is

$$y^{*\prime} = \Lambda_k^{-1} \mu^k$$

is the unique solution to

$$\Lambda_k y' = \mu^k,$$

where

$$\Lambda^k = \begin{bmatrix} \lambda_1^1 & \cdots & \lambda_k^1 \\ \vdots & & \vdots \\ \lambda_1^k & \cdots & \lambda_k^k \end{bmatrix},$$

and

$$\mu^k = \begin{bmatrix} \mu_1 \\ \vdots \\ \mu_k \end{bmatrix}.$$

Consider the linear transformation

$$x' = \Lambda_k y' - \mu^k \tag{11}$$

that maps Y into X (where $Y = R^k$ and $X = R^k$). This transformation maps each hyperplane H^j in Y, $j = 1, ..., k$, into the j^{th} coordinate of X, and y^* into the origin of X. Furthermore, it maps each hyperplane H^h in Y, $h = k+1, ..., m$, into a hyperplane Z^h in X, $Z^h = \{x \in X : \beta^h x' = \gamma_h\}$, where $\beta^h = \lambda^h \Lambda_k^{-1}$ and $\gamma_h = \mu_h - \lambda^h \Lambda_k^{-1} \mu_k$. Without loss of generality, suppose that $\beta^{k+1} > 0$ and $\gamma_{k+1} > 0$. Then, for all $x < 0$, $\beta^{k+1} x' < \gamma_{k+1}$, which implies that the hyperplane Z^{k+1} does not partition the negative orthant of X. This implies that the hyperplane H^{k+1} does not partition the region in Y that corresponds to the negative orthant of X under the linear transformation (11). It follows that for any collection of $k < m$ hyperplanes, there always exists at least a region in Y given by some intersection of the half spaces determined by these hyperplanes that is not partitioned by some other hyperplane. ■

Proof of Proposition 2: Proposition 2 follows from a general result in combinatorial geometry on the maximum number of regions in arrangements of hyperplanes in k-dimensional

Euclidean space. The proof we report here is an adaptation of a proof by Edelsbrunner (1987; pp. 8-10).

Let $H = \{H^1, ..., H^m\}$ denote the collection of the m hyperplanes defined in (2), which defines a partition of R^k into connected objects of dimensions 0 through k, called an arrangement $A(H)$ of H. We use the term vertex to denote a 0-dimensional object in $A(H)$ (that is, a point generated by the intersection of k hyperplanes), and refer to an l-dimensional object in $A(H)$, $1 \leq l \leq k$, as an l-region. We are interested in characterizing the maximum number of k-regions in an arrangement $A(H)$, $\rho(m, k)$.

For $m \leq k$ the first part of the proof of Proposition 1 implies that

$$\rho(m,k) = 2^m = \sum_{t=0}^{m} \binom{m}{t} = \sum_{t=0}^{k} \binom{m}{t}.$$

Hence, we only need to prove the case $m > k$. The proof is by induction on the number of dimensions of the ideological space, k. The assertion is trivial in one dimension, where m points—that is, 0-dimensional hyperplanes—partition R into at most $m+1$ intervals—that is, 1-regions (where the "at most" qualifier follows from the fact that although the positions of all candidates are distinct, the mid-points between any pairs of candidates, one pair in each election, may coincide). Thus, assume that the assertion holds for all dimensions less than k.

Any k hyperplanes intersect in at most one point in R^k (and generically in exactly one point). Hence, $A(H)$ contains $d^{w^e} \leq \binom{m}{k}$ vertices. Consider a new hyperplane

$$h(s) = \left\{y \in R^k : y_1 = s\right\}$$

that sweeps through $A(H)$ as the parameter s varies from $-\infty$ to $+\infty$. Without loss of generality assume that no hyperplane in H is vertical and that no two vertices in $A(H)$ share the same y_1-coordinate. Let $s_1 < s_2 < \cdots < s_{d^{w^e}}$ be the y_1-coordinates of the d^{w^e} vertices in $A(H)$. We say that vertex i, $i = 1, ..., d^{w^e}$, lies behind $h(s)$ if $s_i < s$, and that a k-region lies behind $h(s)$ if the y_1-coordinates of all the points in the region are less than s.

Let $A_s(H)$ denote the intersection of $A(H)$ with $h(s)$. Hence, $A_s(H)$ is an arrangement of m hyperplanes in R^{k-1}, which by induction hypothesis contains at most $\rho(m, k-1)$

$(k-1)$-regions, where

$$\rho(m, k-1) = \sum_{t=0}^{k-1} \binom{m}{t}.$$

Furthermore, each $(k-1)$-region in $A_s(H)$ is contained in a unique k-region of $A(H)$.

To complete the proof we count the number of k-regions in $A(H)$ that either lie behind or intersect the hyperplane $h(s)$ as it sweeps through $A(H)$ (that is, as s varies from $-\infty$ to $+\infty$). Clearly, when $s = -\infty$, no k-region lies behind $h(s)$, when $s_1 < s < s_2$ one k-region lies behind $h(s)$, and as $h(s)$ passes each other vertex in $A(H)$, one more k-region lies behind $h(s)$. During the entire sweep, $h(s)$ passes d^{w^e} vertices, which implies that at most $\binom{m}{k}$ k-regions lie behind $h(s)$, and for $s > s_{d^{w^e}}$ the remaining k-regions in $A(H)$ intersect $h(s)$. It follows that

$$\rho(m, k) = \binom{m}{k} + \sum_{t=0}^{k-1} \binom{m}{t} = \sum_{t=0}^{k} \binom{m}{t}. \blacksquare$$

Proof of Proposition 3: The proof follows directly from the observation that for a generic election e, the set $Y^{j_e}(w^e)$ for each candidate $j_e \in J^e$ is a Voronoi polyhedron, which is always non empty. Hence, an election partitions Y into q_e convex regions, where each region is the ideological support of the vote for a different candidate in the election. $\blacksquare$

Proof of Proposition 4: Since $q_e \geq 2$ for all $e = 1, ..., m$, consider an arbitrary pair of candidates in each election. Given this subset of $2m$ candidates, Proposition 1 implies that if $m > k$, there must exist at least one combination of m candidates, one for each election, such that the voting profile corresponding to that combination of candidates is not consistent with ideological voting. This establishes the result. $\blacksquare$

Proof of Proposition 5: For the case where $k = 1$, the derivation of $\tau_1(q_1, ..., q_m)$ is straightforward and follows directly from the observation that each election $e = 1, ..., m$, with $q_e \in \{2, ..., \overline{q}\}$ candidates implies $(q_e - 1)$ points that partition the line into q_e regions. Hence, starting from the case of no elections, where the number of regions in R is 1, adding each election $e = 1, ..., m$ one at the time increases the number of regions by at most $(q_e - 1)$.

Now consider the case where $k = 2$. Then each election $e \in \{1, ..., m\}$ defines a Voronoi diagram in the plane with q_e regions. Note that, given any collection of Voronoi diagrams that partitions the plane into Q regions, if we superimpose an additional diagram with q_j

regions, the total number of regions becomes $Q + (q_j - 1) + n$, where n is the number of intersection points of the edges of the additional Voronoi diagram with the edges of the other diagrams.

Let the union of the edges of the Voronoi diagram defined by election e be denoted by U_e, $e = 1, ..., m$. Then for each pair of elections, $e, f \in \{1, ..., m\}$, $e \neq f$, the cardinality n of the intersection of U_e and U_f is at most $(q_e - 1)(q_f - 1)$. To see that this is the case, note that the number of regions in the superimposition of the two Voronoi diagrams is at most $q_e q_f$. But, as noted above, it is also equal to $q_e + (q_f - 1) + n$. It follows that $n \leq (q_e - 1)(q_f - 1)$.

Starting with the Voronoi diagram defined by election $e = 1$, superimposing the remaining $m - 1$ Voronoi diagrams defined by elections $2, ..., m$ one at the time, we obtain a number of regions r_m that is at most

$$
\begin{aligned}
& q_1 + (q_2 - 1) + (q_2 - 1)(q_1 - 1) + (q_3 - 1) + (q_3 - 1)(q_1 - 1 + q_2 - 1) \\
& + \cdots + (q_m - 1) + (q_m - 1)(q_1 - 1 + \cdots + q_{m-1} - 1)
\end{aligned}
$$

or, equivalently,

$$
1 + \sum_{e=1}^{m} \left[(q_e - 1) \left(1 + \sum_{f=e+1}^{m} (q_f - 1) \right) \right]. \blacksquare
$$

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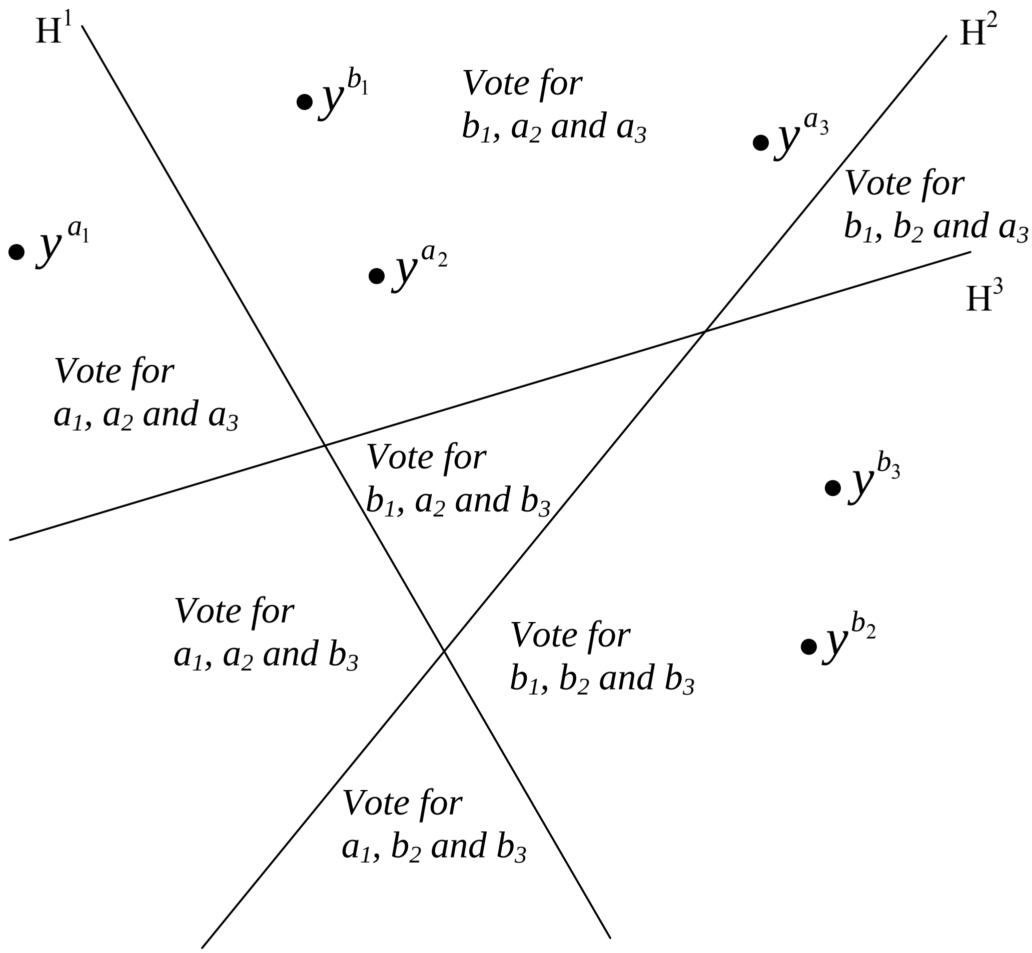

FIGURE 1: Three 2-candidate elections in a two-dimensional ideological space

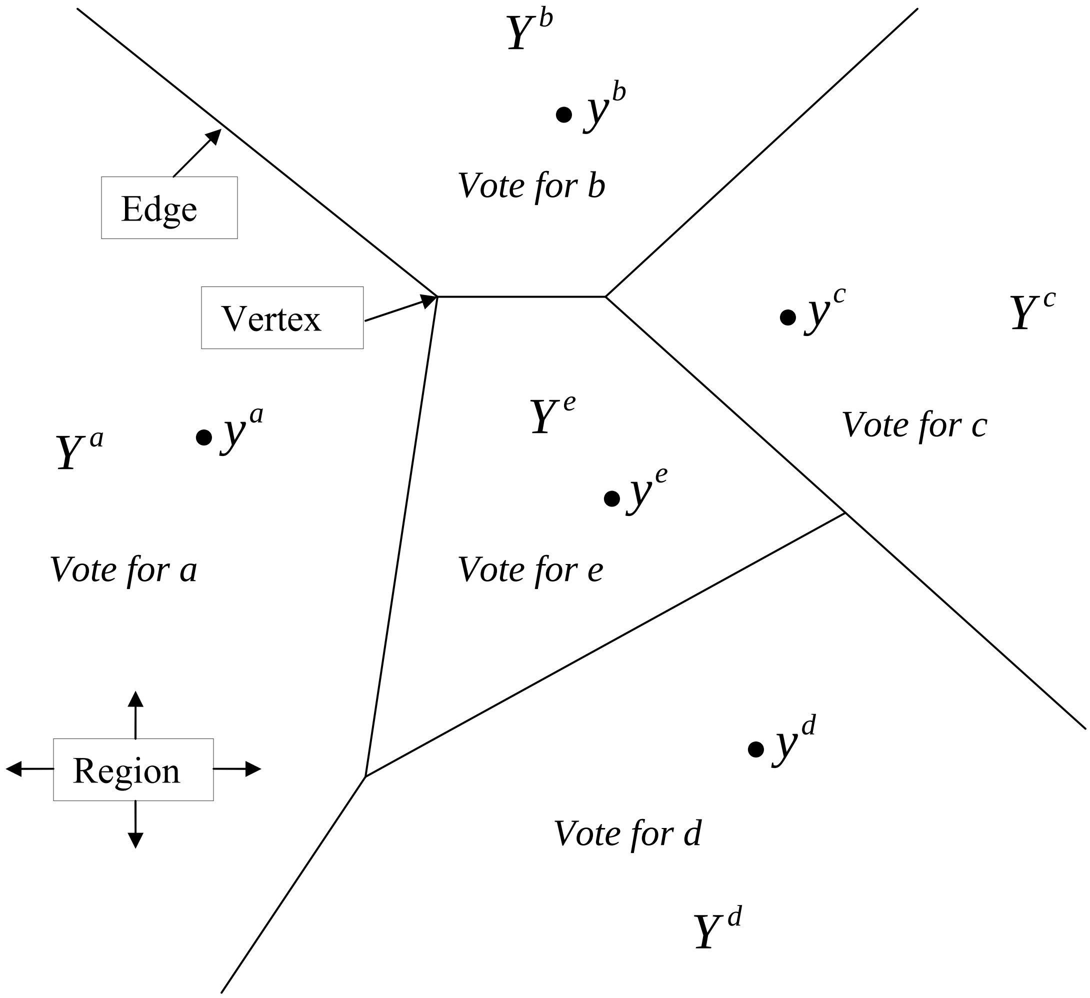

FIGURE 2: The Voronoi tessellation corresponding to a 5-candidate election in a two-dimensional ideological space.

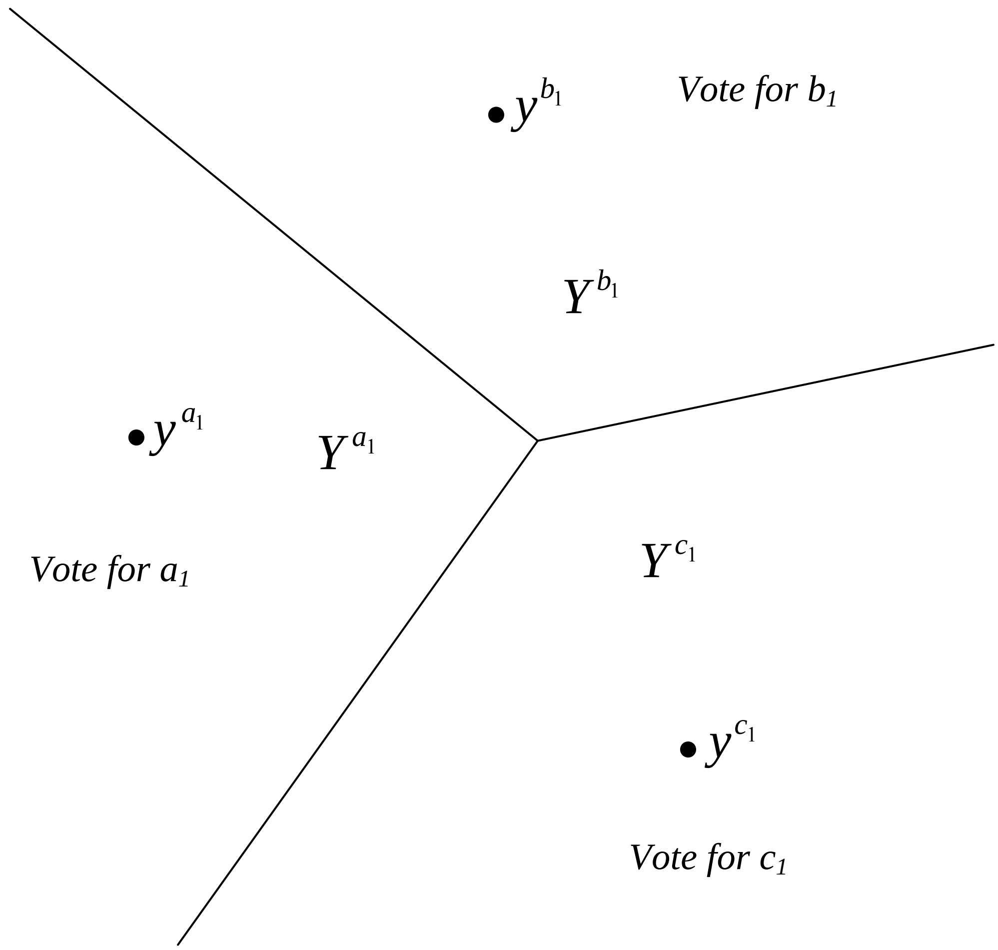

FIGURE 3: A 3-candidate election in a two-dimensional ideological space

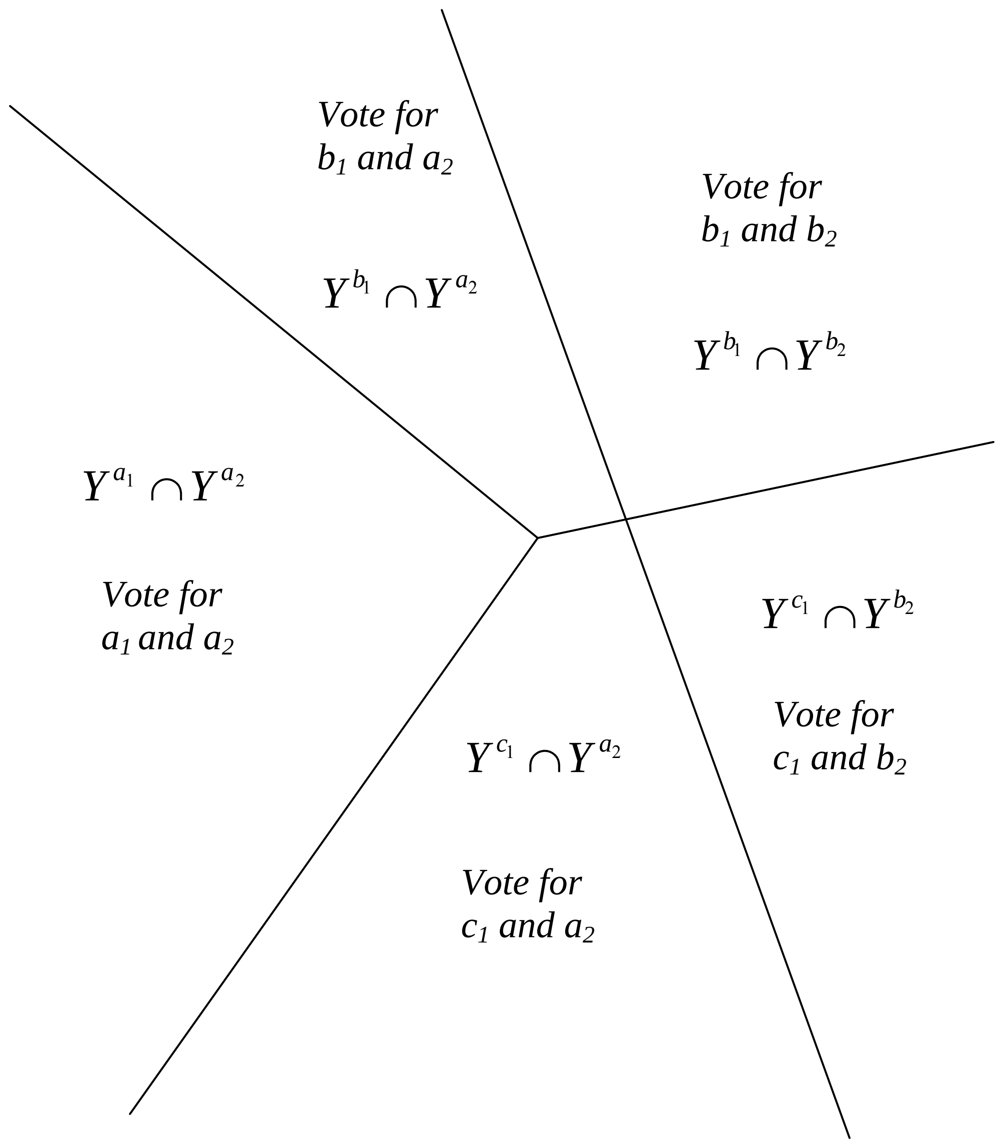

FIGURE 4: An example of a 3-candidate election and a 2-candidate election in a two-dimensional ideological space where the hypothesis that voters vote ideologically is falsifiable

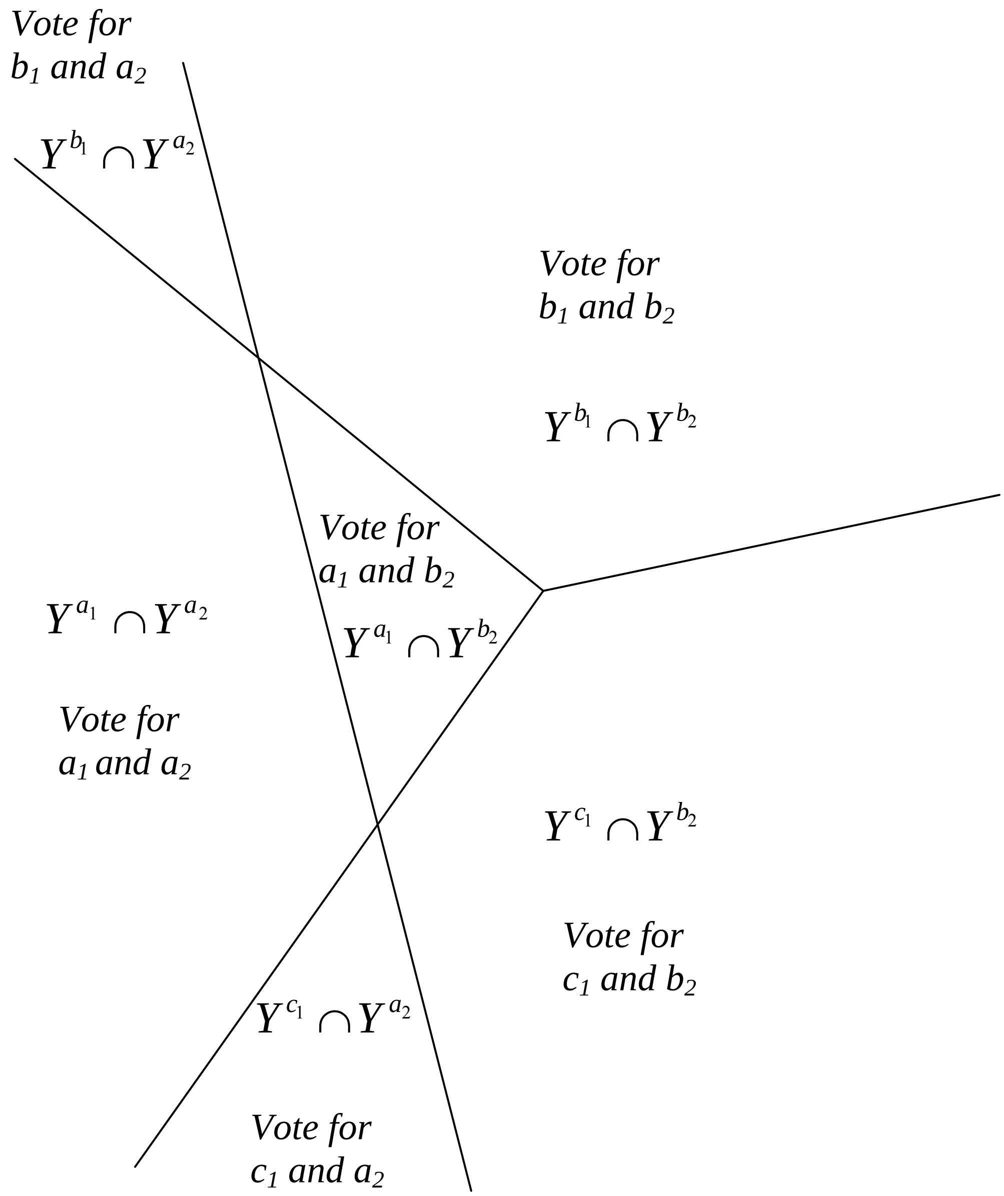

FIGURE 5: An example of a 3-candidate election and a 2-candidate election in a two-dimensional ideological space where the hypothesis that voters vote ideologically is not falsifiable

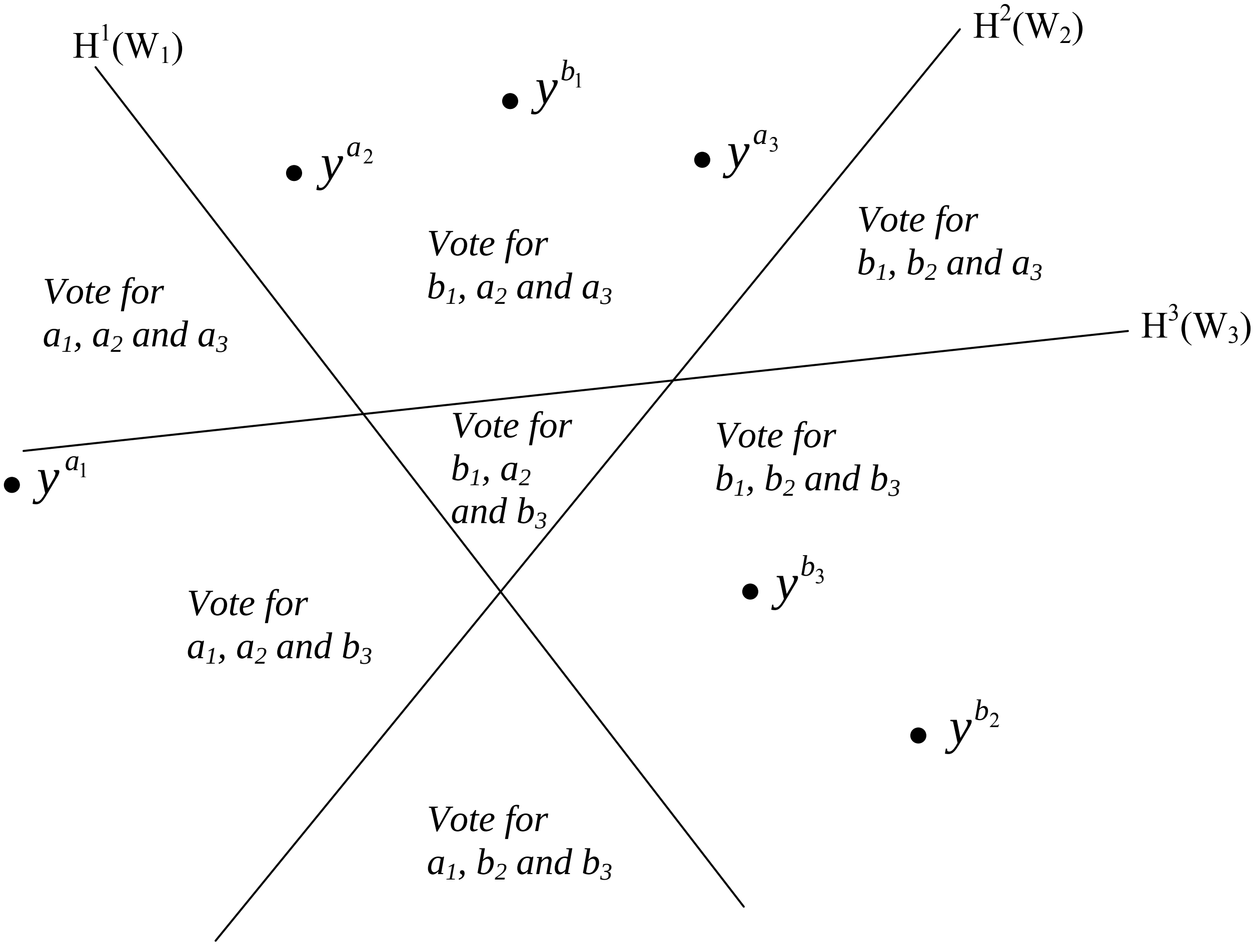

FIGURE 6: Three 2-candidate elections in a two-dimensional ideological space with weighting matrices $W_1 = W_2 = W_3 = I$

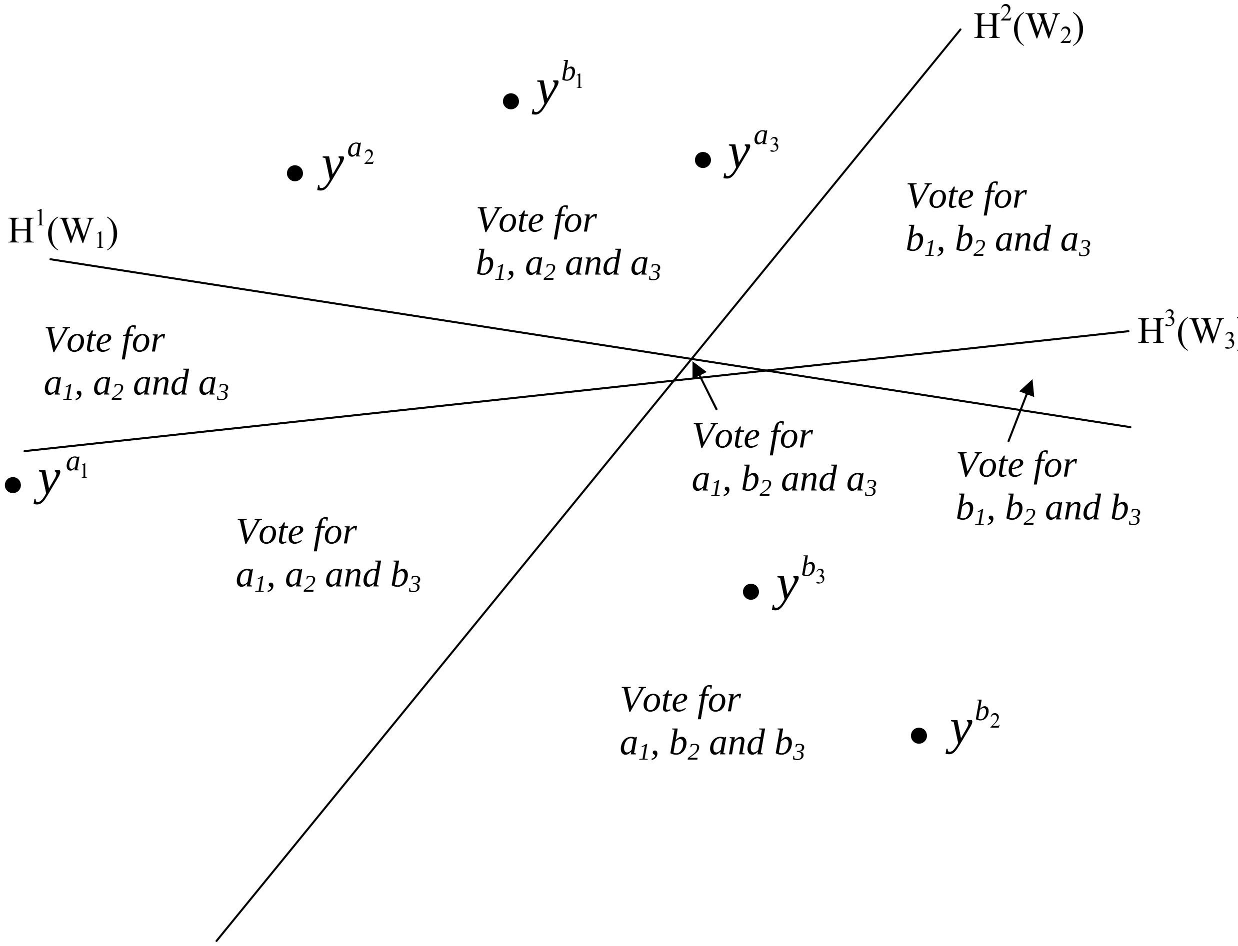

FIGURE 7: The same three 2-candidate elections as in Figure 6 in a two-dimensional ideological space with weighting matrices $W_1 = [10\ 1]'.*I$ and $W_2 = W_3 = I$

TABLE 1: Number of observations

Year	Number of voters in presidential and House elections	Number of voters in presidential, House and Senate elections
1972	1121	515
1976	968	561
1980	641	440
1984	1046	575
1988	797	590
1996	885	490
2000	782	565
Overall	6240	3736

TABLE 2: Percentage of observations consistent with ideological voting

Year	Voters in presidential and House elections (unidimensional space)	Voters in presidential and House, or presidential, House and Senate elections (unidimensional space)	Voters in presidential, House and Senate elections (two-dimensional space)
1972	96.5%	91.4%	99.2%
1976	96.2%	91.0%	99.6%
1980	94.9%	90.7%	99.5%
1984	96.5%	92.3%	99.8%
1988	98.4%	92.1%	99.7%
1996	96.2%	92.7%	100.0%
2000	98.1%	95.6%	99.8%
Overall	96.7%	92.2%	99.7%